M.Sc (INFORMATICS)/1st Semester 2016 **1714**

Paper IT-14- Mathematical Foundation for Computer Science

Time: 3hrs Max.Marks:75

Attempt five questions in all. Question no.1 is compulsory

Q.1(a) Let A,B,and C be the finite sets with $|A| = 6, |B| = 8, |C| = 6, |A \cup B \cup C| = 11, |A \cap B| = 3, |A \cap C| = 2,$, and $|B \cap C| = 5$. Find $|A \cap B \cap C|$. (3)

(b) If n fair coins are tossed and the results recorded, how many (i) record sequences are possible? (ii) sequences contain exactly three tails assuming $n \geq 3$? (iii) sequences contain exactly k heads , assuming $n \geq k$? (3)

(c) A genetics experiment classifies fruit flies according to the following two criteria: (3)

Gender: male (m), female (f)

Wing span: Short (s), medium(m), long (l)

(i) How many categories are there in this classification scheme ?

(ii) List all the categories in this classification scheme.

(d) Let $A = \{a, b, c, d\}$, and R be the relation on A that has the matrix (3)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Construct the digraph of R and list in-degrees and out-degrees of all vertices.

(e) If $G = \{(s, A), (a, b), S, P\}$, where P consists of the productions $\{S \rightarrow aAS, S \rightarrow a, A \rightarrow SbA, A \rightarrow SS, A \rightarrow ba\}$, generate the string $aabbaa$ using (i) a left most derivation , (ii) a right most derivation. (3)

Q.2(a) An airline services the five cities c_1, c_2, c_3, c_4 and c_5. Table:1 gives the cost(in dollars) of oing from c_i to city c_j. WE now define the following relation R on the set of cities $A = \{c_1, c_2, c_3, c_4, c_5\}$: $c_i R c_j$ if and only if the cost of going from c_i to C_j os less than or equal to 180 dollars. Find R. (5)

Table:1

From	To c_1	c_2	c_3	c_4	c_5
c_1	—	140	100	150	200
c_2	190	—	200	160	220
c_3	110	180	—	190	250
c_4	190	200	120	—	150
c_5	200	100	200	150	—

(b) Find the domain, range , matrix and when $A = B$, the digraph of the relation R wh
$A = \{1, 2, 3, 4, 8\}, B = \{1, 4, 6, 9\}; aRb$ if and only if $a|b$.

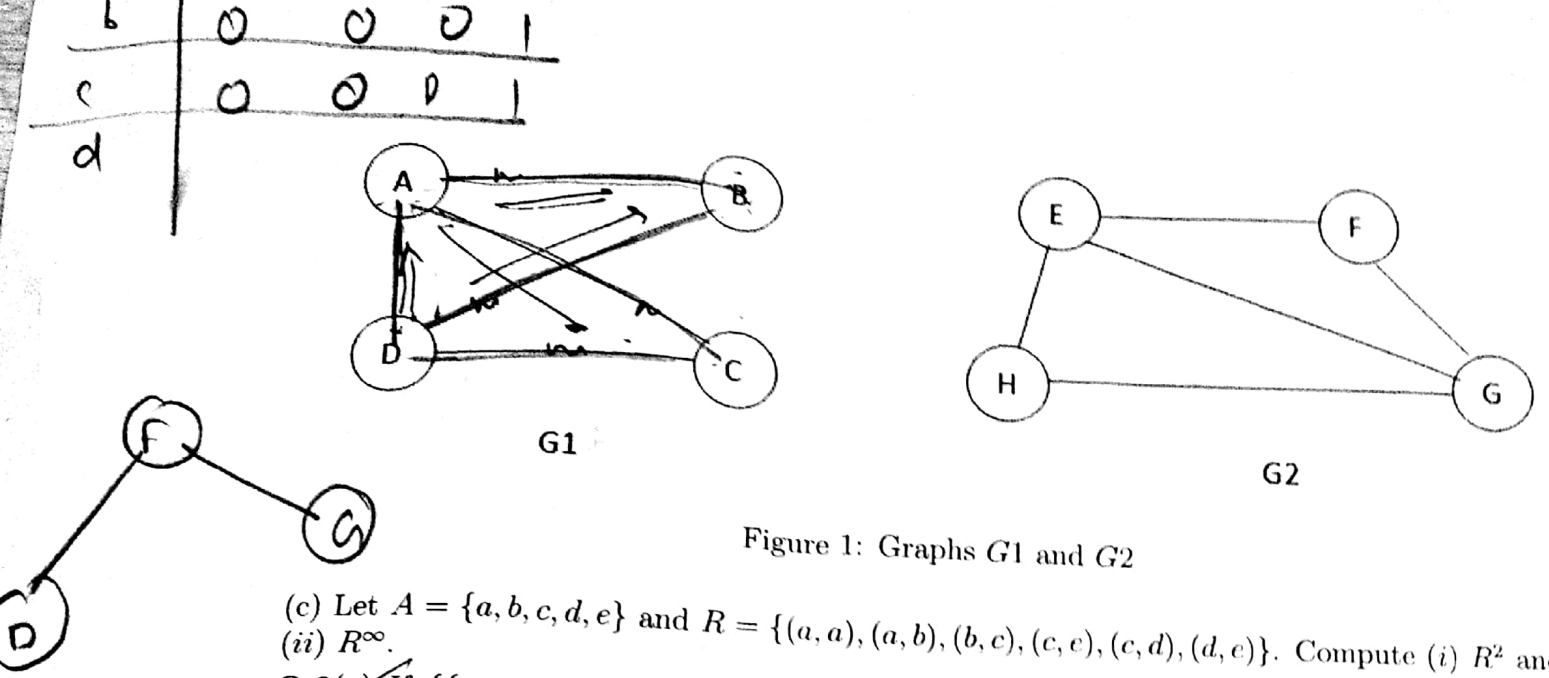

Figure 1: Graphs $G1$ and $G2$

(c) Let $A = \{a, b, c, d, e\}$ and $R = \{(a, a), (a, b), (b, c), (c, c), (c, d), (d, c)\}$. Compute (i) R^2 and (ii) R^∞.

Q.3(a) If $\{\{a, c, e\}, \{b, d, f\}\}$ is a partition of the set $A = \{a, b, c, d, e, f\}$, determine the corresponding equivalence relation R. (5)

(b) Let the matrix of relation R and S be: (5)

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

Find the composition $S \circ R$. (6)

(c) Let $A = \{1, 2, 3, 4\}$, and let $R = \{(1, 2), (2, 3), (3, 4), (2, 1)\}$. Find the transitive closure of R. (4)

Q.4(a) Define simple graph, regular graph, multigraph , pseudograph. Draw $K_{2,3}$ and $K_{3,3}$ graphs. (3)

(b) If all the vertices of an undirected graph are each of odd degree k, show that the number of edges of the graph is a multiple of k. (3)

(c) Establish the isomorphism of the two graphs given in Fig.1 by considering their adjacency matrix. . (4)

(d) Describe Dijkstra's shortest path algorithm. Use it to find the shortest path between the vertex A and the vertex Z in graph G (Fig.2). (5)

Q.5(a) Represent the postfix expression $ab + cd * ef/ - -a*$ as a binary tree and write the corresponding $infix$ and $prefix$ forms. (3)

(b) Use Prim's algorithm to find a minimum spanning tree for the weighted graph G given in Fig3. (

(c) Find an Euler path or an Euler circuit, if it exists in graph $G1$ of Fig.1. Also find a Hamiltonian path or a Hamiltonian circuit , if it exists in this graph G1 (Fig.1).

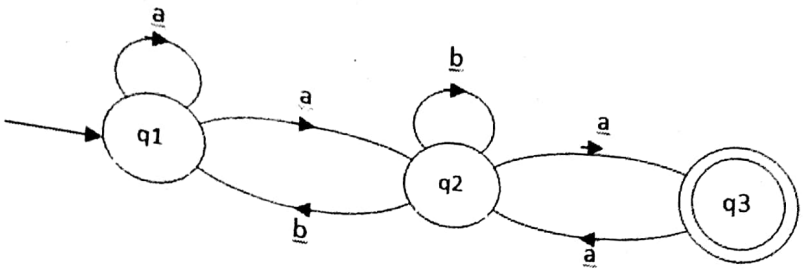

Figure 4: Transition system

(d) Construct an optimal Huffman code for the message "ENGINEERING AND COMPUTER SCIENCE APPLICATION". (4)

Q.6(a) The state table of a finite state machine M is given in Table:2. (5)

Table:2

f, g	a	b
s_0	s_0, b	s_4, b
s_1	s_0, a	s_3, b
s_2	s_0, a	s_2, a
s_3	s_1, b	s_1, b
s_4	s_1, b	s_0, a

(*i*) Find the input set I , the state set S, the output set O and the initial state of M.
(*ii*) Draw the state diagram of M.
(*iii*) Find the output of the word $w = a^2bab^2a$.

(b) Find the DFA equivalent to the $NDFA$ for which the state table is given in Table:3 and s_2 is the accepting state. (3)

Table:3

S	δ $I = a$	$I = b$
s_0	s_0, s_1	s_2
s_1	s_0	s_1
s_2	s_1	s_0, s_1
s_4	s_1, b	s_0, a

(c) Find a DFA that accepts precisely the string generated by the regular grammar $G = \{V_N, V_T, S, P\}$, , where $V_N = \{S, A, B\}$, $V_T = \{a, b\}$, $P = \{S \to bS, S \to a, S \to aA, A \to bB, A \to aS, B \to bA, B \to aS, B \to b\}$ and S is the starting symbol. (3)

(d) Shoe that the transition system given in Fig.4 recognizes the string $(a)+a(b+aa)^*b)^*a(b+aa)^*a$. (4)

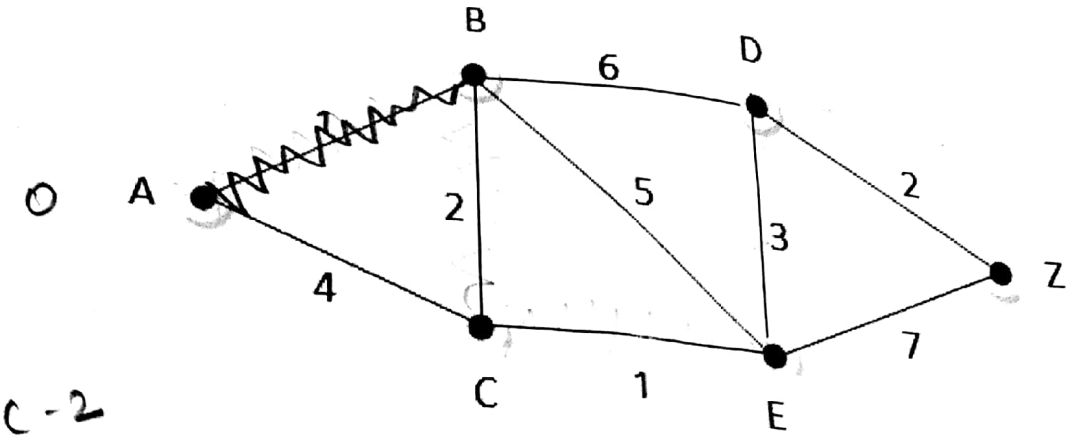

Figure 2: Graph G

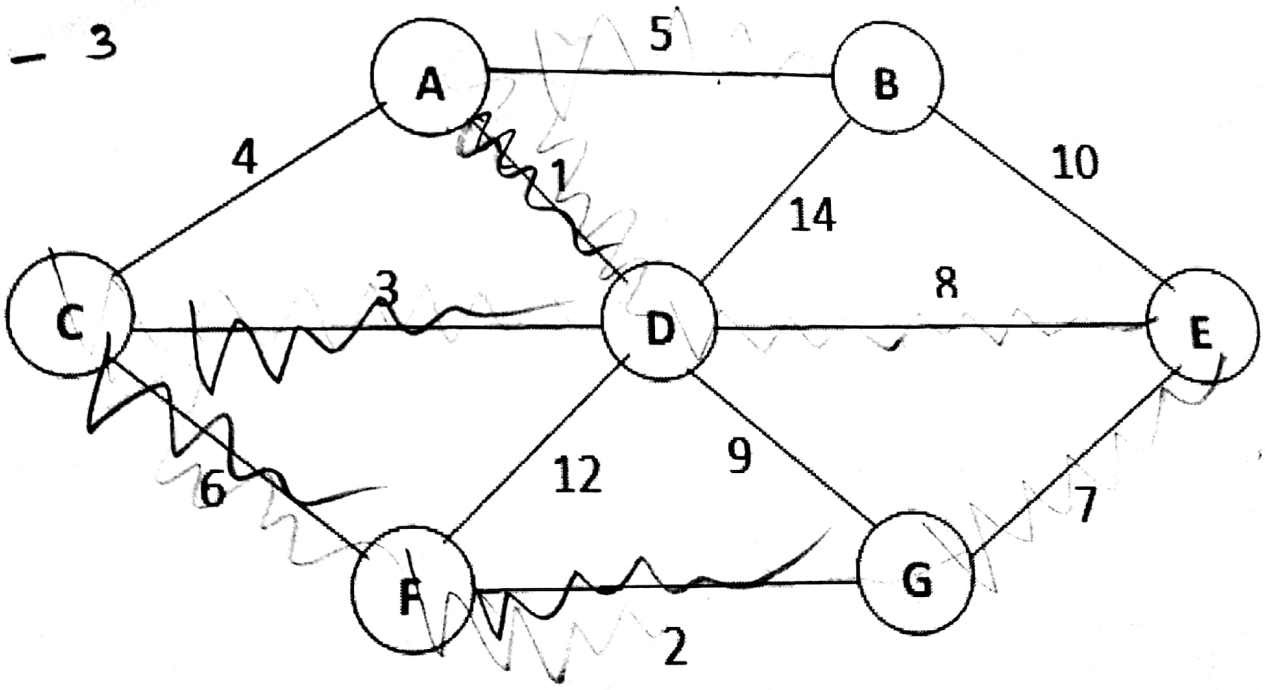

Figure 3: Graph G