A6(b)

We want to solve the system $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} -2 \\ -2 \\ -4 \end{bmatrix}.$$

Form the augmented matrix:

$$\left[\begin{array}{ccc|c} 0 & 2 & 4 & -2 \\ 2 & 4 & 2 & -2 \\ 3 & 3 & 1 & -4 \end{array}\right].$$

Row Operations

$$r_1 \longleftrightarrow r_2 \qquad \Longrightarrow \left[\begin{array}{ccc|c} 2 & 4 & 2 & -2 \\ 0 & 2 & 4 & -2 \\ 3 & 3 & 1 & -4 \end{array}\right],$$

$$r_1 \leftarrow \tfrac{1}{2}r_1, \quad r_2 \leftarrow \tfrac{1}{2}r_2 \qquad \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & 2 & -1 \\ 3 & 3 & 1 & -4 \end{array}\right],$$

$$r_3 \leftarrow r_3 - 3r_1 \qquad \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & -2 & -1 \end{array}\right],$$

$$r_3 \leftarrow r_3 + 3r_2 \qquad \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 4 & -4 \end{array}\right],$$

$$r_3 \leftarrow \tfrac{1}{4}r_3 \qquad \Longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right].$$

Back-Substitution and Solution

From the final augmented matrix, we read off the equations:

$$\begin{cases} x_1 + 2x_2 + x_3 = -1, \\ x_2 + 2x_3 = -1, \\ x_3 = -1. \end{cases}$$

Starting from the bottom row, $x_3 = -1$. Plugging into the second row:

$$x_2 + 2(-1) = -1 \implies x_2 - 2 = -1 \implies x_2 = 1.$$

Finally, from the first row:

$$x_1 + 2(1) + (-1) = -1 \implies x_1 + 1 = -1 \implies x_1 = -2.$$

Hence the solution is

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}.$$