REPORT SUBMITTED FOR THE THIRD YEAR PROJECT AS A PART OF THE RESEARCH EDUCATION ADVANCEMENT PROGRAM (REAP)

---

# Analyzing the onset of hydrodynamic non-equilibrium in Helmet Streamers.

---

UNDER THE GUIDANCE OF- **DR. PARAMESWARAN VENKATAKRISHNAN**

Submitted by- **Rohan S**

BMS College of Engineering, Bangalore.

September 1, 2020

# Contents

# List of Figures

## ACKNOWLEDGMENTS

I would like to express my heartfelt gratitude to my project supervisor, Dr.P.Venkatakrishnan, who was generous enough to clarify multiple doubts and questions I had, and for continuing to motivate me until the end of the project. I would like to express my thanks to Mr.Madhusudhan, helping me get a project on time, and for administering REAP which in-turn made the project possible. I would like to thank all the faculty and students at REAP who, knowingly or not, have helped me throughout the course.

## 0.1 Introduction

The number of fascinating phenomenon on the heliosphere is limitless. The most prominent out of them is the solar wind. Around 1951, Biermann noticed that many comets showed excess ionization and abrupt changes in the outflow of material in their tails, this "abrupt" motion meant there is a causal agent that affects the motion of ions around it. This was later discovered to be Solar wind. Subsequent discoveries by Parker (1958, 1960, 1963, 1965) helped recognize that the solar wind is a consequence of the expansion of coronal material into interplanetary space (Parker, 1958). A number of detailed theoretical studies exist which have examined this expansion assuming spherically symmetric flow, the base of Parker's theory itself is on the assumption that the flow is isothermal, steady, and spherically symmetric, assuming a area function proportional to $r^2$.

Solar wind isn't a separate phenomenon devoid of any interaction, it's brimming with multiple structures as a result of it. One such structure of primary importance for the current problem is "Coronal Streamers". (One more is Polar Plumes) These are also called *Helmet* Streamers, as they supposedly resemble olden days Russian styled helmets. Helmet streamers are large cap-like coronal structures with long pointed peaks that usually overlie sunspots and active regions. We often find a prominence or filament lying at the base of these structures. Helmet streamers are formed by a network of magnetic loops that connect the sunspots in active regions and help suspend the prominence material above the solar surface. The closed magnetic field lines trap the electrically charged coronal gases to form these relatively dense structures. The pointed peaks are formed by the action of the solar wind blowing away from the Sun in the spaces between the streamers.

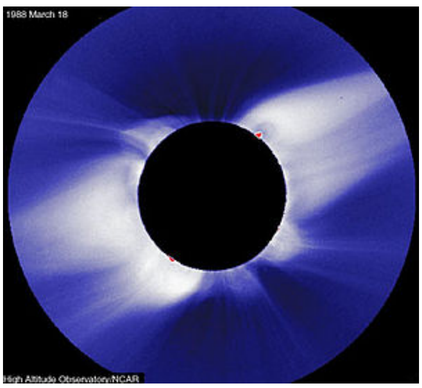


Figure 1: Helmet Streamers. Courtesy: Wikipedia

The importance given to such a structure is partly because of the theory behind the relationship between Coronal streamers and the Coronal Mass Ejections (CME's). The discoveries and theories since then were aimed at answering the physical mechanism responsible for the initiation of CME's in the streamers structure.

The current project models an equilibrium flow in an area resembling that of coronal streamers. The non-equilibrium conditions are computed geometrically, and analyzed. These conditions in turn reveal the dependencies of parameters such as area, the flow speed, density on the streamer structure.

## 0.2 Problem Statement

I started with understanding and analyzing the Parker solution of the Solar wind, keeping the assumptions fixed. Once I got the exact results, the problem was then split into two parts:

1- We approximated an area function that is not exactly spherically symmetric, but symmetric only beyond the neck region of the structure. I had to model the variation of Mach number (M) as a function of the distance from the base of the structure.

2- The above case is still isothermal, and steady-state. I then had to solve the Parker's problem eliminating these assumptions, hence a time-dependent solution to the Solar wind inside a Helmet streamer.

The solution is analytical, and was physically reasonable. The crux of the whole problem is to: "Reduce the assumptions made by Parker, one by one and analyse the difference in the equilibrium condition of the Solar wind using an approximated area function."

## 0.3 Parker's Solution

Parker assumed the flow to be spherically symmetric, isothermal, and steady. The assumption also extends to $P_{Sun} >> P_{Ism}$, driving the flow of the wind.

Chapman in 1957 considered the corona to be in hydrostatic equilibrium obeying:

$$\frac{dP}{dr} = -\rho g \tag{1}$$

and

$$\frac{dP}{dr} + \frac{GM_s\rho}{r^2} = 0 \tag{2}$$

Now, we consider the equations for fluid flow, viz., Continuity, Momentum equations, Considering the area to be proportional to $r^{\alpha}$, ($\alpha$=2 denotes a parabolic area distribution), and independent of time, we get:

$$\frac{d\rho vA}{dr} = 0 \tag{3}$$

There will be a portion of the distribution whose area of cross-section decreases with height, forming a narrow "neck" region, a radius of $r_n$, the area of cross-section then begins to increase with height beyond the neck region.

The outflow equation can then be expressed as:

$$\frac{dP}{dr} + \frac{GM_s\rho}{r^2} + \rho\frac{dv}{dt} = 0 \tag{4}$$

We see that $\frac{dv}{dt}$ can be expressed as $v\frac{dv}{dr}$ (continuity equation), and substituting this back we get:

$$\rho v\frac{dv}{dr} = -\frac{GM_s\rho}{r^2} - \frac{dP}{dr} \tag{5}$$

This is the momentum equation.

Here, $\rho$ is the plasma density, $M_s$ is the solar mass, $v$ is the flow speed. The equation describes the convective acceleration of the flow with due to the pressure gradient and gravity.

Considering the mass flow rate is constant, we can write the continuity equation as:

$$\frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{v}\frac{dv}{dr} - \frac{2}{r} \tag{6}$$

For a complete ionization, we have $P = 2\rho S^2$, where S is the speed of sound, and hence substituting this back in momentum equation, we get:

$$(v - \frac{S}{v})\frac{dv}{dr} - \frac{2S}{r} + \frac{GM_s}{r^2} = 0 \tag{7}$$

We henceforth write the relationships in terms of the Mach Number "M", given by $M = v/S$, where $S$ is the speed of sound at that particular temperature T. (S$\rightarrow$S(T))

Re-arranging, we get:

$$(\frac{1-M^2}{M^2})\frac{dM^2}{dr} = \frac{2r_c - \alpha r}{r^2} \tag{8}$$

where $r_c$ is the Parker radius given by $\frac{GM_s}{2S^2}$, and $\alpha$ is given by $\frac{d(lnA)}{dr}$.

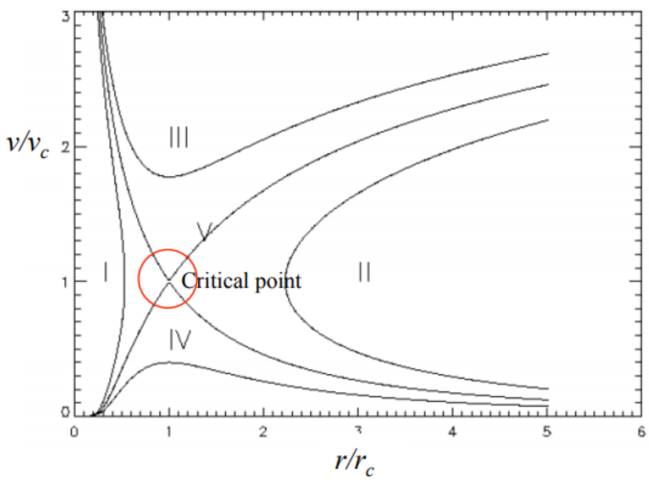


Figure 2: Parker's solution for different values of C.

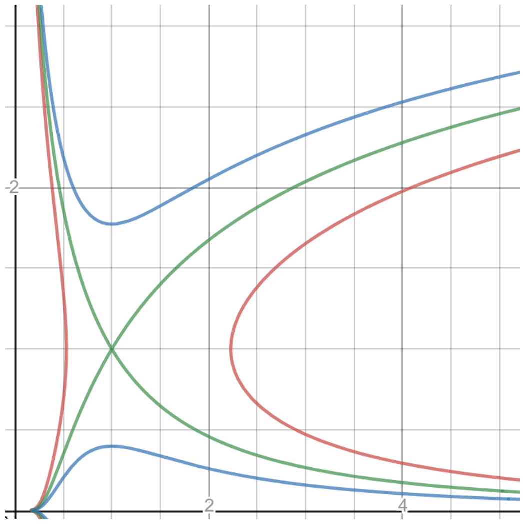

Figure 3: Plot verified.

Finally, integrating the above equation, we arrive at the required relationship:

$$M^2 - ln(M^2) = 4(ln\frac{r}{r_c}) + 4(\frac{r}{r_c}) + C \tag{9}$$

where C is the integration constant to be found using boundary conditions. Different values of C give rise to different solutions, the optimal solution is for C= $-3$. We observe that at any other curve apart from the critical point either has a double solution (in case of the red curves) or supersonic near the sun's surface which isn't what happens. (blue curves) The curves that are in green passes through (1,1), hence passing through $r = r_c$ and hence, $v = S$, called the critical point. The flow is subsonic for $r < r_c$ and supersonic for $r > r_c$.

## 0.4 Modified Area Problem

We then considered an area distribution that was not spherically symmetric throughout to drive a comparatively realistic model. Consider an area function of the form:

$$A(r) = \begin{cases} a_1[r_n(r_1 - r) + r(r - r_n)] & \text{if r} < r_1 \\ a_1[\frac{(r_1 - r_n)}{r_1}]r^2 & \text{if r} > r_1 \end{cases} \tag{10}$$

we observe that at $r = r_1$, both the equations reach the same form. We also observe that since A is only spatially varying, there is little modification to the existing set of equations, except for $\alpha$. Consider an arbitrary distance $r_1$ from the base where $r_1 < r_n$, the neck radius, a linearly varying area segment such that A=f(r) and beyond a certain distance, area is proportional to the square of r, like in case of Parker's solution.

We get two solutions for $\alpha$ based on the distance from $r_1$. For $r < r_1$, we get:

$$\alpha = \frac{2(r - r_n)}{r^2 - 2rr_n + r_n r_1} \tag{11}$$

and for $r > r_1$,

$$\alpha = \frac{2r_1}{r(r_1 - r_n)} \tag{12}$$

hence the equation is modified as; for $r < r_1$:

$$M^2 - ln(M^2) = 4(\frac{r}{r_c}) - \int \frac{2(r - r_n)}{r(r^2 - 2rr_n + r_n r_1)} dr \tag{13}$$

and for $r > r_1$

$$M^2 - ln(M^2) = 4(\frac{r}{r_c}) - \int \frac{2r_1}{r(r_1 - r_n)} dr \tag{14}$$

The integral of the second equation is direct, the first one (13) is split up using partial fractions and each term is integrated. For $r > r_1$, we have:

$$M^2 - ln(M^2) = 4(\frac{r}{r_c}) - \frac{2r_1 ln(r)}{(r_1 - r_n)} \tag{15}$$

and for $r < r_1$,

$$M^2 - ln(M^2) = 4(\frac{r}{r_c}) - \frac{ln(r_1 r_n - 2r_n r + r^2) - \frac{2\sqrt{r_n - r_1} tanh^{-1}(\frac{r_n - r}{\sqrt{r_n(r_n - r_1)}})}{\sqrt{r_n}}}{2r_1} \tag{16}$$

We observe that for $r > r_1$, the solution resembles the parker's solution but with a finite displacement. Initially for T= 1 Million degrees, S= 92km/s and Rc=$(GM_s/2S^2) = 8 * 10^9 m$, we plot for a flow that starts from an arbitrary "r" such that $r_n < r_1 < r_c$, and the observe how the flow propagates for $r < r_1$ and $r > r_1$.

We observe that for $r < r_1$, the flow passes through sub-sonic phase until M=1 which happens lower than critical radius, and then the supersonic flow occurs which (as evident from the slope) faster than Parker's solution. $r > r_1$ case is very similar to the spherically symmetric solution with a rather gradual and realistic increase in the flow speed. In figure 4, the flow only till $r_1$ is needed, and further

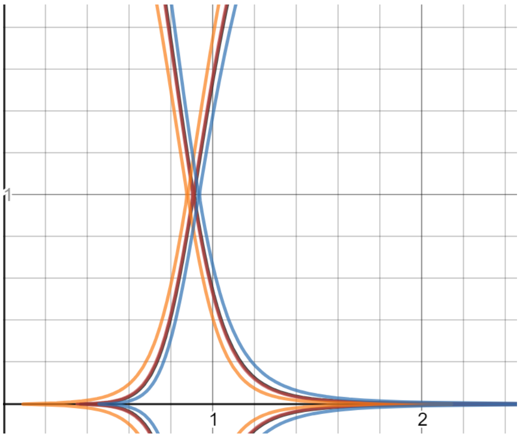


Figure 4: Flow for $r < r_1$.


Figure 5: Flow for $r > r_1$

path is given by Figure 5.

The basic idea for this is that the area function of the helmet streamer represents something similar to a DeLaval Nozzle. Initially the flow is subsonic, accelerating for convergent portion where $M < 1$, then transonic near the neck and finally super-sonic in the divergent region and $M > 1$. Gas dynamics behind both are strikingly similar.

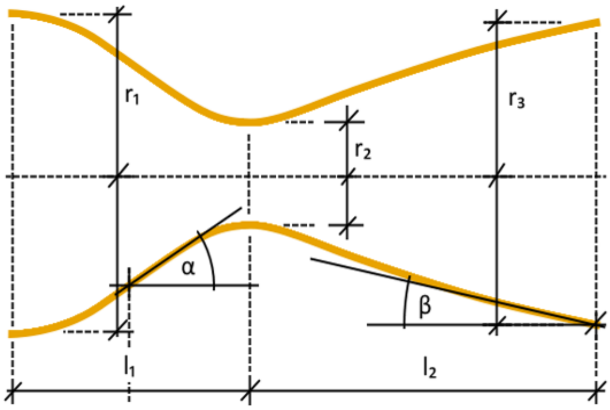


Figure 6: A DeLaval Nozzle.

For various speed of sound (indirectly, varying the temperature, the point of transition varies, which implies that at different places of the corona, the streamer originates having different super-sonic speeds at the given time interval- due to the assumption that flow is steady), we find the sonic point, for constant $r_1$. It is then observed from the plot that unless $r_c$ tends very close to $r_s$, there will always be flows that becomes super-sonic after crossing $r_c$, due to steady conditions, it is therefore summarized that in steady state, if $r_c > r_s$, there will always be an equilibrium and a critical point of transition for helmet streamers, for the said assumptions.

## 0.5 Time-Dependent Solution

Finally, we vary the speed of sound, with a key intention of varying the temperature of the flow, polytropically, instead of isothermally. Let $S = S_0(1 + \beta t)$ where $\beta$ is some constant and $S_0$ is the initial speed of sound. The continuity and momentum equations still remain, with an additional energy equation, assuming a polytropic process. We would have to consider:

$$(\frac{\partial}{\partial t} + v\frac{\partial}{\partial r})\frac{P}{\rho^\gamma} = 0 \tag{17}$$

where $\gamma$ is the ratio of specific heats. The remaining two equations are stated again to help the process:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v A)}{\partial r} = 0 \tag{18}$$

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial r} = \frac{\partial P}{\partial r} - \frac{\rho G M_s}{r^2} \tag{19}$$

The method of characteristics is used to solve the equations and obtain the relationship for Mach number in terms of time and spatial component. Solving the equations, we get one quasi-linear PDE:

$$\frac{\partial M}{\partial t} + S(M - \frac{1}{M})\frac{\partial M}{\partial r} = \frac{2S}{\gamma r} - \frac{2Sr_c}{r^2} - \frac{M}{(1+t)} \tag{20}$$

we should remember that S is a function of time. Here we can convert this into a set of ODE's by considering a parameter "p" and finding the derivative of both "t" and "r" along p.

Hence;

$$\frac{dt}{dp} = 1 \tag{21}$$

$$\frac{dr}{dp} = S(M - \frac{1}{M}) \tag{22}$$

$$\frac{dM}{dp} = \alpha - \delta M \tag{23}$$

where $\alpha = \frac{2S}{\gamma r} - \frac{2Sr_c}{r^2}$ and $\delta = \frac{1}{1+t}$. We then integrate them individually calculating a relationship for "p" and each of the constants of integration. The calculations are not addressed, we directly present the final form of each terms.

We calculate p to be:

$$p = \frac{r - r_c}{S(M - \frac{1}{M})} \tag{24}$$

$$c_2 = r_c \tag{25}$$

$$c_3 = ln(\frac{2S_0(\frac{1}{\gamma} - 1)}{r_c} - 1) + p \tag{26}$$

where $c_3$ and $c_2$ are the constants of integration from equations (22) and (23). Substituting for the constants and "p" in the characteristic equations produces the equation for M. Once we substitute, we observe M=f(r,t) to be of the form $M = (1 + t)(\alpha - \beta^{g(M)})$, hence an implicit function. Considering a value of $\gamma = \frac{5}{3}$, $r_c = 3 * 10^9 m$, and $S_0 = 1.5 * 10^5 m/s$, we get:

$$M = (1 + t)[\frac{2S_0(1 + t)}{r}(0.6 - \frac{r_c}{r}) - e^{-\frac{1}{(1+t)}(\frac{2(r - r_c)}{S_0(1+t)[M - \frac{1}{M}]} + ln(|\alpha|))}] \tag{27}$$

where $\alpha = \frac{2S_0(\frac{1}{\gamma} - 1)}{r_c} - 1$. This equation is then plotted as x vs f(x) where x is $\frac{r}{r_c}$, and for $t = 0, 1, 100, 200..$ seconds, the final equation takes the form:

$$y = (1 + t)[\frac{10^{-4}(1 + t)}{x}(0.6 - \frac{1}{x}) - e^{-\frac{1}{(1+t)}(\frac{2(x - 1)}{10^-4[y - \frac{1}{y}]} + ln(0.8 * 10^{-4}))}] \tag{28}$$

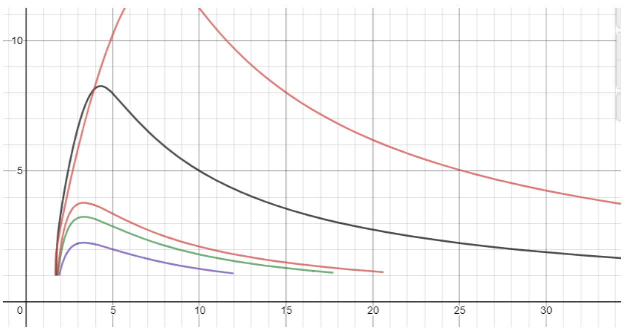


Figure 7: Time-dependent Solution

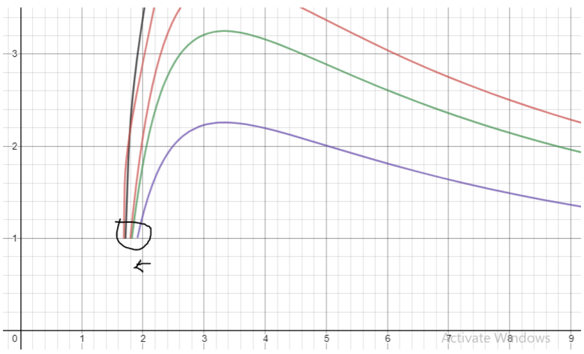


Figure 8: A closer view showing the change in $r_c$

The plot shows how the Mach number changes with distance as time increases, $r_c$ hence, r reduces for $M = 1$ at different times as $S$ increases. As the r increases, $M$ first becomes supersonic and gradually evens out, becoming sub-sonic at large distances as it's observed, when the solar wind speed is measured experimentally from the sun's surface towards the end of the solar system.

## 0.6 Discussion and Conclusion

Parker's solution was initially plotted and verified. The optimal solution was found out and analyzed. Then, a realistic area approximation was introduced, removing the spherically symmetric condition. The variation of Mach number was plotted with distance, the optimal solution had a rather steep and higher accelerating curve for low $r$ values and eventually reached Parker's solution.
Time-dependent solution provided clarity on the flow as time increased in poly-tropic conditions. As the time increased, the Parker's radius (critical radius) changes as it's a function of $S$. The Parker radius reduces, this would mean $r_c$ moves closer to the sun's surface, and the $M = 1$ point shifts towards the left of the graph, which was observed. This also showed that as time increased the variation of flow is not altered but just displaced smoothly, reaching the supersonic speed and eventually dispersing into space.