SET THEORY

A set is a collection of objects

- Each object is called an element of the set.
- The set that contains all the elements of a given collection is called the universal set and is represented by the symbol ‘μ’

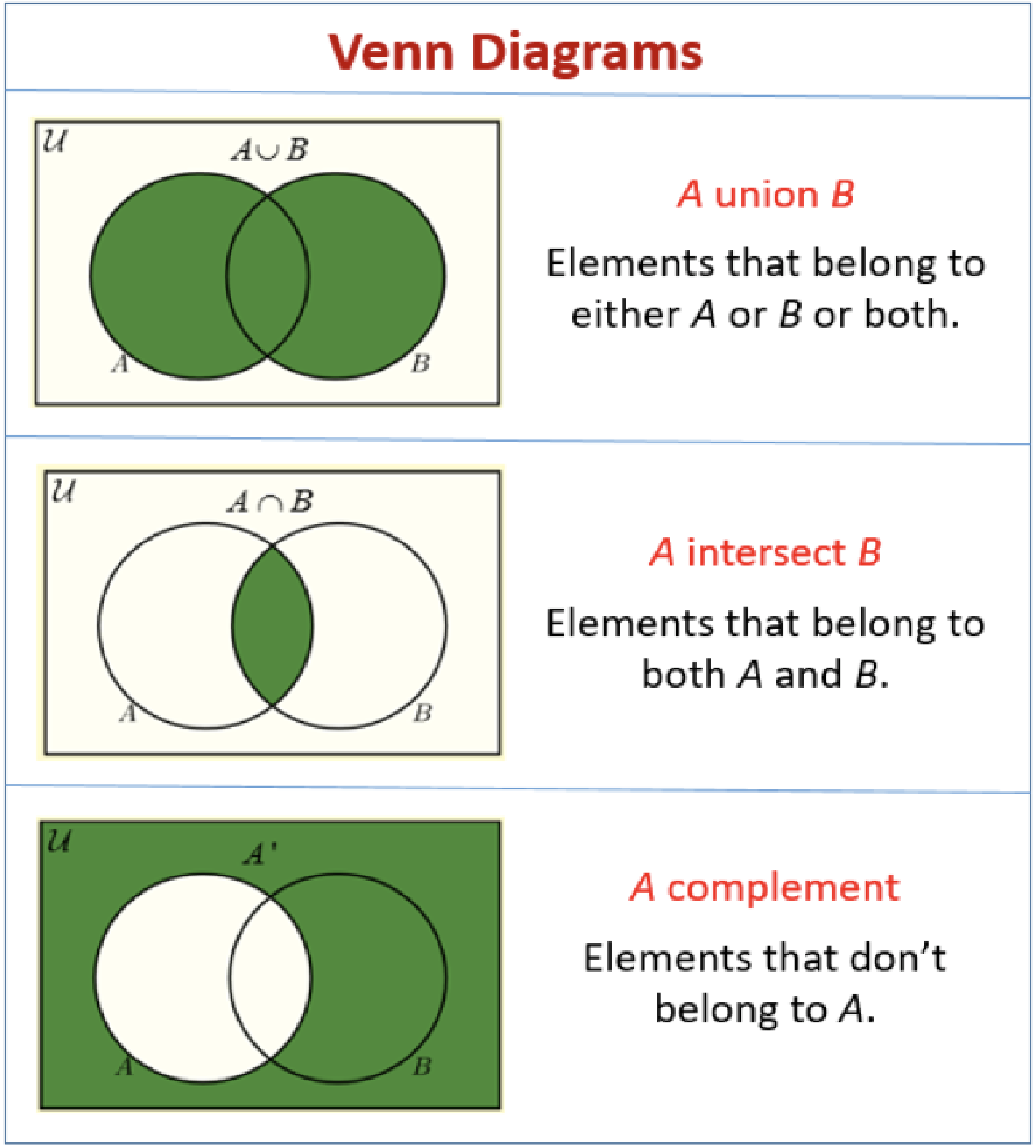
For two sets A and B,

- $n(A \cup B)$ is the number of elements present in either of the sets A or B.
- $n(A \cap B)$ is the number of elements present in both the sets A and B.
- $n(A \cup B) = n(A) + (n(B) - n(A \cap B)$

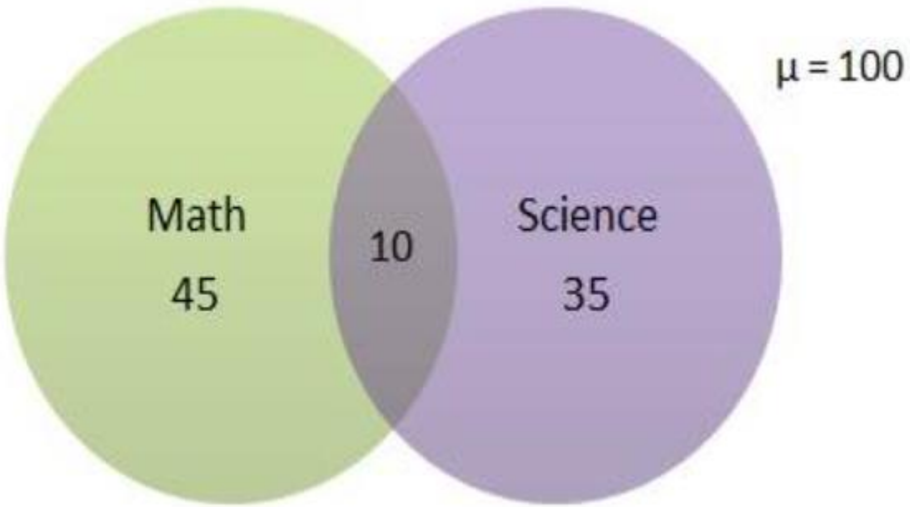
For three sets A, B and C,

- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

The easiest way to solve problems on sets is by drawing Venn diagrams, as shown below.



In a class of 100 students, 35 like science and 45 like math. 10 like both. How many like neither?



Solution:

Total number of students, n(μ) = 100

Number of science students, n(S) = 35

Number of math students, n(M) = 45

Number of students who like both, $n(M \cap S) = 10$

Number of students who like either of them,

$n(M \cup S) = n(M) + n(S) – n(M \cap S)$

→ 45+35-10 = 70

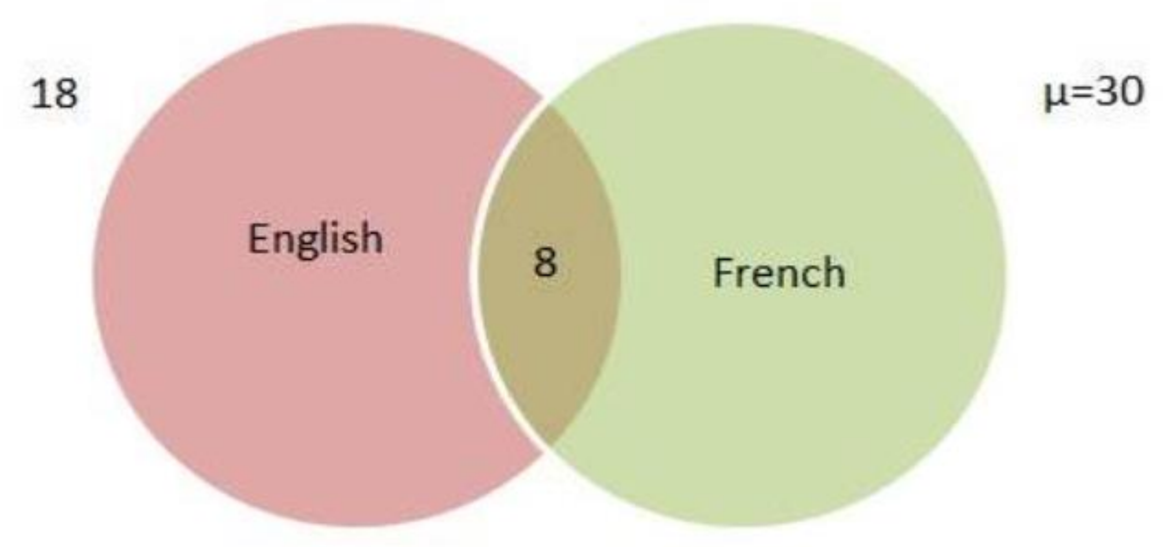
Number of students who like neither = $n(\mu) – n(M \cup S) = 100 – 70 = 30$

either of them and how many like neither?

There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total?

Solution:

From the given question, initially the Venn diagram for this problem looks like this.



Every student is learning at least one language. Hence there is no one who fall in the category ‘neither’.

So in this case, $n(E \cup F) = n(\mu)$.

It is mentioned in the problem that a total of 18 are learning English. This DOES NOT mean that 18 are learning ONLY English. Only when the word ‘only’ is mentioned in the problem should we consider it so.

Now, 18 are learning English and 8 are learning both. This means that $18 - 8 = 10$ are learning ONLY English.

$n(\mu) = 30$, $n(E) = 10$

$n(E \cup F) = n(E) + n(F) - n(E \cap F)$

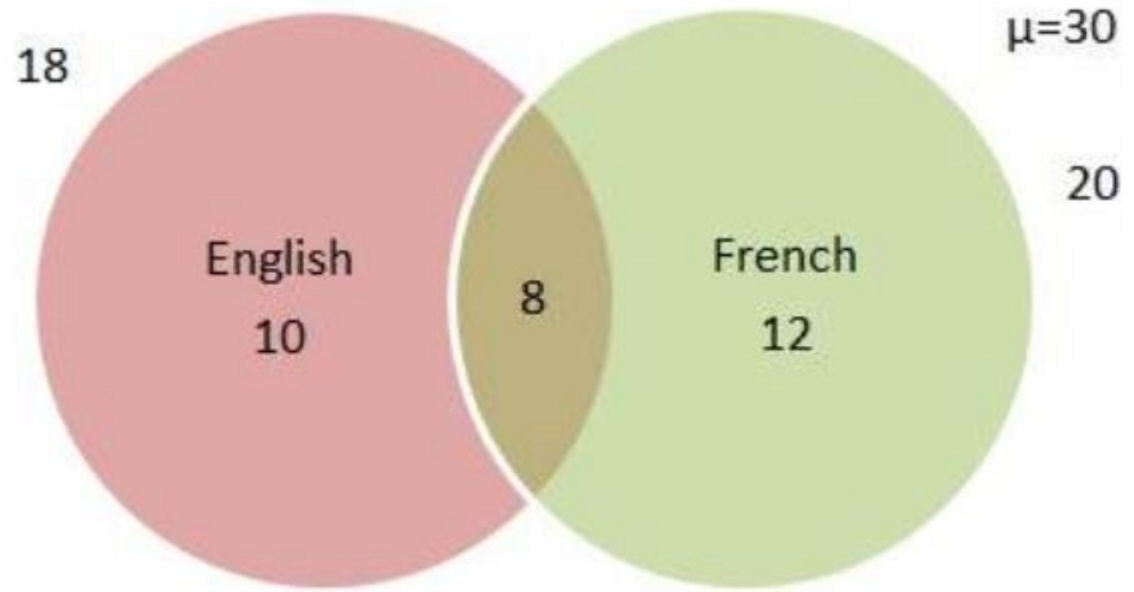
$30 = 18 + n(F) - 8$

$n(F) = 20$

Therefore, total number of students learning French = 20.

Note: The question was only about the total number of students learning French and not about those learning ONLY French, which would have been a different answer, 12.

Finally, the Venn diagram looks like this.



Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 15 played both cricket and hockey, 20 played both hockey and volley ball, 15 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students and how many played only cricket, only hockey and only volley ball?

Solution:

$n(C) = 50$, $n(H) = 50$, $n(V) = 40$

$n(C \cap H) = 15$

$n(H \cap V) = 20$

$n(C \cap V) = 15$

$n(C \cap H \cap V) = 10$

No. of students who played at least one game

$n(C \cup H \cup V) = n(C) + n(H) + n(V) – n(C \cap H) – n(H \cap V) –$

$n(C \cap V) + n(C \cap H \cap V)$

$= 50 + 50 + 40 – 15 – 20 – 15 + 10$

Total number of students = 100.

Let a denote the number of people who played cricket and volleyball only.

Let b denote the number of people who played cricket and hockey only.

Let c denote the number of people who played hockey and volleyball only.

Let d denote the number of people who played all three games.

Accordingly, $d = n(C \cap H \cap V) = 10$

Now, $n(C \cap V) = a + d = 15$

$n(C \cap H) = b + d = 15$

$n(H \cap V) = c + d = 20$

Therefore, $a = 15 – 10 = 5$ [cricket and volleyball only]

$b = 15 – 10 = 5$ [cricket and hockey only]

$c = 20 – 10 = 10$ [hockey and volleyball only]

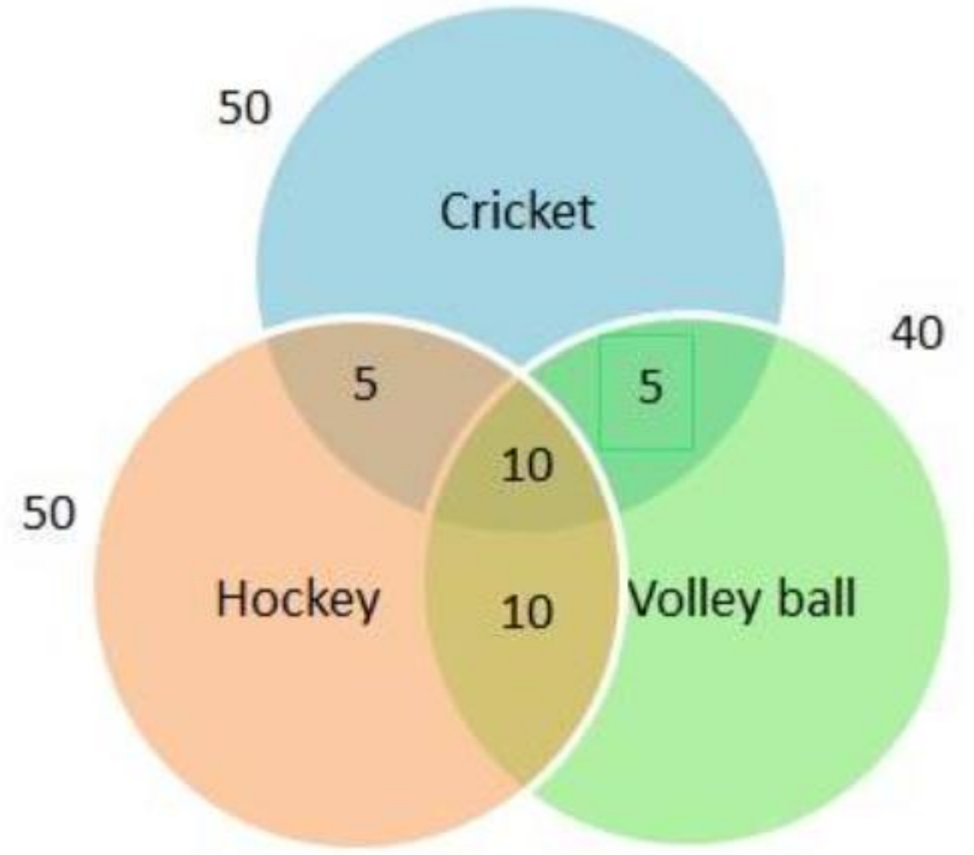
No. of students who played only cricket = $n(C) – [a + b + d] = 50 – (5 + 5 + 10) = 30$

No. of students who played only hockey = $n(H) – [b + c + d] = 50 – (5 + 10 + 10) = 25$

No. of students who played only volley ball = $n(V) – [a + c + d] = 40 – (10 + 5 + 10) = 15$

Alternatively, we can solve it faster with the help of a Venn diagram.

The Venn diagram for the given information looks like this.



PROBLEMS FOR PRACTICE

In a group, there were 115 people whose proofs of identity were being verified. Some had passport, some had voter id and some had both. If 65 had passport and 30 had both, how many had voter id only and not passport?

A. 30
B. 50
C. 80
D. None of the above

Answer B.

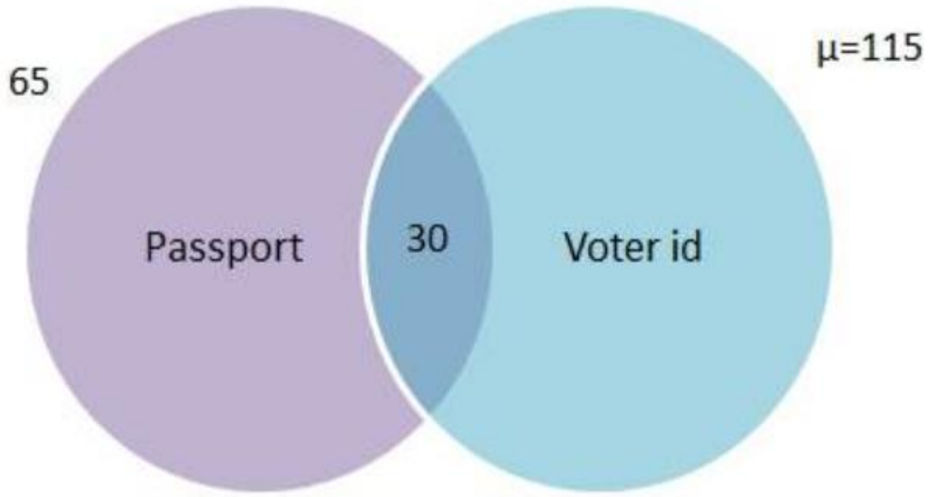
Explanation

$n(P \cup V) = n(P) + n(V) - n(P \cap V)$

$115 = 65 + n(V) - 30$

$n(V) = 80$

People with only voter id = 80-30 = 50



Among a group of people, 40% liked red, 30% liked blue and 30% liked green. 7% liked both red and green, 5% liked both red and blue, 10% liked both green and blue. If 86% of them liked at least one colour, what percentage of people liked all three?

A. 10

B. 6

C. 8

D. None

Answer C.

Explanation:

$n(R \cup B \cup G) = n(R) + n(B) + n(G) - n(R \cap B) - n(B \cap G) - n(R \cap G) + n(R \cap G \cap B)$

$86 = 40+30+30-5-10-7+ n(R \cap G \cap B)$

Solving this gives 8.

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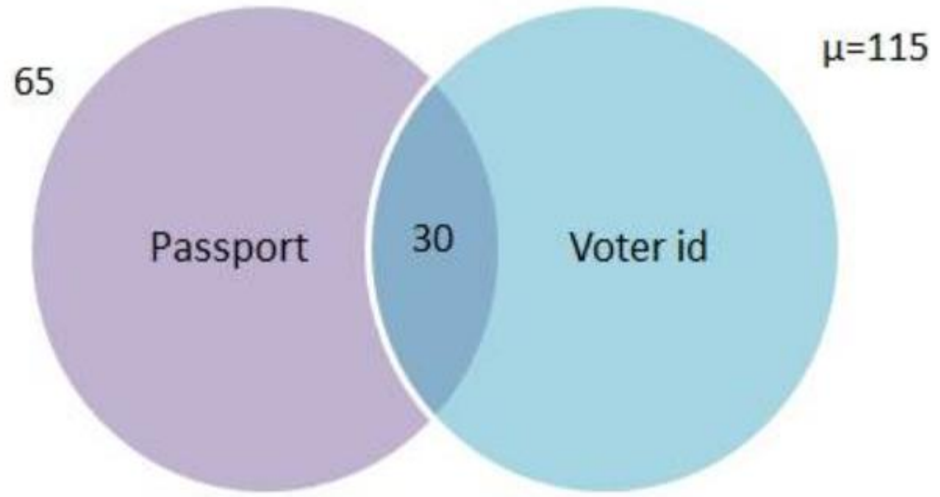

THANK YOU