

**Roll No.**

**Total No. of Pages : 02**

**Total No. of Questions : 08**

**B.Tech (CSE/IT) (Sem.-4)**

# DISCRETE MATHEMATICS

**Subject Code : BTCS-401-18**

**M.Code : 77626**

**Date of Examination : 24-08-21**

**Time : 2 Hrs.**

**Max. Marks : 60**

**INSTRUCTIONS TO CANDIDATES :**

**1. Attempt any FIVE question(s), each question carries 12 marks.**

1. i) If  $S_k = \{\dots, -2, -1, 0, 1, 2, \dots, k\}$ , then find.

a)  $\bigcup_{k=1}^n S_k$  and  $\bigcup_{k=1}^{\infty} S_k$

b)  $\bigcap_{k=1}^n S_k$  and  $\bigcap_{k=1}^{\infty} S_k$

- ii) Let  $R_1$  and  $R_2$  be two antisymmetric relations. Determine whether the  $R_1 \cap R_2$  is antisymmetric?
2.
  - i) Let  $f: \mathbb{Z} \rightarrow \mathbb{Z}$  and  $g: \mathbb{Z} \rightarrow \mathbb{Z}$ . Show that the functions  $f(x) = x^2 - 1$  and  $g(y) = \sqrt{y+1}$  are inverse to each other.
  - ii) Consider the sequence  $x_1, x_2, x_3, \dots, x_n$  of  $n$  natural number defined as  $x_n = x_{n-1} + x_{n-2} + x_{n-3}$ , where  $x_1 = 2, x_2 = 2, x_3 = 3$ . Show that  $x_n < 2^n$  for all  $n \geq 4$ .
3.
  - i) Show that if any eight positive integers are chosen, then two of them will have the same remainder when divided by 7.
  - ii) Show that conclusion T follows logically from the given premises :  $P \rightarrow Q, Q \rightarrow R, R \rightarrow S, \sim S$  and  $P \vee T$ .
4. Determine whether the following argument is valid or not : Some scientists are not doctors. Some biologists are not doctors. Therefore, some scientists are not biologists.
5. Prove that a group  $G$  is abelian if and only if  $(xy)^{-1} = x^{-1}y^{-1}$ , for all  $x, y \in G$ .

6.
  - i) Express the function  $f(x, y, z) = (x + y)(x + y')$  into product-of-sums form.
  - ii) Write the dual of the Boolean equation :  $(x.1)(0 + x') = 0$ .
7. Show that the number of leaf nodes in a rooted tree with  $n$  nodes, with each node having zero or three children, is  $(2n + 1)/3$ .
8. Prove that a planar graph has at most  $3n - 6$  edges if  $n \geq 3$ .

**Note:** Any student found attempting answer sheet from any other person(s), using incriminating material or involved in any wrong activity reported by evaluator shall be treated under UMC provisions.

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