Problem 8:

1. For an input graph $G=(V,E)$, the certificate is the colouring $C: V \rightarrow \{a,b,c\}$

where a,b,c are the 3 colours.

Define a verifier V taking input $\langle G,C \rangle$

→ For each vertex v in G, verify that $C(v) \in \{a,b,c\}$

→ For each edge (u,v) in G, verify that $C(u) \neq C(v)$

→ Accept if all checks pass, reject otherwise

Time taken is $O(|V|+|E|)$, which is polynomial in the size of the input.

2. Given a 3-SAT formula (φ), the aim is to construct a graph (G) such that

φ is satisfiable $\Longleftrightarrow$ G is 3-colorable

→ Use 3 ~~colour~~ colours → True, False, None

→ First, create a triangle in the graph with each vertex with a different colour to force that these colours are different.

→ For each variable x, create two vertices x and $-x$, connect them with each other and connect both with None. If x is coloured True, that means x is true, otherwise $-x$ has to be True-coloured.

~~and~~

Each clause in 3-SAT is of the form $(l_1 \vee l_2 \vee l_3)$

l_1, l_2 and l_3 can be replaced by the vertices where they form a triangle. Since one of them has to be True-coloured, each clause evaluates to true and it gives a satisfying assignment.