Question A2

(b)

We know that $\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$. To show $\mathrm{Cov}(X,Y) = 0$, we need to prove:

$$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y].$$

By the definition of expectation of the joint PDF:

$$\mathbb{E}[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_{X,Y}(x,y)\, dx\, dy.$$

Using independence:

$$f_{X,Y}(x,y) = f_X(x) f_Y(y).$$

Substituting this into the expression for $\mathbb{E}[XY]$, we have:

$$\mathbb{E}[XY] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy f_X(x) f_Y(y)\, dx\, dy.$$

The double integral can be separated:

$$\mathbb{E}[XY] = \left(\int_{-\infty}^{\infty} x f_X(x)\, dx \right) \cdot \left(\int_{-\infty}^{\infty} y f_Y(y)\, dy \right).$$

The first term is $\mathbb{E}[X]$, and the second term is $\mathbb{E}[Y]$. Thus:

$$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y].$$

Therefore:

$$\mathrm{Cov}(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X] \cdot \mathbb{E}[Y] = 0.$$

$\blacksquare$