MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type I : Complementary Functions : **Marks**

1. If the roots $m_1, m_2, m_3, \ldots, m_n$ of auxiliary equation $\phi(D) = 0$ are real and distinct, then solution of $\phi(D)\,y = 0$ is (1)

(A) $c_1e^{m_1x} + c_2e^{m_2x} + \ldots + c_ne^{m_nx}$
(B) $c_1 \cos m_1x + c_2 \cos m_2x + \ldots + c_n \cos m_nx$
(C) $m_1e^{c_1x} + m_2e^{c_2x} + \ldots + m_ne^{c_nx}$
(D) $c_1 \sin m_1x + c_2 \sin m_2x + \ldots + c_n \sin m_nx$

2. The roots $m_1, m_2, m_3, \ldots, m_n$ of auxiliary equation $\phi(D) = 0$ are real. If two of these roots are repeated say $m_1 = m_2$ and the remaining roots $m_3, m_4, \ldots, m_n$ are distinct then solution of $\phi(D)\,y = 0$ is (1)

(A) $c_1e^{m_1x} + c_2^{m_2x} + \ldots + c_n^{m_nx}$
(B) $(c_1x + c_2) \cos m_1x + c_3 \cos m_3x + x \ldots + c_n \cos m_nx$
(C) $(c_1x + c_2)\, e^{m_1x} + c_3e^{m_3x} + \ldots + c_ne^{m_nx}$
(D) $(c_1x + c_2) \sin m_1x + c_3 \sin m_3x + \ldots + c_n \sin m_nx$

3. The roots $m_1, m_2, m_3 \ldots, m_n$ of auxiliary equation $\phi(D) = 0$ are real. If three of these roots are repeated, say, $m_1 = m_2 = m_3$ and the remaining roots $m_4, m_5, \ldots m_n$ are distinct then solution of $\phi(D)\,y = 0$ is (1)

(A) $c_1e^{m_1x} + c_2^{m_2x} + \ldots + c_n e^{m_nx}$
(B) $(c_1x^2 + c_2x + c_3)\, e^{m_1x} + c_4e^{m_4x} + \ldots + c_ne^{m_nx}$
(C) $(c_1x^2 + c_2x + c_3) \cos m_1x + c_4 \cos m_4x + \ldots + c_n \cos m_nx$
(D) $(c_1x^2 + c_2x + c_3) \sin m_1x + c_4 \sin m_4x + \ldots + c_n \sin m_nx$

4. If $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ are two complex roots of auxiliary equation of second order DE $\phi(D)\,y = 0$ then it's solution is (1)

(A) $e^{\beta x} [c_1 \cos \alpha x + c_2 \sin \alpha x]$
(B) $e^{\alpha x} [(c_1x + c_2) \cos \beta x + (c_3x + c_4) \sin \beta x]$
(C) $c_1e^{\alpha x} + c_2e^{\beta x}$
(D) $e^{\alpha x} [c_1 \cos \beta x + c_2 \sin \beta x]$

5. If the complex roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ of auxiliary equation of fourth order DE $\phi(D)\,y = 0$ are repeated twice then it's solution is (1)

(A) $e^{\beta x} [c_1 \cos \alpha x + c_2 \sin \alpha x]$
(B) $e^{\alpha x} [(c_1x + c_2) \cos \beta x + (c_3x + c_4) \sin \beta x]$
(C) $(c_1x + c_2)\, e^{\alpha x} + (c_3x + c_4)\, e^{\beta x}$
(D) $e^{\alpha x} [c_1 \cos \beta x + c_2 \sin \beta x]$

6. The solution of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ is (1)

(A) $c_1e^{2x} + c_2e^{-3x}$
(B) $c_1e^{-2x} + c_2e^{3x}$
(C) $c_1e^{-2x} + c_2e^{-3x}$
(D) $c_1e^{2x} + c_2e^{3x}$

7. The solution of differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 0$ is (1)

(A) $c_1e^{-x} + c_2e^{6x}$
(B) $c_1e^{-2x} + c_2e^{-3x}$
(C) $c_1e^{3x} + c_2e^{2x}$
(D) $c_1e^{-3x} + c_2e^{-2x}$

8. The solution of differential equation $2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 10y = 0$ is (1)

(A) $c_1e^{2x} + c_2e^{\frac{5}{2}x}$
(B) $c_1e^{-2x} + c_2e^{-\frac{5}{2}x}$
(C) $c_1e^{-2x} + c_2e^{\frac{5}{2}x}$
(D) $c_1e^{-2x} + c_2e^{\frac{3}{2}x}$

9. The solution of differential equation $\frac{d^2y}{dx^2} - 4y = 0$ is (1)

(A) $(c_1x + c_2)\, e^{2x}$
(B) $c_1e^{4x} + c_2e^{-4x}$
(C) $c_1 \cos 2x + c_2 \sin 2x$
(D) $c_1e^{2x} + c_2e^{-2x}$

10. The solution of differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ is (1)

(A) $c_1e^{2x} + c_2e^{x}$
(B) $c_1e^{2x} + c_2e^{-x}$
(C) $c_1e^{-2x} + c_2e^{x}$
(D) $c_1e^{-2x} + c_2e^{-x}$

11. The solution of differential equation $2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 3y = 0$ is (1)

(A) $c_1e^{x} + c_2e^{\frac{3}{2}x}$
(B) $c_1e^{2x} + c_2e^{3x}$
(C) $c_1e^{-x} + c_2e^{\frac{3}{2}x}$
(D) $c_1e^{\frac{x}{2}} + c_2e^{\frac{3}{2}x}$

12. The solution of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ is (1)

(A) $c_1e^{2x} + c_2e^{x}$
(B) $c_1e^{x} + c_2e^{-x}$
(C) $(c_1x + c_2)\, e^{-x}$
(D) $(c_1x + c_2)\, e^{x}$

13. The solution of differential equation $4\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$ is (1)

(A) $c_1e^{\frac{x}{2}} + c_2e^{\frac{x}{2}}$ (B) $(c_1 + c_2x)\, e^{-2x}$ (C) $c_1 \cos 2x + c_2 \sin 2x$ (D) $(c_1 + c_2x)\, e^{\frac{x}{2}}$

14. The solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ is (1)

(A) $(c_1x + c_2)\, e^{2x}$ (B) $(c_1x + c_2)\, e^{-2x}$ (C) $c_1e^{4x} + c_2e^{-4x}$ (D) $c_1e^{2x} + c_2e^{-2x}$

15. The solution of differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$ is (1)

(A) $c_1e^{-6x} + c_2e^{-9x}$ (B) $(c_1x + c_2)\, e^{-3x}$ (C) $(c_1x + c_2)\, e^{3x}$ (D) $c_1e^{3x} + c_2e^{2x}$

16. The solution of differential equation $\frac{d^2y}{dx^2} + y = 0$ is (1)

(A) $c_1e^{x} + c_2e^{-x}$

(B) $(c_1x + c_2)\, e^{-x}$

(C) $c_1 \cos x + c_2 \sin x$

(D) $e^{x}(c_1 \cos x + c_2 \sin x)$

17. The solution of differential equation $\frac{d^2y}{dx^2} + 9y = 0$ is (1)

(A) $c_1 \cos 2x + c_2 \sin 2x$

(B) $(c_1x + c_2)\, e^{-3x}$

(C) $c_1e^{3x} + c_2e^{-3x}$

(D) $c_1 \cos 3x + c_2 \sin 3x$

18. The solution of differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 10y = 0$ is (1)

(A) $e^{-3x}(c_1 \cos x + c_2 \sin x)$

(B) $e^{x}(c_1 \cos 3x + c_2 \sin 3x)$

(C) $c_1e^{5x} + c_2e^{2x}$

(D) $e^{x}(c_1 \cos x + c_2 \sin x)$

19. The solution of differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ is (1)

(A) $e^{x}(c_1 \cos x + c_2 \sin x)$

(B) $e^{x/2}\left[c_1 \cos\left(\frac{3}{2}\right)x + c_2 \sin\left(\frac{3}{2}\right)x\right]$

(C) $e^{-\frac{1}{2}x}\left[c_1 \cos\left(\frac{\sqrt{3}}{2}\right)x + c_2 \sin\left(\frac{\sqrt{3}}{2}\right)x\right]$

(D) $c_1e^{x} + c_2e^{-x}$

20. The solution of differential equation $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ is (1)

(A) $e^{-x}(c_1 \cos 2x + c_2 \sin 2x)$

(B) $e^{-x/2}[c_1 \cos x + c_2 \sin x]$

(C) $e^{-2x}(c_1 \cos x + c_2 \sin x)$

(D) $c_1e^{-4x} + c_2e^{-5x}$

21. The solution of differential equation $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$ is (2)

(A) $c_1e^{x} + c_2e^{2x} + c_3e^{3x}$

(B) $c_1e^{-x} + c_2e^{2x} + c_3e^{-3x}$

(C) $c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$

(D) $c_1e^{x} + c_2e^{-2x} + c_3e^{3x}$

22. The solution of differential equation $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0$ is (2)

(A) $c_1e^{x} + c_2e^{2x} + c_3e^{3x}$

(B) $c_1e^{-x} + c_2e^{-2x} + c_33^{6x}$

(C) $c_1e^{-x} + c_2e^{2x} + c_3e^{x}$

(D) $c_1e^{-x} + c_2e^{-2x} + c_3e^{3x}$

23. The solution of differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ is (2)

(A) $c_1 + e^{x}(c_2x + c_3)$

(B) $c_1 + e^{-x}(c_2x + c_3)$

(C) $e^{-x}(c_2x + c_3)$

(D) $c_1 + c_2e^{x} + c_3e^{-x}$

24. The solution of differential equation $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = 0$ is

(A) $c_1e^x + (c_2x + c_3)e^{2x}$
(B) $c_1e^x + c_2e^{2x} + c_3e^{3x}$
(C) $(c_2x + c_3)e^{2x}$
(D) $c_1e^{-x} + (c_2x + c_3)e^{-2x}$

25. The solution of differential equation $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 0$ is

(A) $c_1e^{2x} + c_2e^{-2x}$
(B) $c_1 + c_2\cos 2x + c_3\sin 2x$
(C) $c_1e^x + c_2e^{-2x} + c_3e^{-3x}$
(D) $c_1 + c_2e^{2x} + c_3e^{-2x}$

26. The solution of differential equation $\frac{d^3y}{dx^3} + y = 0$ is

(A) $c_1e^x + e^x\left(c_2\cos\frac{\sqrt{3}}{2}x + c_3\sin\frac{\sqrt{3}}{2}x\right)$
(B) $c_1e^{-x} + e^{\frac{1}{2}x}\left(c_2\cos\frac{1}{2}x + c_3\sin\frac{1}{2}x\right)$
(C) $c_1e^{-x} + e^{\frac{1}{2}x}\left(c_2\cos\frac{\sqrt{3}}{2}x + c_3\sin\frac{\sqrt{3}}{2}x\right)$
(D) $(c_1 + c_2x + c_3x^2)e^{-x}$

27. The solution of differential equation $\frac{d^3y}{dx^3} + 3\frac{dy}{dx} = 0$ is

(A) $c_1 + c_2\cos x + c_3\sin x$
(B) $c_1 + c_2\cos\sqrt{3}x + c_3\sin\sqrt{3}x$
(C) $c_1 + c_2e^{\sqrt{3}x} + c_3e^{-\sqrt{3}x}$
(D) $c_1\cos x + c_2\sin x$

28. The solution of differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 12y = 0$ is

(A) $c_1e^{-3x} + e^x\left(c_2\cos\sqrt{3}x + c_3\sin\sqrt{3}x\right)$
(B) $c_1e^{-3x} + (c_2\cos 3x + c_3\sin 3x)$
(C) $c_1e^{3x} + e^{-x}\left(c_2\cos\sqrt{3}x + c_3\sin\sqrt{3}x\right)$
(D) $c_1e^{-x} + c_2e^{-\sqrt{3}x} + c_3e^{\sqrt{3}x}$

29. The solution of differential equation $(D^3 - D^2 + 3D + 5)y = 0$ where $D = \frac{d}{dx}$ is

(A) $c_1e^{-x} + e^x(c_2\cos 2x + c_3\sin 2x)$
(B) $c_1e^{-x} + (c_2\cos 3x + c_3\sin 3x)$
(C) $c_1e^x + e^{-x}(c_2\cos 2x + c_3\sin 2x)$
(D) $c_1e^{-x} + c_2e^{-2x} + c_3e^{-3x}$

30. The solution of differential equation $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 4y = 0$ is

(A) $(c_1 + c_2x)e^{-2x} + c_3e^{-x}$
(B) $c_1e^x + c_2\cos 4x + c_3\sin 4x$
(C) $c_1e^x + c_2\cos 2x + c_3\sin 2x$
(D) $c_1e^x + c_2e^{2x} + c_3e^{-2x}$

31. The solution of differential equation $\frac{d^4y}{dx^4} - y = 0$ is

(A) $(c_1x + c_2)e^{-x} + c_3\cos x + c_4\sin x$
(B) $(c_1x + c_2)\cos x + (c_3x + c_4)\sin x$
(C) $(c_1 + c_2x + c_3x^2 + c_4x^3)e^x$
(D) $c_1e^x + c_2e^{-x} + c_3\cos x + c_4\sin x$

32. The solution of differential equation $(D^4 + 2D^2 + 1)y = 0$ where $D = \frac{d}{dx}$ is

(A) $(c_1x + c_2)e^x + (c_3x + c_4)e^{-x}$
(B) $(c_1x + c_2)\cos x + (c_3x + c_4)\sin x$
(C) $c_1e^x + c_2e^{-x} + c_3\cos x + c_4\sin x$
(D) $(c_1x + c_2)\cos 2x + (c_3x + c_4)\sin 2x$

33. The solution of differential equation $(D^2 + 9)^2y = 0$, where $D = \frac{d}{dx}$ is

(A) $(c_1x + c_2)e^{3x} + (c_3x + c_4)e^{-3x}$
(B) $(c_1x + c_2)\cos 3x + (c_3x + c_4)\sin 3x$
(C) $(c_1x + c_2)\cos 9x + (c_3x + c_4)\sin 9x$
(D) $(c_1x + c_2)\cos x + (c_3x + c_4)\sin x$

34. The solution of differential equation $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$ is

(A) $c_1e^{2x} + c_2e^{-x} + c_3e^x + c_4e^{-2x}$
(B) $(c_1x + c_2)e^{2x} + (c_3x + c_4)e^{-2x}$
(C) $(c_1x + c_2)\cos 4x + (c_3x + c_4)\sin 4x$
(D) $(c_1x + c_2)\cos 2x + (c_3x + c_4)\sin 2x$

35. The solution of differential equation $\frac{d^6y}{dx^6} + 6\frac{d^4y}{dx^4} + 9\frac{d^2y}{dx^2} = 0$ is (2)

(A) $c_1x + c_2 + (c_3x + c_4)\cos\sqrt{3}x + (c_5x + c_6)\sin\sqrt{3}x$

(B) $c_1x + c_2 + (c_3x + c_4)\cos 3x + (c_5x + c_6)\sin 3x$

(C) $(c_1x + c_2)\cos\sqrt{3}x + (c_3x + c_4)\sin\sqrt{3}x$

(D) $c_1x + c_2 + (c_3x + c_4)e^{\sqrt{3}x}$

Answers

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (D)	7. (A)	8. (C)
9. (D)	10. (B)	11. (A)	12. (C)	13. (D)	14. (A)	15. (B)	16. (C)
17. (D)	18. (A)	19. (C)	20. (B)	21. (C)	22. (D)	23. (B)	24. (A)
25. (D)	26. (C)	27. (B)	28. (A)	29. (A)	30. (C)	31. (D)	32. (B)
33. (B)	34. (D)	35. (A)					

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Particular Integral : **Marks**

1. Particular Integral of linear differential equation with constant coefficient $\phi(D)\,y = f(x)$ is given by (1)

(A) $\frac{1}{\phi(D)} f(x)$ (B) $\frac{1}{\phi(D)\,f(x)}$ (C) $\phi(D)\frac{1}{f(x)}$ (D) $\frac{1}{\phi(D^2)} f(x)$

2. $\frac{1}{D-m} f(x)$, where $D \equiv \frac{d}{dx}$ and m is constant, is equal to (1)

(A) $e^{mx}\int e^{-mx}\,dx$ (B) $\int e^{-mx} f(x)\,dx$

(C) $e^{mx}\int e^{-mx} f(x)\,dx$ (D) $e^{-mx}\int e^{mx} f(x)\,dx$

3. $\frac{1}{D+m} f(x)$, where $D \equiv \frac{d}{dx}$ and m is constant, is equal to (1)

(A) $e^{-mx}\int e^{mx}\,dx$ (B) $\int e^{mx} f(x)\,dx$

(C) $e^{mx}\int e^{-mx} f(x)\,dx$ (D) $e^{-mx}\int e^{mx} f(x)\,dx$

4. Particular Integral $\frac{1}{\phi(D)} e^{ax}$, where $D \equiv \frac{d}{dx}$ and $\phi(a) \neq 0$ is (1)

(A) $\frac{1}{\phi(-a)} e^{ax}$ (B) $x\frac{1}{\phi(a)} e^{ax}$ (C) $\frac{1}{\phi(a^2)} e^{ax}$ (D) $\frac{1}{\phi(a)} e^{ax}$

5. Particular Integral $\frac{1}{(D-a)^r} e^{ax}$ where $D \equiv \frac{d}{dx}$ is

(A) $\frac{1}{r!} e^{ax}$ (B) $\frac{x^r}{r} e^{ax}$ (C) $\frac{x^r}{r!} e^{ax}$ (D) $x^r e^{ax}$

6. Particular Integral $\frac{1}{\phi(D^2)} \sin(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(-a^2) \neq 0$ is

(A) $\frac{1}{\phi(-a^2)} \cos(ax+b)$ (B) $\frac{1}{\phi(-a^2)} \sin(ax+b)$

(C) $x \frac{1}{\phi(-a^2)} \sin(ax+b)$ (D) $\frac{1}{\phi(a^2)} \sin(ax+b)$

7. Particular Integral $\frac{1}{\phi(D^2)} \sin(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(-a^2) = 0$, $\phi'(-a^2) \neq 0$ is

(A) $x \frac{1}{\phi'(-a^2)} \cos(ax+b)$ (B) $x \frac{1}{\phi'(-a^2)} \sin(ax+b)$

(C) $\frac{1}{\phi(-a^2)} \sin(ax+b)$ (D) $\frac{1}{\phi'(-a^2)} \sin(ax+b)$

8. Particular Integral $\frac{1}{\phi(D^2)} \cos(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(-a^2) \neq 0$ is

(A) $\frac{1}{\phi(-a^2)} \cos(ax+b)$ (B) $\frac{1}{\phi(-a^2)} \sin(ax+b)$

(C) $x \frac{1}{\phi'(-a^2)} \cos(ax+b)$ (D) $\frac{1}{\phi(a^2)} \cos(ax+b)$

9. Particular Integral $\frac{1}{\phi(D^2)} \cos(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(-a^2) = 0$, $\phi'(-a^2) \neq 0$ is

(A) $\frac{1}{\phi'(-a^2)} \cos(ax+b)$ (B) $\frac{1}{\phi'(-a^2)} \cos(ax+b)$

(C) $x \frac{1}{\phi'(-a^2)} \sin(ax+b)$ (D) $x \frac{1}{\phi'(-a^2)} \cos(ax+b)$

10. Particular Integral $\frac{1}{\phi(D^2)} \sinh(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(a^2) \neq 0$ is

(A) $\frac{1}{\phi(a^2)} \cosh(ax+b)$ (B) $x \frac{1}{\phi'(a^2)} \sinh(ax+b)$

(C) $\frac{1}{\phi(a^2)} \sinh(ax+b)$ (D) $\frac{1}{\phi(-a^2)} \sinh(ax+b)$

11. Particular Integral $\frac{1}{\phi(D^2)} \cosh(ax+b)$, where $D \equiv \frac{d}{dx}$ and $\phi(a^2) \neq 0$ is

(A) $\frac{1}{\phi(a^2)} \cosh(ax+b)$ (B) $x \frac{1}{\phi'(a^2)} \cosh(ax+b)$

(C) $\frac{1}{\phi(a^2)} \sinh(ax+b)$ (D) $\frac{1}{\phi(-a^2)} \cosh(ax+b)$

12. Particular Integral $\frac{1}{\phi(D)} e^{ax} V$ where V is any function of x and $D \equiv \frac{d}{dx}$ is

(A) $e^{ax} \frac{1}{\phi(D-a)} V$ (B) $e^{ax} \frac{1}{\phi(a)} V$ (C) $e^{ax} \frac{1}{\phi(D+a)} V$ (D) $\frac{1}{\phi(D+a)} V$

13. Particular Integral $\frac{1}{\phi(D)} xV$ where V is a function of x and $D \equiv \frac{d}{dx}$ is

(A) $\left[x - \frac{1}{\phi(D)}\right] \frac{1}{\phi(D)} V$ (B) $\left[x - \frac{\phi'(D)}{\phi(D)}\right] \phi(D) V$

(C) $\left[x + \frac{\phi'(D)}{\phi(D)}\right] V$ (D) $\left[x - \frac{\phi'(D)}{\phi(D)}\right] \frac{1}{\phi(D)} V$

14. Particular integral $\frac{1}{D+1} e^{e^x}$, where $D \equiv \frac{d}{dx}$ is (2)

(A) $e^{-x} e^{e^x}$ (B) e^{e^x} (C) $e^x e^{e^x}$ (D) $e^{-2x} e^{e^x}$

15. Particular Integral $\frac{1}{D+2} e^{-x} e^{e^x}$ where $D \equiv \frac{d}{dx}$ is (2)

(A) $e^{2x} e^{e^x}$ (B) $e^{-2x} e^{e^x}$ (C) e^{e^x} (D) $e^{-x} e^{e^x}$

16. particular Integral $\frac{1}{D+1} \sin e^x$, where $D \equiv \frac{d}{dx}$ is (2)

(A) $-e^{-x} \sin e^x$ (B) $e^x \cos e^x$ (C) $-e^{-x} \cos e^x$ (D) $e^{-x} \cos e^x$

7. Particular Integral $\frac{1}{D+2} e^{-x} \cos e^x$, where $D \equiv \frac{d}{dx}$ is (2)

(A) $e^{-x} \cos e^x$ (B) $e^{-x} \sin e^x$ (C) $e^{-2x} \cos e^x$ (D) $e^{-2x} \sin e^x$

8. Particular Integral $\frac{1}{D+2} e^{-2x} \sec^2 x\,(1 + 2\tan x)$, (use $\tan x = t$ and $D \equiv \frac{d}{dx}$) is (2)

(A) $e^{-2x} (1 + 2\tan^2 x)$

(B) $e^{-2x} (\tan x + \tan^2 x)$

(C) $e^{2x} (\tan x + 2\tan^2 x)$

(D) $e^{-2x} (\tan x + \sec x)$

9. Particular Integral $\frac{1}{D+1}\left(\frac{1}{1+e^x}\right)$ where $D \equiv \frac{d}{dx}$ is (2)

(A) $e^x \log (1 - e^x)$

(B) $\log (1 + e^x)$

(C) $e^x \log (1 + e^x)$

(D) $e^{-x} \log (1 + e^x)$

0. Particular Integral of differential equation $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}$ is (2)

(A) $-\frac{xe^{2x}}{3}$ (B) $-\frac{e^{2x}}{4}$ (C) $\frac{e^{2x}}{4}$ (D) $\frac{e^{2x}}{24}$

. Particular Integral of differential equation $(D^2 - 5D + 6)\, y = 3e^{5x}$ is (2)

(A) $\frac{e^{5x}}{2}$ (B) $\frac{e^{5x}}{6}$ (C) $-\frac{e^{5x}}{14}$ (D) $-\frac{e^{2x}}{2}$

. Particular Integral of differential equation $(D^2 - 9)\, y = e^{3x} + 1$ is (2)

(A) $\frac{3x}{2} e^{3x} - \frac{1}{9}$

(B) $x\frac{e^{3x}}{6} + \frac{3}{8}$

(C) $x\frac{e^{3x}}{6} - \frac{1}{9}$

(D) $xe^{3x} + \frac{1}{8}$

Particular Integral differential equation $(D^2 + 4D + 3)\, y = e^{-3x}$ is (2)

(A) xe^{-3x} (B) $-\frac{1}{2} e^{-3x}$ (C) $-\frac{x}{10} e^{-3x}$ (D) $-\frac{x}{2} e^{-3x}$

Particular Integral of differential equation $(D-2)^3\, y = e^{2x} + 3^x$ is (2)

(A) $\frac{x^3}{3!} e^{2x} + \frac{1}{(\log 3 - 2)^3} 3^x$

(B) $\frac{x^3}{3!} e^{2x} + \frac{1}{(e^3 - 2)^3} 3^x$

(C) $\frac{x}{3!} e^{2x} + \frac{1}{(\log 3 - 2)^3} 3^x$

(D) $\frac{x^3}{3!} e^{2x} + \frac{1}{(\log 3 - 2)^3}$

Particular Integral of differential equation $(D^5 - D)\, y = 12e^x$ is (2)

(A) $3e^x$ (B) $\frac{12}{5} xe^x$ (C) $12xe^x$ (D) $3xe^x$

Particular Integral of differential equation $(D^2 + 1)(D - 1)\, y = e^x$ is (2)

(A) xe^x (B) $\frac{1}{2} x^2 e^x$ (C) $\frac{1}{2} xe^x$ (D) $x^2 e^x$

Particular Integral of differential equation $(D^2 - 4D + 4)\, y = \sin 2x$ is (2)

(A) $-\frac{\cos 2x}{8}$ (B) $\frac{\cos 2x}{8}$ (C) $\frac{\sin 2x}{8}$ (D) $x\frac{\cos 2x}{8}$

28. Particular Integral of differential equation $(D^3 + D)\,y = \cos x$ is (2)

(A) $-\frac{x}{2}\sin x$ (B) $\frac{x}{4}\cos x$ (C) $-\frac{1}{2}\cos x$ (D) $-\frac{x}{2}\cos x$

29. Particular Integral of differential equation $(D^2 + 1)\,y = \sin x$ is (2)

(A) $-\frac{x}{2}\cos x$ (B) $-\frac{x}{4}\cos x$ (C) $-\frac{x}{2}\sin x$ (D) $-\frac{1}{2}\cos x$

30. Particular Integral of differential equation $(D^3 + 9D)\,y = \sin 3x$ is (2)

(A) $-\frac{x}{18}\cos 3x$ (B) $-\frac{x}{18}\sin 3x$ (C) $-x\sin 3x$ (D) $-\frac{1}{18}\sin 3x$

31. Particular integral of differential equation $(D^4 + 10D^2 + 9)\,y = \sin 2x + \cos 4x$ is (2)

(A) $-\frac{1}{23}\sin 2x - \frac{1}{105}\cos 4x$ (B) $\frac{1}{15}\sin 2x + \cos 4x$

(C) $-\frac{1}{15}\sin 2x + \frac{1}{105}\cos 4x$ (D) $-\frac{1}{15}\sin 2x + \frac{1}{87}\cos 4x$

32. Particular Integral of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$ is (2)

(A) $\frac{8}{3}\sin x$ (B) $\sin x - 2\cos x$

(C) $4\sin x + 2\cos x$ (D) $2\sin x + \cos x$

33. Particular Integral of differential equation $(D^4 - m^4)\,y = \cos mx$ is (2)

(A) $\frac{-x}{4m^3}\cos mx$ (B) $\frac{x}{m^3}\sin mx$

(C) $-x\sin mx$ (D) $\frac{-x}{4m^3}\sin mx$

34. Particular Integral of differential equation $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = 2\cosh 2x$ is (2)

(A) $\frac{1}{4}\cosh 2x$ (B) $\frac{x}{8}\cosh 2x$ (C) $\frac{x}{4}\cosh 2x$ (D) $\frac{x}{4}\sinh 2x$

35. Particular Integral of differential equation $(D^2 + 6D - 9)\,y = \sinh 3x$ is (2)

(A) $\frac{1}{18}\cosh 3x$ (B) $\frac{1}{2}\cosh 3x$ (C) $\frac{1}{18}\sinh 3x$ (D) $-\frac{1}{18}\cosh 3x$

36. Particular Integral of differential equation $\frac{d^3y}{dx^3} + 8y = x^4 + 2x + 1$ is (2)

(A) $\frac{1}{8}(x^4 + 5x + 1)$ (B) $\frac{1}{8}(x^3 - 3x^2 + 1)$

(C) $x^4 - x + 1$ (D) $\frac{1}{8}(x^4 - x + 1)$

37. Particular Integral of differential equation $(D^4 + D^2 + 1)\,y = 53x^2 + 17$ is (2)

(A) $53x^2 + 17$ (B) $53x^2 - 89$ (C) $53x^2 + 113$ (D) $3x^2 - 17$

38. Particular integral of differential equation $(D^2 - D + 1)\,y = 3x^2 - 1$ is (2)

(A) $3x^2 + 6x + 5$ (B) $x^2 - 6x + 1$ (C) $3x^2 + 6x - 1$ (D) $x^2 + 18x - 11$

39. Particular Integral of differential equation $(D^2 - 1)\,y = x^3$ is (2)

(A) $-x^3 + 6x$ (B) $x^2 + 6$ (C) $x^3 + 6x$ (D) $-x^3 - 6x$

40. Particular Integral of differential equation $(D^3 + 3D^2 - 4)\,y = x^2$ is (2)

(A) $-\frac{1}{4}\left(x^2 + \frac{3}{2}\right)$ (B) $\frac{1}{4}\left(x^2 + \frac{3}{2}x\right)$ (C) $\left(x^2 + \frac{3}{2}\right)$ (D) $-\frac{1}{4}\left(x^2 - \frac{3}{2}\right)$

41. Particular Integral of differential equation $(D^4 + 25)\, y = x^4 + x^2 + 1$ is (2)

(A) $\left(x^4 + x^2 - \frac{1}{25}\right)$ (B) $\left(x^4 + x^2 + \frac{49}{25}\right)$

(C) $\frac{1}{25}(x^4 + x^2 + 24x + 1)$ (D) $\frac{1}{25}\left(x^4 + x^2 + \frac{1}{25}\right)$

42. Particular Integral of differential equation $(D^2 - 4D + 4)\, y = e^{2x} x^4$ is (2)

(A) $\frac{x^6}{120} e^{2x}$ (B) $\frac{x^6}{60} e^{2x}$ (C) $\frac{x^6}{30} e^{2x}$ (D) $\frac{x^5}{20} e^{2x}$

43. Particular Integral of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x} \cos x$ is (2)

(A) $e^x \cos x$ (B) $-e^{-x} \sin x$ (C) $-e^{-x} \cos x$ (D) $(c_1x + c_2)\, e^{-x}$

44. Particular integral of differential equation $(D^2 + 6D + 9)\, y = e^{-3x} x^{-3}$ is (2)

(A) $\frac{e^{-3x}}{2x}$ (B) $e^{-3x} x$ (C) $\frac{e^{-3x}}{12x}$ (D) $(c_1x + c_2)\, e^{-3x}$

45. Particular Integral of differential equation $(D^2 + 2D + 1)\, y - e^{-x}(1 + x^2)$ is (2)

(A) $e^{-x}\left(\frac{x^2}{2} - \frac{x^4}{12}\right)$ (B) $e^{-x}\left(x + \frac{x^3}{3}\right)$ (C) $e^{-x}\left(\frac{x^2}{2} + \frac{x^4}{12}\right)$ (D) $\left(\frac{x^2}{2} + \frac{x^4}{12}\right)$

46. Particular Integral of differential equation $(D - 1)^4\, y = e^x \sqrt{x}$ is (2)

(A) $\frac{4}{15} e^x x^{5/2}$ (B) $\frac{8}{105} e^x x^{7/2}$ (C) $e^x x^{7/2}$ (D) $\frac{3}{8} e^x x^{-5/2}$

47. Particular integral of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$ is (2)

(A) $-e^x (x \sin x + 2 \cos x)$ (B) $e^x (x \sin x - 2 \cos x)$

(C) $(x \sin x + 2 \cos x)$ (D) $-e^x (x \cos x + 2 \sin x)$

48. Solution of differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{2x}$ is (2)

(A) $e^x\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) - \frac{1}{7}e^{2x}$ (B) $e^{\frac{1}{2}x}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + \frac{1}{5}e^{2x}$

(C) $e^{-\frac{1}{2}x}\left(c_1 \cos \frac{1}{2}x + c_2 \sin \frac{1}{2}x\right) + \frac{1}{7}e^x$ (D) $e^{-\frac{1}{2}x}\left(c_1 \cos \frac{\sqrt{3}}{2}x + c_2 \sin \frac{\sqrt{3}}{2}x\right) + \frac{1}{7}e^{2x}$

49. Solution of differential equation $(D^2 + 1)\, y = x$ is (2)

(A) $c_1 \cos x + c_2 \sin x - x$ (B) $c_1 \cos x + c_2 \sin x + x$

(C) $c_1 \cos x + c_2 \sin x + 2x$ (D) $c_1 \cos x + c_2 \sin x - 2x$

Answers

1. (A)	2. (C)	3. (D)	4. (D)	5. (C)	6. (B)	7. (B)	8. (A)	9. (D)
10. (C)	11. (A)	12. (C)	13. (D)	14. (A)	15. (B)	16. (C)	17. (D)	18. (B)
19. (D)	20. (B)	21. (A)	22. (C)	23. (D)	24. (A)	25. (D)	26. (C)	27. (B)
28. (D)	29. (A)	30.(B)	31. (C)	32. (D)	33.(D)	34. (C)	35. (A)	36. (D)
37. (B)	38. (C)	39. (D)	40. (A)	41. (D)	42. (C)	43. (C)	44. (A)	45. (C)
46. (B)	47. (A)	48. (D)	49. (B)					

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Method of Variation of Parameter : **Marks**

1. Complimentary function of differential equation $a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2y = f(x)$ is $c_1y_1 + c_2y_2$. Then by method of variation of parameters, particular integral is $u(x, y)\, y_1 + v\,(x, y)\, y_2$ where u is obtained from (1)

(A) $\int \frac{f(x)}{y_1y'_2 + y_2y'_1} dx$

(B) $\int \frac{y_1 f(x)}{y_1y'_2 - y_2y'_1} dx$

(C) $\int \frac{y_2 f(x)}{y_1y'_2 - y_2y'_1}$

(D) $\int \frac{-y_2 f(x)}{y_1y'_2 - y_2y'_1} dx$

2. Complementary function of differential equation $a_0 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_2y = f(x)$ is $c_1y_1 + c_2y_2$. Then by method of variation of parameters, particular integral is $u(x, y)\, y_1 + v(x, y)\, y_2$ where v is obtained from (1)

(A) $\int \frac{y_1 f(x)}{y_1y'_2 - y_2y'_1} dx$

(B) $\int \frac{-y_1 f(x)}{y_1y'_2 - y_2y'_1} dx$

(C) $\int \frac{-y_2 f(x)}{y_1y'_2 - y_2y'_1} dx$

(D) $\int \frac{f(x)}{y_1y'_2 + y_2y'_1} dx$

3. In solving differential equation $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ by method of variation of parameters, complimentary function $= c_1 \cos x + c_2 \sin x$, Particular Integral $= u \cos x + v \sin x$ then u is equal to (2)

(A) $-\log \sin x$ (B) x (C) $-x$ (D) $\log \sin x$

4. In solving differential equation $\frac{d^2y}{dx^2} + 4y = \sec 2x$ by method of variation of parameters, complimentary function $= c_1 \cos 2x + c_2 \sin 2x$, Particular Integral $= u \cos 2x + v \sin 2x$ then u is equal to (2)

(A) $-\frac{1}{2}x$

(B) $\frac{1}{4}\log(\cos 2x)$

(C) $-\frac{1}{4}\log(\cos 2x)$

(D) $\left(\frac{1}{2}\right)x$

5. In solving differential equation $\frac{d^2y}{dx^2} - y = (1 + e^{-x})^{-2}$ by method of variation of parameters, complimentary function $= c_1e^x + c_2e^{-x}$, Particular Integral $= ue^x + ve^{-x}$ then u is equal to (2)

(A) $\frac{1}{(1 + e^{-x})}$

(B) $\frac{1}{2(1 + e^{-x})^2}$

(C) $\log(1 + e^x)$

(D) $\frac{1}{2(1 + e^{-x})}$

6. In solving differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin e^x$ by method of variation of parameters, complimentary function $= c_1e^{-x} + c_2e^{-2x}$, Particular Integral $= ue^{-x} + ve^{-2x}$ then u is equal to

(A) $-e^x \cos(e^x) + \sin(e^x)$ (B) $-\cos(e^x)$

(C) $\cos(e^x)$ (D) $e^x \sin(e^x) + \cos(e^x)$

7. In solving differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ by method of variation of parameters, complimentary function $= c_1xe^{3x} + c_2e^{3x}$, Particular integral $= uxe^{3x} + ve^{3x}$ then u is equal to

(A) $-\frac{2}{x^3}$ (B) $\frac{1}{x}$ (C) $-\frac{1}{x}$ (D) $-\log x$

8. In solving differential equation $\frac{d^2y}{dx^2} + y = \tan x$ by method of variation of parameters, complimentary function $= c_1 \cos x + c_2 \sin x$, Particular Integral $= u \cos x + v \sin x$ then v is equal to

(A) $-\cos x$ (B) $[\log(\sec x + \tan x)] - \sin x$

(C) $-[\log(\sec x + \tan x)] + \sin x$ (D) $\cos x$

9. In solving differential equation $\frac{d^2y}{dx^2} + 9y = \frac{1}{1 + \sin 3x}$ by method of variation of parameters, complimentary function $= c_1 \cos 3x + c_2 \sin 3x$, Particular Integral $= u \cos 3x + v \sin 3x$ then v is equal to

(A) $\frac{1}{3}\left(-\frac{1}{3}\sec 3x + \frac{1}{3}\tan 3x - x\right)$ (B) $-\frac{1}{9}\log(1 + \sin 3x)$

(C) $\frac{1}{9}\log(1 + \sin 3x)$ (D) $\frac{1}{3}\log \cos x$

10. In solving differential equation $\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$ by method of variation of parameters, complimentary function $= c_1e^x + c_2e^{-x}$, particular integral $= ue^x + ve^{-x}$ then v is equal to

(A) $e^{-x} - \log(1 + e^{-x})$ (B) $-\log(1 + e^x)$

(C) $\log(1 + e^x)$ (D) $-e^{-x} + \log(1 + e^{-x})$

11. In solving differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$ by method of variation of parameters, complimentary function $c_1e^{-2x} + c_2e^{-x}$, Particular Integral $= ue^{-2x} + ve^{-x}$ then v is equal to

(A) $-e^{e^x}$ (B) $e^{-2x} e^{e^x}$ (C) $e^x e^{e^x}$ (D) e^{e^x}

12. In solving differential equation $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$ by method of variation of parameters, complimentary function $= c_1 \cos 2x + c_2 \sin 2x$,

Particular Integral $= u \cos 2x + v \sin 2x$ then v is equal to

(A) $\log(\sec 2x + \tan 2x)$ (B) $-\sec 2x$

(C) $\sec 2x + \tan 2x$ (D) $\log(\tan 2x)$

Answers

1. (D)	2. (A)	3. (C)	4. (B)	5. (D)	6. (B)	7. (C)	8. (A)
9. (C)	10. (B)	11. (D)	12. (A)				

1.12 EQUATIONS REDUCIBLE TO ... WITH CONSTANT COEFFICIENTS

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Cauchy's and Legendre's Linear Differential Equations : **Marks**

1. The general form of Cauchy's linear differential equation is (1)

(A) $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2, \ldots, a_n$ are constants.

(B) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are functions of x, y, z.

(C) $a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots a_n$ are constants

(D) $a_0 (ax + b)^n \frac{d^n y}{dx^n} + a_1 (ax + b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 (ax + b)^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant.

2. Cauchy's linear differential equation $a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$ can be reduced to linear differential equation with constant coefficients by using substitution (1)

(A) $x = e^z$ (B) $y = e^z$ (C) $x = \log z$ (D) $x = e^{z^2}$

3. The general form of Legendre's linear differential equation is (1)

(A) $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant.

(B) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are functions of x, y, z.

(C) $a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant

(D) $a_0 (ax + b)^n \frac{d^n y}{dx^n} + a_1 (ax + b)^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 (ax + b)^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2, \ldots, a_n$ are constant.

4. Legendre's linear differential equation $a_0 (ax + b)^n \dfrac{d^n y}{dx^n} + a_1 (ax + b)^{n-1} \dfrac{d^{n-1} y}{dx^{n-1}} + a_2 (ax + b)^{n-2} \dfrac{d^{n-2} y}{dx^{n-2}} + \ldots + a_n y = f(x)$ can be reduced to linear differential equation with constant coefficients by using substitution

(A) $x = e^z$ (B) $ax + b = e^z$ (C) $ax + b = \log z$ (D) $ax + b = e^{z^2}$

5. To reduce the differential equation $x^2 \dfrac{d^2y}{dx^2} - 4x \dfrac{dy}{dx} + 6y = x^4$ to linear differential equation with constant coefficients, substitutions is

(A) $x = z^2 + 1$ (B) $x = e^z$ (C) $x = \log z$ (D) $x^2 = \log z$

6. To reduce the differential equation $(x + 2)^2 \dfrac{d^2y}{dx^2} - (x + 2) \dfrac{dy}{dx} + y = 4x + 7$ to linear differential equation with constant coefficients, substitution is (1)

(A) $x + 2 = e^{-z}$ (B) $x = z + 1$ (C) $x + 2 = e^z$ (D) $x + 2 = \log z$

7. To reduce the differential equation $(3x + 2)^2 \dfrac{d^2y}{dx^2} + 3 (3x + 2) \dfrac{dy}{dx} - 36y = x^2 + 3x + 1$ to linear differential equation with constant coefficients, substitution is (1)

(A) $3x + 2 = e^z$ (B) $3x + 2 = z$ (C) $x = e^z$ (D) $3x + 2 = \log z$

8. On putting $x = e^z$ and using $D \equiv \dfrac{d}{dz}$ the differential equation $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = x$ is transformed into (1)

(A) $(D^2 - 1)\, y = e^z$ (B) $(D^2 + 1)\, y = e^z$

(C) $(D^2 + 1)\, y = x$ (D) $(D^2 + D + 1)\, y = e^z$

9. The differential equation $x^2 \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} + 4y = \cos (\log x) + x \sin (\log x)$, on putting $x = e^z$ and using $D \equiv \dfrac{d}{dz}$ is transformed into (1)

(A) $(D^2 - D + 4)\, y = \sin z + e^z \cos z$ (B) $(D^2 - 2D + 4)\, y = \cos (\log x) + x \sin (\log x)$

(C) $(D^2 + 2D + 4)\, y = \cos z + e^{-z} \sin z$ (D) $(D^2 - 2D + 4)\, y = \cos z + e^z \sin z$

10. On putting $x = e^z$ the transformed differential equation of $x^2 \dfrac{d^2y}{dx^2} - 3x \dfrac{dy}{dx} + 5y = x^2 \sin (\log x)$ using $D \equiv \dfrac{d}{dz}$ is

(A) $(D^2 - 4D + 5)\, y = e^{2z} \sin z$ (B) $(D^2 - 4D + 5)\, y = x^2 \sin (\log x)$

(C) $(D^2 - 4D - 4)\, y = e^z \sin z$ (D) $(D^2 - 3D + 5)\, y = e^{z^2} \sin z$

11. The differential equation $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} - y = \dfrac{x^3}{1 + x^2}$, on putting $x = e^z$ and using $D \equiv \dfrac{d}{dz}$ is transformed into

(A) $(D^2 - 1)\, y = \dfrac{x^3}{1 + x^2}$ (B) $(D^2 - 2D - 1)\, y = \dfrac{e^{3z}}{1 + e^{2z}}$

(C) $(D^2 - 1)\, y = \dfrac{e^{3z}}{1 + e^{2z}}$ (D) $(D^2 - 1)\, y = \dfrac{e^{z^3}}{1 + e^{z^2}}$

12. The differential equation $x^2 \dfrac{d^2y}{dx^2} - 5x \dfrac{dy}{dx} + 5y = x^2 \log x$, on putting $x = e^z$ and using $D \equiv \dfrac{d}{dz}$ is transformed into

(A) $(D^2 - 5D + 5)\, y = z\, e^{z^2}$ (B) $(D^2 - 5D - 5)\, y = e^{2z} z$

(C) $(D^2 - 6D + 5)\, y = x^2 \log x$ (D) $(D^2 - 6D + 5)\, y = z\, e^{2z}$

13. The differential equation $(2x + 1)^2 \dfrac{d^2y}{dx^2} - 2 (2x + 1) \dfrac{dy}{dx} - 12y = 6x$, on putting $2x + 1 = e^z$ and putting $D \equiv \dfrac{d}{dz}$ is transformed into

(A) $(D^2 - 2D - 3)\, y = \dfrac{3}{4} (e^z - 1)$ (B) $(D^2 + 2D + 3)\, y = 3 (e^z - 1)$

(C) $(D^2 + 2D - 12)\, y = \dfrac{3}{4} (e^z - 1)$ (D) $(D^2 - 2D - 3)\, y = 6x$

14. The differential equation $(3x + 2)^2 \frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = \frac{1}{3}[(3x + 2)^2 - 1]$. On putting $3x + 2 = e^z$ and using $D \equiv \frac{d}{dz}$ is transformed into (1)

(A) $(D^2 + 3D - 36)\, y = \frac{1}{27}(e^{2z} - 1)$

(B) $(D^2 + 4)\, y = \frac{1}{9}(e^{2z} - 1)$

(C) $(D^2 - 4)\, y = \frac{1}{27}(e^{2z} - 1)$

(D) $(D^2 - 9)\, y = (e^{2z} - 1)$

15. The differential equation $(1 + x)^2 \frac{d^2y}{dx^2} + 3(1 + x)\frac{dy}{dx} - 36y = 4 \cos[\log(1 + x)]$ on putting $1 + x = e^z$ and using $D \equiv \frac{d}{dz}$ is transformed into (1)

(A) $(D^2 + 2D - 36)\, y = 4 \cos[\log(1 + x)]$

(B) $(D^2 + 2D - 36)\, y = 4 \cos z$

(C) $(D^2 + 3D - 36)\, y = 4 \cos z$

(D) $(D^2 - 2D - 36)\, y = 4 \cos(\log z)$

16. The differential equation $(4x + 1)^2 \frac{d^2y}{dx^2} + 2(4x + 1)\frac{dy}{dx} + 2y - 2x + 1$ on putting $4x + 1 = e^z$ and using $D \equiv \frac{d}{dz}$ is transformed into (1)

(A) $(D^2 + D + 2)\, y = \frac{1}{2}(e^z + 1)$

(B) $(16D^2 + 8D + 2)\, y = (e^z + 1)$

(C) $(16D^2 - 8D + 2)\, y = \frac{1}{2}(e^z + 1)$

(D) $(D^2 + 2D + 2)\, y = (e^z - 1)$

17. The differential equation $(x + 2)^2 \frac{d^2y}{dx^2} + 3(x + 2)\frac{dy}{dx} + y = 4 \sin[\log(x + 2)]$ on putting $x + 2 = e^z$ and using $D \equiv \frac{d}{dz}$ is transformed into (1)

(A) $(D^2 + 3D + 1)\, y = 4 \sin(\log z)$

(B) $(D^2 + 1)\, y = 4 \sin z$

(C) $(D^2 + 2D + 1)\, y = 4 \sin[\log(x + 2)]$

(D) $(D^2 + 2D + 1)\, y = 4 \sin z$

18. For the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x^2 + x^{-2}$, complimentary function is given by (2)

(A) $c_1x + c_2$

(B) $c_1 \log x + c_2$

(C) $c_1 \cos x + c_2 \sin x$

(D) $c_1 \cos(\log x) + c_2 \sin(\log x)$

19. For the differential equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = A + B \log x$, complimentary function is given by (2)

(A) $c_1x + c_2$

(B) $c_1x^2 + c_2$

(C) $c_1 \log x + c_2$

(D) $\frac{c_1}{x} + c_2$

20. For the differential equation $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x^5$, complimentary function is given by (2)

(A) $c_1x^2 + c_2x^3$

(B) $c_1x^2 + c_2x$

(C) $c_1x^{-2} + c_2x^{-3}$

(D) $c_1x^5 + c_2x$

21. For the differential equation $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$, complimentary function is given by (2)

(A) $[c_1 \cos\sqrt{3}(\log x) + c_2 \sin\sqrt{3}(\log x)]$

(B) $x\,[c_1 \cos\sqrt{2}(\log x) + c_2 \sin\sqrt{2}(\log x)]$

(C) $x\,[c_1 \cos(\log x) + c_2 \sin(\log x)]$

(D) $x\,[c_1 \cos\sqrt{3}(\log x) + c_2 \sin\sqrt{3}(\log x)]$

22. For the differential equation $r^2 \frac{d^2u}{dr^2} + r\frac{du}{dr} - u = -kr^3$, complimentary function is given by (2)

(A) $(c_1 \log r + c_2)\, r$

(B) $c_1r + \frac{c_2}{r}$

(C) $[c_1 \cos(\log r) + c_2 \sin(\log r)]$

(D) $c_1r^2 + \frac{c_2}{r^2}$

23. For the differential equation $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + y = x$, particular integral is given by (2)

(A) x (B) $\dfrac{x}{2}$ (C) $\dfrac{x}{3}$ (D) $2x$

24. For the differential equation $x^2 \dfrac{d^2y}{dx^2} - 4x \dfrac{dy}{dx} + 6y = x^5$, particular integral is given by (2)

(A) $\dfrac{x^5}{6}$ (B) $\dfrac{x^5}{56}$ (C) $\dfrac{x^4}{6}$ (D) $-\dfrac{x^5}{44}$

25. Solution of differential equation $x \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x$ is (2)

(A) $(c_1 x + c_2) - \dfrac{x^2}{4}$ (B) $(c_1 x^2 + c_2) + \dfrac{x^2}{4}$

(C) $(c_1 \log x + c_2) - \dfrac{x^2}{4}$ (D) $(c_1 \log x + c_2) + \dfrac{x^2}{4}$

26. Solution of differential equation $x^2 \dfrac{d^2y}{dx^2} + 2x \dfrac{dy}{dx} = \dfrac{1}{x^2}$ is (2)

(A) $(c_1 x + c_2) - \dfrac{x^2}{4}$ (B) $(c_1 x^2 + c_2) + \dfrac{x^2}{4}$

(C) $c_1 + c_2 \dfrac{1}{x} + \dfrac{1}{2x^2}$ (D) $(c_1 \log x + c_2) + \dfrac{x^2}{4}$

27. For the differential equation $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} + y = 2 \sin[\log(x+1)]$, complimentary function is given by (2)

(A) $c_1 (x+1) + c_2 (x+1)^{-1}$ (B) $c_1 \cos[\log(x+1)] + c_2 \sin[\log(x+1)]$

(C) $[c_1 \log(x+1) + c_2](x+1)$ (D) $c_1 \cos(\log x) + c_2 \sin(\log x)$

28. For the differential equation $(2x+3)^2 \dfrac{d^2y}{dx^2} - 2(2x+3)\dfrac{dy}{dx} - 12y = 6x$, complimentary function is given by (2)

(A) $c_1 (2x+3)^3 + c_2 (2x+3)^{-1}$ (B) $c_1 (2x+3)^{-3} + c_2 (2x+3)$

(C) $c_1 (2x+3)^3 + c_2 (2x+3)^2$ (D) $c_1 (2x-3)^2 + c_2 (2x-3)^{-1}$

29. For the differential equation $(3x+2)^2 \dfrac{d^2y}{dx^2} + 3(3x+2)\dfrac{dy}{dx} - 36y = (3x+2)^2$, complimentary function is given by (2)

(A) $c_1 (3x+2)^3 + c_2 (3x+2)^{-3}$ (B) $[c_1 \log(3x+2) + c_2](3x+2)^{-2}$

(C) $c_1 (3x+2)^2 + c_2 (3x+2)^{-2}$ (D) $c_1 (3x-2)^2 + c_2 (3x-2)^{-2}$

30. For the differential equation $(x+2)^2 \dfrac{d^2y}{dx^2} - (x+2)\dfrac{dy}{dx} + y = (3x+6)$, complimentary function is given by (2)

(A) $c_1 (x+2) + c_2 (x+2)^{-1}$ (B) $c_1 \log(x+2) + c_2$

(C) $c_1 (x-2) + c_2 (x-2)^{-1}$ (D) $[c_1 \log(x+2) + c_2](x+2)$

Answers

1. (C)	2. (A)	3. (D)	4. (B)	5. (B)	6. (C)	7. (A)	8. (B)
9. (D)	10. (A)	11. (C)	12. (D)	13. (A)	14. (C)	15. (B)	16. (C)
17. (D)	18. (D)	19. (C)	20. (A)	21. (D)	22. (B)	23. (B)	24. (A)
25. (D)	26. (C)	27. (B)	28. (A)	29. (C)	30. (D)		

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Simultaneous Linear Differential Equations :

Mark

1. For the simultaneous linear differential equations

$\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dx} - 3x + 2y = e^{2t}$ solution of x using $D \equiv \frac{d}{dt}$ is obtain from

(A) $(D^2 + 4D - 5)\, x = 1 + 2t + 3e^{2t}$

(B) $(D^2 - 4D - 5)\, x = 1 + 2t - 3e^{2t}$

(C) $(D^2 + 4D - 5)\, x = 3t + 3e^{2t}$

(D) $(D^2 + 4D - 5)\, y = 3t + 4e^{2t}$

2. For the system of linear differential equations $\frac{dx}{dt} + 2x - 3y = t$, $\frac{dy}{dt} - 3x + 2y = e^{2t}$ elimination of x results in $\left(\text{use } D \equiv \frac{d}{dt}\right)$ (2)

(A) $(D^2 + 4D - 5)\,x = 1 + 2t + 3e^{2t}$
(B) $(D^2 - 4D - 5)\,y - t - 4e^{2t}$
(C) $(D^2 - 4D + 5)\,y = 3t - 2e^{2t}$
(D) $(D^2 + 4D - 5)\,y = 3t + 4e^{2t}$

3. For the simultaneous Linear DE $\frac{du}{dx} + v = \sin x$, $\frac{dv}{dx} + u = \cos x$ solution of u using $D \equiv \frac{d}{dx}$ is obtain from

(A) $(D^2 + 1)\,u = 2\cos x$
(B) $(D^2 - 1)\,u = 0$
(C) $(D^2 - 1)\,u = \sin x - \cos x$
(D) $(D^2 - 1)\,v = -2\sin x$

4. For the simultaneous Linear DE $\frac{du}{dx} + v = \sin x$, $\frac{dv}{dx} + u = \cos x$ eliminating u results in $\left(\text{use } D \equiv \frac{d}{dx}\right)$ (2)

(A) $(D^2 + 1)\,v = 0$
(B) $(D^2 - 1)\,u = 0$
(C) $(D^2 - 1)\,v = -2\sin x$
(D) $(D^2 + 1)\,v = \sin x + \cos x$

5. For the simultaneous Linear DE $\frac{dx}{dt} - 3x - 6y = t^2$, $\frac{dy}{dt} + \frac{dx}{dt} - 3y = e^t$ solution of x using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(D^2 + 9)\,x = 6e^t - 3t^2 + 2t$
(B) $(D^2 + 9)\,y = -2e^t - 2t$
(C) $(D^2 - 9)\,x = 6e^t - 3t^2$
(D) $(D^2 + 12D + 9)\,x = 6e^t + 3t^2 + 2t$

6. For the simultaneous Linear DE $L\frac{dx}{dt} + Rx + R(x - y) = E$, $L\frac{dy}{dt} + Ry - R(x - y) = 0$ where L, R and E are constants, solution of x using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(L^2D^2 + 4RLD + 5R^2)\,x = 2RE + 2R$
(B) $(L^2D^2 + 4RLD + 3R^2)\,y = RE$
(C) $(L^2D^2 + 4RLD + 3R^2)\,x = 2RE$
(D) $(L^2D^2 + 2RLD + 5R^2)\,x = 2RE$

7. For the simultaneous Liner DE $L\frac{dx}{dt} + Rx + R(x - y) = E$, $L\frac{dy}{dt} + Ry - R(x - y) = 0$ where L, R and E are constants, solution of y using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(L^2D^2 + 4RLD + 5R^2)\,y = RE + 2R$
(B) $(L^2D^2 + 4RLD + 3R^2)\,y = RE$
(C) $(L^2D^2 + 4RLD + 3R^2)\,x = 2RE$
(D) $(L^2D^2 + 2RLD + 5R^2)\,y = 2RE$

8. For the simultaneous Linear DE $\frac{dx}{dt} + y = e^t$, $\frac{dy}{dt} + x = e^{-t}$ solution of x using $D = \frac{d}{dt}$ is obtain from (2)

(A) $(D^2 - 1)\,x = 2e^t$
(B) $(D^2 - 1)\,y = -e^t - e^{-t}$
(C) $(D^2 + 1)\,x = e^{-t} + e^t$
(D) $(D^2 - 1)\,x = e^t - e^{-t}$

9. From the simultaneous Linear DE $\frac{dx}{dt} + y = e^t$, $\frac{dy}{dt} + x = e^{-t}$, solution of y using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(D^2 - 1)\,y = 2e^t$
(B) $(D^2 - 1)\,y = -e^t - e^{-t}$
(C) $(D^2 + 1)\,y = e^{-t} + e^t$
(D) $(D^2 - 1)\,x = e^t - e^{-t}$

10. For the simultaneous Linear DE $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$, solution of x using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(D^2 + 6D + 9)\,x = 1 + t$
(B) $(D^2 - 6D + 9)\,x = 2t$
(C) $(D^2 + 6D + 1)\,x = t$
(D) $(D^2 + 6D + 9)\,y = 2t$

11. For the simultaneous Linear DE $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$, solution of y using $D \equiv \frac{d}{dt}$ is obtain from (2)

(A) $(D^2 - 6D - 9)\,y = 2t$
(B) $(D^2 + 6D + 9)\,x = 1 + t$
(C) $(D^2 + 6D + 1)\,y = t$
(D) $(D^2 + 6D + 9)\,y = -2t$

Answers

1. (A)	2. (D)	3. (B)	4. (C)	5. (A)	6. (C)	7. (B)	8. (D)
9. (B)	10. (A)	11. (D)					

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Mar

Type : Symmetrical Simultaneous Differential Equations :

1. The general form of symmetric simultaneous DE is

(A) $a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant

(B) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, where P, Q, R are function of x, y, z

(C) $a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant

(D) $a_0 (ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1}y}{dx^{n-1}} + a_2 (ax+b)^{n-2} \frac{d^{n-2}y}{dx^{n-2}} + \ldots + a_n y = f(x)$, where $a_0, a_1, a_2 \ldots, a_n$ are constant

2. Solution of symmetric simultaneous DE $\frac{dx}{1} = \frac{dy}{1} = \frac{dz}{1}$ is

(A) $x + y = 0,\ y + z = 0$

(B) $x - y = c_1,\ y + z = c_2$

(C) $x + y = c_1,\ y - z = c_2$

(D) $x - z = c_1,\ y - z = c_2$

3. Solution of symmetric simultaneous DE $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is

(A) $x = c_1 y,\ y = c_2 z$

(B) $xy = c_1 z,\ yz = c_2 x$

(C) $x + y = c_1,\ y + z = c_2$

(D) $x + y = c_1,\ y - z = c_2$

4. Considering the first two ratio of the symmetrical simultaneous DE $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$, one of the relation in the solution is DE

(A) $\frac{1}{x} - \frac{1}{y} = c$

(B) $x - y = c$

(C) $x^2 - y^2 = c$

(D) $x^3 - y^3 - c$

5. Considering the first two ratio of the symmetrical simultaneous DE $\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z - 2y)}$, one of the relation in the solution of DE is (2)

(A) $x^2 + y^2 = c$ (B) $x^3 + y^3 = c$ (C) $-\frac{x^2}{2} = \frac{y^3}{3} + c$ (D) $x^2 - y^2 = c$

6. Considering the first two ratio of the symmetrical simultaneous DE $\frac{dx}{y^2z} = \frac{dy}{x^2z} = \frac{dz}{y^2x}$, one of the relation in the solution of DE is (2)

(A) $x^2 - y^2 = c$ (B) $x - y = c$ (C) $x^3 - y^3 = c$ (D) $x^3 + y^3 = c$

7. Considering the first and third ratio of the symmetrical simultaneous DE $\frac{xdx}{y^3z} = \frac{dy}{x^2z} = \frac{dz}{y^3}$, one of the relation in the solution of DE is (2)

(A) $x^2 - z^2 = c$ (B) $x^4 - y^4 = c$ (C) $x^3 - z^3 = c$ (D) $x - z = c$

8. Considering the second and third ratio of the symmetrical simultaneous DE $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$, one of the relation in the solution of DE is (2)

(A) $\frac{1}{y^2} - \frac{1}{z^2} = c$ (B) $y^2 - z^2 = c$ (C) $y = cz$ (D) $x - z = c$

9. Using a set of multiplier as 1, 1, 1 the solution of DE $\frac{dx}{y - z} = \frac{dy}{z - x} = \frac{dz}{x - y}$ is (2)

(A) $x^2 + y^2 + z^2 = c$ (B) $x - y - z = c$

(C) $x + y + z = c$ (D) $-x + y - z = c$

10. Using a set of multiplier as x, y, z the solution of DE $\frac{dx}{3z - 4y} = \frac{dy}{4x - 2z} = \frac{dz}{2y - 3x}$ is (2)

(A) $x^3 + y^3 + z^3 = c$ (B) $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = c$

(C) $x + y + z = c$ (D) $x^2 + y^2 + z^2 = c$

11. Using a set of multiplier as x^3, y^3, z^3 the solution of DE $\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}$ is (2)

(A) $x^3 + y^3 + z^3 = c$ (B) $x^4 + y^4 + z^4 = c$

(C) $x + y + z = c$ (D) $xyz = c$

12. Using a set of multiplier as 3, 2, 1 the solution of DE $\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{2x - 3y}$ is (2)

(A) $3x^2 + 2y^2 + z^2 = c$ (B) $\frac{3}{x} + \frac{2}{y} + \frac{1}{z} = c$

(C) $3x - 2y - z = c$ (D) $3x + 2y + z = c$

13. Using a set of multiplier as 1, y, z the solution of DE $\frac{dx}{z^2 - 2yz - y^2} = \frac{dy}{y + z} = \frac{dz}{y - z}$ is (2)

(A) $x^2 + y^2 + z^2 = c$ (B) $x + \frac{y^2}{2} + \frac{z^2}{2} = c$

(C) $x + y + z = c$ (D) $x + y^2 + z^2 = c$

Answers

1. (B)	2. (D)	3. (A)	4. (D)	5. (A)	6. (C)	7. (A)	8. (C)
9. (C)	10. (D)	11. (B)	12. (D)	13. (B)			

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type I : Fourier Integral Representation Fourier Transform and Inverse Fourier Transform **Marks**

1. The fourier integral representation of f(x) defined in the interval $-\infty < x < \infty$ is (1)

(A) $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda(u-x)}\, du\, d\lambda$ (B) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda(u-x)}\, du\, d\lambda$

(C) $\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{i\lambda u}\, du\, dx$ (D) $\frac{2}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)\, e^{i\lambda(u-x)}\, du\, d\lambda$

2. The Fourier transform $F(\lambda)$ of function f(x) defined in the interval $-\infty < x < \infty$ is (1)

(A) $\int_{-\infty}^{\infty} f(u)\, e^{iu}\, du$ (B) $\int_{-\infty}^{\infty} f(u)\, e^{-\lambda u}\, du$

(C) $\int_{-\infty}^{\infty} f(u)\, e^{-i\lambda u}\, du$ (D) $\int_{0}^{\infty} f(u)\, e^{-i\lambda u}\, du$

3. The inverse Fourier transform f(x) defined in $-\infty < x < \infty$ of $F(\lambda)$ is (1)

(A) $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{i\lambda x}\, d\lambda$ (B) $\frac{2}{\pi}\int_{-\infty}^{\infty} F(\lambda)\, e^{-i\lambda x}\, d\lambda$

(C) $\frac{1}{2\pi}\int_{-\infty}^{0} F(\lambda)\, e^{ix}\, d\lambda$ (D) $\frac{1}{2\pi}\int_{0}^{\infty} F(\lambda)\, e^{i\lambda x}\, dx$

4. In the Fourier integral representation of $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1-i\lambda}{1+\lambda^2}\right) e^{i\lambda x}\, d\lambda = \begin{cases} 0 & , x < 0 \\ e^{-x} & , x > 0 \end{cases}$, $F(\lambda)$ is (1)

(A) $\frac{1+\lambda^2}{1-i\lambda}$ (B) $\frac{\sin\lambda}{1+\lambda^2}$ (C) $\frac{\cos\lambda}{1+\lambda^2}$ (D) $\frac{1-i\lambda}{1+\lambda^2}$

5. In the Fourier integral representation of $\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{e^{-i\lambda\pi}+1}{1-\lambda^2}\right) e^{i\lambda x}\, d\lambda = \begin{cases} \sin x, & 0 < x < \pi \\ 0 & , x < 0 \text{ and } x > \pi \end{cases}$, $F(\lambda)$ is (1)

(A) $\frac{1+\lambda^2}{1-i\lambda}$ (B) $\frac{e^{-i\lambda}}{1-\lambda^2}$ (C) $\frac{e^{-i\lambda\pi}+1}{1-\lambda^2}$ (D) $\frac{\sin\lambda}{1-\lambda^2}$

6. In the Fourier integral representation $\frac{1}{2\pi}\int_{-\infty}^{\infty}\pi\left(\frac{1-i\lambda}{1+\lambda^2}\right) e^{i\lambda x}\, d\lambda = \begin{cases} 0 & , x < 0 \\ e^{-\pi} & , x > 0 \end{cases}$, $F(\lambda)$ is (2)

(A) $\frac{1+\lambda^2}{1-i\lambda}$ (B) $\frac{\sin\lambda}{1+\lambda^2}$ (C) $\frac{\cos\lambda}{1+\lambda^2}$ (D) $\pi\frac{1-i\lambda}{1+\lambda^2}$

7. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $i\lambda$ (B) $\frac{1}{i\lambda}$ (C) $\frac{1}{\lambda}$ (D) λ

8. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ is (2)

(A) $\frac{2 \sin \lambda a}{\lambda}$ (B) $\frac{e^{-i\lambda a}}{\lambda}$ (C) $\frac{e^{i\lambda a}}{\lambda}$ (D) $\frac{2 \cos \lambda a}{\lambda}$

9. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $\frac{1 - \lambda}{1 + \lambda^2}$ (B) $\frac{1 - i\lambda}{1 + \lambda^2}$ (C) $\frac{1 - i\lambda}{1 - \lambda^2}$ (D) $\frac{1}{1 + \lambda^2}$

10. The Fourier transform $F(\lambda)$ of $f(x) = e^{-|x|}$ is given by (2)

(A) $\frac{3}{1 + \lambda^2}$ (B) $\frac{1}{1 - \lambda^2}$ (C) $\frac{2}{1 - \lambda^2}$ (D) $\frac{2}{1 + \lambda^2}$

11. If $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x < 0 \text{ and } x > \pi \end{cases}$ then Fourier transform $F(\lambda)$ of $f(x)$ is

(A) $\frac{e^{i\lambda\pi} + 1}{1 + \lambda^2}$ (B) $\frac{e^{i\lambda\pi} + 1}{1 - \lambda^2}$ (C) $\frac{e^{-i\lambda\pi} + 1}{1 - \lambda^2}$ (D) $\frac{e^{-i\lambda\pi} + 1}{1 + \lambda^2}$

12. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} \cos x, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $\frac{i\lambda}{1 - \lambda^2}$ (B) $-\frac{i\lambda}{1 - \lambda^2}$ (C) $-\frac{i\lambda}{1 + \lambda^2}$ (D) $\frac{i\lambda}{1 + \lambda^2}$

13. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} \sin x, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $\frac{1}{1 - \lambda^2}$ (B) $\frac{1}{1 + \lambda^2}$ (C) $\frac{i\lambda}{1 - \lambda^2}$ (D) $\frac{i\lambda}{1 + \lambda^2}$

14. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) 0 (B) $\frac{1}{\lambda^2}$ (C) λ^2 (D) $-\frac{1}{\lambda^2}$

15. If $f(x) = \begin{cases} 2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ then Fourier transform $F(\lambda)$ of $f(x)$ is given by (2)

(A) $\frac{4 \cos \lambda}{\lambda^2}$ (B) $\frac{4 \sin \lambda}{\lambda}$ (C) $\frac{2 \sin 2\lambda}{\lambda}$ (D) $\frac{\sin \lambda}{\lambda}$

16. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x^2, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $-\frac{2i}{\lambda^3}$ (B) $\frac{1}{i\lambda^3}$ (C) $\frac{2i}{\lambda^3}$ (D) $-\frac{1}{i\lambda^3}$

17. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} x - x^2, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $\frac{2}{\lambda^2} + i\frac{1}{\lambda^3}$ (B) $\frac{1}{\lambda^2} - i\frac{2}{\lambda^3}$ (C) $\frac{1}{\lambda^2} + i\frac{2}{\lambda^3}$ (D) $-\frac{1}{\lambda^2} - i\frac{2}{\lambda^3}$

18. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ is (2)

(A) $-\frac{4}{\lambda^3}(\sin \lambda - \lambda \cos \lambda)$ (B) $\frac{4}{\lambda^3}(\sin \lambda - \lambda \cos \lambda)$

(C) $\frac{4}{\lambda^2}(\sin \lambda - \lambda \cos \lambda)$ (D) $\frac{4}{\lambda^3}(\sin \lambda + \lambda \cos \lambda)$

19. The Fourier transform $F(\lambda)$ of $f(x) = \begin{cases} 2 + x, & x > 0 \\ 0, & x < 0 \end{cases}$ is (2)

(A) $-\frac{1}{\lambda^2} - i\frac{2}{\lambda}$ (B) $\frac{1}{\lambda^2} - i\frac{2}{\lambda}$ (C) $\frac{1}{\lambda^2} + i\frac{2}{\lambda}$ (D) $-\frac{1}{\lambda^2} + i\frac{2}{\lambda}$

20. The inverse Fourier transform, f(x) defined in $-\infty < x < \infty$ of $F(\lambda) = \left[\frac{1 - i\lambda}{1 + \lambda^2}\right]$ is

(A) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 + \lambda^2} \right] d\lambda$

(B) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\cos \lambda x - \lambda \sin \lambda x}{1 + \lambda^2} + i \frac{-\lambda \cos \lambda x - \sin \lambda x}{1 + \lambda^2} \right] d\lambda$

(C) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\cos \lambda x + \lambda \sin \lambda x}{1 + \lambda^2} + i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 + \lambda^2} \right] d\lambda$

(D) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\cos \lambda x + \lambda \sin \lambda x}{1 - \lambda^2} + i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 - \lambda^2} \right] d$

21. The inverse Fourier transform f(x) defined in $-\infty < x < \infty$ of $F(\lambda) = \pi \left[\frac{1 - i\lambda}{1 + \lambda^2}\right]$ is

(A) $\frac{1}{2} \int_{0}^{\infty} \left[\frac{\cos \lambda x + \lambda \sin \lambda x}{1 + \lambda^2} + i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 + \lambda^2} \right] d\lambda$

(B) $\frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{\cos \lambda x + \lambda \sin \lambda x}{1 + \lambda^2} + i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 + \lambda^2} \right] d$

(C) $\frac{1}{2} \int_{-\infty}^{\infty} \left[i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 + \lambda^2} \right] d\lambda$

(D) $\frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{\cos \lambda x + \lambda \sin \lambda x}{1 - \lambda^2} + i \frac{-\lambda \cos \lambda x + \sin \lambda x}{1 - \lambda^2} \right]$

22. The inverse Fourier transform f(x) defined in $-\infty < x < \infty$ of $F(\lambda) = \frac{e^{-i\lambda\pi} + 1}{1 - \lambda^2}$ is

(A) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{1 + \cos \lambda x}{1 - \lambda^2} \right] (\cos \lambda x + i \sin \lambda x)\, d\lambda$

(B) $\frac{1}{2\pi} \int_{0}^{\infty} \left[\frac{(1 + \cos \lambda x) - i \sin \lambda\pi}{1 - \lambda^2} \right] (\cos \lambda x + i \sin \lambda$

(C) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{(1 + \cos \lambda\pi) - i \sin \lambda\pi}{1 - \lambda^2} \right] (\cos \lambda x + i \sin \lambda x)\, d\lambda$

(D) $\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\sin \lambda\pi}{1 - \lambda^2} \right] (\cos \lambda x + i \sin \lambda x)\, d\lambda$

23. If the Fourier integral representation of f(x) is $\frac{2}{\pi} \int_0^{\infty} \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ then value of integral $\int_0^{\infty} \frac{\sin \lambda}{\lambda} d\lambda$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) 0 (D) 1

24. If the Fourier integral representation of f(x) is

$$\frac{1}{\pi} \int_0^{\infty} \frac{\cos \lambda x + \cos [\lambda(\pi - x)]}{1 - \lambda^2} d\lambda = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x < 0 \text{ and } x > \pi \end{cases}$$ then value of the integral $\int_0^{\infty} \frac{\cos \frac{\lambda\pi}{2}}{1 - \lambda^2} d\lambda$ is

(A) $\frac{\pi}{4}$ (B) 1 (C) 0 (D) $\frac{\pi}{2}$

Answers

1. (A)	2. (C)	3. (A)	4. (D)	5. (C)	6. (D)	7. (B)	8. (
9. (B)	10. (D)	11. (C)	12. (A)	13. (A)	14. (D)	15. (B)	16.
17. (D)	18. (B)	19. (A)	20. (C)	21. (B)	22. (C)	23. (B)	24.

ype II : Fourier Sine and Cosine Integral Representations, Transform and Inverse Transform

.. The Fourier consine integral representation of an even function f(x) defined in the interval $-\infty < x < \infty$ is

(A) $\int_0^{\infty} \int_0^{\infty} f(u) \cos \lambda u \sin \lambda x \, du \, d\lambda$

(B) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(u) \cos \lambda u \cos \lambda x \, du \, d\lambda$

(C) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(u) \sin \lambda u \cos \lambda x \, du \, d\lambda$

(D) $\frac{2}{\pi} \int_0^{\infty} \int_0^{\infty} f(u) \sin \lambda u \sin \lambda x \, du \, d\lambda$

(2)

2. The Fourier sine integral representation of an odd function f(x) defined in the interval $-\infty < x < \infty$ is

(A) $\int_0^\infty \int_0^\infty f(u) \sin \lambda u \cos \lambda x \, du \, d\lambda$ (B) $\int_0^\infty \int_0^\infty f(u) \cos \lambda u \sin \lambda x \, du \, d\lambda$

(C) $\frac{2}{\pi} \int_0^\infty \int_0^\infty f(u) \cos \lambda u \cos \lambda x \, du \, d\lambda$ (D) $\frac{2}{\pi} \int_0^\infty \int_0^\infty f(u) \sin \lambda u \sin \lambda x \, du \, d\lambda$

3. The Fourier cosine transform $F_c(\lambda)$ of an even function f(x) defined in the interval $-\infty < x < \infty$ is

(2)

(A) $\int_0^\infty f(u) \sec \lambda u \, du$ (B) $\int_0^\infty f(u) \cos \lambda u \, d\lambda$

(C) $\int_0^\infty f(u) \cos \lambda u \, du$ (D) $\int_0^\infty f(u) \sin \lambda u \, du$

4. The Fourier sine transform $F_s(\lambda)$ of an odd function f(x) defined in the interval $-\infty < x < \infty$ is

(2)

(A) $\int_0^\infty f(u) \sin \lambda u \, du$ (B) $\int_0^\infty f(u) \operatorname{cosec} \lambda u \, du$

(C) $\int_0^\infty f(u) \sin \lambda u \, d\lambda$ (D) $\int_0^\infty f(u) \cos \lambda u \, du$

5. The inverse Fourier cosine transform f(x) of $F_c(\lambda)$ is

(A) $\int_0^\infty F_c(\lambda) \sin \lambda x \, d\lambda$ (B) $\frac{2}{\pi} \int_0^\infty F_c(\lambda) \cos \lambda x \, dx$

(2)

(C) $\int_0^\infty F_c(\lambda) \sec \lambda x \, d\lambda$ (D) $\frac{2}{\pi} \int_0^\infty F_c(\lambda) \cos \lambda x \, d\lambda$

6. The inverse Fourier sine transform f(x) of $F_s(\lambda)$ is

(A) $\frac{2}{\pi} \int_0^\infty F_s(\lambda) \sin \lambda x \, d\lambda$ (B) $\frac{2}{\pi} \int_0^\infty F_s(\lambda) \cos \lambda x \, d\lambda$

(2)

(C) $\frac{2}{\pi} \int_0^\infty F_s(\lambda) \operatorname{cosec} \lambda x \, d\lambda$ (D) $\int_0^\infty F_s(\lambda) \sin \lambda x \, dx$

7. For the Fourier sine integral representation $e^{-x} \cos x = \frac{2}{\pi} \int_0^\infty \frac{\lambda^3}{\lambda^4 + 4} \sin \lambda x \, d\lambda$, $F_s(\lambda)$ is

(A) $\frac{\lambda}{\lambda^4 + 4}$ (B) $\frac{\lambda^3}{\lambda^4 + 4}$ (C) $\frac{\lambda^4 + 4}{\lambda^4}$ (D) $\frac{1}{\lambda^4 + 4}$

8. For the Fourier cosine integral representation $\frac{2}{\pi} \int_0^\infty \frac{\cos \frac{\pi\lambda}{2}}{1 - \lambda^2} \cos \lambda x \, d\lambda = \begin{cases} \cos x, & |x| \le \frac{\pi}{2} \\ 0, & |x| > \frac{\pi}{2} \end{cases}$, then Fourier cosine transform $F_c(\lambda)$ is

(1)

(A) $\frac{1 - \lambda^2}{\cos \frac{\pi\lambda}{2}}$ (B) $\frac{\sin \frac{\pi\lambda}{2}}{1 - \lambda^2}$ (C) $\frac{\cos \frac{\pi\lambda}{2}}{1 - \lambda^2}$ (D) $\frac{\cos \frac{\pi\lambda}{2}}{1 + \lambda^2}$

9. For the Fourier sine integral representation $\frac{2}{\pi}\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda}\sin\lambda x\, d\lambda = \begin{cases}1, & 0<x<\pi\\ 0, & x>\pi\end{cases}$, $F_s(\lambda)$ is (1)

(A) $\frac{1-\cos\pi\lambda}{\lambda^2}$ (B) $\frac{\lambda}{1-\cos\pi\lambda}$ (C) $\frac{1-\sin\pi\lambda}{\lambda}$ (D) $\frac{1-\cos\pi\lambda}{\lambda}$

10. For the Fourier sine integral representation $\frac{2}{\pi}\int_0^\infty \frac{\sin\pi\lambda}{1-\lambda^2}\sin\lambda x\, d\lambda = \begin{cases}\sin x, & |x|\le\pi\\ 0, & |x|>\pi\end{cases}$, $F_c(\lambda)$ is (1)

(A) $\frac{\sin\pi\lambda}{1-\lambda^2}$ (B) $\frac{1-\cos\pi\lambda}{1-\lambda^2}$ (C) $\frac{\sin\pi\lambda}{1+\lambda^2}$ (D) $\frac{1-\lambda^2}{\sin\lambda\pi}$

11. For the Fourier sine integral representation $\frac{6}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x}{(\lambda^2+1)(\lambda^2+4)}\, d\lambda = e^{-x}-e^{-2x}$, $x>0$, $F_s(\lambda)$ is (1)

(A) $\frac{(\lambda^2+1)(\lambda^2+4)}{3\lambda}$ (B) $\frac{\lambda}{(\lambda^2+1)(\lambda^2+4)}$

(C) $\frac{3\lambda}{(\lambda^2+1)(\lambda^2+4)}$ (D) $\frac{\lambda\sin\lambda x}{(\lambda^2+1)(\lambda^2+4)}$

12. For the Fourier sine integral representation $\frac{2}{\pi}\int_0^\infty \frac{2\lambda\sin\lambda x}{\lambda^4+4}\, d\lambda = e^{-x}\sin x$, $x>0$, $F_s(\lambda)$ is (1)

(A) $\frac{\lambda^4+4}{2\lambda\sin\lambda x}$ (B) $\frac{2\lambda}{\lambda^4+4}$ (C) $\frac{2\lambda\sin\lambda x}{\lambda^4+4}$ (D) $\frac{2\lambda\cos\lambda x}{\lambda^4+4}$

13. For the Fourier sine integral representation $\frac{12}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x}{(\lambda^2+4)(\lambda^2+16)}\, d\lambda = e^{-3x}\sinh x$, $x>0$, $F_s(\lambda)$ is (1)

(A) $\frac{6\lambda}{(\lambda^2+4)(\lambda^2+16)}$ (B) $\frac{\lambda}{(\lambda^2+4)(\lambda^2+16)}$ (C) $\frac{6\cos\lambda x}{(\lambda^2+4)(\lambda^2+16)}$ (D) $\frac{1}{(\lambda^2+4)(\lambda^2+16)}$

14. For the Fourier cosine integral representation $\frac{2}{\pi}\int_0^\infty \frac{\lambda\sin\pi\lambda}{1-\lambda^2}\cos\lambda x\, d\lambda = \begin{cases}\cos x, & |x|\le\pi\\ 0, & |x|>\pi\end{cases}$, $F_c(\lambda)$ is (1)

(A) $\frac{\sin\pi\lambda}{1-\lambda^2}$ (B) $\frac{\lambda\sin\pi\lambda}{1-\lambda^2}$ (C) $\frac{\lambda\cos\pi\lambda}{1-\lambda^2}$ (D) $\frac{1-\lambda^2}{\sin\lambda\pi}$

15. For the Fourier cosine integral representation $\frac{20}{\pi}\int_0^\infty \left(\frac{1}{\lambda^2+5}+\frac{1}{\lambda^2+4}\right)\cos\lambda x\, d\lambda = 2e^{-5x}+5e^{-2x}$, $F_c(\lambda)$ is (1)

(A) $2e^{-5\lambda}+5e^{-2\lambda}$ (B) $\left(\frac{1}{\lambda^2+5}+\frac{1}{\lambda^2+4}\right)\cos\lambda x$

(C) $\left(\frac{1}{\lambda^2+5}+\frac{1}{\lambda^2+4}\right)$ (D) $10\left(\frac{1}{\lambda^2+5}+\frac{1}{\lambda^2+4}\right)$

16. For the Fourier sine transform of $f(x) = e^{-mx}$, $m>0$, $x>0$ is $F_s(\lambda) = \frac{\lambda}{\lambda^2+m^2}$ then its inverse Fourier sine transform is (1)

(A) $\frac{2}{\pi}\int_0^\infty \frac{\lambda}{\lambda^2+m^2}\sin\lambda x\, dm$ (B) $\frac{2}{\pi}\int_0^\infty \frac{\lambda}{\lambda^2+m^2}\sin\lambda x\, dx$

(C) $\frac{2}{\pi}\int_0^\infty \frac{\lambda}{\lambda^2+m^2}\cos\lambda x\, d\lambda$ (D) $\frac{2}{\pi}\int_0^\infty \frac{\lambda}{\lambda^2+m^2}\sin\lambda x\, d\lambda$

17. If the Fourier cosine integral representation of $f(x) = \begin{cases}1, & |x|<1\\ 0, & |x|>1\end{cases}$ is

$f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda$ then the value of integral $\int_0^\infty \frac{\sin\lambda}{\lambda}\, d\lambda$ is equal to (1)

(A) $\frac{\pi}{2}$ (B) $\frac{2}{\pi}$ (C) 1 (D) 0

18. The Fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\frac{\pi}{2}\left(\frac{1 - \sin \lambda\pi}{\lambda}\right)$ (B) $\frac{\pi}{2}\left(\frac{\cos \lambda\pi - 1}{\lambda}\right)$ (C) $\frac{\pi}{2}\left(\frac{1 - \cos \lambda\pi}{\lambda}\right)$ (D) $\left(\frac{\cos \lambda\pi}{\lambda}\right)$

19. The Fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$ is (2)

(A) $\left(\frac{\cos \lambda\pi - 1}{\lambda}\right)$ (B) $\left(\frac{1 - \cos \lambda}{\lambda}\right)$ (C) $\left(\frac{1 - \sin \lambda}{\lambda}\right)$ (D) $\left(\frac{\cos \lambda\pi}{\lambda}\right)$

20. If $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ then Fourier cosine transform $F_c(\lambda)$ of f(x) is given by (2)

(A) $\frac{\lambda \sin \lambda + \cos \lambda - 1}{\lambda^2}$ (B) $\frac{\cos \lambda - \lambda \sin\lambda - 1}{\lambda^2}$

(C) $\frac{\cos \lambda - \lambda \sin \lambda + 1}{\lambda^2}$ (D) $\frac{\lambda \sin \lambda + 1}{\lambda^2}$

21. If $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ then Fourier sine transform $F_s(\lambda)$ of f(x) is given by (2)

(A) $\frac{\lambda \cos \lambda + \sin \lambda}{\lambda^2}$ (B) $\frac{-\lambda \cos \lambda - \sin \lambda}{\lambda^2}$

(C) $\frac{-\lambda \cos \lambda + \sin \lambda}{\lambda^2}$ (D) $\frac{\cos \lambda}{\lambda^2}$

22. If $f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ then Fourier cosine transform $F_c(\lambda)$ of f(x) is given by (2)

(A) $\frac{-\lambda^2 \sin \lambda + 2\lambda \cos \lambda - 2 \sin \lambda}{\lambda^3}$ (B) $\frac{\lambda^2 \sin \lambda - 2\lambda \cos \lambda - 2 \sin \lambda}{\lambda^3}$

(C) $\frac{\lambda^2 \sin \lambda - 2\lambda \cos \lambda + 2 \sin \lambda}{\lambda^3}$ (D) $\frac{\lambda^2 \sin \lambda + 2\lambda \cos \lambda - 2 \sin \lambda}{\lambda^3}$

23. If $f(x) = \begin{cases} x^2, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ then Fourier sine transform $F_s(\lambda)$ of f(x) is given by (2)

(A) $\frac{-\lambda^2 \cos \lambda + 2\lambda \sin\lambda + 2(\cos \lambda - 1)}{\lambda^3}$ (B) $\frac{\lambda^2 \cos \lambda + 2\lambda \sin \lambda + 2(\cos \lambda - 1)}{\lambda^3}$

(C) $\frac{\lambda^2 \cos \lambda - 2\lambda \sin \lambda + 2(\cos \lambda - 1)}{\lambda^3}$ (D) $\frac{\lambda^2 \cos \lambda - 2\lambda \sin \lambda - 2(\cos \lambda - 1)}{\lambda^3}$

24. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ is (2)

(A) $-\frac{2}{\lambda^3}(\sin \lambda - \lambda \cos \lambda)$ (B) $\frac{2}{\lambda^3}(\sin \lambda - \lambda \cos \lambda)$

(C) $\frac{2}{\lambda^2}(\sin \lambda - \lambda \cos \lambda)$ (D) $\frac{2}{\lambda^3}(\sin \lambda + \lambda \cos \lambda)$

25. The Fourier cosine transform $f_c(\lambda)$ of $f(x) = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\frac{\pi}{2}\left(\frac{1 - \sin \lambda\pi}{\lambda}\right)$ (B) $\left(\frac{1 - \sin \lambda\pi}{\lambda}\right)$ (C) $\left(\frac{\pi \sin \lambda\pi}{2\lambda}\right)$ (D) $\left(\frac{\sin \lambda\pi}{\lambda}\right)$

26. The Fourier sine transform $F_s(\lambda)$ of $f(x) = e^{-x}$, $x > 0$ is given by (2)

(A) $\frac{3\lambda}{1 + \lambda^2}$ (B) $\frac{\lambda}{1 - \lambda^2}$ (C) $\frac{\lambda}{1 + \lambda^2}$ (D) $\frac{\lambda}{1 - \lambda^2}$

27. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = e^{-x}$, $x > 0$ is given by (2)

(A) $\frac{2}{1 - \lambda^2}$ (B) $\frac{1}{1 - \lambda^2}$ (C) $\frac{2}{1 + \lambda^2}$ (D) $\frac{1}{1 + \lambda^2}$

28. If $f(x) = e^{kx}$, $x > 0$, $k > 0$ then Fourier sine transform $F_s(\lambda)$ of $f(x)$ is given by (2)

(A) $\dfrac{\lambda}{k^2+\lambda^2}$ (B) $\dfrac{k}{k^2+\lambda^2}$ (C) $\dfrac{1}{k^2+\lambda^2}$ (D) $-\dfrac{k}{k^2+\lambda^2}$

29. If $f(x) = e^{-kx}$, $x > 0$ then Fourier cosine transform $F_c(\lambda)$ of $f(x)$ is given by (2)

(A) $-\dfrac{k}{k^2+\lambda^2}$ (B) $\dfrac{k}{k^2+\lambda^2}$ (C) $\dfrac{\lambda}{k^2+\lambda^2}$ (D) $\dfrac{1}{k^2+\lambda^2}$

30. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = e^{-|x|}$, $-\infty < x < \infty$ is (2)

(A) $\dfrac{\lambda}{1+\lambda^2}$ (B) $\dfrac{1}{1+\lambda^2}$ (C) $\dfrac{1}{1-\lambda^2}$ (D) $-\dfrac{1}{1+\lambda^2}$

31. The Fourier sine transform $F_s(\lambda)$ of $f(x) = e^{-|x|}$, $0 < x < \infty$ is (2)

(A) $\dfrac{\lambda}{1+\lambda^2}$ (B) $\dfrac{1}{1+\lambda^2}$ (C) $\dfrac{1}{1-\lambda^2}$ (D) $-\dfrac{1}{1+\lambda^2}$

32. If $f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$ then Fourier cosine transform $F_c(\lambda)$ of $f(x)$ is given by (2)

(A) $\dfrac{\cos\lambda}{\lambda}$ (B) $\dfrac{\cos 2\lambda}{\lambda}$ (C) $\dfrac{\sin\lambda}{\lambda}$ (D) $\dfrac{\sin 2\lambda}{\lambda}$

33. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a, \end{cases}$ is (2)

(A) $\dfrac{1-\cos\lambda a}{\lambda}$ (B) $\dfrac{\cos\lambda a - 1}{\lambda}$ (C) $\dfrac{\sin\lambda a}{a}$ (D) $\dfrac{\sin\lambda a}{\lambda}$

34. The Fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} 1, & 0 < x < 2 \\ 0, & x > 2 \end{cases}$ is (2)

(A) $\dfrac{1-\cos\lambda a}{\lambda}$ (B) $\dfrac{\sin\lambda a}{\lambda}$ (C) $\dfrac{\cos\lambda a - 1}{\lambda}$ (D) $\dfrac{\sin\lambda a}{a}$

35. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\dfrac{1}{2}\left[-\dfrac{\sin(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (B) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

(C) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (D) $\dfrac{1}{2}\left[-\dfrac{\sin(1+\lambda)u}{1+\lambda} - \dfrac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

36. The Fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} \sin x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (B) $\dfrac{1}{2}\left[\dfrac{\sin(1-\lambda)u}{1-\lambda} - \dfrac{\sin(1+\lambda)u}{1+\lambda}\right]_0^{\pi}$

(C) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (D) $\dfrac{1}{2}\left[-\dfrac{\sin(1+\lambda)u}{1+\lambda} - \dfrac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

37. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\dfrac{1}{2}\left[\dfrac{\sin(1-\lambda)u}{1-\lambda} - \dfrac{\cos(1+\lambda)u}{1+\lambda}\right]_0^{\pi}$ (B) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

(C) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\cos(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (D) $\dfrac{1}{2}\left[\dfrac{\sin(1+\lambda)u}{1+\lambda} + \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

38. The Fourier sine transform $F_s(\lambda)$ of $f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$ is (2)

(A) $\dfrac{1}{2}\left[\dfrac{\sin(1-\lambda)u}{1-\lambda} - \dfrac{\cos(1+\lambda)u}{1+\lambda}\right]_0^{\pi}$ (B) $\dfrac{1}{2}\left[-\dfrac{\cos(\lambda+1)u}{\lambda+1} - \dfrac{\cos(\lambda-1)u}{\lambda-1}\right]_0^{\pi}$

(C) $\dfrac{1}{2}\left[-\dfrac{\cos(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$ (D) $\dfrac{1}{2}\left[\dfrac{\sin(1+\lambda)u}{1+\lambda} - \dfrac{\sin(1-\lambda)u}{1-\lambda}\right]_0^{\pi}$

39. The Fourier cosine transform $F_c(\lambda)$ of $f(x) = \begin{cases} \cos x, & 0 < x < a \\ 0 & , x > a \end{cases}$ is

(A) $\frac{1}{2}\left[\frac{\sin(\lambda+1)a}{\lambda+1} - \frac{\sin(\lambda-1)a}{\lambda-1}\right]$

(B) $\frac{1}{2}\left[\frac{\sin(\lambda-1)a}{\lambda-1} - \frac{\sin(\lambda+1)a}{\lambda+1}\right]$

(C) $\frac{1}{2}\left[\frac{\sin(\lambda+1)a}{\lambda+1} + \frac{\sin(\lambda-1)a}{\lambda-1}\right]$

(D) $\frac{\sin(\lambda+1)a}{\lambda+1}$

40. The solution f(x) of integral equation $\int_0^\infty f(x)\cos\lambda x\,dx = e^{-\lambda}, \lambda > 0$ is

(A) $\frac{2}{\pi}\left(\frac{e^{-x}}{1+x^2}\right)$ (B) $\frac{2}{\pi}\left(\frac{x}{1+x^2}\right)$ (C) $\frac{2}{\pi}\left(\frac{1}{1-x^2}\right)$ (D) $\frac{2}{\pi}\left(\frac{1}{1+x^2}\right)$

41. The solution of integral equation $\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0 & , \lambda \ge 1 \end{cases}$ is

$f(x) = \frac{2}{\pi}\int_0^1 (1-\lambda)\sin\lambda x\,d\lambda$ then the value of f(x) is equal to

(A) $\frac{2}{\pi}\left(\frac{1}{x} - \frac{\sin x}{x^2}\right)$ (B) $\frac{2}{\pi}\left(\frac{1}{x} - \frac{\cos x}{x^2}\right)$ (C) $\frac{2}{\pi}\left(\frac{1}{x} + \frac{\sin x}{x^2}\right)$ (D) $\frac{2}{\pi}\left(-\frac{1}{x} + \frac{\sin x}{x^2}\right)$

42. The solution of integral equation $\int_0^\infty f(x)\cos\lambda x\,dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0 & , \lambda \ge 1 \end{cases}$ is

$f(x) = \frac{2}{\pi}\int_0^1 (1-\lambda)\sin\lambda x\,d\lambda$ then the value of f(x) is equal to

(A) $\frac{2}{\pi}\left(\frac{1+\cos x}{x^2}\right)$ (B) $\frac{2}{\pi}\left(\frac{1-\cos x}{x^2}\right)$ (C) $\frac{2}{\pi}\left(\frac{1+\sin x}{x^2}\right)$ (D) $\frac{2}{\pi}\left(\frac{1-\sin x}{x^2}\right)$

43. The solution f(x) of integral $\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1, & 0 \le \lambda \le 1 \\ 2, & 1 \le \lambda < 2 \\ 0, & \lambda \ge 2 \end{cases}$ is

(A) $\frac{2}{\pi}\left[\left(\frac{1-\sin x}{x}\right) + 2\left(\frac{\sin x - \sin 2x}{x}\right)\right]$

(B) $\frac{2}{\pi}\left[\left(\frac{-1+\cos x}{x}\right) + 2\left(\frac{-\cos x + \cos 2x}{x}\right)\right]$

(C) $\frac{2}{\pi}\left[\left(\frac{1-\cos x}{x}\right) + 2\left(\frac{\cos x - \cos 2x}{x}\right)\right]$

(D) $\frac{2}{\pi}\left[\left(\frac{1-\cos x}{x^2}\right) + 2\left(\frac{\cos x - \cos 2x}{x^2}\right)\right]$

44. The solution f(x) of integral equation $\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$ is

(A) $\frac{2}{\pi}\left(\frac{1+\cos x}{x}\right)$ (B) $\frac{2}{\pi}\left(\frac{1+\sin x}{x}\right)$ (C) $\frac{2}{\pi}\left(\frac{1-\sin x}{x}\right)$ (D) $\frac{2}{\pi}\left(\frac{1-\cos x}{x}\right)$

45. The solution f(x) of integral equation $\int_0^\infty f(x)\cos\lambda x\,dx = \begin{cases} 1, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}$ is

(A) $\frac{2}{\pi}\left(\frac{\sin x}{x}\right)$ (B) $\frac{2}{\pi}\left(\frac{\cos x}{x}\right)$ (C) $\frac{2}{\pi}\left(\frac{1-\cos x}{x}\right)$ (D) $\frac{2}{\pi}\left(\frac{1+\sin x}{x}\right)$

46. The inverse Fourier cosine transform f(x) of $F_c(\lambda) = \frac{\sin a\lambda}{\lambda}$ is

(A) $\frac{1}{\pi}\int_0^\infty \frac{\cos(a+x)\lambda + \sin(a-x)\lambda}{\lambda}\,d\lambda$

(B) $\frac{1}{\pi}\int_0^\infty \frac{\cos(a+x)\lambda + \cos(a-x)\lambda}{\lambda}\,d\lambda$

(C) $\frac{1}{\pi}\int_0^\infty \frac{\sin(a+x)\lambda + \sin(a-x)\lambda}{\lambda}\,d\lambda$

(D) $\frac{1}{\pi}\int_0^\infty \frac{\sin(a+x)\lambda + \cos(a-x)\lambda}{\lambda}\,d\lambda$

47. If the Fourier cosine integral representation of $f(x) = \begin{cases} 1 - x^2, & 0 < x < 1 \\ 0 & , x > 1 \end{cases}$ is (2)

$f(x) = \frac{4}{\pi} \int_0^\infty \left(\frac{\sin \lambda - \lambda \cos \lambda}{\lambda^3}\right) \cos \lambda x \, d\lambda$ then the value of integral $\int_0^\infty \left(\frac{\sin \lambda - \lambda \cos \lambda}{\lambda^3}\right) \cos \frac{\lambda}{2} \, d\lambda$ is equal to (2)

(A) $-\frac{3\pi}{16}$ (B) $\frac{3\pi}{16}$ (C) $\frac{3\pi}{8}$ (D) $\frac{3\pi}{4}$

48. Given that $\int_0^\infty \frac{\sin t}{t} dt = \frac{\pi}{2}$, then Fourier sine transform $F_s(\lambda)$ of $f(x) = \frac{1}{x}$, $x > 0$ is given by (2)

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $-\pi$

49. For the Fourier cosine transform $\int_0^\infty \left(\frac{1 - \cos u}{u^2}\right) \cos \lambda u \, du = \begin{cases} \frac{\pi}{2}(1 - \lambda), & 0 < \lambda < 1 \\ 0 & , \lambda > 1 \end{cases}$ the value of integral $\int_0^\infty \frac{\sin^2 z}{z^2} dz$ is (2)

(A) 1 (B) $\frac{\pi}{2}$ (C) 0 (D) $\frac{\pi}{4}$

50. For the Fourier sine integral representation

$\frac{2}{\pi} \int_0^\infty \left(\frac{1 - \cos \lambda}{\lambda}\right) \sin \lambda x \, d\lambda = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$, the value of integral $\int_0^\infty \frac{\sin^3 t}{t} dt$ is (2)

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) $\frac{\pi}{4}$

51. Given that $F_c(\lambda) = \int_0^\infty u^{m-1} \cos \lambda u \, du = \frac{\overline{|m}}{\lambda^m} \cos \frac{m\pi}{2}$, then Fourier cosine transform $F_c(\lambda)$ of $f(x) = x^3$, $x > 0$ is given by (2)

(A) $\frac{6}{\lambda^4}$ (B) $\frac{3}{\lambda^3}$ (C) $\frac{4}{\lambda^2}$ (D) $\frac{1}{\lambda^2}$

52. Given that $F_s(\lambda) = \int_0^\infty u^{m-1} \sin \lambda u \, du = \frac{\overline{|m}}{\lambda^m} \sin \frac{m\pi}{2}$, then Fourier sine transform $F_s(\lambda)$ of $f(x) = x^2$, $x > 0$ is given by (2)

(A) $\frac{2}{\lambda^3}$ (B) $-\frac{2}{\lambda^3}$ (C) $\frac{3}{\lambda^2}$ (D) $-\frac{3}{\lambda^2}$

Answers

1. (B)	2. (D)	3. (C)	4. (A)	5. (D)	6. (A)	7. (B)	8. (C)
9. (D)	10. (A)	11. (C)	12. (B)	13. (A)	14. (B)	15. (D)	16. (D)
17. (A)	18. (C)	19. (B)	20. (A)	21. (C)	22. (D)	23. (A)	24. (B)
25. (C)	26. (C)	27. (D)	28. (A)	29. (B)	30. (B)	31. (A)	32. (C)
33. (D)	34. (A)	35. (C)	36. (B)	37. (D)	38. (B)	39. (C)	40. (D)
41. (A)	42. (B)	43. (C)	44. (D)	45. (A)	46. (C)	47. (B)	48. (C)
49. (B)	50. (D)	51. (A)	52. (B)				

3.9 DISCRETE FOURIER TRANSFORM

3.9.1 Sampling of Signals in the Time and Frequency Domains

Introduction : Sampling is a process by which a signal that is a function of a continuous variable is converted into a signal that is a function of discrete variable. In this section we treat time-domain sampling of continuous-time signals and frequency-domain sampling of discrete-time signals having a continuous spectrum. Of particular importance is the introduction of the discrete Fourier transform (DFT) and its inverse, the IDFT. The DFT is described as the sampled version of the spectrum X (ω) [finite duration discrete frequency sequence [X(k)] of a discrete time sequence [x(n)] of finite duration.

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Z-transform **Marks**

1. Z-transform of sequence $\{f(k)\}$ is defined as (1)

(A) $\sum_{k=-\infty}^{\infty} f(k)\, z^{-k}$ (B) $\sum_{k=-\infty}^{\infty} f(k)\, z^{k}$ (C) $\sum_{k=-\infty}^{\infty} f(k)\, z^{-2k}$ (D) $\sum_{k=-\infty}^{\infty} f(k)\, z^{2k}$

Z-transform of causal sequence $\{f(k)\}$, $k \geq 0$ is defined as (1)

(A) $\sum_{k=0}^{\infty} f(k)\, z^{k}$ (B) $\sum_{k=0}^{\infty} f(k)\, z^{-k}$ (C) $\sum_{k=0}^{\infty} f(-k)\, z^{-k}$ (D) $\sum_{k=0}^{\infty} f(-k)\, z^{k}$

3. If $U(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$, then Z-transform of U(k) is given by (2)

(A) $-\frac{z}{z-1}, |z| > 1$ (B) $\frac{1}{z-1}, |z| > 1$ (C) $\frac{z}{z-1}, |z| > 1$ (D) $\frac{2}{z-1}, |z| > 1$

If $\delta(k) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}$, then Z-transform of $\delta(k)$ is given by (2)

(A) $\frac{1}{z}$ (B) $\frac{1}{z-1}$ (C) $\frac{2}{z-2}$ (D) 1

If $f(k) = a^k$, $k \geq 0$, then Z-transform of $\{a^k\}$ is given by (1)

(A) $\frac{z}{z-a}, |z| < |a|$ (B) $\frac{z}{z-a}, |z| > |a|$

(C) $\frac{1}{z-a}, |z| > |a|$ (D) $-\frac{z}{z-a}, |z| > |a|$

If $f(k) = a^k$, $k < 0$, then Z-transform of $\{a^k\}$ is given by (1)

(A) $\frac{z}{a-z}, |z| < |a|$ (B) $\frac{z}{z-a}, |z| < |a|$

(C) $\frac{1}{a-z}, |z| > |a|$ (D) $\frac{z}{a-z}, |z| > |a|$

If $f(k) = 2^k$, $k \geq 0$, then Z-transform of $\{2^k\}$ is given by (1)

(A) $\frac{z}{z-2}, |z| < |2|$ (B) $\frac{1}{z-2}, |z| > |2|$

(C) $\frac{z}{z-2}, |z| > |2|$ (D) $-\frac{z}{z-2}, |z| > |2|$

If $f(k) = 3^k$, $k < 0$, then Z-transform of $\{3^k\}$ is given by (1)

(A) $\frac{z}{3-z}, |z| > |3|$ (B) $\frac{z}{z-3}, |z| < |3|$

(B) $\frac{1}{3-z}, |z| > |3|$ (D) $\frac{z}{3-z}, |z| < |3|$

If $f(k) = \cos \alpha k$, $k \geq 0$, then Z-transform of $\{\cos \alpha k\}$ is given by (1)

(A) $\frac{z(z + \cos\alpha)}{z^2 - 2z\cos\alpha + 1}, |z| > 1$ (B) $\frac{z(z - \cos\alpha)}{z^2 - 2z\cos\alpha + 1}, |z| < 1$

(C) $\frac{z(z - \cos\alpha)}{z^2 - 2z\cos\alpha + 1}, |z| > 1$ (D) $\frac{z\cos\alpha}{z^2 + 2z\cos\alpha + 1}, |z| > 1$

10. If f(k) = sin αk, k ≥ 0, then Z-transform of {sin αk} is given by (1)

(A) $\frac{z \sin \alpha}{z^2 - 2z \cos \alpha + 1}, |z| > 1$ (B) $\frac{z \sin \alpha}{z^2 + 2z \cos \alpha + 1}, |z| > 1$

(C) $\frac{z(z - \sin \alpha)}{z^2 - 2z \cos \alpha + 1}, |z| > 1$ (D) $\frac{z \sin \alpha}{z^2 + 2z \cos \alpha + 1}, |z| < 1$

11. If f(k) = cosh αk, k ≥ 0, then Z-transform of {cosh αk} is given by (1)

(A) $\frac{z(z - \sinh \alpha)}{z^2 - 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$ (B) $\frac{z(z - \cosh \alpha)}{z^2 - 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$

(C) $\frac{z(z + \cosh \alpha)}{z^2 + 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$ (D) $\frac{z(z - \cosh \alpha)}{z^2 - 2z \cosh \alpha + 1}, |z| < \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$

12. If f(k) = sinh αk, k ≥ 0, then Z-transform of {sinh αk} is given by (1)

(A) $\frac{z \sinh \alpha}{z^2 - 2z \cosh \alpha + 1}, |z| < \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$ (B) $\frac{z(z - \sinh \alpha)}{z^2 - 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$

(C) $\frac{z(z + \sinh \alpha)}{z^2 + 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$ (D) $\frac{z \sinh \alpha}{z^2 - 2z \cosh \alpha + 1}, |z| > \max(|e^{\alpha}| \text{ or } |e^{-\alpha}|)$

13. If f(k) = cosh 2k, k ≥ 0, then Z-transform of {cosh 2k} is given by (1)

(A) $\frac{z \sinh 2}{z^2 - 2z \cosh 2 + 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$ (B) $\frac{z(z - \cosh 2)}{z^2 - 2z \cosh 2 + 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$

(C) $\frac{z(z + \cosh 2)}{z^2 + 2z \cosh 2 + 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$ (D) $\frac{z(z - \cosh 2)}{z^2 - 2z \cosh 2 + 1}, |z| < \max(|e^{2}| \text{ or } |e^{-2}|)$

14. If f(k) = sinh 2k, k ≥ 0, then Z-transform of {sinh 2k} is given by (1)

(A) $\frac{z \sinh 2}{z^2 + 2z \cosh 2 - 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$ (B) $\frac{z(z - \cosh 2)}{z^2 - 2z \cosh 2 + 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$

(C) $\frac{z \sinh 2}{z^2 - 2z \cosh 2 + 1}, |z| > \max(|e^{2}| \text{ or } |e^{-2}|)$ (D) $\frac{z(z - \cosh 2)}{z^2 - 2z \cosh 2 + 1}, |z| < \max(|e^{2}| \text{ or } |e^{-2}|)$

15. If f(k) = cos 2k, k ≥ 0, then Z-transform of {cos 2k} is given by (1)

(A) $\frac{z(z + \cos 2)}{z^2 - 2z \cos 2 + 1}, |z| > 1$ (B) $\frac{z \cos 2}{z^2 + 2z \cos 2 + 1}, |z| > 1$

(C) $\frac{z(z - \cos 2)}{z^2 - 2z \cos 2 + 1}, |z| < 1$ (D) $\frac{z(z - \cos 2)}{z^2 - 2z \cos 2 + 1}, |z| > 1$

16. If f(k) = sin 2k, k ≥ 0, then Z-transform of {sin 2k} is given by (1)

(A) $\frac{z \sin 2}{z^2 - 2z \cos 2 + 1}, |z| > 1$ (B) $\frac{z \sin 2}{z^2 + 2z \cos 2 + 1}, |z| > 1$

(C) $\frac{z(z - \sin 2)}{z^2 - 2z \cos 2 + 1}, |z| > 1$ (D) $\frac{z \sin 2}{z^2 + 2z \cos 2 + 1}, |z| < 1$

17. If Z {f(k)} = F(z), then Z $\{a^k f(k)\}$, a constant, is equal to (1)

(A) $F\left(\frac{a}{z}\right)$ (B) $F\left(\frac{z}{a}\right)$ (C) F(az) (D) $\frac{F(z)}{a}$

18. If Z {f(k)} = F(z), then Z $\{e^{-ak} f(k)\}$, a constant, is equal to (1)

(A) $F\left(\frac{z}{e^a}\right)$ (B) $F(e^{-a} z)$ (C) $F(e^a z)$ (D) $\frac{F(z)}{e^a}$

19. If Z {f(k)} = F(z), then Z $\{k^n f(k)\}$, is equal to (1)

(A) $\left(-z\frac{d}{dz}\right)^n F(z)$ (B) $\left(z\frac{d}{dz}\right)^n F(z)$ (C) $(-z)^n \frac{d}{dz} F(z)$ (D) $\left(z\frac{d}{dz}\right)^{n-1} F(z)$

20. Z-transform of $\{f(k)\} = \frac{a^k}{k!}$, k ≥ 0 is given by (1)

(A) $e^{z/a}$ (B) e^{az} (C) ze^{a} (D) $e^{a/z}$

21. If $Z\{f(k)\} = F(z)$, $k \geq 0$ then $Z\{f(k+1)\}$ is given by (1)

(A) $zF(z) + zf(0)$ (B) $zF(z) - zf(0)$ (C) $zF(z) - f(0)$ (D) $z^2F(z) - zf(0)$

22. If $Z\{f(k)\} = F(z)$, $k \geq 0$ then $Z\{f(k+2)\}$ is given by (1)

(A) $z^2F(z) - zf(0) - f(1)$ (B) $z^2F(z) + z^2f(0) + zf(1)$

(C) $z^2F(z) + zf(0) + f(1)$ (D) $z^2F(z) - z^2f(0) - zf(1)$

23. If $Z\{f(k)\} = F(z)$, $k \geq 0$ then $Z\{f(k-1)\}$ is given by (1)

(A) $z^{-1}F(z)$ (B) $z^{-1}(F) - f(0)$

(C) $zF(z)$ (D) $z^{-2}F(z) - z^{-1}(0)$

24. If $Z\{f(k)\} = F(z)$, $k \geq 0$ then $Z\{f(k-2)\}$ is given by (1)

(A) $z^2F(z) - zf(0)$ (B) $z^{-1}F(z) - f(0)$

(C) $z^{-2}F(z)$ (D) $z^{-2}F(z) - z^{-1}f(0)$

25. Convolution of two sequences $\{f(k)\}$ and $\{g(k)\}$ is $\{h(k)\} = \{f(k)\} \cdot \{g(k)\}$. Then $Z[\{h(k)\}]$ is given by (1)

(A) $F(z)\,G(z)$ (B) $F(z) + G(z)$ (C) $F(z) - G(z)$ (D) $\dfrac{F(z)}{G(z)}$

26. For $\{f(k)\} = \{-2, \underset{\uparrow}{-1}, 2\}$, $F(z)$ is given by (2)

(A) $2z + 1 + 2z^{-1}$ (B) $-2z - 1 + 2z^{-1}$ (C) $2z + 1 - 2z^{-1}$ (D) $2z - 1 + 2z^{-1}$

$-2z - 1 + 2z^{-1}$

27. For $\{f(k)\} = \{2, 1, \underset{\uparrow}{3}, 2, -4\}$, $F(z)$ is given by (2)

(A) $2z^2 - z - 3 + 2z^{-1} - 4z^{-2}$ (B) $2z^2 + z + 3 - 2z^{-1} + 4z^{-2}$

(C) $2z^2 + z + 3 + 2z^{-1} - 4z^{-2}$ (D) $2z^2 + z + 3 + 2z^{-1} + 4z^{-2}$

28. If $f(k) = a^{|k|}$, $\forall$ k, then Z-transofrm of $\{a^{|k|}\}$ is given by (2)

(A) $\left(\dfrac{az}{1+az} + \dfrac{z}{z-a}\right)$, $|a| < |z| < \dfrac{1}{|a|}$ (B) $\left(\dfrac{az}{1-az} - \dfrac{z}{z-a}\right)$, $|a| < |z| < \dfrac{1}{|a|}$

(C) $\left(\dfrac{az}{1+az} + \dfrac{z}{z+a}\right)$, $|a| < |z| < \dfrac{1}{|a|}$ (D) $\left(\dfrac{az}{1-az} + \dfrac{z}{z-a}\right)$, $|a| < |z| < \dfrac{1}{|a|}$

29. Z-transform of $\{f(k)\} = \dfrac{2^k}{k!}$, $k \geq 0$ is given by (1)

(A) $e^{z/2}$ (B) e^{2z} (C) e^{z} (D) $e^{2/z}$

30. If $f(k) = \cos \pi k$, $k \geq 0$, then Z-transform of $\{\cos \pi k\}$ is given by (2)

(A) $\dfrac{z(z-1)}{(z+1)^2}$, $|z| > 1$ (B) $\dfrac{z-1}{z+1}$, $|z| > 1$

(C) $\dfrac{z(z+1)}{(z-1)^2}$, $|z| > 1$ (D) $\dfrac{z}{z+1}$, $|z| > 1$

31. If $f(k) = \cos \dfrac{\pi}{2} k$, $k \geq 0$, then Z-transform of $\left\{\cos \dfrac{\pi}{2} k\right\}$ is given by (2)

(A) $\dfrac{z^2}{z^2+1}$, $|z| > 1$ (B) $\dfrac{z^2}{z^2-1}$, $|z| > 1$

(C) $\dfrac{z}{z+1}$, $|z| > 1$ (D) $\dfrac{z}{z-1}$, $|z| < 1$

32. If $f(k) = \sin \dfrac{\pi}{2} k$, $k \geq 0$, then Z-transform of $\left(\sin \dfrac{\pi}{2} k\right)$ is given by (2)

(A) $\dfrac{z}{z^2-1}$, $|z| < 1$ (B) $\dfrac{z^2}{z^2+1}$, $|z| > 1$

(C) $\dfrac{z}{z^2+1}$, $|z| > 1$ (D) $\dfrac{z}{z^2-1}$, $|z| > 1$

33. If $f(k) = \left(\frac{\pi}{2}\right)^k \cos\frac{\pi}{2}k,\ k \geq 0$, then Z-transform of $\left\{\left(\frac{\pi}{2}\right)^k \cos\frac{\pi}{2}k\right\}$ is given by (2)

(A) $\dfrac{z^2}{z^2 + \frac{\pi^2}{4}},\ |z| > \frac{\pi}{2}$ (B) $\dfrac{z^2}{z^2 - \frac{\pi^2}{4}},\ |z| < \frac{\pi}{2}$

(C) $\dfrac{z}{z^2 + \frac{\pi^2}{4}},\ |z| > \frac{\pi}{2}$ (D) $\dfrac{z}{z^2 - \frac{\pi^2}{4}},\ |z| > \frac{\pi}{2}$

34. If $f(k) = 2^k \sin\frac{\pi}{2}k,\ k \geq 0$, then Z-transform of $\left\{2^k \sin\frac{\pi}{2}k\right\}$ is given by (2)

(A) $\dfrac{2z}{z^2 - 4},\ |z| > 2$ (B) $\dfrac{2z}{z^2 - 4},\ |z| < 2$ (C) $\dfrac{2z}{z^2 + 4},\ |z| < 2$ (D) $\dfrac{2z}{z^2 + 4},\ |z| > 2$

35. If $f(k) = 2^k \sin\frac{\pi}{3}k,\ k \geq 0$, then Z-transform of $\left\{2^k \sin\frac{\pi}{3}k\right\}$ is given by (2)

(A) $\dfrac{\sqrt{3}z}{z^2 - 2z + 4},\ |z| > 2$ (B) $\dfrac{\sqrt{3}z}{z^2 - 2z + 4},\ |z| < 2$

(C) $\dfrac{\sqrt{3}z}{z^2 + 2z + 4},\ |z| > 2$ (D) $\dfrac{\sqrt{3}z}{z^2 + 2z + 4},\ |z| < 2$

36. If $f(k) = 2^k \cosh 3k,\ k \geq 0$, then Z-transform of $\{2^k \cosh 3k\}$ is given by (2)

(A) $\dfrac{z(z - 2\cosh 3)}{z^2 - 4z\cosh 3 + 4},\ |z| > \max(|e^{3}| \text{ or } |e^{-3}|)$ (B) $\dfrac{z(z - 2\cosh 3)}{z^2 - 4z\cosh 3 + 4},\ |z| > \max(|e^3| \text{ or } |e^{-3}|)$

(C) $\dfrac{z(z + 2\cosh 3)}{z^2 + 4z\cosh 3 + 4},\ |z| < \max(|e^3| \text{ or } |e^{-3}|)$ (D) $\dfrac{z(z - 2\sinh 3)}{z^2 - 4z\sinh 3 + 4},\ |z| > \max(|e^3| \text{ or } |e^{-3}|)$

37. If $f(k) = 3^k \sinh 2k,\ k \geq 0$, then Z-transform of $\{3^k \sinh 2k\}$ is given by (2)

(A) $\dfrac{3z\sinh 2}{z^2 + 6z\cosh 2 - 9},\ |z| > \max(|e^2| \text{ or } |e^{-2}|)$ (B) $\dfrac{3z\sinh 2}{z^2 - 6z\cosh 2 + 9},\ |z| > \max(|e^3| \text{ or } |e^{-3}|)$

(C) $\dfrac{3z\sinh 2}{z^2 - 6z\cosh 2 + 9},\ |z| > \max(|e^2| \text{ or } |e^{-2}|)$ (D) $\dfrac{3z(z - \sinh 2)}{z^2 - 6z\cosh 2 + 9},\ |z| < \max(|e^2| \text{ or } |e^{-2}|)$

38. If $f(k) = k,\ k \geq 0$ then Z-transform of $\{k\}$ is given by (2)

(A) $\dfrac{z}{(z-1)^2},\ |z| > 1$ (B) $\dfrac{(z-1)^2}{z^2},\ |z| > 1$

(C) $\dfrac{(z+1)^2}{z^2},\ |z| > 1$ (D) $\dfrac{z^2}{(z+1)^2},\ |z| > 1$

39. If $f(k) = k5^k,\ k \geq 0$ then Z-transform of $\{k5^k\}$ is given by (2)

(A) $\dfrac{(z-5)^2}{5z},\ |z| > 5$ (B) $\dfrac{(z-5)^2}{z},\ |z| > 5$

(C) $\dfrac{5z}{(z-5)^2},\ |z| > 5$ (D) $\dfrac{5z}{(z+5)^2},\ |z| > 5$

40. If $f(k) = (k+1)\,2^k,\ k \geq 0$, then Z-transform of $\{(k+1)\,2^k\}$ is given by (2)

(A) $\dfrac{2}{(z+2)^2} + \dfrac{z}{z-2},\ |z| > 2$ (B) $-\dfrac{2z}{(z-2)^2} - \dfrac{z}{z-2},\ |z| > 2$

(C) $-\dfrac{2z}{(z-2)^2} + \dfrac{z}{z-2},\ |z| > 2$ (D) $\dfrac{2z}{(z-2)^2} + \dfrac{z}{z-2},\ |z| > 2$

41. $Z\{3^k e^{-2k}\},\ k \geq 0$ is given by (2)

(A) $\dfrac{z}{(z-3e)^2}$ (B) $\dfrac{z}{z - 3e^{-2}}$ (C) $\dfrac{z}{z - 2e^3}$ (D) $\dfrac{z}{z + 3e^2}$

42. $Z\{ke^{-k}\}$, $k \geq 0$ is given by (2)

(A) $\dfrac{ez}{(ez+1)^2}$ (B) $\dfrac{e^{-1}z}{(z-e^{-1})}$ (C) $\dfrac{e^{-1}z}{(z-e^{-1})^2}$ (D) $\dfrac{e^{-1}z}{(z+e^{-1})^2}$

43. $Z\{\cos(2k+3)\}$, $k \geq 0$ is given by (2)

(A) $\cos 3 \dfrac{z(z-\cos 2)}{z^2-2z\cos 2+1} + \sin 3 \dfrac{z\sin 2}{z^2-2z\cos 2+1}$ (B) $\cos 3 \dfrac{z(z-\cos 2)}{z^2-2z\cos 2+1} - \sin 3 \dfrac{z\sin 2}{z^2-2z\cos 2+1}$

(C) $\sin 3 \dfrac{z(z-\cos 2)}{z^2-2z\cos 2+1} - \cos 3 \dfrac{z\sin 2}{z^2-2z\cos 2+1}$ (D) $\cos 3 \dfrac{z(z+\cos 2)}{z^2+2z\cos 2+1} + \sin 3 \dfrac{z\sin 2}{z^2+2z\cos 2+1}$

44. $Z\{\sinh(bk+c)\}$, $k \geq 0$ is given by (2)

(A) $\cosh c \dfrac{z\sinh b}{z^2-2z\cosh b+1} + \sinh c \dfrac{z(z-\cosh b)}{z^2-2z\cosh b+1}$ (B) $\cosh c \dfrac{z(z-\cosh b)}{z^2-2z\cosh b+1} + \sinh c \dfrac{z\sinh b}{z^2-2z\cosh b+1}$

(C) $\cosh c \dfrac{z(z-\cosh b)}{z^2-2z\cosh b+1} - \sinh c \dfrac{z\sinh b}{z^2-2z\cosh b+1}$ (D) $\cosh c \dfrac{z\sinh b}{z^2+2z\cosh b+1} + \sinh c \dfrac{z(z+\cosh b)}{z^2+2z\cosh b+1}$

45. $Z\{e^{-2k}\sin 3k\}$, $k \geq 0$ is given by (2)

(A) $\dfrac{(ze^3)\sin 2}{(ze^3)^2+2(ze^3)\cos 2-1}$ (B) $\dfrac{(ze^2)(ze^2-\cos 3)}{(ze^2)^2-2(ze^2)\cos 3+1}$

(C) $\dfrac{(ze^3)\sin 2}{(ze^3)^2-2(ze^3)\cos 2+1}$ (D) $\dfrac{(ze^2)\sin 3}{(ze^2)^2-2(ze^2)\cos 3+1}$

46. If $f(k) = {}^2C_k$, $0 \leq k \leq 2$ then $Z\{{}^2C_k\}$ is given by (2)

(A) $(1-z^{-1})^2$, $|z| > 0$ (B) $(1+z^{-1})^2$, $|z| > 0$

(C) $(1+z^{-1})$, $|z| > 0$ (D) $(1-z^{-1})$, $|z| > 0$

47. If $f(k) = a^k U(k)$ then $Z\{f(k)\}$ is given by (2)

(A) $\dfrac{z}{z-1}$, $|z| > |a|$ (B) $\dfrac{z-1}{z}$, $|z| > |a|$

(C) $\dfrac{z^2}{z-1}$, $|z| > |a|$ (D) $\dfrac{z}{z-a}$, $|z| > |a|$

48. If $\{x(k)\} = \left\{\dfrac{1}{1^k}\right\} \cdot \left\{\dfrac{1}{2^k}\right\}$ then $Z\{x(k)\}$ is given by (2)

(A) $\left(\dfrac{z}{z-1}\right)\left(\dfrac{2z}{2z-1}\right)$, $|z| > 1$ (B) $\left(\dfrac{z}{z-1}\right) + \left(\dfrac{2z}{2z-1}\right)$, $|z| > 1$

(C) $\left(\dfrac{z}{z-1}\right) - \left(\dfrac{2z}{2z-1}\right)$, $|z| > 1$ (d) $\left(\dfrac{z}{z-1}\right) \div \left(\dfrac{2z}{2z-1}\right)$, $|z| > 1$

Answers

1. (A)	2. (B)	3. (C)	4. (D)	5. (B)	6. (A)	7. (C)	8. (D)
9. (C)	10. (A)	11. (B)	12. (D)	13. (B)	14. (C)	15. (D)	16. (A)
17. (B)	18. (C)	19. (A)	20. (D)	21. (B)	22. (D)	23. (A)	24. (C)
25. (A)	26. (B)	27. (C)	28. (D)	29. (D)	30. (D)	31. (A)	32. (C)
33. (A)	34. (D)	35. (A)	36. (B)	37. (C)	38. (A)	39. (C)	40. (D)
41. (B)	42. (C)	43. (B)	44. (A)	45. (D)	46. (B)	47. (D)	48. (A)

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{e^{3i\theta} + e^{-3i\theta}}{2} \right) e^{in\theta} \, d\theta$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} \left[e^{i(n+3)\theta} + e^{i(n-3)\theta} \right] d\theta$$

$$= 0 \quad \text{if} \quad n \neq 3, -3 = \frac{1}{4\pi} (2\pi) \quad \text{if} \quad n = 3, -3$$

$$f(n) = \frac{1}{2} \quad \text{if } n = 3, -3$$

$$= 0, \quad \text{otherwise.}$$

i.e. $$f = \left\{ \ldots\ldots \frac{1}{2}, 0, 0, \underset{k=0}{0}, 0, 0, \frac{1}{2}, \ldots\ldots \right\}.$$

Here $f(-3) = \frac{1}{2}$, $f(-2) = f(-1) = f(0) = f(1) = f(2) = 0$, $f(3) = \frac{1}{2}$ and so on.

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Marks

Type : Inverse Z-transform and Difference Equation :

1. If $|z| > |a|$, inverse Z-transform of $\frac{z}{z-a}$ is given by (1)

(A) $a^k, k \geq 0$ (B) $a^k, k < 0$ (C) $a^{k-1}, k \geq 0$ (D) $-a^k, k \geq 0$

2. If $|z| < |a|$, inverse Z-transform of $\frac{z}{z-a}$ is given by (1)

(A) $a^k, k \geq 0$ (B) $a^k, k < 0$ (C) $a^{k-1}, k \geq 0$ (D) $-a^k, k < 0$

3. If $|z| > |a|$, inverse Z-transform of $\frac{1}{z-a}$ is given by

(A) $a^{k-1}, k \geq 0$ (B) $a^{k-1}, k < 0$ (C) $a^{k-1}, k \geq 1$ (D) $-a^k, k \geq 0$

4. If $|z| < |a|$, inverse Z-transform of $\frac{1}{z-a}$ is given by (1)

(A) $a^{k-1}, k \geq 0$ (B) $-a^{k-1}, k \leq 0$ (C) $a^{k-1}, k \geq 1$ (D) $-a^k, k \geq 0$

5. If $|z| > 2$, inverse Z-transform of $\frac{z}{z-2}$ is given by (1)

(A) $2^k, k \leq 0$ (B) $2^{k-1}, k > 0$ (C) $2^k, k \geq 0$ (D) $-2^k, k \geq 0$

6. If $|z| < 3$, inverse Z-transform of $\frac{z}{z-3}$ is given by (1)

(A) $-3^k, k < 0$ (B) $3^{k-1}, k < 0$ (C) $-3^{k-1}, k \geq 0$ (D) $3^k, k \geq 0$

7. If $|z| > 5$, inverse Z-transform of $\frac{1}{z-5}$ is given by (1)

(A) $5^{k-1}, k \leq 1$ (B) $5^{k-1}, k \geq 1$ (C) $5^k, k \geq 0$ (D) $-5^k, k \geq 1$

8. If $|z| < 5$, inverse Z-transform of $\frac{1}{z-5}$ is given by (1)

(A) $5^{k+1}, k \geq 0$ (B) $5^k, k \leq 0$ (C) $5^{k+1}, k \geq 1$ (D) $-5^{k-1}, k \leq 0$

9. If $|z| > |a|$, inverse Z-transform of $\frac{z}{(z-a)^2}$ is given by (1)

(A) $k\,a^{k-1}, k \geq 0$ (B) $a^{k-1}, k \geq 0$ (C) $k\,a^{k-1}, k < 0$ (D) $(k-1)\,a^k, k \leq 0$

10. If $|z| > 1$, $k \geq 0$, $Z^{-1}\left[\frac{z}{z-1}\right]$ is given by (1)

(A) $U(-k)$ (B) $U(k)$ (C) $U(k+1)$ (D) $\delta(k)$

11. $Z^{-1}[1]$ for all k is given by (1)

(A) $\delta(k+1)$ (B) $U(k)$ (C) $\delta(k)$ (D) $U(k-1)$

12. Inverse Z-transform of F(z) by inversion integral method is (1)

(A) $f(k) = \sum$ [Residues of $z^k F(z)$ at the poles of F(z)]
(B) $f(k) = \sum$ [Residues of $z^{k+2} F(z)$ at the poles of F(z)]
(C) $f(k) = \sum$ [Residues of $z^{k+1} F(z)$ at the poles of F(z)]
(D) $f(k) = \sum$ [Residues of $z^{k-1} F(z)$ at the poles of F(z)]

. If $|z| > 10$, $k \geq 0$, inverse Z-transform of $\frac{z(z-\cosh 2)}{z^2 - 2z\cosh 2 + 1}$ is given by (1)

(A) $\cosh 2k$ (B) $\cosh 3k$ (C) $\sinh 2k$ (D) $\sinh 3k$

14. If $|z| > 21$, $k \geq 0$, inverse Z-transform of $\frac{z \sinh 3}{z^2 - 2z\cosh 3 + 1}$ is given by (1)

(A) $\cosh 2k$ (B) $\cosh 3k$ (C) $\sinh 2k$ (D) $\sinh 3k$

15. If $|z| < 2$, inverse Z-transform $Z^{-1}\left(\frac{3}{(z-2)^2}\right)$ is given by (2)

(A) $\left(\frac{-k}{2^{-k+1}}\right)$, $k \leq 0$
(B) $\left(\frac{-k+1}{2^{-k+2}}\right)$, $k \leq 0$
(C) $3\left(\frac{-k+1}{2^{-k+2}}\right)$, $k \leq 0$
(D) $\left(\frac{-k+1}{2^{-k+2}}\right)$, $k \geq 0$

. If $|z| > 3$, $k \geq 0$, inverse Z-transform $Z^{-1}\left[\frac{z^2}{(z-3)^2}\right]$ is given by (2)

(A) $-(k+1)\,3^k$ (B) $(k+1)\,3^k$ (C) $(k+1)\,3^{-k}$ (D) $(k-1)\,3^k$

17. If $|z| < 2$, $Z^{-1}\left[\frac{1}{(z-3)(z-2)}\right]$ is given by (2)

(A) $2^{k-1} + 3^{k-1}$, $k \leq 0$
(B) $-2^{k-1} - 3^{k-1}$, $k \leq 0$
(C) $-2^{k-1} + 3^{k-1}$, $k \leq 0$
(D) $2^{k-1} - 3^{k-1}$, $k \leq 0$

. If $2 < |z| < 3$, $Z^{-1}\left[\frac{1}{(z-3)(z-2)}\right]$ is given by (2)

(A) $-3^{k-1} - 2^{k-1}$
$(k \leq 0)$ $(k \geq 1)$
(B) $3^{k-1} + 2^{k-1}$
$(k \leq 0)$ $(k \geq 2)$
(C) $3^{k+1} - 2^{k+1}$
$(k \leq 0)$ $(k \leq 0)$
1
(D) $\left(\frac{1}{3}\right)^{k+1} - \left(\frac{1}{2}\right)^{k+1}$
$(k \leq 1)$ $(k \leq 2)$

. If $|z| > 2$, $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$ is given by (2)

(A) $1 - 2^k$, $k \geq 0$ (B) $2^k - 1$, $k \geq 0$ (C) $\frac{1^k}{2} - 1$, $k \geq 0$ (D) $k - 1$, $k \geq 0$

20. If $|z| < 1$, $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$ is given by (2)

(A) $2^k - 1$, $k \geq 0$ (B) $2^{k+1} - 1$, $k > 1$ (C) $1 - 2^k$, $k < 0$ (D) $2 - 3^k$, $k < 0$

. If $1 < |z| < 2$, $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$ is given by (2)

(A) $1 + 2^k$, $k > 0$
(B) $3^k + 2^k$, $k < 0$
(C) $3^k - 1$, $k < 0$
(D) -2^k $-$ 1
$(k \leq 0)$ $(k > 0)$

22. If $|z| > 1$, $k \geq 0$, $Z^{-1}\left[\frac{z^2}{z^2+1}\right]$ is given by (2)

(A) $\cos \pi k$ (B) $\sin \frac{\pi}{2} k$ (C) $\cos \frac{\pi}{2} k$ (D) $\sin \pi k$

23. If $|z| > 1$, $k \geq 0$, $Z^{-1}\left[\frac{z}{z^2+1}\right]$ is given by (2)

(A) $\sin \frac{\pi}{2} k$ (B) $\sin \frac{\pi}{4} k$ (C) $\cos \frac{\pi}{2} k$ (D) $\cos \frac{\pi}{4} k$

24. For finding inverse Z-transform by inversion integral method of $F(z) = \frac{z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{5}\right)}$ the residue of $z^{k-1} F(z)$ at the pole $z = \frac{1}{4}$ is (2)

(A) $-\frac{1}{20}\left(\frac{1}{4}\right)^k$ (B) $20\left(\frac{1}{4}\right)^k$ (C) $-20\left(\frac{1}{4}\right)^k$ (D) $\frac{1}{20}\left(\frac{1}{4}\right)^k$

25. For finding inverse Z-transform by inversion integral method of $F(z) = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$ the residue of $z^{k-1} F(z)$ at the pole $z = \frac{1}{2}$ is (2)

(A) $-\frac{1}{2}\left(\frac{1}{2}\right)^k$ (B) $\frac{1}{6}\left(\frac{1}{2}\right)^k$ (C) $-3\left(\frac{1}{2}\right)^k$ (D) $6\left(\frac{1}{2}\right)^k$

26. For finding inverse Z-transform by inversion integral method of $F(z) = \frac{10z}{(z-1)(z-2)}$ the residue of $z^{k-1} F(z)$ at the pole $z = 1$ is (2)

(A) 10 (B) 10^{k-1} (C) -10 (D) 10^k

27. For finding inverse Z-transform by inversion integral method of $F(z) = \frac{1}{(z-2)(z-3)}$ the residue of $z^{k-1} F(z)$ at the pole $z = 2$ is (2)

(A) -2^{k-1} (B) 2^{k-1} (C) -1 (D) -2^k

28. For the difference equation $f(k+1) + \frac{1}{2} f(k) = \left(\frac{1}{2}\right)^k$, $k \geq 0$, $f(0) = 0$, $F(z)$ is given by (2)

(A) $\frac{1}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)}$ (B) $\frac{z}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)}$ (C) $\frac{z}{\left(z+\frac{1}{3}\right)\left(z+\frac{1}{2}\right)}$ (D) $\frac{z}{\left(z-\frac{1}{2}\right)^2}$

29. For the difference equation $12f(k+2) - 7f(k+1) + f(k) = 0$, $f(0) = 0$, $f(1) = 3$, $F(z)$ is given by (2)

(A) $\frac{36z}{12z^2 - 7z - 1}$ (B) $\frac{36z}{12z^2 + 7z + 1}$ (C) $\frac{36z}{12z^2 - 7z + 1}$ (D) $\frac{36z}{12z^2 + 7z - 1}$

30. For the difference equation $y_k - 4y_{k-2} = 1$, $k \geq 0$, $Y(z)$ is given by (2)

(A) $\frac{z}{(z-1)(z^2-4)}$ (B) $\frac{1}{(1-4z^2)}$ (C) $\frac{z}{(z-1)(1-4z^2)}$ (D) $\frac{z^3}{(z-1)(z^2-4)}$

Answers

1. (A)	2. (D)	3. (C)	4. (B)	5. (C)	6. (A)	7. (B)	8. (D)
9. (A)	10. (B)	11. (C)	12. (D)	13. (A)	14. (D)	15. (C)	16. (B)
17. (D)	18. (A)	19. (B)	20. (C)	21. (D)	22. (C)	23. (A)	24. (B)
25. (D)	26. (C)	27. (A)	28. (B)	29. (C)	30. (D)		

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2. Following table gives the Marks obtained in a paper of statistics out of 50, by the students of two divisions :

C.I.	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50
Div. A f	2	6	8	8	15	18	12	11	9	4
Div. B f	3	5	7	9	12	16	11	5	6	2

Find out which of the two divisions show greater variability.

Also find the common mean and standard deviation.

Ans. B has greater variability $\bar{x} = 26.1458, \sigma = 11.1267$

3. Calculate the first four moments about the mean of the following distribution. Find the coefficient of Skewness and Kurtosis.

x	1	2	3	4	5	6	7	8	9	10
f	6	15	23	42	62	60	40	24	13	5

Ans. $\mu_1 = 0, \mu_2 = 3.703, \mu_3 = 0.04256, \mu_4 = 37.5, \beta_1 = 0.00005572, \beta_2 = 2.8411$

4. The Mean and Standard deviation of 25 items is found to be 11 and 3 respectively. It was observed that one item 9 was incorrect. Calculate the Mean and Standard deviation if :

(i) the wrong item is omitted. (ii) it is replaced by 13.

Ans. (i) $\bar{x} = 11.08, \sigma = 3.345$; (ii) $\bar{x} = 11.16, \sigma = 2.9915$

5. Age distribution of 150 life insurance policy-holders is as follows :

Age as on Nearest Birthday	Number
15 – 19.5	10
20 – 24.5	20
25 – 29.5	14
30 – 34.5	30
35 – 39.5	32
40 – 44.5	14
45 – 49.5	15
50 – 54.5	10
55 – 59.5	5

Calculate mean deviation from median age.

Ans. M.D. = 8.4284

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Measures of Central Tendencies and Dispersion :

Marks

1. If the data is presented in the forms of frequency distribution then arithmetic mean $\bar{x}$ is given by $(N = \sum f)$ (1)

(A) $\dfrac{\sum fx}{N}$ (b) $\dfrac{1}{N}\Sigma f|x - A|$ (C) $N\sum fx$ (D) $\dfrac{\sum fx^2}{N}$

2. For the data presented in the form of frequency distribution, mean deviation (M.D.) from the average A is given by $(N = \sum f)$ (1)

(A) $\dfrac{\sum fx}{N}$ (B) $\sum f|x - A|$ (C) $\dfrac{1}{N}\sum f|x - A|$ (D) $\dfrac{1}{N}\Sigma f|x - A|^2$

3. If the data is presented in the form of frequency distribution then standard deviation σ is given by ($\bar{x}$ is arithmetic mean $N = \sum f$)

(A) $\frac{1}{N}\sum f(x-\bar{x})^2$ (B) $\sqrt{\frac{1}{N}\sum f(x-\bar{x})^2}$

(C) $\frac{\sum fx}{N}$ (D) $\frac{1}{N}\sum f|x-\bar{x}|$

4. If the data is presented in the form of frequency distribution then variance V is given by ($\bar{x}$ is arithmetic mean and $N = \sum f$)

(A) $\frac{1}{N}\sum f|x-\bar{x}|$ (B) $\sqrt{\frac{1}{N}\sum f(x-\bar{x})^2}$

(C) $\frac{\sum fx}{N}$ (D) $\frac{1}{N}\sum f(x-\bar{x})^2$

5. To compare the variability of two or more than two series, coefficient of variation (C.V.) is obtained using ($\bar{x}$ is arithmetic mean an σ is standard deviation).

(A) $\frac{\bar{x}}{\sigma}\times 100$ (B) $\frac{\sigma}{\bar{x}}\times 100$ (C) $\sigma\times\bar{x}\times 100$ (D) $\frac{\bar{x}}{\sigma^2}\times 100$

6. If the data is presented in the form of frequency distribution then r^{th} moment μ_r about the arithmetic mean $\bar{x}$ of distribution i given by ($N = \sum f$)

(A) $\frac{1}{N}\sum f^r(x+\bar{x})^r$ (B) $N\times\sum f(x-\bar{x})^r$

(C) $\frac{1}{N}\sum f^r(x-\bar{x})$ (D) $\frac{1}{N}\sum f(x-\bar{x})^r$

7. If the data is presented in the form of frequency distribution then 1^{st} moment μ_1 about the arithmetic mean $\bar{x}$ of distribution is ($N = \sum f$) (1)

(A) 1 (B) σ^2 (C) 0 (D) $\frac{1}{N}\sum f(x-\bar{x})^3$

8. If μ_1' and μ_2' are the first two moments of the distribution about certain number then second moment μ_2 of the distribution about the arithmetic mean is given by (1)

(A) $\mu_2' - (\mu_1')^2$ (B) $2\mu_2' - \mu_1'$

(C) $\mu_2' + (\mu_1')^2$ (D) $\mu_2' + 2(\mu_1')^2$

9. If μ_1', μ_2', μ_3' are the first three moments of the distribution about certain number then third moment μ_3 of the distribution about the arithmetic mean is given by (1)

(A) $\mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$ (B) $\mu_3' - 3\mu_1' + (\mu_2')^3$

(C) $\mu_3' + 2\mu_2'\mu_1' + (\mu_3')^3$ (D) $\mu_3' + 3\mu_2'\mu_1' + (\mu_1')^2$

10. If $\mu_1', \mu_2', \mu_3', \mu_4'$ are the first four moments of the distribution about certain number then fourth moment μ_4 of the distribution about the arithmetic mean is given by (1)

(1) $\mu_4' + 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^4 + 3(\mu_1')^4$ (B) $\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

(C) $\mu_4' + 4\mu_3'\mu_1' - 6\mu_2'(\mu_1')^4 - 3(\mu_1')^4$ (D) $\mu_4' + 2\mu_3'\mu_1' - 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

11. If μ_1' be the first moment of the distribution about any number A then arithmetic mean $\bar{x}$ is given by (1)

(A) $\mu_1' + A$ (B) μ_1' (C) $\mu_1' - A$ (D) $\mu_1' A$

2. Second moment μ_2 about mean is (1)

(A) Mean (B) Standard deviation

(C) Variance (D) Mean deviation

3. Coefficient of skewness β_1 is given by (1)

(A) $\frac{\mu_2^3}{\mu_3^2}$ (B) $\frac{\mu_1^2}{\mu_2^3}$ (C) $\frac{\mu_2^2}{\mu_3^2}$ (D) $\frac{\mu_3^2}{\mu_2^3}$

4. Coefficient of kurtosis β_2 is given by (1)

(A) $\frac{\mu_4}{\mu_3}$ (B) $\frac{\mu_4}{\mu_2^2}$ (C) $\frac{\mu_3}{\mu_2^2}$ (D) $\frac{\mu_4}{\mu_2^3}$

5. For a distribution coefficient of kurtosis $\beta_2 = 2.5$, this distribution is (1)

(A) Leptokurtic (B) Mesokurtic (C) Platykurtic (D) None of these

6. For a distribution coefficient of kurtosis $\beta_2 = 3.9$, this distribution is (1)

(A) Leptokurtic (B) Mesokurtic (C) Platykurtic (D) None of these

7. The first four moments of a distribution about the mean are 0, 16, −64 and 162. Standard deviation of a distribution is (1)

(A) 21 (B) 12 (C) 16 (D) 4

8. Standard deviation of three numbers 9, 10, 11 is (2)

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{2}$

9. Standard deviation of four numbers 9, 11, 13, 15 is (2)

(A) 2 (B) 4 (C) $\sqrt{6}$ (D) $\sqrt{5}$

0. From the given information $\sum x = 235$, $\sum x^2 = 6750$, $n = 10$. Standard deviation of x is (2)

(A) 11.08 (B) 13.08 (C) 8.08 (D) 7.6

1. Coefficient of variation of the data 1, 3, 5, 7, 9 is (2)

(A) 54.23 (B) 56.57 (C) 55.41 (D) 60.19

2. The standard deviation and arithmetic mean of the distribution are 12 and 45.5 respectively. Coefficient of variation of the distribution is (2)

(A) 26.37 (B) 32.43 (C) 12.11 (D) 22.15

3. The Standard Deviation and Arithmetic Mean of three distribution x, y, z are as follow:

	Arithmetic mean	Standard deviation
x	18.0	5.4
y	22.5	4.5
z	24.0	6.0

The more stable distribution is (2)

(A) x (B) y (C) z (D) x and z

4. The standard deviation and arithmetic mean of scores of three batsman x, y, z in ten inning during a certain season are

	Arithmetic mean	Standard deviation
x	50	24.43
y	46	25.495
z	40	27

The more consistent batsman is (2)

(A) y and z (B) y (C) z (D) x

25. The standard deviation and arithmetic mean of aggregate marks obtained three group of students x, y, z are as follow :

	Arithmetic mean	Standard deviation
x	532	11
y	831	9
z	650	10

The more variable group is (2)

(A) y and z (B) z (C) y (D) x

26. Arithmetic mean of four numbers is 16, one item 20 is replaced by 24, what is the new arithmetic mean (2)

(A) 15 (B) 17 (C) 18 (D) 16

27. The first moment of the distribution about the value 5 is 2. Arithmetic mean of the distribution is (2)

(A) 5 (B) 2 (C) 4 (D) 7

28. The first and second moments of the distribution about the value 3 are 2 and 20. Second moment about the mean is (2)

(A) 12 (B) 14 (C) 16 (D) 20

29. The first three moments of a distribution about the value 5 are 2, 20 and 40. Third moment about the mean is (2)

(A) −64 (B) 64 (C) 32 (D) −32

30. The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. Fourth moment about the mean is (2)

(A) 160 (B) 162 (C) 210 (D) 180

31. The first moments of a distribution about the value 2 are −2, 12, −20 and 100. Fourth moment about the mean is (2)

(A) 200 (B) 190 (C) 170 (D) 180

32. The first three moments of a distribution about the value 2 are −2, 12, −20. Third moment about the mean is (2)

(A) 36 (B) 30 (C) 22 (D) 8

33. The first and second moments of the distribution about the value 2 are 1 and 16. Variance of the distribution is (2)

(A) 12 (B) 3 (C) 15 (D) 17

34. The second and third moments of a distribution about the arithmetic mean are 16 and −64 respectively. Coefficient of skewness β_1 is given by (2)

(A) −0.25 (B) 1 (C) 4 (D) −1

35. The second and fourth moments of a distribution about the arithmetic mean are 16 and 162 respectively. Coefficient of kurtosis β_2 is given by (2)

(A) 1 (B) 1.51 (C) 0.63 (D) 1.69

Answers

1. (A)	2. (C)	3. (B)	4. (D)	5. (B)	6. (D)	7. (C)	8. (A)
9. (A)	10. (B)	11. (A)	12. (C)	13. (D)	14. (B)	15. (C)	16. (A)
17. (D)	18. (C)	19. (D)	20. (A)	21. (B)	22. (A)	23. (B)	24. (D)
25. (D)	26. (B)	27. (D)	28. (C)	29. (A)	30. (B)	31. (D)	32. (A)
33. (C)	34. (B)	35. (C)					

Using value of b in equation (3)

$$a + 7.37997 \times 0.29105 = 0.2288$$

$$a = -1.91914$$

∴ Equation of the required curve is,

$$-1.91914x^2 + 0.29105y^2 = x$$

Exercise 5.2

1. Fit a straight line of the form y = mx + c to the following data, using least square criteria

x	0	1	2	3	4	5	6
y	–4	–1	2	5	8	11	14

Ans. y = 3x –

2. If a curve of the form $x = ay^2 + by + c$ satisfies the data :

x	–6	–8	–4	6	22	44	72
y	0	1	2	3	4	5	6

Ans. a = 3, b = – 5, c =

Find the best values of a, b, c.

3. Find the best values of a, b, c assuming that the following values of x, y are connected by the relation

$$y = ax^2 + bx + c$$

x	1	2	3	4	5
y	3.38	8.25	16.6	28.5	44

Ans. a = 1.772, b = – 0.383, c = 2.

...(1)

4. Find the law of the form by = 10^{cx} where x, y are tabulated as

x	1	1.2	1.4	1.6	1.8
y	3.67	3.01	2.46	2.02	1.65

Ans. b = 0.1, c = – 0.4

5. If x and y are connected by the relation $ax^2 + by^2 = x$, find the values of a and b by using least square criteria

... (2)

x	1	2	3	4	5
y	3.35	5.92	8.43	10.93	13.45

Ans. a = – 1.25, b –

MULTIPLE CHOICE QUESTIONS (MCQ'S)

1. For least square fit of the straight line y = ax + b with n points, the normal equations are

(A) $a \sum x + nb = \sum y$,
$a \sum x^2 + b \sum x = \sum xy$

(B) $a \sum x^2 + nb = \sum x$,
$a\sum x + nb = \sum y$

(C) $a \sum y^2 + nb = \sum y$,
$a \sum y + b\sum x = \sum x$

(D) $a \sum y + b \sum x = nb$,
$a \sum x^2 + nb = \sum y$

2. For least square fit of the straight line x = ay + b with n points, the normal equations are

... (3)

(A) $a \sum x + nb = \sum y$,
$a \sum x^2 + b\sum x = \sum xy$

(B) $a\sum y + nb = \sum x$,
$a\sum y^2 + b \sum y = \sum xy$

... (4)

(C) $a \sum x^2 + nb = \sum xy$,
$a \sum y^2 + n\sum x = \sum x^2$

(D) $a \sum x + b\sum y = \sum x$,
$a \sum x^2 + b\sum y^2 = \sum y$

3. For least square fit of the straight line ax + by = c with n points, the normal equation are (1)

(A) $\frac{c}{b}\Sigma x - n\frac{c}{b}\Sigma x,$ $-\frac{a}{b}\Sigma x^2 + n\frac{c}{b} = \Sigma xy$

(B) $\frac{c}{b}\Sigma x + n\frac{c}{b} = \Sigma y,$ $\frac{a}{b}\Sigma y + \frac{c}{b}\Sigma y^2 = \Sigma x$

(C) $\frac{c}{b}\Sigma x^2 + \frac{a}{b}\Sigma y = \Sigma x^2,$ $\frac{c}{b}\Sigma x^2 + \frac{a}{b}\Sigma x = \Sigma y^2$

(D) $-\frac{a}{b}\Sigma x + n\frac{c}{b} = \Sigma y,$ $\frac{a}{b}\Sigma x^2 + \frac{c}{b}\Sigma x - \Sigma xy$

4. Least square fit for the straight line y = ax + b to the data (2)

x	1	2	3
y	5	7	9

is

(A) y = 2x + 4 (B) y = 2x – 3 (C) y = 2x + 3 (D) y = 3x – 4

5. Least square fit for the straight line x = ay + b to the data (2)

y	1	2	3
x	–1	1	3

is

(A) x = y + 1 (B) x = y + 5 (C) x = y – 5 (D) x = 2y – 3

6. Least square fit for the straight line y = ax + b to the data (2)

x	2	3	4
y	1	4	7

is

(A) y = 2x – 5 (B) y = 3x – 5 (C) y = 2x + 3 (D) y = 2x – 3

7. Least square fit for the straight line x = ay + b to the data (2)

y	0	1	2
x	2	5	8

is

(A) x = 3y – 1 (B) x = 3y + 1 (C) x = 3y + 2 (D) x = 3y – 4

8. Least square fit for the straight line y = ax + b to the data (2)

x	0	1	2
y	–1	1	3

is

(A) y = 2x – 1 (B) y = 2x + 3 (C) y = 2x – 4 (D) y = x + 3

Least square fit for the straight line x = ay + b to the data (2)

y	1	2	3
x	–1	3	7

is

(A) x = 2y – 5 (B) x = 4y + 4 (C) x = 4y – 5 (D) x = y + 2

10. Least square fit for the straight line ax + by = c to the data (2)

x	0	1	2
y	$-\frac{4}{3}$	$-\frac{2}{3}$	0

is

(A) 2x + 3y = 4 (B) x – 3y = 4 (C) 2x + y = 4 (D) 2x – 3y = 4

11. For least square fit of the straight line $y = ax + b$ to the data

x	0	1	2
y	−1	1	3

the normal equations are

(A) $3a + 3b = 3$
$5a + 3b = 7$

(B) $3a + 3b = 3$
$3a + 5b = 7$

(C) $3a + 3b = 3$
$5a + 7b = 3$

(D) $3a + 3b = 7$
$5a + 3b = 3$

12. For least square fit of the straight line $y = ax + b$ to the data

x	2	3	4
y	1	4	7

the normal equations are

(A) $9a + 3b = 42$
$29a + 9b = 12$

(B) $9a + 3b = 12$
$9a + 29b = 42$

(C) $9a + 3b = 12$
$29a + 9b = 42$

(D) $9a + 3b = 12$
$29a + 42b = 9$

13. For least square fit of the straight line $x = ay + b$ to the data

y	1	4	7
x	2	3	4

the normal equations are

(A) $12a + 3b = 9$
$12a + 66b = 42$

(B) $12a + 3b = 9$
$66a + 12b = 42$

(C) $12a + 3b = 9$
$66a + 42b = 12$

(D) $12a + 3b = 42$
$66a + 12b = 9$

14. For least square fit of the straight line $x = ay + b$ to the data

y	1	3	5
x	5	9	13

the normal equations are

(A) $9a + 3b = 27$
$9a + 35b = 97$

(B) $9a + 3b = 97$
$35a + 9b = 27$

(C) $9a + 3b = 27$
$35a + 97b = 9$

(D) $9a + 3b = 27$
$35a + 9b = 97$

15. Least square fit for the curve $y = ax^b$ to the data

y	1	2	3
x	2	16	54

is

(A) $y = 2x^3$ (B) $y = 2x^2$ (C) $y = 3x^2$ (D) $y = 4x^3$

16. Least square fit for the curve $y = ax^b$ to the data

x	1	2	3
y	3	12	27

is

(A) $y = 3x^4$ (B) $y = 2x^3$ (C) $y = 3x^2$ (D) $y = 2x^2$

17. Least square fit for the curve $y = ax^b$ to the data (2)

x	2	4	6
y	2	16	54

is

(A) $y = \frac{1}{4}x^3$ (B) $y = \frac{1}{4}x^2$ (C) $y = 2x^3$ (D) $y = \frac{1}{2}x^3$

18. Least square fit for the curve $y = ax^b$ to the data (2)

x	1	3	5
y	2	18	50

is

(A) $y = 2x^3$ (B) $y = 2x^2$ (C) $y = 3x^2$ (D) $y = 4x^2$

19. Least square fit for the curve $x = ay^b$ to the data (2)

y	2	4	6
x	8	32	72

is

(A) $x = 3y^2$ (B) $x = 2y^3$ (C) $x = y^3$ (D) $x = 2y^2$

20. Least square fit for the curve $x = ay^b$ to the data (2)

y	1	2	3
x	3	12	27

is

(A) $x = 2y^3$ (B) $x = 3y^3$ (C) $x = 3y^2$ (D) $x = 2y^2$

21. Least square fit for the curve $x = ay^b$ to the data (2)

y	1	3	5
y	4	36	100

is

(A) $x = 3y^2$ (B) $x = 2y^4$ (C) $x = 4y^2$ (D) $x = 4y^3$

22. Least square fit for the curve $x = ay^b$ to the data (2)

y	2	4	6
x	2	16	54

is

(A) $x = \frac{1}{4}y^3$ (B) $x = \frac{1}{4}y^4$ (C) $x = \frac{1}{2}y^3$ (D) $x = \frac{1}{4}y^2$

23. For the least square fit of the parabola $y = ax^2 + bx + c$ with n points, the normal equations are (1)

(A) $a\Sigma x^2 + b\Sigma x + nc = y$
$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = xy$
$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = x^2y$

(B) $a\Sigma x^2 + b\Sigma x + nc = \Sigma y$
$a\Sigma y^3 + b\Sigma y^2 + c\Sigma y = \Sigma xy$
$a\Sigma y^4 + b\Sigma y^3 + c\Sigma y^2 = \Sigma y^2x$

(C) $a\Sigma x^2 + b\Sigma x + nc = \Sigma y$
$a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma xy$
$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2y$

(D) $a\Sigma x^2 + b\Sigma x + nc = \Sigma y$
$a\Sigma x^3 + b\Sigma x^2 + ny = \Sigma x$
$a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2 = x^2y$

24. For the least square fit of the parabola $x = ay^2 + by + c$ with n points, the normal equations are

(A) $a\sum x^2 + b\sum x + nc = \sum y$

$a\sum x^3 + b\sum x^2 + c\sum x = \sum xy$

$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum x^2 y$

(B) $a\sum y^2 + b\sum y + nc = \sum x$

$a\sum y^3 + b\sum y^2 + c\sum y = \sum xy$

$a\sum y^4 + b\sum y^3 + c\sum y^2 = \sum y^2 x$

(C) $a\sum y^2 + b\sum y + nc = x$

$a\sum y^3 + b\sum y^2 + c\sum y = xy$

$a\sum y^4 + b\sum y^3 + c\sum y^2 = y^2 x$

(D) $a\sum y^2 + b\sum y + nc = \sum x$

$a\sum y^3 + b\sum y^2 + c\sum y = \sum y$

$a\sum y^4 + b\sum y^3 + nc = \sum y^2$

25. For least square fit of the parabola $y = ax^2 + bx + c$ to the data

x	0	1	2
y	4	3	6

the normal equations are

(A) $5a + 3b + 3c = 0$

$9a + 5b + 3c = 0$

$17a + 9b + 5c = 0$

(B) $5a + 3b + 3c = 15$

$9a + 5b + 3c = 27$

$17a + 9b + 5c = 13$

(C) $13a + 3b + 3c = 13$

$9a + 13b + 3c = 15$

$17a + 9b + 13c = 27$

(D) $5a + 3b + 3c = 13$

$9a + 5b + 3c = 15$

$17a + 9b + 5c = 27$

26. For least square fit of the parabola $y = ax^2 + bx + c$ to the data

x	0	1	2
y	2	2	4

the normal equations are

(A) $5a + 3b + 3c = 8$

$9a + 5b + 3c = 10$

$17a + 9b + 5c = 18$

(B) $5a + 3b + 3c = 18$

$9a + 5b + 3c = 8$

$17a + 9b + 5c = 10$

(C) $17a + 3b + 3c = 8$

$9a + 17b + 3c = 10$

$17a + 9b + 17c = 18$

(D) $5a + 3b + 3c = 0$

$9a + 5b + 3c = 0$

$17a + 9b + 5c = 0$

27. For least square fit of the parabola $x = ay^2 + by + c$ to the data

y	1	2	3
x	3	7	13

the normal equations are

(A) $3a + 6b + 3c = 23$

$36a + 3b + 6c = 56$

$98a + 36b + 3c = 148$

(B) $14a + 6b + 3c = 0$

$36a + 14b + 6c = 0$

$98a + 36b + 14c = 0$

(C) $14a + 6b + 3c = 23$

$36a + 14b + 6c = 56$

$98a + 36b + 14c = 148$

(D) $14a + 6b + 3c = 148$

$36a + 14b + 6c = 23$

$98a + 36b + 14c = 56$

8. For least square fit of the parabola $x = ay^2 + by + c$ to the data

y	0	1	3
x	3	6	24

is normal equations are (2)

(A) $10a + 4b + 3c = 0$
$28a + 10b + 4c = 0$
$82a + 28b + 10c = 0$

(B) $4a + 10b + 3c = 33$
$28a + 4b + 10c = 78$
$82a + 28b + 4c = 222$

(C) $10a + 4b + 3c = 78$
$28a + 10b + 4c = 33$
$82a + 28b + 10c = 222$

(D) $10a + 4b + 3c = 33$
$28a + 10b + 4c = 78$
$82a + 28b + 10c = 222$

9. For the least square parabolic fit of the parabola $y = ax^2 + bx + c$ with 3 points data given as $\Sigma x = 3, \Sigma x^2 = 5, \Sigma x^3 = 9, \Sigma x^4 = 17, \Sigma y = 13, \Sigma xy = 15, \Sigma x^2y = 27$, the normal equations are (2)

(A) $5a + 3b + 3c = 13$
$9a + 5b + 3c = 15$
$9a + 17b + 5c = 27$

(B) $5a + 3b + 3c = 13$
$9a + 5b + 3c = 15$
$17a + 9b + 5c = 27$

(C) $5a + 3b + 3c = 13$
$9a + 5b + 3c = 15$
$17a + 9b + 27c = 5$

(D) $5a + 3b + 3c = 13$
$9a + 5b + 3c = 15$
$9a + 17b + 5c = 27$

30. For the least square parabolic fit of the parabola $y = ax^2 + bx + c$ with 3 points data given as $\Sigma x = 3, \Sigma x^2 = 5, \Sigma x^3 = 9, \Sigma x^4 = 17, \Sigma y = 8, \Sigma xy = 10, \Sigma xy^2 = 18$, the normal equations are (2)

(A) $5a + 3b + 3c = 8$
$9a + 5b + 3c = 10$
$17a + 9b + 5c = 18$

(B) $5a + 3b + 3c = 8$
$9a + 5b + 3c = 10$
$9a + 17b + 5c = 18$

(C) $5a + 3b + 3c = 8$
$9a + 5b + 3c = 10$
$17a + 9b + 18c = 5$

(D) $5a + 3b + 3c = 8$
$9a + 5b + 3c = 10$
$9a + 17b + 5c = 18$

31. For the least square parabolic fit of the parabola $x = ay^2 + by + c$ with 3 points data given as $\Sigma y = 9, \Sigma y^2 = 35, \Sigma y^3 = 153, \Sigma y^4 = 707, \Sigma x = 82, \Sigma xy = 350, \Sigma y^2x = 1602$, the normal equations are (2)

(A) $35a + 9b + 3c = 82$
$153a + 35b + 9c = 350$
$707a + 153b + 35c = 1602$

(B) $35a + 9b + 3c = 82$
$153a + 35b + 9c = 350$
$707a + 153b + 1602c = 35$

(C) $35a + 9b + 3c = 350$
$153a + 35b + 9c = 1602$
$707a + 153b + 35c = 82$

(D) $35a + 9b + 3c = 82$
$153a + 35b + 9c = 350$
$153a + 707b + 35c = 1602$

2. For the least square parabolic fit of the parabola $x = ay^2 + by + c$ with 3 points data given as $\Sigma y = 6, \Sigma y^2 = 14, \Sigma y^3 = 36, \Sigma y^4 = 96, \Sigma x = 30, \Sigma xy = 70, \Sigma y^2x = 180$, the normal equations are (2)

(A) $14a + 6b + 3c = 30$
$36a + 14b + 6c = 70$
$96a + 14b + 36c = 180$

(B) $14a + 6b + 3c = 70$
$36a + 14b + 6c = 180$
$96a + 36b + 14c = 30$

(C) $14a + 6b + 3c = 30$
$36a + 14b + 6c = 70$
$96a + 36b + 14c = 180$

(D) $14a + 6b + 3c = 30$
$36a + 14b + 6c = 70$
$36a + 96b + 14c = 180$

Answers

1. (A)	2. (B)	3. (D)	4. (C)	5. (D)	6. (B)	7. (C)	8. (A)
9. (C)	10. (D)	11. (A)	12. (C)	13. (B)	14. (D)	15. (A)	16. (C)
17. (A)	18. (B)	19. (D)	20. (C)	21. (C)	22. (A)	23. (C)	24. (B)
25. (D)	26. (A)	27. (C)	28. (D)	29. (B)	30. (A)	31. (A)	32. (C)

5.8 CORRELATION

We have already considered distributions involving one variable or what we call as univariate distributions. In many problems practical nature, we are required to deal with two or more variables. If we consider the marks obtained by a group of students in two more subjects, the distribution will involve two or more variables. Distributions using two variables are called *Bivariate distributions*. such distributions, we are often interested in knowing whether there exists some kind of relationship between the two variables involve In language of statistics, this means whether there is correlation or co-variance between the two variables. If the change in one variab affects the change in the other variable, the variables are said to be correlated. For example, change in rainfall will affect the crop outp and thus the variables 'Rainfall recorded' and 'crop output' are correlated. Similarly for a group of workers, the variables 'income' ar 'expenditure' would be correlated. If the increase (or decrease) in one variable causes corresponding increase (or decrease) in the oth the correlation is said to be *positive* or *direct*. On the other hand, if increase in the value of one variable shows a corresponding decrea in the value of the other or vice versa, the correlation is called *negative* or *inverse*. As the income of a worker increases, as a natu course his expenditure also increases, hence the correlation between income and expenditure is positive or direct. Correlation betwe heights and weights of a group of students will also be positive. If we consider the price and demand of a certain commodity then c experience tells us that as the price of a commodity rises, its demand falls and thus the correlation between these variables is negat or inverse. Several such examples can be given. Correlation can also be classified as linear and non-linear. It is based upon the constar of the ratio of change between the two variables. As an example, consider the values assumed by variables x and y.

x	5	8	11	15	17	19	20
y	10	16	22	30	34	38	40

Here the ratio $\frac{y}{x}$ is equal to 2 for all the values of x and y.

Correlation in such case is called *linear*.

When the amount of change in one variable is not in a constant ratio to the amount of change in other variable, the correlatior called *non-linear*. In such a case, the relationship between the variables x and y is not of the form $y = mx$ (or of the form $y = mx + c$) practical situations, the correlation is generally non-linear, but its analysis is quite complicated. Usually, it is assumed that the relat between x and y is linear and further analysis is made. There are different methods to determine whether the two variables correlated. Some of these methods such as 'Scatter Diagram' are graphical methods and give rough idea about the correlation. Th methods are not suitable if the number of observations is large. There are mathematical methods such as '*Karl Pearson's* Coefficient Correlation', 'Concurrent Deviation Method' etc. which are more suitable. We shall discuss '*Karl Pearson's* Coefficient of Correlati which is widely used in practice.

5.9 KARL PEARSON'S COEFFICIENT OF CORRELATION

To measure the intensity or degree of linear relationship between two variables, Karl Pearson developed a formula called *correla coefficient*.

Correlation coefficient between two variables x and y denoted by r (x, y) is defined as

$$r(x, y) = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

In bivariate distribution if (x_i, y_i) take the values (x_1, y_1) (x_2, y_2) ... (x_n, y_n)

$$\text{cov}(x, y) = \frac{1}{n} \sum (x_i - \bar{x})(y_i - \bar{y})$$

where $\bar{x}$, $\bar{y}$ are arithmetic means for x and y series respecti

4. Determine the equations [cut off]

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

and obtain an estimate of y for $x = 4.5$. (M

Ans. $0.95x + 7.25$, $x = 0.957 - 6.4$

5. Determine the reliability of estimates for the data :

x	10	14	19	26	30	34	39
y	12	16	18	26	29	35	38

Ans. $r^2 = 0$

6. The following marks have been obtained by a group of students in Engineering Mathematics.

Paper I	80	45	55	56	58	60	65	68	70	75	85
Paper II	82	56	50	48	60	62	64	65	70	74	90

Calculate the coefficient of correlation.

A

7. Coefficient of correlation between two variables X and Y is 0.8. Their covariance is 20. The variance of X is 16. Find the deviation of Y series.

Ans.

8. Find the coefficient of correlation for the following table : **(Dec. 06, May 17)**

x	10	14	18	22	26	30
y	18	12	24	6	30	36

Ans.

9. The two regression equations of the variables x and y are

$$x = 19.13 - 0.87\,y \qquad y = 11.64 - 0.50\,x$$

Find (i) $\bar{x}$, $\bar{y}$, (ii) The correlation coefficient between x and y. **(Dec. 2006)**

Ans. $\bar{x} = 15.935$, $\bar{y} = 3.673$, $r =$

10. If θ is the acute angle between the two regression lines in the case of two variables x and y, show that

$$\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Correlation and Regression :

1. Covariance between two variables x and y is given by

(A) $\frac{1}{n}\sum (x - \bar{x})(y - \bar{y})$ (B) $\frac{1}{n}\sum (x + \bar{x})(y + \bar{y})$

(C) $n\sum (x - \bar{x})(y - \bar{y})$ (D) $\frac{1}{n}\sum [(x - \bar{x}) + (y - \bar{y})]$

2. Correlation coefficient r between two variables x and y is given by

(A) $\frac{\text{cov}(x, y)}{\sigma_x^2 \sigma_y^2}$ (B) $\frac{\sigma_y}{\sigma_x}$ (C) $\frac{\sigma_x}{\sigma_y}$ (D) $\frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$

3. Range of coefficient of correlation r is

(A) $-\infty < \frac{1}{r} < \infty$ (B) $-\infty < r < \infty$ (C) $-1 \le r \le 1$ (D) $0 \le r \le 1$

Probable error of coefficient of correlation r is (1)

(A) $0.6745\left(\frac{1+r^2}{\sqrt{N}}\right)$ (B) $0.6745\left(\frac{1-r^2}{\sqrt{N}}\right)$ (C) $0.6745\left(\frac{1-r^2}{N}\right)$ (D) $0.6547\left(\frac{1-r^2}{n}\right)$

5. Line of regression y on x is (1)

(A) $y+\bar{y}=r\frac{\sigma_x}{\sigma_y}(x+\bar{x})$ (B) $x-\bar{x}=r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

(C) $y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$ (D) $y-\bar{y}=r\frac{\sigma_x}{\sigma_y}(x-\bar{x})$

6. Line of regression x on y is (1)

(A) $y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$ (B) $x+\bar{x}=r\frac{\sigma_x}{\sigma_y}(y+\bar{y})$

(C) $x-\bar{x}=r\frac{\sigma_y}{\sigma_x}(y-\bar{y})$ (D) $x-\bar{x}=r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

7. Slope of regression line of y on x is (1)

(A) $r(x, y)$ (B) $r\frac{\sigma_y}{\sigma_x}$ (C) $r\frac{\sigma_x}{\sigma_y}$ (D) $\frac{\sigma_y}{\sigma_x}$

8. Slope of regression line of x on y is (1)

(A) $r\frac{\sigma_x}{\sigma_y}$ (B) $r(x, y)$ (C) $\frac{\sigma_x}{\sigma_y}$ (D) $r\frac{\sigma_y}{\sigma_x}$

In regression line y on x, b_{yx} is given by (1)

(A) cov (x, y) (B) r (x, y) (C) $\frac{\text{cov}(x, y)}{\sigma_x^2}$ (D) $\frac{\text{cov}(x, y)}{\sigma_y^2}$

10. In regression line x on y, b_{xy} is given by (1)

(A) cov (x, y) (B) r (x, y) (C) $\frac{\text{cov}(x, y)}{\sigma_x^2}$ (D) $\frac{\text{cov}(x, y)}{\sigma_y^2}$

11. If b_{xy} and b_{yx} are the regression coefficient x on y and y on x respectively then the coefficient of correlation r(x, y) is given by (1)

(A) $\sqrt{b_{xy}+b_{yx}}$ (B) $b_{xy}\,b_{yx}$ (C) $\sqrt{\frac{b_{xy}}{b_{yx}}}$ (D) $\sqrt{b_{xy}\,b_{yx}}$

12. If θ is the acute angle between the regression line of y on x and the regression line of x on y, then tan θ is (1)

(A) $\frac{(1-r^2)}{|r|}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$ (B) $\frac{|r|}{(1-r^2)}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$

(C) $|r|\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$ (D) $\frac{1}{|r|}\frac{\sigma_x^2+\sigma_y^2}{\sigma_x\sigma_y}$

13. If $\sum xy = 2638$, $\bar{x} = 14$, $\bar{y} = 17$, n = 10 then cov (x, y) is (1)

(A) 24.2 (B) 25.8 (C) 23.9 (D) 20.5

14. If $\sum xy = 1242$, $\bar{x} = -5.1$, $\bar{y} = -10$, n = 10, then cov (x, y) is (2)

(A) 67.4 (B) 83.9 (C) 58.5 (D) 73.2

15. If $\sum x^2 = 2291$, $\sum y^2 = 3056$, $\sum (x+y)^2 = 10623$, n = 10, $\bar{x} = 14.7$, $\bar{y} = 17$ then cov (x, y) is (2)

(A) 1.39 (B) 13.9 (C) 139 (D) – 13.9

16. If the two regression coefficient are 0.16 and 4 then the correlation coefficient is (2)

(A) 0.08 (B) – 0.8 (C) 0.8 (D) 0.64

17. If the two regression coefficient are $\frac{8}{15}$ and $\frac{5}{6}$ then the correlation coefficient is

(A) – 0.667 (B) 0.5 (C) – 1.5 (D) [illegible]

18. If covariance between x and y is 10 and the variance of x and y are 16 and 9 respectively then coefficient of correlation r(x, y) is

(A) 0.833 (B) 0.633 (C) 0.527 (D) 0.745

19. If cov (x, y) = 25.8 σ_x = 6, σ_y = 5 then correlation coefficient r(x, y) is equal to

(A) 0.5 (B) 0.75 (C) 0.91 (D) 0.86

20. If $\sum xy = 190$, $\bar{x} = 4$, $\bar{y} = 4$, n = 10, $\sigma_x = 1.732$, $\sigma_y = 2$ then correlation coefficientr(x, y) is equal to

(A) 0.91287 (B) 0.8660 (C) 0.7548 (D) 0.5324

21. If $\sum xy = 2800$, $\bar{x} = 16$, $\bar{y} = 16$, n = 10, variance of x is 36 and variance of y is 25 then correlation coefficient r(x, y) is equal to

(A) 0.95 (B) 0.73 (C) 0.8 (D) 0.65

22. The correlation coefficient for the following data

$n = 10, \sum x = 140, \sum y = 150, \sum x^2 = 1980, \sum y^2 = 2465, \sum xy = 2160$ is

(A) 0.753 (B) 0.4325 (C) 0.556 (D) 0.9013

23. You are given the following information related to a distribution comprising 10 observation $\bar{x} = 5.5$, $\bar{y} = 4$, $\sum x^2 = 385$, $\sum y^2 = 192$, $\sum (x + y)^2 = 947$. The correlation coefficient r(x, y) is

(A) – 0.924 (B) – 0.681 (C) – 0.542 (D) – 0.813

24. Given the following data

$r = 0.022$, $\sum xy = 33799$, $\sigma_x = 4.5$, $\sigma_y = 64.605$, $\bar{x} = 68$, $\bar{y} = 62.125$. The value of n (number of observation) is

(A) 5 (B) 7 (C) 8 (D) 10

25. Given the following data $r = 0.5$, $\sum xy = 350$, $\sigma_x = 1$, $\sigma_y = 4$, $\bar{x} = 3$, $\bar{y} = 4$. The value of n (number of observation) is

(A) 25 (B) 5 (C) 20 (D) 15

26. Coefficient of correlation between the variables x and y is 0.8 and their covariance is 20, the variance of x is 16. Standard deviation of y is

(A) 6.75 (B) 6.25 (C) 7.5 (D) 8.25

27. Line of regression y on x is 8x – 10y + 66 = 0. Lie of regression x on y is 40x – 18y – 214 = 0. Mean values of x and y are

(A) $\bar{x} = 12$, $\bar{y} = 15$ (B) $\bar{x} = 10$, $\bar{y} = 11$

(C) $\bar{x} = 13$, $\bar{y} = 17$ (D) $\bar{x} = 9$, $\bar{y} = 8$

28. If the two lines of regression of 9x + y – λ – 0 and 4x + y – μ and the mean of x and y are 2 and –3 respectively then the values of λ and μ are

(A) λ = 15 and μ = 5 (B) λ = –15 and μ = –5

(C) λ = 5 and μ = 15 (D) λ = 15 and μ = –5

29. Line of regression y on x is 8x – 10y + 66 = 0. Line of regression x on y is 40x – 18y – 214 = 0. Correlation coefficient r(x, y) is given by (2)

(A) 0.6 (B) 0.5 (C) 0.75 (D) 0.45

30. The regression lines are 9x + y = 15 and 4x + y = 5. Correlation r(x, y) is given by (2)

(A) 0.444 (B) – 0.11 (C) 0.663 (D) 0.7

31. Line of regression y on x is 8x – 10y + 66 = 0. Line of regression x on y is 40x – 18y – 214 = 0. The value of variance of x is 9. The standard deviation of y is equal to

(A) 2 (B) 5 (C) 6 (D) 4

32. Line of regression y on x is $8x - 10y + 66 = 0$ Line of regression x on y is $40x - 18y - 214 - 0$. The value of variance of y is 16. The standard deviation of x is equal to (2)

(A) 3 (B) 2 (C) 6 (D) 7

33. Line of regression y on x is $3x + 2y = 26$, line of regression x on y is $6x + y = 31$. The value of variance of x is 25. Then the standard deviation of y is (2)

(A) – 15 (B) 15 (C) 1.5 (D) 1.5

34. The correlation coefficient between two variable x and y is 0.6. If $\sigma_x = 1.5$, $\sigma_y = 2.00$, $\bar{x} = 10$, $\bar{y} = 20$ then the lines of regression are (2)

(A) $x = 0.45y + 12$ and $y = 0.8x + 1$ (B) $x = 0.45y + 1$ and $y = 0.8x + 12$

(C) $x = 0.65y + 10$ and $y = 0.4x + 12$ (D) $x = 0.8y + 1$ and $y = 0.45x + 12$

35. The correlation coefficient between two variable x and y is 0.711. If $\sigma_x = 4$, $\sigma_y = 1.8$, $\bar{x} = 5$, $\bar{y} = 4$ then the lines of regression are (2)

(A) $x - 5 = 1.58 (y - 4)$ and $y - 4 = 0.32 (x - 5)$ (B) $x + 5 - 1.58 (y + 4)$ and $y + 4 = 0.32 (x + 5)$

(C) $x - 5 = 0.32 (y - 4)$ and $y - 4 = 1.58 (x - 5)$ (D) $x - 4 = 1.58 (y - 5)$ and $y - 5 = 0.32 (x - 4)$

36. Your are given below the following information about advertisement expenditure and sales

	Adv. Expenditure (X) ₹ (Crore)	Sales (Y) ₹ (Crore)
Mean	10	90
Standard Deviation	3	12

Correlation coefficient = 0.8

The two lines of regression are (2)

(A) $x = 58 + 3.2y$ and $y = -8 + 0.2x$ (B) $x = -8 + 2.2y$ and $y = 8 + 1.2x$

(C) $x = -8 + 3.2y$ and $y = 58 + 0.2x$ (D) $x = -8 + 0.2y$ and $y = 58 + 3.2x$

7. You are given below the following information about rainfall and production of rice

	Rainfall (X) in inches	Production of Rice (Y) in Kg
Mean	30	500
Standard Deviation	5	100

Correlation coefficient = 0.8

The two lines of regression are (2)

(A) $x + 30 = 0.04 (y + 500)$ and $y + 500 = 6 (x + 30)$ (B) $x - 30 = 0.4 (y - 500)$ and $y - 500 = 1.6 (x - 30)$

(C) $x - 30 = 0.04 (y - 500)$ and $y - 500 = 16 (x - 30)$ (D) $x - 30 = 16 (y - 500)$ and $y - 500 = 0.04 (x - 30)$

8. Given $b_{xy} = 0.85$, $b_{yx} = 0.89$ and the standard deviation of x is 6 then the value of correlation coefficient r(x, y) and standard deviation of y is (2)

(A) $r = 0.87$, $\sigma_y = 6.14$ (B) $r = -0.87$, $\sigma_y = 0.614$

(C) $r = 0.75$, $\sigma_y = 6.14$ (D) $r = 0.89$, $\sigma_y = 4.64$

9. Given $b_{xy} = 0.8411$, $b_{yx} = 0.4821$ and the standard deviation of y is 1.7916 then the value of correlation coefficient r(x, y) and standard deviation of x is (2)

(A) $r = -0.6368$ and $\sigma_x = -2.366$ (B) $r = 0.63678$ and $\sigma_x = 2.366$

(C) $r = 0.40549$ and $\sigma_x = 2.366$ (D) $r = 0.63678$ and $\sigma_x = 5.6$

. For a given set of Bivariate data $\bar{x} = 53.2$, $\bar{y} = 27.9$ Regression coefficient of y on x = –1.5. By using line of regression y on x the most probable value of y when x is 60 is (2)

(A) 15.7 (B) 13.7 (C) 17.7 (D) 21.7

41. Given the following data $\bar{x} = 36$, $\bar{y} = 85$, $\sigma_x = 11$, $\sigma_y = 8$, $r = 0.66$. By using line of regression x on y, the most probable val x when y = 75 is

(A) 29.143 (B) 24.325 (C) 31.453 (D) 26.925

42. For a given set of Bivariate data $\bar{x} = 2$, $\bar{y} = -3$ Regression coefficient of x on y = −0.11. By using line of regression x on y the probable value of x when y is 10 is

(A) 0.77 (B) 0.57 (C) 1.77 (D) 0.87

Answers

1. (A)	2. (D)	3. (C)	4. (B)	5. (C)	6. (D)	7. (B)	8. (A)
9. (C)	10. (D)	11. (D)	12. (A)	13. (B)	14. (D)	15. (B)	16. (C)
17. (A)	18. (A)	19. (D)	20. (B)	21. (C)	22. (D)	23. (B)	24. (C)
25. (A)	26. (B)	27. (C)	28. (A)	29. (A)	30. (C)	31. (D)	32. (A)
33. (B)	34. (B)	35. (A)	36. (D)	37. (C)	38. (A)	39. (B)	40. (C)
41. (D)	42. (B)						

the second one is white. What is the probability that the first is also white ? **Ans.** $\frac{1}{\ }$

9. Box A contains 3 red and 2 blue marbles. The box B contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin shows Head a marble is chosen from box A, if it shows Tail, a marble is chosen from box B. Find the probability that a red marble is chosen.

Ans. $\frac{\ }{5}$

10. One shot is fired from each of the three guns. E_1, E_2, E_3 denote the events that the target is hit by the first, second and third gu respectively. If $P(E_1) = 0.5$, $P(E_2) = 0.6$, $P(E_3) = 0.7$ and E_1, E_2, E_3 are independent events, then find the probability that at least two hits are registered.

Ans. 0.

11. A problem on computer mathematics is given to the three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved ?

Ans. $\frac{2}{32}$

12. Urn I contains 6 white and 4 black balls and urn II contains 4 white and 5 black balls. From urn I, two balls are transferred to urn without noticing the colour. Sample of size 2 is then drawn without replacement from urn II. What is the probability that the sampl contains exactly 1 white ball ?

Ans. $\frac{4}{\ }$

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Probability **Mark**

1. A throw is made with two dice. The probability of getting a score of 10 points is -

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{2}{3}$

2. A throw is made with two dice. The probability of getting a score of at least 10 points is

(A) $\frac{1}{12}$ (B) $\frac{5}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

3. In a single throw of two dice, the probability of getting more than 7 points is

(A) $\frac{7}{36}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{5}{36}$

4. In a single throw of two dice, the probability that the total score is a prime number is

(A) $\frac{1}{6}$ (B) $\frac{5}{12}$ (C) $\frac{1}{2}$ (D) $\frac{5}{36}$

5. A throw is made with two dice. The probability of getting score a perfect square is

(A) $\frac{11}{36}$ (B) $\frac{7}{36}$ (C) $\frac{10}{36}$ (D) $\frac{1}{4}$

6. A card is drawn from a well shuffled a pack of 52 cards, the probability of getting a club card is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

7. Two cards are drawn from a well shuffled a pack of 52 cards, the probability that both the cards are spade is

(A) $\frac{1}{26}$ (B) $\frac{1}{4}$ (C) $\frac{1}{17}$ (D) $\frac{1}{13}$

8. Three cards are drawn from a well shuffled a pack of 52 cards, the probability of getting all of them red is

(A) $\frac{3}{17}$ (B) $\frac{5}{17}$ (C) $\frac{4}{17}$ (D) $\frac{2}{17}$

9. A card is drawn from a well shuffled a pack of 52 cards. The probability of getting a queen of club or king of heart is (2)

(A) $\frac{1}{52}$ (B) $\frac{1}{26}$ (C) $\frac{1}{18}$ (D) $\frac{1}{12}$

10. Two cards are drawn from a well shuffled a pack of 52 cards. If the first card drawn is replaced, the probability that they are both kings is (2)

(A) $\frac{1}{15}$ (B) $\frac{1}{442}$ (C) $\frac{1}{169}$ (D) $\frac{2}{221}$

11. Two cards are drawn from a well shuffled a pack of 52 cards. If the first card drawn is not replaced, the probability that they are both kings is (2)

(A) $\frac{1}{221}$ (B) $\frac{1}{17}$ (C) $\frac{1}{15}$ (D) $\frac{2}{221}$

12. If A and B are two events such that $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(A \cap B) = 0.2$ then $P(B) =$ (2)

(A) 0.1 (B) 0.3 (C) 0.7 (D) 0.5

13. If A and B are any two mutually exclusive events such that $P(A) = 0.4$, $P(B) = 0.2$ then $P(A \cup B) =$ (2)

(A) 0.8 (B) 0.4 (C) 0.6 (D) 0.7

14. A ball is drawn from a box containing 6 red balls, 4 white balls and 5 black balls. The probability that it is not red is (2)

(A) $\frac{4}{15}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

15. The probability of drawing a white ball from a bag containing 3 black and 4 white balls is (2)

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{1}{7}$ (D) $\frac{2}{7}$

16. The chances to fail in physics are 20% and the chances to fail in mathematics are 10%. The chances to fail in at least one subject is(2)

(A) 28% (B) 38% (C) 52% (D) 62%

17. Probability that a leap year selected at random will contain 53 Sunday is (2)

(A) $\frac{1}{7}$ (B) $\frac{6}{7}$ (C) $\frac{3}{7}$ (D) $\frac{2}{7}$

18. Probability that a non leap year (ordinary year) has 53 Sunday is (2)

(A) $\frac{6}{7}$ (B) $\frac{1}{7}$ (C) $\frac{3}{7}$ (D) $\frac{2}{7}$

19. In a simultaneous throw of three coins the probability of getting at least two tail is (2)

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

20. Three coins are tossed simultaneously. The probability of getting at most two head is (2)

(A) $\frac{7}{8}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{1}{8}$

21. A coin is tossed and a dice is rolled. The probability that the coin shows the head and dice shows 6 is (2)

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$

22. An envelope contains six tickets with numbers 1, 2, 3, 5, 6, 7. Another envelope contains four tickets with numbers 1, 3, 5, 7. An envelope is chosen at random and ticket is drawn from it. Probability that the ticket bears the numbers 2 or 7 is (2)

(A) $\frac{1}{6}$ (B) $\frac{7}{24}$ (C) $\frac{1}{8}$ (D) $\frac{5}{24}$

23. There are six married couples in a room. If two persons are chosen at random, the probability that they are of different sex is (2)

(A) $\frac{3}{11}$ (B) $\frac{1}{11}$ (C) $\frac{5}{11}$ (D) $\frac{6}{11}$

24. A, B play a game of alternate tossing a coin, one who gets head first wins the game. The probability of B winning the game if A has start is (2)

(A) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots$ (B) $\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\ldots$

(C) $\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\ldots$ (D) $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots$

25. A, B play a game of alternate tossing a coin, one who gets head first wins the game. The probability of A winning the game if A has start is (2)

(A) $\frac{1}{2}+\frac{1}{2^3}+\frac{1}{2^5}+\ldots$ (B) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots$

(C) $\frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\ldots$ (D) $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots$

26. If A and B are two independent events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{5}$ then $P(A\cap B)=$ (2)

(A) $\frac{1}{15}$ (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{1}{10}$

27. If A and B are two independent events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ then $P(A\cup B)=$ (2)

(A) $\frac{3}{5}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$

28. If A and B are two independent events such that $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ then $P(\bar{A}\cap\bar{B})$ [i.e. P (neither A nor B)] = (2)

(A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

29. A can hit the target 2 out of 5 times, B can hit the target 1 out of 3 times, C can hit the target 3 out of 4 times. The probability that all of them hit the target is (2)

(A) $\frac{9}{10}$ (B) $\frac{4}{10}$ (C) $\frac{1}{10}$ (D) $\frac{7}{10}$

30. A can hit the target 3 out of 5 times, B can hit the target 1 out of 3 times. The probability that no one can hit the target is (2)

(A) $\frac{7}{15}$ (B) $\frac{3}{5}$ (C) $\frac{1}{15}$ (D) $\frac{4}{15}$

31. A problem in statistics is given to three students A, B, C whose chance of solving it are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ respectively. The probability that all of them can solved the problem is (2)

(A) $\frac{1}{8}$ (B) $\frac{1}{24}$ (C) $\frac{1}{12}$ (D) $\frac{1}{6}$

32. The probability that A can solve a problem is $\frac{2}{3}$ and B can solve it is problem is $\frac{3}{4}$. If both attempt the problem, then the probability that the problem get solved is (2)

(A) $\frac{11}{12}$ (B) $\frac{7}{12}$ (C) $\frac{5}{12}$ (D) $\frac{9}{12}$

33. If A and B are any two events with $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$ then $P(A|B)=$ (2)

(A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{1}{4}$ (D) $\frac{2}{3}$

34. If A and B are any two events with $P(A)=\frac{1}{4}$, $P(B)=\frac{1}{3}$ and $P(A\cup B)=\frac{1}{2}$ then $P(A|B)=$ (2)

(A) $\frac{1}{2}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{4}$

35. If A and B are any two events with P(A) = 0.25, P(B) = 0.15 and $P(A \cup B) = 0.3$ then P (B|A) =

(A) 0.1 (B) 0.6 (C) 0.4 (D) 0.5 (2)

36. In a class 40% students read statistics, 25% read mathematics and 15% read both statistics and mathematics. One student is selected at random. The probability that he read statistics if it is known that he read mathematics is

(A) 0.6 (B) 0.7 (C) 0.5 (D) 0.4 (2)

Answers

1. (A)	2. (D)	3. (C)	4. (B)	5. (B)	6. (A)	7. (C)	8. (D)
9. (B)	10. (C)	11. (A)	12. (D)	13. (C)	14. (D)	15. (B)	16. (A)
17. (D)	18. (B)	19. (C)	20. (A)	21. (A)	22. (B)	23. (D)	24. (C)
25. (A)	26. (D)	27. (B)	28. (C)	29. (C)	30. (D)	31. (B)	32. (A)
33. (B)	34. (D)	35. (C)	36. (A)				

6.6 PROBABILITY DISTRIBUTION

In Chapter 5, we have seen that statistical data can be presented in the form of frequency distribution, giving tabulated values of variate x and corresponding frequencies. Probability distribution for a variate x can be presented in a similar manner.

6.6.1 Random Variable, Probability Density Function Sample Space

If a trial or an experiment is conducted, the set S of all possible outcomes is called sample space.

In an experiment of tossing a fair coin, which results in Head H or Tail T, the sample space S = {H, T}. If a coin is tossed two times successively, all possible outcomes are HH, TT, HT, TH. Sample space in this case is the set S = {HH, TT, HT, TH}.

If a die is thrown two times successively, sample space

$$S = \{(1, 1), (1, 2) \ldots (1, 6)\}$$

$$\ldots\ldots$$

$$= \{(6, 1)\ (6, 2) \ldots (6, 6)\}$$

Random Variable : It is a real valued function defined over the sample space of an experiment. A variable whose value is a number determined by the out come of an experiment, associated with a sample space is called random variable. It is usually denoted by capital letter X or Y etc. If out comes are 0_i or x_i i = 1, 2, 3, ... then $x(0_i)$ or $X(x_i)$ or $f(x_i)$ stands for the value x at $x = x_i$.

Probability Function : X is random variable with values x_i, i = 0, 1, 2, ... n and associated probabilities $p(x_i)$. The set p with elements $[x_i, p(x_i)]$ is called the probability function or probability distribution function of X. It can also be called probability density function of x.

Illustration

Ex. 1 : *A coin is tossed which results in Head or Tail. Let X be the random variable whose value for any outcome is the number of Heads obtained. Find the probability function of x and contruct a probability distribution table.*

Sol. : Let H denote a head and T a tail

Sample space is $S = \{H, T\}$

$$X(H) = 1, \; X(T) = 1$$

x is number of Heads which takes the values 0 and 1

$$f(x) = p(X = x)$$

$$f(0) = \frac{1}{2}, \; f(1) = \frac{1}{2}$$

Probability distribution table is

x(x)	0	1
f(x)	$\frac{1}{2}$	$\frac{1}{2}$

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Marks

1. Three coins are tossed together, x the random variable which denote the number of heads with distribution give (2)

x	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

the mathematical expectation E (x) is

(A) $\frac{13}{8}$ (B) $\frac{3}{2}$ (C) $\frac{9}{8}$ (D) $\frac{2}{3}$

2. The probability distribution of x is (2)

x	1	2	3	4
P(x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

the mathematical expectation E (x) is

(A) $\frac{11}{8}$ (B) $\frac{13}{8}$ (C) $\frac{15}{8}$ (D) $\frac{9}{8}$

3. The probability distribution of x is (2)

x	1	2	3	4
P(x)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

the mathematical expectation E (x) is

(A) 2 (B) 3 (C) 5 (D) 7

4. If x is random variable with distribution given below (2)

x	0	1	2	3
P(x)	k	3k	3k	k

the value of k is

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{8}$ (D) $\frac{2}{3}$

5. If x is random variable with distribution given below (2)

x	2	3	4	5
P(x)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

the value of k is

(A) 16 (B) 8 (C) 48 (D) 32

6. Let f (x) be the continuous probability density function of random variable x then $P(a \le x \le b)$ is (1)

(A) $\int_a^b f(x)\,dx$ (B) f (b) – f (a) (C) f (b – a) (D) $\int_a^b x\,f(x)\,dx$

7. If probability density function f (x) of a continuous random variable x is defined by $f(x) = \begin{cases} \frac{1}{4}, & -2 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$, then $P(x \le 1)$ is (2)

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{3}{4}$

. If probability density function f (x) of a continuous random variable x is defined by $f(x) = \begin{cases} \frac{3}{2}x^2, & -1 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$, then $P\left(-\frac{1}{3} \le x \le \frac{1}{3}\right)$ is (2)

(A) $\frac{2}{27}$ (B) $\frac{1}{27}$ (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

. If probability density function f (x) of a continuous random variable x is defined by $f(x) = \begin{cases} \frac{A}{x^3}, & 5 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$, then the value of A is (2)

(A) $\frac{50}{3}$ (B) $\frac{250}{3}$ (C) $\frac{100}{3}$ (D) $\frac{200}{3}$

Answers

1. (B)	2. (C)	3. (B)	4. (C)	5. (D)	6. (A)	7. (D)	8. (B)	9. (D)

.7 BINOMIAL PROBABILITY DISTRIBUTIONS

Consider the experiment or a trial which has only two outcomes, a success or failure with p as the probability of success and q as e probability of failure. Since there are only two outcomes, $p + q = 1$.

Let us consider series of n such trials each of which either results in success or failure.

) find the probability of r successes in n trials, consider one run of outcomes.

$$\underbrace{SSS \ldots\ldots S}_{r} \quad \underbrace{FFF \ldots\ldots F}_{n-r}$$

In which there are r consecutive successes and n − r failures.

Probability of this event is given by

$$\begin{aligned} P(SSS \ldots S\ FFF \ldots F) &= P(S)\,P(S) \ldots (r \text{ times}) \times P(F)\,P(F) \ldots \{(n-r) \text{ times}\} \\ &= pp \ldots p\ (r \text{ times}) \times qq \ldots q\ (n - r \text{ times}) \\ &= p^r q^{n-r} \end{aligned}$$

r success and n − r failures can occur in ${}^nC_r$ mutually exclusive cases each of which has the probability $p^r q^{n-r}$.

∴ Probability of r success in n trials is ${}^nC_r \cdot p^r q^{n-r}$. This formula gives probability of r = 0, 1, 2, 3, ... n success in n trials.

Putting it in tabular form,

r	0	1	2	3	......	n
p (r)	${}^nC_0 p^0 q^n$	${}^nC_1 p^1 q^{n-1}$	${}^nC_2 p^2 q^{n-2}$	${}^nC_3 p^3 q^{n-3}$	......	${}^nC_n p^n q^{n-n}$

$${}^nC_0 = 1, \quad {}^nC_n = 1$$

Consider now the Binomial expansion of

$$(q + p)^n = q^n + {}^nC_1 q^{n-1} p + {}^nC_2 q^{n-2} p^2 + \ldots + p^n$$

Terms on R.H.S. of this expansion give probability of r = 0, 1, 2, ..., n success. This is the reason for above probability distribution to called Binomial probability distribution. It is denoted by B (n, p, r).

Thus, $$B(n, p, r) = {}^nC_r\, p^r q^{n-r}$$

Illustrations

***Ex. 1 :** An unbiased coin is thrown 10 times. Find the probability of getting exactly 6 Heads, at least 6 Heads.*

Sol. : Here $p = q = \frac{1}{2}$ and $n = 10$. Here occurrence of Head is treated as success.

Probability of getting exactly 6 Heads is

$$P(6) = {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4$$

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Probability Distributions.

Marks

1. In binomial probability distribution, probability of r successes in n trials is (where p probability of successes and q probability of failure in a single trial) (1)

   (A) $p^r q^{n-r}$ (B) $^nC_r p^r q^{n+r}$ (C) $^nC_r p^r q^{n-r}$ (D) $^rC_n p^n q^{n-r}$

2. Mean of binomial probability distribution is (1)

   (A) nq (B) n^2p (C) npq (D) np

3. Variance of binomial probability distribution is (1)

   (A) npq (B) np (C) np^2q (D) npq^2

4. Standard deviation of binomial probability distribution is (1)

   (A) $\sqrt{pq}$ (B) $\sqrt{npq}$ (C) $\sqrt{np}$ (D) np

5. An unbiased coin is thrown five times. Probability of getting three heads is (1)

   (A) $\frac{1}{16}$ (B) $\frac{3}{16}$ (C) $\frac{5}{16}$ (D) $\frac{5}{8}$

. 20% of bolts produced by machine are defective. The probability that out of three bolts chosen at random 1 is defective is (2)

(A) 0.384 (B) 0.9728 (C) 0.5069 (D) 0.6325

7. Probability of man now aged 60 years will live upto 70 years of age is 0.65. The probability that out of 10 men 60 years old 2 men will live upto 70 is (2)

(A) 0.5 (B) 0.002281 (C) 0.003281 (D) 0.004281

. The probability that a person hit a target in shooting practice is 0.3. If the shoots 10 times, the probability that he hits the target is (2)

(A) 1 (B) $1-(0.7)^{10}$ (C) $(0.7)^{10}$ (D) $(0.3)^{10}$

9. An unbiased coin is tossed five times. The probability of getting at least one head is (2)

(A) $\frac{1}{32}$ (B) $\frac{31}{32}$ (C) $\frac{16}{32}$ (D) $\frac{13}{32}$

10. A box contains 100 bulbs out of which 10 are defective. A sample of 5 bulbs is drawn. The probability that none is defective is (2)

(A) $\left(\frac{1}{10}\right)^5$ (B) $\left(\frac{1}{2}\right)^5$ (C) $\left(\frac{9}{10}\right)^5$ (D) $\frac{9}{10}$

11. On an average a packet containing 10 blades is likely to have two defective blades. In a box containing 100 packets, number of packets expected to contain less than two defective blades is (2)

(A) 38 (B) 52 (C) 26 (D) 47

2. Out of 2000 families with 4 children each, the number of families you would expect to have no girls is

p = probability of having a boy = $\frac{1}{2}$, q = probability of having a girl = $1-\frac{1}{2}=\frac{1}{2}$ (2)

(A) 300 (B) 150 (C) 200 (D) 125

13. In 100 set of 10 tosses of a coin, the number of cases you expect 7 head and 3 tail is (2)

(A) 8 (B) 12 (C) 15 (D) 17

14. 20% of bolts produced by machine are defective. The mean and standard deviation of defective bolts in total of 900 bolts are respectively (2)

(A) 180 and 12 (B) 12 and 180 (C) 90 and 12 (D) 9 and 81

15. The mean and variance of binomial probability distribution are $\frac{5}{4}$ and $\frac{15}{16}$ respectively. Probability of success in a single trial p is equal to (2)

(A) $\frac{1}{2}$ (B) $\frac{15}{16}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

16. The mean and variance of binomial probability distribution are 6 and 4 respectively. Number of trials n is given by (2)

(A) 14 (B) 10 (C) 12 (D) 18

7. The mean and standard derivation of binomial probability distribution are 36 and 3 respectively. Number of trials n is given by (2)

(A) 42 (B) 36 (C) 48 (D) 24

. The mean and variance of binomial probability distribution are 6 and 2 respectively. p $(r \geq 2)$ is (2)

(A) 0.66 (B) 0.88 (C) 0.77 (D) 0.99

. If X follows the binomial distribution with parameter n = 6 and p and 9P (X = 4) = P(X = 2), then p is equal to (2)

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{2}{3}$

. If X follows the binomial distribution with parameter n and p = $\frac{1}{2}$ and P(X = 6) = P(X = 8), then n is equal to (2)

(A) 10 (B) 14 (C) 12 (D) 7

1. If X follows the binomial distribution with parameter n and p = $\frac{1}{2}$ and P (X = 4) = P(X = 5), then P(X = 2) is equal to (2)

(A) ${}^{7}C_2\left(\frac{1}{2}\right)^7$ (B) ${}^{11}C_2\left(\frac{1}{2}\right)^{11}$ (C) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$ (D) ${}^{9}C_2\left(\frac{1}{2}\right)^9$

. The mean and variance of binomial probability distribution are 1 and $\frac{2}{3}$ respectively. Then p(r < 1) is (2)

(A) $\frac{4}{27}$ (B) $\frac{8}{27}$ (C) $\frac{5}{27}$ (D) $\frac{1}{27}$

23. In a binomial probability distribution, the probability of getting a success is $\frac{1}{4}$ and standard deviation is 3. Then its mean is (2)

(A) 6 (B) 8 (C) 12 (D) 10

24. A dice is thrown 10 times. If getting even number is considered as success, then the probability of getting four successes is (2)

(A) ${}^{10}C_4\left(\frac{1}{2}\right)^{10}$ (B) ${}^{10}C_4\left(\frac{1}{2}\right)^{4}$ (C) ${}^{10}C_4\left(\frac{1}{2}\right)^{8}$ (D) ${}^{10}C_4\left(\frac{1}{2}\right)^{6}$

25. A fair coin is tossed n number of times. In a binomial probability distribution, if the probability of getting 7 heads is equal to that of getting 9 then n is equal to (2)

(A) 7 (B) 2 (C) 9 (D) 16

26. If z = np where n the number of trials is very large and p the probability of success at each trial, then in Poisson's probability distribution p(r) the probability of r successes is given by (1)

(A) $\frac{e^{z} z}{r!}$ (B) $\frac{e^{-z} z^{r}}{r}$ (C) $\frac{e^{-z} z^{r}}{r!}$ (D) $\frac{e^{z} z^{r}}{r!}$

27. In a Poisson's probability distribution if n – 100, p – 0.01, p(r – 0) is given by (2)

(A) $\frac{1}{e}$ (B) $\frac{2}{e}$ (C) $\frac{3}{e}$ (D) $\frac{4}{e}$

28. In a Poisson's probability distribution if n = 100, p = 0.02, p(r = 1) is given by (2)

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{2}{e}$ (D) $\frac{1}{e}$

29. For a tabular data (2)

x	0	1	2	3
F	2	4	6	8

Poisson's fit p(r) is given by

(A) $\frac{e^{-1} 2^{r}}{r!}$ (B) $\frac{e^{-2} 2^{r}}{r!}$ (C) $\frac{e^{-2} 2^{3}}{r!}$ (D) $\frac{e^{-3} 3^{r}}{r!}$

30. For a tabulated data : (2)

x	0	1	2	3
f	1	4	15	24

Poisson's fit p(r) is given by

(A) $\frac{e^{-4.609} (4.609)^{r}}{r!}$ (B) $\frac{e^{-6.709} (6.709)^{r}}{r!}$ (C) $\frac{e^{-3.509} (3.509)^{r}}{r!}$ (D) $\frac{e^{-2.409} (2.409)^{r}}{r!}$

31. In a Poisson's probability distribution if p (r = 1) = 2p (r = 2) and p (r = 3) is given by (2)

(A) $\frac{1}{6e}$ (B) $\frac{2}{3e}$ (C) $\frac{1}{8e}$ (D) $\frac{1}{9e}$

32. In a Poisson's probability distribution if 3p (r = 4) = p (r = 5) and p (r = 6) is given by (2)

(A) $\frac{e^{-12} (12)^{6}}{6!}$ (B) $\frac{e^{-18} (18)^{6}}{6!}$ (C) $\frac{e^{-15} (15)^{6}}{6!}$ (D) $\frac{e^{-10} (10)^{6}}{6!}$

33. In a Poisson's probability distribution if p (r = 2) = 9p (r = 4) + 90p (r = 6) then mean of the distribution is (2)

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

34. Number of road accidents on a highway during a month follows a Poisson distribution with mean 2. Probability that in a certain month number of accidents on the highway will be equal to 2 is (2)

(A) 0354 (B) 0.2707 (C) 0.435 (D) 0.521

35. Between 2 P.M. and 3 P.M. the average number of phone calls per minute coming into company are 2. Using Poisson's probability distribution, the probability that during one particular minute there will be no phone call at all, is given by (2)

(A) 0.354 (B) 0.356 (C) 0.135 (D) 0.457

36. Average number of phone calls per minute coming into company are 3, during certain period. These calls follows Poisson's probability distribution. Probability that during one particular minute there will be less than two calls, is given by (2)

(A) 0.299 (B) 0.333 (C) 0.444 (D) 0.199

37. In a certain factory turning out razor blades, there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are supplied in a packets of 10. Using Poisson distribution, the probability that a packet contain one defective blade is (2)
(A) 0.0196 (B) 0.0396 (C) 0.0596 (D) 0.0496

38. The average number of misprints per page of a book is 1.5. Assuming the distribution of number of misprints to be Poisson. The probability that a particular book is free from misprints, is (2)
(A) 0.329 (B) 0.435 (C) 0.549 (D) 0.2231

39. The probability density function of normal variable x with mean μ and variance σ^2 is (2)
(A) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$ (B) $f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$ (C) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ (D) $f(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

40. Normal distribution curve is given by the equation $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Integral $\int_{\mu}^{\infty} y\,dx$ has the value. (1)
(A) 0.025 (B) 1 (C) 0.5 (D) 0.75

41. Normal distribution curve is given by the equation $y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Integral $\int_{-\infty}^{\infty} y\,dx$ has the value (1)
(A) 0.025 (B) 1 (C) 0.5 (D) 0.75

42. X is normally distributed. The mean of X is 15 and standard deviation 3. Given that for z = 1, A = 0.3413, p (X ≥ 18) is given by (2)
(A) 0.1587 (B) 0.4231 (C) 0.2231 (D) 0.3413

43. X is normally distributed. The mean of X is 15 and standard deviation 3. Given that for z = 1, A = 0.3413, p (X ≥ 12) is given by (2)
(A) 0.6587 (B) 0.8413 (C) 0.9413 (D) 0.7083

44. X is normally distributed. The mean of X is 15 and standard deviation 3. Given that for z = 1.666, A = 0.4515, p (x ≤ 10) is given by (2)
(A) 0.0585 (B) 0.0673 (C) 0.0485 (D) 0.1235

45. X is normally distributed. The mean of X is 30 and variance 25. The probability p (26 ≤ x ≤ 40) is (Given : Area corresponding to z = 0.8 is 0.2881 and Area corresponding to z = 2 is 0.4772). (2)
(A) 0.8562 (B) 0.6574 (C) 0.3745 (D) 0.7653

46. In a sample of 1000 candidates, the mean of certain test is 14 and standard deviation is 2.5. Assuming Normal distribution, the probability of candidates getting less than eight marks i.e. p (x ≤ 8) is (Given : Area corresponding to z = 2.4 is 0.4918) (2)
(A) 0.0054 (B) 0.0075 (C) 0.0082 (D) 0.0035

47. In a normally distributed group of 450 students with mean 42 and standard deviation 8, the number of students scoring less than 48 marks is (Given : Area corresponding to z = 0.75 is 0.2734). (2)
(A) 348 (B) 102 (C) 127 (D) 250

48. In a certain examination test 10000 students appeared in a subject of mathematics. Average marks obtained were 50% with standard deviation 5%. Marks are normally distributed. Number of students expected to get more than 60% marks is equal to (2)
(z = 2, A = 0.4772)
(A) 200 (B) 300 (C) 325 (D) 228

49. For normal variable x with probability density function $f(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{18}(x-6)^2}$ the mean μ and standard deviation σ are (2)
(A) 3, 9 (B) 9, 6 (C) 6, 3 (D) 18, 6

Answers

1. (C)	2. (D)	3. (A)	4. (B)	5. (C)	6. (A)	7. (D)	8. (B)	9. (B)	10. (C)
11. (A)	12. (D)	13. (B)	14. (A)	15. (C)	16. (D)	17. (C)	18. (D)	19. (A)	20. (B)
21. (D)	22.(B)	23.(C)	24.(A)	25.(D)	26. (C)	27. (A)	28. (B)	29. (B)	30. (D)
31. (A)	32. (C)	33. (A)	34. (B)	35. (A)	36.(D)	37. (A)	38. (D)	39. (C)	40. (C)
41. (B)	42. (A)	43. (B)	44. (C)	45. (D)	46. (C)	47. (A)	48. (D)	49. (C)	

Type II : Chi-square Distribution :

1. A bank utilizes three teller windows to render service to the customer. On a particular day 600 customer were served. If the customers are uniformly distributed over the counters. Expected numbers of customer served on each counter is
   (A) 100 (B) 200 (C) 300 (D) 150
2. 200 digits are chosen at random from a set of tables. The frequencies of the digits are as follows :

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	22	20	21	15

   The expected frequency and degree of freedom for uniform distribution is
   (A) 20 and 10 (B) 21 and 9 (C) 20 and 9 (D) 15 and 8
3. In experiment on pea breeding, the observed frequencies are 222, 120, 32, 150 and expected frequencies are 323, 81, 81, 40, then χ_3^2 has the value (2)
   (A) 382.502 (B) 380.50 (C) 429.59 (D) 303.82
4. If observed frequencies O_1, O_2, O_3 are 5, 10, 15 and expected frequencies e_1, e_2, e_3 are each equal to 10, then χ_2^2 has the value (2)
   (A) 20 (B) 10 (C) 15 (D) 5
5. Number of books issued on six days of the week, excluding Sunday which is holiday are given as 120, 130, 110, 115, 135, 110 and expectation is 120 books on each day, then χ_5^2 is
   (A) 2.58 (B) 3.56 (C) 6.56 (D) 4.58
6. A coin is tossed 160 times and following are expected and observed frequencies for number of heads

No. of heads	0	1	2	3	4
Observed frequency	17	52	54	31	6
Expected Frequency	10	40	60	40	10

   Then χ_4^2 is
   (A) 12.72 (B) 9.49 (C) 12.8 (D) 9.00
7. Among 64 offspring's of a certain cross between guinea pig 34 were red, 10 were black and 20 were white. Acceding to genetic model, these number should in the ratio 9 :3 : 4. Expected frequencies in the order
   (A) 36, 12, 16 (B) 12, 36, 16 (C) 20, 12, 16 (D) 36, 12, 25
8. A sample analysis of examination results of 500 students was made. The observed frequencies are 220, 170, 90 and 20 and the numbers are in the ratio 4 : 3 : 2 : 1 for the various categories. Then the expected frequencies are
   (A) 150, 150, 50, 25 (B) 200, 100, 50, 10 (C) 200, 150, 100, 50 (D) 400, 300, 200, 100
9. In experiment on pea breeding, the observed frequencies are 222, 120, 32, 150 and the theory predicts that the frequencies should be in proportion 8 : 2 : 2 : 1. Then the expected frequencies are (2)
   (A) 323, 81, 40, 81 (B) 81, 323, 40, 81 (C) 323, 81, 81, 40 (D) 433, 81, 81, 35

Answers

1. (B)	2. (C)	3. (A)	4. (D)	5. (D)	6. (A)	7. (A)	8. (C)	9. (C)

◈◈◈

16. If $\bar{r}(t) = t^2\,\bar{i} + t\,\bar{j} - 2t^3\,\bar{k}$, then evaluate $\int_1^2 \bar{r} \times \frac{d^2\bar{r}}{dt^2}\,dt$. **(May 2012)** **Ans.** $-28\,\bar{i} + 30\,\bar{j} - 3\,\bar{k}$

17. If $\bar{r} = \bar{a}\sinh t + \bar{b}\cosh t$, then prove that

(i) $\frac{d^2\bar{r}}{dt^2} = \bar{r}$ (ii) $\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} = \text{constant}$ (iii) $\bar{r} \cdot \frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} = 0$

18. The position vector of a particle at time t is

$$\bar{r} = \cos(t-1)\,\bar{i} + \sinh(t-1)\,\bar{j} + mt^3\,\bar{k}$$

Find the condition imposed on m by requiring that at time t = 1, the acceleration is normal to the position vector.

(Dec. 2005, May 2006) **Ans.** $m = \frac{1}{\sqrt{6}}$

19. Prove that if a particle moves always on the surface of the sphere

(i) $\bar{r} \cdot \bar{a} + \bar{V} \cdot \bar{V} = 0$ (ii) $\bar{r} \cdot \bar{a} \le 0$

20. If a particle P moves along the curve $r = ae^{\theta}$ with constant angular velocity ω, then show that the radial and transverse components of its velocity are equal and its acceleration is always perpendicular to radius vector and is equal to $2r\omega^2$.

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Type : Vector Differentiation : **Marks**

1. If $\bar{r}(t)$ is position vector of a point on the curve C where t is a scalar variable then $\frac{d\bar{r}}{dt}$ represents (1)

(A) Tangent vector (B) Normal vector (C) Radius vector (D) Orthogonal vector

2. If $\bar{r}(t) = x(t)\,\bar{i} + y(t)\,\bar{j} + z(t)\,\bar{k}$ be the position vector of a particle moving along the curve at time t then $\frac{d\bar{r}}{dt}$ represents (1)

(A) Acceleration vector (B) Velocity vector (C) Radius vector (D) Normal vector

3. If $\bar{r}(t) = x(t)\,\bar{i} + y(t)\,\bar{j} + z(t)\,\bar{k}$ be the position vector of a particle moving along the curve at time t then $\frac{d^2\bar{r}}{dt^2}$ represents (1)

(A) Radius vector (B) Velocity vector (C) Acceleration vector (D) Orthogonal vector

4. For vector function $\bar{u}(t)$ and $\bar{v}(t)$, $\frac{d}{dt}(\bar{u} \cdot \bar{v}) =$ (1)

(A) $\bar{u} \cdot \frac{d\bar{v}}{dt} - \frac{d\bar{u}}{dt} \cdot \bar{v}$ (B) $\bar{u} \times \frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt} \times \bar{v}$ (C) $\bar{u} \cdot \frac{d\bar{u}}{dt} + \frac{d\bar{v}}{dt} \cdot \bar{v}$ (D) $\bar{u} \cdot \frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt} \cdot \bar{v}$

5. For vector functiosn $\bar{u}(t)$ and $\bar{v}(t)$, $\frac{d}{dt}(\bar{u} \times \bar{v}) =$ (1)

(A) $\bar{v} \times \frac{d\bar{u}}{dt} + \frac{d\bar{v}}{dt} \times \bar{u}$ (B) $\frac{d\bar{u}}{dt} \times \bar{v} + \bar{u} \times \frac{d\bar{v}}{dt}$ (C) $\frac{d\bar{v}}{dt} \times \bar{v} - \bar{u} \times \frac{d\bar{v}}{dt}$ (D) $\bar{u} \cdot \frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt} \cdot \bar{v}$

6. For vector functions $\bar{u}(t)$, $\bar{v}(t)$ and $\bar{w}(t)$, $\frac{d}{dt}[\bar{u} \cdot (\bar{v} \times \bar{w})] =$ (1)

(A) $\frac{d\bar{u}}{dt} \cdot (\bar{v} \times \bar{w}) - \bar{u} \cdot \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) - \bar{u} \cdot \left(\bar{v} \times \frac{d\bar{w}}{dt}\right)$

(B) $\frac{d\bar{u}}{dt} \times (\bar{v} \times \bar{w}) + \bar{u} \times \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) + \bar{u} \times \left(\bar{v} \times \frac{d\bar{w}}{dt}\right)$

(C) $\frac{d\bar{u}}{dt} \cdot (\bar{v} \times \bar{w}) + \bar{u} \cdot \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) + \bar{u} \cdot \left(\bar{v} \times \frac{d\bar{w}}{dt}\right)$

(D) $\frac{d\bar{u}}{dt} \cdot (\bar{w} \times \bar{v}) + \bar{u} \cdot \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) + \bar{u} \cdot \left(\frac{d\bar{w}}{dt} \times \bar{v}\right)$

7. For vector functions $\bar{u}(t)$, $\bar{v}(t)$ and $\bar{w}(t)$, $\frac{d}{dt}[\bar{u} \times (\bar{v} \times \bar{w})] =$ (1)

(A) $\frac{d\bar{u}}{dt} \times (\bar{v} \times \bar{w}) + \bar{u} \times \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) + \bar{u} \times \left(\bar{v} \times \frac{d\bar{w}}{dt}\right)$

(B) $\frac{d\bar{u}}{dt} \cdot (\bar{v} \times \bar{w}) + \bar{u} \cdot \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) + \bar{u} \cdot \left(\bar{v} \times \frac{d\bar{w}}{dt}\right)$

(C) $\frac{d\bar{u}}{dt} \times (\bar{w} \times \bar{v}) + \bar{u} \times \left(\bar{w} \times \frac{d\bar{v}}{dt}\right) + \bar{u} \times \left(\frac{d\bar{w}}{dt} \times \bar{v}\right)$

(D) $\frac{d\bar{u}}{dt} \times (\bar{w} \times \bar{v}) + \left(\frac{d\bar{v}}{dt} \times \bar{w}\right) \times \bar{u} + \left(\frac{d\bar{w}}{dt} \times \bar{v}\right) \times \bar{u}$

8. For scalar function s(t) and vector functions $\bar{u}(t)$, $\frac{d}{dt}[s(t)\,\bar{u}(t)] =$ (1)

(A) $\frac{ds}{dt} \cdot \bar{u} + s \cdot \frac{d\bar{u}}{dt}$ (B) $\frac{ds}{dt}\bar{u} - s\frac{d\bar{u}}{dt}$ (C) $\dfrac{\frac{ds}{dt}\bar{u} - s\frac{d\bar{u}}{dt}}{s^2}$ (D) $\frac{ds}{dt}\bar{u} + s\frac{d\bar{u}}{dt}$

9. If $\bar{r} = r\cos\theta\,\bar{i} + r\sin\theta\,\bar{j}$, then $\hat{r}$ is given by (1)

(A) $\cos\theta\,\bar{i} + \sin\theta\,\bar{j}$ (B) $\sin\theta\,\bar{i} + \sec\theta\,\bar{j}$ (C) $\cos\theta\,\bar{i} + \operatorname{cosec}\theta\,\bar{j}$ (D) $\tan\theta\,\bar{i} + \cos\theta\,\bar{j}$

10. A curve is given by $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$. Tangent vectors to the curve at t = 1 and t = 2 are (2)

(A) $2\bar{i} + 4\bar{j} + 2\bar{k}$, $2\bar{i} + 4\bar{j} + \bar{k}$ (B) $2\bar{i} + 4\bar{j} - 2\bar{k}$, $4\bar{i} + 4\bar{j} + 2\bar{k}$

(C) $2\bar{i} + 4\bar{j} - 2\bar{k}$, $2\bar{i} + 4\bar{j} - 2\bar{k}$ (D) $3\bar{i} + 4\bar{j} + 2\bar{k}$, $5\bar{i} + 4\bar{j} - 2\bar{k}$

11. A curve is given by $\bar{r} = (t^3 + 2)\,\bar{i} + (4t - 5)\,\bar{j} + (2t^2 - 6t)\,\bar{k}$. Tangent vectors to the curve at t = 0 and t = 2 are (2)

(A) $3\bar{i} + 4\bar{j} - 6\bar{k}$, $6\bar{i} + 4\bar{j} + 2\bar{k}$ (B) $3\bar{i} - 6\bar{k}$, $12\bar{i} + 4\bar{j} + 2\bar{k}$

(C) $4\bar{j} - 6\bar{k}$, $12\bar{i} + 4\bar{j} + 2\bar{k}$ (D) $4\bar{j} - 6\bar{k}$, $12\bar{i} + 2\bar{k}$

12. A curve is given by $\bar{r} = 2t^2\,\bar{i} + (t^2 - 4t)\,\bar{j} + (2t - 5)\,\bar{k}$. Tangent vectors to the curve at t = 1 and t = 3 are (1)

(A) $2\bar{i} - 2\bar{j} + 2\bar{k}$, $3\bar{i} + 2\bar{j} + 2\bar{k}$ (B) $4\bar{i} + 2\bar{j} + 2\bar{k}$, $12\bar{i} - 2\bar{j} + 2\bar{k}$

(C) $4\bar{i} - 2\bar{j}$, $12\bar{i} + 2\bar{j}$ (D) $4\bar{i} - 2\bar{j} + 2\bar{k}$, $12\bar{i} + 2\bar{j} + 2\bar{k}$

13. The tangent vector to the curve $x = a\cos t$, $y = a\sin t$, $z = at\tan\alpha$ at $t = \frac{\pi}{4}$, where a and α are constants is (2)

(A) $-\frac{a}{\sqrt{2}}\bar{i} + \frac{a}{\sqrt{2}}\bar{j} + a\tan\alpha\,\bar{k}$ (B) $\frac{a}{\sqrt{2}}\bar{i} - \frac{a}{\sqrt{2}}\bar{j} + a\tan\alpha\,\bar{k}$

(C) $-\frac{a}{2}\bar{i} + \frac{a}{2}\bar{j} + a\tan\alpha\,\bar{k}$ (D) $-\frac{a}{\sqrt{2}}\bar{i} + \frac{a}{\sqrt{2}}\bar{j} + \alpha\,\bar{k}$

14. A curve is given by $\bar{r} = (e^t\cos t)\,\bar{i} + (e^t\sin t)\,\bar{j} + (e^t)\,\bar{k}$. Tangent vector to the curve at t = 0 is (2)

(A) $-\bar{i} - \bar{j} - \bar{k}$ (B) $\bar{j} + \bar{k}$ (C) $2\bar{i} + 2\bar{j} + \bar{k}$ (D) $\bar{i} + \bar{j} + \bar{k}$

15. For the curve $\bar{r} = e^{-t}\,\bar{i} + \log(t^2 + 1)\,\bar{j} - \tan t\,\bar{k}$, velocity and acceleration vectors at t = 0 are (2)

(A) $\bar{i} + 2\bar{j} - \bar{k}$, $\bar{i} + 2\bar{j}$ (B) $\bar{i} + \bar{k}$, $\bar{i} + 2\bar{j}$ (C) $-\bar{i} - \bar{k}$, $\bar{i} + 2\bar{j}$ (D) $-\bar{i} - \bar{k}$, $\bar{i} - 2\bar{k}$

16. For the curve $x = t^3 + 1$, $y = t^2$, $z = t$, velocity and acceleration vectors at t = 1 are (2)

(A) $4\bar{i} + 2\bar{j}$, $6\bar{i} + 2\bar{j}$ (B) $3\bar{i} + 2\bar{j} + \bar{k}$, $6\bar{i} + 2\bar{j}$

(C) $2\bar{i} + 2\bar{j} + \bar{k}$, $3\bar{i} + 2\bar{j}$ (D) $3\bar{i} + 2\bar{j}$, $6\bar{i} + \bar{j}$

17. For the curve $x = t, y = t^2, z = t^3$, angle between tangents at $t = 0$ and $t = 1$ is given by (2)

(A) $\frac{\pi}{2}$ (B) $\cos^{-1}\frac{1}{\sqrt{5}}$ (C) $\cos^{-1}\frac{1}{3}$ (D) $\cos^{-1}\left(\frac{1}{\sqrt{14}}\right)$

18. Angle between tangents $\bar{T}_1 = 2\bar{i} + 4\bar{j} - 2\bar{k}$, $\bar{T}_2 = 4\bar{i} + 4\bar{j} + 2\bar{k}$ to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ at $t = 1$ and $t = 2$ is (2)

(A) $\cos^{-1}\left(\frac{5}{\sqrt{6}}\right)$ (B) $\cos^{-1}\left(\frac{1}{3\sqrt{6}}\right)$ (C) $\cos^{-1}\left(\frac{5}{3\sqrt{6}}\right)$ (D) $\tan^{-1}\left(\frac{5}{3\sqrt{6}}\right)$

19. Angle between tangents to the curve $x = 2t^2, y = t^2 - 4t, z = 2t - 5$ at $t = 0$ and $t = 1$ is (2)

(A) $\cos^{-1}\left(\frac{12}{\sqrt{6}\sqrt{5}}\right)$ (B) $\cos^{-1}\left(\frac{3}{\sqrt{6}\sqrt{5}}\right)$ (C) $\cos^{-1}\left(\frac{3}{\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{3}{\sqrt{6}\sqrt{5}}\right)$

20. Angle between tangent to the curve $\bar{r} = (e^t \cos t)\,\bar{i} + (e^t \sin t)\,\bar{j} + (e^t)\,\bar{k}$ at $t = 0$ and z axis is given by (2)

(A) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (B) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (C) $\cos^{-1}(\sqrt{3})$ (D) $\frac{\pi}{2}$

21. If $\bar{r} = \bar{a}\,e^{5t} + \bar{b}\,e^{-5t}$ where $\bar{a}$ and $\bar{b}$ are constant vectors then $\frac{d^2\bar{r}}{dt^2} - 25\bar{r}$ is equal to (2)

(A) 1 (B) 2 (C) zero (D) 5

22. If $\bar{r} = \bar{a}\cos 2t + \bar{b}\sin 2t$ where $\bar{a}$ and $\bar{b}$ are constant vectors then $\frac{d^2\bar{r}}{dt^2}$ is equal to (2)

(A) $-4\bar{r}$ (B) $4\bar{r}$ (C) $-\bar{r}$ (D) $\bar{r}$

23. If $\bar{r} = at\cos t\,\bar{i} + bt\sin t\,\bar{j}$ where a and b are constants then $\frac{d^2\bar{r}}{dt^2}$ at $t = 0$ is equal to (2)

(A) $2b\bar{j}$ (B) $-2a\bar{i}$ (C) $a\bar{i} + b\bar{j}$ (D) $\bar{0}$

24. If $\bar{r} = \bar{a}\cosh t + \bar{b}\sinh t$ where $\bar{a}$ and $\bar{b}$ are constant vectors then $\frac{d^2\bar{r}}{dt^2}$ is equal to (2)

(A) $-2\bar{r}$ (B) $2\bar{r}$ (C) $-\bar{r}$ (D) $\bar{r}$

25. If acceleration vector $\frac{d^2\bar{r}}{dt^2} = -\bar{i} + 6m\bar{k}$, m is constant, is normal to the position vector $\bar{r} = \bar{i} + m\bar{k}$ then value of m is (1)

(A) $\pm\sqrt{6}$ (B) $\pm\frac{1}{\sqrt{6}}$ (C) 0 (D) ± 1

26. If $\bar{r} = \cos(t-1)\,\bar{i} + \sinh(t-1)\,\bar{j} + t^3\bar{k}$ then $\bar{r}\cdot\frac{d^2\bar{r}}{dt^2}$ at $t = 1$ is given by (1)

(A) 4 (B) 5 (C) 2 (D) 1

27. If $\bar{r}(t) = t^2\,\bar{i} + t\,\bar{j} - 2t^3\,\bar{k}$ then the value of $\bar{r}\times\frac{d^2\bar{r}}{dt^2}$ is (2)

(A) $12t^2\,\bar{i} + 8t^3\,\bar{j} + 2t\,\bar{k}$ (B) $-12t^2\,\bar{i} + 8t^3\,\bar{j}$

(C) $-12t^2\,\bar{i} + 16t^3\,\bar{j} + (t^2 - 2t)\,\bar{k}$ (D) $-12t^2\,\bar{i} + 8t^3\,\bar{j} - 2t\,\bar{k}$

28. If $\bar{r} = \bar{a}\cosh t + \bar{b}\sinh t$ where $\bar{a}$ and $\bar{b}$ are constant vectors then $\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}$ is equal to (2)

(A) $\bar{b}\times\bar{a}$ (B) $\bar{a}\times\bar{b}$ (C) $\bar{r}$ (D) zero

29. If $\bar{r} = t\,\bar{i} + 2t\,\bar{j} + t^2\,\bar{k}$ then $\bar{r}\cdot\left(\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}\right)$ is equal to (1)

(A) 1 (B) -1 (C) 0 (D) $\bar{k}$

30. If $\bar{r} \cdot \dfrac{d\bar{r}}{dt} = 0$ then $\bar{r}$ has

(A) Constant direction (B) Constant magnitude

(C) Both constant magnitude and direction (D) None of these

31. An electron moves such that its velocity is always perpendicular to its radius vector then its path is (2)

(A) Ellipse (B) Hyperbola (C) Straight line (D) Circle

32. $\dfrac{d}{dt}\left[\bar{r} \cdot \left(\dfrac{d\bar{r}}{dt} \times \dfrac{d^2\bar{r}}{dt^2}\right)\right] =$ (2)

(A) $\left(\dfrac{d\bar{r}}{dt} \times \dfrac{d^2\bar{r}}{dt^3}\right)$ (B) $\bar{r} \cdot \left(\dfrac{d^2\bar{r}}{dt^2} \times \dfrac{d^3\bar{r}}{dt^3}\right)$

(C) $\bar{r} \cdot \left(\dfrac{d\bar{r}}{dt} \times \dfrac{d^3\bar{r}}{dt^3}\right)$ (D) 0

33. If $\dfrac{d\bar{u}}{dt} = \bar{w} \times \bar{u}$ and $\dfrac{d\bar{v}}{dt} = \bar{w} \times \bar{v}$ then $\dfrac{d}{dt}(\bar{u} \times \bar{v}) =$ (2)

(A) $(\bar{v} \cdot \bar{w})\,\bar{u} - (\bar{u} \cdot \bar{w})\,\bar{v}$ (B) $(\bar{v} \cdot \bar{w})\,\bar{u} + (\bar{v} \cdot \bar{w})\,\bar{u}$

(C) $(\bar{u} \cdot \bar{w})\,\bar{v} - (\bar{u} \cdot \bar{v})\,\bar{w}$ (D) $(\bar{v} \cdot \bar{w})\,\bar{u} + (\bar{u} \cdot \bar{v})\,\bar{w}$

34. If a is a constant vector then $\dfrac{d}{dt}\left[r^3\bar{r} + \bar{a} \times \dfrac{d^2\bar{r}}{dt^2}\right] =$ (2)

(A) $r^3\dfrac{d\bar{r}}{dt} + \bar{a} \times \dfrac{d^2\bar{r}}{dt^2}$ (B) $3r^2\dfrac{dr}{dt}\,\bar{r} + r^3\dfrac{d\bar{r}}{dt} + \bar{a} \times \dfrac{d^3\bar{r}}{dt^3}$

(C) $3r^2\,\bar{r} + r^3\dfrac{d\bar{r}}{dt}$ (D) $r^2\,\bar{r} + r^2\dfrac{d\bar{r}}{dt} + \bar{a} \times \dfrac{d^2\bar{r}}{dt^2}$

35. If $\bar{v} = t^2\,\bar{i} + 2t\,\bar{j} + (4t-5)\,\bar{k}$ then the value of $\bar{v} \cdot \left(\dfrac{d\bar{v}}{dt} \times \dfrac{d^2\bar{v}}{dt^2}\right)$ is (2)

(A) $t^2 - 4t + 5$ (B) 10 (C) $16t + 10$ (D) 20

36. If $\bar{r} = t^2\,\bar{i} + t\,\bar{j}$, value of $\displaystyle\int_0^1 \left(\bar{r} \times \dfrac{d\bar{r}}{dt}\right) dt$ is given by (1)

(A) $\bar{i} + \bar{j}$ (B) $-\dfrac{1}{3}\bar{k}$ (C) $\dfrac{2}{3}(\bar{i} + \bar{k})$ (D) $(\bar{i} - \bar{k})$

Answers

1. (A)	2. (b)	3. (C)	4. (D)	5. (B)	6. (C)	7. (A)	8. (D)
9. (A)	10. (B)	11. (C)	12. (D)	13. (A)	14. (D)	15. (C)	16. (B)
17. (D)	18. (C)	19. (B)	20. (A)	21. (C)	22. (A)	23. (A)	24. (D)
25. (B)	26.(B)	27. (D)	28. (A)	29. (C)	30. (B)	31. (D)	32. (C)
33. (A)	34. (B)	35. (D)	36. (B)				

8.3 GRADIENT, DIVERGENCE AND CURL

Before we define these quantities which are so often encountered in vector analysis, we shall introduce certain terms.

(i) **Scalar Point Function :** If a scalar quantity ϕ depends for its value on its position say (x, y, z) in space, then ϕ (x, y, z) is called scalar point function. Pressure in a fluid usually varies according to its depth, hence p (x, y, z) is a scalar point function. Temperature, density, potential etc. are other examples of a scalar point functions, as these quantities usually take different values at different points.

(ii) **Vector Point Function :** If a vector quantity $\bar{F}$ depends for its value on its position (x, y, z) in space, then $\bar{F}$ (x, y, z) is called vector point function. Velocity, Force, Electric Intensity etc. are examples of vector point functions.

... be a position vector such that $r = |\bar{r}|$ and $\bar{u}$ be a differentiable vector function, then using vector identities, prove that,

$\nabla \int r^n \, dr = r^{n-1} \, \bar{r}$

$\nabla^2 (r^n \log r) = [n(n+1) \log r + 2n + 1] \, r^{n-2}$ **(May 2005)**

...r scalars ϕ and ψ, show that $\nabla \times (\phi \nabla \psi) = \nabla \phi \times \nabla \psi = -\nabla \times (\psi \nabla \phi)$.

... $= (y+z)\,\bar{i} + (z+x)\,\bar{j} + (x+y)\,\bar{k}$ then show that curl curl curl curl $\bar{F} = \nabla^4 [(y+z)\,\bar{i} + (z+x)\,\bar{j} + (x+y)\,\bar{k}]$

...$\bar{w}$ is constant vector and $\bar{v} = \bar{w} \times \bar{r}$, prove that div $\bar{v} = 0$.

Prove that $\bar{F} = \dfrac{1}{(x^2+y^2)} (x\,\bar{i} + y\,\bar{j})$ is solenoidal.

Find the function f(r) so that $f(r)\,\bar{r}$ is solenoidal.

...$\bar{u}$ and $\bar{v}$ are irrotational vectors then prove that $\bar{u} \times \bar{v}$ is solenoidal vector. **(Dec. 2005, 2006, 2007)**

...ϕ, ψ satisfy Laplace equation, then prove that the vector $(\phi \nabla \psi - \psi \nabla \phi)$ is solenoidal.

...ow that $\bar{F} = \dfrac{\bar{a} \times \bar{r}}{r^n}$ is solenoidal field.

...$\bar{F}_1 = yz\,\bar{i} + zx\,\bar{j} + xy\,\bar{k}$, $\bar{F}_2 = (\bar{a} \cdot \bar{r})\,\bar{a}$ then show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. **(May 2006)**

...erify whether following fields are irrotational and if so, find corresponding potential ϕ.

... $(y \sin z - \sin x)\,\bar{i} + (x \sin z + 2yz)\,\bar{j} + (xy \cos z + y^2)\,\bar{k}$. **(May 09, Nov. 14, 15)**

... $\dfrac{\bar{r}}{r^2}$ (iii) $(\bar{a} \cdot \bar{r})\,\bar{a}$.

...ow that the vector field given by $\bar{F} = (y^2 \cos x + z^2)\,\bar{i} + (2y \sin x)\,\bar{j} + 2xz\,\bar{k}$ is conservative and find scalar field such that ... $= \nabla \phi$. **(Dec. 04, 12, 16; May 12)**

...the vector field $\bar{F} = (x + 2y + az)\,\bar{i} + (bx - 3y - z)\,\bar{j} + (4x + cy + 2z)\,\bar{k}$ is irrotational, find a, b, c and determine ϕ such that ... $= \nabla \phi$. **(Dec. 05, 06)**

...ow that $\bar{F} = r^2 \bar{r}$ is conservative and obtain the scalar potential associated with it. **(May 2015)**

...ow that $\bar{F} = (2xz^3 + 6y)\,\bar{i} + (6x - 2yz)\,\bar{j} + (3x^2z^2 - y^2)\,\bar{k}$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla \phi$.

Ans. $\phi = 6xy + x^2z^3 - y^2z$

...ow that vector field $\bar{F} = (x^2 - yz)\,\bar{i} + (y^2 - zx)\,\bar{j} + (z^2 - xy)\,\bar{k}$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla \phi$. **(May 2014)**

Ans. $\phi = x^3/3 + y^3/3 + z^3/3 - xyz + c$

MULTIPLE CHOICE QUESTIONS (MCQ'S)

Marks

...e : Gradient, Divergence, Curl and Directional Derivative

...tor differential oprator ∇ is defined by (1)

... $\bar{i}\dfrac{\partial}{\partial x} + \bar{j}\dfrac{\partial}{\partial y} + \bar{k}\dfrac{\partial}{\partial z}$ (B) $\dfrac{\partial}{\partial x} + \dfrac{\partial}{\partial y} + \dfrac{\partial}{\partial z}$

... $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ (D) $\bar{i}\dfrac{\partial^2}{\partial x^2} + \bar{j}\dfrac{\partial^2}{\partial y^2} + \bar{k} + \dfrac{\partial^2}{\partial z^2}$

...dient of scalar point function $\phi(x, y, z)$ is (1)

... $\dfrac{\partial^2\phi}{\partial x^2} + \dfrac{\partial^2\phi}{\partial y^2} + \dfrac{\partial^2\phi}{\partial z^2}$ (B) $\dfrac{\partial\phi}{\partial x} + \dfrac{\partial\phi}{\partial y} + \dfrac{\partial\phi}{\partial z}$ (C) $\dfrac{\partial\phi}{\partial x}\dfrac{\partial\phi}{\partial y}\dfrac{\partial\phi}{\partial z}$ (D) $\dfrac{\partial\phi}{\partial x}\bar{i} + \dfrac{\partial\phi}{\partial y}\bar{j} + \dfrac{\partial\phi}{\partial z}\bar{k}$

...the level surface $\phi(x, y, z) = c$, gradient of ϕ represents (1)

... unive vector (B) tangent vector (C) normal vector (D) radius vector

4. For the scalar point functions ϕ and ψ, $\nabla(\phi\psi)$ = (1)

(A) $\phi\nabla\psi - \psi\nabla\phi$ (B) $\phi\nabla\psi + \psi\nabla\phi$ (C) $\phi(\nabla^2\psi) + \psi(\nabla^2\phi)$ (D) $\dfrac{\phi\nabla\psi - \psi\nabla\phi}{\psi^2}$

5. For the scalar point function ϕ and ψ, $\nabla\left(\dfrac{\phi}{\psi}\right)$ = (1)

(A) $\phi\nabla\psi + \psi\nabla\phi$ (B) $\dfrac{\phi\nabla\psi - \psi\nabla\phi}{\psi^2}$ (C) $\dfrac{\psi\nabla\phi + \phi\nabla\psi}{\psi^2}$ (D) $\dfrac{\psi\nabla\phi - \phi\nabla\psi}{\psi^2}$

(1)

6. If $\bar{F} = F_1(x, y, z)\,\bar{i} + F_2(x, y, z)\,\bar{j} + F_3(x, y, z)\,\bar{k}$ is a vector field then divergence of $\bar{F}$ is

(A) $\dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$ (B) $\dfrac{\partial F_1}{\partial x}\bar{i} + \dfrac{\partial F_2}{\partial y}\bar{j} + \dfrac{\partial F_3}{\partial z}\bar{k}$

(C) $\dfrac{\partial F_1}{\partial x}\dfrac{\partial F_2}{\partial y}\dfrac{\partial F_3}{\partial z}$ (D) $\left(\bar{i}\dfrac{\partial}{\partial x} + \bar{j}\dfrac{\partial}{\partial y} + \bar{k}\dfrac{\partial}{\partial z}\right) \times (F_1\bar{i} + F_2\bar{j} + F_3\bar{k})$

(1)

7. If $\bar{F} = F_1(x, y, z)\,\bar{i} + F_2(x, y, z)\,\bar{j} + F_3(x, y, z)\,\bar{k}$ is a vector field then curl of $\bar{F}$ is

(A) $\dfrac{\partial F_1}{\partial x}i + \dfrac{\partial F_2}{\partial y}j + \dfrac{\partial F_3}{\partial z}k$ (B) $\dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$

(C) $\left(\bar{i}\dfrac{\partial}{\partial x} + \bar{j}\dfrac{\partial}{\partial y} + \bar{k}\dfrac{\partial}{\partial z}\right) \times (F_1\bar{i} + F_2\bar{j} + F_3\bar{k})$ (D) $\dfrac{\partial F_1}{\partial x}\dfrac{\partial F_2}{\partial y}\dfrac{\partial F_3}{\partial z}$

8. A rigid body rotating with costant angular velocity $\bar{\omega}$ about a fixed axis, if $\bar{v}$ is the linear velocity of a point of the body then curl $\bar{v}$ is (1)

(A) $\bar{\omega}$ (B) $2\bar{\omega}$ (C) $\dfrac{\bar{\omega}}{2}$ (D) $3\bar{\omega}$

(1)

9. Vector field $\bar{F}$ is solenoidal if

(A) $\nabla \times \bar{F} = 0$ (B) $\nabla \cdot \bar{F} = 0$ (C) $\nabla^2 \bar{F} = 0$ (D) $\bar{F} \cdot \nabla = 0$

(1)

10. Vector field $\bar{F}$ is irrotational if

(A) $\nabla \cdot \bar{F} = 0$ (B) $\bar{F} \times \nabla = 0$ (C) $\nabla^2 \bar{F} = 0$ (D) $\nabla \times \bar{F} = \bar{0}$

(1)

11. Directional derivative of scalar point function $\phi(x, y, z)$ at a point $P(x_1, x_2, x_3)$ in the direction of vector $\bar{u}$ is

(A) $\nabla \cdot (\phi\,\hat{u})(x_1, x_2, x_3)$ (B) $(\nabla\phi)(x_1, x_2, x_3) \times \hat{u}$

(C) $(\nabla\phi)(x_1, x_2, x_3) \cdot \hat{u}$ (D) $(\nabla^2\phi)(x_1, x_2, x_3) \cdot \hat{u}$

(1)

12. Magnitude of maximum directional derivative of scalar point function $\phi(x, y, z)$ in the given direction is

(A) $|\nabla\phi|$ (B) $|\nabla^2\phi|$ (C) $|\phi\nabla\phi|$ (D) zero

(1)

13. Maximum directional derivative of scalar point function $\phi(x, y, z)$ is in the direction of

(A) tangent vector (B) $\bar{i} + \bar{j} + \bar{k}$ (C) radius vector (D) normal vector

(1)

14. If $\phi = xy^2 + yz^2$ and $(\nabla\phi)_{(1,-1,1)} = \bar{i} - \bar{j} - 3\bar{k}$ then the value of maximum directional derivative is

(A) $\dfrac{\bar{i} - \bar{j} - 3\bar{k}}{\sqrt{11}}$ (B) $\dfrac{1}{\sqrt{11}}$ (C) $\sqrt{4}$ (D) $\sqrt{11}$

(2)

15. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$ then ∇r is given by

(A) $\dfrac{\bar{r}}{r}$ (B) $\bar{r}$ (C) $\dfrac{\bar{r}}{r^2}$ (D) $\dfrac{1}{r^3}$

16. If $\phi = x + y + z$, $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ then $\nabla\phi \cdot \hat{a}$ is equal to (2)

(A) $\dfrac{3}{2}$ (B) $\sqrt{3}$ (C) 0 (D) $-\dfrac{5}{2}$

17. If $\phi = mx^2 + y + z$, $\bar{b} = 2\bar{i} + 3\bar{j} - \bar{k}$ and $\nabla\phi$ at the point (1, 0, 1) is perpendicular to $\bar{b}$ then m is equal to (2)

(A) 0 (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{5}{2}$

18. The divergence of vector field $\bar{F} = 3xz\,\bar{i} + 2xy\,\bar{j} - yz^2\,\bar{k}$ at a point (1, 1, 1) is (2)

(A) 3 (B) 4 (C) 7 (D) 0

19. The divergence of vector field $\bar{F} = x^2y\,\bar{i} + y^2\,\bar{j} + z^2x\,\bar{k}$ at a point (1, 2, 1) is (2)

(A) 5 (B) 8 (C) 10 (D) 12

20. If vector field $\bar{v} = (x + 3y)\,\bar{i} + (y - 2z)\,\bar{j} + (x + az)\,\bar{k}$ is solenoidal then value of a is (2)

(A) 0 (B) 3 (C) 2 (D) – 2

21. The value of λ so that the vector field $\bar{u} = (2x + 3y)\,\bar{i} + (4y - 2z)\,\bar{j} + (3x - \lambda 6z)\,\bar{k}$ is solenoidal is (2)

(A) – 6 (B) 1 (C) 0 (D) – 1

22. The curl of vector field $\bar{F} = x^2y\,\bar{i} + xyz\,\bar{j} + z^2y\,\bar{k}$ at the point (0, 1, 2) is (2)

(A) $4\bar{i} - 2\bar{j} + 2\bar{k}$ (B) $4\bar{i} + 2\bar{j} + 2\bar{k}$ (C) $4\bar{i} + 2\bar{k}$ (D) $2\bar{i} + 4\bar{k}$

23. If the vector field $\bar{F} = (x + 2y + az)\,\bar{i} + (2x - 3y - z)\,\bar{j} + (4x - y + 2z)\,\bar{k}$ is irrotational then the value of a is (2)

(A) – 4 (B) 3 (C) – 3 (D) 4

24. If $\bar{u} = x^2y\,\bar{i} + y^2x^3\,\bar{j} - 3x^2z^2\,\bar{k}$ and $\phi = x^2yz$, then $(\bar{u} \cdot \nabla)\,\phi$ at the point (1, 2, 1) is (2)

(A) 6 (B) 9 (C) 18 (D) 5

25. If $u = x + y + z$, $v = x + y$, $w = -2xz - 2yz - z^2$ then $\nabla u \cdot (\nabla v \times \nabla w)$ is (2)

(A) $-2y - 2z$ (B) 0 (C) $-4x - 4y - 4z$ (D) $-2x - 2y - 2z$

26. Unit vector in the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1, 2, 2) is (2)

(A) $\frac{1}{3}(\bar{i} + 2\bar{j} + 2\bar{k})$ (B) $\frac{1}{3}(\bar{i} - 2\bar{j} - 2\bar{k})$ (C) $\frac{1}{3}(\bar{i} + \bar{j} + \bar{k})$ (D) $\frac{1}{9}(\bar{i} + 2\bar{j} + 2\bar{k})$

27. Unit vector in the direction normal to the surface $xy = z^2$ at (1, 1, 1) is (2)

(A) $\frac{1}{\sqrt{6}}(2\bar{i} + \bar{j} + 2\bar{k})$ (B) $\frac{1}{\sqrt{6}}(\bar{i} - \bar{j} + 2\bar{k})$ (C) $\frac{1}{6}(\bar{i} - \bar{j} - 2\bar{k})$ (D) $\frac{1}{\sqrt{6}}(\bar{i} + \bar{j} - 2\bar{k})$

28. Unit vector in the direction normal to the surfae $2x + 3y + 4z = 7$ at (1, –1, 2) is (2)

(A) $\frac{1}{\sqrt{29}}(2\bar{i} + \bar{j} - 4\bar{k})$ (B) $\frac{1}{\sqrt{29}}(2\bar{i} + 3\bar{j} + 4\bar{k})$

(C) $\frac{1}{29}(2\bar{i} - 3\bar{j} + 4\bar{k})$ (D) $\frac{1}{\sqrt{29}}(8\bar{i} + 6\bar{j} + 48\bar{k})$

29. Unit vector in the direction of tangent to the curve $x = \sin t$, $y = \cos t$, $z = t$ at $t = \frac{\pi}{4}$ is (2)

(A) $\frac{1}{2}(\bar{i} - \bar{j} + \bar{k})$ (B) $-\frac{1}{2}\bar{i} + \frac{1}{2}\bar{j} + \frac{1}{\sqrt{2}}\bar{k}$

(C) $\frac{1}{2}\bar{i} - \frac{1}{2}\bar{j} + \frac{1}{\sqrt{2}}\bar{k}$ (D) $\frac{1}{4}\bar{i} - \frac{1}{4}\bar{j} + \frac{1}{\sqrt{2}}\bar{k}$

30. Unit vector in the direction of tangent to the curve $x = e^{-t}$, $y = 2\sin t + 1$, $z = t - \cos t$ at $t = 0$ is (2)

(A) $\frac{1}{\sqrt{6}}(-\bar{i} + 2\bar{j} + \bar{k})$ (B) $\frac{1}{6}(-\bar{i} + 2\bar{j} + \bar{k})$ (C) $\frac{1}{\sqrt{6}}(-2\bar{i} + \bar{j} + \bar{k})$ (D) $\frac{1}{\sqrt{6}}(-\bar{i} + \bar{j} - \bar{k})$

31. Unit vector in the direction of tangent to the curve $x = t^3 - 1$, $y = 3t - 1$, $z = t^2 - 1$ at $t = 1$ is (2)

(A) $\frac{1}{22}(3\bar{i} + 3\bar{j} + \bar{k})$ (B) $\frac{1}{\sqrt{22}}(3\bar{i} + \bar{j} + \bar{k})$ (C) $\frac{1}{\sqrt{22}}(\bar{i} - 3\bar{j} + 2\bar{k})$ (D) $\frac{1}{\sqrt{22}}(3\bar{i} + 3\bar{j} + 2\bar{k})$

32. Unit vector along the line equally inclined with co-ordinate axes is

(A) $\frac{1}{\sqrt{3}}(\bar{i} + \bar{j} + \bar{k})$ (B) $\frac{1}{\sqrt{3}}(\bar{i} - \bar{j} - \bar{k})$ (C) $\frac{1}{3}(\bar{i} + \bar{j} + \bar{k})$ (D) $\frac{1}{\sqrt{3}}(-\bar{i} + \bar{j} - \bar{k})$

33. Unit vector along the direction of line $2(x - 2) = (y + 1) = (z - 1)$ is

(A) $\frac{1}{\sqrt{3}}(\bar{i} + 2\bar{j} - 2\bar{k})$ (B) $\frac{1}{3}(\bar{i} + 2\bar{j} + 2\bar{k})$ (C) $\frac{1}{3}(\bar{i} - 2\bar{j} + 2\bar{k})$ (D) $\frac{1}{3}(2\bar{i} + \bar{j} + 2\bar{k})$

34. Unit vector along the direction of line $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-3}{5}$ is

(A) $\frac{1}{\sqrt{14}}(\bar{i} - 2\bar{j} - 3\bar{k})$ (B) $\frac{1}{\sqrt{30}}(\bar{i} + 2\bar{j} + 5\bar{k})$ (C) $\frac{1}{30}(2\bar{i} + \bar{j} - 5\bar{k})$ (D) $\frac{1}{\sqrt{30}}(2\bar{i} + \bar{j} + 5\bar{k})$

35. The directional derivative of $\phi = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) in the direction of vector $\bar{u} = \bar{i} - 2\bar{j} + 2\bar{k}$ is

(A) $\frac{8}{3}$ (B) 8 (C) $\frac{4}{3}$ (D) $\frac{16}{3}$

36. The directional derivative of $\phi = xy^2 + yz^3$ at the point (1, –1, 1) in the direction of vector $\bar{u} = 2\bar{i} + 4\bar{j} + 4\bar{k}$ is

(A) $\frac{7}{3}$ (B) $-\frac{7}{3}$ (C) –7 (D) $-\frac{7}{6}$

37. The directional derivative of $\phi = xy + yz + xz$ at the point (1, 2, 0) in the direction of vector $\bar{u} = 2\bar{i} + \bar{j} + 3\bar{k}$ is

(A) $\frac{14}{\sqrt{6}}$ (B) $\frac{10}{\sqrt{14}}$ (C) $\sqrt{14}$ (D) $\frac{8}{\sqrt{14}}$

38. The directional derivative of $\phi = e^{2x-y-z}$ at the point (1, 1, 1) in the direction of vector $\bar{u} = -\bar{i} + 2\bar{j} + \bar{k}$ is (2)

(A) $-\frac{5}{2}$ (B) $-\frac{1}{\sqrt{6}}$ (C) $-\frac{5}{\sqrt{6}}$ (D) $\frac{5}{\sqrt{6}}$

39. The directional derivative of $\phi = e^{2x}\cos(yz)$ at origin in the direction of vector $\bar{u} = \bar{i} + \bar{j} + \bar{k}$ is (2)

(A) $\frac{4}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) 0 (D) $\frac{5}{\sqrt{3}}$

40. The directional derivative of $\phi = xy^2 + yz^3$ at (1, –1, 1) in the direction towards the point (2, 1, –1) is [Given : $(\nabla\phi)_{(1,-1,1)} = \bar{i} - \bar{j} - 3\bar{k}$] (2)

(A) $\frac{5}{3}$ (B) 5 (C) 3 (D) $\frac{5}{\sqrt{3}}$

41. If the partial derivatives of certain function $\phi(x, y)$ are given by the equations $-\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y} = 6$, $\frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial y} = -4$ then the directional derivative of $\phi(x, y)$, along the direction of the vector $\bar{i} + \bar{j}$ is given by (2)

(A) $2\sqrt{2}$ (B) $3\sqrt{2}$ (C) $\sqrt{2}$ (D) $-2\sqrt{2}$

42. For what values of a, b, c the directional derivative of $\phi = axy + byz + czx$ at (1, 1, 1) has maximum magnitude 4 in a direction parallel to x-axis (2)

[Given : $(\nabla\phi)_{(1,1,1)} = (a + c)\,\bar{i} + (a + b)\,\bar{j} + (b + c)\,\bar{k}$]

(A) a = – 2, b = 2, c = – 2 (B) a = 1, b = – 1, c = 1

(C) a = 2, b = – 2, c = 2 (D) a = 2, b = 2, c = 2

43. For what values of a, b, c the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, –1) has maximum magnitude 64 in a direction parallel to z-axis (2)

[Given : $(\nabla\phi)_{(1,2,-1)} = (4a + 3c)\,\bar{i} + (4a - b)\,\bar{j} + (2b - 2c)\,\bar{k}$]

(A) a = 24, b = 6, c = – 8 (B) a = – 6, b = – 24, c = 8

(C) a = 4, b = 16, c = 16 (D) a = 6, b = 24, c = – 8

44. The directional derivative of $\phi = x^2yz^3$ at (2, 1, –1) has maximum value in the direction of vector (2)

(A) $-4\bar{i} - 4\bar{j} - 2\bar{k}$ (B) $-4\bar{i} - 4\bar{j} + 12\bar{k}$ (C) $-\bar{i} + 4\bar{j} + 12\bar{k}$ (D) $4\bar{i} - 4\bar{j} - 12\bar{k}$

45. The dierctional derivative of $\phi = xy + yz + xz$ at (1, 2, 0) has maximum value in the direction of vector (2)

(A) $2\bar{i} + \bar{j} + 3\bar{k}$ (B) $\bar{i} + 2\bar{j} + 3\bar{k}$ (C) $2\bar{i} + 3\bar{j}$ (D) $2\bar{j} + 3\bar{j} + \bar{k}$

46. The directional derivative of $f = x^2y + 2y^2x$ at (1, 3) has maximum value in the direction of vector (2)

(A) $42\bar{i} + 13\bar{j}$ (B) $24\bar{i} + 31\bar{j}$ (C) $13\bar{i} + 24\bar{j}$ (D) $24\bar{i} + 13\bar{j}$

47. If the directional derivatived of $\phi = ax + by$ has maximum magnitude 2 along x-axis, then a, b are respectively given by (2)

(A) 1, 0 (B) 0, 1 (C) 2, 0 (D) 1, 1

48. Maximum value of directional derivative of $\phi = 4xy^2 - 16yz + 2z^2x^2$ at (2, 1, 1) is (2)

(A) 12 (B) 8 (C) 16 (D) 4

49. Maximum value of directional derivative of $\phi = xyz^2$ at (1, 0, 3) is. (2)

(A) 12 (B) 9 (C) 3 (D) 17

50. Maximum value of directional derivative of $\phi = 2xy - 2yz + 2xz$ at (1, 1, 1) is (2)

(A) 2 (B) 13 (C) 4 (D) 11

51. The angle between the surfaces $\phi = x \log z - y^2 - 1 = 0$ and $\psi = x^2y + z + 2 = 0$ at (1, 1, 1) is (2)

[Given : $\nabla\phi = \log z\,\bar{i} + (-2y)\,\bar{j} + \frac{x}{z}\bar{k}$ and $\nabla\psi = 2xy\,\bar{i} + x^2\,\bar{j} + \bar{k}$]

(A) $\cos^{-1}\left(-\frac{3}{\sqrt{10}}\right)$ (B) $\cos^{-1}\left(-\frac{1}{\sqrt{30}}\right)$ (C) $\cos^{-1}\left(-\frac{1}{2\sqrt{3}}\right)$ (D) $\cos^{-1}-\left(\frac{2}{\sqrt{30}}\right)$

52. The angle between the surfaces $\phi = \frac{5}{2}x^2 - yz - \frac{9}{2}x = 0$ and $\psi = 4x^2y + z^3 - 4 = 0$ at (1, 1, 1) is (2)

[Given : $\nabla\phi = \left(5x - \frac{9}{2}\right)\bar{i} + (-z)\,\bar{j} + (-y)\,\bar{k}$ and $\nabla\psi = 8xy\,\bar{i} + 4x^2\,\bar{j} + 3z^2\,\bar{k}$]

(A) $\cos^{-1}\left(-\frac{2}{\sqrt{89}}\right)$ (B) $\cos^{-1}\left(-\frac{9}{2\sqrt{89}}\right)$ (C) $\cos^{-1}\left(\frac{2}{\sqrt{89}}\right)$ (D) $\cos^{-1}\left(-\frac{10}{3\sqrt{89}}\right)$

53. If the surfaces $\phi_1 = xyz - 1 = 0$ and $\phi_2 = x^2 + ay^2 + z^2 = 0$ are orthogonal at (1, 1, 1) then a is equal to (2)

(A) – 1 (B) 2 (C) 1 (D) – 2

Answers

1. (A)	2. (D)	3. (C)	4. (B)	5. (D)	6. (A)	7. (C)	8. (B)
9. (B)	10. (D)	11.(C)	12. (A)	13. (D)	14. (D)	15. (A)	16. (B)
17. (C)	18. (A)	19. (C)	20. (D)	21. (B)	22. (C)	23. (D)	24.(A)
25. (B)	26. (A)	27. (D)	28. (B)	29. (C)	30. (A)	31. (D)	32. (A)
33. (B)	34. (D)	35. (A)	36. (B)	37. (C)	38. (C)	39. (B)	40. (A)
41. (D)	42. (C)	43. (D)	44. (B)	45. (A)	46. (D)	47. (C)	48. (A)
49. (B)	50. (C)	51. (B)	52. (A)	53. (D)			

Type : Vector Identities :

1. $\nabla f(r)$ is equal to (1)

(A) $\frac{f(r)}{r}\bar{r}$ (B) $\frac{f'(r)}{r}\bar{r}$ (C) $\frac{r}{f'(r)}\bar{r}$ (D) $f'(r)\,\bar{r}$

2. For a constant vector $\bar{a}$, $\nabla(\bar{a}\cdot\bar{r})$ is equal to (1)

(A) $\bar{a}$ (B) $3\bar{a}$ (C) $\bar{r}$ (D) 0

3. For constant vectors $\bar{a}$ and $\bar{b}$, $\nabla(\bar{a}\cdot\bar{b})$ is equal to (1)

(A) $\bar{a}\cdot\bar{b}$ (B) $\bar{a}$ (C) $\bar{b}$ (D) 0

44. The directional derivative of $\phi = x^2yz^3$ at $(2, 1, -1)$ has maximum value in the direction of vector (2)

(A) $-4\bar{i} - 4\bar{j} - 2\bar{k}$ (B) $-4\bar{i} - 4\bar{j} + 12\bar{k}$ (C) $-\bar{i} + 4\bar{j} + 12\bar{k}$ (D) $4\bar{i} - 4\bar{j} - 12\bar{k}$

45. The dierctional derivative of $\phi = xy + yz + xz$ at $(1, 2, 0)$ has maximum value in the direction of vector (2)

(A) $2\bar{i} + \bar{j} + 3\bar{k}$ (B) $\bar{i} + 2\bar{j} + 3\bar{k}$ (C) $2\bar{i} + 3\bar{j}$ (D) $2\bar{j} + 3\bar{j} + \bar{k}$

46. The directional derivative of $f = x^2y + 2y^2x$ at $(1, 3)$ has maximum value in the direction of vector (2)

(A) $42\bar{i} + 13\bar{j}$ (B) $24\bar{i} + 31\bar{j}$ (C) $13\bar{i} + 24\bar{j}$ (D) $24\bar{i} + 13\bar{j}$

47. If the directional derivatived of $\phi = ax + by$ has maximum magnitude 2 along x-axis, then a, b are respectively given by (2)

(A) 1, 0 (B) 0, 1 (C) 2, 0 (D) 1, 1

48. Maximum value of directional derivative of $\phi = 4xy^2 - 16yz + 2z^2x^2$ at $(2, 1, 1)$ is (2)

(A) 12 (B) 8 (C) 16 (D) 4

49. Maximum value of directional derivative of $\phi = xyz^2$ at $(1, 0, 3)$ is. (2)

(A) 12 (B) 9 (C) 3 (D) 17

50. Maximum value of directional derivative of $\phi = 2xy - 2yz + 2xz$ at $(1, 1, 1)$ is (2)

(A) 2 (B) 13 (C) 4 (D) 11

51. The angle between the surfaces $\phi = x \log z - y^2 - 1 = 0$ and $\psi = x^2y + z + 2 = 0$ at $(1, 1, 1)$ is (2)

[Given : $\nabla\phi = \log z\,\bar{i} + (-2y)\,\bar{j} + \frac{x}{z}\bar{k}$ and $\nabla\psi = 2xy\,\bar{i} + x^2\,\bar{j} + \bar{k}$]

(A) $\cos^{-1}\left(-\frac{3}{\sqrt{10}}\right)$ (B) $\cos^{-1}\left(-\frac{1}{\sqrt{30}}\right)$ (C) $\cos^{-1}\left(-\frac{1}{2\sqrt{3}}\right)$ (D) $\cos^{-1} - \left(\frac{2}{\sqrt{30}}\right)$

52. The angle between the surfaces $\phi = \frac{5}{2}x^2 - yz - \frac{9}{2}x = 0$ and $\psi = 4x^2y + z^3 - 4 = 0$ at $(1, 1, 1)$ is (2)

[Given : $\nabla\phi = \left(5x - \frac{9}{2}\right)\bar{i} + (-z)\,\bar{j} + (-y)\,\bar{k}$ and $\nabla\psi = 8xy\,\bar{i} + 4x^2\,\bar{j} + 3z^2\,\bar{k}$]

(A) $\cos^{-1}\left(-\frac{2}{\sqrt{89}}\right)$ (B) $\cos^{-1}\left(-\frac{9}{2\sqrt{89}}\right)$ (C) $\cos^{-1}\left(\frac{2}{\sqrt{89}}\right)$ (D) $\cos^{-}\left(-\frac{10}{3\sqrt{89}}\right)$

53. If the surfaces $\phi_1 = xyz - 1 = 0$ and $\phi_2 = x^2 + ay^2 + z^2 = 0$ are orthogonal at $(1, 1, 1)$ then a is equal to (2)

(A) −1 (B) 2 (C) 1 (D) −2

Answers

1. (A)	2. (D)	3. (C)	4. (B)	5. (D)	6. (A)	7. (C)	8. (B)
9. (B)	10. (D)	11.(C)	12. (A)	13. (D)	14. (D)	15. (A)	16. (B)
17. (C)	18. (A)	19. (C)	20. (D)	21. (B)	22. (C)	23. (D)	24.(A)
25. (B)	26. (A)	27. (D)	28. (B)	29. (C)	30. (A)	31. (D)	32. (A)
33. (B)	34. (D)	35. (A)	36. (B)	37. (C)	38. (C)	39. (B)	40. (A)
41. (D)	42. (C)	43. (D)	44. (B)	45. (A)	46. (D)	47. (C)	48. (A)
49. (B)	50. (C)	51. (B)	52. (A)	53. (D)			

Type : Vector Identities :

1. $\nabla f(r)$ is equal to (1)

(A) $\frac{f(r)}{r}\bar{r}$ (B) $\frac{f'(r)}{r}\bar{r}$ (C) $\frac{r}{f'(r)}\bar{r}$ (D) $f'(r)\,\bar{r}$

For a constant vector $\bar{a}$, $\nabla(\bar{a} \cdot \bar{r})$ is equal to (1)

(A) $\bar{a}$ (B) $3\bar{a}$ (C) $\bar{r}$ (D) 0

For constant vectors $\bar{a}$ and $\bar{b}$, $\nabla(\bar{a} \cdot \bar{b})$ is equal to (1)

(A) $\bar{a} \cdot \bar{b}$ (B) $\bar{a}$ (C) $\bar{b}$ (D) 0

4. $\nabla \cdot \bar{r}$ is equal to (1)

(A) 0 (B) $\frac{1}{r}\bar{r}$ (C) 3 (D) 1

5. $\nabla \times \bar{r} =$ (1)

(A) $\bar{r}$ (B) 3 (C) $\frac{1}{r}\bar{r}$ (D) $\bar{0}$

6. For a constant vector $\bar{a}$, $(\bar{a} \cdot \nabla)\,\bar{r}$ is equal to (1)

(A) $\bar{a}$ (B) $\bar{a} \cdot \bar{r}$ (C) $\bar{a} \cdot \frac{1}{r}\bar{r}$ (D) 3

7. For scalar function ϕ and vector function $\bar{u}$, $\nabla \cdot (\phi\bar{u})$ is equal to (1)

(A) $\phi(\nabla \times \bar{u}) + \nabla\phi \times \bar{u}$ (B) $\phi(\nabla \cdot \bar{u}) + \nabla\phi \cdot \bar{u}$ (C) $\phi(\nabla \cdot \bar{u}) - \nabla\phi \cdot \bar{u}$ (D) $\phi(\bar{u} \cdot \nabla) + \bar{u} \cdot \nabla\phi$

8. For scalar function ϕ and vector function $\bar{u}$, $\nabla \times (\phi\bar{u})$ is equal to (1)

(A) $\phi(\nabla \times \bar{u}) + \bar{u} \times \nabla\phi$ (B) $\phi(\nabla \cdot \bar{u}) - \nabla\phi \cdot \bar{u}$ (C) $\phi(\nabla \times \bar{u}) + \nabla\phi \times \bar{u}$ (D) $\phi(\nabla \cdot \bar{u}) + \nabla\phi \cdot \bar{u}$

9. For the vector function $\bar{u}$ and $\bar{v}$, $\nabla \cdot (\bar{u} \times \bar{v})$ is equal to (1)

(A) $\bar{v} \cdot (\nabla \times \bar{u}) - \bar{u} \cdot (\nabla \times \bar{v})$

(B) $\bar{v} \times (\nabla \cdot \bar{u}) - \bar{u} \times (\nabla \cdot \bar{v})$

(C) $\bar{u} \cdot (\nabla \times \bar{v}) - \bar{v} \cdot (\nabla \times \bar{u})$

(D) $\bar{v} \cdot (\bar{u} \times \nabla) + \bar{u} \cdot (\bar{v} \times \nabla)$

10. For the scalar function ϕ, div (grad ϕ) is equal to (1)

(A) 1

(B) $\frac{\partial\phi}{\partial x}\bar{i} + \frac{\partial\phi}{\partial y}\bar{j} + \frac{\partial\phi}{\partial z}\bar{k}$

(C) $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

(D) 0

11. For the scalar function ϕ, curl (grad ϕ) is equal to (1)

(A) $\frac{\partial^2\phi}{\partial x^2}\bar{i} + \frac{\partial^2\phi}{\partial y^2}\bar{j} + \frac{\partial^2\phi}{\partial z^2}\bar{k}$

(B) $\frac{\partial\phi}{\partial x}\bar{i} + \frac{\partial\phi}{\partial y}\bar{j} + \frac{\partial\phi}{\partial z}\bar{k}$

(C) $\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z}$

(D) $\bar{0}$

12. For vector function $\bar{u}$, div (curl $\bar{u}$) is equal to (1)

(A) $(\nabla \cdot \bar{u}) - \nabla^2\bar{u}$ (B) 0 (C) $\nabla(\nabla \cdot \bar{u}) - \nabla^2\bar{u}$ (D) $\nabla(\nabla \cdot \bar{u}) + \nabla^2\bar{u}$

13. For vector function $\bar{u}$, curl (curl $\bar{u}$) is equal to (1)

(A) $\nabla(\nabla \cdot \bar{u}) - \nabla^2\bar{u}$ (B) $\nabla(\nabla \cdot \bar{u}) + \nabla^2\bar{u}$ (C) $\nabla(\nabla \times \bar{u}) - \nabla \cdot \bar{u}$ (D) $\nabla \cdot (\nabla \times \bar{u}) + \nabla^2\bar{u}$

14. $\nabla^2 f(r)$ is equal to (1)

(A) $\frac{f'(r)}{r}\bar{r}$ (B) $\frac{d^2f}{dr^2} + \frac{df}{dr}$ (C) $\frac{d^2f}{dr^2} - \frac{2}{r}\frac{df}{dr}$ (D) $\frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$

15. If $\bar{F}$ is irrotational vector field then there exists scalar potential ϕ such that (1)

(A) $\bar{F} = \nabla^2\phi$ (B) $\bar{F} = \nabla\phi$ (C) $\phi = \nabla \cdot \bar{F}$ (D) $\nabla \times \bar{F} = \nabla\phi$

16. ∇e^r is equal to

(A) $e^r\,\bar{r}$ (B) $\frac{e^r}{r}$ (C) $\frac{e^r}{r}\bar{r}$ (D) $\frac{r}{e^r}\bar{r}$

17. $\nabla \log r$ is equal to (1)

(A) $\frac{\log r}{r}\bar{r}$ (B) $\frac{1}{r^2}\bar{r}$ (C) $\bar{r}$ (D) $\frac{1}{r}\bar{r}$

18. ∇r^n is equal to (1)

(A) $n\,r^{n-1}$ (B) $\frac{r^{n+1}}{n+1}\bar{r}$ (C) $\frac{3r^{n-2}}{r}$ (D) $n\,r^{n-2}\,\bar{r}$

19. $\nabla (r^2 e^{-r})$ is given by (2)

(A) $(2 - r)\,\bar{r}\,e^{-r}$ (B) $(2 + r^2)\,\bar{r}\,e^{-r}$ (C) $(2 - r)\,e^{-r}$ (D) $\bar{r}\,e^{-r}$

20. $\nabla (r^2 \log r)$ is equal to (2)

(A) $(2 \log r + 1)\,r\,\bar{r}$ (B) $(2r + 1)\log r\,\bar{r}$ (C) $(2 \log r + 1)\,\bar{r}$ (D) $(2 \log r + 1)$

21. For constant vector $\bar{a}$, $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right)$ is equal to (2)

(A) $\frac{\bar{a}\cdot\bar{r}}{r^n} - \frac{1}{r^{n+2}}\bar{r}$ (B) $\frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})}{r^{n+2}}\bar{r}$ (C) $\frac{\bar{a}}{r^n} + \frac{(\bar{a}\cdot\bar{r})}{r^{n+2}}\bar{r}$ (D) $\frac{\bar{a}}{r^n} - \frac{n(\bar{a}\cdot\bar{r})}{r^{n+1}}$

22. $\nabla \cdot (r^n\,\bar{r})$ is equal to (2)

(A) $(n + 3)\,r^n$ (B) $3r^n + \frac{n}{r^{-n-2}}$ (C) $(n - 3)\,r^n$ (D) $(n + 3)\,r^{-n}$

23. For constant vector $\bar{a}$, $\nabla \cdot [(\bar{a}\cdot\bar{r})\,\bar{a}]$ is equal to (2)

(A) $\bar{a}\cdot\bar{r}$ (B) 0 (C) $\bar{a}\cdot\bar{a}$ (D) $|\bar{a}|$

24. $\nabla \cdot [(\log r)\,\bar{r}]$ is equal to (2)

(A) $3 \log r + \frac{1}{r}$ (B) $3 \log r + \frac{1}{r^2}\bar{r}$ (C) $5 + 6 \log r$ (D) $1 + 3 \log r$

25. $\nabla \cdot \left[r\nabla\left(\frac{1}{r^3}\right)\right]$ is equal to (2)

(A) $\frac{3}{r^4}$ (B) $\frac{3}{r^2}$ (C) $\frac{1}{r^4}$ (D) $3r^4$

26. If $\nabla^2\phi = 0$ and $\nabla^2\psi = 0$ then $\nabla \cdot [\phi\nabla\psi - \psi\nabla\phi]$ is equal to (2)

(A) 0 (B) $2\nabla\phi \cdot \nabla\psi$ (C) $\nabla\phi + \nabla\psi$ (D) $[\phi\nabla\psi - \psi\nabla\phi]$

27. $\nabla\left[\bar{b}\cdot\nabla\left(\frac{1}{r}\right)\right] =$ (2)

(A) $\frac{\bar{b}}{r^3} - \frac{3}{r^4}(\bar{b}\cdot\bar{r})\,\bar{r}$ (B) $-\frac{\bar{b}}{r^3} + \frac{3}{r^5}\bar{r}$ (C) $\frac{\bar{b}}{r^3} - \frac{3}{r^5}(\bar{b}\cdot\bar{r})$ (D) $-\frac{\bar{b}}{r^3} + \frac{3}{r^5}(\bar{b}\cdot\bar{r})\,\bar{r}$

28. $\nabla\,[\bar{a}\cdot\nabla \log r] =$ (2)

(A) $\frac{\bar{a}}{r^2} + \frac{2}{r^4}\bar{r}$ (B) $\frac{\bar{a}}{r} + \frac{1}{r^3}(\bar{a}\cdot\bar{r})\,\bar{r}$

(C) $\frac{\bar{a}}{r^2} - \frac{2}{r^4}(\bar{a}\cdot\bar{r})\,\bar{r}$ (D) $\frac{\bar{a}}{r^2} - \frac{2}{r^3}(\bar{a}\cdot\bar{r})$

29. $\nabla \times \left(\frac{\bar{r}}{r^3}\right)$ is equal to (2)

(A) $\frac{3}{r^2}$ (B) $\bar{0}$ (C) $-\frac{2}{r^2}$ (D) $\frac{1}{r^2}\bar{r}$

30. $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n}\right) =$

(A) $\frac{2+n}{r^n}\bar{a} + \frac{1}{r^{n+2}}(\bar{a} \quad \bar{r})\bar{r}$

(B) $\frac{2-n}{r^n} + \frac{n}{r^n}(\bar{a} \cdot \bar{r})\bar{r}$

(C) $\frac{2-n}{r^n}\bar{a} + \frac{n}{r^{n+2}}(\bar{a} \cdot \bar{r})\bar{r}$

(D) $\frac{2-n}{r^n}\bar{a} + \frac{n}{r^{-n-2}}(\bar{a} \quad \bar{r})$

31. $\nabla \times \left((\bar{a} \cdot \bar{r})\frac{\bar{r}}{r}\right) =$

(A) $\bar{a} \times \frac{\bar{r}}{r}$

(B) $\frac{\bar{r}}{r} \times \bar{a}$

(C) $\bar{a} \times \bar{r}$

(D) $\frac{\bar{r}}{r} + \frac{1}{r^2}(\bar{a} \cdot \bar{r})$

32. Given $\bar{v} = 2y^2z\,\bar{i} + (3xy \quad yz^4)\,\bar{j} + 2x^3z\,\bar{k}$, the value of $\nabla(\nabla \cdot \bar{v})$ at (1, 1, 2) is

(A) $7\bar{i} + 8\bar{j} \quad 32\bar{k}$

(B) $2\bar{i} + 3\bar{j} + 2\bar{k}$

(C) $9\bar{i} + 32\bar{k}$

(D) $9\bar{i} - 32\bar{k}$

33. $\nabla^2\left(\frac{1}{r^2}\right)$ is equal to

(A) $\frac{1}{r^3}$

(B) $\frac{2}{r^4}$

(C) $-\frac{2}{r^4}\bar{r}$

(D) $\frac{6}{r^4}$

34. $\nabla^2 e^r$ is equal to

(A) $e^r + \frac{2}{r}e^r$

(B) $e^r + \frac{1}{r}e^r$

(C) $\frac{e^r}{r}\bar{r}$

(D) $e^r - \frac{2}{r}e^r$

35. $\nabla^2 (r^2 \log r)$ is equal to

(A) $\frac{(1 + \log r)}{r}\bar{r}$

(B) $(3 + 2 \log r)$

(C) $(5 + 6 \log r)$

(D) $(5 + 6 \log r)\, r$

36. $\nabla^2\left(\frac{\bar{a} \cdot \bar{b}}{r}\right)$ is equal to

(A) $-(\bar{a} \cdot \bar{b})\frac{1}{r^2}\bar{r}$

(B) $\frac{4}{r^3}(\bar{a} \cdot \bar{b})$

(C) $(\bar{a} \cdot \bar{b})\left(\frac{2}{r^3} - \frac{1}{r^2}\right)$

(D) 0

37. If $\nabla^2 (r^2 \log r) = 5 + 6 \log r$ then $\nabla^4 (r^2 \log r) =$

(A) $\frac{18}{r^2}$

(B) $\frac{6}{r^2}$

(C) $-\frac{6}{r^2}$

(D) $-\frac{6}{r^2} + \frac{6}{r}$

38. If $\phi = 2xz + 2yz + z^2$ then $\nabla^2\phi$ is

(A) $2(x + y + z)$

(B) 2

(C) 0

(D) 6z

39. For constant vector $\bar{a}$, $\nabla \times (\bar{a} \times \bar{r}) =$ (2)

(A) $3\bar{a}$

(B) $\bar{a}$

(C) 0

(D) $2\bar{a}$

40. div (grad r^3) = $\nabla \cdot (\nabla r^3) =$ (2)

(A) 12r

(B) 8r

(C) 2r

(D) 4r

41. If $\phi = 2x^2 - 3y^2 + 4z^2$ then curl (grad ϕ) is (2)

(A) 3

(B) $4x\,\bar{i} - 6y\,\bar{j} + 8z\,\bar{k}$

(C) 0

(D) $4x - 6y + 2z$

42. If $\bar{F}$ is a solenoidal vector field then curl curl $\bar{F}$ is (2)

(A) $\nabla^2\bar{F}$

(B) $-\nabla^2\bar{F}$

(C) $\nabla^4\bar{F}$

(D) $\nabla(\nabla \cdot \bar{F})$

43. If $\bar{F}$ is a solenoidal vector field and curl curl $\bar{F} = -\nabla^2\bar{F}$ then curl curl curl curl F is (2)

(A) $\nabla^2\bar{F}$ (B) $\nabla^4\bar{F}$ (C) $-\nabla^4\bar{F}$ (D) 0

44. For the vector field $\bar{F} = (6xy + z^3)\,\bar{i} + (3x^2 - z)\,\bar{j} + (3xz^2 - y)\,\bar{k}$, $\nabla \times \bar{F}$ is (2)

(A) $6y\,\bar{i} + 6xz\,\bar{k}$ (B) $-2\bar{i} + 6z^2\,\bar{j} + 12x\,\bar{k}$

(C) $\bar{0}$ (D) $6y + 6xz$

45. For the vector field $\bar{F} = (2xz^3 + 6y)\,\bar{i} + (6x - 2yz)\,\bar{j} + (3x^2z^2 - y^2)\,\bar{k}$, $\nabla \times \bar{F}$ is (2)

(A) $2z^3\bar{i} - 2z\bar{j} + 6xz^2\,\bar{k}$ (b) $4y\,\bar{i} - 12xz^2\,\bar{j} + 12\bar{k}$

(C) $2z^3 - 2z + 6xz^2$ (D) $\bar{0}$

46. If for the vector field $\bar{u}$ and $\bar{v}$ are irrotational vectors then the value of $\nabla \cdot (\bar{u} \times \bar{v})$ is (2)

(A) 2 (B) 1 (C) 3 (D) 0

47. The vector field $\bar{F} = (6xy + z^3)\,\bar{i} + (3x^2 - z)\,\bar{j} + (3xz^2 - y)\,\bar{k}$ is irrotational. Corresponding scalar function ϕ satisfying $\bar{F} = \nabla\phi$ is (2)

(A) $3x^2y + z^3x - yz + c$ (B) $3x^2y + z^2x + c$

(C) $6x^2y + x^3 + xy - yz + c$ (D) $x^2y + z^3x - y^3 + c$

48. For irrotational vector field $\bar{F} = (x + 2y + 4z)\,\bar{i} + (2x - 3y - z)\,\bar{j} + (4x - y + 2z)\,\bar{k}$, scalar function ϕ such that $\bar{F} = \nabla\phi$ is (2)

(A) $\frac{x^2}{2} + 2xy + 4xz - \frac{3}{2}y^2 - yz + z^2 + c$ (B) $x^2 + xy + xz - y^2 - yz + z^2 + c$

(C) $\frac{x^2}{2} + 2xy + 4xz - \frac{1}{2}y^2 - yz + c$ (D) $\frac{x^2}{2} + y^2 + 4xz - yz + 2z^2 + c$

49. For irrotational vector field $\bar{F} = (2xz^3 + 6y)\,\bar{i} + (6x - 2yz)\,\bar{j} + (3x^2z^2 - y^2)\,\bar{k}$, scalar function ϕ such that $\bar{F} = \nabla\phi$ is (2)

(A) $x^2z^3 + 3y^2 + 3x^2 - \frac{y^3}{3} + c$ (B) $x^2z^3 + 6xy + 3x^2 - 2y^2z + x^2z^3 + c$

(C) $xz^3 + 6xy + y^2z + \frac{y^3}{3} + c$ (D) $x^2z^3 + 6xy - y^2z + c$

50. For irrotational vector field $\bar{F} = (y^2 \cos x + z^2)\,\bar{i} + (2y \sin x - 4)\,\bar{j} + (2xz + 2)\,\bar{k}$, scalar function ϕ such that $\bar{F} = \nabla\phi$ is (2)

(A) $-y^2 \sin x + z^2x + y^2 \sin x + xz^2 + c$ (B) $y^2 \sin x + z^2x - 4y + 2z + c$

(C) $y^2 \cos x + z^2x + y^2 \sin x - 4y + xz^3 + c$ (D) $\frac{y^2}{3} \sin x + z^3y + 2y \cos x - 4x + c$

51. If $\bar{F} = yz\,\bar{i} + zx\,\bar{j} + xy\,\bar{k}$ and $\bar{F} = \nabla\phi$, then ϕ is given by (2)

(A) $x + y + z + c$ (B) $x^2 + y^2 + z^2 + c$

(C) $xyz + c$ (D) $x^2 + y + z + c$

52. If $\nabla\phi = (y^2 + 2y + z)\,\bar{i} + (2xy + 2x)\,\bar{j} + x\,\bar{k}$ and $\phi(1, 1, 0) = 5$ then ϕ is (2)

(A) $xy^2 + 4xy + 2zx + xy^2 - 5$ (B) $xy^2 + 2xy + zx + 2$

(C) $xy^2 + xy + zx + 2$ (D) $xy^2 + 2xy + 2zx + y^2 - 2$

53. If $\bar{F} = r^2$ ...

(A) $\frac{r^4}{4}$

54. If ∇ (f(...

(A)

53. If $\bar{F} = r^2 \bar{r}$ is conservative, then scalar ϕ associated with it is given by

(A) $\frac{r^4}{4} + c$ (B) $\frac{r^2}{2} + c$ (C) $\frac{r^3}{3} + c$ (D) $r + c$

54. If $\nabla \cdot \{f(r)\,\bar{r}\} = 0$, then f(r) is given by (c is constant)

(A) $\frac{c}{r^2}$ (B) $\frac{c}{r}$ (C) $\frac{c}{r^4}$ (D) $\frac{c}{r^3}$

Answers

1. (B)	2. (A)	3. (D)	4. (C)	5. (D)	6. (A)	7. (B)	8. (C)
9. (A)	10. (C)	11. (D)	12. (B)	13. (A)	14. (D)	15. (B)	16. (C)
17. (B)	18. (D)	19. (A)	20. (C)	21. (B)	22. (A)	23. (C)	24. (D)
25. (A)	26. (A)	27. (D)	28. (C)	29. (B)	30. (C)	31. (A)	32. (D)
33. (B)	34. (A)	35. (C)	36. (D)	37. (B)	38. (B)	39. (D)	40. (A)
41. (C)	42. (B)	43. (B)	44. (C)	45. (D)	46. (D)	47. (A)	48. (A)
49. (D)	50. (B)	51. (C)	52. (B)	53. (A)	54. (D)		

MODEL QUESTION PAPERS

Online Examination (Phase-I)

Time : 30 Min. Marks : 25

Unit 1 : Linear Differential Equations :

1. The solution of differential equation $\frac{d^3y}{dx^3} + 3\frac{dy}{dx} = 0$ is (2)

(A) $c_1 + c_2 \cos x + c_3 \sin x$ (B) $c_1 + c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x$

(C) $c_1 + c_2 e^{\sqrt{3}x} + c_3 e^{-\sqrt{3}x}$ (D) $c_1 \cos x + c_2 \sin x$

2. Particular integral of differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}\cos x$ is (2)

(A) $e^x \cos x$ (B) $-e^{-x} \sin x$ (C) $-e^{-x} \cos x$ (D) $(c_1x + c_2)e^{-x}$

3. Particular integral of differential equation $(D^4 + D^2 + 1)y = 53x^2 + 17$ is (2)

(A) $53x^2 + 17$ (B) $3x^2 - 17$ (C) $53x^2 + 113$ (D) $53x^2 - 89$

4. For the differential equation $(2x + 3)^2\frac{d^2y}{dx^2} - 2(2x + 3)\frac{dy}{dx} - 12y = 6x$, complimentary function is given by (2)

(A) $c_1(2x + 3)^3 + c_2(2x + 3)^{-1}$ (B) $c_1(2x + 3)^{-3} + c_2(2x + 3)$

(C) $c_1(2x + 3)^3 + c_2(2x + 3)^2$ (D) $c_1(2x - 3)^3 + c(2x - 3)^{-1}$

5. For the simultaneous linear DE $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$, solution of y using $D \equiv \frac{d}{dt}$ is obtained from (1)

(A) $(D^2 + 6D + 9)y = -2t$ (B) $(D^2 + 6D + 9)x = 1 + t$

(C) $(D^2 + 6D + 1)y = t$ (D) $(D^2 - 6D - 9)y = 2t$

6. The solution of differential equation $\frac{d^2y}{dx^2} - 4y = 0$ is (1)

(A) $(c_1x + c_2)e^{2x}$ (B) $c_1e^{4x} + c_2e^{-4x}$

(C) $c_1 \cos 2x + c_2 \sin 2x$ (D) $c_1e^{2x} + c_2e^{-2x}$

7. Particular integral $\frac{1}{\phi(D^2)}\cos(ax + b)$, where $D \equiv \frac{d}{dx}$ and $\phi(-a^2) = 0$, $\phi'(-a^2) \neq 0$ is (1)

(A) $\frac{1}{\phi'(-a^2)}\cos(ax + b)$ (B) $\frac{1}{\phi'(-a^2)}\cos(ax + b)$

(C) $x\frac{1}{\phi'(-a^2)}\cos(ax + b)$ (D) $x\frac{1}{\phi'(-a^2)}\sin(ax + b)$

8. To reduce the differential equation $(x + 2)^2\frac{d^2y}{dx^2} - (x + 2)\frac{dy}{dx} + y = 4x + 7$ to linear differential equation with constant coefficients, substitution is (1)

(A) $x + 2 = e^{-z}$ (B) $x = z + 1$ (C) $x + 2 = e^z$ (D) $x + 2 = \log z$

Unit II : Fourier and Z-Transform

9. If $f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$ then Fourier cosine transform $F_c(\lambda)$ of f(x) is given by (2)

(A) $\frac{\lambda \sin\lambda + \cos\lambda - 1}{\lambda^2}$ (B) $\frac{\cos\lambda - \lambda\sin\lambda - 1}{\lambda^2}$ (C) $\frac{\cos\lambda - \lambda\sin\lambda + 1}{\lambda^2}$ (D) $\frac{\lambda\sin\lambda + 1}{\lambda^2}$

10. The solution f(x) of integral equation $\int_0^\infty f(x)\sin\lambda x\,dx = \begin{cases} 1, & 0 \le \lambda < 1 \\ 0, & \lambda \ge 2 \end{cases}$ is (2)

(A) $\frac{2}{\pi}\left(\frac{1 + \cos x}{x}\right)$ (B) $\frac{2}{\pi}\left(\frac{1 + \sin x}{x}\right)$ (C) $\frac{2}{\pi}\left(\frac{1 - \sin x}{x}\right)$ (D) $\frac{2}{\pi}\left(\frac{1 - \cos x}{x}\right)$

11. $Z\{3^k e^{-2k}\}, k \geq 0$ is given by (2)

(A) $\frac{z}{(z-3e)^2}$ (B) $\frac{z}{z-3e^{-2}}$ (C) $\frac{z}{z-2e^3}$ (D) $\frac{z}{z+3e^2}$

12. If $|z| > 2$, $Z^{-1}\left[\frac{z}{(z-1)(z-2)}\right]$ is given by (2)

(A) $1-2^k, k>0$ (B) $2^k-1, k \geq 0$ (C) $\frac{1^k}{2}-1, k \geq 0$ (D) $k-1, k \geq 0$

13. For the difference equation $12f(k+2) - 7f(k+1) + f(k) = 0, k \geq 0, f(0) = 0, f(1) = 3$, F(z) is given by (1)

(A) $\frac{36z}{12z^2-7z-1}$ (B) $\frac{36z}{12z^2+7z+1}$ (C) $\frac{36z}{12z^2-7z+1}$ (D) $\frac{36z}{12z^2+7z-1}$

14. The inverse Fourier transform f(x) defined in $-\infty < x < \infty$ of $F(\lambda)$ is (1)

(A) $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\lambda)e^{i\lambda x}\, d\lambda$ (B) $\frac{2}{\pi}\int_{-\infty}^{\infty} F(\lambda)e^{-i\lambda x}\, d\lambda$

(C) $\frac{1}{2\pi}\int_{-\infty}^{0} F(\lambda)e^{ix}\, d\lambda$ (D) $\frac{1}{2\pi}\int_{0}^{\infty} F(\lambda)e^{i\lambda x}\, dx$

15. If $f(k) = 3^k, k < 0$, then Z-transform of $\{3^k\}$ is given by

(A) $\frac{z}{3-z}, |z| > |3|$ (B) $\frac{z}{z-3}, |z| < |3|$

(C) $\frac{1}{3-z}, |z| > |3|$ (D) $\frac{z}{3-z}, |z| < |3|$

Answers

1. (B)	2. (C)	3. (D)	4. (A)	5. (A)	6. (D)	7. (C)	8. (C)
9. (A)	10. (D)	11. (B)	12. (B)	13. (C)	14. (A)	15. (D)	

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Online Examination (Phase-II)

Time : 30 Min. **Marks : 25**

Unit III : Statistics and Probability

1. The standard deviation and Arithmetic mean of three distributions x, y, z are as follow :

	Arithmetic mean	Standard deviation
x	18.0	5.4
y	22.5	4.5
z	24.0	6.0

The more stable distribution is (2)

(A) x (B) y (C) z (D) x and z

2. If $\sum xy = 2800$, $\overline{x} = 16$, $\overline{y} = 16$, $n = 10$, variance of x is 36 and variance of y is 25 then correlation coefficient r(x, y) is equal to (2)

(A) 0.95 (B) 0.73 (C) 0.8 (D) 0.65

3. Line of regression y on x is $8x - 10y + 66 = 0$. Line of regression x on y is $40x - 18y - 214 = 0$. The value of variance of y is 16. The standard deviation of x is equal to (2)

(A) 3 (B) 2 (C) 6 (D) 7

4. The probability that a person hit a target in shooting practice is 0.3. If he shoots 10 times, the probability that he hits the target is (2)

(A) 1 (B) $(0.3)^{10}$ (C) $(0.7)^{10}$ (D) $1 - (0.7)^{10}$

5. X is normally distributed. The mean of X is 15 and standard deviation 3. Given that for z = 1, A = 0.3413, $p(X \geq 18)$ is given by (2)

(A) 0.1587 (B) 0.4231 (C) 0.2231 (D) 0.3413

6. Line of regression y on x is (1)

(A) $y + \overline{y} = r\frac{\sigma_x}{\sigma_y}(x + \overline{x})$ (B) $x - \overline{x} = r\frac{\sigma_x}{\sigma_y}(y - \overline{y})$

(C) $y - \overline{y} = r\frac{\sigma_y}{\sigma_x}(x - \overline{x})$ (D) $y - \overline{y} = r\frac{\sigma_x}{\sigma_y}(x - \overline{x})$

7. If z = np where n the number of trials is very large and p the probability of success at each trial, then in Poisson's probability distribution, p(r) the probability of r successes is given by (

(A) $\frac{e^{z}\,z}{r!}$ (B) $\frac{e^{-z}\,z^{r}}{r!}$ (C) $\frac{e^{-z}\,z^{r}}{r}$ (D) $\frac{e^{z}\,z^{r}}{r!}$

Unit IV : Vector Differential Calculus

8. A curve is given by $\overline{r} = 2t^2\,\overline{i} + (t^2 - 4t)\overline{j} + (2t - 5)\overline{k}$. Tangent vectors to the curve at t = 1 and t = 3 are (

(A) $2\overline{i} - 2\overline{j} + 2\overline{k},\ 3\overline{i} + 2\overline{j} + 2\overline{k}$ (B) $4\overline{i} + 2\overline{j} + 2\overline{k},\ 12\overline{i} - 2\overline{j} + 2\overline{k}$

(C) $4\overline{i} - 2\overline{j},\ 12\overline{i} + 2\overline{j}$ (D) $4\overline{i} - 2\overline{j} + 2\overline{k},\ 12\overline{i} + 2\overline{j} + 2\overline{k}$

9. The value of λ so that the vector field $\overline{u} = (2x + 3y)\overline{i} + (4y - 2z)\overline{j} + (3x - \lambda 6z)\overline{k}$ is solenoidal is (

(A) −6 (B) 1 (C) 0 (D) −1

10. The directional derivative of $\phi = e^{2x-y-z}$ at the point (1, 1, 1) in the direction of vector $\overline{u} = -\overline{i} + 2\overline{j} + \overline{k}$ is (

(A) $-\frac{5}{2}$ (B) $-\frac{1}{\sqrt{6}}$ (C) $\frac{5}{\sqrt{6}}$ (D) $\frac{5}{\sqrt{6}}$

11. $\nabla \cdot [(\log r)\bar{r}]$ is equal to (2)

(A) $3 \log r + \frac{1}{r}$ (B) $3 \log r + \frac{1}{r^2}\bar{r}$ (C) $5 + 6 \log r$ (D) $1 + 3 \log r$

12. For irrotational vector field $\bar{F} = (x + 2y + 4z)\bar{i} + (2x - 3y - z)\bar{j} + (4x - y + 2z)\bar{k}$, scalar function ϕ such that $\bar{F} = \nabla\phi$ is (2)

(A) $\frac{x^2}{2} + 2xy + 4xz - \frac{3}{2}y^2 - yz + z^2 + c$ (B) $x^2 + xy + xz - y^2 - yz + z^2 + c$

(C) $\frac{x^2}{2} + 2xy + 4xz - \frac{1}{2}y^2 - yz + c$ (D) $\frac{x^2}{2} + y^2 + 4xz - yz + 2z^2 + c$

13. For vector functions $\bar{u}(t)$ and $\bar{v}(t)$, $\frac{d}{dt}(\bar{u} \times \bar{v}) =$ (1)

(A) $\bar{v} \times \frac{d\bar{u}}{dt} + \frac{d\bar{v}}{dt} \times \bar{u}$ (B) $\frac{d\bar{u}}{dt} \times \bar{v} + \bar{u} \times \frac{d\bar{v}}{dt}$

(C) $\frac{d\bar{u}}{dt} \times \bar{v} - \bar{u} \times \frac{d\bar{v}}{dt}$ (D) $\bar{u} \cdot \frac{d\bar{v}}{dt} + \frac{d\bar{u}}{dt} \cdot \bar{v}$

14. Magnitude of maximum directional derivative of scalar point function $\phi(x, y, z)$ in the given direction is (1)

(A) $|\nabla\phi|$ (B) $|\nabla^2\phi|$ (C) $|\phi\nabla\phi|$ (D) Zero

15. $\nabla^2 f(r)$ is equal to (1)

(A) $\frac{f'(r)}{r}\bar{r}$ (B) $\frac{d^2f}{dr^2} + \frac{df}{dr}$ (C) $\frac{d^2f}{dr^2} - \frac{2}{r}\frac{df}{dr}$ (D) $\frac{d^2f}{dr^2} + \frac{2}{r}\frac{df}{dr}$

Answers

1. (B)	2. (C)	3. (A)	4. (D)	5. (A)	6. (C)	7. (B)	8. (D)
9. (B)	10. (C)	11. (D)	12. (A)	13. (B)	14. (A)	15. (D)	

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