27 (8)

[This question paper contains 6 printed pages.]

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Your Roll No. ....

M.Sc. (INFORMATICS) / MIT / I Sem. – 2010

Paper IT14 – MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

Time : 3 hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** *questions in all.*

Q.1(a) If A , B and C are any finite set , then draw the Venn diagram for (i) $(A \cup B) \cap C$, (ii) $A \cap (B \oplus C)$ (2)

(b) Let $A = \{a, b, c, d, e\}$ and R be the relation on A defined by

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Compute A/R. (4)

(c) Let $A = \{1, 2, 3, 4\}$ and let R and S be the relations on A described by

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, M_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Use warshall's algorithm to compute the transitive closure of $R \cup S$. (4)

(d) Analyze the operation performed by the given piece of pseudocode and write a function that describes the number of steps required. Give the Θ-class of the function

1. $A \leftarrow 1$
2. $B \leftarrow 1$
3. $UNTIL(B > 100)$

a. $B \leftarrow 2A - 2$

b. $A \leftarrow A + 3$ (3)

(e) Is the permutation

$$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$

even or odd? (2)

P.T.O.

Q.2(a) Determine the Hasse diagram of the relation on $A = \{1, 2, 3, 4, 5\}$ whose matrix is shown below:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(3)

(b) Let $A = \{1, 2, 3, 4, 5, \cdots, 11\}$ be the poset whose Hasse diagram is shown in Fig.1. Find the LUB and GLB of $B = \{6, 7, 10\}$, if they exist. (3)

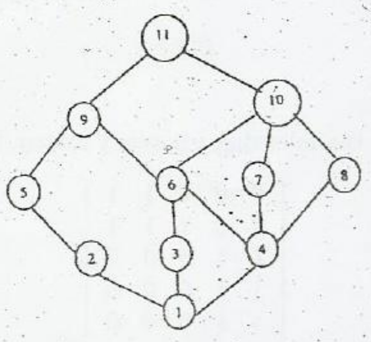

Fig.1

(c) Show that in a Boolean algebra, for any a, b and c (2)

$$(a \wedge b \wedge c) \vee (b \wedge c) = b \wedge c.$$

(d) Construct a linear $(6, 3)$ code given that

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(4)

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(e) Solve the recurrence relation

$$a_n = 4a_{n-1} - 4a_{n-2}, \quad n \geq 2$$

(3)

with initial conditions $a_0 = 6$, $a_1 = 8$.

Table:1

Character	a	b	c	d	e	f
Frequency	4	1	2	3	5	2

Q.3(a) Find all the paths of length 3 in the graph G of Fig.2. Also find the length of the shortest path, if any, that connects the vertex 1 and vertex 4. (4)

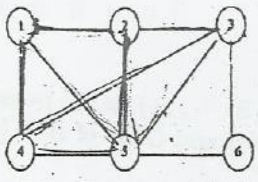

Fig.2

1, 2, 5, 4

(b) Is the graph with the following adjacency matrix Eulerian?

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(3)

(c) Describe the Dijkstra's shortest path algorithm. Using it Find the shortest path from a to z in the graph G (Fig.3). (8)

Q.4(a) Show that the maximum number of vertices in a binary tree of height h is $2^{h+1} - 1$. (2)

(b) Construct a Huffman code for each character in the alphabet $\{a, b, c, d, e, f\}$ using Table:1?
Using the codes, decode the message 001110101110 (5)

(c) Describe Prim's algorithm to find a minimal spanning tree. Using it find the minimal spanning tree of the graph shown in Fig.3. (8)

Q.5(a) Consider the finite automaton M_3 (Fig.4). Find the language accepted by this automaton. (2)

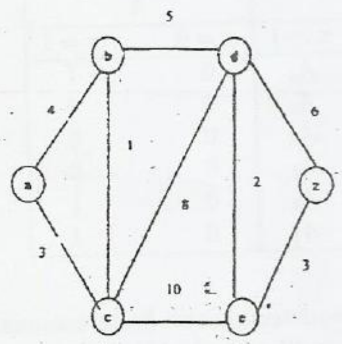

Fig.3

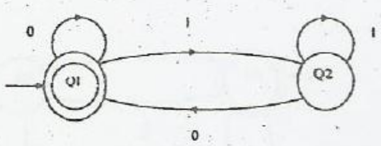

Fig.4

(b) Design an equivalent machine for an FSM M whose state transition is shown in Table:2. (5)

Table:2

PS	δ		λ	
	$x=0$	$x=1$	$x=0$	$x=1$
A_0	A_5	A_3	0	1
A_1	A_1	A_4	0	0
A_2	A_1	A_3	0	0
A_3	A_1	A_2	0	0
A_4	A_5	A_2	0	1
A_5	A_4	A_1	0	1

(c) Construct a nondeterministic finite automaton accepting the set of all strings over $\{a, b\}$ ending in aba. Use it to construct a DFA accepting the same set of strings. (4)

(d) Prove that the FA whose transition diagram is given in Fig.5 accepts the set of all strings over the alphabet $\{a, b\}$ with an equal number of a's and b's such that each prefix has at most one more a than b's and at most one more b than a's. (4)

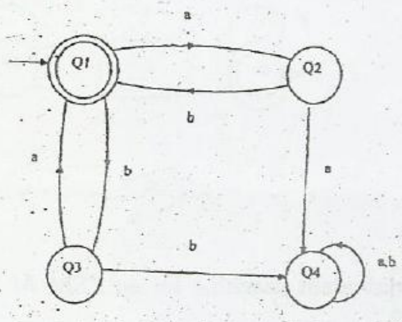

Fig.5

Q.6(a) Construct a right linear grammar for $L(001^*0)$. (3)

(b) Show that $L = \{a^n b^n : n \geq 1\}$ is not regular. (3)

(c) Consider CFG, $G = (V, T, S, P)$ with $V = \{E, I\}$ and $T = \{+, *, (,), x, y\}$ and productions P such that $E \to E + E$, $E \to E * E$, $E \to I|(E)$ and $I \to x|y$. Show that the grammar is ambiguous. (4)