Elementary Dynamical Simulation of Hurricane Winds using Rankine Vortex

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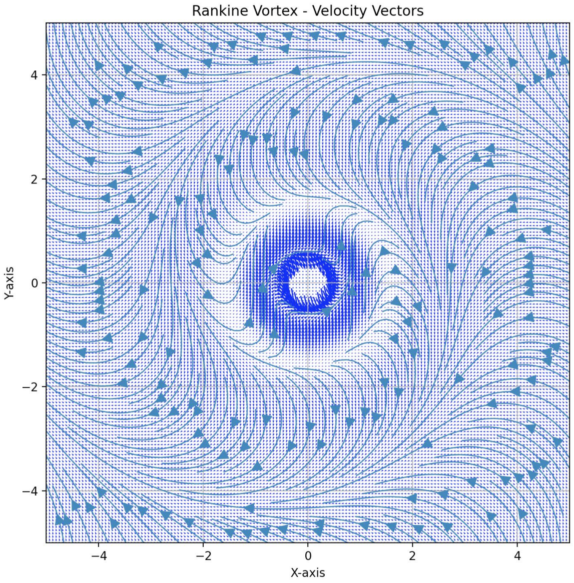
What is a Rankine Vortex?

Simple model of rotational fluid flow is a surprisingly accurate for intense wind vortices

Named for the Scottish engineer William John Rankine, the Rankine vortex is characterized by a central core where the fluid velocity is constant and equal to the vortex's maximum velocity. Outside the core, the velocity decreases radially until it becomes zero at a certain distance from the center.

This Rankine vortex function gives a velocity vector at a given radius r from the center of the vortex. Below, ‘a’ refers to the radius of the vortex-core (see image on bottom-right), ‘r’ is the distance from the center, and Gamma is the circulation strength.



Why is this vortex a good representation of hurricane winds?

“The use of such a simple approach is made possible by the remarkable consistency and relative uniformity of hurricane circulations, and indeed those of all intense vortices.” (Holland et al., 2010)

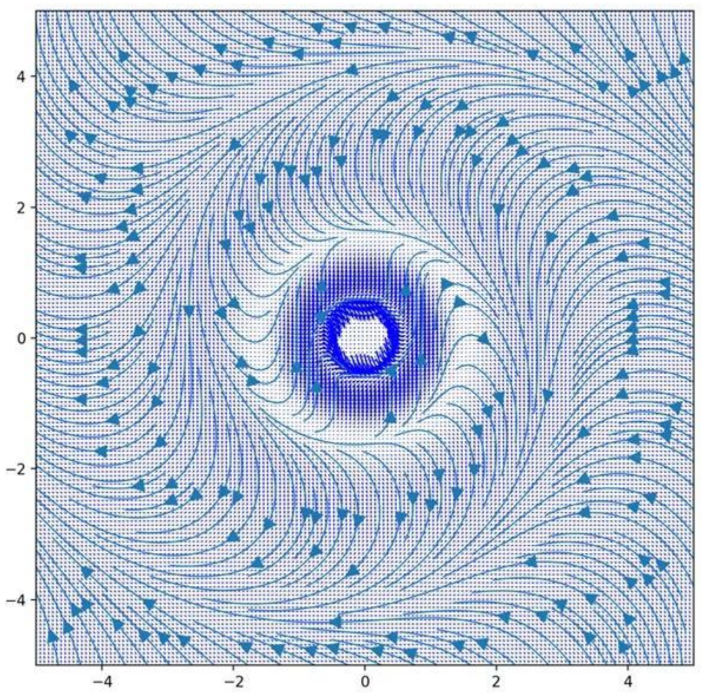
$$v_r = 0, \quad v_\theta(r) = \frac{\Gamma}{2\pi}\begin{cases} r/a^2 & r \le a, \\ 1/r & r > a \end{cases}, \quad v_z = 0$$

```python
def rankine_vortex(u_inf, gamma, R, x, y):
    """
    Rankine vortex model for circular flow.

    Parameters:
    - u_inf: Free stream velocity
    - gamma: Circulation strength (positive for clockwise circulation, negative
    - R: Radius of the vortex
    - x, y: Meshgrid coordinates

    Returns:
    - u, v: Velocity components
    """

    r = np.sqrt(x**2 + y**2)

    u = u_inf * (1 - R**2 / r**2) * np.sin(gamma / (2 * np.pi) * np.log(r / R))
    v = u_inf * (1 + R**2 / r**2) * np.cos(gamma / (2 * np.pi) * np.log(r / R))

    return u, v
```

Animation

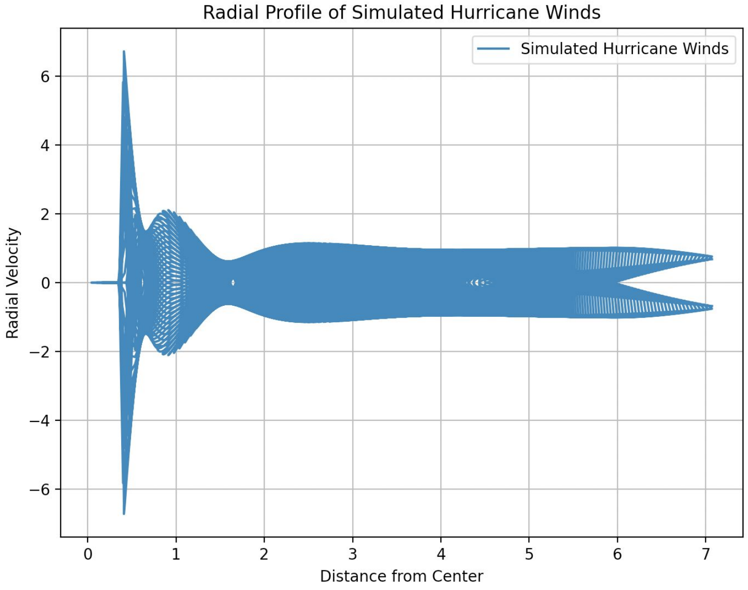
Selected animation follows direct leftward path. Motion of eye can be modeled via random walks as well

```python
# Define parameters
u_inf = 1.0
gamma = 20.0
R = 1.0
center_mask_radius = 0.4 # radius of the eye

# Create meshgrid
x = np.linspace(-5, 5, 160)
y = np.linspace(-5, 5, 160)
X, Y = np.meshgrid(x, y)


# Compute velocity field using Rankine vortex model
u, v = rankine_vortex(u_inf, gamma, R, X, Y)

center_mask = X**2 + Y**2 < center_mask_radius**2
u[center_mask] = 0
v[center_mask] = 0

# Plot velocity vectors
plt.title("Leftward Motion of Rankine Vortex")
fig, ax = plt.subplots(figsize=(8, 8))
ax.set_xlim(-5, 5)
ax.set_ylim(-5, 5)
streamplot = ax.streamplot(X, Y, u, v, density=2, linewidth=1, arrowsize=2)
q = ax.quiver(X, Y, u, v, scale=20, color='blue', units='xy', pivot='mid')
title = ax.set_title('Rankine Vortex - Velocity Vectors and Colormap')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')

def update(frame):
    new_u, new_v = rankine_vortex(u_inf, gamma, R, X + frame/30, Y)
    center_mask = np.sqrt((X + frame/30)**2 + Y**2) < center_mask_radius

    # Set zero velocity at the center in the new field
    new_u[center_mask] = 0
    new_v[center_mask] = 0

    ax.clear()

    plt.title("Leftward Motion of Rankine Vortex")
    ax.set_xlabel('X-position')
    ax.set_ylabel('Y-position')

    q = ax.quiver(X, Y, new_u, new_v, scale=20, color='blue', units='xy', pivot='mid')
    streamplot = ax.streamplot(X, Y, new_u, new_v, density=2, linewidth=1, arrowsize=2)

    return q, streamplot.lines


# Create animation
animation = FuncAnimation(fig, update, frames=200, interval=50, blit=False)

animation.save('animation.mp4', writer='ffmpeg', fps=10)

# Show the animation (this may take some time to render)
plt.show()
```



Verification

Ensuring accuracy of generated simulation by comparison to findings in relevant literature



What can explain the difference between these two curves? 1) Parameter selections for circulation strength and 2) my addition of an “eye” at center of vortex

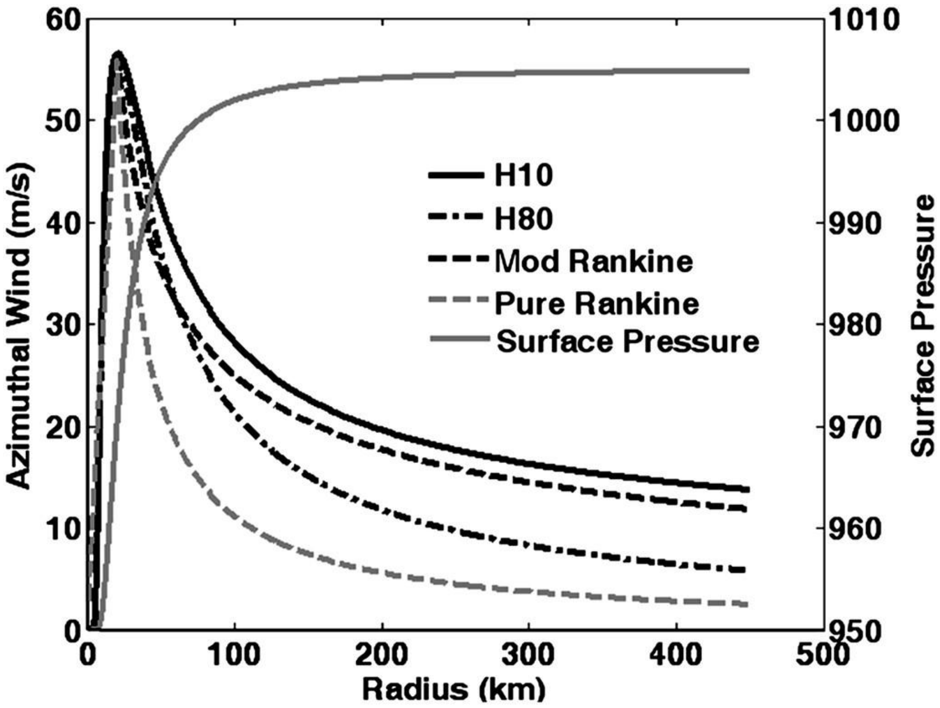


Figure from Holland et al. (2010)

References

Thank you for listening!

1. Holland, G. J., J. I. Belanger, and A. Fritz, 2010: A Revised Model for Radial Profiles of Hurricane Winds. Mon. Wea. Rev., 138, 4393–4401, https://doi.org/10.1175/2010MWR3317.1.
2. Dario Giaiotti and Fulvio Stel: The Rankine Vortex Model. Regional Meteorological Observatory, via Oberdan, 16/A I-33040 Visco (UD), ITALY (4 October 2006)
3. Maciej Dutkiewicz 2019 IOP Conf. Ser.: Models of strong wind acting on buildings and infrastructure. Mater. Sci. Eng. 471 052030