

Supplementary materials

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1 Customized models in the literature

1.1 PLSP 1

Indices

j, j'	Index of jobs ($j, j' = 1, \dots, J$)
o, o'	Index of operations
h_j	Last operation of job j
m	Index of machines ($m = 1, \dots, M$)
t	Index of macro-periods ($t = 1, \dots, T$)
τ, τ'	Index of micro-periods
ζ_t	Index of the last micro-period belonging to the macro-period t

Parameters

D_{jt}	Demand of job j at the end of period t
p_{jo}	Processing time needed to produce one unit of product related to operation O_{jo}
C_{mt}	Capacity of machine m during period t
c_τ	Available time in micro-period τ
$\delta_{joj'o'}$	Setup time of $O_{j'o'}$ if processed immediately after O_{jo} on the same machine
a_{jo}	Machine that is assigned to process O_{jo}
sc_{jot}	Setup cost needed to run O_{jo} in period t
pc_{jot}	Production cost needed to produce one unit of product related to O_{jo} in period t
hc_{jot}	Inventory holding cost for one unit of product related to O_{jo} at the end of period t
ϑ_t	Set of micro-periods belonging to the macro-period t
G	A big positive number

Variables

$x_{jo\tau}$	Quantity of O_{jo} that is processed in micro-period τ (lot-size)
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$I_{jo\tau}$ Amount of inventory related to O_{jo} at the end of micro-period τ
 $z_{jo\tau}$ 1 iff a setup for O_{jo} is occurred in micro-period τ ; 0, otherwise
 $y_{jo\tau}$ 1 iff a machine is set up for O_{jo} at the end of micro-period τ ; 0, otherwise
 $w_{joj'o'\tau}$ 1 iff $O_{j'o'}$ produced immediately after O_{jo} in micro-period τ on the same machine; 0, otherwise
 TSC Total system cost

$$\text{Min } TSC = \left(\sum_{\forall t} \sum_{\forall(j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \quad (1)$$

$$I_{jh_j\zeta_{t-1}} + x_{jh_j\zeta_t} - I_{jh_j\zeta_t} = D_{jt} \quad \forall t, \forall j \quad (2)$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j - 1 \quad (3)$$

$$I_{jo\tau-1} \geq x_{j(o+1)\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j - 1 \quad (4)$$

$$x_{jo\tau} \cdot p_{jo} \leq (y_{jo\tau-1} + y_{jo\tau}) \cdot c_\tau \quad \forall \tau, \forall(j, o) \quad (5)$$

$$\sum_{\forall(j,o)|a_{jo}=m} \left(x_{jo\tau} \cdot p_{jo} + \sum_{\forall(j',o')|a_{j'o'}=m} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \right) \leq c_\tau \quad \forall \tau, \forall m \quad (6)$$

$$\sum_{\tau \in \vartheta_t} \sum_{\forall(j,o)|a_{jo}=m} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \quad \forall t, \forall m \quad (7)$$

$$\sum_{\forall(j,o)|a_{jo}=m} y_{jo\tau} \leq 1 \quad \forall \tau, \forall m \quad (8)$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \quad \forall \tau, \forall m, \forall j, o = 1, \dots, h_j | a_{jo} = m \quad (9)$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \quad \forall \tau, \forall m \quad (10)$$

$$\forall \tau, \forall(j, o), \forall(j', o') | (j, o) \neq (j', o') \& a_{jo} = a_{j'o'} = m$$

$$\{x_{jo\tau}, I_{jo\tau}\} \in \mathbb{R}^+; \{z_{jo\tau}, y_{jo\tau}, \omega_{joj'o'\tau}\} \in \{0, 1\} \quad \forall t, \forall(j, o), \forall(j', o') \quad (11)$$

1.2 PLSP 2

New variables

$xb_{jo\tau}$ Quantity of O_{jo} that is processed in micro-period τ for first campaign
 $xe_{jo\tau}$ Quantity of O_{jo} that is processed at the end of micro-period τ (if a second campaign is started in micro-period τ)

$$\text{Min } TSC = \left(\sum_{\forall t} \sum_{\forall (j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \quad (12)$$

$$I_{jh_j\tau-1} + x_{jh_j\tau} - I_{jh_j\tau} = D_{jt} \quad \forall t, \forall j, \tau = \zeta_t \quad (13)$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j - 1 \quad (14)$$

$$x_{jo\tau} = xb_{jo\tau} + xe_{jo\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (15)$$

$$\sum_{\forall (j,o) | a_{jo}=m} y_{jo\tau} \leq 1 \quad \forall \tau, \forall m \quad (16)$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \quad \forall \tau, \forall m, \forall j, o = 1, \dots, h_j | a_{jo} = m \quad (17)$$

$$xb_{jo\tau} \leq y_{jo\tau-1} \cdot G \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (18)$$

$$xe_{jo\tau} \leq z_{jo\tau} \cdot G \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (19)$$

$$\sum_{\tau \in \vartheta_t} \sum_{\forall (j,o) | a_{jo}=m} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \quad \forall t, \forall m \quad (20)$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \quad \forall \tau, \forall m \quad (21)$$

$$\forall (j, o), \forall (j', o') | (j, o) \neq (j', o') \& a_{jo} = a_{j'o'} = m$$

$$xb_{jo\tau} \leq I_{jo\tau-1} \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (22)$$

$$xe_{jo\tau} \leq I_{jo\tau-1} + xb_{jo\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (23)$$

$$z_{jo\tau} \leq y_{jo\tau} \quad \forall \tau, \forall j, o = 1, \dots, h_j \quad (24)$$

$$z_{jo\tau} \geq y_{jo\tau} - \sum_{\tau' \in \vartheta_t | \tau' < \tau} \sum_{\forall (j', o') | a_{j'o'} = a_{jo}} y_{j'o'\tau'} \quad \forall t, \forall \tau \in \vartheta_t, \forall j, o = 1, \dots, h_j \quad (25)$$

$$\sum_{j=1}^J \sum_{o=1|a_{jo}=m}^{h_j} \left(x_{jot} \cdot p_{jo} + \sum_{\forall(j',o')|a_{j'o'}=m} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \right) \leq c_\tau \quad \forall \tau, \forall m \quad (26)$$

$$\{x_{jot}, xb_{jot}, xe_{jot}, I_{jot}\} \in \mathbb{R}^+; \{z_{joj'o'\tau}, y_{joj'o'\tau}, \omega_{joj'o'\tau}\} \in \{0, 1\} \quad \forall t, \forall(j, o), \forall(j', o') \quad (27)$$

1.3 GLSP

New parameters

R Number of microperiod per machine in each period

Variables

u_{rt}	Starting time of microperiod r in period t (lot-size)
I_{jort}	Inventory of product of operation O_{jo} at the end of microperiod r (units)
x_{jort}	Quantity of O_{jo} that is processed in microperiod r of period t (lot-size)
\hat{x}_{jort}	Share of x_{jort} that can be used by successors in the same microperiod r of period t (units)
\tilde{x}_{jort}	Share of x_{jort} that can as WIP-stock first be used by successors in the following microperiod $r + 1$ of period t (units)
y_{jomrt}	Setup state: $y_{jomrt} = 1$, if machine m is set up for O_{jo} in microperiod r (0 otherwise)
$z_{joj'o'rt}$	Takes on 1, if a changeover from operation O_{jo} to operation $O_{j'o'}$ takes place on their eligible machine during microperiod r (0 otherwise)
x_{mrt}^b	Fractional setup time for changeover at the beginning of microperiod r on machine m
x_{mrt}^e	Fractional setup time for changeover at the end of microperiod r on machine m
\bar{x}_{mrt}^b	Standby time on machine m in microperiod r before production
\bar{x}_{mrt}^e	Standby time on machine m in microperiod r after production

$$\begin{aligned} \text{Min } TSC = & \left(\sum_{\forall t} \sum_{\forall r|r>1} \sum_{\forall(j,o)} \sum_{\substack{\forall(j',o') \\ |a_{jo}=a_{j'o'}}} z_{j'o'jort} \cdot sc_{jot} + \sum_{\forall t} \sum_{\forall(j,o)} y_{jom1t} \cdot sc_{jot} \right. \\ & \left. + \sum_{\forall t} \sum_{\forall r} \sum_{\forall(j,o)} x_{jort} \cdot pc_{jot} + \sum_{\forall t} \sum_{\forall r|r=R} \sum_{\forall(j,o)} I_{jort} \cdot hc_{jot} \right) \quad (28) \end{aligned}$$

$$I_{jort} = I_{jo(r-1)t} + \hat{x}_{jort} + \tilde{x}_{jo(r-1)t} - x_{j(o+1)rt} \quad \forall t, \forall r, \forall j, o = 1, \dots, h_j - 1 \quad (29)$$

$$I_{jort} = I_{jo(r-1)t} + x_{jort} - D_{jt} \quad \forall t, \forall r, \forall j, o = h_j | r = R \quad (30)$$

$$I_{jort} = I_{jo(r-1)t} + x_{jort} \quad \forall t, \forall r, \forall j, o = h_j | r < R \quad (31)$$

$$I_{jort} = I_{jor'(t-1)} + \hat{x}_{jort} - x_{j(o+1)rt} \quad \forall t, \forall r, r' \forall j, o = 1, \dots, h_j - 1 | r = 1, r' = R \quad (32)$$

$$I_{jort} = I_{jor'(t-1)} + x_{jort} \quad \forall t, \forall r, r' \forall j, o = h_j | r = 1, r' = R \quad (33)$$

$$x_{jort} = \hat{x}_{jort} + \check{x}_{jort} \quad \forall t, \forall r, \forall (j, o) \quad (34)$$

$$x_{jort} - y_{jomrt} \cdot G \leq 0 \quad \forall t, \forall r, \forall (j, o), \forall m | a_{jo} = m \quad (35)$$

$$\sum_{j=1}^J \sum_{o=1|a_{jo}=m}^{h_j} y_{jomrt} = 1 \quad \forall t, \forall r, \forall m \quad (36)$$

$$y_{jom(r-1)t} + y_{j'o'mrt} - 1 \leq z_{joj'o'rt} \quad \forall t, \forall r, \forall m, \quad (37)$$

$$\forall (j, o), \forall (j', o') | (j, o) \neq (j', o') \& a_{jo} = a_{j'o'} = m$$

$$\sum_{\forall (j,o)} \sum_{\substack{\forall (j',o') \\ |a_{jo}=a_{j'o'}=m}} z_{joj'o'rt} = 1 \quad \forall t, \forall r, \forall m \quad (38)$$

$$x_{mrt}^b + \bar{x}_{mrt}^b \geq x_{m'rt}^b + \bar{x}_{m'rt}^b - G \cdot (2 - y_{jom'rt} - y_{j(o+1)mrt}) \quad \forall j, o = 1, \dots, h_j - 1 \quad (39)$$

$$\forall t, \forall r, \forall m, m' | a_{jo+1} = m \& a_{jo} = m'$$

$$\check{x}_{jort} \cdot p_{jo} + x_{m'rt}^e + \bar{x}_{m'rt}^e \geq x_{mrt}^e + \bar{x}_{mrt}^e - G \cdot (2 - y_{jom'rt} - y_{j(o+1)mrt}) \quad \forall j, o = 1, \dots, h_j - 1 \quad (40)$$

$$\forall t, \forall r, \forall m, m' | a_{jo+1} = m \& a_{jo} = m'$$

$$x_{m(r-1)t}^e + x_{mrt}^b = \sum_{\forall (j,o)} \sum_{\substack{\forall (j',o') \\ |a_{jo}=a_{j'o'}=m}} \delta_{joj'o'} \cdot z_{joj'o'rt} \quad \forall t, \forall r, \forall m \quad (41)$$

$$u_{r+1t} \geq u_{rt} + x_{mrt}^b + \bar{x}_{mrt}^b + x_{mrt}^e + \bar{x}_{mrt}^e + \sum_{\forall (j,o)|a_{jo}=m} x_{jort} \cdot p_{jo} \quad \forall t, \forall r, \forall m | r < R \quad (42)$$

$$x_{mrt}^b = \sum_{\forall (j,o)|a_{jo}=m} \delta_{0jo} \cdot y_{jomrt} \quad \forall t, \forall r, \forall m | r = 1 \quad (43)$$

$$\sum_{\forall (j,o)|a_{jo}=m} \sum_{\forall r} x_{jort} \cdot p_{jo} \leq C_{mt} \quad \forall t, \forall m \quad (44)$$

$$u_{rt} \geq t \cdot L \quad \forall t, \forall r | r = 1 \quad (45)$$

$$u_{rt} + x_{mrt}^b + \bar{x}_{mrt}^b + x_{mrt}^e + \bar{x}_{mrt}^e + \sum_{\forall (j,o)|a_{jo}=m} x_{jort} \cdot p_{jo} \leq (t+1) \cdot L \quad \forall t, \forall r, \forall m | r = R \quad (46)$$

$$\{u_{rt}, I_{jort}, x_{jort}, \hat{x}_{jort}, \check{x}_{jort}, x_{mrt}^b, \bar{x}_{mrt}^b, x_{mrt}^e, \bar{x}_{mrt}^e\} \in \mathbb{R}^+; \{y_{jomrt}, z_{joj'o'rt}\} \in \{0, 1\} \quad (47)$$

$$\forall t, \forall r, \forall m \forall (j, o), \forall (j', o')$$

1.4 CTLSP

Variables

x_{jot}	Quantity of O_{jo} that is processed in macro-period t (lot-size)
I_{jot}	Amount of inventory related to O_{jo} at the end of macro-period t
s_{jot}	Start time of O_{jo} in macro-period t
z_{jot}	1 iff O_{jo} is produced in macro-period t ; 0, otherwise
$y_{joj'o't}$	1 iff $O_{j'o'}$ produced immediately after O_{jo} in macro-period t on the same machine; 0, otherwise

$$\text{Min } TSC = \left(\sum_{\forall t} \sum_{\forall (j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jot} \cdot hc_{jot} \right) \quad (48)$$

$$I_{jh_jt-1} + x_{jh_jt} - I_{jh_jt} = D_{jt} \quad \forall t, \forall j \quad (49)$$

$$I_{jot-1} + x_{jot} - I_{jot} = x_{j(o+1)t} \quad \forall t, \forall j, o = 1, \dots, h_j - 1 \quad (50)$$

$$x_{jot} - z_{jot} \cdot G \leq 0 \quad \forall t, \forall (j, o) \quad (51)$$

$$\sum_{\forall (j,o) | a_{jo}=m} x_{jot} \cdot p_{jo} \leq C_{mt} \quad \forall t, \forall m \quad (52)$$

$$s_{jot} \geq s_{j'o't} + \delta_{j'o'jo} + x_{j'o't} \cdot p_{j'o'} + (1 - y_{j'o'jot}) \cdot G \quad (53)$$

$$\forall t, \forall (j, o), \forall (j', o') \mid (j, o) \neq (j', o') \& a_{jo} = a_{j'o'}$$

$$s_{j(o+1)t} \geq s_{jot} + x_{jot} \cdot p_{jo} \quad \forall t, \forall j, o = 1, \dots, h_j - 1 \quad (54)$$

$$s_{jot} \geq t \cdot L + \delta_{0jo} \cdot z_{jot} \quad \forall t, \forall (j, o) \quad (55)$$

$$s_{jot} + x_{jot} \cdot p_{jo} \leq (t + 1) \cdot L \quad \forall t, \forall (j, o) \quad (56)$$

$$y_{j'o'jot} + y_{joj'o't} \geq 1 - (2 - z_{jot} - z_{j'o't}) \cdot G \quad \forall t, \forall (j, o), \forall (j', o') \mid a_{jo} = a_{j'o'} \quad (57)$$

$$y_{j'o'jot} + y_{joj'o't} \leq 1 \quad \forall t, \forall (j, o), \forall (j', o') \mid a_{jo} = a_{j'o'} \quad (58)$$

$$\{x_{jot}, I_{jot}, s_{jot}\} \in \mathbb{R}^+; \{z_{jot}, y_{joj'o't}\} \in \{0, 1\} \quad \forall t, \forall (j, o), \forall (j', o') \quad (59)$$