Roll No. 1609

M.Sc.(Informatics)/I-Sem.-2013

Paper:IT-14

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Time:3 hours *Maximum Marks:75*

Write your Roll No. on the top immediately on receipt of this question paper

Attempt five *questions in all.*

Q.1 (*a*) Let $A = \{1,2,3,4,5\}$, $B = \{1,2\}$, and $C = \{3,4\}$. Show that $A - (B \times C) \neq (A - B) \times (A - C)$. (3)

(*b*) Let $A = \{0,1,2,3,4\}$. $B = \{0,1,2,3\}$ and aRb if and only if $a + b = 4$. Find R? (3)

(*c*) If R and S are relations on a set A represented by the matrices

10

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

show that $(i) M_{R\cup S} = M_R \vee M_S$ and $(ii) M_{R\cap S} = M_R \wedge M_S$ (6)

(*d*) If R is a relation on the set of real numbers such that aRB if and only if $(a - b)$ is an integer, the show that R is an equivalence relation. (3)

Q.2(*a*) If R and S are relations on $A = 1,2,3$ represented by the matrices

10

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

find the matrices that represent $(i) R \cap S$ and $(ii) R \circ S$ (5)

(*b*) Draw the Hasse diagram for the divisibility relation on $\{2,4,5,10,12,20,25\}$ starting from the digraph. (4)

(*c*) Simplify the Boolean function $f(a,b,c,d) = \sum(0,1,2,3,4,5,6,7,8,9,11)$. by Karnaugh map method. (6)

Q.3(*a*) Given the generator matrix G:

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

corresponding to the encoding function $e : B^3 \to B^6$, find the corresponding parity check matrix and use it to decode the following received words and hence to find the original message: (7)

$$111101, 100100, 111100, 010100$$

(b) Draw a complete graph on 5 vertices and verify that $\sum_i deg(\nu_i) = 2|E|$. (2)

(c) The adjacency matrix A_1 and A_2 of two graphs G_1 and G_2 respectively are given by:

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Show that the two graphs are isomorphic? (6)

Q.4(a) Describe the Dijkstra's algorithm of finding the shortest path between two vertices, say a and z. For the following graph (Fig.1), using Dijkstra's algorithm, find the shortest path between the vertex A and F. (5)

(b) Use Prim's algorithm to find a minimum spanning tree for the weighted graph shown in Fig.1. (5)

(c) Construct the binary tree whose inorder and postorder traversal are respectively $DCEBFAHGI$ and $DECFBHIGA$. (2)

(d) Represent the expression $((a-c)*d)/(a+(b-d))$ as a binary tree and write prefix and postfix forms of the expression. (3)

Q.5(a) Design an FSM that performs serial binary addition. (4)

(b) design an FSM that outputs 1, if k 1's have been input, where k is a multiple of 3 and output 0 otherwise. (2)

(c) Draw the state diagram for the NDFA for which the state table is given in Table:1. Characterize the strings accepted by this NDFA, for which the accepting states are s_1 and s_2. Also find the DFA equivalent to this NDFA. (7)

(d) Draw the parse tree for the sentence " Delhi is a beautiful city". (2)

Q.6(a) State Arden's theorem. Find the regular expression corresponding to the automaton of Fig.2? (7)

(b) Construct the transition graph for the regular expression $R = (0+11)^*$. (3)

10

20

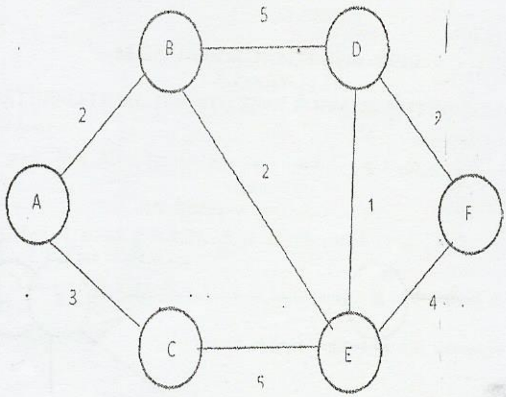

Fig.1

(c)Prove that the string $abbaab$ is accepted by the PDA
$M = [\{s_0, s_f\}, \{a, b\}, \{0, 1, z_0\}, f, s_0, z_0, \{s_f\}]$, where f is given by

$$\begin{aligned}
f(s_0, \lambda, z_0) &= (s_f, z_0); \quad f(s_0, a, z_0) = (s_0, 0z_0); \\
f(s_0, b, z_0) &= (s_0, 1z_0); \quad f(s_0, a, 0) = (s_0, 00); \\
f(s_0, b, 0) &= (s_0, \lambda); \quad f(s_0, a, 1) = (s_0, \lambda); \\
f(s_0, b, 1) &= (s_0, 11)
\end{aligned}$$

(5)

Table:1 Transition table.

$I \rightarrow$	δ	
$S \downarrow$	a	b
s_0	s_0, s_1	s_2
s_1	ϕ	s_1
s_2	s_1, s_2	ϕ

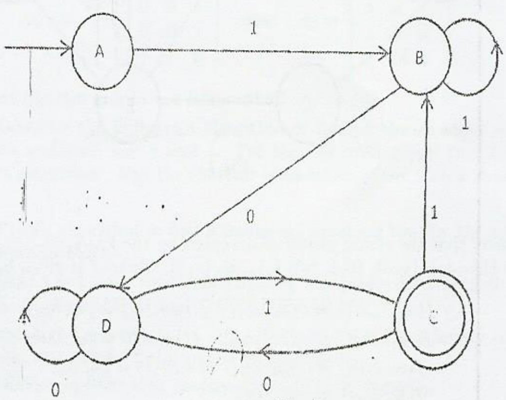

Fig.2