**Step 1: Implement Counting Sort with n = 10^5 and Key Range 0 to 60,000**

1. **Counting Sort** is efficient when the range of key values is not significantly larger than the input size n. It is known to have a time complexity of O(n+k), where k is the range of keys.
2. For this scenario:
   * Implement Counting Sort in C++.
   * Run the algorithm with an array of size n = 10^5 where each key falls within 0 to 60,000.
   * Record the execution time.

**Step 2: Implement Quick Sort with n = 10^5 and Same Key Range 0 to 60,000**

1. **Quick Sort** is a general-purpose comparison sort with an average time complexity of O(nlogn).
2. Use a single processor for the implementation.
3. Run the Quick Sort algorithm on the same data array (or generate similar random arrays within the key range).
4. Record the execution time.

**Step 3: Analyze Complexity Based on Observations**

After running both Counting Sort and Quick Sort, compare their run times:

* **Counting Sort** should theoretically perform faster due to its linear complexity, as long as k remains comparable to n.
* **Quick Sort** will likely be slower, with average O(nlogn) complexity, but it can sometimes perform faster if implemented with optimizations like random pivot selection.

**Step 4: Extend Counting Sort for Larger Key Range (0 to 200,000+)**

1. Increase the key range to 0 to 200,000 or even larger.
2. Run Counting Sort on an array of the same size (n = 10^5) with this larger key range.
3. Observe and record the running time. With a significant increase in k, Counting Sort’s performance may degrade, as it requires additional memory and time to handle larger arrays of counts.

**Step 5: Implement Radix Sort as an Improvement for Large Key Ranges**

1. **Radix Sort** can serve as an improvement when k is significantly larger than n.
   * Radix Sort has a time complexity of O(d⋅(n+b)), where d is the number of digits in the largest key (or number of passes needed), and b is the base of the numbers.
   * For integers in base-10, d can be minimized by choosing an optimal base (e.g., base-256).
2. Implement Radix Sort and run it on the same array.
3. Record the execution time and compare it with both Counting Sort and Quick Sort.

**Step 6: Comment on Time Complexities and Observations**

1. **Counting Sort**:
   * Time complexity: O(n+k)
   * Effective when k is close to n, but may slow down if k becomes significantly larger.
2. **Quick Sort**:
   * Time complexity: O(nlogn) on average.
   * It performs well across various cases, but not as efficient as Counting Sort when k is low.
3. **Radix Sort**:
   * Time complexity: O(d⋅(n+b)).
   * Performs better than Counting Sort for large key ranges by breaking down the range into manageable passes.