A8(d)

Let λ be an eigenvalue of C, and let v be the corresponding eigenvector. By definition of eigenvalues and eigenvectors, we have:

$$Cv = \lambda v.$$

Taking the quadratic form $v^\top Cv$, we substitute $Cv = \lambda v$:

$$v^\top Cv = v^\top(\lambda v).$$

Since λ is a scalar, we can factor it out:

$$v^\top Cv = \lambda(v^\top v).$$

Since C is PSD, it satisfies:

$$v^\top Cv \geq 0 \quad \text{for all vectors } v.$$

Substituting $v^\top Cv = \lambda(v^\top v)$, we get:

$$\lambda(v^\top v) \geq 0.$$

The term $v^\top v$ is the squared norm of v, which is strictly positive since $v \neq 0$ (by definition of an eigenvector). Thus:

$$\lambda \geq 0.$$

Thus, all eigenvalues of C are non-negative.

■