ASSIGN-
MENT
FIRST

2023

MCSC-202

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1.Compute and find the absolute, relative and percentage errors.:

i) $1.3254 + 0.56 + 27.2879604 + 0.0375$

Solution:

$$\begin{aligned}
\text{Exact Value} &= 29.2108604\\
\text{Approximate Value} &= 29.12\\
\text{Absolute Error} &= |x - x_1|\\
&= |29.2108604 - 29.12|\\
&= 0.0008608\\
\text{Relative Error} &= \frac{\text{Absolute Error}}{x}\\
&= \frac{0.0008608}{29.2108604}\\
&= 0.0000294\\
\text{Percentage Error} &= E_R * 100\%\\
&= 0.00294\%
\end{aligned}$$

$\square$

ii) $4.6 * 0.128$

Solution:

$$\begin{aligned}
\text{Exact Value} &= 0.5888\\
\text{Approximate Value} &= 0.59\\
\text{Absolute Error} &= |x - x_1|\\
&= |0.5888 - 0.59|\\
&= 0.0012\\
\text{Relative Error} &= \frac{\text{Absolute Error}}{x}\\
&= \frac{0.0012}{0.5888}\\
&= 0.002038\\
\text{Percentage Error} &= E_R * 100\%\\
&= 0.2038\%
\end{aligned}$$

$\square$

iii) $\frac{0.995*1.53}{1.592}$

Solution:

$$\begin{aligned}
\text{Exact Value} &= 0.95625\\
\text{Approximate Value} &= \frac{0.995 * 1.53}{1.592} = \frac{1.52235}{1.592} = \frac{1.522}{1.592} = 0.9560301508 = 0.9560\\
\text{Absolute Error} &= |x - x_1|\\
&= |0.95625 - 0.9560|\\
&= 0.00025\\
\text{Relative Error} &= \frac{\text{Absolute Error}}{x}\\
&= \frac{0.00025}{0.95625}\\
&= 0.0002614\\
\text{Percentage Error} &= E_R * 100\%\\
&= 0.02614\%
\end{aligned}$$

$\square$

iv) $\sqrt{2.01} - \sqrt{2}$

Solution:

$$\begin{aligned}
\text{Exact Value} &= 0.003531125 = 0.003531 \\
\text{Approximate Value} &= 0.0036 \\
\text{Absolute Error} &= |x - x_1| \\
&= |0.003531 - 0.0036| \\
&= 0.000069 \\
\text{Relative Error} &= \frac{\text{Absolute Error}}{x} \\
&= \frac{0.000069}{0.003531} \\
&= 0.01954 \\
\text{Percentage Error} &= E_R * 100\% \\
&= 1.95\%
\end{aligned}$$

$\square$

2.Compute the maximum absolute and relative error in $u = \frac{5x^3y^2}{z^5}$ **when** $x = 3.25, y = 45.129$ **and** $z = 0.577$.

Solution:

$$\begin{aligned}
u &= \frac{5x^3y^2}{z^5} \\
u &= \frac{5 * (3.25)^3 * (45.929)^2}{0.577^5} \\
&= 5465783.71 \\
\delta u_{max} &= ?
\end{aligned}$$

$$\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{15x^2y^2}{z^5} = \frac{15 * (3.25)^2 * (45.929)^2}{0.577^5} = 5045338.809 \\
\frac{\partial u}{\partial y} &= \frac{10x^3y}{z^5} = \frac{10 * (3.25)^3 * (45.929)}{0.577^5} = 242229.3297 \\
\frac{\partial u}{\partial z} &= \frac{-25x^3y^2}{z^6} = \frac{-25 * (3.25)^3 * (45.929)^2}{0.577^6} = -47363810.31
\end{aligned}$$

$\delta x = \delta y = \delta z = 0.0005$

$$\begin{aligned}
\delta u &= |\frac{\partial u}{\partial x}\delta x| + |\frac{\partial u}{\partial y}\delta y| + |\frac{\partial u}{\partial z}\delta z| \\
&= |5045338.809 * 0.0005| + |242229.3297 * 0.0005| + |-47363810.31 * 0.0005| \\
&= 26325.68922
\end{aligned}$$

$$\begin{aligned}
\text{Relative Error} &= \frac{\delta u}{u} = \frac{26325.68922}{5465783.71} \\
&= 0.004816
\end{aligned}$$

$\square$

3. Calculate a real root correct to four significant figures of the following equations:
a)Bisection method:
i)$x^3 - 4x - 9 = 0, (2, 3)$
Solution: $f(x) = x^3 - 4x - 9 = 0$
$Tolerance = 4$ decimal places
$\epsilon = 1/2 * 10 - 4 = 0.0005$

	$f(a) < 0(-)$	$f(b) > 0(+)$			
n	b	b	x	f(x)	Remarks
1	2.5	3	2.5	-3.375	$f(x) < 0$
2	2.5	3	2.75	0.79688	$f(x) > 0$
3	2.625	2.75	2.625	-1.4121	$f(x) < 0$
4	2.6875	2.75	2.6875	-0.33911	$f(x) < 0$
5	2.6875	2.75	2.7188	0.22092	$f(x) > 0$
6	2.7031	2.7188	2.7031	-0.061077	$f(x) < 0$
7	2.7031	2.7188	2.7109	0.079423	$f(x) > 0$
8	2.7031	2.7109	2.707	0.0090492	$f(x) > 0$
9	2.7051	2.707	2.7051	-0.026045	$f(x) < 0$
10	2.7061	2.707	2.7061	-0.0085056	$f(x) < 0$
11	2.7061	2.707	2.7065	0.0002699	$f(x) > 0$
12	2.7063	2.7065	2.7063	-0.0041183	$f(x) < 0$
13	2.7064	2.7065	2.7064	-0.0019243	$f(x) < 0$
14	2.7065	2.7065	2.7065	-0.00082725	$f(x) < 0$

$\square$

ii)$2xcos(2x) - (x + 1)2 = 0, (-3, -2)$
Solution: $f(x) = 2xcos(2x) - (x + 1)2 = 0, (-3, -2)$
$Tolerance = 4$ decimal places
$\epsilon = 1/2 * 10 - 4 = 0.0005$

	$f(a) < 0(-)$	$f(b) > 0(+)$			
n	b	b	x	f(x)	Remarks
1	-3	-2	-2.5	-3.6683	$f(x) < 0$
2	-2.5	-2	-2.25	-0.61392	$f(x) < 0$
3	-2.25	-2	-2.125	0.63025	$f(x) > 0$
4	-2.25	-2.125	-2.1875	0.038076	$f(x) > 0$
5	-2.25	-2.1875	-2.2188	-0.28084	$f(x) < 0$
6	-2.2188	-2.1875	-2.2031	-0.11956	$f(x) < 0$
7	-2.2031	-2.1875	-2.1953	-0.040279	$f(x) < 0$
8	-2.1953	-2.1875	-2.1914	-0.00098519	$f(x) < 0$
9	-2.1914	-2.1875	-2.1895	0.018574	$f(x) > 0$
10	-2.1914	-2.1895	-2.1904	0.0088019	$f(x) > 0$

$\square$

b)False Position:
i) $xlog_{10}x - 1.2 = 0, (2, 3)$
Solution:

$$\begin{aligned}
f(2) &= xlog_{10}x - 1.2 = 0 \\
&= 2 * log_{10} * 2 - 1.2 \\
&= -0.597 < 0 \\
f(3) &= xlog_{10}x - 1.2 \\
&= 3 * log_{10} * 3 - 1.2 \\
&= 0.23136 > 0
\end{aligned}$$

$Tolerance = 4$ decimal places
$\epsilon = 1/2 * 10^{-4} = 0.0005$

a(-)	b(+)	f(a)	f(b)	c $c = \frac{af(b)-bf(a)}{f(b)-f(a)}$	f(c)
2	3	-0.59794	0.23136	2.72102	-0.01709
2.72102	3	-0.01709	0.23136	2.74021	-0.00038
2.74021	3	-0.00038	0.23136	2.74063	-0.000014
2.74063	3	-0.000014	0.23136	2.74064	-0.000005

∴The required root is 2.74064.

□

ii) $x^3 - 4x - 9 = 0, (2, 3)$
Solution:

$$\begin{aligned} f(2) &= x^3 - 4x - 9 = 0 \\ &= 2^3 - 4 * 2 - 9 \\ &=< 0 \\ f(3) &= x^3 - 4x - 9 \\ &= 3^3 - 4 * 3 - 9 \\ &=> 0 \end{aligned}$$

n	a(-)	b(+)	c $c = \frac{af(b)-bf(a)}{f(b)-f(a)}$	f(a)	f(b)	f(c)
1	2	3	2.6	-9	6	-1.824
2	2.6	3	2.84	-1.824	6	2.5463
3	2.6	2.84	2.656	-1.824	2.5463	-0.88853
4	2.656	2.84	2.7328	-0.88853	2.5463	0.47726
5	2.656	2.7328	2.6758	-0.88853	0.47726	-0.54435
6	2.6758	2.7328	2.7129	-0.54435	0.47726	0.11428
7	2.6758	2.7129	2.6956	-0.54435	0.11428	-0.19617
8	2.6956	2.7129	2.7099	-0.19617	0.11428	0.060066
9	2.6956	2.7099	2.7046	-0.19617	0.060066	-0.034645
10	2.7046	2.7099	2.7086	-0.034645	0.060066	0.037824
11	2.7046	2.7086	2.7061	-0.034645	0.037824	-0.0081667
12	2.7061	2.7086	2.7073	-0.0081667	0.037824	0.013806
13	2.7061	2.7073	2.7063	-0.0081667	0.013806	-0.0042666
14	2.7063	2.7073	2.7067	-0.0042666	0.013806	0.0024487
15	2.7063	2.7067	2.7064	-0.0042666	0.0024487	-0.0026815

□

d)Newton-Raphson: i)$e^x - cotx = 0, x_0 = 0.8$
Solution: $a = 0.8$

$$\begin{aligned} f(a) &= e^x - cotx \\ &= e^{0.8} - cot(0.8) \\ &= 1.2543 \\ f'(a) &= e^x + (cosec(x))^2 \\ &= e^{0.8} + (cosec(0.8))^2 \\ &= 4.1688 \end{aligned}$$

a	f(a)	f'(a)	c	f(c)
0.8	1.2543	4.1688	0.49912	-0.18711
0.49912	-0.18708	6.0121	0.53023	-0.00650
0.53023	-0.0064888	5.6091	0.53139	-0.000004

□

ii)$1.05 - 1.04x + lnx = 0, x_0 = -1$
Solution: $a = -1$

$$
\begin{aligned}
f(a) &= 1.05 - 1.04x + lnx \\
&= 1.05 - 1.04 * -1 + ln(-1) \\
&= Undefined \\
f'(a) &= -1.04 + \frac{1}{x} \\
&= -2.04
\end{aligned}
$$

There is no solution.

□

e)Fixed point iteration:
i)$xe^x = 1, x0 = 1$
Solution:

$$
\begin{aligned}
f(x)xe^x - 1 \\
x = \frac{1}{e^x} \\
\text{we choose} g(x) = \frac{1}{e^x}
\end{aligned}
$$

x	g(x)
1	0.367879
0.367879	0.692201
0.692201	0.500474
0.500474	0.606244
0.606244	0.545396
0.545396	0.579612
0.579612	0.560115
0.560115	0.571143
0.571143	0.564879
0.564879	0.568429
0.568429	0.566415
0.566415	0.567557
0.567557	0.566909
0.566909	0.567276
0.567276	0.567068
0.567068	0.567186
0.567186	0.567119

$\therefore$ The required root is 0.5671.

□

ii) $cosx = 3x - 1, x_0 = 2$
Solution:

$$x = \frac{cosx + 1}{3}$$
$$\text{If } g(x) = \frac{cosx + 1}{3}$$
$$g'(x) = \frac{-1}{3} sinx < 1$$

$$x = cos^{-1}(3x - 1)$$
$$\text{If } g(x) = cos^{-1}(3x - 1)$$
$$g'(x) = \frac{-1}{\sqrt{1 - (3x - 1)^2}} * 3$$
$$= \frac{-3}{\sqrt{6x - 9x^2}}$$

So we consider, $g(x) = \frac{cosx+1}{3}$

x	g(x)
2	0.194618
0.194618	0.660374
0.660374	0.596588
0.596588	0.609086
0.609086	0.606724
0.606724	0.607173
0.607173	0.607088
0.607088	0.607104
0.607104	0.607101
0.607101	0.607102
0.607102	0.607102

□

4. Use the Newton method and Generalized newton method to obtain a real root of the equation $(x + 2)3(x - 2) = 0$ correct to 0.0001 with initial guess $x_0 = 3$.
Solution:

a	f(a)	f'(a)	c
3	125	200	2.375
2.375	31.403	105.27	2.0767
2.0767	5.197	71.577	2.0041
2.0041	0.2631	64.394	2
2	0.00080294	64.001	2

x = 2.00000

□

5. Solve the following systems of nonlinear equations using Newton-Raphson method accurate to 3 decimal places:
i) $x^2 + 4y^2 - 16 = 0, xy^2 - 4 = 0, x_0 = y_0 = 1.5$
Solution:

$$f(x, y) = x^2 + 4y^2 - 16$$
$$g(x, y) = xy^2 - 4$$

Then ,

$$\frac{\partial f}{\partial x} = 2x$$
$$\frac{\partial f}{\partial y} = 8y$$
$$\frac{\partial g}{\partial x} = y^2$$
$$\frac{\partial g}{\partial y} = 2xy$$

x	y	f	g	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$	$\frac{\partial g}{\partial x}$	$\frac{\partial g}{\partial y}$	D	D_1
1.5	1.5	-4.75	-0.625	3	12	2.25	4.5	-13.5	13.875
0.472222	2.15278	2.7608	-1.81151	0.944444	17.2222	4.63445	2.03318	-77.8953	-36.8114
0.944797	1.96656	0.362039	-0.346138	1.88959	15.7325	3.86735	3.716	-53.8212	-6.79094
1.07097	1.92839	0.0217473	-0.0173808	2.14195	15.4271	3.71869	4.13051	-48.5214	-0.357963
1.07835	1.92596	7.81236e-05	-6.2866e-05	2.1567	15.4077	3.70931	4.15371	-48.1934	-0.00129312
1.07838	1.92595	1.03156e-09	-8.28222e-10	2.15676	15.4076	3.70928	4.1538	-48.1922	-1.70458e-08

D_2	nx	ny
-8.8125	0.472222	2.15278
14.5057	0.944797	1.96656
2.05419	1.07097	1.92839
0.1181	1.07835	1.92596
0.000425368	1.07838	1.92595
5.61262e-09	1.07838	1.92595

□

6. Solve the following system of equations using iteration method correct to 3 significant figures.

i)$x^2 - 2x - y + 0.5 = 0, x^2 + 4y^2 - 4 = 0, x_0 = 0, y_0 = 1$

Solution:

$$f(x, y) = x^2 - 2x - y + 0.5$$
$$g(x, y) = x^2 + 4y^2 - 4$$
$$x = \frac{x^2 - y + 0.5}{2}$$
$$y = \frac{4 - x^2}{4}$$
$$F(x, y) = \frac{x^2 - y + 0.5}{2}$$
$$G(x, y) = \sqrt{\frac{4 - x^2}{4}} = \frac{1}{2}(4 - x^2)^1/2$$
$$\frac{\partial F}{\partial x} = x$$
$$\frac{\partial F}{\partial y} = -0.5$$
$$\frac{\partial G}{\partial y} = \frac{x}{2\sqrt{4 - x^2}}$$
$$\frac{\partial G}{\partial y} = 0$$

At (0,1)

$$|\frac{\partial F}{\partial x}| + |\frac{\partial F}{\partial y}| < 1$$
$$|\frac{\partial G}{\partial x}| + |\frac{\partial G}{\partial y}| < 1$$

So, Now,

x	y	F(x,y)	G(x,y)
0	1	-0.25	1
-0.25	1	-0.21875	0.9921
-0.2187	0.9921	-0.2221	0.9940
-0.2221	0.9940	-0.2223	0.9938
-0.2223	0.9938	-0.2222	0.9938
-0.2222	0.9938	-0.2222	0.9938

$$\therefore x = -0.2222$$
$$\therefore y = 0.9938$$

□

ii)$x^2 + y^2 + z^2 + 10x - 4 = 0, x^2 - y^2 + z^2 + 10y - 5 = 0, x^2 + y^2 - z^2 + 10z - 6 = 0$ ***Solution:***

$$f(x, y, z) = x^2 + y^2 + z^2 + 10x - 4 = 0$$
$$g(x, y, z) = x^2 - y^2 + z^2 + 10y - 5 = 0$$
$$h(x, y, z) = x^2 + y^2 - z^2 + 10z - 6 = 0$$
$$x = F(x, y, z) = \frac{4 - x^2 - y^2 - z^2}{10}$$
$$y = G(x, y, z) = \frac{5 - x^2 + y^2 - z^2}{10}$$
$$z = H(x, y, z) = \frac{6 - x^2 - y^2 + z^2}{10}$$

$$\frac{\partial F}{\partial x} = -0.2x \qquad \frac{\partial F}{\partial y} = -0.2y$$
$$\frac{\partial F}{\partial z} = -0.2z \qquad \frac{\partial G}{\partial x} = -0.2x$$
$$\frac{\partial G}{\partial y} = 0.2y \qquad \frac{\partial G}{\partial z} = 0.2z$$
$$\frac{\partial H}{\partial x} = -0.2x \qquad \frac{\partial H}{\partial y} = -0.2y$$
$$\frac{\partial H}{\partial z} = 0.2z$$

At (0,1)

$$|\frac{\partial F}{\partial x}| + |\frac{\partial F}{\partial y}| + |\frac{\partial F}{\partial z}| < 1$$
$$|\frac{\partial G}{\partial x}| + |\frac{\partial G}{\partial y}| + |\frac{\partial G}{\partial z}| < 1$$
$$|\frac{\partial H}{\partial x}| + |\frac{\partial H}{\partial y}| + |\frac{\partial H}{\partial z}| < 1$$

x	y	z	F(x,y)	G(x,y)	H
0	0	0	0.4	0.5	0.6
0.4	0.5	0.6	0.323	0.473	0.595
0.323	0.473	0.595	0.331792	0.476537	0.602597
0.331792	0.476537	0.602597	0.32997	0.475388	0.602595
0.32997	0.475388	0.602595	0.330201	0.475399	0.602825
0.330201	0.475399	0.602825	0.330157	0.475357	0.602836

$$x = 0.330201$$
$$y = 0.475399$$
$$z = 0.602825$$

□

7. Solve the following systems using LU decomposition method taking $l_{ii} = 1$

x + 2y + 3z = 14
2x + 20y + 26z = 120
3x + 26y + 70z = 265

Solution:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 20 & 26 \\ 3 & 26 & 70 \end{bmatrix} B = \begin{bmatrix} 14 \\ 120 \\ 265 \end{bmatrix}$$

Then $A = LU$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 20 & 26 \\ 3 & 26 & 70 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ L_{21} & L_{21}U_{21} + U_{22} & L_{21}U_{13} + U_{23} \\ L_{31}U_{11} & L_{31}U_{12} + L_{32}U_{22} & L_{31}U_{13} + L_{32}U_{23} + U_{33} \end{bmatrix}$$

$$U_{11} = 1 \qquad U_{12} = 2$$
$$U_{13} = 3 \qquad L_{21} = 2$$
$$L_{31} = \frac{3}{U_{11}} = \frac{3}{1} = 3$$

2nd row,

$$\begin{aligned} U_{22} &= a_{22} - L_{21}U_{12} \\ &= 20 - 2 * 2 \\ &= 16 \\ U_{23} &= a_{23} - L_{21}U_{13} \\ &= 26 - 2 * 3 \\ &= 20 \end{aligned}$$

2nd col,

$$\begin{aligned} U_{22}L_{32} &= a_{32} - L_{31}U_{12} \\ &= \frac{26 - 3 * 2}{16} \\ L_{32} &= 5/4 \end{aligned}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{5}{4} & 1 \end{bmatrix} U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 16 & 20 \\ 0 & 0 & 36 \end{bmatrix}$$

Now,

$$L\vec{Y} = \vec{B}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{5}{4} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 120 \\ 265 \end{bmatrix}$$

$$y_1 = 14$$
$$y_2 = 92$$
$$y_3 = 108$$

Then

$$L\vec{Y} = \vec{B}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 16 & 20 \\ 0 & 0 & 36 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 120 \\ 265 \end{bmatrix}$$

$$x = 1$$
$$y = 2$$
$$z = 3$$

□

8. Solve the following systems using LU decomposition method taking $u_{ii} = 1$.

4x + 8y + 4z = 28
x + 5y + 4z- 3w = 13
x + 4y + 7z + 2w = 23
x + 3y - 2w = 4

Solution:

$$A = \begin{bmatrix} 4 & 8 & 4 & 0 \\ 1 & 5 & 4 & -3 \\ 1 & 4 & 7 & 2 \\ 1 & 3 & 0 & -2 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} B = \begin{bmatrix} 28 \\ 13 \\ 23 \\ 4 \end{bmatrix}$$

If A = LU
Then equation 1 becomes
$LU\vec{X} = \vec{B}$
If we set

$$U\vec{X} = \vec{Y} \tag{1}$$

$$L\vec{Y} = \vec{B} \tag{2}$$

$$\begin{bmatrix}4 & 8 & 4 & 0\\1 & 5 & 4 & -3\\1 & 4 & 7 & 2\\1 & 3 & 0 & -2\end{bmatrix}$$

$R_1 \to R_1 * 1/4$

$$\begin{bmatrix}1 & 2 & 1 & 0\\1 & 5 & 4 & -3\\1 & 4 & 7 & 2\\1 & 3 & 0 & -2\end{bmatrix}$$

$R_2 = R_2 - R_1, R_3 = R_3 - R_1, R_4 \to R_4 - R_1$

$$\begin{bmatrix}1 & 2 & 1 & 0\\0 & 3 & 3 & -3\\0 & 2 & 6 & 2\\0 & 1 & -1 & -2\end{bmatrix}$$

$R_2 \to R_2 * 1/3$

$$\begin{bmatrix}1 & 2 & 1 & 0\\0 & 1 & 1 & -1\\0 & 2 & 6 & 2\\0 & 1 & -1 & -2\end{bmatrix}$$

$R_3 \to R_3 - 2 * R_2, R_4 \to R_4 - R_2$

$$\begin{bmatrix}1 & 2 & 1 & 0\\0 & 1 & 1 & -1\\0 & 0 & 4 & 4\\0 & 0 & -2 & -1\end{bmatrix}$$

$R_3 \to R_3 * 1/4$

$$\begin{bmatrix}1 & 2 & 1 & 0\\0 & 1 & 1 & -1\\0 & 0 & 1 & 1\\0 & 0 & -2 & -1\end{bmatrix}$$

$R_4 \to R_4 + 2R_3$

$$U = \begin{bmatrix}1 & 2 & 1 & 0\\0 & 1 & 1 & -1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 1\end{bmatrix}$$

Now ,
A=LU

$$\begin{bmatrix}4 & 8 & 4 & 0\\1 & 5 & 4 & -3\\1 & 4 & 7 & 2\\1 & 3 & 0 & -2\end{bmatrix} = \begin{bmatrix}l_{11} & 0 & 0 & 0\\l_{21} & l_{22} & 0 & 0\\l_{31} & l_{32} & l_{33} & 0\\l_{41} & l_{42} & l_{43} & l_{44}\end{bmatrix}\begin{bmatrix}1 & 2 & 1 & 0\\0 & 1 & 1 & -1\\0 & 0 & 1 & 1\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}4 & 8 & 4 & 0\\1 & 5 & 4 & -3\\1 & 4 & 7 & 2\\1 & 3 & 0 & -2\end{bmatrix} = \begin{bmatrix}l_{11} & 2l_{11} & l_{11} & 0\\l_{21} & 2l_{21} + l_{22} & l_{21} + l_{22} & -l_{22}\\l_{31} & 2l_{32} + l_{32} & l_{31} + l_{32} + l_{33} & -l_{32} + l_{33}\\l_{41} & 2l_{41} + l_{42} & l_{41} + l42 + l_{43} & -l_{42} + l_{43} + l_{44}\end{bmatrix}$$

On equating,

$$
\begin{aligned}
&l_{11} = 4, l_{21} = 1, l_{31} = 1, l_{41} = 1\\
&l_{22} = 3\\
&2l_{31} + l_{32} = 4, 2l_{41} + l_{42} = 3\\
&l_{32} = 2, l_{42} = 1
\end{aligned}
$$

$$
\begin{aligned}
&l_{31} + l_{32} + l_{33} = 7\\
&1 + 2 + l_{33} = 7\\
&l_{33} = 4
\end{aligned}
$$

$$
\begin{aligned}
&l_{41} + l42 + l_{43} = 0\\
&1 + 1 + l_{43} = 0\\
&l_{43} = -2
\end{aligned}
$$

$$
\begin{aligned}
&- l_{42} + l_{43} + l_{44} = -2\\
&l_{44} = 1
\end{aligned}
$$

So,

$$
L = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 2 & 4 & 0 \\ 1 & 1 & -2 & 1 \end{bmatrix}
$$

Using forward substitution,

$$
\begin{aligned}
&4y_1 = 8 \Rightarrow y_1 = 2\\
&y_1 + 3y_2 = 13 \Rightarrow 2 + 3y_2 = 13 \Rightarrow y_2 = 11/3\\
&y_1 + 2y_2 + 4y_3 = 23\\
\Rightarrow &2 + 2 * 11/3 + 4y_3 = 23 \Rightarrow y_3 = 41/12\\
&y_1 + y_2 - 2y_3 + y_4 = 4\\
\Rightarrow &2 + 11/3 - 2 * 41/12 + y_4 = 4\\
\Rightarrow &y_4 = 31/6
\end{aligned}
$$

Again,

$$
U\vec{X} = \vec{Y}
$$

$$
\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 11/3 \\ 41/12 \\ 31/6 \end{bmatrix}
$$

Using backward substitution,

$$
\begin{aligned}
&w = 31/6\\
&z + w = 41/12 \Rightarrow z = 41/12 - 31/6 = -7/4\\
&y + z - w = 11/3 \Rightarrow y = 11/7 + 7/4 + 31/6 = 127/12\\
&x + 2y + z = 2 \Rightarrow x = 2 - 2 * \frac{127}{12} + \frac{7}{4} = \frac{-209}{12}
\end{aligned}
$$

$\square$

9. Solve the following tidiagonal systems using Thomas Algorithm:

$$4x + 8y = 8$$
$$8x + 18y + 2z = 18$$
$$2y + 5z + 1.5w = 0.5$$
$$1.5z + 1.75w = -1.75$$

Solution: The augmented matrix is:

$$A = \begin{bmatrix} 4 & 8 & 0 & 0 & : & 8 \\ 8 & 18 & 2 & 0 & : & 18 \\ 0 & 2 & 5 & 1.5 & : & 0.5 \\ 0 & 0 & 1.5 & 1.75 & : & -1.75 \end{bmatrix}$$

Given i = 4,

$$\begin{aligned} \alpha_1 &= b_1 = 4 \\ \alpha_2 &= b_2 - \frac{a_2 c_1}{\alpha_1} \\ &= 18 - \frac{8 * 8}{4} \\ &= 2 \\ \alpha_3 &= b_3 - \frac{a_3 c_2}{\alpha_2} = 5 - \frac{2 * 2}{2} = 3 \\ \alpha_4 &= b_4 - \frac{a_4 c_3}{\alpha_3} = 175 - \frac{1.5 * 1.5}{3} = 1 \end{aligned}$$

$$\begin{aligned} \beta_1 &= \frac{b_1}{b_1} = 2 \\ \beta_2 &= \frac{d_2 - a_2\beta_1}{\alpha_2} = \frac{18 - 8 * 2}{2} = 1 \\ \beta_3 &= \frac{d_3 - a_3\beta_2}{\alpha_3} = -\frac{1}{2} \\ \beta_4 &= \frac{d_4 - a_4\beta_3}{\alpha_4} = \frac{-175 - 1.5 * -1/2}{1} = -1 \end{aligned}$$

Let $x_n = b_n$ i.e $x_4 = b_4 = -1$

$$\begin{aligned} x_3 &= \beta_3 - \frac{c_3 x_4}{\alpha_3} \\ &= -1/2 - \frac{-1.5 * -1}{3} \\ &= 0 \\ x_2 &= \beta_2 - \frac{c_2 x_3}{\alpha_2} \\ &= 1 - \frac{2 * 0}{2} = 1 \\ x_1 &= \beta_1 - \frac{c_1 x_2}{\alpha_1} \\ &= 2 - \frac{8 * 1}{4} = 0 \end{aligned}$$

$\therefore x = 0, y = 1, z = x_3 = 0, w = x_4 = -1$

$\square$