SRM Institute of Science and Technology
College of Engineering and Technology
DEPARTMENT OF MATHEMATICS
Academic Year: 2023-2024 EVEN

**HANDS-ON PRACTICE**

**Test: CLA1-T1** **Deadline:** 14 /02/2024
**Course Code & Title: 21MAB204T / Probability and Queueing Theory**
**Year & Sem: II & IV** **Max. Marks: 5**

**Course Articulation Matrix:**

| At the end of this course, learners will be able to: | | BL | Program Outcomes (PO) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Course Outcomes (CO) | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| CO1 | *evaluate the characteristics of discrete and continuous random variables and apply them in science and engineering.* | 4 | 3 | 3 | | | | | | | | | | |
| CO2 | *identify the random variables and model them using various distributions.* | 4 | 3 | 3 | | | | | | | | | | |
| CO3 | *infer results from two-dimensional random variables which describe the real-life phenomenon* | 4 | 3 | 3 | | | | | | | | | | |
| CO4 | *examine the significant results of various queueing models.* | 4 | 3 | 3 | | | | | | | | | | |
| CO5 | *determine the transition probabilities and classify the states of the Markov chain.* | 4 | 3 | 3 | | | | | | | | | | |

**Each question carries 1 mark**

| Question 1 | BL | CO | PO | PI Code |
|---|---|---|---|---|
| | 4 | 1 | 2 | 2.8.1 |

Suppose the life in hours of a certain part of a radio tube is a continuous random variable $X$ with PDF given by $f(x) = \begin{cases} \frac{100}{x^2}, & x \geq 100 \\ 0, & elsewhere \end{cases}$ (i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first 150 hours of operation? (ii) What is the probability that none of three of the original tubes will have to be replaced during the first 150 hours of operation? (iii) What is the probability that a tube will last less than 200 hours if it is known that the tube is still functioning after 150 hours of service?

| Question 2 | BL | CO | PO | PI Code |
|---|---|---|---|---|
| | 3 | 1 | 2 | 2.8.1 |

The following is the Cumulative Distribution Function of a discrete Random Variable $X$:

| $x$ | $-3$ | $-1$ | 0 | 1 | 2 | 3 | 5 | 8 |
|---|---|---|---|---|---|---|---|---|
| $F(x)$ | 0.10 | 0.30 | 0.45 | 0.5 | 0.75 | 0.90 | 0.95 | 1.00 |

Find (i) the probability distribution of $X$. (ii) $P(X \text{ is even})$ (iii) $P(X = -3|X < 0)$ (iv) $P(|X| \geq -1)$

| Question 3 | BL | CO | PO | PI Code |
|---|---|---|---|---|
| | 3 | 1 | 2 | 2.8.1 |

A Random Variable $X$ has a probability density function $f(x) = K(x - x^2),\ 0 \leq x \leq 1$. Find $K$, $\mu_r'$ and hence find the first four central moments.

| Question 4 | BL | CO | PO | PI Code |
|---|---|---|---|---|
| | 3 | 1 | 2 | 2.8.1 |

Find the MGF of the random variable $X$ if its probability density function is given by $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$. Hence find the first four central moments.

| Question 5 | BL | CO | PO | PI Code |
|---|---|---|---|---|
| | 4 | 1 | 2 | 2.8.1 |

A Random Variable $X$ takes the values $-1, 1,\ 3,\ 5$ with associated probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{6},\ \&\ \frac{1}{2}$. Find by direct computation $P(|X - 3| \geq 3)$. Also, find an upper bound to this probability by applying Tchebycheff's inequality.