Roll No. 4014

M.Sc.(Informatics)/I-Sem.-2014

Paper:IT-14

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Time:3 hours *Maximum Marks:75*

Write your Roll No. on the top immediately on receipt of this question paper

Attempt **five** *questions in all.* **Q.1** *is compulsary*

Q.1(a)If A, B and C are sets, prove analytically or graphically that $A \cap (B - C) = (A \cap B) - (A \cap C)$. (3)

(b)If $A = \{1,2,3,4\} \times \{1,2,3,4\}$ and the relation R is defined on A by $(a,b)R(c,d)$ if $a + b = c + d$, verify that A is an equivalence relation on A and also find the quotient set of A by R. (5)

(c) If R and S be relations on a set A represented by the matrices (3)

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

find the matrices that represent

$$M_{R \oplus S} = M_{R \cup S} - M_{R \cap S}$$

(d) What type of graph does the following adjacency matrix represent? Name the graph. (2)

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(e)If $G = \{(S, A), (a, b), S, P\}$, where P consists of the production

$$\{S \rightarrow aAS, S \rightarrow a, A \rightarrow SbA, A \rightarrow SS, A \rightarrow ba\}$$

, generate the string $aabbaa$ using a left most derivation. (2)

Q.2(a)R and S are "Congruent modulo 3" and "Congruent modulo 4" relations respectively on the set of integers. That is $R = \{(a,b) | a \equiv b(mod3)\}$ and $S = \{(a,b) | a \equiv b(mod4)\}$. Find (i) $R \cup S$ and (ii) $R \oplus S$. (4)

(b)Using Warshal algorithm , find the transitive closure of a relation R such that (3)

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

(c)Determine the Hasse diagram of the partial order having the digraph shown in Fig.1. (3)

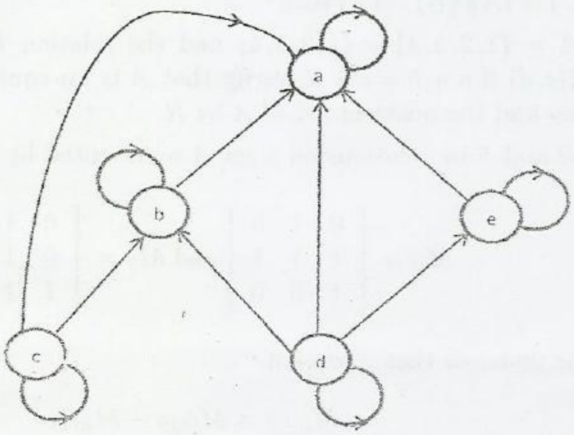

Fig.1

(d)Simplify the Boolean function $f(a,b,c,d) = \sum(0,1,2,3,4,5,6,7,8,9,11)$, by Kaurnaugh map method. (3)

(e) In any Boolean algebra , show that (2)

$$(x+y)(x'+z) = xz + x'y + yz = xz + x'y$$

Q.3(a) Given the following adjacency matrix A_1 and A_2 that corresponds to the graph G_1 and G_2 respectively.

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

If P is the appropriate permutation matrix, write the condition for the two graphs to be isomorphic. Draw the two graphs G_1 and G_2 and using the foregoing condition show that they are isomorphic. (6)

(b)Label the vertices of the undirected graph G_1 as A, B, C and D in any order. Find the number of paths of length 3 from the vertex D to A. (3)

(d) Write down the criterion used to find Eulerian path and Eulerian circuits in a given graph. Draw the complete graph K_5 and show that it is possible to have an Euler circuit in it. Write one such circuit. (3)

(e) Find the value of : (3)

(*i*) the prefix expression $+ - \uparrow 32 \uparrow 23/8 - 42$.

(*ii*) the postfix expression $72 - 3 + 232 + -13 - */$.

Q.4(a)Describe the Prim's algorithm to find a minimum spanning tree. Use it to find a minimum spanning tree for the weighted graph shown in Fig.2. (6)

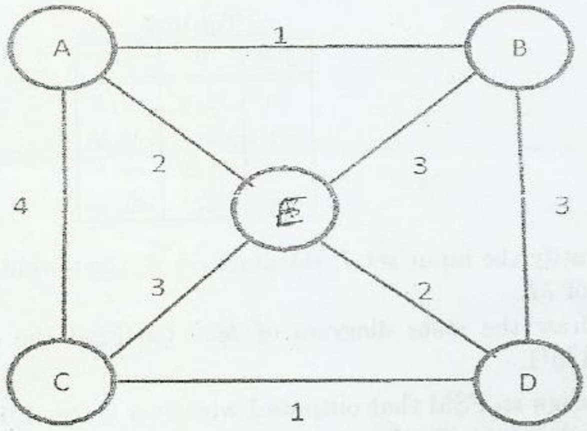

Fig.2

(b) Sketch the 11-vertex binary trees with minimum and maximum heights. Find also the path length of both the trees. (3)

(c) Construct a binary tree whose inorder and postorder traversals are respectively $DCEBFAHGI$ and $DECFBHIGA$. (3)

(d) We give below (Table:1), in array form , the doubly -linked -list representation of a labeled tree T (not binary). Draw the digraph of both the labeled binary tree $B(T)$ actually stored in the arrays and labelled tree T of which $B(T)$ is the binary representation. (3)

Table:1

INDEX	LEFT	DATA	RIGHT
1	2		0
2	3	a	0
3	4	b	5
4	6	c	7
5	8	d	0
6	0	e	10
7	0	f	0
8	0	g	11
9	0	h	0
10	0	i	9
11	0	j	12
12	0	k	0

Q.5(a) The state table of a finite state machine M is given in Table:2 (3)

Table:2

f, g	0	1
s_0	s_2, y	s_1, z
s_1	s_2, x	s_3, y
s_2	s_2, y	s_1, z
s_3	s_2, z	s_0, x

(i)Identify the input set I, the state set S, the output set O and the initial state of M.

(ii) Draw the state diagram of M. (iii)Find the output of the string $0^2 1^2 0 1^2 0^2 1$.

(b) Design an FSM that outputs 1 whenever it sees 101 as consecutive input bits and outputs 0 otherwise. (4)

(c) Find the DFA equivalent to the $NDFA$ for which the state table is given in Table:3 and the accepting state is s_2. (3)

Table:3

PS↓	Next State	
	a	b
s_0	ϕ	s_0, s_1
s_1	ϕ	s_2
s_2	s_0, s_1, s_2	ϕ

(d) Find a DFA that accepts precisely the string generated by the regular grammar $G = \{V_N, V_T, S, P\}$, where $V_N = \{S, A, B\}$, $V_T = \{a, b\}$,

$$P = \{S \to bS, S \to aA, S \to a, A \to aS, A \to bB, B \to bA, B \to aS, B \to b\}$$

and S is the starting symbol. (5)

Q.6(a) Find the language generated by each of the following grammars: (3)

(i) $G = \{(S, A, B), (a, b), S, P\}$, where P is the set of production

$$\{S \to AB, S \to AA, A \to aB, A \to ab, B \to b\}$$

(ii) $G = \{(S), (0, 1), S, P\}$, where P consists of the production

$$\{S \to aSb, Sb \to bA, abA \to C\}$$

(b) Construct a transition graph for the regular expression

$$\mathrm{R} = (0 + 11)^*(10^* + \lambda)$$

. Convert the graph into a graph without λ-transition. (3)

(c) Using Arden's lemma, find the regular expression accepted by the following automata (Fig.3). (4)

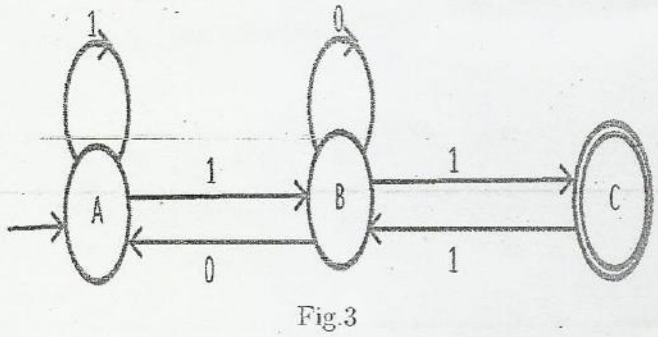

Fig.3

(d) Using the following transition rules in a Turing machine:

$$\delta(q_0, 1) = (q_0, 1, R), \delta(q_0, b) = (q_1, 1, R)$$

$$\delta(q_1, 1) = (q_1, 1, R), \delta(q_1, b) = (q_2, b, L)$$

$$\delta(q_2, 1) = (q_3, b, L), \delta(q_3, 1) = (q_3, 1, L)$$
$$\delta(q_3, b) = (q_f, b, R)$$

show that if $w_1 = 11$ and $w_2 = 111$, then $q_0w_1bw_2 \vdash *bq_fw_1w_2bb$. Here b represent the blank symbol. Further $q_i, i = 1, 2, 3$ and q_f represents the different states and the final state respectively in a Turing machine. (5)