

INTRODUCTION TO COMMUNICATION AND SYSTEMS

Time: 3 hours

Maximum Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt five questions in all.

Q.1(a) A square wave may be represented by

$$f(t) = \begin{cases} -1 & \text{for } -\frac{1}{2}T \leq t < 0, \\ +1 & \text{for } 0 \leq t < \frac{1}{2}T. \end{cases}$$

(b) Show that the Fourier series of $f(t)$ comprises only odd harmonics. (6)(c) The exponential pulse may be defined as $g(t) = u(t) \exp(-t)$ where $u(t)$ defines the unit step function:

$$u(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0. \end{cases} \quad \text{or } u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

Show that

$$u(t) \exp(-t) \Rightarrow \frac{1}{1 + j2\pi f}$$

(d) If $g(t) \Rightarrow G(f)$, then for a constant frequency shift f_c , (5)

$$\exp(j2\pi f_c t) g(t) \Rightarrow G(f - f_c)$$

where symbols have their usual meanings. (4)

Q.2(a) Show that

$$\sin(2\pi f_c t) \Rightarrow \frac{1}{2j} [\delta(f - f_c) - \delta(f + f_c)]$$

(b) Show that a band-limited signal of finite energy, which has no frequency components higher than W Hz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$ seconds. (6)(c) Consider a linear time-invariant system of impulse response $h(t)$ driven by a complex exponential input $x(t) = \exp(j2\pi f t)$ of frequency f . If $y(t)$ is the response of the system then show that

$$Y(f) = H(f)X(f)$$

where $Y(f)$, $H(f)$ and $X(f)$ refers to Fourier transform of $y(t)$, $h(t)$ and $x(t)$ respectively. (5)

Q.3(a) Draw a schematic diagram of (i) spectrum of a message signal, (ii) Spectrum of AM wave. (3)

(b) Describe the time domain and frequency domain characteristic of different modulated waves produced by a single-tone $m(t) = \sin(2\pi f_m t)$. (6)

What would be the power of (i) carrier wave, (ii) Upper side-frequency and (iii) Lower side-frequency.

(c) What do you understand by double-sideband suppressed-carrier (DSBSC) modulation? Describe the coherent detection of DSBSC modulated wave. (6)

Q.4(a) Distinguish between frequency modulation (FM) and phase modulation (PM). (3)

(b) A sinusoidal modulating wave of amplitude 5V and frequency 1kHz is applied to a frequency modulator. The frequency sensitivity of the modulator is 40Hz/V. The carrier frequency is 100kHz. Calculate (i) the frequency deviation, and (ii) the modulation index. (5)

(c) Describe the generation of a narrow-band FM wave. (3)

(d) Describe the phase-locked loop (PLL) as a frequency demodulator. (4)

Q.5(a) Consider a random variable X defined by (assuming $b > a$) (5)

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

Show that

$$E(X^2) = \frac{1}{3}(b^2 + ab + a^2)$$

$$\sigma_X^2 = \frac{1}{12}(b-a)^2$$

(b) Consider a sinusoidal process with random phase. The process is denoted by

$$X(t) = A \cos(2\pi f_c t - \Theta)$$

where A and f_c are constants and the random variable Θ denotes the phase. Show that the autocorrelation function of $X(t)$ is

$$R_X(\tau) = \frac{A^2}{2} \cos(2\pi f_c \tau)$$

(c) Show that the mean-square value of the output of a stable linear time-invariant filter in response to a wide-sense stationary input process is equal to the integral over all frequencies of the power spectral density of the input random process multiplied by the squared magnitude of the transfer function of the filter. (5)

Q.6(a) Explain the generation of pulse code modulation (PCM). Contrast the difference between the delta modulation from PCM. (6)

(b) A PCM system is to have a signal-to-noise ratio 40dB. The signals are speech, and an rms-to-peak ratio of -10dB is allowed for. Find the number of bits per code word required. (3)

(c) Explain briefly the principal involved in frequency shift keying (FSK). How is this signal detected. (4)

(d) A digital signal utilizes 8-bit codewords. Calculate the probability of a received codeword containing three errors, given that the bit-error probability in transmission is 0.01. (2)