Question A2

(a)

Since $\mathbb{E}[Y] = \mathbb{E}[\mathbb{E}[Y \mid X]]$ (by the Law of Total Expectation), we know:

$$\mathbb{E}[Y \mid X] = X \implies \mathbb{E}[Y] = \mathbb{E}[X].$$

By the definition of covariance:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])].$$

Expanding this out:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X] \cdot \mathbb{E}[Y].$$

Substituting $\mathbb{E}[Y \mid X] = X$, we get:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[X \cdot X] - \mathbb{E}[X] \cdot \mathbb{E}[X].$$

This simplifies to:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2.$$

Now, using the expansion:

$$\mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \mathbb{E}[(X - \mathbb{E}[X])^2].$$

Thus:

$$\mathrm{Cov}(X, Y) = \mathbb{E}[(X - \mathbb{E}[X])^2].$$

■