

Exercise 9.1. [5pts] Let F_1, F_2 be subfields of a field E . Prove that $F = F_1 \cap F_2$ is a subfield of E .

Exercise 9.2. [16pts] Let $f(x) = x^2 + x + 2 \in \mathbb{Z}_3[x]$.

- (a) Show that $f(x)$ is irreducible. Hence, $E = F[x]/f(x)$ is a field.
- (b) Is $x^3 - x^2 - 1$ trivial in E , or not? Why?
- (c) $x^3 + 2x = 2x^2$ in E , or not? Why?
- (d) Find the multiplicative inverse of $x + 1$ in E .
- (e) $\chi(E) =$
- (f) $|E| =$
- (g) Find the order of $x + 2$ in E .
- (h) Is x a primitive root in E ?