

A1

Define the probabilities:

$P(D)$ = The probability of having the disease,

$P(\text{Positive})$ = The probability of testing positive.

Using Bayes' Theorem:

$$P(D \mid \text{Positive}) = \frac{P(\text{Positive} \mid D)P(D)}{P(\text{Positive})}.$$

Known probabilities:

$$P(D) = 0.0001,$$

$$P(\neg D) = 0.9999,$$

$$P(\text{Positive} \mid D) = 0.99,$$

$$P(\text{Positive} \mid \neg D) = 0.01.$$

Calculate $P(\text{Positive})$:

$$P(\text{Positive}) = P(\text{Positive} \mid D)P(D) + P(\text{Positive} \mid \neg D)P(\neg D).$$

$$P(\text{Positive}) = (0.99)(0.0001) + (0.01)(0.9999).$$

$$P(\text{Positive}) = 0.000099 + 0.009999 = 0.010098.$$

Calculate $P(D \mid \text{Positive})$:

$$P(D \mid \text{Positive}) = \frac{(0.99)(0.0001)}{0.010098}.$$

$$P(D \mid \text{Positive}) = \frac{0.000099}{0.010098}.$$

$$P(D \mid \text{Positive}) \approx 0.0098.$$

Final result:

$$P(D \mid \text{Positive}) \approx 0.98\%.$$