7_keras

December 11, 2021

1 French Motor Third-Party Liability Claims

2 Fitting a GLM using keras

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2021-12-12

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4 1. Introduction

This notebook was created for the course "Deep Learning with Actuarial Applications in R" of the Swiss Association of Actuaries (https://www.actuaries.ch/).

This notebook serves as accompanion to the tutorial "Insights from Inside Neural Networks", available on SSRN.

The code is similar to the code used in above tutorial and combines the raw R code in the scripts, available on GitHub along with some more comments. Please refer to the tutorial for explanations.

Note that the results might vary depending on the R and Python package versions, see last section for the result of `sessionInfo()` and corresponding info on the Python setup.

5 2. Data Preparation

The tutorial uses the French MTPL data set available on openML (ID 41214).

5.1 2.1 Load packages

[1]:
```
library(mgcv)
library(keras)
library(magrittr)
library(dplyr)
library(tibble)
library(purrr)
library(ggplot2)
library(gridExtra)
library(splitTools)
library(tidyr)
library(OpenML)
library(farff)
```

```
Loading required package: nlme

This is mgcv 1.8-33. For overview type 'help("mgcv-package")'.
```

```
Attaching package: 'dplyr'

The following object is masked from 'package:nlme':

    collapse

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union


Attaching package: 'purrr'

The following object is masked from 'package:magrittr':

    set_names


Attaching package: 'gridExtra'

The following object is masked from 'package:dplyr':

    combine


Attaching package: 'tidyr'

The following object is masked from 'package:magrittr':

    extract
```

5.2 2.2 Set global parameters

[2]:
```
options(encoding = 'UTF-8')
```

[3]:
```
# set seed to obtain best reproducibility. note that the underlying␣
↪architecture may affect results nonetheless, so full reproducibility cannot␣
↪be guaranteed across different platforms.
seed <- 100
Sys.setenv(PYTHONHASHSEED = seed)
set.seed(seed)
reticulate::py_set_seed(seed)
tensorflow::tf$random$set_seed(seed)
```

The results below will not exactly match the results in the paper, since the underlying dataset and some packages are different. In addition the split into training and testing data is different as well. However, the general conclusions remain the same.

5.3 2.3 Helper functions

Subsequently, for ease of reading, we provide all the helper functions which are used in this tutorial in this section.

[4]:
```
# Poisson deviance
PoissonDeviance <- function(pred, obs) {
    200 * (sum(pred) - sum(obs) + sum(log((obs/pred)^(obs)))) / length(pred)
}
```

6 2.4 Load data

We consider the data `freMTPL2freq` included in the R package `CASdatasets` for claim frequency modeling. This data comprises a French motor third-party liability (MTPL) insurance portfolio with corresponding claim counts observed in one accounting year. We do not incorporate claim sizes which would also be available through freMTPL2sev.

As the current package version provides a slightly amended dataset, we use an older dataset available on openML (ID 41214) to allow replication of the results in the tutorial.

There are two options to load this dataset: - Option 1: Manually download the data from github in case of firewalls or other issues - Option 2: Download from openML (ID 41214)

[5]:
```
# Local loading
# you can download it from https://github.com/JSchelldorfer/
↪DeepLearningWithActuarialApplications/blob/master/freMTPL2freq.RData
load(freMTPL2freq.RData")

# download (only if company firewall allows it)
#freMTPL2freq <- getOMLDataSet(data.id = 41214)$data
```

6.1 2.5 General data preprocessing

A priori, there is not sufficient information about this data to do a sensible decision about the best consideration of the exposure measure, either as feature or as offset. In the following we treat the exposure always as an offset.

Data preprocessing includes a couple of transformations. We ensure that ClaimNb is an integer, VehAge, DrivAge and BonusMalus have been capped for the plots at age 20, age 90 and bonus-malus level 150, respectively, to improve visualization. Density is logarithmized and VehGas is a categorical variable. We leave away the rounding used in the first notebook, which were mainly used for nicer visualizations of the data.

We are adding a group_id identifying rows possibly referring to the same policy. Respecting group_id in data splitting techniques (train/test, cross-validation) is essential. This is different to the tutorial where another splitting has been used. As a consequence, the figures in this notebook do not match the figures in the tutorial, but the conclusions drawn are the same.

In addition to the previous tutorial, we decide to truncate the ClaimNb and the Exposure in order to correct for unreasonable data entries and simplifications for the modeling part.

[6]:
```r
# Grouping id
distinct <- freMTPL2freq %>%
  distinct_at(vars(-c(IDpol, Exposure, ClaimNb))) %>%
  mutate(group_id = row_number())
```

[7]:
```r
dat <- freMTPL2freq %>%
  left_join(distinct) %>%
  mutate(ClaimNb = pmin(as.integer(ClaimNb), 4),
         VehAge = pmin(VehAge,20),
         DrivAge = pmin(DrivAge,90),
         BonusMalus = pmin(BonusMalus,150),
         Density = round(log(Density),2),
         VehGas = factor(VehGas),
         Exposure = pmin(Exposure, 1))
```

```
Joining, by = c("Area", "VehPower", "VehAge", "DrivAge", "BonusMalus",
"VehBrand", "VehGas", "Density", "Region", "ClaimTotal")
```

[8]:
```r
# Group sizes of suspected clusters
table(table(dat[, "group_id"]))
```

```
     1      2      3      4      5      6      7      8      9     10     11
429576  84201  13940   2437    966    754    720    475    400    269    142
    12     13     14     15     18     22
   191      3      1      2      1      1
```

6.2 2.6 Store model results

As we are going to compare two models, we create a table which stores the metrics we are going to use for the comparison and the selection of the best model.

[9]:
```
# initialize table to store all model results for comparison
df_cmp <- tibble(
 model = character(),
 epochs = numeric(),
 run_time = numeric(),
 parameters = numeric(),
 in_sample_loss = numeric(),
 out_sample_loss = numeric(),
 avg_freq = numeric(),
)
```

7 3. Pre-processing Neural Networks

7.1 3.1 Introduction

In this chapter, we explain how the data need to be pre-processed to be used in neural networks. It can not be processed in the same way as shown above for GLMs. Further details can be found in this tutorial on SSRN, chapter 2.

We are going to highlight a few important points in data pre-processing that are necessary for a successful application of networks.

In network modeling the choice of the scale of the feature components may substantially influence the fitting procedure of the predictive model. Therefore, data pre-processing requires careful consideration. We treat unordered categorical (nominal) feature components and continuous (or ordinal) feature components separately. Ordered categorical feature components are treated like continuous ones, where we simply replace the ordered categorical labels by integers. Binary categorical feature components are coded by 0's and 1's for the two binary labels (for binary labels we do not distinguish between ordered and unordered components). Remark that if we choose an anti-symmetric activation function, i.e. $-\phi(x) = \phi(-x)$, we may also set binary categorical feature components to $\pm 1/2$, which may simplify initialization of optimization algorithms.

7.1.1 3.1.1 Unordered (nominal) categorical feature components

We need to transform (nominal) categorical feature components to numerical values. The most commonly used transformations are the so-called **dummy coding** and the **one-hot encoding**. Both methods construct binary representations for categorical labels. For dummy coding one label is chosen as reference level. Dummy coding then uses binary variables to indicate which label a particular policy possesses if it differs from the reference level. In our example we have two unordered categorical feature components, namely VehBrand and Region. We use VehBrand as illustration. It has 11 different labels $\{B_1, B_{10}, B_{11}, B_{12}, B_{13}, B_{14}, B_2, B_3, B_4, B_5, B_6\}$. We choose B_1 as reference label. Dummy coding then provides the coding scheme below (left). We observe that the 11 labels are replaced by 10-dimensional feature vectors $\{0, 1\}^{10}$, with components summing up to either 0 or 1.

label	feature components $x^\star \in \{0,1\}^{10}$									
B1	0	0	0	0	0	0	0	0	0	0
B10	1	0	0	0	0	0	0	0	0	0
B11	0	1	0	0	0	0	0	0	0	0
B12	0	0	1	0	0	0	0	0	0	0
B13	0	0	0	1	0	0	0	0	0	0
B14	0	0	0	0	1	0	0	0	0	0
B2	0	0	0	0	0	1	0	0	0	0
B3	0	0	0	0	0	0	1	0	0	0
B4	0	0	0	0	0	0	0	1	0	0
B5	0	0	0	0	0	0	0	0	1	0
B6	0	0	0	0	0	0	0	0	0	1

label	feature components $x^\star \in \{0,1\}^{11}$										
B1	1	0	0	0	0	0	0	0	0	0	0
B10	0	1	0	0	0	0	0	0	0	0	0
B11	0	0	1	0	0	0	0	0	0	0	0
B12	0	0	0	1	0	0	0	0	0	0	0
B13	0	0	0	0	1	0	0	0	0	0	0
B14	0	0	0	0	0	1	0	0	0	0	0
B2	0	0	0	0	0	0	1	0	0	0	0
B3	0	0	0	0	0	0	0	1	0	0	0
B4	0	0	0	0	0	0	0	0	1	0	0
B5	0	0	0	0	0	0	0	0	0	1	0
B6	0	0	0	0	0	0	0	0	0	0	1

In contrast to dummy coding, one-hot encoding does not choose a reference level, but uses an indicator for each label. In this way the 11 labels of VehBrand are replaced by the 11 unit vectors. The main difference between dummy coding and one-hot encoding is that the former leads to full rank design matrices, whereas the latter does not. This implies that under one-hot encoding there are identifiability issues in parametrizations. In network modeling identifiability is less important because we typically work in over-parametrized nonconvex optimization problems (with multiple equally good models/parametrizations); on the other hand, identifiability in GLMs is an important feature because one typically tries to solve a convex optimization problem, where the full rank property is important to efficiently and the (unique) solution.

Remark that other coding schemes could be used for categorical feature components such as Helmert's contrast coding. In classical GLMs the choice of the coding scheme typically does not influence the prediction, however, interpretation of the results may change by considering a different contrast. In network modeling the choice of the coding scheme may influence the prediction: typically, we exercise an early stopping rule in network calibrations. This early stopping rule and the corresponding result may depend on any chosen modeling strategy, such as the encoding scheme of categorical feature components.

Remark that dummy coding and one-hot encoding may lead to very high-dimensional input layers in networks, and it provides sparsity in input features. Moreover, the Euclidean distance between any two labels in the one-hot encoding scheme is that same. From natural language processing (NLP) we have learned that there are more efficient ways of representing categorical feature components, namely, by embedding them into lower-dimensional spaces so that proximity in these spaces has a useful meaning in the regression task. In networks this can be achieved by so-called embedding layers. In the context of our French MTPL example we refer to our next notebook.

7.1.2 3.1.2 Continuous feature components

In theory, continuous feature components do not need pre-processing if we choose a sufficiently rich network, because the network may take care of feature components living on different scales. This statement is of purely theoretical value. In practice, continuous feature components need pre-processing such that they all live on a similar scale and such that they are sufficiently equally distributed across this scale. The reason for this requirement is that the calibration algorithms mostly use gradient descent methods (GDMs). These GDMs only work properly, if all components live on a similar scale and, thus, all directions contribute equally to the gradient. Otherwise, the optimization algorithms may get trapped in saddle points or in regions where the gradients are at (also known as vanishing gradient problem). Often, one uses $[-1, +1]$ as the common scale because the/our choice of activation function is focused to that scale.

A popular transformation is the so-called MinMaxScaler. For this transformation we fix each continuous feature component of x, say x_l, at a time. Denote the minimum and the maximum of the domain of x_l by m_l and M_l, respectively. The MinMaxScaler then replaces

$$x_l \ \mapsto \ x_l^* = \frac{2(x_l - m_l)}{M_l - m_l} - 1 \ \in \ [-1, 1].$$

In practice, it may happen that the minimum ml or the maximum Ml is not known. In this case one chooses the corresponding minimum and/or maximum of the features in the observed data. For prediction under new features one then needs to keep the original scaling of the initially observed data, i.e. the one which has been used for model calibration.

Remark that if we have outliers, the above transformations may lead to very concentrated transformed feature components x_l^*, $i = 1, ..., n$, because the outliers may, for instance, dominate the maximum in the MinMaxScaler. In this case, feature components should be transformed first by a log-transformation or by a quantile transformation so that they become more equally spaced (and robust) across the real line.

7.1.3 3.1.3 Binary feature components

We observe that binary feature components (e.g. gender) are often embedded in the ML literature into a higher-dimensional space. However, we are of the opinion that this does not make sense. Hence, we suggest to set binary categorical feature components to $\pm 1/2$.

7.1.4 3.1.4 Summary

As a rule of thumb one could formulate it as follows: - continuous features $\Rightarrow$ scale to $[-1, +1]$ (if no outliers) - binary features $\Rightarrow$ set to $\{-1/2, +1/2\}$ - categorical features: * make them numerical $\Rightarrow$ scale to $[-1, +1]$ * One-hot encoding $\Rightarrow$ no scaling * dummy encoding $\Rightarrow$ no scaling * embedding $\Rightarrow$ made numerical and no scaling

7.2 3.2 Pre-processing functions

In our example we use dummy coding for the feature components VehBrand and Region. We use the MinMaxScaler for Area (after transforming $\{A, ..., F\} \mapsto \{1, ..., 6\}$), VehPower, VehAge (after capping at age 20), DrivAge (after capping at age 90), BonusMalus (after capping at level 150) and Density (after first taking the log-transform). VehGas we transform to $\pm 1/2$ and the volume Exposure $\in (0, 1]$ we keep untransformed.

Below the corresponding pre-processing functions:

[10]:
```
# MinMax scaler
preprocess_minmax <- function(varData) {
  X <- as.numeric(varData)
  2 * (X - min(X)) / (max(X) - min(X)) - 1
```

```
}

# Dummy coding
preprocess_catdummy <- function(data, varName, prefix) {
  varData <- data[[varName]]
  X <- as.integer(varData)
  n0 <- length(unique(X))
  n1 <- 2:n0
  addCols <- purrr::map(n1, function(x, y) {as.integer(y == x)}, y = X) %>%
    rlang::set_names(paste0(prefix, n1))
  cbind(data, addCols)
}

# Feature pre-processing using MinMax Scaler and Dummy Coding
preprocess_features <- function(data) {
  data %>%
    mutate_at(
      c(AreaX = "Area", VehPowerX = "VehPower", VehAgeX = "VehAge",
        DrivAgeX = "DrivAge", BonusMalusX = "BonusMalus", DensityX = "Density"),
      preprocess_minmax
    ) %>%
    mutate(
      VehGasX = as.integer(VehGas) - 1.5
    ) %>%
    preprocess_catdummy("VehBrand", "Br") %>%
    preprocess_catdummy("Region", "R")
}
```

7.3 3.3 Execute pre-processing

[11]:
```
dat2 <- preprocess_features(dat)
```

7.4 3.4 Inspect the pre-processed data

[12]:
```
head(dat2)
```

A data.frame: 6 × 52

	IDpol <dbl>	Exposure <dbl>	Area <fct>	VehPower <dbl>	VehAge <dbl>	DrivAge <dbl>	BonusMalus <dbl>	VehBranc <fct>
1	1	0.10	D	5	0	55	50	B12
2	3	0.77	D	5	0	55	50	B12
3	5	0.75	B	6	2	52	50	B12
4	10	0.09	B	7	0	46	50	B12
5	11	0.84	B	7	0	46	50	B12
6	13	0.52	E	6	2	38	50	B12

[13]:
```
str(dat2)
```

```
'data.frame':   678007 obs. of  52 variables:
 $ IDpol      : num  1 3 5 10 11 13 15 17 18 21 …
 $ Exposure   : num  0.1 0.77 0.75 0.09 0.84 0.52 0.45 0.27 0.71 0.15 …
 $ Area       : Factor w/ 6 levels "A","B","C","D",..: 4 4 2 2 2 5 5 3 3 2 …
 $ VehPower   : num  5 5 6 7 7 6 6 7 7 7 …
 $ VehAge     : num  0 0 2 0 0 2 2 0 0 0 …
 $ DrivAge    : num  55 55 52 46 46 38 38 33 33 41 …
 $ BonusMalus : num  50 50 50 50 50 50 50 68 68 50 …
 $ VehBrand   : Factor w/ 11 levels "B1","B2","B3",..: 9 9 9 9 9 9 9 9 9 9 …
 $ VehGas     : Factor w/ 2 levels "Diesel","Regular": 2 2 1 1 1 2 2 1 1 1 …
 $ Density    : num  7.1 7.1 3.99 4.33 4.33 8.01 8.01 4.92 4.92 4.09 …
 $ Region     : Factor w/ 22 levels "R11","R21","R22",..: 18 18 3 15 15 8 8 20
20 12 …
 $ ClaimTotal : num  0 0 0 0 0 0 0 0 0 0 …
 $ ClaimNb    : num  0 0 0 0 0 0 0 0 0 0 …
 $ group_id   : int  1 1 2 3 3 4 4 5 5 6 …
 $ AreaX      : num  0.2 0.2 -0.6 -0.6 -0.6 0.6 0.6 -0.2 -0.2 -0.6 …
 $ VehPowerX  : num  -0.818 -0.818 -0.636 -0.455 -0.455 …
 $ VehAgeX    : num  -1 -1 -0.8 -1 -1 -0.8 -0.8 -1 -1 -1 …
 $ DrivAgeX   : num  0.0278 0.0278 -0.0556 -0.2222 -0.2222 …
 $ BonusMalusX: num  -1 -1 -1 -1 -1 -1 -1 -0.64 -0.64 -1 …
 $ DensityX   : num  0.392 0.392 -0.218 -0.151 -0.151 …
 $ VehGasX    : num  0.5 0.5 -0.5 -0.5 -0.5 0.5 0.5 -0.5 -0.5 -0.5 …
 $ Br2        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br3        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br4        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br5        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br6        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br7        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br8        : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br9        : int  1 1 1 1 1 1 1 1 1 1 …
 $ Br10       : int  0 0 0 0 0 0 0 0 0 0 …
 $ Br11       : int  0 0 0 0 0 0 0 0 0 0 …
 $ R2         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R3         : int  0 0 1 0 0 0 0 0 0 0 …
 $ R4         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R5         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R6         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R7         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R8         : int  0 0 0 0 0 1 1 0 0 0 …
 $ R9         : int  0 0 0 0 0 0 0 0 0 0 …
 $ R10        : int  0 0 0 0 0 0 0 0 0 0 …
 $ R11        : int  0 0 0 0 0 0 0 0 0 0 …
 $ R12        : int  0 0 0 0 0 0 0 0 0 1 …
 $ R13        : int  0 0 0 0 0 0 0 0 0 0 …
 $ R14        : int  0 0 0 0 0 0 0 0 0 0 …
 $ R15        : int  0 0 0 1 1 0 0 0 0 0 …
 $ R16        : int  0 0 0 0 0 0 0 0 0 0 …
```

```
 $ R17         : int  0 0 0 0 0 0 0 0 0 0 ...
 $ R18         : int  1 1 0 0 0 0 0 0 0 0 ...
 $ R19         : int  0 0 0 0 0 0 0 0 0 0 ...
 $ R20         : int  0 0 0 0 0 0 0 1 1 0 ...
 $ R21         : int  0 0 0 0 0 0 0 0 0 0 ...
 $ R22         : int  0 0 0 0 0 0 0 0 0 0 ...
```

[14]:
```
summary(dat2)
```

```
     IDpol            Exposure           Area          VehPower     
 Min.   :      1   Min.   :0.002732   A:103957   Min.   : 4.000  
 1st Qu.:1157948   1st Qu.:0.180000   B: 75459   1st Qu.: 5.000  
 Median :2272153   Median :0.490000   C:191880   Median : 6.000  
 Mean   :2621857   Mean   :0.528547   D:151590   Mean   : 6.455  
 3rd Qu.:4046278   3rd Qu.:0.990000   E:137167   3rd Qu.: 7.000  
 Max.   :6114330   Max.   :1.000000   F: 17954   Max.   :15.000  
                                                                  
     VehAge          DrivAge       BonusMalus        VehBrand     
 Min.   : 0.000   Min.   :18.0   Min.   : 50.00   B12    :166024  
 1st Qu.: 2.000   1st Qu.:34.0   1st Qu.: 50.00   B1     :162730  
 Median : 6.000   Median :44.0   Median : 50.00   B2     :159861  
 Mean   : 6.976   Mean   :45.5   Mean   : 59.76   B3     : 53395  
 3rd Qu.:11.000   3rd Qu.:55.0   3rd Qu.: 64.00   B5     : 34753  
 Max.   :20.000   Max.   :90.0   Max.   :150.00   B6     : 28548  
                                                  (Other): 72696  
     VehGas          Density            Region         ClaimTotal     
 Diesel :332136   Min.   : 0.000   R24     :160601   Min.   :      0  
 Regular:345871   1st Qu.: 4.520   R82     : 84752   1st Qu.:      0  
                  Median : 5.970   R93     : 79315   Median :      0  
                  Mean   : 5.982   R11     : 69791   Mean   :     88  
                  3rd Qu.: 7.410   R53     : 42122   3rd Qu.:      0  
                  Max.   :10.200   R52     : 38751   Max.   :4075401  
                                   (Other):202675                    
    ClaimNb           group_id           AreaX            VehPowerX      
 Min.   :0.00000   Min.   :     1   Min.   :-1.00000   Min.   :-1.0000  
 1st Qu.:0.00000   1st Qu.:149318   1st Qu.:-0.60000   1st Qu.:-0.8182  
 Median :0.00000   Median :273211   Median :-0.20000   Median :-0.6364  
 Mean   :0.03891   Mean   :275320   Mean   :-0.08412   Mean   :-0.5537  
 3rd Qu.:0.00000   3rd Qu.:404072   3rd Qu.: 0.20000   3rd Qu.:-0.4545  
 Max.   :4.00000   Max.   :534079   Max.   : 1.00000   Max.   : 1.0000  
                                                                          
    VehAgeX           DrivAgeX          BonusMalusX          DensityX      
 Min.   :-1.0000   Min.   :-1.00000   Min.   :-1.0000   Min.   :-1.0000  
 1st Qu.:-0.8000   1st Qu.:-0.55556   1st Qu.:-1.0000   1st Qu.:-0.1137  
 Median :-0.4000   Median :-0.27778   Median :-1.0000   Median : 0.1706  
 Mean   :-0.3024   Mean   :-0.23620   Mean   :-0.8049   Mean   : 0.1729  
 3rd Qu.: 0.1000   3rd Qu.: 0.02778   3rd Qu.:-0.7200   3rd Qu.: 0.4529  
 Max.   : 1.0000   Max.   : 1.00000   Max.   : 1.0000   Max.   : 1.0000  
```

```
    VehGasX               Br2              Br3               Br4         
 Min.   :-0.50000   Min.   :0.0000   Min.   :0.00000   Min.   :0.00000  
 1st Qu.:-0.50000   1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.00000  
 Median : 0.50000   Median :0.0000   Median :0.00000   Median :0.00000  
 Mean   : 0.01013   Mean   :0.2358   Mean   :0.07875   Mean   :0.03714  
 3rd Qu.: 0.50000   3rd Qu.:0.0000   3rd Qu.:0.00000   3rd Qu.:0.00000  
 Max.   : 0.50000   Max.   :1.0000   Max.   :1.00000   Max.   :1.00000  

      Br5               Br6               Br7               Br8         
 Min.   :0.00000   Min.   :0.00000   Min.   :0.00000   Min.   :0.00000  
 1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.00000  
 Median :0.00000   Median :0.00000   Median :0.00000   Median :0.00000  
 Mean   :0.05126   Mean   :0.04211   Mean   :0.02612   Mean   :0.02004  
 3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.00000  
 Max.   :1.00000   Max.   :1.00000   Max.   :1.00000   Max.   :1.00000  

      Br9              Br10              Br11                R2          
 Min.   :0.0000   Min.   :0.00000   Min.   :0.000000   Min.   :0.000000  
 1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:0.000000   1st Qu.:0.000000  
 Median :0.0000   Median :0.00000   Median :0.000000   Median :0.000000  
 Mean   :0.2449   Mean   :0.01796   Mean   :0.005969   Mean   :0.004463  
 3rd Qu.:0.0000   3rd Qu.:0.00000   3rd Qu.:0.000000   3rd Qu.:0.000000  
 Max.   :1.0000   Max.   :1.00000   Max.   :1.000000   Max.   :1.000000  

       R3                R4                R5               R6         
 Min.   :0.00000   Min.   :0.00000   Min.   :0.0000   Min.   :0.00000  
 1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.0000   1st Qu.:0.00000  
 Median :0.00000   Median :0.00000   Median :0.0000   Median :0.00000  
 Mean   :0.01179   Mean   :0.01296   Mean   :0.2369   Mean   :0.01607  
 3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.0000   3rd Qu.:0.00000  
 Max.   :1.00000   Max.   :1.00000   Max.   :1.0000   Max.   :1.00000  

       R7                R8                R9               R10          
 Min.   :0.00000   Min.   :0.00000   Min.   :0.00000   Min.   :0.000000  
 1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.000000  
 Median :0.00000   Median :0.00000   Median :0.00000   Median :0.000000  
 Mean   :0.01547   Mean   :0.04024   Mean   :0.01916   Mean   :0.003245  
 3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.000000  
 Max.   :1.00000   Max.   :1.00000   Max.   :1.00000   Max.   :1.000000  

      R11                 R12                R13                R14         
 Min.   :0.000000   Min.   :0.00000   Min.   :0.00000   Min.   :0.00000  
 1st Qu.:0.000000   1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.00000  
 Median :0.000000   Median :0.00000   Median :0.00000   Median :0.00000  
 Mean   :0.001956   Mean   :0.05715   Mean   :0.06213   Mean   :0.02809  
 3rd Qu.:0.000000   3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.00000  
 Max.   :1.000000   Max.   :1.00000   Max.   :1.00000   Max.   :1.00000  
```

```
      R15              R16              R17               R18        
 Min.   :0.00000   Min.   :0.00000   Min.   :0.000000   Min.   :0.000  
 1st Qu.:0.00000   1st Qu.:0.00000   1st Qu.:0.000000   1st Qu.:0.000  
 Median :0.00000   Median :0.00000   Median :0.000000   Median :0.000  
 Mean   :0.04621   Mean   :0.02528   Mean   :0.006736   Mean   :0.125  
 3rd Qu.:0.00000   3rd Qu.:0.00000   3rd Qu.:0.000000   3rd Qu.:0.000  
 Max.   :1.00000   Max.   :1.00000   Max.   :1.000000   Max.   :1.000  

      R19                R20              R21             R22          
 Min.   :0.000000   Min.   :0.0000   Min.   :0.000   Min.   :0.000000  
 1st Qu.:0.000000   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.000000  
 Median :0.000000   Median :0.0000   Median :0.000   Median :0.000000  
 Mean   :0.007798   Mean   :0.0528   Mean   :0.117   Mean   :0.006661  
 3rd Qu.:0.000000   3rd Qu.:0.0000   3rd Qu.:0.000   3rd Qu.:0.000000  
 Max.   :1.000000   Max.   :1.0000   Max.   :1.000   Max.   :1.000000  
```

7.5 3.5 Split train and test data

First, we split the dataset into train and test. Due to the potential grouping of rows in policies we can not just do a random split. For this purpose, we use the function `partition(...)` from the `splitTools` package.

[15]:
```
ind <- partition(dat2[["group_id"]], p = c(train = 0.8, test = 0.2),
                 seed = seed, type = "grouped")
train <- dat2[ind$train, ]
test <- dat2[ind$test, ]
```

[16]:
```
# size of train/test
sprintf("Number of observations (train): %s", nrow(train))
sprintf("Number of observations (test): %s", nrow(test))

# Claims frequency of train/test
sprintf("Empirical frequency (train): %s", round(sum(train$ClaimNb) /␣
 ↪sum(train$Exposure), 4))
sprintf("Empirical frequency (test): %s", round(sum(test$ClaimNb) /␣
 ↪sum(test$Exposure), 4))
```

'Number of observations (train): 542331'

'Number of observations (test): 135676'

'Empirical frequency (train): 0.0736'

'Empirical frequency (test): 0.0736'

7.6 3.6 Common neural network specifications

In this section, we define objects and parameters which are used for all subsequent neural networks considered, independent of their network structure.

We need to define which components in the pre-processed dataset `dat2` are used as input features. As we have added the pre-processed features appropriate for the neural networks to the original features, we only must use the relevant ones.

[17]:
```
# select the feature space
col_start <- ncol(dat) + 1
col_end <- ncol(dat2)
features <- c(col_start:col_end) # select features, be careful if␣
↪pre-processing changes
print(colnames(train[, features]))
```

```
 [1] "AreaX"       "VehPowerX"   "VehAgeX"     "DrivAgeX"    "BonusMalusX"
 [6] "DensityX"    "VehGasX"     "Br2"         "Br3"         "Br4"
[11] "Br5"         "Br6"         "Br7"         "Br8"         "Br9"
[16] "Br10"        "Br11"        "R2"          "R3"          "R4"
[21] "R5"          "R6"          "R7"          "R8"          "R9"
[26] "R10"         "R11"         "R12"         "R13"         "R14"
[31] "R15"         "R16"         "R17"         "R18"         "R19"
[36] "R20"         "R21"         "R22"
```

The input to keras requires the train and testing data to be of matrix format, including all features used in the matrix and already correctly pre-processed.

[18]:
```
# feature matrix
Xtrain <- as.matrix(train[, features])  # design matrix training sample
Xtest <- as.matrix(test[, features])    # design matrix test sample
```

8 4. Designing neural networks

The choice of a particular network architecture and its calibration involve many steps. In each of these steps the modeler has to make certain decisions, and it may require that each of these decisions is revised several times in order to get the best (or more modestly a good) predictive model.

In this section we provide a short explanation on the ones explicitly used below. We refer to this tutorial on SSRN and further literature (provided below in the last chapter) to learn more about these hyperparameters and the way to choose the right architecture.

The tutorial covers the choice of a particular network architecture and its calibration. In each of these steps the modeler has to make certain decisions, and it may require that each of these decisions is revised several times in order to get the best (or more modestly a good) predictive model. The choices involve:

(a) data cleaning and data pre-processing

(b) choice of loss function (objective function) and performance measure for model calibration;

(c) number of hidden layers K;

(d) number of neurons $q_1, \ldots, q_K$ in the hidden layers;

(e) choice of activation function ϕ;

(f) optimization algorithm used for calibration which may include further choices of

  (i) initialization of algorithm,
  (ii) random (mini-)batches of data,
  (iii) stopping, number of iterations, number of epochs, etc.,
  (iv) parameters like learning rates, momentum parameters, etc.;

(g) normalization layers, dropout rates;

(h) regularization like LASSO or ridge regression, etc.

These choices correspond to the modeling cycle that is typically performed in statistical applications, the final validation step is not mentioned above.

8.1 4.1 Gradient descent methods

There are seval optimizers availble to find the solution to a neural networks, a short description as follows:

- The stochastic gradient descent method, called 'sgd', can be fine-tuned for the speed of convergence by using optimal learning rates, momentum-based improvements, the Nesterov acceleration and optimal batches. 'stochastic' gradient means that in contrast to (steepest) gradient descent, we explore locally optimally steps on random sub-samples
- 'adagrad' chooses learning rates that differ in all directions of the gradient and that consider the directional sizes of the gradients ('ada' stands for adapted);
- 'adadelta' is a modi
ed version of 'adagrad' that overcomes some deficiencies of the latter, for instance, the sensitivity to hyperparameters;
- 'rmsprop' is another method to overcome the deficiencies of 'adagrad' ('rmsprop' stands for root mean square propagation);
- 'adam' stands for adaptive moment estimation, similar to 'adagrad' it searches for directionally optimal learning rates based on the momentum induced by past gradients measured by an '2-norm;
- 'adamax' considers optimal learning rates as 'adam' but based on the l_1-norm;
- 'nadam' is a Nesterov accelerated version of 'adam'.

In the tutorial on SSRN, chapter 4 the performance of the various optimizers are analyzed. The study shows that `nadam` is a good candiate to be used.

[19]:
```
# available optimizers for keras
# https://keras.io/optimizers/
```

```
optimizers <- c('sgd', 'adagrad', 'adadelta', 'rmsprop', 'adam', 'adamax', 'nadam')
```

8.2 4.2 Epochs and batches

Epochs indicates how many times we go through the entire learning data $\mathcal{D}$, and batch size indicates the size of the subsamples considered in each Gradient Descent Method (GDM) step. Thus, if the batch size is equal to the number of observations n we do exactly one GDM step in one epoch, if the batch size is equal to 1 then we do n GDM steps in one epoch until we have seen the entire learning data $\mathcal{D}$. Note that smaller batches are needed for big data because it is not feasible to simultaneously calculate the gradient on all data efficiently if we have many observations. Therefore, we partition the entire data at random into (mini-) batches in the application of the GDM. Note that this partitioning of the data is of particular interest if we work with big data because it allows us to explore the data sequentially.

Concretely, for the maximal batch size n we can do exactly one GDM step in one epoch, for batch size k we can do n/k GDM steps in one epoch. For the maximal batch size we need to calculate the gradient on the entire data $\mathcal{D}$. The latter is, of course, much faster but on the other hand we need to calculate n/k gradients to run through the entire data (in an epoch).

The partitioning of the data $\mathcal{D}$ into batches is done at random, and it may happen that several potential outliers lie in the same batch. This happens especially if the chosen batch size is small and the expected frequency is low (class imbalance problem).

8.3 4.3 Initialization

A simple way to bring the initial network onto the right price level is to embed the homogeneous model into the neural network. This can be achieved by setting the output weights of the neurons equal to zero, and by initializing the output intercept to the homogeneous model. This is obtained subsequently by the code part defining the `weights`.

[20]:
```
# homogeneous model (train)
lambda_hom <- sum(train$ClaimNb) / sum(train$Exposure)
```

8.4 4.4 Activation function

Next we discuss the choice of activation function $\phi(\cdot)$ Typical choices of activation functions are:

> $\phi : \mathbb{R} \to \mathbb{R}$ is a (non-linear) activation function, which models the strengths of the activations in the neurons. Often, one of the following four choices is made
>
> $$\phi(x) = \begin{cases} \frac{1}{1+e^{-x}} & \text{sigmoid activation function,} \\ \tanh(x) & \text{hyperbolic tangent activation function,} \\ \mathbb{1}_{\{x \geq 0\}} & \text{step function activation,} \\ x\mathbb{1}_{\{x \geq 0\}} & \text{rectified linear unit (ReLU) activation function.} \end{cases} \tag{1.6}$$

The particular choice of the activation function may matter: calibration of deep networks may be slightly simpler if we choose the hyperbolic tangent activation because this will guarantee that all hidden neurons are in $(-1, +1)$, which is the domain of the main activation of the neurons in the next layer.

The step function activation is useful for theoretical considerations. From a practical point of view it is less useful because it is not differentiable and difficult to calibrate. Moreover, the discontinuity also implies that neighboring feature components may have rather different regression function responses: if the step function jumps, say, between driver's ages 48 and 49, then these two driver's ages may have a rather different insurance premium. For these reasons we do not pursue with the step function activation here.

We remark that the ReLU activation function often leads to sparsity in deep network activations because some neurons remain unactivated for the entire input. Such an effect may be wanted because it reduces the complexity of the regression model, but it can also be an undesired side effect because it may increase the diffculty in model calibration because of more vanishing gradients. Moreover, ReLU may lead to arbitrarily large activations in neurons because it is an unbounded activation function, this may be an unwanted effect because it may need re-scaling of activations to the main domain around the origin.

9 5. Generalized Linear Model using `keras(...)`

9.1 5.1 Definition

[21]:
```
# define network and load pre-specified weights
q0 <- length(features)                    # dimension of features

sprintf("Neural network with K=0 hidden layer")
sprintf("Input feature dimension: q0 = %s", q0)
sprintf("Output dimension: %s", 1)
```

'Neural network with K=0 hidden layer'

'Input feature dimension: q0 = 38'

'Output dimension: 1'

[22]:
```
# set seeds for reproducability of model fit
# needs to be set before the keras_model function!
Sys.setenv(PYTHONHASHSEED = seed)
set.seed(seed)
reticulate::py_set_seed(seed)
tensorflow::tf$random$set_seed(seed)
```

[23]:
```
Design  <- layer_input(shape = c(q0), dtype = 'float32', name = 'Design')
LogVol  <- layer_input(shape = c(1), dtype = 'float32', name = 'LogVol')

Network <- Design %>%
  layer_dense(units = 1, activation = 'linear', name = 'Network',
```

```
              weights = list(array(0, dim = c(q0, 1)), array(log(lambda_hom),␣
↪dim = c(1))))

Response <- list(Network, LogVol) %>%
  layer_add(name = 'Add') %>%
  layer_dense(units = 1, activation = k_exp, name = 'Response', trainable =␣
↪FALSE,
              weights = list(array(1, dim = c(1, 1)), array(0, dim = c(1))))

model_glm <- keras_model(inputs = c(Design, LogVol), outputs = c(Response))
```

9.2 5.2 Compilation

[24]:
```
model_glm %>% compile(
  loss = 'poisson',
  optimizer = optimizers[7]
)

summary(model_glm)
```

```
Model: "model"
________________________________________________________________________________
Layer (type)                 Output Shape         Param #   Connected to
================================================================================
Design (InputLayer)          [(None, 38)]         0
________________________________________________________________________________
Network (Dense)              (None, 1)            39        Design[0][0]
________________________________________________________________________________
LogVol (InputLayer)          [(None, 1)]          0
________________________________________________________________________________
Add (Add)                    (None, 1)            0         Network[0][0]
                                                            LogVol[0][0]
________________________________________________________________________________
Response (Dense)             (None, 1)            2         Add[0][0]
================================================================================
Total params: 41
Trainable params: 39
Non-trainable params: 2
________________________________________________________________________________
```

9.3 5.3 Fitting

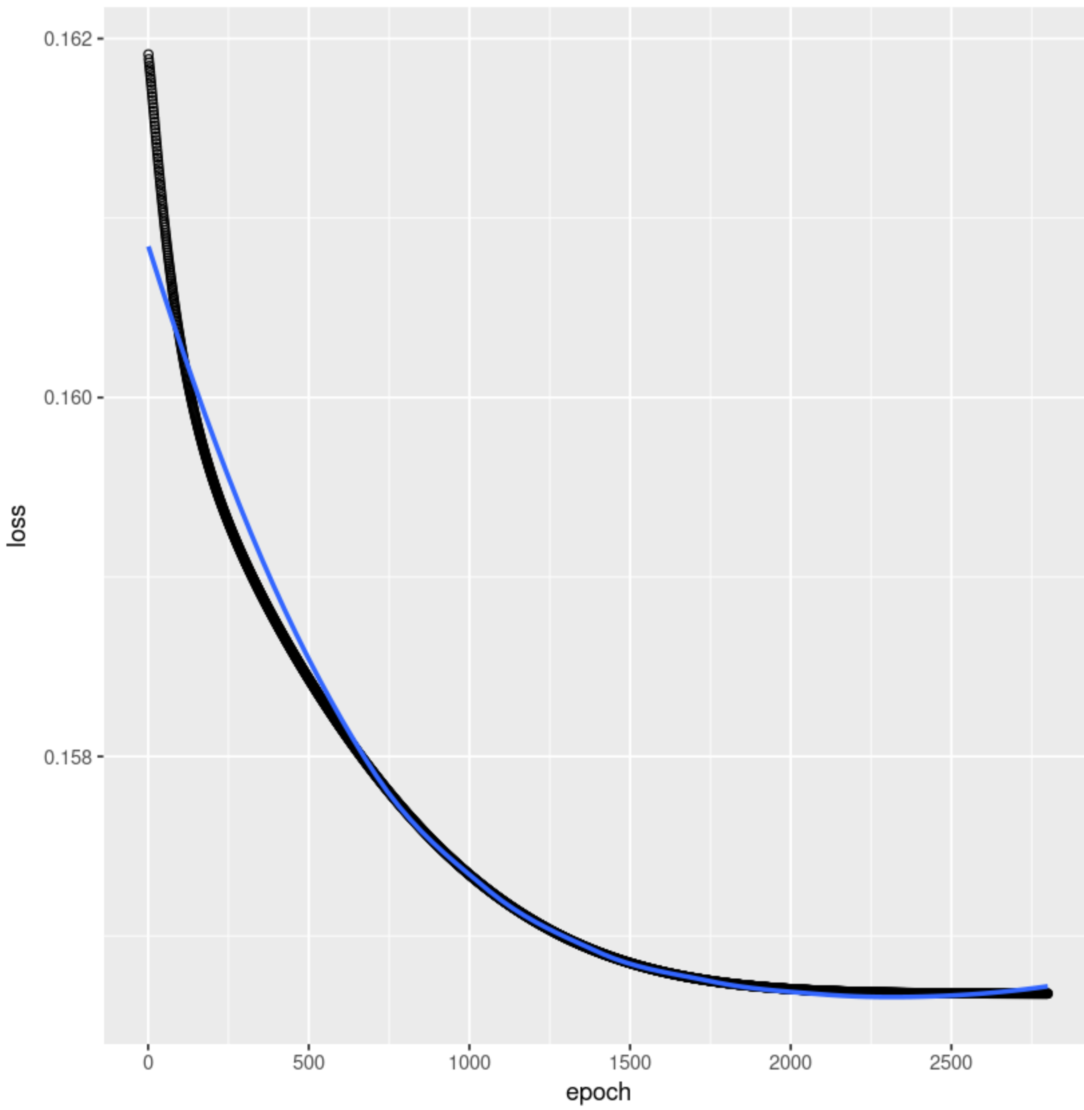
[25]:
```
# set hyperparameters
epochs <- 2800
batch_size <- nrow(Xtrain)
validation_split <- 0 # set to >0 to see train/validation loss in plot(fit)
verbose <- 1
```

[26]:
```
# expected run-time on Renku 8GB environment around 70 seconds
exec_time <- system.time(
  fit <- model_glm %>% fit(
    list(Xtrain, as.matrix(log(train$Exposure))), as.matrix(train$ClaimNb),
    epochs = epochs,
    batch_size = batch_size,
    validation_split = validation_split,
    verbose = verbose,
    callbacks = list(callback_early_stopping(patience=5))
  )
)
exec_time[1:5]
```

user.self 1167.459 **sys.self** 199.457 **elapsed** 244.315 **user.child** 0 **sys.child** 0

[27]:
```
plot(fit)
```

```
`geom_smooth()` using formula 'y ~ x'
```



9.4 5.4 Validation

[28]:
```
# calculating the predictions
train$fitglmNN <- as.vector(model_glm %>% predict(list(Xtrain, as.
↪matrix(log(train$Exposure)))))
test$fitglmNN <- as.vector(model_glm %>% predict(list(Xtest, as.
↪matrix(log(test$Exposure)))))

# average in-sample and out-of-sample losses (in 10^(-2))
sprintf("100 x Poisson deviance shallow network (train): %s",␣
↪PoissonDeviance(train$fitglmNN, train$ClaimNb))
```

```r
sprintf("100 x Poisson deviance shallow network (test): %s",␣
↪PoissonDeviance(test$fitglmNN, test$ClaimNb))

# average frequency
sprintf("Average frequency (test): %s", round(sum(test$fitglmNN) /␣
↪sum(test$Exposure), 4))
```

'100 x Poisson deviance shallow network (train): 24.1578577334396'

'100 x Poisson deviance shallow network (test): 24.2342830372812'

'Average frequency (test): 0.0737'

[29]:
```r
trainable_params <- sum(unlist(lapply(model_glm$trainable_weights,␣
↪k_count_params)))
df_cmp %<>% bind_rows(
  data.frame(model = "GLM with keras", epochs = epochs,
             run_time = round(exec_time[[3]], 0), parameters = trainable_params,
             in_sample_loss = round(PoissonDeviance(train$fitglmNN,␣
↪train$ClaimNb), 4),
             out_sample_loss = round(PoissonDeviance(test$fitglmNN,␣
↪test$ClaimNb), 4),
             avg_freq = round(sum(test$fitglmNN) / sum(test$Exposure), 4)
  ))

df_cmp
```

A tibble: 1 × 7

model	epochs	run_time	parameters	in_sample_loss	out_sample_loss	avg_
<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl
GLM with keras	2800	244	39	24.1579	24.2343	0.073

10 6. Generalized Linear Model using glm(...)

10.1 6.1 Fitting

[30]:
```r
exec_time <- system.time(
  glm1 <- glm(ClaimNb ~ AreaX + VehPowerX + VehAgeX + DrivAgeX + BonusMalusX +␣
↪DensityX + VehGasX + VehBrand + Region,
              data = train, offset = log(Exposure), family = poisson())
)
exec_time[1:5]
summary(glm1)
```

user.self 11.425 **sys.self** 1.46599999999998 **elapsed** 12.906 **user.child** 0 **sys.child** 0

```
Call:
glm(formula = ClaimNb ~ AreaX + VehPowerX + VehAgeX + DrivAgeX +
    BonusMalusX + DensityX + VehGasX + VehBrand + Region, family = poisson(),
```

```
    data = train, offset = log(Exposure))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.3729  -0.3255  -0.2472  -0.1396   6.9156

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.497636   0.041189 -36.360  < 2e-16 ***
AreaX        0.116014   0.053222   2.180  0.02927 *
VehPowerX    0.188582   0.020202   9.335  < 2e-16 ***
VehAgeX     -0.173527   0.014874 -11.666  < 2e-16 ***
DrivAgeX     0.229850   0.018739  12.266  < 2e-16 ***
BonusMalusX  1.345937   0.019390  69.413  < 2e-16 ***
DensityX     0.193388   0.080645   2.398  0.01648 *
VehGasX     -0.157057   0.014246 -11.024  < 2e-16 ***
VehBrandB2  -0.015839   0.019244  -0.823  0.41049
VehBrandB3   0.051440   0.026613   1.933  0.05325 .
VehBrandB4   0.050664   0.036061   1.405  0.16003
VehBrandB5   0.093294   0.030544   3.054  0.00225 **
VehBrandB6   0.011863   0.034837   0.341  0.73346
VehBrandB10  0.006627   0.044026   0.151  0.88035
VehBrandB11  0.151385   0.046946   3.225  0.00126 **
VehBrandB12 -0.304520   0.024674 -12.342  < 2e-16 ***
VehBrandB13  0.066317   0.049774   1.332  0.18274
VehBrandB14 -0.165573   0.097736  -1.694  0.09025 .
RegionR21   -0.014688   0.132675  -0.111  0.91185
RegionR22    0.162787   0.066761   2.438  0.01476 *
RegionR23   -0.034192   0.081020  -0.422  0.67301
RegionR24    0.008005   0.031079   0.258  0.79674
RegionR25   -0.025784   0.059395  -0.434  0.66421
RegionR26    0.047735   0.065073   0.734  0.46322
RegionR31    0.012498   0.043880   0.285  0.77579
RegionR41   -0.150341   0.058231  -2.582  0.00983 **
RegionR42    0.026830   0.117882   0.228  0.81996
RegionR43   -0.123807   0.190739  -0.649  0.51628
RegionR52    0.044481   0.037889   1.174  0.24040
RegionR53    0.031803   0.036623   0.868  0.38518
RegionR54    0.047086   0.048205   0.977  0.32867
RegionR72    0.109974   0.041949   2.622  0.00875 **
RegionR73   -0.174751   0.062161  -2.811  0.00493 **
RegionR74    0.426908   0.082940   5.147 2.64e-07 ***
RegionR82    0.234995   0.029306   8.019 1.07e-15 ***
RegionR83    0.016626   0.095972   0.173  0.86246
RegionR91    0.008749   0.043069   0.203  0.83903
RegionR93    0.156534   0.031163   5.023 5.09e-07 ***
RegionR94    0.157873   0.099689   1.584  0.11327
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 136692  on 542330  degrees of freedom
Residual deviance: 131015  on 542292  degrees of freedom
AIC: 171669

Number of Fisher Scoring iterations: 6
```

10.2 6.2 Validation

[31]:
```
# Predictions
train$fitGLM1 <- fitted(glm1)
test$fitGLM1 <- predict(glm1, newdata = test, type = "response")
dat$fitGLM1 <- predict(glm1, newdata = dat2, type = "response")
```

[32]:
```
# in-sample and out-of-sample losses (in 10^(-2))
sprintf("100 x Poisson deviance GLM (train): %s",␣
↪PoissonDeviance(train$fitGLM1, train$ClaimNb))
sprintf("100 x Poisson deviance GLM (test): %s", PoissonDeviance(test$fitGLM1,␣
↪test$ClaimNb))

# Overall estimated frequency
sprintf("average frequency (test): %s", round(sum(test$fitGLM1) /␣
↪sum(test$Exposure), 4))
```

'100 x Poisson deviance GLM (train): 24.1578035936711'

'100 x Poisson deviance GLM (test): 24.2344045952532'

'average frequency (test): 0.0737'

[33]:
```
df_cmp %<>% bind_rows(
  data.frame(model = "GLM with glm", epochs = NA, run_time =␣
↪round(exec_time[[3]], 0), parameters = length(coef(glm1)),
             in_sample_loss = round(PoissonDeviance(train$fitGLM1, as.
↪vector(unlist(train$ClaimNb))), 4),
             out_sample_loss = round(PoissonDeviance(test$fitGLM1, as.
↪vector(unlist(test$ClaimNb))), 4),
             avg_freq = round(sum(test$fitGLM1) / sum(test$Exposure), 4))
)
df_cmp
```

A tibble: 2 × 7

model	epochs	run_time	parameters	in_sample_loss	out_sample_loss	avg_
<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl
GLM with keras	2800	244	39	24.1579	24.2343	0.073
GLM with glm	NA	13	39	24.1578	24.2344	0.073

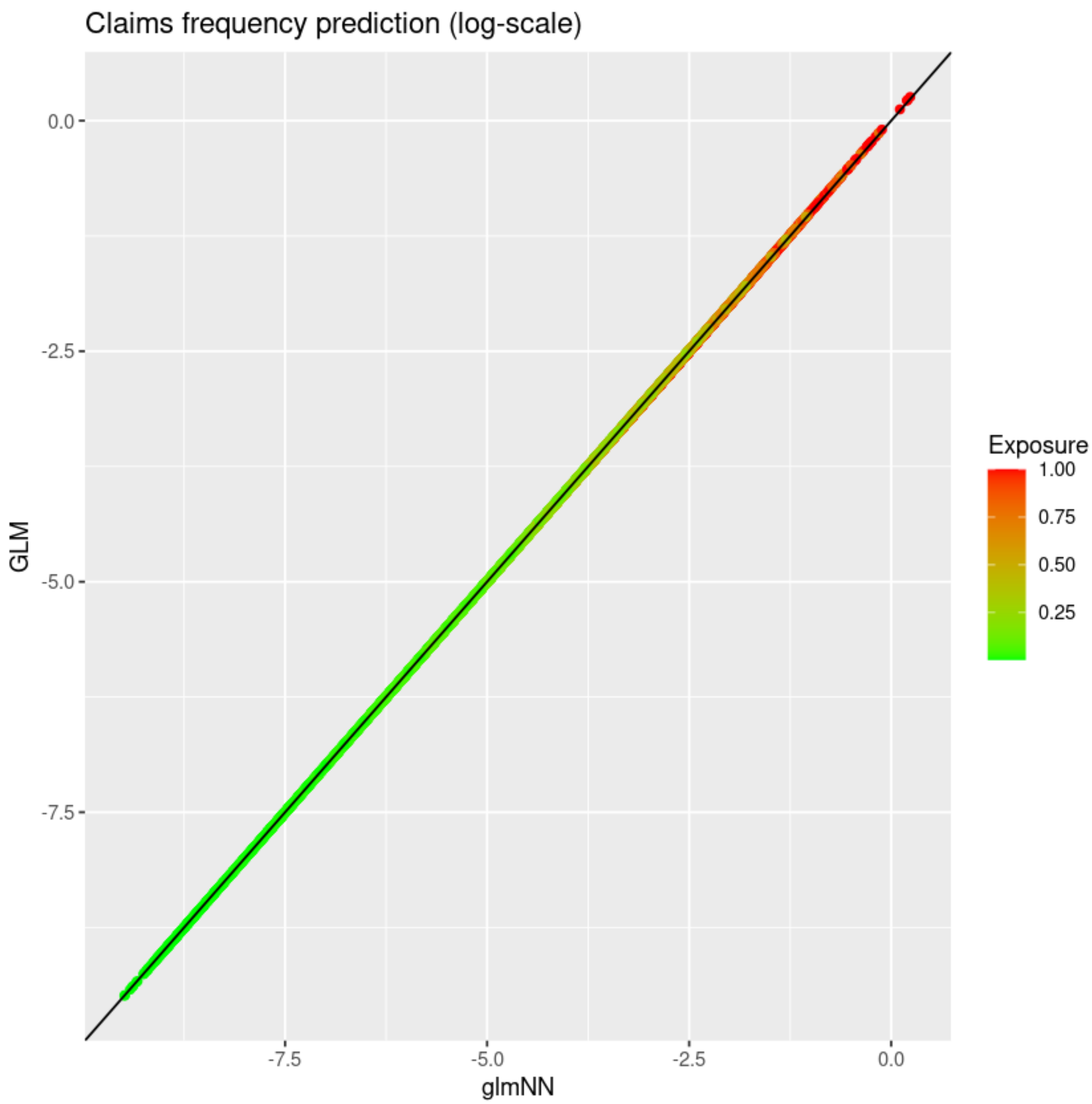
11 7. Model Comparison

11.0.1 7.1 Compare fitted values

[34]:
```
plot_claims_freq <- function(xvar, yvar, xlab, ylab) {
  axis_min <- log(max(test[[xvar]], test[[yvar]]))
  axis_max <- log(min(test[[xvar]], test[[yvar]]))

  ggplot(test, aes(x = log(!!sym(xvar)), y = log(!!sym(yvar)), colour =␣
 ↪Exposure)) + geom_point() +
    geom_abline(colour = "#000000", slope = 1, intercept = 0) +
    xlim(axis_max, axis_min) + ylim(axis_max, axis_min) +
    labs(x = xlab, y = ylab, title = "Claims frequency prediction (log-scale)")␣
 ↪+
    scale_colour_gradient(low = "green", high = "red")
}
```

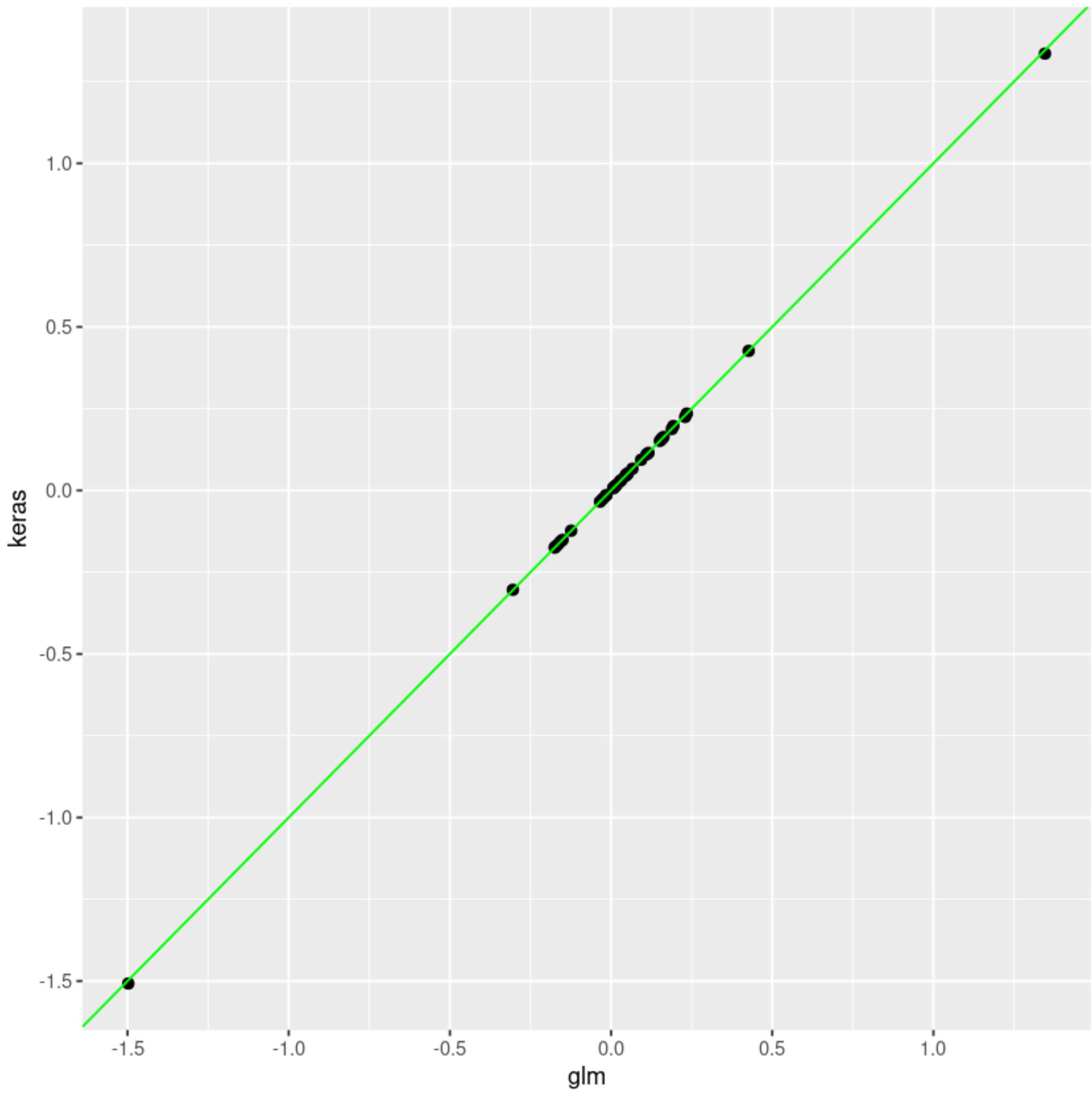
[35]:
```
plot_claims_freq("fitglmNN", "fitGLM1", "glmNN", "GLM")
```



11.0.2 7.2 Compare coefficients/weights

[36]:
```
df_coef <- data.
↪frame(glm=coef(glm1),keras=c(get_weights(model_glm)[[2]],get_weights(model_glm)[[1]]))
```

[37]:
```
ggplot(df_coef,aes(x=glm,y=keras)) + geom_point(size=2) +␣
↪geom_abline(intercept=0, slope=1, col="green")
```



[38]:
```
head(df_coef, n=10)
```

A data.frame: 10 × 2

	glm <dbl>	keras <dbl>
(Intercept)	-1.49763558	-1.50749433
AreaX	0.11601366	0.11477026
VehPowerX	0.18858234	0.18776892
VehAgeX	-0.17352658	-0.17288700
DrivAgeX	0.22984962	0.22471580
BonusMalusX	1.34593742	1.33544743
DensityX	0.19338782	0.19686709
VehGasX	-0.15705729	-0.15676792
VehBrandB2	-0.01583881	-0.01561991
VehBrandB3	0.05144019	0.05155322

12 8. Remarks

Some final remarks as follows: - A GLM is a special case of a neural network - A GLM can be fit using `keras()` - The number of epochs needs to be high to reach convergence - There is no reason to fit a glm with `keras()`

13 9. Session Info

The html is generated with the follow packages (which migth be slightly newer than the ones used in the published tutorial).

[39]:
```
sessionInfo()
```

```
R version 4.0.3 (2020-10-10)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 20.04.3 LTS

Matrix products: default
BLAS:   /usr/lib/x86_64-linux-gnu/blas/libblas.so.3.9.0
LAPACK: /usr/lib/x86_64-linux-gnu/lapack/liblapack.so.3.9.0

locale:
 [1] LC_CTYPE=en_US.UTF-8       LC_NUMERIC=C
 [3] LC_TIME=en_US.UTF-8        LC_COLLATE=en_US.UTF-8
 [5] LC_MONETARY=en_US.UTF-8    LC_MESSAGES=en_US.UTF-8
 [7] LC_PAPER=en_US.UTF-8       LC_NAME=C
 [9] LC_ADDRESS=C               LC_TELEPHONE=C
[11] LC_MEASUREMENT=en_US.UTF-8 LC_IDENTIFICATION=C

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] farff_1.1        OpenML_1.10      tidyr_1.1.2      splitTools_0.3.1
 [5] gridExtra_2.3    ggplot2_3.3.3    purrr_0.3.4      tibble_3.0.5
 [9] dplyr_1.0.3      magrittr_2.0.1   keras_2.3.0.0    mgcv_1.8-33
[13] nlme_3.1-151

loaded via a namespace (and not attached):
 [1] pbdZMQ_0.3-4      reticulate_1.18   tidyselect_1.1.0  repr_1.1.3
 [5] splines_4.0.3     lattice_0.20-41   colorspace_2.0-0  vctrs_0.3.6
 [9] generics_0.1.0    htmltools_0.5.1   base64enc_0.1-3   XML_3.99-0.5
[13] rlang_0.4.10      pillar_1.4.7      glue_1.4.2        withr_2.4.0
[17] DBI_1.1.1         rappdirs_0.3.1    uuid_0.1-4        lifecycle_0.2.0
[21] tensorflow_2.2.0  munsell_0.5.0     gtable_0.3.0      memoise_1.1.0
[25] evaluate_0.14     labeling_0.4.2    tfruns_1.4        curl_4.3
[29] IRdisplay_1.0     Rcpp_1.0.6        readr_1.4.0       scales_1.1.1
[33] backports_1.2.1   checkmate_2.0.0   IRkernel_1.1.1    jsonlite_1.7.2
```

```
[37] farver_2.0.3     hms_1.0.0        digest_0.6.27     stringi_1.5.3
[41] BBmisc_1.11      getPass_0.2-2    grid_4.0.3        tools_4.0.3
[45] crayon_1.3.4     whisker_0.4      pkgconfig_2.0.3   zeallot_0.1.0
[49] ellipsis_0.3.1   Matrix_1.3-2     data.table_1.14.0 httr_1.4.2
[53] assertthat_0.2.1 R6_2.5.0         compiler_4.0.3
```

[40]:

```
reticulate::py_config()
```

```
python:         /usr/bin/python3
libpython:      /usr/lib/python3.8/config-3.8-x86_64-linux-gnu/libpython3.8.so
pythonhome:     //usr://usr
version:        3.8.10 (default, Sep 28 2021, 16:10:42)  [GCC 9.3.0]
numpy:          /usr/local/lib/python3.8/dist-packages/numpy
numpy_version:  1.20.1
tensorflow:     /usr/local/lib/python3.8/dist-packages/tensorflow

python versions found:
 /usr/bin/python3
 /usr/bin/python
```

[41]:

```
tensorflow::tf_version()
```

```
[1] '2.4'
```