

Problem 5:

$$A_{TM} = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$$

\hookrightarrow undecidable

Construct M' such that on input string s

- \rightarrow simulate M on input w
- \rightarrow accept all inputs x if M ever accepts w
- \rightarrow if M never accepts w , accept x if $x \in \{0^n 1^n \mid n \geq 0\}$

~~Assume~~

\hookrightarrow If M accepts w , $L(M')$ is regular (Σ^*)
Otherwise it's not.

$$\langle M, w \rangle \in A_{TM} \Leftrightarrow \langle M' \rangle \in \text{REGULAR}_{TM}$$

Assume a decider D exists for REGULAR_{TM} ,

\Rightarrow a decider must also exist for A_{TM}

\therefore There's a contradiction, REGULAR_{TM} is undecidable