A3(b)

We proved in (a) that:

$$h(z) = \int_{-\infty}^{\infty} f(x)g(z-x)\,dx.$$

Since f and g are nonzero only on $[0,1]$, we note that $g(z-x)$ is:

$$g(z-x) = \begin{cases} 1 & \text{for } 0 \leq z-x \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Plugging in values:

$$h(z) = \int_0^1 g(z-x)\,dx.$$

For $0 \leq z \leq 1$, $g(z-x) = 1$ for $x \in [0, z]$:

$$h(z) = \int_0^z 1\,dx = z.$$

For $1 < z \leq 2$, $g(z-x) = 1$ for $x \in [z-1, 1]$:

$$h(z) = \int_{z-1}^1 1\,dx = 1-(z-1) = 2-z.$$

Thus:

$$h(z) = \begin{cases} z & \text{for } 0 \leq z \leq 1, \\ 2-z & \text{for } 1 < z \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

■