Roll No. 1407

M.Sc (Informatics) Ist Semester-2011.

Paper: IT-14 - Mathematical Foundation for Computer Science

Time: 3hrs. Max.Marks:75

(Write your Roll No. on the top immediately on receipt of the paper.)

Attempt five questions in all. Q.1 is compulsory.

Q.1 (a) A survey has been taken on methods of commuter travel. Each respondent was asked to check BUS, TRAIN, or AUTOMOBILE as a major method of travelling to work. More than one answer was permitted. The results reported were as follows: BUS, 30 people; TRAIN, 35 people; AUTOMOBILE, 100 people; BUS and TRAIN, 15 people; BUS and AUTOMOBILE, 15 people; TRAIN and AUTOMOBILE, 20 people; and all three methods, 5 people. How many people completed a survey form? (3)

(b) Let p, q, and r be the following statements:

p: I will study discrere structure; q: I will go to movie; r: I am in a good mood.

Write English sentences corresponding to the following statement.

(a) $r \Rightarrow (p \vee q)$; (b) $((\sim p) \wedge q) \Rightarrow r$. (3)

(c) If A={1,2,3,4}, B={1,4,6,8,9} and aRb if and only if $b = a^2$, then find the domain, range, metrix, and, when A=B, the digraph of the relation R. (3)

(d) Let A={1,2,3,4} and let R={(1,2), (1,3), (4,2)}. Is R transitive? (3)

(e) Write the permutation

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$$

of the set A={1, 2, 3, 4, 5, 6, 7, 8} as a product of disjoint cycle. (3)

Q.2 (a) Let S = {1, 2, 3, 4, and A = S × S. define the following relation R on A :

(a, b) R (a', b') if and only b' = a' b. Show that R is an equivalent relation. Compute A/R. (7)

(b) Describe the Warsh algorithm to compute the transitive closure of a relation. Let A = {a, b, c, d, e} let R and S be the relations on A described by

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

and

$$M_S = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Use Warshall's algorithm to compute the transitive closure of R∪S. (8)

Q.3 (a) Determine the Hasse diagram of the relation on A = { 1 , 2 , 3 , 4 , 5 } whose matrix is shown below:

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3)

(b) Is the poset A = {2 , 3 , 6 , 12, 24 , 36 , 72 } under the relation of divisibility a lattice? (3)

(c) Consider a (3,8) encoding function such that

e(000) = 00000 , e(001) = 11110 , e(010) = 01101, e(011) = 10011,

e(100) = 01010 , e(101) = 10100 , e(110) = 00111 , e(111)=11001.

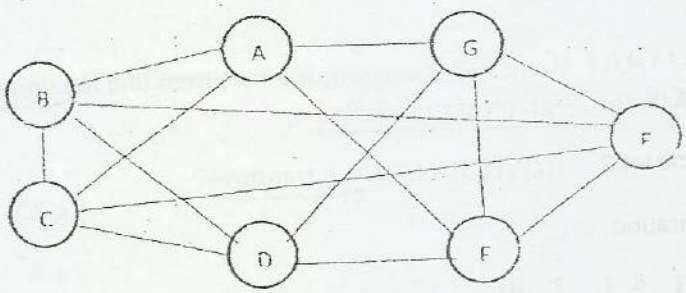
Find (i) its minimum distance , (ii) the number of errors it can detect. (3)

(d) Solve the recurrence relation

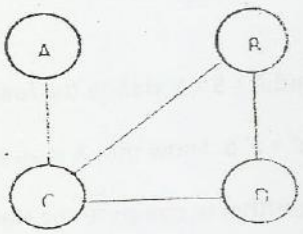
$$f_n = f_{n-1} + f_{n-2}$$

with initial conditions $f_1 = 1$ and $f_2 = 1$. (3)

(e) Explain why the graph shown below is Eulerian and find an Eulerian circuit. (3)

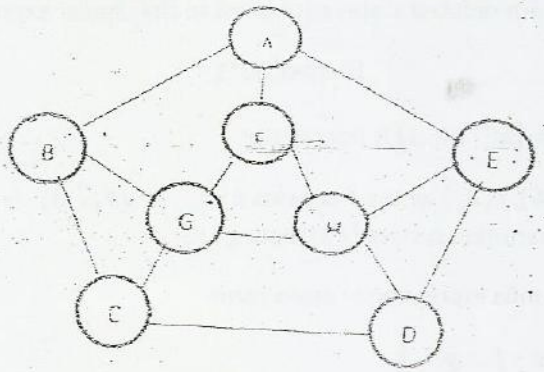


Q.4(a) Write the adjacency matrix for the following graph. (5)

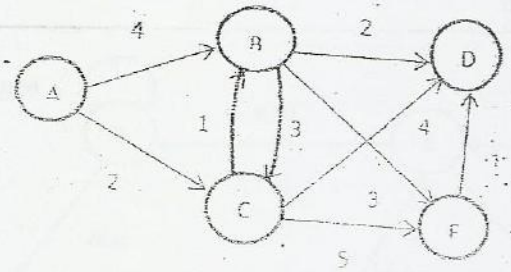


How many paths of length 3 are there?

(b)Determine ; with reasons, whether the following graph is Hamiltonian. (3)

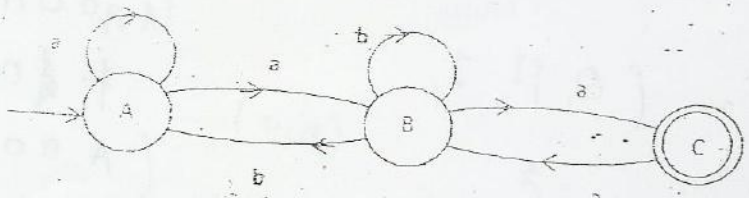


(c) Describe Dijkstra's shortest path algorithm. Use it to find the shortest path between *A* to *D* in the following graph. (7)

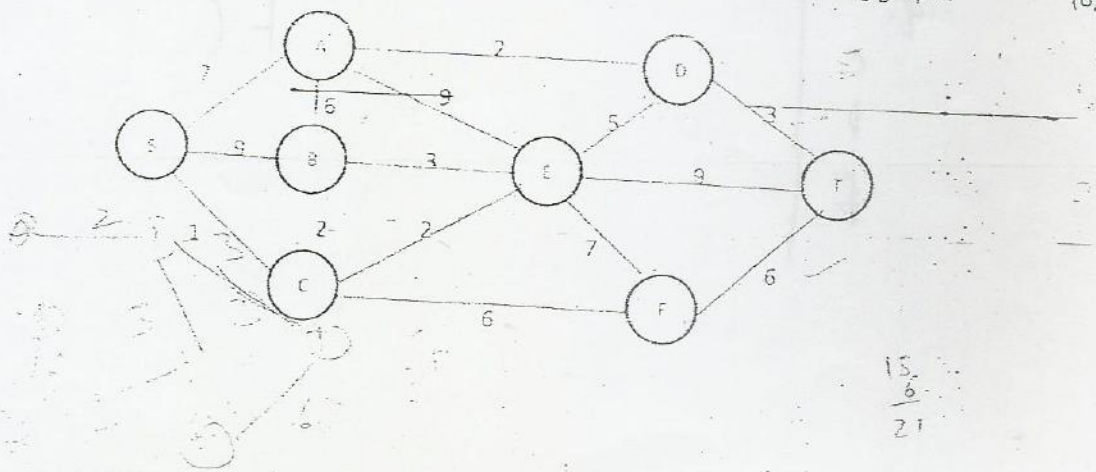


Q.5(a) If G is the grammar $\rightarrow SbS \mid a$, show that G is ambiguous. (3)

(b) Consider the transition system shown below. Find the string accepted/ recognized by this automata. (4)



(c) Explain the Kruskal algorithm to obtain the spanning tree of a weighted connected graph. Use it to obtain a spanning tree of minimum total weight for the following graph. (8)



Q.6 (a) Construct a DFA with reduced states equivalent to the regular expression (3)

$$10+(0+11)0^*1$$

(b) Show that the set $L = \{a^{i^2} \mid i \geq 1\}$ is not regular. (3)

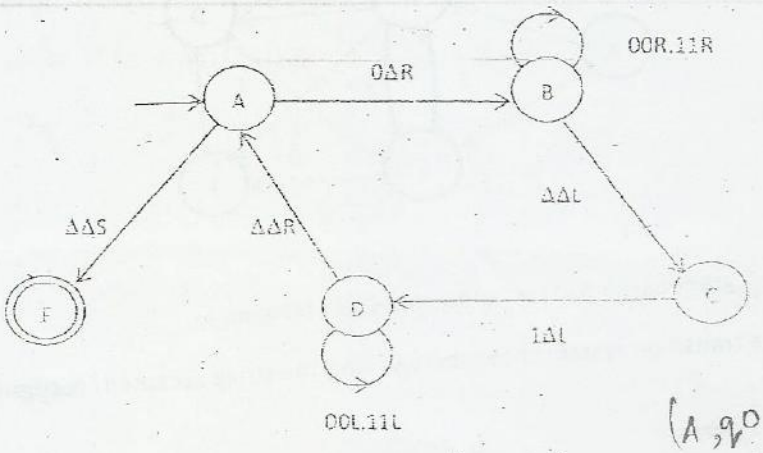
(c) Let $G = (\{A_0, A_1\}, \{a, b\}, P, A_0)$ where P consists of $A_0 \rightarrow aA_1$, $A_1 \rightarrow bA_1$, $A_1 \rightarrow a$, $A_1 \rightarrow bA_0$. Construct a transition system M accepting L(G). (3)

(d) Rewrite the following infix expression in prefix form:

$$a - (b * c + d) \mid e * f - g \uparrow h$$

and hence construct a binary expression tree. (3)

(e) Define a Turing Machine (TM). Using the transition graph of the following TM, show that the string 00111 is rejected. (3)



	0
A	
B	
C	
D	
E	