IC1514

M.Sc. (Informatics)/I- Sem. - 2015

Paper : IT-14

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Time: 3 hours *Maximum Marks: 75*

(*Write your Roll No. on the top immediately on receipt of this question paper*)

Attempt **five** *questions in all.* **Q.1** *is compulsary*

Q.1(a) If A , B , and C are sets , prove that $A-(B\cap C)=(A-B)\cup(A-C)$ analytically or graphically. (3)

(b) If **R** is the relation on the set of real numbers such that $a\mathbf{R}b$ if and only if $a-b$ is an integer, show that **R** is an equivalence relation. (3)

(c) Let $A=\{a,b,c,d,e\}$ and R be the relation on A defined by (3)

$$M_R=\begin{bmatrix}1&1&1&0&1\\1&1&1&0&1\\1&1&1&0&1\\0&0&0&1&0\\1&1&1&0&1\end{bmatrix}$$

Computr A/R?

(d) Let $A=\{1,2,3,4\}$ and $R=\{(2,1),(2,3),(3,2),(3,3),(2,2),(4,2)\}$. Find (i) reflexive closure of R and (ii) the symmetric closure of R. (3)

(e) Let $A=\{1,2,3,4,5\}$ and let R and S be the equivalence relations on A whose matrices are given. Compute the matrix of the smallest equivalence relation containing R and S, and list the elements of this relation. (3)

$$M_R=\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&1&1\end{bmatrix},M_S=\begin{bmatrix}1&0&0&0&0\\0&1&1&1&0\\0&1&1&1&0\\0&1&1&1&0\\0&0&0&0&1\end{bmatrix}.$$ $R\circ S=$ $R^{\circ}S$

Q.2(a) Are the following networks bipartite? Why? (5)

No Yes No No Yes.

(i)a chain, (ii) a tree, (iii)a ring, (iv)a lattice and(v)a star

$2^{h+1}-1$

(b) For a binary tree with m layers , (i) what is the number of nodes in the network? (ii) what is the total number of edges in the network? (3)

$2(2^{100}-2)$

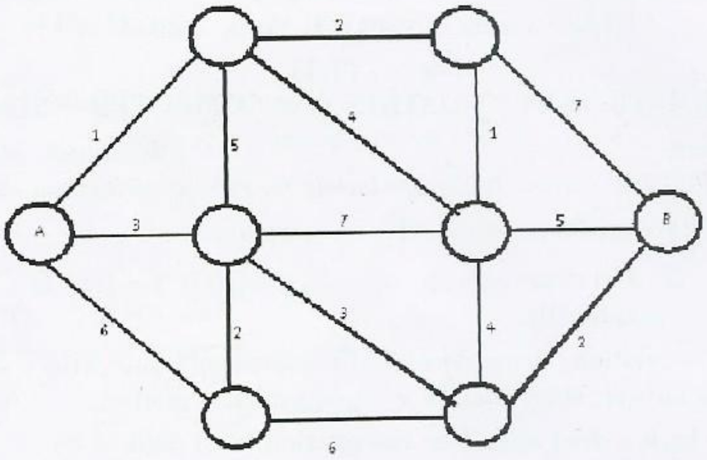

Fig. 1

(c) Apply Dijkstra's algorithm to solve the minimum connector problem from point A to point B in the graph shown in Fig.1. (7)

Q.3(a) Define Eulerian path and Eulerian circuit of a graph, with an example for each. When is a graph called an Eulerian graph? (3)

(b) Find a Hamiltonian path or a Hamiltonian circuit, if it exists, in each of the graphs in Fig.2 ? (3)

(c) Find the number of paths of length 4 from the vertex A to D in the simple graph G (Fig.3) ? (4)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$0 \quad 1 \quad 1 \quad 0,$

A - D - E - B - C

No. path.

No. Rf. 1

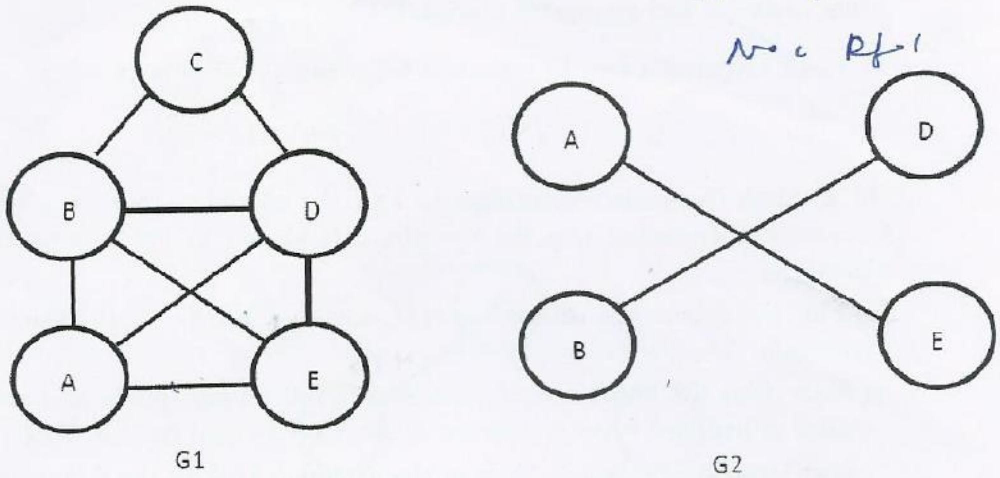

Fig.2

Fig 3

(d) List the order in which the vertices of the tree given in Fig.4 are processed using preorder and postorder traversal. (5)

Q.4(a) Construct a tree to represent following algebraic expression: (2)

$$((x*y-2)/(y+x))/((x+y)-(x-y))$$

(b) Explain the Prim's algorithm to find the spanning tree for a graph . Construct a spanning tree for the graph G shown in Fig.1 using Prim's algorithm. (6)

(c) Find the value of (i) the prefix expression $+-\uparrow^{3}2\uparrow^{2}3/8-42$, ($ii$) the postfix expression $72-3+232+-13-*/$. 4 (3 (4) 3↑

(d) Show that the number n of vertices of a full binary tree is odd and the number of pendant vertices (leaves) of the tree is equal to $(n+1)/2$. (3)

Q.5(a) Determine whether, each of the grammar G with the following production is context-sensitve , context-free , regular or none of these. Give the reason also: (3)

(i)

$$S\rightarrow;S\rightarrow AAB;Aa\rightarrow Aba;A\rightarrow aa;Bb\rightarrow ABb;AB\rightarrow ABB;B\rightarrow b.$$

(ii)

$$S\rightarrow BAB;S\rightarrow ABA;A\rightarrow AB;B\rightarrow BA;A\rightarrow aA;A\rightarrow ab;B\rightarrow b.$$

(iii)

$$<S>::=b<S>|a<A>|a;<A>::=a<S>|b<B>;$$

$$<B>::=b<A>|a<S>|b.$$

(b)Construct derivation trees for the words (i) $ababbbba$, (ii) $bbbcbba$ using the grammar G_1 and G_2, respectively, where G_1 consists of the productions $\{S\rightarrow AbS,\quad A\rightarrow aS,\quad S\rightarrow ba,\quad \text{and}A\rightarrow b\}$ and G_2 connsists of the productions $\{S\rightarrow bcS,\quad S\rightarrow bbS,\quad S\rightarrow cb,\ quadS\rightarrow a\}$. (3)

(c) Find a grammar that generates the set of words $\{a^nb^nc^n|n|ge1\}$. (3)

(d) Examine whether the grammar $G=\{(S,A),(a,b),S,P\}$, where $P=\{S\rightarrow aSb,\quad S\rightarrow aASb,\quad S\rightarrow ab,\quad A\rightarrow\lambda\}$ is ambiguous or not. (3)

(e) Design an FSM that performs serial binary addition. (3)

Q.6(a) Find the DFA equivalent to the NDFA for which the state table is given in Table:1 and s_2 is the accepting state. (4)

ABDPILMGCEMJK

LMIFGDBJKMECA

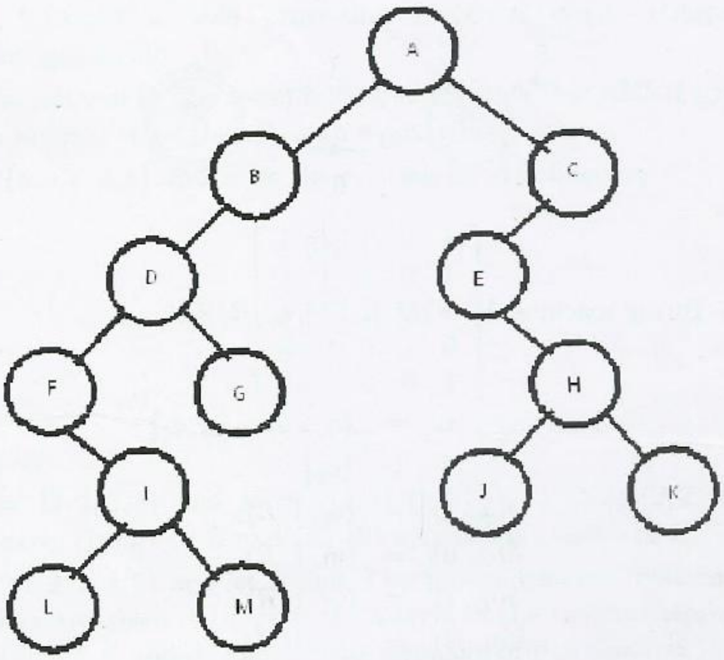

Fig.4

Table:1

PS	δ	
	a	b
s_0	s_0, s_1	s_2
s_1	s_0	s_1
s_2	s_1	s_0, s_1

(b)State Arden's theorem. Find the regular expression corresponding to the automaton of Fig.5? (6)

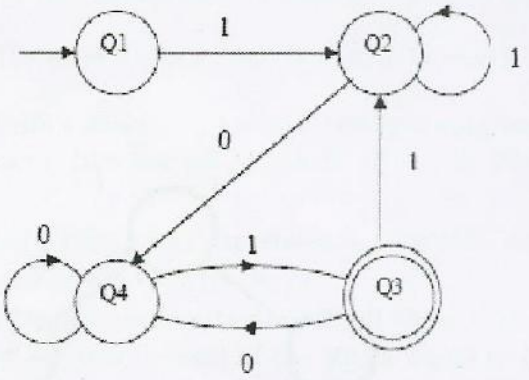

Fig.5

(c) In a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, (5)

$$
\begin{aligned}
Q &= \{q-0, q_1, q_3, q_4\} \\
F &= \{q_4\} \\
\delta(q_0, 1) &= (q_0, 1, R), \\
\delta(q_0, 0) &= (q_1, 1, R), \\
\delta(q_1, 1) &= (q_1, 1, R), \\
\delta(q_1, B) &= (q_2, B, L), \\
\delta(q_2, 1) &= (q_3, 0, L), \\
\delta(q_3, 1) &= (q_3, 1, L), \\
\delta(q_3, B) &= (q_4, B, R).
\end{aligned}
$$

Using the convention of unary notation in which positive integer x is represented by $w(x) \in \{1\}^+$, such that $|w(x)| = x$, show that $q_0w(x)0w(y) \to q_4w(x+y)0$ by taking $x = 111$ and $y = 11$.