

Time: 3hrs

Max.Marks:75

Attempt five questions in all. Question no.1 is compulsory

Q.1(a) Find the Fourier series expansion of the periodic function

$$f(x) = x, \quad -\pi \leq x \leq \pi, \quad f(x+2\pi) = f(x)$$

(5)

(b) Define even and odd functions. Draw graph of the functions (i)  $f(x) = |x|$ , (ii)  $f(x) = x^3$  where  $x$  is any real number and explain why the functions are odd or even.

(5)

(c) Find the complex Fourier series of the function

(5)

$$f(x) = e^{-x}, \quad -\pi < x < \pi.$$

Q.2(a) Find the Fourier transform if the function  $f(t) = e^{-a|t|}$ ,  $-\infty < t < \infty$ ,  $a > 0$ . Write the inverse transform.

(b) Let  $g(t) \Rightarrow G(f)$ . Then

$$g(at) \Rightarrow \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

Let  $g(t) \Rightarrow G(f)$  (5)

where  $a$  is a time-scaling factor that may be positive or negative.

(5)

(c) Consider an exponentially damped sinusoidal wave defined by

$$g(t) = \begin{cases} \exp(-t) \sin(2\pi f_c t), & t > 0 \\ 0, & t \leq 0 \end{cases}$$

$$e^{-t} \sin(2\pi f_c t) \quad (5)$$

Using the expression

$$\sin(2\pi f_c t) = \frac{1}{2j} [\exp(j2\pi f_c t) - \exp(-j2\pi f_c t)]$$

$$j2\pi f_c t$$

$$= e^{j2\pi f_c t} - e^{-j2\pi f_c t}$$

and applying the frequency -shifting property, find the Fourier transform of  $g(t)$ .

Q.3(a) Write down the properties of Dirac delta function. If  $g(t) = \sin(2\pi f_c t)$  then show that (5)

$$g(t) \Rightarrow \frac{1}{2j} [\delta(f - f_c) - \delta(f + f_c)].$$

↑ area = unity

(b) Consider a linear time-invariant system of impulse response  $h(t)$  driven by a complex exponential input of unit amplitude i.e.,  $x(t) = \exp(j2\pi ft)$ . Deduce an expression for the output  $y(t)$  and its Fourier transform. Explain the term (i) amplitude response and (ii) phase response of a linear system

(5)

(c) Consider a linear time-invariant device with a transfer function defined by

(5)

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ j, & f < 0 \end{cases}$$

Find the amplitude response and phase response of the device and explain the results along with their schematic diagrams.

Q.4(a) Explain the process of amplitude modulation. Describe the (i) time-domain and (ii) frequency domain process of amplitude modulation with suitable diagrams. (5)

(b) If a message signal is denoted by  $m(t) = A_m \cos(2\pi f_m t)$  and the carrier signal by  $c(t) = A_c \cos(2\pi f_c t)$ , then deduce an expression for the Fourier spectrum,  $S(f)$ , of the amplitude modulated signal  $s(t)$ . (5)

(c) Explain the process of generating double-sideband suppressed-carrier (DSBSC) modulation. Explain with schematic diagram, the coherent detection of DSBSC modulated waves. (3)

(d) How Costas loop is used to obtain a synchronous receiving system suitable for use with DSBSC modulated waves? (2)

Q.5(a) What do you understand by single-sideband modulation (SSB)? For a message signal  $m(t) = A_m \cos(2\pi f_m t)$ , explain the suppression of lower side-frequency  $f_c - f_m$  of the corresponding DSBSC wave. Here  $f_c$  corresponds to frequency of the carrier wave. Using a block diagram explain the phase-discrimination method for generating SSB modulated wave. (10)

(b) Draw a block diagram of a two stage SSB modulator. The input signal consists of voice signal occupying the frequency band 0.3 to 3.4 kHz. The two oscillator frequencies have the value  $f_1 = 100 \text{ kHz}$  and  $f_2 = 10 \text{ MHz}$ . Specify (i) sidebands of DSBSC modulated wave appearing at the two modulator output, (ii) sidebands of SSB modulated wave appearing at the two band-pass filter outputs. (5)

Q.6(a) Describe the (i) the angle modulation and (ii) frequency modulation of waves. Draw the block diagram for generating frequency modulated (FM) and phase modulated (PM) wave. Show that for a sinusoidal modulating wave,  $m(t) = A_m \cos(2\pi f_m t)$ , the spectrum is given by

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{+\infty} J_n(\beta) [\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)]$$

where  $\beta$  is the modulation index,  $A_c$  and  $f_c$  refers to the carrier wave amplitude and frequency respectively and further  $J_n(\beta)$  is the Bessel function of order  $n$ .

(b) Write short note on (i) Pulse amplitude modulation, (ii) Pulse code modulation

(c) Determine the Nyquist rate for a continuous-time signal

$$x(t) = 6 \cos 50\pi t + 20 \sin 300\pi t - 10 \cos 100\pi t$$

Handwritten notes:  $2\pi\omega t$ ,  $2\pi 300\pi$ ,  $50\pi$ ,  $\frac{1}{2\pi 300}$ ,  $a \sin 2\omega t$ ,  $2 \times 25 \times \pi \times t$ ,  $\omega_c$ ,  $25$ ,  $150$ ,  $50$ ,  $= 300$ ,  $2$

$$m(t) = A_m \cos(2\pi f_m t)$$

$$c(t) = A_c \cos(2\pi f_c t)$$