Supplementary materials

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1 Customized models in the literature

1.1 PLSP 1

Indices

j, j'	Index of jobs ($j, j' = 1, ..., J$)
o, o'	Index of operations
h_j	Last operation of job j
m	Index of machines ($m = 1, ..., M$)
t	Index of macro-periods ($t = 1, ..., T$)
τ, τ'	Index of micro-periods
ζ_t	Index of the last micro-period belonging to the macro-period t

Parameters

D_{jt}	Demand of job j at the end of period t
p_{jo}	Processing time needed to produce one unit of product related to operation O_{jo}
C_{mt}	Capacity of machine m during period t
c_τ	Available time in micro-period τ
$\delta_{joj'o'}$	Setup time of $O_{j'o'}$ if processed immediately after O_{jo} on the same machine
a_{jo}	Machine that is assigned to process O_{jo}
sc_{jot}	Setup cost needed to run O_{jo} in period t
pc_{jot}	Production cost needed to produce one unit of product related to O_{jo} in period t
hc_{jot}	Inventory holding cost for one unit of product related to O_{jo} at the end of period t
ϑ_t	Set of micro-periods belonging to the macro-period t
G	A big positive number

Variables

$x_{jo\tau}$	Quantity of O_{jo} that is processed in micro-period τ (lot-size)

$I_{jo\tau}$ Amount of inventory related to O_{jo} at the end of micro-period τ

$z_{jo\tau}$ 1 iff a setup for O_{jo} is occurred in micro-period τ; 0, otherwise

$y_{jo\tau}$ 1 iff a machine is set up for O_{jo} at the end of micro-period τ; 0, otherwise

$w_{joj'o'\tau}$1 iff $O_{j'o'}$ produced immediately after O_{jo} in micro-period τ on the same machine; 0, otherwise

TSC Total system cost

$$Min\ TSC = \left(\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{o=1}^{h_j} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \tag{1}$$

$$I_{jh_j\zeta_t-1} + x_{jh_j\zeta_t} - I_{jh_j\zeta_t} = D_{jt} \qquad \forall t,\ \forall j \tag{2}$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \qquad \forall \tau,\ \forall j, o = 1, ..., h_j - 1 \tag{3}$$

$$I_{jo\tau-1} \geq x_{j(o+1)\tau} \qquad \forall \tau,\ \forall j, o = 1, ..., h_j - 1 \tag{4}$$

$$x_{jo\tau} \cdot p_{jo} \leq (y_{jo\tau-1} + y_{jo\tau}) \cdot c_\tau \qquad \forall \tau,\ \forall (j, o) \tag{5}$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} \left(x_{jo\tau} \cdot p_{jo} + \sum_{j'=1}^{J} \sum_{o'=1|a_{j'o'}=m}^{h_{j'}} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \right) \leq c_\tau \qquad \forall \tau,\ \forall m \tag{6}$$

$$\sum_{\tau\in\vartheta_t} \sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \qquad \forall t,\ \forall m \tag{7}$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} y_{jo\tau} \leq 1 \qquad \forall \tau,\ \forall m \tag{8}$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \qquad \forall \tau,\ \forall m,\ \forall j, o = 1, ..., h_j | a_{jo} = m \tag{9}$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \qquad \forall \tau,\ \forall m \tag{10}$$
$$\forall \tau,\ \forall (j, o), \forall (j', o') \,|\, (j, o) \neq (j', o')\, \&\, a_{jo} = a_{j'o'} = m$$

$$\{x_{jo\tau}, I_{jo\tau}\} \in \mathbb{R}^+; \{z_{jo\tau}, y_{jo\tau}, \omega_{joj'o'\tau}\} \in \{0, 1\} \qquad \forall t,\ \forall (j, o),\ \forall (j', o') \tag{11}$$

1.2 PLSP 2

New variables

$xb_{jo\tau}$ Quantity of O_{jo} that is processed in micro-period τ for first campaign

$xe_{jo\tau}$ Quantity of O_{jo} that is processed at the end of micro-period τ (if a second campaign is started in micro-period τ)

$$Min\ TSC = \left(\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{o=1}^{h_j} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jo\zeta_t} \cdot hc_{jot} \right) \tag{12}$$

$$I_{jh_j\tau-1} + x_{jh_j\tau} - I_{jh_j\tau} = D_{jt} \qquad \forall t, \forall j, \tau = \zeta_t \tag{13}$$

$$I_{jo\tau-1} + x_{jo\tau} - I_{jo\tau} = x_{j(o+1)\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j - 1 \tag{14}$$

$$x_{jo\tau} = xb_{jo\tau} + xe_{jo\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{15}$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} y_{jo\tau} \leq 1 \qquad \forall \tau, \forall m \tag{16}$$

$$z_{jo\tau} \geq y_{jo\tau} - y_{jo\tau-1} \qquad \forall \tau, \forall m, \forall j, o = 1, ..., h_j | a_{jo} = m \tag{17}$$

$$xb_{jo\tau} \leq y_{jo\tau-1} \cdot G \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{18}$$

$$xe_{jo\tau} \leq z_{jo\tau} \cdot G \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{19}$$

$$\sum_{\tau \in \vartheta_t} \sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} x_{jo\tau} \cdot p_{jo} \leq C_{mt} \qquad \forall t, \forall m \tag{20}$$

$$\omega_{joj'o'\tau} \geq y_{jo\tau-1} + y_{j'o'\tau} - 1 \qquad \forall \tau, \forall m \tag{21}$$
$$\forall \tau, \forall (j, o), \forall (j', o') \,|\, (j, o) \neq (j', o') \,\&\, a_{jo} = a_{j'o'} = m$$

$$xb_{jo\tau} \leq I_{jo\tau-1} \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{22}$$

$$xe_{jo\tau} \leq I_{jo\tau-1} + xb_{jo\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{23}$$

$$z_{jo\tau} \leq y_{jo\tau} \qquad \forall \tau, \forall j, o = 1, ..., h_j \tag{24}$$

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$$z_{jo\tau} \geq y_{jo\tau} - \sum_{\tau' \in \vartheta_t | \tau' < \tau} \sum_{j'=1}^{J} \sum_{o'=1 | a_{j'o'}=a_{jo}}^{h_{j'}} y_{j'o'\tau'} \qquad \forall t, \forall \tau \in \vartheta_t, \forall j, o = 1, ..., h_j \qquad (25)$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} \left(x_{jo\tau} \cdot p_{jo} + \sum_{j'=1}^{J} \sum_{o'=1|a_{j'o'}=m}^{h_{j'}} \omega_{joj'o'\tau} \cdot \delta_{joj'o'} \right) \leq c_\tau \qquad \forall \tau, \forall m \qquad (26)$$

$$\{x_{jo\tau}, xb_{jo\tau}, xe_{jo\tau}, I_{jo\tau}\} \in \mathbb{R}^+; \{z_{jo\tau}, y_{jo\tau}, \omega_{joj'o'\tau}\} \in \{0,1\} \quad \forall t, \forall(j,o), \forall(j',o') \qquad (27)$$

1.3 GLSP

New variables

x_{jot}

1.4 CTLSP

Variables

x_{jot} Quantity of O_{jo} that is processed in macro-period t (lot-size)
I_{jot} Amount of inventory related to O_{jo} at the end of macro-period t
s_{jot} Start time of O_{jo} in macro-period t
z_{jot} 1 iff O_{jo} is produced in macro-period t; 0, otherwise
$y_{joj'o't}$ 1 iff $O_{j'o'}$ produced immediately after O_{jo} in macro-period t on the same machine; 0, otherwise

$$Min\ TSC = \left(\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{o=1}^{h_j} z_{jot} \cdot sc_{jot} + x_{jot} \cdot pc_{jot} + I_{jot} \cdot hc_{jot} \right) \qquad (28)$$

$$I_{jh_jt-1} + x_{jh_jt} - I_{jh_jt} = D_{jt} \qquad \forall t, \forall j \qquad (29)$$

$$I_{jot-1} + x_{jot} - I_{jot} = x_{j(o+1)t} \qquad \forall t, \forall j, o = 1, ..., h_j - 1 \qquad (30)$$

$$x_{jot} - z_{jot} \cdot G \leq 0 \qquad \forall t, \forall(j,o) \qquad (31)$$

$$\sum_{j=1}^{J} \sum_{o=1|a_{jo}=m}^{h_j} x_{jot} \cdot p_{jo} \leq C_{mt} \qquad \forall t, \forall m \qquad (32)$$

$$s_{jot} \geq s_{j'o't} + \delta_{j'o'jo} + x_{j'o't} \cdot p_{j'o'} + (1 - y_{j'o'jot}) \cdot G \qquad (33)$$

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$$\forall t,\, \forall(j,o),\, \forall(j',o') \mid (j,o) \neq (j',o') \,\&\, a_{jo} = a_{j'o'}$$

$$s_{j(o+1)t} \geq s_{jot} + x_{jot} \cdot p_{jo} \qquad \forall t,\, \forall j,\, o = 1, ..., h_j - 1 \tag{34}$$

$$s_{jot} \geq t \cdot L + \delta_{0jo} \cdot z_{jot} \qquad \forall t,\, \forall(j,o) \tag{35}$$

$$s_{jot} + x_{jot} \cdot p_{jo} \leq (t+1) \cdot L \qquad \forall t,\, \forall(j,o) \tag{36}$$

$$y_{j'o'jot} + y_{joj'o't} \geq 1 - (2 - z_{jot} - z_{j'o't}) \cdot G \qquad \forall t,\, \forall(j,o),\, \forall(j',o')|a_{jo} = a_{j'o'} \tag{37}$$

$$y_{j'o'jot} + y_{joj'o't} \leq 1 \qquad \forall t,\, \forall(j,o),\, \forall(j',o')|a_{jo} = a_{j'o'} \tag{38}$$

$$\{x_{jot}, I_{jot}, s_{jot}\} \in \mathbb{R}^+;\, \{z_{jot}, y_{joj'o't}\} \in \{0,1\} \qquad \forall t,\, \forall(j,o),\, \forall(j',o') \tag{39}$$