Supplementary materials

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1 Customized models in the literature

1.1 PLSP 1

Indices

j, j'	Index of jobs ($j, j' = 1, ..., J$)
o, o'	Index of operations
h_j	Last operation of job j
m	Index of machines ($m = 1, ..., M$)
t	Index of macro-periods ($t = 1, ..., T$)
τ, τ'	Index of micro-periods
ζ_t	Index of the last micro-period belonging to the macro-period t

Parameters

D_{jt}	Demand of job j at the end of period t
p_{jo}	Processing time needed to produce one unit of product related to operation O_{jo}
C_{mt}	Capacity of machine m during period t
c_τ	Available time in micro-period τ
$\delta_{joj'o'}$	Setup time of $O_{j'o'}$ if processed immediately after O_{jo} on the same machine
a_{jo}	Machine that is assigned to process O_{jo}
sc_{jot}	Setup cost needed to run O_{jo} in period t
pc_{jot}	Production cost needed to produce one unit of product related to O_{jo} in period t
hc_{jot}	Inventory holding cost for one unit of product related to O_{jo} at the end of period t
ϑ_t	Set of micro-periods belonging to the macro-period t
G	A big positive number

Variables

$x_{jo\tau}$	Quantity of O_{jo} that is processed in micro-period τ (lot-size)

$I_{jo\tau}$ Amount of inventory related to O_{jo} at the end of micro-period τ

$z_{jo\tau}$ 1 iff a setup for O_{jo} is occurred in micro-period τ; 0, otherwise

$y_{jo\tau}$ 1 iff a machine is set up for O_{jo} at the end of micro-period τ; 0, otherwise

$w_{joj'o'\tau}$ 1 iff $O_{j'o'}$ produced immediately after O_{jo} in micro-period τ on the same machine; 0, otherwise

TSC Total system cost

$$\textit{Min } TSC = \left(\sum_{\forall t} \sum_{\forall (j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \tag{1}$$

$$I_{jh_j\zeta_t - 1} + x_{jh_j\zeta_t} - I_{jh_j\zeta_t} = D_{jt} \qquad \forall t, \forall j \tag{2}$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j - 1 \tag{3}$$

$$I_{jo\tau-1} \geq x_{j(o+1)\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j - 1 \tag{4}$$

$$x_{jo\tau} \cdot p_{jo} \leq (y_{jo\tau-1} + y_{jo\tau}) \cdot c_\tau \qquad \forall \tau, \forall (j,o) \tag{5}$$

$$\sum_{\forall (j,o) | a_{jo} = m} \left(x_{jo\tau} \cdot p_{jo} + \sum_{\forall (j',o') | a_{j'o'} = m} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \right) \leq c_\tau \qquad \forall \tau, \forall m \tag{6}$$

$$\sum_{\tau \in \vartheta_t} \sum_{\forall (j,o) | a_{jo} = m} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \qquad \forall t, \forall m \tag{7}$$

$$\sum_{\forall (j,o) | a_{jo} = m} y_{jo\tau} \leq 1 \qquad \forall \tau, \forall m \tag{8}$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \qquad \forall \tau, \forall m, \forall j, o = 1, ..., h_j | a_{jo} = m \tag{9}$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \qquad \forall \tau, \forall m \tag{10}$$
$$\forall \tau, \forall (j,o), \forall (j',o') \,|\, (j,o) \neq (j',o') \,\&\, a_{jo} = a_{j'o'} = m$$

$$\{x_{jo\tau}, I_{jo\tau}\} \in \mathbb{R}^+; \{z_{jo\tau}, y_{jo\tau}, \omega_{joj'o'\tau}\} \in \{0,1\} \quad \forall t, \forall (j,o), \forall (j',o') \tag{11}$$

1.2 PLSP 2

New variables

$xb_{jo\tau}$ Quantity of O_{jo} that is processed in micro-period τ for first campaign

$xe_{jo\tau}$ Quantity of O_{jo} that is processed at the end of micro-period τ (if a second campaign is started in micro-period τ)

$$\textit{Min } TSC = \left(\sum_{\forall t} \sum_{\forall (j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \tag{12}$$

$$I_{jh_j\tau-1} + x_{jh_j\tau} - I_{jh_j\tau} = D_{jt} \qquad \forall t,\, \forall j,\, \tau = \zeta_t \tag{13}$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \qquad \forall \tau,\, \forall j, o = 1, ..., h_j - 1 \tag{14}$$

$$x_{jo\tau} = xb_{jo\tau} + xe_{jo\tau} \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{15}$$

$$\sum_{\forall (j,o) | a_{jo} = m} y_{jo\tau} \leq 1 \qquad \forall \tau,\, \forall m \tag{16}$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \qquad \forall \tau,\, \forall m,\, \forall j, o = 1, ..., h_j | a_{jo} = m \tag{17}$$

$$xb_{jo\tau} \leq y_{jo\tau-1} \cdot G \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{18}$$

$$xe_{jo\tau} \leq z_{jo\tau} \cdot G \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{19}$$

$$\sum_{\tau \in \vartheta_t} \sum_{\forall (j,o) | a_{jo} = m} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \qquad \forall t,\, \forall m \tag{20}$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \qquad \forall \tau,\, \forall m$$
$$\forall (j,o), \forall (j',o') \mid (j,o) \neq (j',o') \,\&\, a_{jo} = a_{j'o'} = m \tag{21}$$

$$xb_{jo\tau} \leq I_{jo\tau-1} \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{22}$$

$$xe_{jo\tau} \leq I_{jo\tau-1} + xb_{jo\tau} \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{23}$$

$$z_{jo\tau} \leq y_{jo\tau} \qquad \forall \tau,\, \forall j, o = 1, ..., h_j \tag{24}$$

$$z_{jo\tau} \geq y_{jo\tau} - \sum_{\tau' \in \vartheta_t | \tau' < \tau} \sum_{\forall (j',o') | a_{j'o'} = a_{jo}} y_{j'o'\tau'} \qquad \forall t,\, \forall \tau \in \vartheta_t,\, \forall j, o = 1, ..., h_j \tag{25}$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} \Big(x_{jo\tau} \cdot p_{jo} + \sum_{\forall (j',o')|a_{j'o'}=m} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \Big) \leq c_\tau \qquad \forall \tau, \forall m \tag{26}$$

$$\{x_{jo\tau}, xb_{jo\tau}, xe_{jo\tau}, I_{jo\tau}\} \in \mathbb{R}^+; \{z_{jo\tau}, y_{jo\tau}, \omega_{joj'o'\tau}\} \in \{0,1\} \quad \forall t, \forall (j,o), \forall (j',o') \tag{27}$$

1.3 GLSP

New parameters

R	Number of microperiod per machine in each period

Variables

u_{rt}	Starting time of microperiod r in period t (lot-size)
I_{jort}	Inventory of product of operation O_{jo} at the end of microperiod r (units)
x_{jort}	Quantity of O_{jo} that is processed in microperiod r of period t (lot-size)
$\hat{x}_{jort}$	Share of x_{jort} that can be used by successors in the same microperiod r of period t (units)
$\check{x}_{jort}$	Share of x_{jort} that can as WIP-stock first be used by successors in the following microperiod $r+1$ of period t (units)
y_{jomrt}	Setup state: $y_{jomrt} = 1$, if machine m is set up for O_{jo} in microperiod r (0 otherwise)
$z_{joj'o'rt}$	Takes on 1, if a changeover from operation O_{jo} to operation $O_{j'o'}$ takes place on their eligible machine during microperiod r (0 otherwise)
x^b_{mrt}	Fractional setup time for changeover at the beginning of microperiod r on machine m
x^e_{mrt}	Fractional setup time for changeover at the end of microperiod r on machine m
$\bar{x}^b_{mrt}$	Standby time on machine m in microperiod r before production
$\bar{x}^e_{mrt}$	Standby time on machine m in microperiod r after production

$$\textit{Min } TSC = \Big(\sum_{\forall t} \sum_{\forall r|r>1} \sum_{\forall (j,o)} \sum_{\substack{\forall (j',o') \\ |a_{jo}=a_{j'o'}}} z_{j'o'jort} \cdot sc_{jot} + \sum_{\forall t} \sum_{\forall (j,o)} y_{jom1t} \cdot sc_{jot} + \sum_{\forall t} \sum_{\forall r} \sum_{\forall (j,o)} x_{jort} \cdot pc_{jot} + \sum_{\forall t} \sum_{\forall r|r=R} \sum_{\forall (j,o)} I_{jort} \cdot hc_{jot} \Big) \tag{28}$$

$$I_{jort} = I_{jo(r-1)t} + \hat{x}_{jort} + \check{x}_{jo(r-1)t} - x_{j(o+1)rt} \qquad \forall t, \forall r, \forall j, o = 1, ..., h_j - 1 \tag{29}$$

$$I_{jort} = I_{jo(r-1)t} + x_{jort} - D_{jt} \qquad \forall t, \forall r, \forall j, o = h_j | r = R \tag{30}$$

$$I_{jort} = I_{jo(r-1)t} + x_{jort} \qquad \forall t, \forall r, \forall j, o = h_j | r < R \tag{31}$$

$$I_{jort} = I_{jor'(t-1)} + \hat{x}_{jort} - x_{j(o+1)rt} \qquad \forall t, \forall r, r' \forall j, o = 1, ..., h_j - 1 | r = 1, r' = R \tag{32}$$

$$I_{jort} = I_{jor'(t-1)} + x_{jort} \qquad \forall t, \forall r, r' \, \forall j, o = h_j | r = 1, r' = R \tag{33}$$

$$x_{jort} = \hat{x}_{jort} + \check{x}_{jort} \qquad \forall t, \forall r, \, \forall (j, o) \tag{34}$$

$$x_{jort} - y_{jomrt} \cdot G \leq 0 \qquad \forall t, \forall r, \, \forall (j, o), \forall m \, | a_{jo} = m \tag{35}$$

$$\sum_{j=1}^{J} \sum_{o=1 | a_{jo}=m}^{h_j} y_{jomrt} = 1 \qquad \forall t, \forall r, \forall m \tag{36}$$

$$y_{jom(r-1)t} + y_{j'o'mrt} - 1 \leq z_{joj'o'rt} \qquad \forall t, \forall r, \forall m, \tag{37}$$
$$\forall (j, o), \forall (j', o') \, | \, (j, o) \neq (j', o') \, \& \, a_{jo} = a_{j'o'} = m$$

$$\sum_{\forall (j,o)} \sum_{\substack{\forall (j',o') \\ | a_{jo} = a_{j'o'} = m}} z_{joj'o'rt} = 1 \qquad \forall t, \forall r, \forall m \tag{38}$$

$$x^b_{mrt} + \bar{x}^b_{mrt} \geq x^b_{m'rt} + \bar{x}^b_{m'rt} - G \cdot (2 - y_{jom'rt} - y_{j(o+1)mrt}) \qquad \forall j, o = 1, ..., h_j - 1 \tag{39}$$
$$\forall t, \forall r, \forall m, m' | a_{jo+1} = m \, \& \, a_{jo} = m'$$

$$\check{x}_{jort} \cdot p_{jo} + x^e_{m'rt} + \bar{x}^e_{m'rt} \geq x^e_{mrt} + \bar{x}^e_{mrt} - G \cdot (2 - y_{jom'rt} - y_{j(o+1)mrt}) \qquad \forall j, o = 1, ..., h_j - 1 \tag{40}$$
$$\forall t, \forall r, \forall m, m' | a_{jo+1} = m \, \& \, a_{jo} = m'$$

$$x^e_{m(r-1)t} + x^b_{mrt} = \sum_{\forall (j,o)} \sum_{\substack{\forall (j',o') \\ | a_{jo} = a_{j'o'} = m}} \delta_{joj'o'} \cdot z_{joj'o'rt} \qquad \forall t, \forall r, \forall m \tag{41}$$

$$u_{r+1t} \geq u_{rt} + x^b_{mrt} + \bar{x}^b_{mrt} + x^e_{mrt} + \bar{x}^e_{mrt} + \sum_{\forall (j,o) | a_{jo} = m} x_{jort} \cdot p_{jo} \qquad \forall t, \forall r, \forall m | r < R \tag{42}$$

$$x^b_{mrt} = \sum_{\forall (j,o) | a_{jo} = m} \delta_{0jo} \cdot y_{jomrt} \qquad \forall t, \forall r, \forall m | r = 1 \tag{43}$$

$$\sum_{\forall (j,o) | a_{jo} = m} \sum_{\forall r} x_{jort} \cdot p_{jo} \leq C_{mt} \qquad \forall t, \forall m \tag{44}$$

$$u_{rt} \geq t \cdot L \qquad \forall t, \forall r | r = 1 \tag{45}$$

$$u_{rt} + x^b_{mrt} + \bar{x}^b_{mrt} + x^e_{mrt} + \bar{x}^e_{mrt} + \sum_{\forall (j,o) | a_{jo} = m} x_{jort} \cdot p_{jo} \leq (t+1) \cdot L \qquad \forall t, \forall r, \forall m | r = R \tag{46}$$

$$\{u_{rt}, I_{jort}, x_{jort}, \hat{x}_{jort}, \check{x}_{jort}, x^b_{mrt}, \bar{x}^b_{mrt}, x^e_{mrt}, \bar{x}^e_{mrt}\} \in \mathbb{R}^+; \{y_{jomrt}, z_{joj'o'rt}\} \in \{0, 1\} \tag{47}$$
$$\forall t, \forall r, \forall m \, \forall (j, o), \, \forall (j', o')$$

1.4 CTLSP

Variables

x_{jot} Quantity of O_{jo} that is processed in macro-period t (lot-size)
I_{jot} Amount of inventory related to O_{jo} at the end of macro-period t
s_{jot} Start time of O_{jo} in macro-period t
z_{jot} 1 iff O_{jo} is produced in macro-period t; 0, otherwise
$y_{joj'o't}$ 1 iff $O_{j'o'}$ produced immediately after O_{jo} in macro-period t on the same machine; 0, otherwise

$$\textit{Min } TSC = \left(\sum_{\forall t} \sum_{\forall (j,o)} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jot} \cdot hc_{jot} \right) \tag{48}$$

$$I_{jh_jt-1} + x_{jh_jt} - I_{jh_jt} = D_{jt} \qquad \forall t,\, \forall j \tag{49}$$

$$I_{jot-1} + x_{jot} - I_{jot} = x_{j(o+1)t} \qquad \forall t,\, \forall j, o = 1, ..., h_j - 1 \tag{50}$$

$$x_{jot} - z_{jot} \cdot G \leq 0 \qquad \forall t,\, \forall (j,o) \tag{51}$$

$$\sum_{\forall (j,o) | a_{jo} = m} x_{jot} \cdot p_{jo} \leq C_{mt} \qquad \forall t,\, \forall m \tag{52}$$

$$s_{jot} \geq s_{j'o't} + \delta_{j'o'jo} + x_{j'o't} \cdot p_{j'o'} + (1 - y_{j'o'jot}) \cdot G \tag{53}$$
$$\forall t,\, \forall (j,o), \forall (j',o') \,|\, (j,o) \neq (j',o') \,\&\, a_{jo} = a_{j'o'}$$

$$s_{j(o+1)t} \geq s_{jot} + x_{jot} \cdot p_{jo} \qquad \forall t,\, \forall j,\, o = 1, ..., h_j - 1 \tag{54}$$

$$s_{jot} \geq t \cdot L + \delta_{0jo} \cdot z_{jot} \qquad \forall t,\, \forall (j,o) \tag{55}$$

$$s_{jot} + x_{jot} \cdot p_{jo} \leq (t+1) \cdot L \qquad \forall t,\, \forall (j,o) \tag{56}$$

$$y_{j'o'jot} + y_{joj'o't} \geq 1 - (2 - z_{jot} - z_{j'o't}) \cdot G \qquad \forall t,\, \forall (j,o),\, \forall (j',o') | a_{jo} = a_{j'o'} \tag{57}$$

$$y_{j'o'jot} + y_{joj'o't} \leq 1 \qquad \forall t,\, \forall (j,o),\, \forall (j',o') | a_{jo} = a_{j'o'} \tag{58}$$

$$\{x_{jot}, I_{jot}, s_{jot}\} \in \mathbb{R}^+; \{z_{jot}, y_{joj'o't}\} \in \{0,1\} \quad \forall t,\, \forall (j,o),\, \forall (j',o') \tag{59}$$

2 Comparison of the number of variables and constraints

In this dection a comparison between our proposed model, CTLSP, PLSP1, PLSP2 and GLSP models has been made in terms of the number of variables and constraints for four instances with different dimensions. The table 1 shows the results.

Table 1: Comparison of the number of variables and constraints

instances	Proposed Model			CTLSP			PLSP1			PLSP2			GLSP		
	No. Constraint	No. Binary variables	No. Positive variables	No. Constraint	No. Binary variables	No. Positive variables	No. Constraint	No. Binary variables	No. Positive variables	No. Constraint	No. Binary variables	No. Positive variables	No. Constraint	No. Binary variables	No. Positive variables
3-10-5-5	2300	974	450	2159	908	450	36390	29671	9000	61115	30450	18000	15331	8550	6345
10-5-5-6	6970	3150	900	6930	3150	900	180158	161242	30000	258804	161700	60000	66320	46500	19890
20-5-5-5	21545	10220	1875	21465	10220	1875	1095431	1037319	100000	1361317	1039500	200000	368020	305000	63150
10-20-15-4	24128	11164	2400	22883	10484	2400	795509	723389	105600	1117630	754000	211200	363638	267600	82656