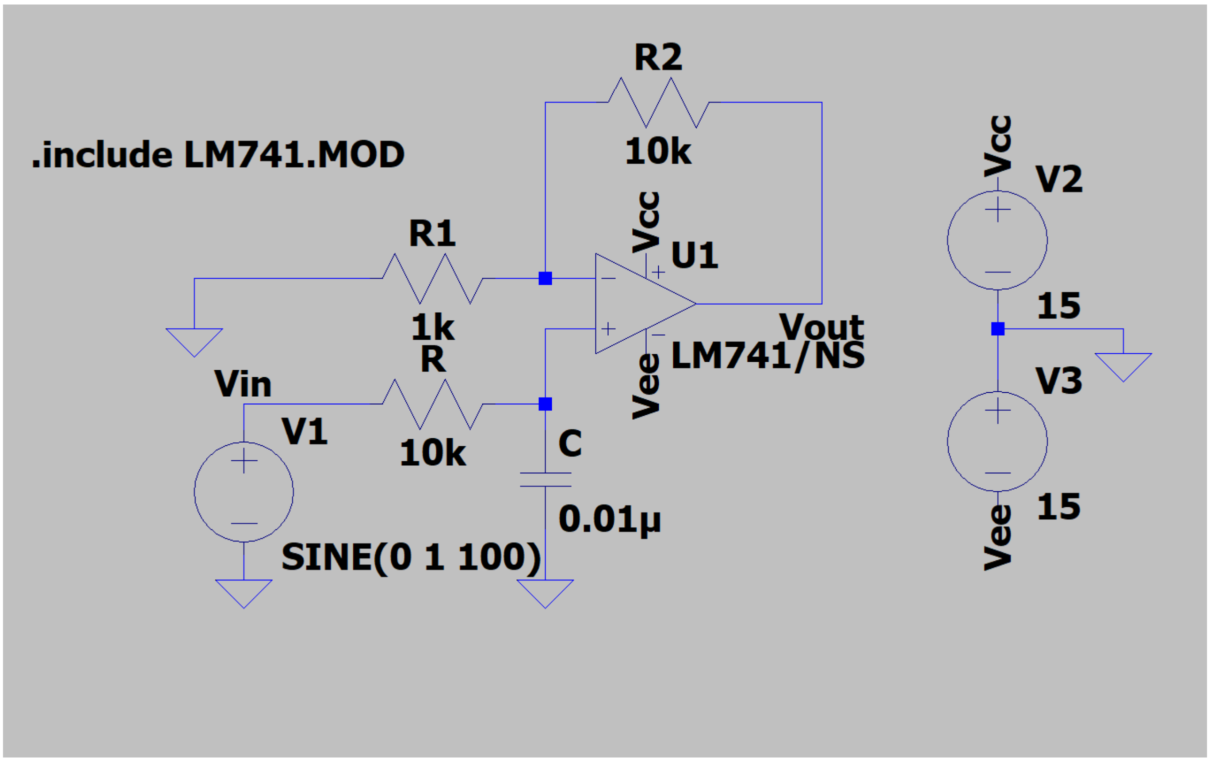
Aim: To study the working of Active filters.

Software used: LTspice

<u>Butterworth low-pass filter:</u>

Circuit:



1. Determining transfer function and cut-off frequency for the above circuit:

$$V_+ = V_{in}\frac{\left(\frac{1}{sC}\right)}{R + \frac{1}{sC}} = \frac{V_{in}}{1 + sRC}$$

$$V_- = V_+ = \frac{V_{in}}{1+sRC} \quad \text{(virtually short)}$$

$$V_o = \frac{V_-}{R_1} \times (R_1 + R_2)$$

$$\Rightarrow V_o = \frac{V_{in}}{1+sRC} \times \left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow \text{transfer function}, \quad \frac{V_o(s)}{V_{in}(s)} = \frac{1 + \frac{R_2}{R_1}}{1 + sRC}$$

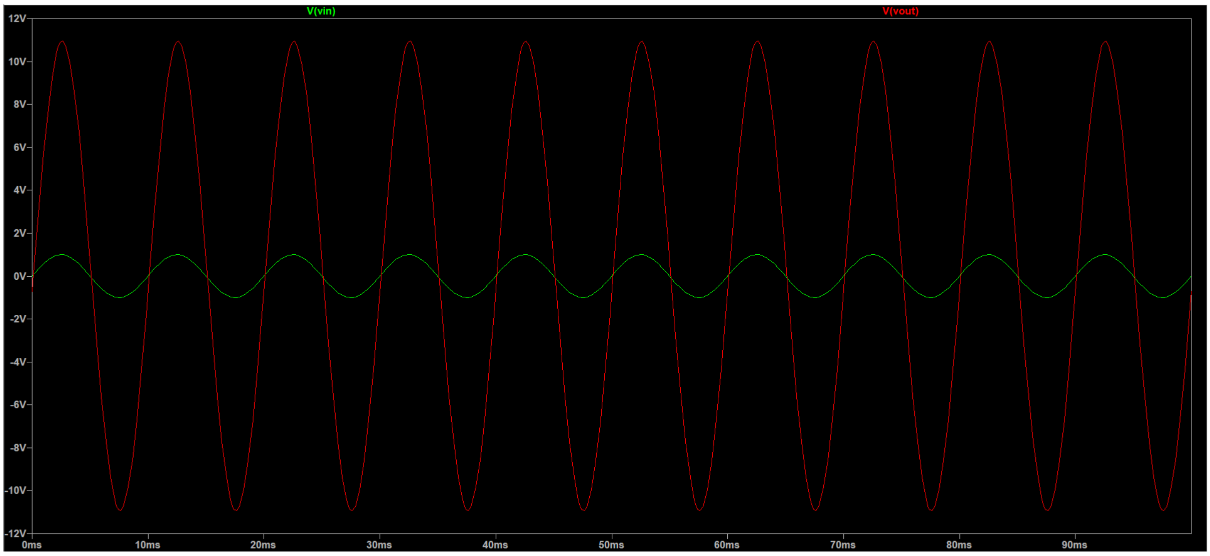
In this circuit, $R_1 = 1k\Omega$, $R_2 = 10k\Omega$, $R = 10k\Omega$ and $C = 0.01\mu F$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{1 + 10}{1 + s(10\times10^3 \times 0.01\times10^{-6})} = \frac{11}{1 + (s\times10^{-4})}$$

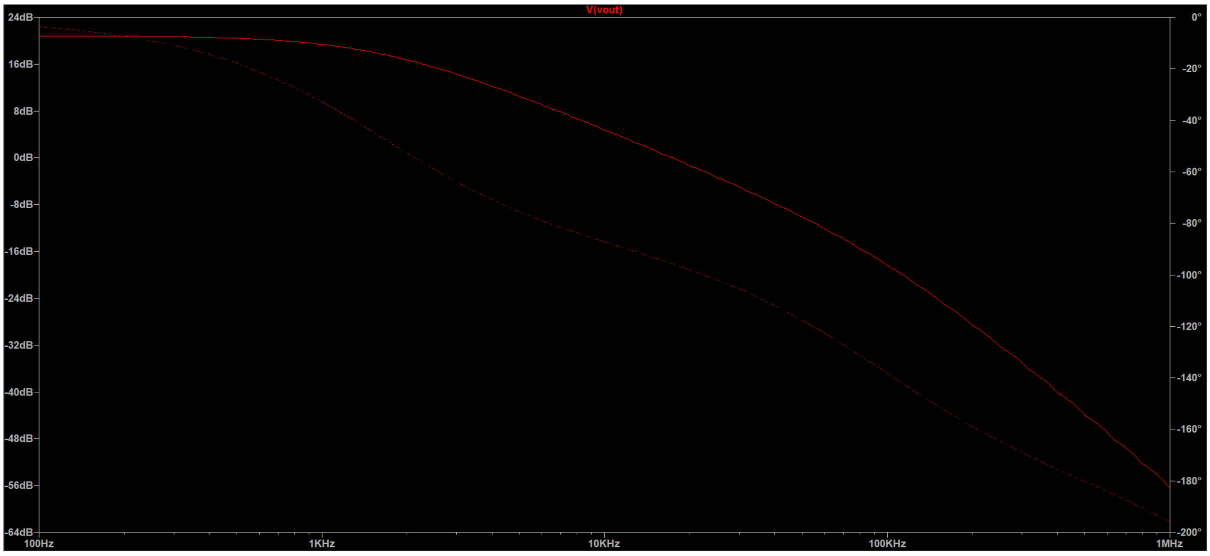
$$\text{cut off frequency } (f_c) = \frac{1}{2\pi \times RC} = \frac{1}{2\pi \times (10\times10^3)\cdot(0.01\times10^{-6})}$$

$$\Rightarrow \boxed{f_c = 1.59 \text{ kHz}}$$

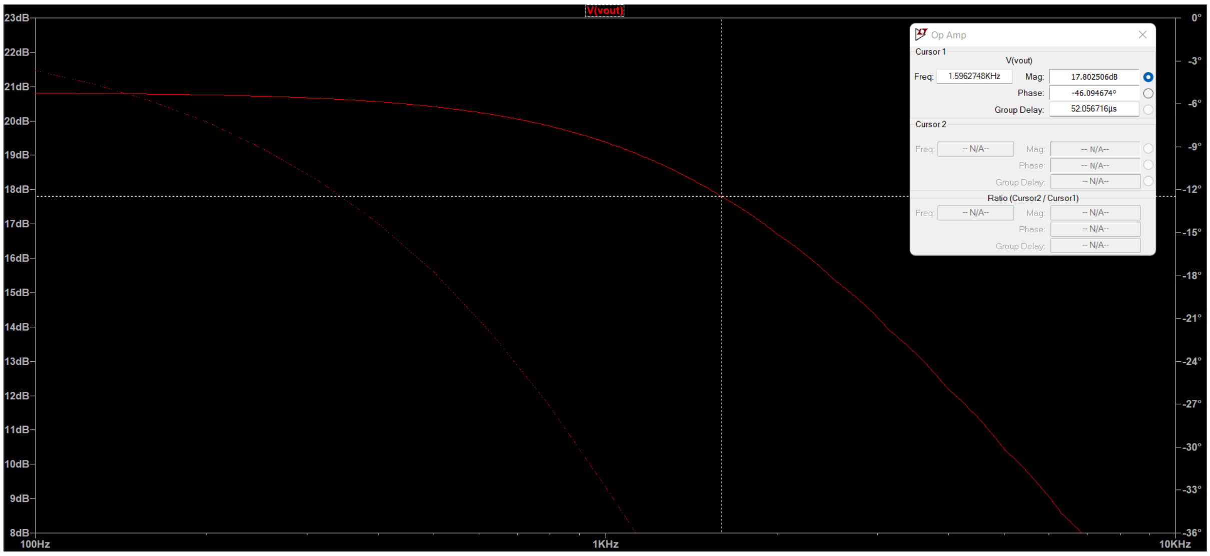
2. Input and Output voltages on the scope when a 1V peak, 100Hz sine wave is applied at its input:



3. Gain as a function of frequency (input frequency varies from 100Hz to 1MHz)

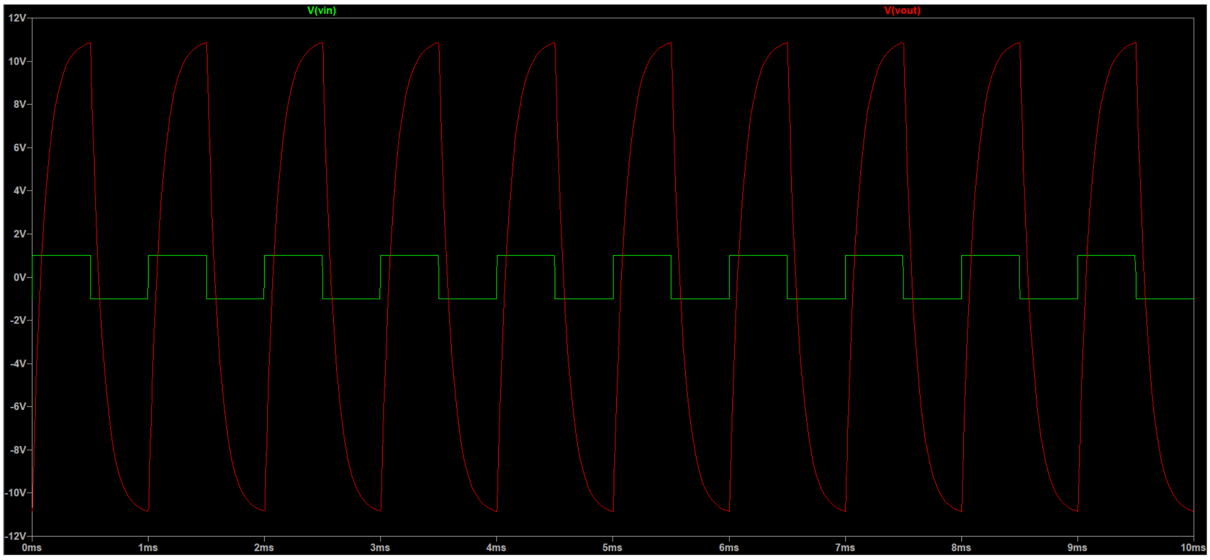


The above plot is magnified to determine the cut-off frequency



- The observed cut-off frequency on the plot is approximately 1.59kHz which is equal to the calculated value (i.e., 1.59kHz)

4. Input and Output voltages on the scope when a 1V peak, 1kHz square wave is applied at its input:



Explanation:

When a dc voltage of V_0 is applied across an RC-circuit the voltage across the capacitor will be

$$V_c(t) = V_0\left(1 - e^{-t/RC}\right) + V_i e^{-t/RC}$$

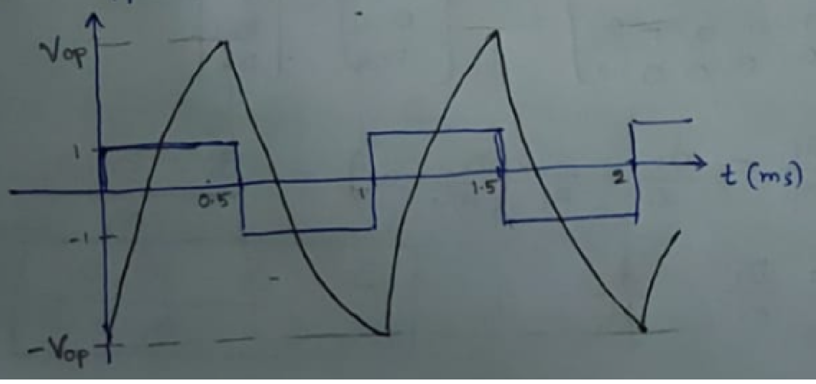
where V_i is initial voltage across capacitor (i.e., at $t=0$)

$$\Rightarrow V_+ = V_c(t) = V_0\left(1 - e^{-t/RC}\right) + V_i e^{-t/RC}$$

$$\Rightarrow V_{out} = V_c(t) \times \left(1 + \frac{R_2}{R_1}\right) \qquad \left(\because V_- = V_+ = V_c(t)\right)$$

$$\Rightarrow V_{out} = \left(1 + \frac{R_2}{R_1}\right) \times \left[V_0\left(1 - e^{-t/RC}\right) + V_i e^{-t/RC}\right]$$

plot appears as shown below,



Lets analyse for positive half cycle,

Let initial voltage across the capacitor be $-V_p$
and at the steady state at the end of positive cycle
the voltage across capacitor will be $+V_p$

Since the duration of positive and negative half cycles is equal.

$$\Rightarrow \cancel{V_c(0.5)}\ V_c(0.5\ ms) = (1)\left(1 - e^{\frac{-0.5}{0.1}}\right) + (-V_p)\, e^{\frac{-0.5}{0.1}} \quad (\because RC = 0.1$$

$$\Rightarrow V_p = 1 - e^{-5} - V_p e^{-5}$$

$$\Rightarrow V_p\left(1 + e^{-5}\right) = 1 - e^{-5}$$

$$\Rightarrow V_p = \frac{1 - e^{-5}}{1 + e^{-5}} \approx 0.987\ V$$

$$\Rightarrow V_{op} = \left(1 + \frac{R_2}{R_1}\right) \times V_p$$

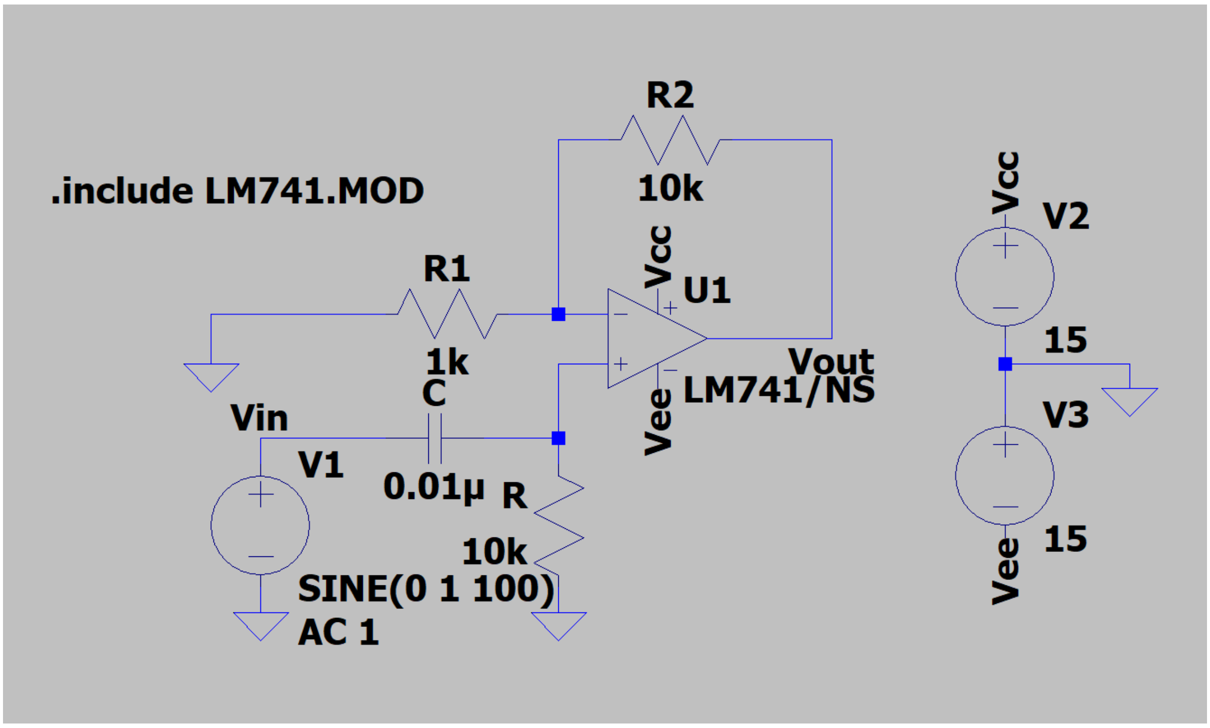
$$V_{op} = \left(1 + \frac{10}{1}\right) \times 0.987$$

$$\Rightarrow V_{op} \approx 10.85\ V$$

~~the~~ similar process applies in negative half cycle too
and the output voltage oscillates between -10.85 V and 10.85 V

Butterworth High-pass filter:

Circuit:



1. Determining transfer function and cut-off frequency for the above circuit:

$$V_+ = V_{in}\left(\frac{R}{R + \frac{1}{sC}}\right) = V_{in}\left(\frac{sRC}{1+sRC}\right)$$

$$V_o = \frac{V_-}{R_1} \times (R_1 + R_2)$$

$$\Rightarrow V_o = V_{in}\left(\frac{sRC}{1+sRC}\right)\left(1+\frac{R_2}{R_1}\right)$$

$\Rightarrow$ transfer function, $$\frac{V_o(s)}{V_{in}(s)} = \frac{\left(1+\frac{R_2}{R_1}\right)\times sRC}{1+sRC}$$

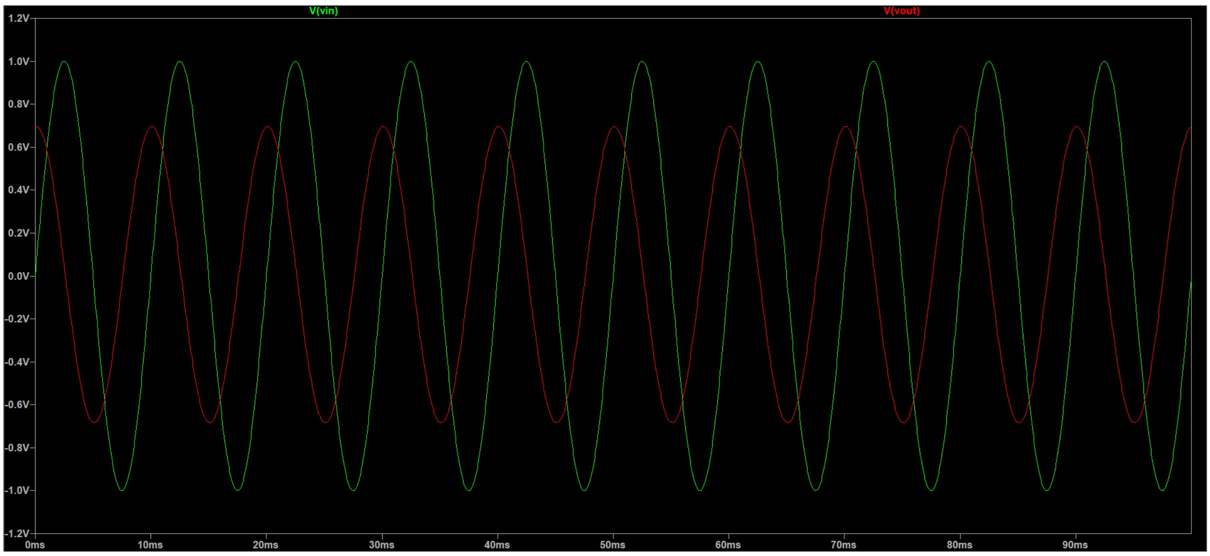
In this circuit, $R_1 = 1k\Omega$, $R_2 = 10k\Omega$, $R = 10k\Omega$ and $C = 0.01\mu F$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{11 \times s \times 10^{-4}}{1 + (s \times 10^{-4})}$$

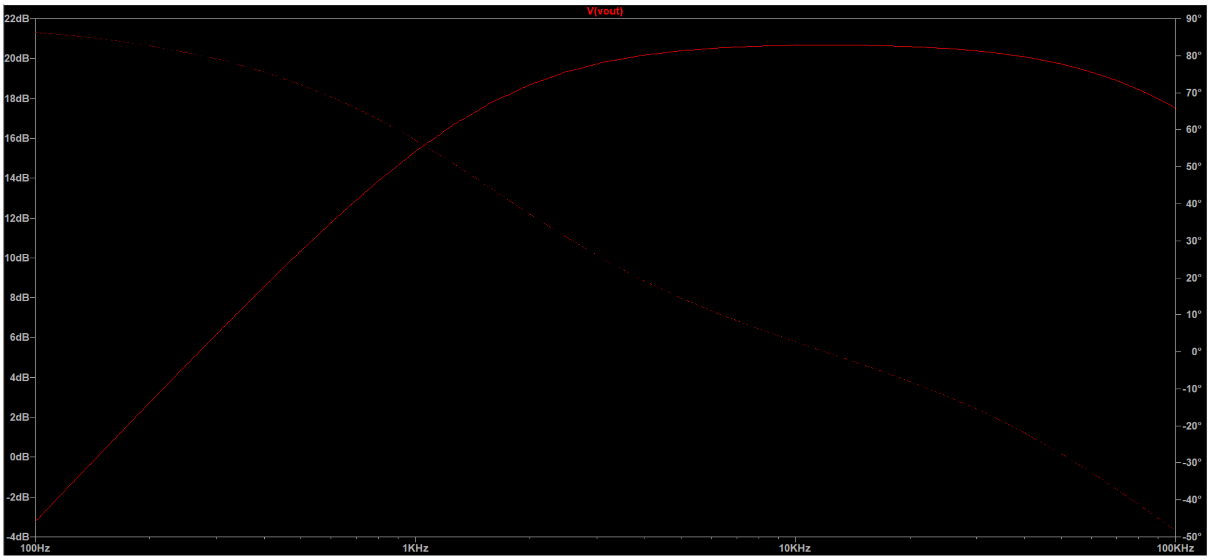
cut-off frequency $(f_c) = \frac{1}{2\pi \times RC} = \frac{1}{2\pi \times (10\times10^3)\cdot(0.01\times10^{-6})}$

$$\Rightarrow \boxed{f_c = 1.59\ kHz}$$

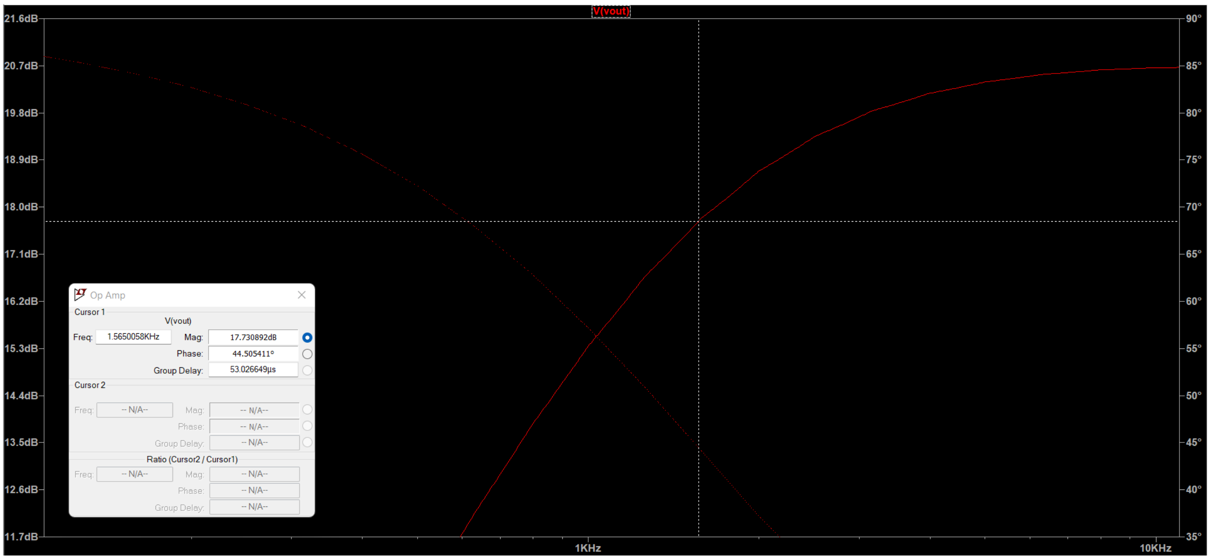
2. Input and Output voltages on the scope when a 1V peak, 100Hz sine wave is applied at its input:



3. Gain as a function of frequency (input frequency varies from 100Hz to 100kHz)

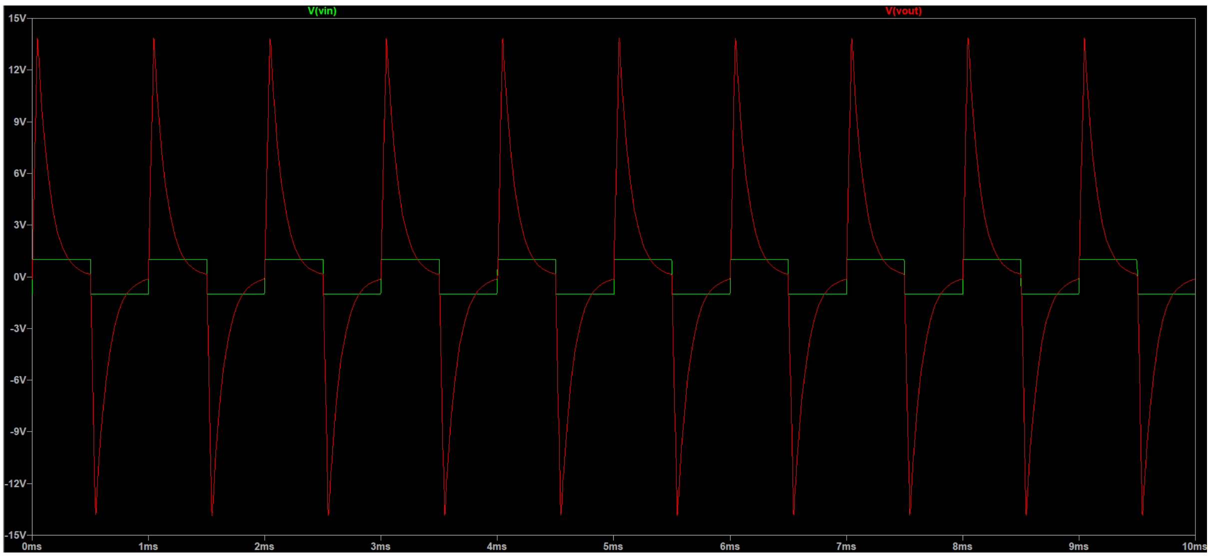


The above plot is magnified to determine the cut-off frequency



- The observed cut-off frequency on the plot is approximately 1.56KHz which is almost equal to the calculated value (1.59 kHz)

4. Input and Output voltages on the scope when a 1V peak, 1kHz square wave is applied at its input:



Explanation:

When a dc voltage of V_0 is applied across an RC-circuit, voltage across the capacitor will be

$$V_C(t) = V_0\left(1 - e^{-t/RC}\right) + V_{iC}e^{-t/RC}$$

where V_{iC} is initial voltage across capacitor (i.e., at $t=0$)

$\Rightarrow$ Voltage across the resistor will be

$$V_R(t) = V_0 - V_C(t)$$

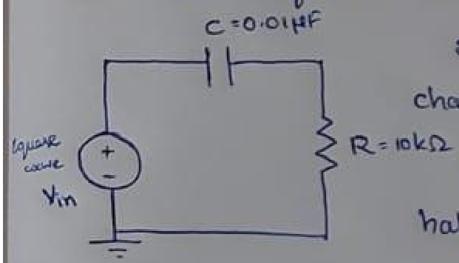
$$\Rightarrow \cancel{V_R(t) = V_0 e^{-t/RC} - V_{iC} e^{-t/RC}}$$

~~let the initial voltage (i.e., at t=0) across resistor be~~ V_{iR}

~~then~~ $\cancel{V_{iR} = V_0 - V_{iC}} \Rightarrow \cancel{V_{iC} = V_0 - V_{iR}}$

$\Rightarrow \cancel{V_R(t) = V_0 e^{-t/RC} - (V_0 - V_{iR})\, e^{-t/RC}}$

when a square wave is applied at its input



$\tau = RC = 0.1\ ms$

Since the voltage across the capacitor can't change rapidly.

So, the voltage at the end of negative half cycle will be same as the voltage across it in the beginning of positive half cycle.

At the steady state, ~~thus~~ the magnitude of the voltage at the end of negative half cycle will be same as the voltage at the end of positive half cycle.

$$\Rightarrow V_p = (1)\left(1 - e^{-\frac{0.5}{0.1}}\right) + (-V_p)\left(e^{-\frac{0.5}{0.1}}\right)$$

$$\Rightarrow V_p \approx 0.987\ V$$

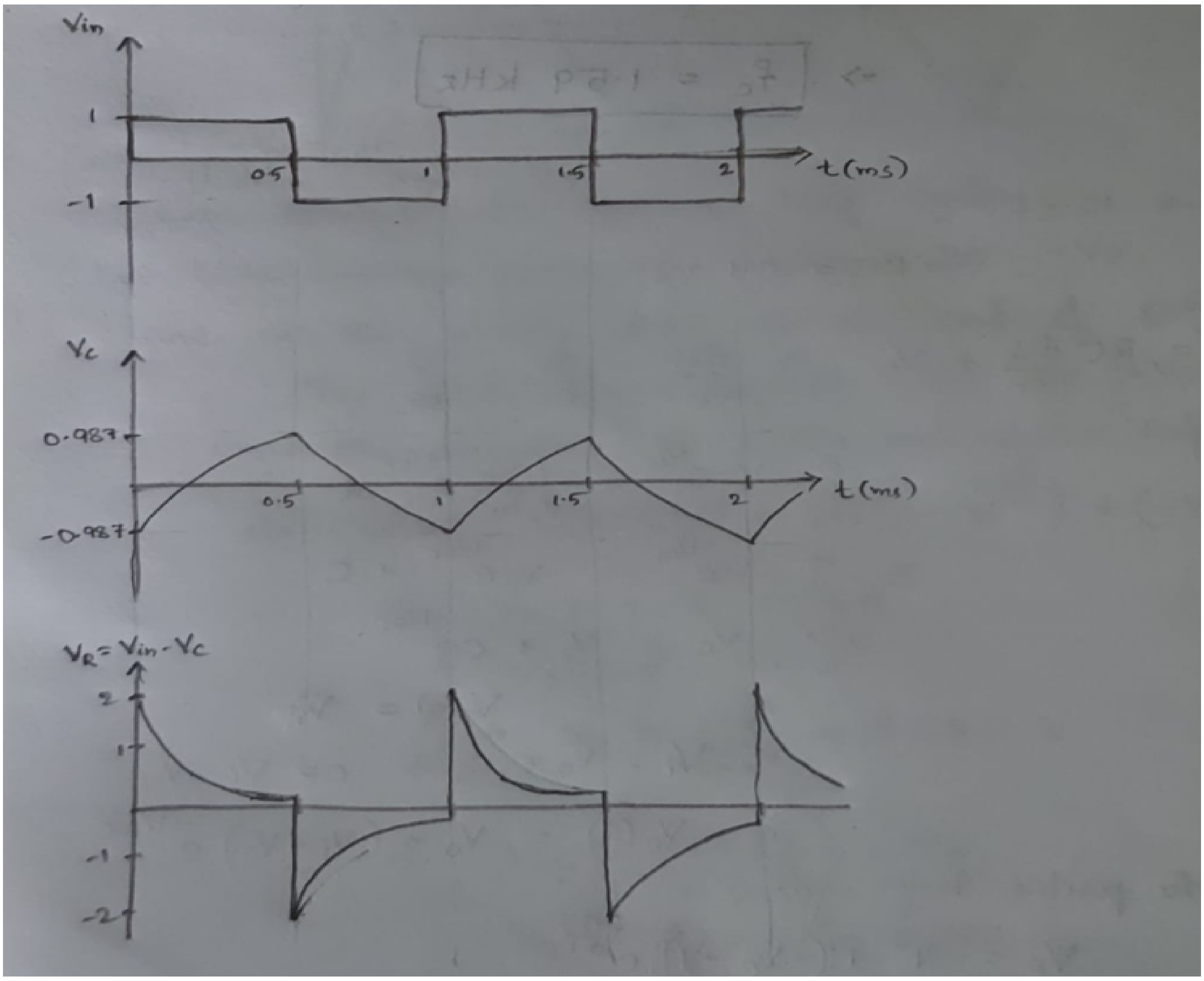
$$\Rightarrow V_C(t) = (1)\left(1 - e^{-t/0.1}\right) - (0.987)\, e^{-t/0.1} \quad \text{for positive half cycle}$$

$$\text{and } V_C(t) = (-1)\left(1 - e^{-t/0.1}\right) + (0.987)\, e^{-t/0.1} \quad \text{for negative half cycle}$$

plots for $V_{in}(t)$, $V_C(t)$ and $V_R(t)$ are plotted on the next page

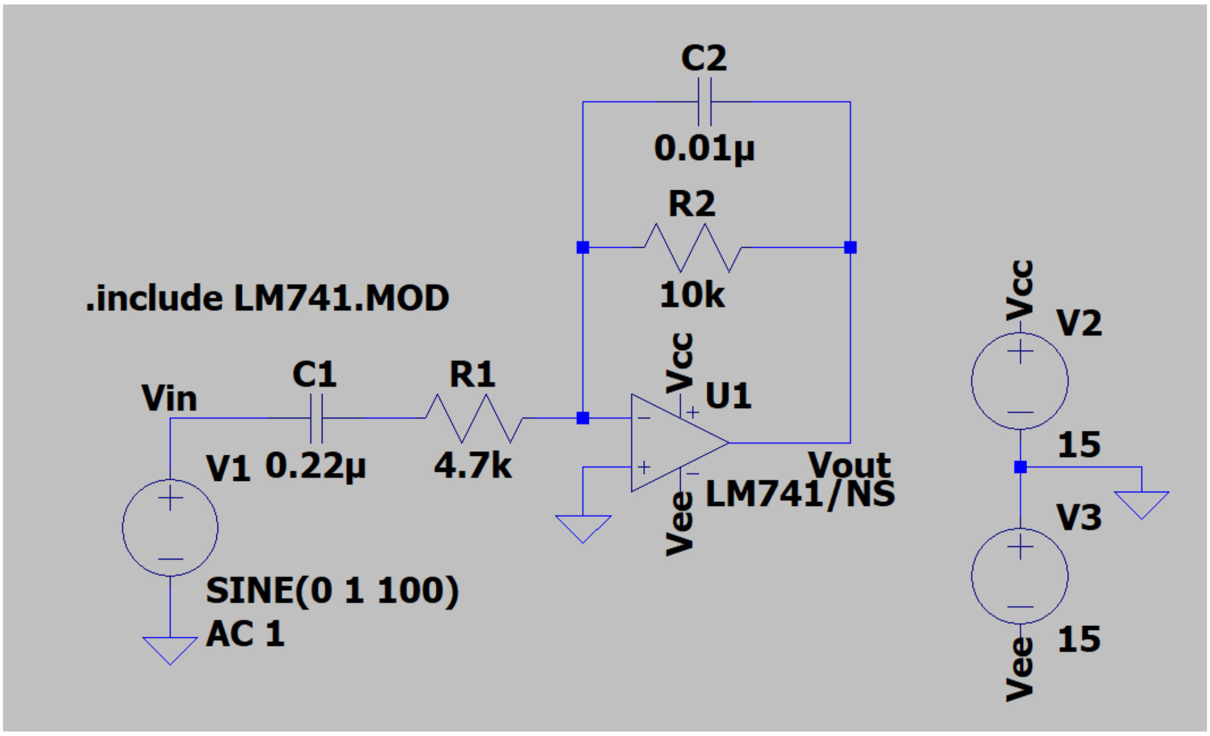
$$V_{out}(t) = \left(1 + \frac{R_2}{R_1}\right) \times V_R(t)$$

$$\Rightarrow \boxed{V_{out}(t) = 11 \times V_R(t)}$$



Wide band-pass filter:

Circuit:



1. Determining transfer function and cut-off frequencies for the above circuit:

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{\left(R_2 \,||\, \frac{1}{sC_2}\right)}{R_1 + \frac{1}{sC_1}}$$

$$\Rightarrow \frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_2}{1+sR_2C_2} \times \frac{sC_1}{1+sR_1C_1}$$

$$\Rightarrow \cancel{\frac{V_{out}(s)}{V_{in}(s)} = -\frac{sR_2C_1}{(1+sR_2C_2)(1+sR_1C_1)}}$$

$$\Rightarrow \boxed{\frac{V_{out}(s)}{V_{in}(s)} = \frac{-R_2/R_1}{1+sR_2C_2} \times \frac{sR_1C_1}{1+sR_1C_1}}$$

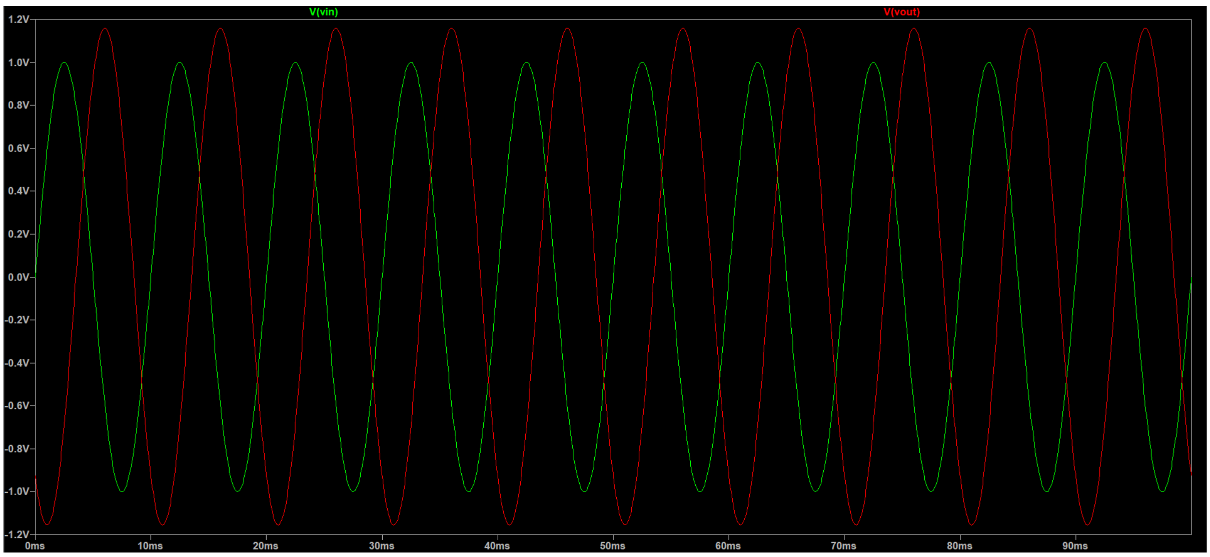
low cut off frequency $(f_{cL}) = \frac{1}{2\pi \times R_1C_1} = \frac{1}{2\pi \times (4.7\times10^3)\times(0.22\times10^{-6})}$

$$\Rightarrow \boxed{f_{cL} = 154 \text{ Hz}}$$

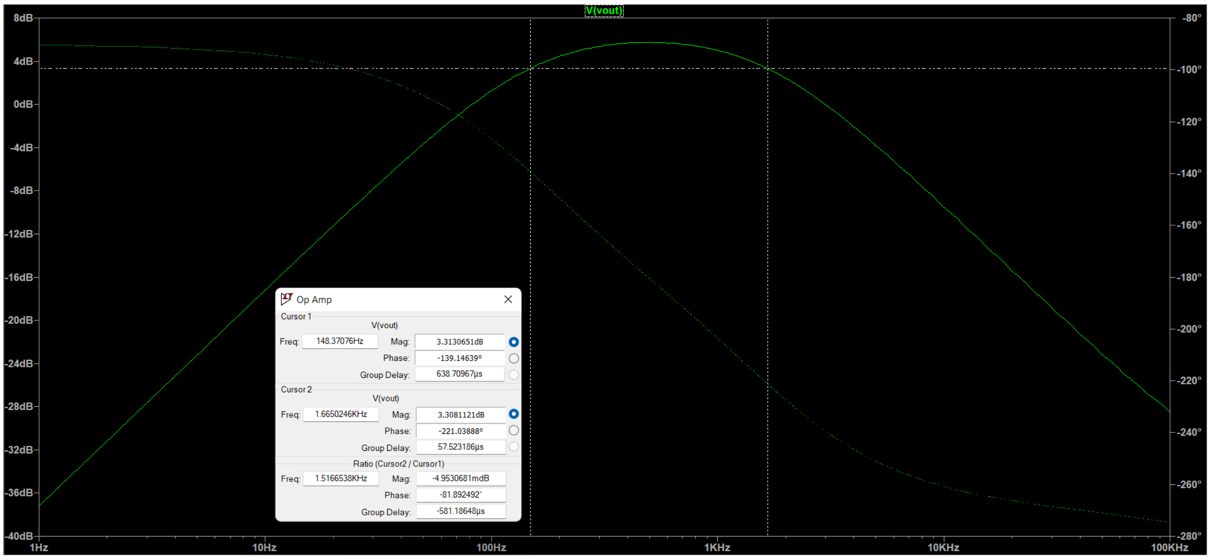
high cut off frequency $(f_{cH}) = \frac{1}{2\pi \times R_2C_2} = \frac{1}{2\pi \times (10\times10^3)\times(0.01\times10^{-6})}$

$$\Rightarrow \boxed{f_{cH} = 1.59 \text{ kHz}}$$

2. Input and Output voltages on the scope when a 1V peak, 100Hz sine wave is applied at its input:

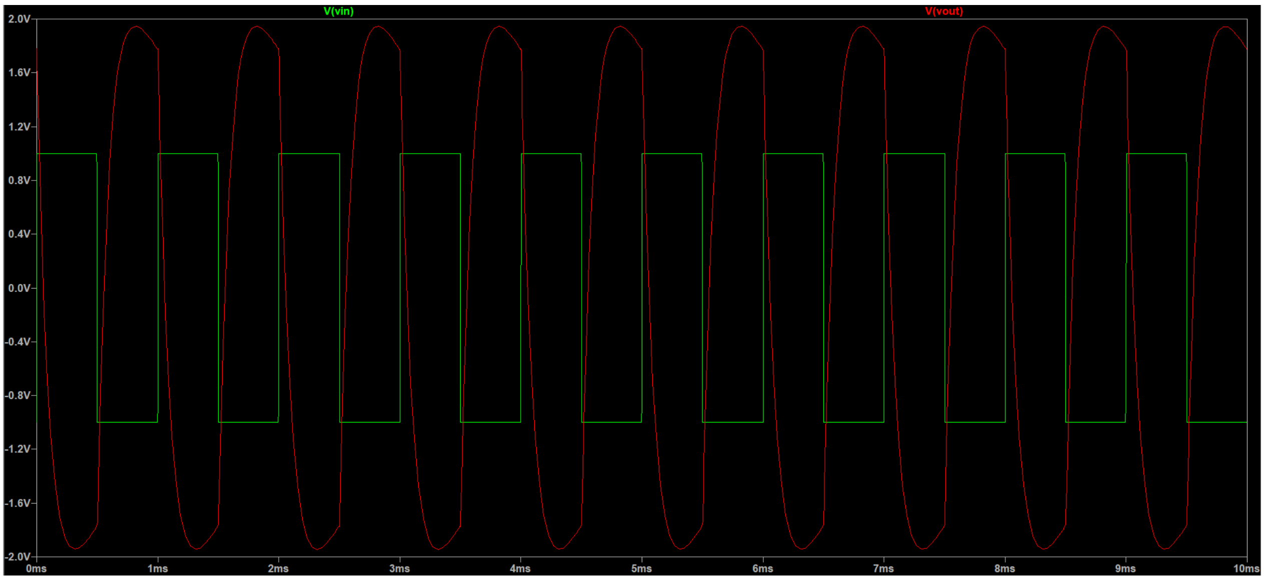


3. Gain and phase as functions of frequency (input frequency varies from 1Hz to 100kHz)



- The observed low cut-off frequency on the plot is approximately 148 Hz which is almost equal to the calculated value (i.e., 154 Hz)
- The observed high cut-off frequency on the plot is approximately 1.66 kHz which is almost equal to the calculated value (i.e., 1.59 kHz)

4. Input and Output voltages on the scope when a 1V peak, 1kHz square wave is applied at its input:



Explanation:

$V_- = V_+ = 0$ (virtually short)

lets analyse for positive half cycle of square wave

let the voltage at the end of negative half cycle be $-V_1$ across the capacitor C_1

$$v_{C_1}(t) = V_0\left(1 - e^{-t/R_1C_1}\right) + V_{ic}\left(e^{-t/R_1C_1}\right)$$ where, V_s = amplitude of [illegible]

intial voltage (i.e, at $t=0$) across capacitor, $V_{ic} = -V_p$

voltage at the end of positive half cycle, $V_{C_1}(0.5\ ms) = V_p$

$$R_1C_1 = (4.7\times10^3)\cdot(0.22\times10^{-6}) = 1.034\ ms$$

$$\Rightarrow V_p = (1)\left(1 - e^{\frac{-0.5}{1.034}}\right) - V_p\left(e^{\frac{-0.5}{1.034}}\right)$$

$$\Rightarrow V_p = \frac{1 - e^{\frac{-0.5}{1.034}}}{1 + e^{\frac{-0.5}{1.034}}} \approx 0.237\ V$$

$$\Rightarrow V_{C_1}(t) = \left(1 - e^{-t/R_1C_1}\right) - 0.237\,e^{-t/R_1C_1}$$

$$\Rightarrow \boxed{v_{C_1}(t) = 1 - 1.237\,e^{-t/R_1C_1}}$$

$$\Rightarrow i_1(t) = C_1\frac{dv_{C_1}}{dt} = \cancel{C_1}\times1.237\times\frac{1}{R_1\cancel{C_1}}\times e^{-t/R_1C_1}$$

$$\Rightarrow \boxed{i_1(t) = 0.263\times e^{-t/R_1C_1}\ mA}$$

$i_2(t) = i_1(t)$ ($\because$ zero current flows into non-inverting terminal)

$$i_2(t) = C_2\frac{dv_{C_2}}{dt} + \frac{v_{C_2}}{R_2}$$

$$\Rightarrow \frac{dv_{C_2}}{dt} + \frac{v_{C_2}}{R_2C_2} = \frac{i_2(t)}{C_2}$$

$$\Rightarrow v_{C_2}e^{t/R_2C_2} = \int\frac{1}{C_2}\cdot0.263\times e^{-t/R_1C_1}\times10^{-3}\times e^{t/R_2C_2}\,dt$$

$$\Rightarrow v_{C_2}e^{t/R_2C_2} = \frac{0.263\times10^{-3}}{C_2}\times\frac{e^{\left(\frac{1}{R_2C_2}-\frac{1}{R_1C_1}\right)t}}{\left(\frac{1}{R_2C_2}-\frac{1}{R_1C_1}\right)} + k$$

$R_2C_2 = 0.1\ ms$ $\quad$ k is integration const

$$\Rightarrow v_{C_2}(t) = 2.91\,e^{-t/R_1C_1} + k\,e^{-t/R_2C_2}$$

Let $v_{C_2}(0) = -V_{P_2} \Rightarrow k = -V_{P_2} - 2.91$

$$v_{C_2}(t) = 2.91\,e^{-t/R_1C_1} - (V_{P_2}+2.91)\,e^{-t/R_2C_2}$$

$$v_{C_2}(0.5\ ms) = V_{P_2} = 2.91\times e^{\frac{-0.5}{1.034}} - (V_{P_2}+2.91)\times e^{\frac{-0.5}{0.1}}$$

$$\Rightarrow V_{P_2} \approx 1.76\ V$$

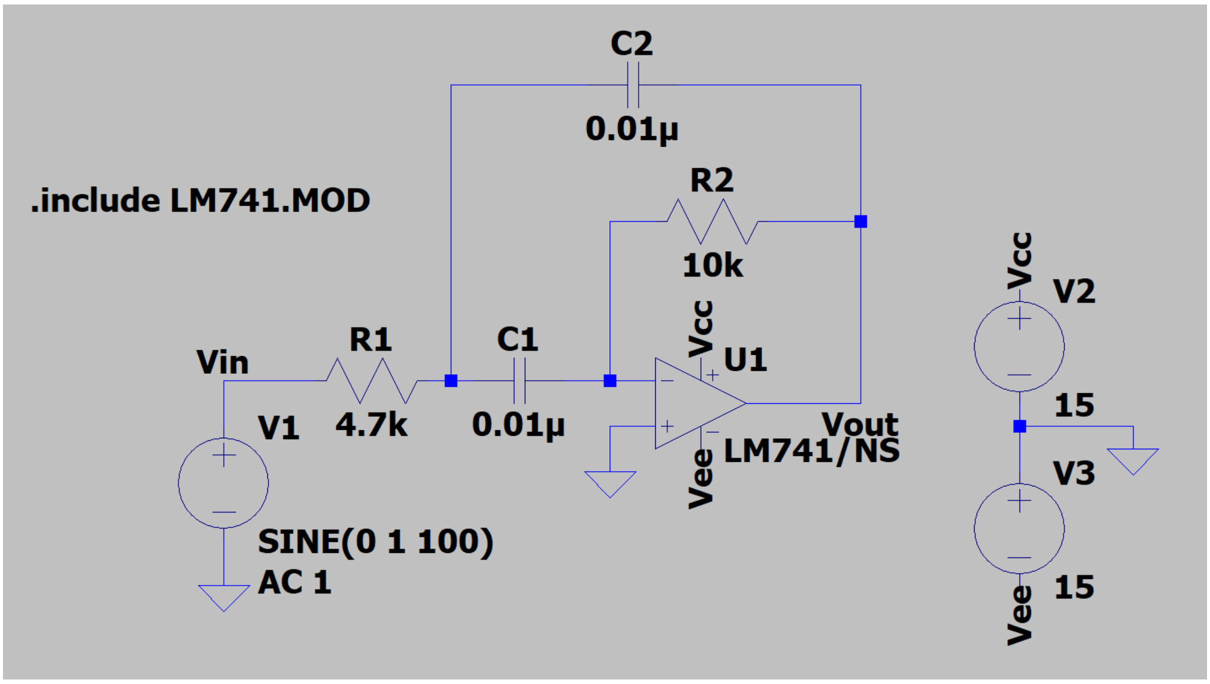
$$\Rightarrow v_{C_2}(t) = 2.91\,e^{-t/R_1C_1} - 4.67\,e^{-t/R_2C_2}$$

$$\Rightarrow v_{out}(t) = -V_{C_2}(t) = 4.67\,e^{-t/R_2C_2} - 2.91\,e^{-t/R_1C_1}$$

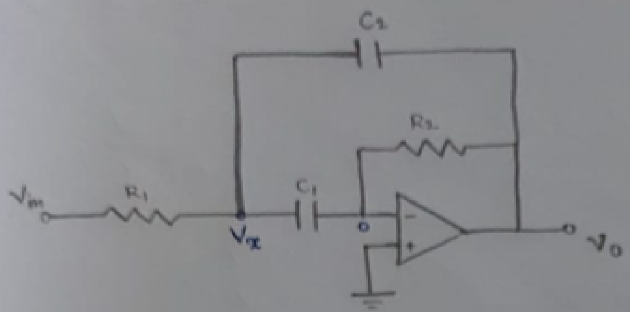
This is for positive half cycle. similarly, we get negative of this for negative half cycle

Multiple Feedback band-pass filter:

Circuit:



1. Determining transfer function for the above circuit:



$$V_- = V_+ = 0$$

Applying KCL at node x

$$\frac{V_x - V_{in}}{R_1} + \frac{V_x - V_o}{\left(\frac{1}{sC_2}\right)} + \frac{V_x - 0}{\left(\frac{1}{sC_1}\right)} = 0$$

$$\Rightarrow V_x\left(\frac{1}{R_1} + sC_2 + sC_1\right) = \frac{V_{in}}{R_1} + V_o s C_2 \longrightarrow (1)$$

$$\frac{V_x}{\left(\frac{1}{sC_1}\right)} = \frac{-V_o}{R_2}$$ ($\because$ current flowing into non-inverting terminal is zero)

$$\Rightarrow V_x = \frac{-V_o}{sR_2C_1}$$

Substituting $V_x = \frac{-V_o}{sR_2C_1}$ in eq^n (1) we get,

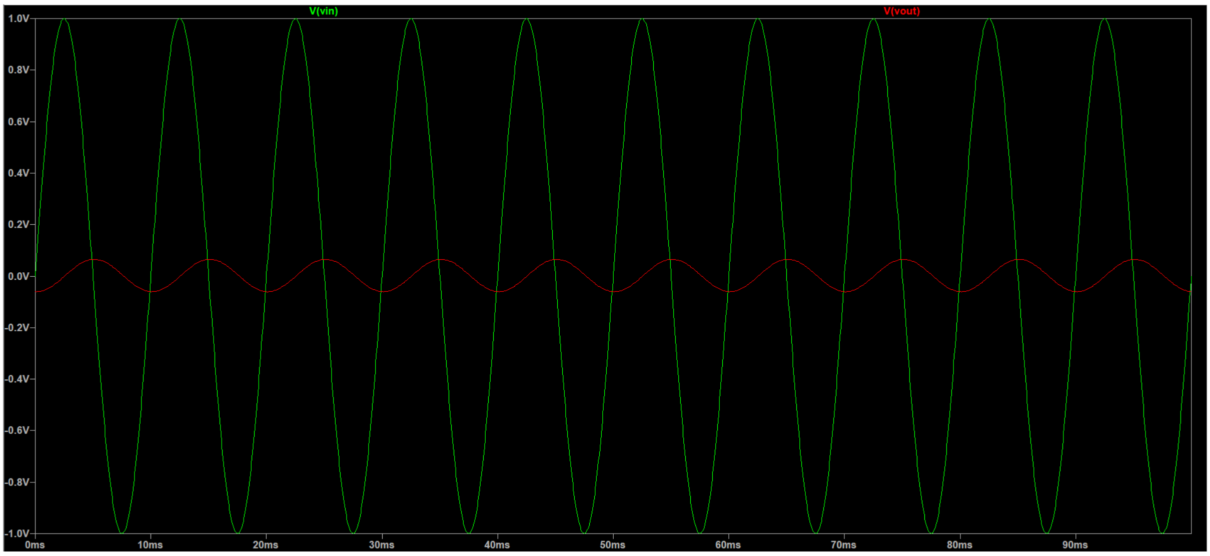
$$\frac{-V_o}{sR_2C_1} \times \left(\frac{1}{R_1} + sC_2 + sC_1\right) = \frac{V_{in}}{R_1} + V_o s C_2$$

$$\Rightarrow \frac{V_o}{sR_1R_2C_1} \times \left(1 + sR_1(C_1 + C_2)\right) = -\frac{V_{in}}{R_1} - V_o s C_2$$

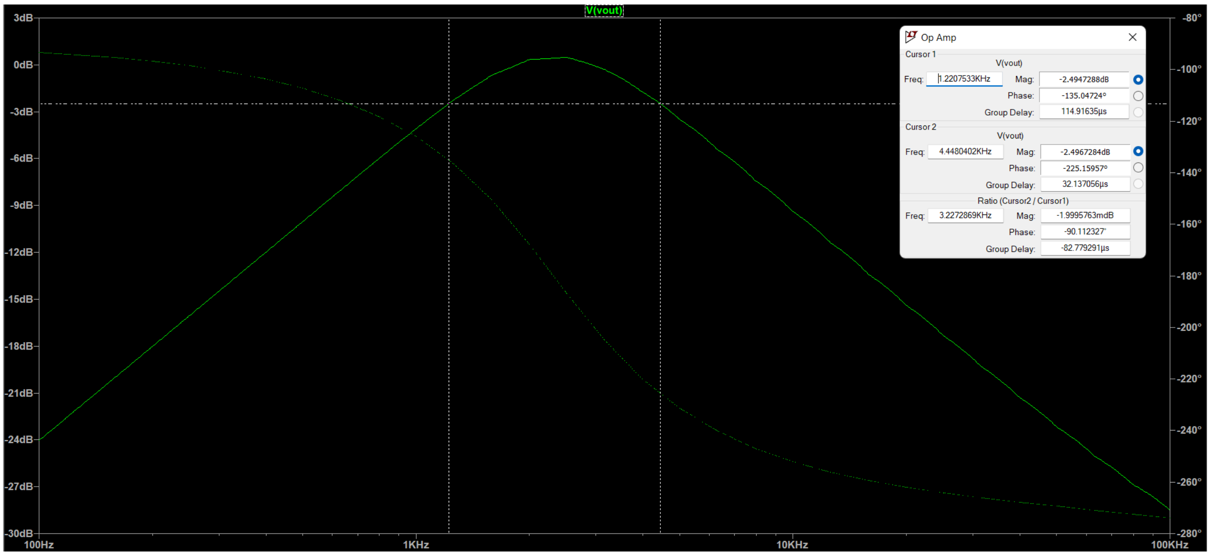
$$\Rightarrow V_o\left(s^2 R_1R_2C_1C_2 + sR_1(C_1+C_2) + 1\right) = -V_{in}\left(sR_2C_1\right)$$

$$\Rightarrow \frac{V_o(s)}{V_{in}(s)} = \frac{-R_2C_1 s}{R_1R_2C_1C_2 s^2 + R_1(C_1+C_2)s + 1}$$

2. Input and Output voltages on the scope when a 1V peak, 100Hz sine wave is applied at its input:



3. Gain and phase as functions of frequency (input frequency varies from 100Hz to 100kHz)



- The observed low cut-off frequency on the plot is approximately 1.22 kHz which is almost equal to the calculated value
- The observed high cut-off frequency on the plot is approximately 4.44 kHz which is almost equal to the calculated value

4. Input and Output voltages on the scope when a 1V peak, 1kHz square wave is applied at its input: