A4(a)

Let $Y = aX_1 + b$. Since $Y \sim \mathcal{N}(0, 1)$, we need to determine a and b such that Y has mean 0 and variance 1.

Mean

The mean of Y is:

$$\mathbb{E}[Y] = \mathbb{E}[aX_1 + b] = a\mathbb{E}[X_1] + b.$$

Since $\mathbb{E}[X_1] = \mu$ and $\mathbb{E}[Y] = 0$ (as given), this becomes:

$$a\mu + b = 0.$$

Solve for b:

$$b = -a\mu. \tag{1}$$

Variance

The variance of Y is:

$$\text{Var}(Y) = \text{Var}(aX_1 + b) = a^2\text{Var}(X_1).$$

Since $\text{Var}(X_1) = \sigma^2$ and $\text{Var}(Y) = 1$ (as given), we have:

$$a^2\sigma^2 = 1.$$

Solve for a:

$$a = \frac{1}{\sigma}. \tag{2}$$

Substituting Values

Substitute $a = \frac{1}{\sigma}$ from (2) into (1):

$$b = -\frac{\mu}{\sigma}.$$

Final Results

The values of a and b are:

$$a = \frac{1}{\sigma}, \quad b = -\frac{\mu}{\sigma}.$$

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