M.Sc. (Informatics)/I- Sem. - 2012

Paper : IT-14

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Time: 3 hours *Maximum Marks: 75*

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt five *questions in all.* Q.1 *is compulsary*

Q.1(a) Let $f : R \to R$ be a function defined by $f(x) = x^3$. If $g : R \to R$ is such that $g(x) = x - 1$, find $g \circ f$ and $f \circ g$. Is $g \circ f$ equals $f \circ g$? (3)

(b) Let $A = \{1, 2, 3, 9, 18\}$. Consider a partial order of divisibility ($\leq$) on A. Draw the Hesse diagram of the poset $(A, \leq)$. (3)

(c) The set of all 2×2 matrices of the form

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

over reals where $ad - bc \neq 0$, forms a non abelian group under matrix multiplication. (3)

(d) Construct an FA that accepts all strings over $\{a, b, c\}$ that contain an odd number of a's. (3)

(e) Draw the Venn diagram for $(A - B) \cup (B - A)$ and $(A \cup B) - (A \cap B)$. (3)

Q.2(a) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$, $n \geq 2$ with initial condition $a_0 = 6$, $a_1 = 8$. (5)

(b)(a) Consider the plane graph shown in Fig.1. How many regions are there? List the edges that form the boundary of each region. Which region is exterior? (6)

Fig.1

(c)Find the adjacency matrix and incidence matrix of the following graph (Fig.2). Find the vertices for which there are two paths of length 2. (4)

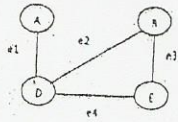

Fig.2

Q.3(a) Determine , with reasons, whether the following graph (Fig.3) is Hamiltonian. (5)

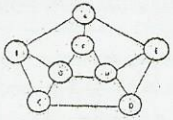

Fig.3

(b) Write down the steps involved in Floyd-Warshall algorithm to find the shortest distances between all pairs in a weighted graph. Hence use it to find the shortest distances between vertices in the following graph (Fig.4). (10)

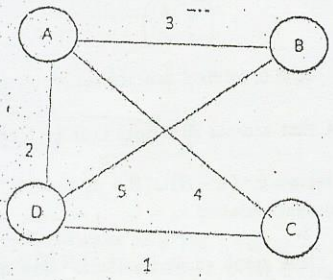

Fig.4

Q.4(a) Describe the Kruskal algorithm to find a minimum spanning tree in a connected weighted graph $G = (V, E)$. Hence use it to find the minimum spanning tree of the following graph (Fig.5). (6)

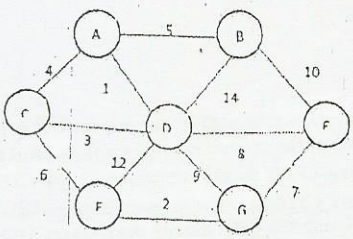

Fig.5

(b) Find the code words generated by the parity check matrix, when the encoding function is $e : B^3 \rightarrow B^6$. (5)

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Simplify the Boolean function $f(a, b, c, d) = \sum(0, 2, 6, 7, 8, 9, 13, 15)$ using Karnaugh map method. (4)

Q.5(a) Find the DFA (deterministic finite automata) equivalent to the NDFA for which the state table is: (8)

$\mapsto$ / S↓	f	
	a	b
s_0	s_0, s_1	s_2
s_1	s_0	s_1
s_2	s_1	s_0, s_2

Fig.6

and s_2 is the accepting state.

(b) Find the language generated by the grammar $G = \{(S, A), (a, b, c), S, P\}$ where P consists of the production $\{S \to aSb, \quad Sb \to bA, \quad abA \to C\}$. (4)

(c) Find the value of the postfix expression $72 - 3 + 232 + -13 - */$. (3)

Q.6(a) Construct a Huffman code for each character in the alphabet $\{a, b, c, d, e, f\}$ using Table.2. Using the code, decode the message 1101010110. (5)

Character	a	b	c	d	e	f
Frequency	4	1	2	3	5	4

Fig.7

(b)State Arden's theorem. Find the regular expression corresponding to the automaton of Fig.8? (6)

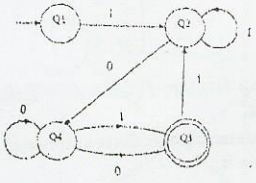

Fig.8

(c) Given the following transition rules of a TM (Turing machine), $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \{a, b, X, Y, \sharp\}, \delta, q_0, \sharp, \{q_4\})$. Using the transition rules find the computation sequence of the string $aabb$. (4)

$$\delta(q_0, a) = (q_1, X, R); \quad \delta(q_0, Y) = (q_3, Y, R);$$

$$\delta(q_1, a) = (q_1, a, R); \quad \delta(q_1, b) = (q_2, Y, L); \quad \delta(q_1, Y) = (q_1, Y, R)$$

$$\delta(q_2, a) = (q_2, a, L); \quad \delta(q_2, X) = (q_0, X, R); \quad \delta(q_2, Y) = (q_2, Y, L)$$

$$\delta(q_3, Y) = (q_3, Y, R); \quad \delta(q_3, \sharp) = (q_4, \sharp, R).$$