M.Sc./II Sem. - 2015

INFORMATICS - Paper IT-22

Data structure and design of Algorithms

Time: 3 hours *Maximum Marks: 75*

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt five questions in all. Question No. **1** *is compulsory*

Q.1 (a) Write an algorithm that finds the largest number in a list (an array) of n numbers. (5)

(b) Write an algorithm that prints out all the subsets of three elements of a set of n elements. The elements of this set set are stored in a list that is input to the algorithm. (5)

(c) Show directly that $f(n) = n^2 + 3n^3 \in \Theta(n^3)$. That is, use the definition of O and Ω to show that $f(n)$ is in both $O(n^3)$ and $\Omega(n^3)$. (5)

Q.2 (a) Consider the following algorithm: (5)

$$\textbf{for}(i = 1; i <= 1.5n; i++)$$

$$\textbf{cout} << i;$$

$$\textbf{for}(i = n; i >= 1; i--)$$

$$\textbf{cout} << i.$$

(*i*) What is the output when $n = 2$, $n = 4$, and $n = 6$?
(*ii*) What is the time complexity $T(n)$? You may assume that the input n is divisible by 2.

(b) Use Binary Search to search for the integer 120 in the following list (array) of integers. Show the action step by step. (5)

12 34 37 45 57 82 99 120 134

(c) What is the time complexity $T(n)$ of the nested loop below? For simplicity , you may assume that n is a power of 2. That is , $n = 2^k$ for some positive integer k. (5)

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for$(i = 1; i <= n; i++)\{$

$j = n;$

while$(j >= 1)\{$

< body of the **while** *loop* > //Needs $\Theta(1)$.

$j = \lfloor j/2 \rfloor;$

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...

Q.3(a)Consider you have two sorted arrays of size n and m. Write an algorithm to merge the two sorted arrays and hence produce a third sorted array. What is the time complexity of your algorithm? Mention any assumptions that you have made. (8 , 2)

b) Write a function that merges two singly linked lists into one. You may consider that the node is consisting of an integer element. Mention clearly what should be the prototype of the function that you have written. (5)

Q.4 Suppose you have a list of numbers. You need to remove duplicates from that list. Which data structure will be most efficient for doing this task. (3)

After identifying the data structure, write pseudo codes to (4,4,4)

(*i*) Add an item in the data structure
(*ii*) Search an item
(*iii*) Delete an item from the Data Structure.

Q.5 (a) Solve the recurrence relation (4)

$$T(n) = 7T(n/2), \quad \text{for} \quad n > 1, \quad n \quad \text{a power of 2}$$

$$T(1) = 1$$

(b) Explain the Strassen's matrix multiplication algorithm (3)

(c)(*i*) Using Quicksort (Partition Exchange Sort) , sort the following (4)

15 22 13 27 12 10 20 25

(*ii*) Write the pseudo algorithm for the procedure *Quicksort* and show that the worst case time complexity is (4)

$$W(n) = \frac{n(n-1)}{2} \in \Theta(n^2)$$

Q.6 (a) The matrix of a relation M_R is given as: (2)

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 1 & 5 \\ 9 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 2 & 0 & 3 \\ 3 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Draw a digraph of the relation.

(b) Describe briefly the Floyd's algorithm for shortest path and use it to find the shortest path between any pair of vertices (v_i, v_j) in the diagraph of Q.6(a). (6)

(c) What do you undestand by Dynamic Programming? Consider the multiplication of four matrices : (2,3)

$$A(20 \times 2) \quad , B(2 \times 30) \quad , C(30 \times 12) \quad \text{and} \quad D(12 \times 8)$$

Find the optimal order for multiplying these matrices?

22 < 15 13 < 15

i, j	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
2, 1	15	22	13	27	12	10	20	25
3	1	15	22	13	27	12	10	20	25
4	2	13		15				
5	2	13	12	15	27	22		
6	3	13	12	10	27	22	15	25
7	4							
8	4	13 < 15						

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22 7 15

22 < 15

12 <

22 7 15

25 < 1

27 , 10 < 15