

### Problem 1:

Assume that  $L$  is a CFL  $\rightarrow p$  is the pumping length

Pumping lemma for CFL states that for any string in  $L$  with length  $\geq p$  can be divided into 5 pieces  $s = uvxyz$  where

$\rightarrow$  for each  $i \geq 0$ ,  $uv^i xy^i z \in L$

$\rightarrow |vy| > 0$

$\rightarrow |vxy| \leq p$

Choose  $s = 0^p 1^p 0^p 1^p \in L$

$vxy$  must contain the midpoint of the string

$\rightarrow$  If  $vxy$  was purely in the left half, pumping up to  $uv^2xy^2z$  puts a 1 in the first position of the second half

$\rightarrow$  If  $vxy$  was purely in the right half, pumping up to  $uv^2xy^2z$  puts a 0 in the ~~last~~ <sup>lost</sup> position of the first half

Pumping down to  $uzz$  gives  $0^p 1^i 0^j 1^p$

$i$  and  $j$  cannot both be  $p$  as  $|vxy| \leq p$

This string is not of the form  $ww$ , and therefore  $s$  cannot be pumped

$\therefore L$  is not a CFL