4^{th} Semester :

Unit - 1 Tutorial - 1

1) Let, $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4) = k$

$P(X=1) = \frac{k}{2}$, $P(X=2) = \frac{k}{3}$, $P(X=3) = k$ & $P(X=4) = \frac{k}{5}$

Now,

$\therefore \sum_{i=1}^{4} P(X=x_i) = 1$

or, $P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1$

or, $\frac{k}{2} + \frac{k}{3} + 1 + \frac{k}{5} = 1$

or $k\left(\frac{1}{2} + \frac{1}{3} + 1 + \frac{1}{5}\right) = 1$

or, $k \times \frac{61}{30} = 1$

$\therefore k = \frac{30}{61}$

$\Rightarrow P(X=1) = \frac{15}{61}$, $P(X=2) = \frac{10}{61}$, $P(X=3) = \frac{30}{61}$, $P(X=4) = \frac{6}{61}$

$\therefore P(X \leq 2) = P(X=1) + P(X=2)$

$= \frac{15}{61} + \frac{10}{61} = \frac{25}{61}$

$\therefore P(X > 2) = P(X=3) + P(X=4)$

$= \frac{30}{61} + \frac{6}{61}$

$= \frac{36}{61}$

2) i) $4k + k + 2k + 3k = 1$

or, $10k = 1$

$\therefore k = \frac{1}{10}$

ii) $P(\frac{1}{2} < x < 5 / x > 1) = \frac{P(\frac{1}{2} < x < 5) \cap P(X > 1)}{P(X > 1)}$

$= \frac{P(X=2) + P(X=3) + P(X=4)}{P(X=2) + P(X=3) + P(X=4)}$

$= 1$

iii) $P(X \leq 3) = P(X=1) + P(X=2) + P(X=3)$

$= 4 \times \frac{1}{10} + \frac{1}{10} + 2 \times \frac{1}{10}$

$= \frac{7}{10}$

iv) $P(1 \leq X < 2) = P(X=1)$

$= 4 \times \frac{1}{10} = \frac{2}{5}$

3) i) $P(|X| \geq 2) = P(-\infty \leq X \leq -2) + P(2 \leq X \leq \infty)$

$= P(X=-2) + P(X=5)$

$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

ii) $P(0 \leq X \leq 10) = P(X=0) + P(X=5)$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4}$

CamScanner

iii) $P(X \leq 0) = P(X=-2) + P(X=0)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

iv) C.D.F :

X	-2	0	5
CDF	1/4	1/2	1

iv) ~~$P(X \leq 1) = \frac{1}{2} + \frac{1}{4} + 0 = \frac{3}{4}$~~

4) Given X is a discrete r.v and X take the values are 0, 1, 2

Since, $P(X \leq 0) = P(X=0) + P(X<0)$

$\therefore P(X=0) = P(X \leq 0) - P(X<0)$

$= 1/4 - 0 = 1/4$

The, $P(X \leq 1) = P(X=1) + P(X<1)$

$\therefore P(X=1) = P(X \leq 1) - P(X<1)$

$= 3/4 - 1/4 = 1/2$

$\therefore P(X=2) = P(X \leq 2) - P(X<2)$

$= 1 - 3/4 = 1/4$

$X = x$	0	1	2
$P(X=x)$	1/4	1/2	1/4

ii) $P(0<X<2) = P(X=1) = 1/2$

iii) $P(1/2 < X < 2 \mid X \geq 1) = \frac{P(X=1)}{P(X=1)+P(X=2)} = \frac{1/2}{1/2+1/4} = \frac{2}{3}$

iv) $P(X \leq 1) = P(X=0) + P(X=1)$

$= \frac{1}{4} + \frac{1}{2}$

$= \frac{3}{4}$

5) i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

or, $\int_{0}^{5} kx\,dx + \int_{5}^{10} k(10-x) = 1$

or, $k\left[\frac{x^2}{2}\right]_0^5 + \left[10kx - \frac{kx^2}{2}\right]_5^{10} = 1$

or, $k\,\frac{25}{2} + \left[(100k - 50k) - \left\{50k - \frac{25k}{2}\right\}\right] = 1$

or, $\frac{25k}{2} + 50k - 50k + \frac{25k}{2} = 1$

or $25k = 1$

$\therefore$ $k = 1/25$

ii) $P(X \le 6 \mid X > 5) = \frac{P(X \le 6) \cap P(X > 5)}{P(X > 5)}$

$= \frac{\int_5^6 1/25\,(10-x)\,dx}{\int_5^{10} 1/25\,(10-x)\,dx}$

$= \frac{1/25\left[10x - \frac{x^2}{2}\right]_5^6}{\frac{1}{25}\left[10x - \frac{x^2}{2}\right]_5^{10}}$

$= \frac{\left(60 - \frac{36}{2}\right) - \left(50 - \frac{25}{2}\right)}{\left(100 - \frac{100}{2}\right) - \left(50 - \frac{25}{2}\right)}$

$= \frac{9}{25}$

iii) $P(X \leq 6) = \int_0^5 \frac{1}{25} x\,dx + \int_5^6 \frac{1}{25} (10-x)\,dx$

$= \frac{1}{25} \left[\frac{x^2}{2}\right]_0^5 + \frac{1}{25} \left[10x - \frac{x^2}{2}\right]_5^6$

$= \frac{1}{25} \times \frac{25}{2} + \frac{1}{25}\left[\left(60 - \frac{36}{2}\right) - \left(50 - \frac{25}{2}\right)\right]$

$= \frac{1}{2} + \frac{1}{25} \frac{9}{2}$

$= \frac{17}{25}$

5) i) PDF :

$\therefore f(x) = \frac{d}{dx} F(x) = \begin{cases} 4x - 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$\therefore P\left(\frac{1}{2} < X < \frac{2}{3}\right) = \int_{1/2}^{2/3} (4x - 3x^2)\,dx$

$= \left[\frac{4x^2}{2} - \frac{3x^3}{3}\right]_{1/2}^{2/3}$

$= \left\{2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3\right\} - \left\{2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^3\right\}$

$= \frac{8}{9} - \frac{8}{27} - \frac{2}{4} + \frac{1}{8}$

$= \frac{47}{216}$

7)

i) $3c^3 + 4c - 10c^2 + 5c - 1 = 1$

or, $3c^3 - 10c^2 + 9c - 1 = 1$

or, $3c^3 - 10c^2 + 9c - 2 = 0$

or, $\left(c - \frac{1}{3}\right)(c-2)(c-1) = 0$

$\therefore c = \frac{1}{3}, 2, 1$

Since, $0 \le P(X = x_i) \le 1$

So, $c = \frac{1}{3}$

ii) CDF

X	0	1	2
F(x)	1/9	1/3	1

$\Rightarrow 3c^3 = 1/9, \quad 4c - 10c^2 = 2/9, \quad 5c - 1 = 2/3$

iii) $X = 1$

iv) $X = 1$

v) $P(X < 2) = P(X=0) + P(X=1)$

$= \frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

vi) $P(|X| \le 2) = P(-2 < X < 2)$

$= P(X=0) + P(X=1)$

$= \frac{1}{9} + \frac{2}{9} = \frac{1}{3}$

8) $\int_{-\infty}^{\infty} f(x)dx = 1$

or, $\int_{-\infty}^{1} 0\,dx + \int_{1}^{2} k(x-1)\,dx + \int_{2}^{3} k(3-x) + \int_{3}^{\infty} 0\,dx = 1$

or, $k\left[\frac{x^2}{2} - x\right]_1^2 + k\left[3x - \frac{x^2}{2}\right]_2^3 = 1$

or, $k\left[(2-2) - \left(\frac{1}{2} - 1\right)\right] + k\left[\left(9 - \frac{9}{2}\right) - (6-2)\right] = 1$

or $k\left(0 - \frac{1}{2} + 1 + 9 - \frac{9}{2} - 4\right) = 1$

or, $k \cdot 1 = 1$

$\therefore k = 1$

ii) CDF :

If $x < 1$, $F(x) = 0$

If $1 \le x \le 2$, $F(x) = \int_1^x k(x-1)\,dx = \left[\frac{kx^2}{2} - x\right]_1^x$

$= \left(1 \cdot \frac{x^2}{2} - x\right) - \left(\frac{1}{2} - 1\right) = \frac{x^2}{2} - x + \frac{1}{2} = \frac{x^2 - 2x + 1}{2}$

$= \frac{(x-1)^2}{2}$

If $2 \le x \le 3$, $F(x) = \int_1^2 (x-1)\,dx + \int_2^x (3-x)\,dx$

$= \left[\frac{x^2}{2} - x\right]_1^2 + \left[3x - \frac{x^2}{2}\right]_2^x$

$= \left[(2-2) - \left(\frac{1}{2} - 1\right)\right] + \left[\left(3x - \frac{x^2}{2}\right) - (6-2)\right]$

$= \frac{1}{2} + 3x - \frac{x^2}{2} - 4 = 3x - \frac{x^2}{2} - \frac{7}{2} = \frac{-x^2 + 6x - 7}{2}$

If $x > 3$, $F(x) = 1$

iii) $P(1 < X < 5/2) = \int_1^2 (x-1)\,dx + \int_2^{5/2} (3-x)\,dx$

$$= \left[\frac{x^2}{2} - x\right]_1^2 + \left[3x - \frac{x^2}{2}\right]_2^{5/2}$$

$$= (2-2) - \left(\frac{1}{2}-1\right) + \left(\frac{15}{2} - \frac{25}{8}\right) - (6-2)$$

$$= \cancel{\frac{1}{2}}\ \frac{7}{8}$$

iv) $P(|X| < 2)$

$= P(-2 < X < 2)$

$= \int_1^2 (x-1)\,dx$

$= \left[\frac{x^2}{2} - x\right]_1^2$

$= (2-2) - \left(\frac{1}{2} - 1\right)$

$= \frac{1}{2}$