Paper Code: IC4317

M.Sc (Informatics), IV-Semester, 2017

IT-43, Modeling, Simulation and Performance Evaluation

Time: 3hrs

Max. Marks: 75

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt ~~six~~ five questions in all. Q.1 is compulsory.

Q.1

(a)State Bernoulli's theorem.

(b) A fair coin is tossed 4 times. What is the probability of getting more heads than tails.? Give reasons for your answer.

(c) Write down the probability distribution of the outcome when 2 fair dice are tossed.

(d) If the probability distribution of X is given as:

x:	1	2	3	4
p_X	0.4	0.3	0.2	0.1

Find $P\left(\frac{1}{2} < X < \frac{7}{2} \mid X > 1\right)$.

(e) If X and Y are two RVs where $Y = g(X)$, how are the density functions of X and Y related?

$(5 \times 3 = 15)$

Q.2

(a) If A , B and C are any 3 events such that $P(A) = P(B) = P(C) = 1/4$; $P(A \cap B) = P(B \cap C) = 0$; $P(C \cap A) = 1/8$. Find the probability that at least 1 of the events A, B and C occurs. **(3)**

(b) In a coin tossing experiment , if the coin shows head, 1 dice is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2? **(3)**

(c) If the random variable X takes the values 1, 2 , 3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$, find the probability distribution and cumulative distribution function of X. **(3)**

(d) A continuous RV X has a pdf $f(x) = kx^2e^{-x}; x \geq 0$. Find k, mean and variance. **(6)**

Q.3

(a) Write down the probability mass function of a binomial RV. Hence find an expression for the mean and standard deviation of the binomial distribution. **(4)**

(b) Fit a binomial distribution for the following data: **(4)**

x:	0	1	2	3	4	5	6	TOTAL
$f(x)$	5	18	28	12	7	6	4	80

(c) Find an expression for the mean and variance of Poisson distribution. Fit a Poisson distribution for following distribution:

x:	0	1	2	3	4	5	total
$f(x)$	142	156	69	27	5	1	400

(d) Show that the largest value of the variance of a binomial distribution is $^n/_4$. **(3)**

Q.4

(a)The following data give the number of aircraft accident that occurred during the various days of a week. **(3)**

Day	Mon	Tues	Wed	Thu	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. Given that $\chi^2(0.05,5) = 11.07$.

(b) The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3, \dots\dots$, having 3 states 1,2 and 3 is

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$. Find $P\{X_2 = 3\}$. **(4)**

(c) The three –state Markov chain is given by the transition probability matrix

$$P = \begin{pmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

Prove that the chain is irreducible and all the states are aperiodic and non-null persistent. **(8)**

Q.5

(a)Deduce the difference equations related to Poisson queue system (M / M / 1). Show that the probability of finding n jobs in a system is given by

$$p_n = p_0\left(\frac{\lambda}{\mu}\right)^n, n = 1,2,3,\dots; \; p_0 = \left(1 - \frac{\lambda}{\mu}\right).$$

where the symbols have their usual meanings. Find also the (i)average number L_q of jobs in the queue and (ii) probability that the number of jobs in the system exceeds k. **(7)**

(b) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the next. The length of a phone call assumed to be distributed exponentially with mean 4 min. (i) What is the probability that a person arriving at the booth will have to wait in the queue? (ii) What is the probability that it will take him more than 10min. altogether to wait for the phone and complete his call? (iii) the telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 min. for phone. By how much the flow of arrivals should increase in order to justify a second booth? (iv) What is the average length of the queue that forms from time to time? **(8)**

Q.6

(a) In a multi-server Poisson queue model (M/M/c : ∞ /FIFO) , the probability of finding n jobs in a system are given by: **(4)**

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & , 0 \le n < c \\ \frac{1}{c!}\frac{1}{c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0 & , n \ge c, \end{cases}$$

where $P_0 = \left[\left\{\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right\} + \left\{\frac{1}{c!}\frac{1}{\left(1-\frac{\lambda}{\mu c}\right)}\left(\frac{\lambda}{\mu}\right)^c\right\}\right]^{-1}$ and other symbols have their usual meanings. Show that the probability that an arrival has to wait is given by

$$P(W_s > 0) = P(N \ge c) = \frac{\left(\frac{\lambda}{\mu}\right)^c P_0}{c!\left(1-\frac{\lambda}{\mu c}\right)}.$$

(b) A telephone company is planning to install telephone booths in a new airport. It has established the policy that a person should not have to wait more than 10% of the times he tries to use a phone. The demand for use is estimated to be Poisson with an average of 30 per hour. The average phone call has an exponential distribution with a mean time of 5 min. How many phone booths should be installed? **(5)**

(c) Find the eigenvalues and eigenvectors of the stochastic matrix **(6)**

$$T = \begin{pmatrix} 1/4 & 1/2 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/4 \end{pmatrix}$$

Construct a formula for T^n, and find $\log_{n\to\infty} T^n$.