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Assignment 1 (Q1 chapter 2)
1. Derive the laplace transform of the following time functions: [section2.2]
a. $u(t)$
so here $f(t) = u(t)$ where

$$\begin{aligned} u(t) &= 1 \quad t > 0 & (1) \\ &= 0 \quad t = 0 & (2) \end{aligned}$$

since the time function doesn't contain impulse function,we can replace the lower limit of laplace transform with 0

$$\begin{aligned} F(s) &= \int_0^\infty f(t)e^{-st}dt \\ &= \int_0^\infty u(t)e^{-st}dt \\ \text{using equation (1)} \\ &= \int_0^\infty e^{-st}dt \\ &= -1/s \times [e^{-st}]_0^\infty \\ &= -1/s \times [e^{-s\times\infty} - e^{-s\times 0}] \\ &= -1/s \times [0-1] \\ &= -1/s \times -1 \\ &= 1/s \end{aligned}$$

b. $tu(t)$
so here $f(t) = tu(t)$ where

$$\begin{aligned} u(t) &= 1 \quad t > 0 & (3) \\ &= 0 \quad t = 0 & (4) \end{aligned}$$

since the time function doesn't contain impulse function,we can replace the lower limit of laplace transform with 0

$$\begin{aligned} F(s) &= \int_0^\infty f(t)e^{-st}dt \\ &= \int_0^\infty tu(t)e^{-st}dt \\ \text{using equation (3)} \\ &= \int_0^\infty t \cdot e^{-st}dt \\ &= [(t\int e^{-st}dt) - (\int(\int e^{-st}dt)dt)]_0^\infty \end{aligned}$$

$$
\begin{aligned}
&= 1/s[-t\cdot e^{-st} + \int e^{-st}dt]_0^\infty \\
&= 1/s[-t\cdot e^{-st} - 1/s\cdot e^{-st}]_0^\infty \\
&= 1/s[0-0+0+(1/s)] \\
&= 1/s^2
\end{aligned}
$$

c. $sin(\omega t)u(t)$
so here $f(t) = sin(\omega t)u(t)$ where

$$
\begin{aligned}
u(t) &= 1 \quad t > 0 && (5)\\
&= 0 \quad t = 0 && (6)
\end{aligned}
$$

$sin(\omega t)$ can be written as following

$$sin(\omega t) = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

using following properties

$$L[f(t_1) + f(t_2)] = L[f(t_1)] + L[f(t_2)]$$

$$L[af(t)] = aL[f(t)]$$

where a = constant

$$
\begin{aligned}
L[sin(\omega t)u(t)] &= L[\frac{e^{i\omega t}u(t) - e^{-i\omega t}u(t)}{2i}] && (7)\\
&= \frac{1}{2i} \times (L[e^{i\omega t}u(t)] - L[e^{-i\omega t}u(t)]) && (8)\\
& && (9)
\end{aligned}
$$

so, first we will find the $L[e^{i\cdot\omega t}u(t)]$

$$L[e^{i\omega t}u(t)] = \int_0^\infty e^{i\omega t}u(t)e^{-st}dt$$

using equation(5)

$$
\begin{aligned}
&= \int_0^\infty e^{i\omega t}(1)e^{-st}dt \\
&= \int_0^\infty e^{(i\omega - s)t}dt \\
&= \frac{1}{i\omega - s} \times [e^{(i\omega - s)t}]_0^\infty
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{i\omega - s} \times (e^{(i\omega-s)\infty} - e^{(i\omega-s)0}) \\
&= \frac{1}{i\omega - s} \times (0-1) \\
&= \frac{-1}{i\omega - s}
\end{aligned}
$$

so, using $L[e^{i\cdot\omega t}u(t)]$ we can find the $L[e^{-i\cdot\omega t}u(t)]$

$$
\begin{aligned}
L[e^{-i\omega t}u(t)] &= \frac{-1}{i(-1)\omega - s} \\
&= \frac{1}{i\omega + s}
\end{aligned}
$$

As we know

$$
\begin{aligned}
L[sin(\omega t)u(t)] &= \frac{1}{2i} \times (L[e^{i\omega t}u(t)] - L[e^{-i\omega t}u(t)]) && (10)\\
&= \frac{1}{2i} \times \left[\left(\frac{-1}{i\omega - s}\right) - \left(\frac{1}{i\omega + s}\right)\right] && (11)\\
&= \frac{-1}{2i} \times \left[\left(\frac{1}{i\omega - s}\right) + \left(\frac{1}{i\omega + s}\right)\right] && (12)\\
&= \frac{-1}{2i} \times \left(\frac{i\omega + s + i\omega - s}{(i\omega - s)(i\omega + s)}\right) && (13)\\
&= \frac{-1}{2i} \times \left(\frac{2i\omega}{(i^2\omega^2 - s^2)}\right) && (14)\\
&= \frac{-1}{2i} \times \left(\frac{2i\omega}{((-1)\omega^2 - s^2)}\right) && (15)\\
&= \frac{-1}{2i} \times \left(\frac{-2i\omega}{(\omega^2 + s^2)})\right) && (16)\\
&= \frac{\omega}{(\omega^2 + s^2)} && (17)
\end{aligned}
$$

d. $cos(\omega t)u(t)$
so here $f(t) = cos(\omega t)u(t)$ where

$$
\begin{aligned}
u(t) &= 1 \quad t > 0 && (18)\\
&= 0 \quad t = 0 && (19)
\end{aligned}
$$

$cos(\omega t)$ can be written as following

$$
cos(\omega t) = \frac{e^{i\omega t} + e^{-i\omega t}}{2}
$$

using following properties

$$L[f(t_1)+f(t_2)] = L[f(t_1)] + L[f(t_2)]$$

$$L[af(t)] = aL[f(t)]$$

where a = constant

$$\begin{aligned}
L[cos(\omega t)u(t)] &= L[\frac{e^{i\omega t}u(t)+e^{-i\omega t}u(t)}{2}] && (20)\\
&= \frac{1}{2} \times (L[e^{i\omega t}u(t)] + L[e^{-i\omega t}u(t)]) && (21)\\
& && (22)
\end{aligned}$$

so, first we will find the $L[e^{i\cdot\omega t}u(t)]$

$$\begin{aligned}
L[e^{i\omega t}u(t)] &= \int_0^\infty e^{i\omega t}u(t)e^{-st}dt\\
\text{using equation(5)} &= \int_0^\infty e^{i\omega t}(1)e^{-st}dt\\
&= \int_0^\infty e^{(i\omega - s)t}dt\\
&= \frac{1}{i\omega - s} \times [e^{(i\omega - s)t}]_0^\infty\\
&= \frac{1}{i\omega - s} \times (e^{(i\omega - s)\infty} - e^{(i\omega - s)0})\\
&= \frac{1}{i\omega - s} \times (0-1)\\
&= \frac{-1}{i\omega - s}
\end{aligned}$$

so, using $L[e^{i\cdot\omega t}u(t)]$ we can find the $L[e^{(-i\cdot\omega t}u(t)]$

$$\begin{aligned}
L[e^{-i\omega t}u(t)] &= \frac{-1}{i(-1)\omega - s}\\
&= \frac{1}{i\omega + s}
\end{aligned}$$

As we know

$$L[sin(\omega t)u(t)] = \frac{1}{2} \times (L[e^{i\omega t}u(t)] + L[e^{-i\omega t}u(t)]) \quad (23)$$

$$
\begin{aligned}
&= \frac{1}{2} \times \left[\left(\frac{-1}{i\omega - s} \right) + \left(\frac{1}{i\omega + s} \right) \right] && (24) \\
&= \frac{1}{2} \times \left[\left(\frac{-1}{i\omega - s} \right) + \left(\frac{1}{i\omega + s} \right) \right] && (25) \\
&= \frac{1}{2} \times \left(\frac{-i\omega - s + i\omega - s}{(i\omega - s)(i\omega + s)} \right) && (26) \\
&= \frac{1}{2} \times \left(\frac{-2s}{(i^2\omega^2 - s^2)} \right) && (27) \\
&= \frac{1}{2} \times \left(\frac{-2s}{((-1)\omega^2 - s^2)} \right) && (28) \\
&= \frac{1}{2} \times \left(\frac{2s}{(\omega^2 + s^2)}) \right) && (29) \\
&= \frac{s}{(\omega^2 + s^2)} && (30)
\end{aligned}
$$