A2(a)

First, we find the likelihood function of the parameter λ for the given data. Let $x_1, x_2, \ldots, x_n$ represent the observed goal counts for n games, where the number of goals in each game follows a Poisson distribution with parameter λ. The probability mass function for a single observation is:

$$P(x_i \mid \lambda) = \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}.$$

Since the observations are independent, the likelihood function for the entire dataset is the product of the probabilities:

$$L(\lambda) = \prod_{i=1}^{n} P(x_i \mid \lambda) = \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!}.$$

Next, we take the natural logarithm of the likelihood function to simplify the optimization process. The log-likelihood function is:

$$\log L(\lambda) = \log \left(\prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right).$$

Expanding the logarithm of the product:

$$\log L(\lambda) = \sum_{i=1}^{n} \log \left(\frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right).$$

Now, apply the logarithm of a quotient:

$$\log \left(\frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right) = \log \left(\lambda^{x_i} e^{-\lambda} \right) - \log(x_i!).$$

For the term $\log \left(\lambda^{x_i} e^{-\lambda} \right)$, apply the logarithm of a product:

$$\log \left(\lambda^{x_i} e^{-\lambda} \right) = \log \left(\lambda^{x_i} \right) + \log \left(e^{-\lambda} \right).$$

Simplify each term: 1. $\log \left(\lambda^{x_i} \right) = x_i \log \lambda$, 2. $\log \left(e^{-\lambda} \right) = -\lambda$.
Substitute these back:

$$\log \left(\frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \right) = x_i \log \lambda - \lambda - \log(x_i!).$$

Combining everything:

$$\log L(\lambda) = \sum_{i=1}^{n} \left(x_i \log \lambda - \lambda - \log x_i! \right).$$

To find the maximum likelihood estimate of λ, we differentiate $\log L(\lambda)$ with respect to λ:

$$\frac{\partial}{\partial \lambda} \log L(\lambda) = \frac{\partial}{\partial \lambda} \left(\sum_{i=1}^{n} x_i \log \lambda - \sum_{i=1}^{n} \lambda - \sum_{i=1}^{n} \log x_i! \right).$$

Taking derivatives term by term:

$$\frac{\partial}{\partial \lambda} \log L(\lambda) = \sum_{i=1}^{n} \frac{x_i}{\lambda} - \sum_{i=1}^{n} 1 + \frac{\partial}{\partial \lambda} \left(- \sum_{i=1}^{n} \log x_i! \right).$$

Since $\sum_{i=1}^{n} \log x_i!$ does not depend on λ, its derivative is zero:

$$\frac{\partial}{\partial \lambda} \log L(\lambda) = \sum_{i=1}^{n} \frac{x_i}{\lambda} - n.$$

Set the derivative equal to zero:

$$\sum_{i=1}^{n} \frac{x_i}{\lambda} - n = 0.$$

Rearranging:

$$\frac{\sum_{i=1}^{n} x_i}{\lambda} = n.$$

Multiply through by λ:

$$\sum_{i=1}^{n} x_i = n\lambda.$$

Solve for λ:

$$\lambda = \frac{\sum_{i=1}^{n} x_i}{n}.$$

Thus, the maximum likelihood estimate for λ is:

$$\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} x_i.$$

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