Problem 3:

A and B are regular, so there exist 2 DFA's

$$M_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$$
$$M_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$$

To construct: $M = (Q, \Sigma, \delta, q_0, F)$

→ Σ is the same

→ $Q = Q_A \times Q_B \times \{0,1\}$

↳ To decide if it's M_A's turn or M_B's turn

↳ Track M_A and M_B simultaneously

→ $q_0 = (q_A, q_B, 0)$

→ If current state is $(m, n, 0)$

$$\delta((m,n,0),a) = (\delta_A(m,a), n, 1)$$

If current state is $(m, n, 1)$

$$\delta((m,n,1),a) = (m, \delta_B(n,a), 0)$$

→ $F = F_A \times F_B \times \{0\}$

↳ Machine must end after a symbol intended for B

∴ DFA has been constructed, so regular languages are closed under perfect shuffle