DAA Mid-2 Assignment Q&A

1) Explain travelling sales person problem using LCBB procedure with the following instance and draw the portion of the state space tree and find an optimal tour.

$$\begin{pmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{pmatrix}$$

Ans:-

→ # Travelling sales person problem using branch & bound :- (LCBB) (least cost)

If there are 'n' cities and cost of travelling from one city to another city, a salesman start from any one of the state & has to visit all the cities exactly once and has to return to the starting place with shortest distance with minimum cost.

Let $G = (V, E)$ be a directed graph defining an instance of the travelling sales person problem, C_{ij} be the cost of the edge (i, j). $C_{ij} = \infty$ (infinity) if (i, j) does not belongs to given graph 'G'.

Procedure :-

STEP 1 :- Let 'A' be the reduced cost matrix for |R| and 'S' be the child of 'R' such that edge of (R,S) corresponds to including edge (i,j).

STEP 2 :- Change all the entities in a row 'i' and column 'j' of A to ∞. and set $(j,1)=\infty$

STEP 3 :- Apply row reduction & column reduction except for rows & columns containing '∞'.

STEP 4 :- The total cost for node is $\boxed{\hat{c}(S) = \hat{c}(R) + A(i,j) + r}$

Here r = sum of row & column reduction.

⇒ Solve the instance of travelling sales person problem by using branch & bound technique. (LCBB)

$$\begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix}$$

Sol :- Row reduction :-

$$\begin{array}{c} \\ \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{array} \begin{array}{c} \text{least value in row} \\ 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline r = 21 \end{array} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

column reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 1 & 0 & 3 & 0 & 0 \end{matrix} \ (r=4) \end{array} \xrightarrow{\text{Column reduction}} \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

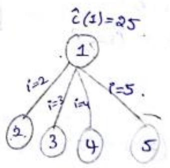
∴ 1st reduced matrix $\boxed{A = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}}$

∴ $r = 21 + 4 = 25$

Initially

$\boxed{\hat{c}(1) = 25 = r}$

∴ The state space tree is



⇒ Consider path (1,2) :-

It means that set '∞' to 1st row & 2nd column and set $(2,1)=\infty$ of reduced matrix 'A'. i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ 0 \\ - \end{matrix} \\ \underline{r=0} \end{array} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}.$$

Column reduction:-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 0 & - & 0 & 0 & 0 \end{matrix}\ (r=0) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

$\therefore r=0.$

from A reduced matrix.

$$\hat{C}(2) = \hat{C}(1) + A(1,2) + r. \qquad \Big(\hat{C}(S) = \hat{C}(R) + (A(R,S)) + r \Big)$$

child   parent.

$$= 25+10+0$$

$$\boxed{\hat{C}(2) = 35}$$

consider path (1,3):-

It means set '∞' to 1st row, 3rd column and set (3,1) = ∞ of reduced matrix A' i.e.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ \underline{r=0} \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix}$$

column reduction:-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 11 & 0 & \infty & 12 & \infty \end{bmatrix} \\ \begin{matrix} 11 & 0 & - & 0 & 0 \end{matrix}\ (r=11) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

$\therefore r = 11+0 = 11.$

$\therefore \hat{c}(3) = \hat{c}(1) + A(1,3) + r.$

$= 25 + 17 + 11$

$\boxed{\hat{c}(3) = 53}$

25
11
17
53

$\Rightarrow$ consider path $(1,4)$:-

It means set '∞' to 1st row, 4th column & set $(4,1) = \infty$ of reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \\ (r=0) \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Column reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & - & 0 \end{matrix} \; (r=0) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$\therefore r = 0 + 0 = 0.$

$\therefore \hat{c}(4) = \hat{c}(1) + A(1,4) + r.$

$= 25 + 0 + 0$

$\boxed{\hat{c}(4) = 25}$

$\Rightarrow$ consider path $(1,5)$:-

It means set '∞' to 1st row, 5th column & set $(5,1) = \infty$ of reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} - \\ 2 \\ 0 \\ 3 \\ 0 \end{matrix} \\ r = 5 \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

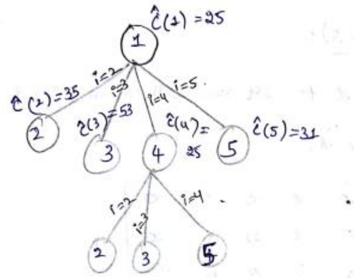
Column reduction :-

$$\begin{array}{c} \begin{matrix} 0 & 0 & 0 & 0 & - \end{matrix} \; (r=0) \\ \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$(r = 5+0 = 5)$

$$\therefore \hat{c}(5) = \hat{c}(1) + A(1,5) + r$$
$$= 25 + 1 + 5$$
$$\boxed{\hat{c}(5) = 31}$$

$\therefore$ state space tree is:



Compare (1,2), (1,3), (1,4), (1,5). Here (1,4) has minimum cost then consider 2nd reduced cost matrix for path (1,4) i.e

$$\boxed{A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}}$$

→ consider path (4,2):-

Consider path (4,2) means that set '∞' to 4th row & 2nd column & set (2,1) = ∞ of 2nd reduced matrix of A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

Row reduction-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 0 \\ - \\ 0 \end{matrix} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

0 —— 0 —— 0 (r=0)

Column reduction:-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \xrightarrow{\text{Column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

0 — 0 — 0 (r=0)

$$\hat{c}(2) = \hat{c}(4) + A(4,2) + r.$$
$$= 25 + 3 + 0$$
$$\boxed{\hat{c}(2) = 28}$$

→ consider path (4,3) =

Consider path (4,3) means that set '∞' to 4th row, 3rd column and set $(3,1) = \infty$ to the reduced 2nd matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ 0 \\ 2 \\ - \\ 0 \end{matrix} \\ (r=2) \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

column reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \\ \begin{matrix} 11 & 0 & - & - & 0 \end{matrix} \; (r=11) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & \infty & 0 \\ \infty & 1 & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & 0 & \infty & \infty & \infty \end{bmatrix}$$

$\therefore \boxed{r = 13.}$

$$\hat{c}(3) = \hat{c}(4) + A(4,3) + r = 25 + 12 + 13 = 50.$$

$\boxed{\hat{c}(3) = 50}$

$\Rightarrow$ consider path $(4,5)$:-

It means set 4th row, 5th column as '∞' & set $(5,1) = \infty$' of 2nd reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

Row reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ 11 \\ 0 \\ - \\ 0 \end{matrix} \\ (r=11) \end{array} \xrightarrow{\text{row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

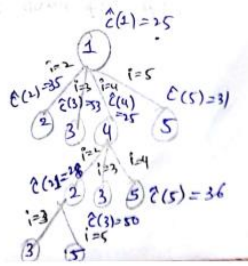
Column reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \\ \begin{matrix} 0 & 0 & 0 & - & - \end{matrix} \; (r=0) \end{array} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix}$$

$$\hat{c}(5) = \hat{c}(4) + A(4,5) + r$$
$$= 25 + 0 + 11 = 36$$

$\boxed{\hat{c}(5) = 36}$

The state space tree is :



Here compare $(4,2)\,(4,3)\,(4,5)$ & $(4,2)$ has minimum cost then consider 3rd reduced matrix of 'A' i.e $(4,2)$ i.e.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

⇒ consider path $(2,3)$:-

It means set '∞' to 2nd row, 3rd column and set $(3,1) = \infty$ of 3rd reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix}$$

Row Reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ 2 \\ - \\ 11 \end{matrix} \\ (r=13) \end{array} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

Column Reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix} \\ 0 \;\; - \;\; - \;\; - \;\; 0 \;\; (r=0) \end{array} \xrightarrow{\text{Column Reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 0 \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\therefore r = 13 + 0 = 13.$$

$$\therefore \hat{c}(3) = \hat{c}(2) + A(2,3) + r$$

$$= 28 + 11 + 13$$

$$\boxed{\hat{c}(3) = 52}$$

consider path $(2,5)$:-

It means set '∞' to 2nd row, 5th column & also $(5,1) = \infty$ of 3rd reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

Row Reduction :-

$$\begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ 0 \\ - \\ 0 \end{matrix} \\ (r=0) \end{array} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix}$$

0 — 0 — 0 $(r=0)$

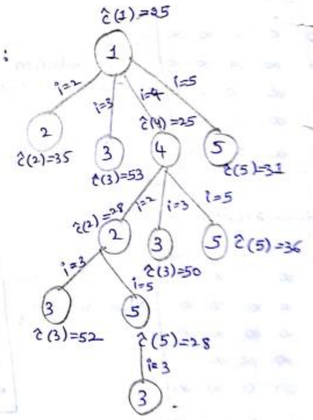
$\therefore r = 0 + 0 = 0$

$\therefore \hat{c}(5) = \hat{c}(2) + A(2,5) + r$

$= 28 + 0 + 0$

$\boxed{\hat{c}(5) = 28}$

$\therefore$ State space tree is :



Here compare (2,3), (2,5) and (2,5) has minimum cost then consider 4th reduced matrix (2,5) of 'A' i.e

Consider path $(5,3)$:-

It means set ∞ to 5th row, 3rd column & set $(3,1) = \infty$ of 4th reduced matrix 'A'.

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

Row Reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} - \\ - \\ - \\ - \\ - \end{matrix} \xrightarrow{\text{Row reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

$(r=0)$

Column Reduction :-

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix} \xrightarrow{\text{column reduction}} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \end{bmatrix}$$

— — — — $(r=0)$

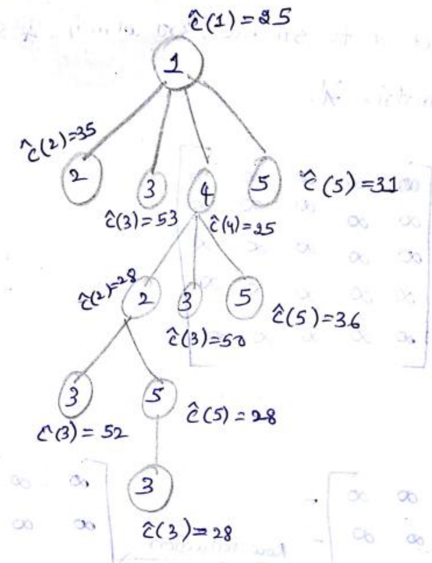
$\therefore r = 0$.

$\hat{c}(3) = \hat{c}(5) + A(5,3) + r$

$= 28 + 0 + 0$

$\boxed{\hat{c}(3) = 28}$

∴ state space tree is:



2) Construct OBST for the following data n=4, (a1,a2,a3,a4) = (do,if,int,while) and P(1:4) = (3,3,1,1) and Q(0:4) = (2,3,1,1,1).

Ans:-

→ Find the OBST by using dynamic programming

$n=4$, $(a_1, a_2, a_3, a_4) = $ (do if read while)

$P(1:4) = (3,3,1,1)$, $Q(0:4) = (2,3,1,1,1)$.

Sol:- Given that $n=4$

a_1 = do	$P_1 = 3$	$Q_0 = 2$
a_2 = if	$P_2 = 3$	$Q_1 = 3$
a_3 = read	$P_3 = 1$	$Q_2 = 1$
a_4 = while	$P_4 = 1$	$Q_3 = 1$
		$Q_4 = 1$.

Step 1:- Initially $C(i,i) = 0$ i.e

$C(1,1) = 0$, $C(2,2) = 0$, $C(3,3) = 0$, $C(4,4) = 0$

$\omega(i,i) = Q(i)$ i.e

$\omega(0,0) = Q(0) = 2$

$\omega(1,1) = Q(1) = 3$

$\omega(2,2) = Q(2) = 1$

$\omega(3,3) = Q(3) = 1$

$\omega(4,4) = Q(4) = 1$.

$R(i,i) = 0$ i.e

$R(1,1) = 0$

$R(2,2) = 0$, $R(3,3) = 0$, $R(4,4) = 0$.

∴

	0			1			2			3			4		
	w	c	R	w	c	R	w	c	R	w	c	R	w	c	R
0	2,	0,	0	8,	8,	1	12,	19,	1	14,	25,	2	16,	32,	2
1				3,	0,	0	7,	7,	2	9,	12,	2	11,	19,	2
2							1,	0,	0	3,	3,	3	5,	8,	3
3										1,	0,	0	3,	3,	4
4													1,	0,	0

Compute all $C(i,j)$ such that

1) $j-i=1$ i.e $j=1+i$ Now $0 \le i < 4$ i.e $i=0,1,2,3$ and $i<k\le j$

for calculating 'k' value.

(i) $i=0$, $j=i+1$, $i<k\le j$

$j=0+1$ $\quad 0<k\le 1$

$\boxed{j=1}$ $\quad \boxed{k=1}$

$w(i,j) = p(j)+q(j)+w(i,j-1)$

$= p(1)+q(1)+w(0,0)$

$= 3+3+2$

$\boxed{w(0,1) = 8}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1)+c(k,j)\}$

$cost(0,1) = w(0,1) + \min\{c(0,0)+c(1,1)\}$

$= 8+\min\{0+0\}$

$\boxed{cost(0,1) = 8}$

$\boxed{R(0,1)=1}$ means that value of 'k' is minimum in the above equation.

(ii) $i=1$, $j=i+1$, $i<k\le j$

$j=1+1$ $\quad 1<k\le 2$

$\boxed{j=2}$ $\quad \boxed{k=2}$

$w(i,j) = p(j)+q(j)+w(i,j-1)$

$= p(2)+q(2)+w(1,1)$

$= 3+1+3$

$\boxed{w(1,2) = 7}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1)+c(k,j)\}$

$cost(1,2) = w(1,2) + \min\{c(1,1)+c(2,2)\}$

$= 7+\min\{0+0\}$

$\boxed{cost(1,2) = 7}$

$\boxed{R(1,2) = 2}$ k value

(iii) $i = 2,\ j = i+1,\ i < k \le j$

$j = 2+1 \quad 2 < k \le 3$

$\boxed{j = 3} \quad \boxed{k = 3}$

$w(i,j) = p(j) + q(j) + w(i,j-1)$

$w(2,3) = p(3) + q(3) + w(2,2)$

$= 1 + 1 + 1$

$\boxed{w(2,3) = 3}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1) + c(k,j)\}$

$cost(2,3) = w(2,3) + \min\{c(2,2) + c(3,3)\}$

$= 3 + \min\{0+0\}$

$\boxed{cost(2,3) = 3}$

$\boxed{R(2,3) = 3}$

(iv) $i = 3,\ j = i+1,\ i < k \le j$

$j = 3+1 \quad 3 < k \le 4$

$\boxed{j = 4} \quad \boxed{k = 4}$

$w(i,j) = p(j) + q(j) + w(i,j-1)$

$w(3,4) = p(4) + q(4) + w(3,3)$

$= 1 + 1 + 1$

$\boxed{w(3,4) = 3}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1) + c(k,j)\}$

$cost(3,4) = w(3,4) + \min\{c(3,3) + c(4,4)\}$

$= 3 + \min\{0+0\}$

$\boxed{cost(3,4) = 3}$

$\boxed{R(3,4) = 4}$

Step 2 :- $j - i = 2,\ j = i+2$

$0 \le i < 3$

$i = 0, 1, 2$

(i) $i = 0,\ j = i+2,\ i < k \le j$

$= 0+2 \quad 0 < k \le 2$

$j = 2 \quad \boxed{k = 1, 2}$

$w(i,j) = p(j) + q(j) + w(i,j-1)$

$= p(2) + q(2) + w(0,1)$

$= 3 + 1 + 8$

$\boxed{w(0,2) = 12}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1) + c(k,j)\}$

$cost(0,2) = w(0,2) + \min\{\underbrace{c(0,0) + c(1,2)}_{k=1},\ \underbrace{c(0,1) + c(2,2)}_{k=2}\}$

$cost(0,2) = 12 + \min\{0+7, 8+0\}$

$= 12 + \min\{7, 8\}$

$= 12 + 7$

$\boxed{c(0,2) = 19}$

$\boxed{R(0,2) = 1}$ → in 'k' take min

(ii) $\boxed{i=1}$, $j = i+2$, $\quad i < k \le j$

$j = 1+2 \quad 1 < k \le 3$

$\boxed{j=3} \quad \boxed{k = 2,3}$

$\omega(i,j) = p(j) + Q(j) + \omega(i, j-1)$

$\omega(1,3) = p(3) + Q(3) + \omega(1,2)$

$= 1+1+7$

$\boxed{\omega(1,3) = 9}$

$cost(i,j) = \omega(i,j) + \min\{C(i,k-1) + C(k,j)\}$

$cost(1,3) = \omega(1,3) + \min\{C(1,1) + C(2,3),\ C(1,2) + C(3,3)\}$

$= 9 + \min\{(0+3), (7+0)\}$

$= 9 + \min\{3,7\}$

$= 9+3$

$\boxed{cost(1,3) = 12}$

$\boxed{R(1,3) = 2}$

(iii) $\boxed{i=2}$, $j = i+2$, $\quad i < k \le j$

$j = 2+2 \quad 2 < k \le 4$

$\boxed{j=4} \quad \boxed{k = 3,4}$

$\omega(i,j) = p(j) + Q(j) + \omega(i, j-1)$

$\omega(2,4) = p(4) + Q(4) + \omega(2,3)$

$= 1+1+3$

$\boxed{\omega(2,4) = 5}$

$cost(i,j) = \omega(i,j) + \min\{C(i,k-1) + C(k,j)\}$

$cost(2,4) = \omega(2,4) + \min\{C(2,2) + C(3,4),\ C(2,3) + C(4,4)\}$

$cost(2,4) = 5 + \min\{(0+3), (3+0)\}$

$= 5 + \min\{3,3\}$

$= 5+3$

$\boxed{C(2,4) = 8}$

$\boxed{R(2,4) = 3}$

STEP 3:- $j - i = 3$, $j = 3+i$, $0 \le i < 2$

$i = 0, 1$.

(i) $\boxed{i=0}$, $j = i+3$, $\quad i < k \le j$

$j = 0+3 \quad 0 < k \le 3$

$\boxed{j=3} \quad \boxed{k = 1,2,3}$

$\omega(i,j) = p(j) + Q(j) + \omega(i, j-1)$

$\omega(0,3) = p(3) + Q(3) + \omega(0,2)$

$= 1+1+12$

$\boxed{\omega(0,3) = 14}$

$cost(i,j) = \omega(i,j) + \min\{C(i,k-1) + C(k,j)\}$

$cost(0,3) = \omega(0,3) + \min\{C(0,0) + C(1,3),\ C(0,1) + C(2,3),$

$C(0,2) + C(3,3)\}$

$= 14 + \min\{(0+12), (8+3), C(19+0)\}$

$= 14 + \min\{12, \underline{11}, 19\}$ which it is at 2nd place so

$cost(0,3) = 14+11$

$\boxed{cost(0,3) = 25}$

$\boxed{R(0,3) = 2}$

(ii) $\boxed{i=1}$, $j=i+3$, $i<k\leq j$

$j = 1+3$ $\quad 1<k\leq 4$

$\boxed{j=4}$ $\quad \boxed{k=2,3,4}$

$w(i,j) = p(j) + q(j) + w(i,j-1)$

$w(1,4) = p(4) + q(4) + w(1,3)$

$= 1+1+9$

$\boxed{w(1,4) = 11}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1)+c(k,j)\}$

$cost(1,4) = w(1,4) + \min\{c(1,1)+c(2,4),\ c(1,2)+c(3,4),$
$c(1,3)+c(4,4)\}$

$cost(1,4) = 11+\min\{(0+8),(7+3),(12+0)\}$

$= 11+\min\{8,10,12\}$

8 is 1st position so in I write k value 1st position value.

$= 11+8$

$\boxed{cost(1,4) = 19}$

$\boxed{R(1,4) = 2}$

STEP4:- $j-i=4$, $j=4+i$, $0\leq i<1$

$i=0$

(i) $\boxed{i=0}$, $j=i+4$, $i<k\leq j$

$j = 0+4$ $\quad 0<k\leq 4$

$\boxed{j=4}$ $\quad \boxed{k=1,2,3,4}$

$w(i,j) = p(j) + q(j) + w(i,j-1)$

$w(0,4) = p(4) + q(4) + w(0,3)$

$w(0,4) = 1+1+14$

$\boxed{w(0,4) = 16}$

$cost(i,j) = w(i,j) + \min\{c(i,k-1)+c(k,j)\}$

$cost(0,4) = w(0,4) + \min\{c(0,0)+c(1,4),\ c(0,1)+c(2,4),$
$c(0,2)+c(3,4),\ c(0,3)+c(4,4)\}$

$cost(0,4) = 16+\min\{(0+19),(8+8),(19+3),(25+0)\}$

$= 16+\min\{19,16,22,25\}$

$= 16+16$

$\boxed{cost(0,4) = 32}$

$\boxed{R(0,4) = 2}$

	0	1	2	3	4
0	2,0,0	8,8,1	12,19,1	14,25,2	16,32,2
1		3,0,0	7,7,2	9,12,2	11,19,2
			4,0,0	3,3,3	5,8,3.
				1,0,0	3,3,4
					1,0,0.

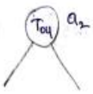
From the table we see that cost(0,4)=32 of OBST for (a_1, a_2, a_3, a_4) i.e the root tree.

T_{04} i.e $R_{04} = 2$ i.e a_2

$\therefore R_{04} = 2$

$i=0, j=4, k=2$.

$\therefore$ The tree is.



Now evaluate left node by using $R_{i\,k-1}$ and evaluate right node by using R_{kj}

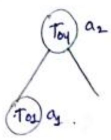
⇒ Left node :-

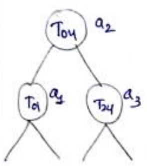
$R_{i\,k-1}$

From root node $i=0, j=4, k=2$

Sub. in above eqn R_{ik-1}

$R_{01} = 1$ i.e a_1.





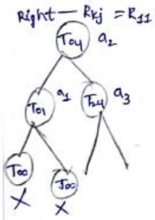
From root node T_{01}

$i=0, j=1, k=1$

$R_{00} = 0 = a_0$

But a_0 is not their.

Right — $R_{kj} = R_{11} = 0$.

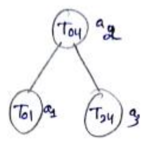


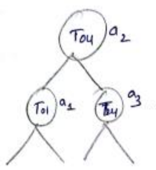
Right node

R_{kj}

From root node $k=2$, $j=4$

$R_{24} = 3$ i.e a_3



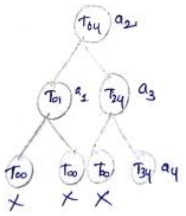


From root node T_{24}

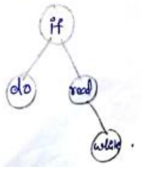
$i=2, j=4, k=3$.

$R_{22} = 0$ (discard.

$R_{kj} = R_{34} = 4 = a_4$.



$\therefore$ The optimal binary search tree is.



3) Explain P, NP, NP-Hard and NP-Complete.

Ans:-

Definition of P :-

A problem that can be solved in polynomial time is called "P" and 'P' stands for polynomial time

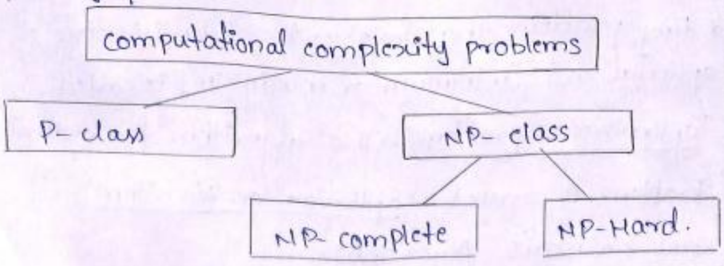
ex: 1. Searching of key element
2. Sorting of elements
3. All pair shortest path.

Definition of NP :

A problem that can be solved in NON-Deterministic polynomial time is called "NP" and NP stands for NON-Deterministic polynomial

ex:- 1. Travelling salesperson problem
2. Graph coloring problem
3. Knapsack problem
4. Hamiltonian circuit problem.

There are two types of computational complexity problems as follows.



A problem is said to be a NP-complete if

i) It belongs to class N-P
ii) Every problem in NP can also be solved in Polynomial time.

* If an NP-Hard problem can be solved in polynomial time then all NP-complete problems can also be solved in polynomial time.

* All NP-complete problems are NP-Hard but All NP-Hard problems cannot be NP-complete.

* The NP-class problem are the Decision problem that can be solved by non-deterministic polynomial Algorithms.

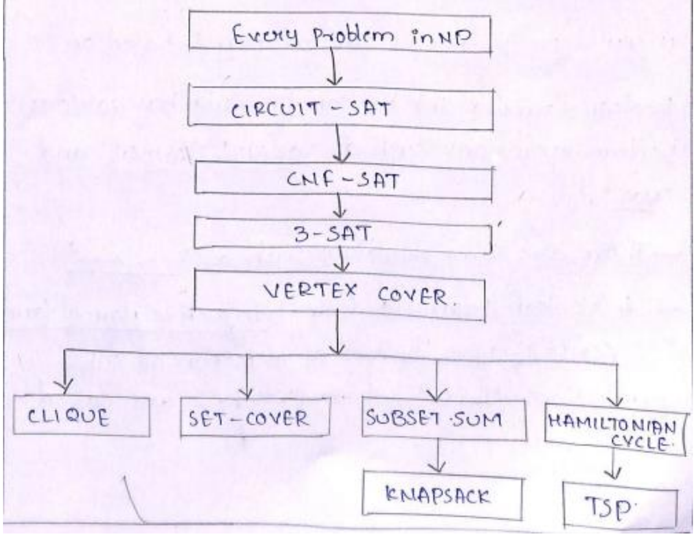
NP - Complete problems:

To prove wether particular problem is NP complete or Not; we use polynomial time reducibility that means if

$$A \xrightarrow{Poly} B \text{ and } B \xrightarrow{Poly} C \text{ then}$$

$$A \xrightarrow{Poly} C$$

→ The reduction is an important task in NP completeness proofs. This can be illustrated by figure as show in below.

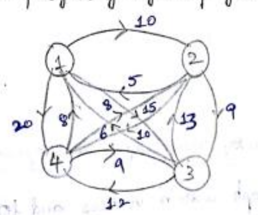


4) Find the shortest tour of travelling sales person problem of the following cost matrix using dynamic programming.

0	10	15	20
5	0	9	10
6	13	0	12
8	8	9	0

Ans:-

Find the minimum cost of a travelling sales person problem of graph G by using dynamic programming.



Sol:- From the graph the cost adjacency matrix is

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix} \end{array}$$

Let us start the tours from vertex '1'.

The general formula is

$$\boxed{g(1,S) = \min\{ C_{1j} + g(j, S-\{j\})\}} \quad (1)$$

And calculate $g(i,\phi) = C_{i1} \quad 1 \le i \le n$

i.e $g(1,\phi) = C_{11} = 0$

$g(2,\phi) = C_{21} = 5$

$g(3,\phi) = C_{31} = 6$

$g(4,\phi) = C_{41} = 8$

Using eq (1) we obtain

$$g(1,\{2,3,4\}) = \min\{ C_{12} + g(2,(3,4)),\ C_{13} + g(3,(2,4)),$$
$$C_{14} + g(4,(2,3))\} \quad (2)$$

Now compute $g(2,(3,4)) = \min\{C_{23} + g(3,4)\},\ C_{24} + g(4,3)$ (3)

$g(3,4) = g(3,\{4\}) = \min\{C_{34} + g(4,\phi)\}$

$= \min\{12+8\}$

$= \min\{20\}$

$\boxed{g(3,\{4\}) = 20}$

$\therefore g(4,3) = g(4,\{3\}) = \min\{C_{43} + g(3,\phi)\}$

$= \min\{9+6\}$

$\boxed{g(4,\{3\}) = 15}$

Now substitute $g(3,\{4\})$ & $g(4,\{3\})$ in eq (3) we get.

$g(2,(3,4)) = \min\{(9+20),\ 10+15\}$

$= \min\{29, 25\}$

$\boxed{g(2,(3,4)) = 25}$

Now compute

$g(3,\{2,4\}) = \min\{C_{32} + g(2,4)\},\ C_{34} + g(4,2)\}$ (4)

Now compute $g(2,\{4\}) = \min\{C_{24} + g(4,\phi)\}$

$= \min\{10+8\}$

$\boxed{g(2,\{4\}) = 18}$

$g(4,\{2\}) = \min\{C_{42} + g(2,\phi)\}$

$= \min\{8+5\}$

$\boxed{g(4,\{2\}) = 13}$

$\therefore$ Sub in (4)

$g(3,\{2,4\}) = \min\{13+18,\ 12+13\}$

$= \min\{31, 25\}$

$\boxed{g(3,\{2,4\}) = 25}$

Now compute $g(4,\{2,3\}) = \min\{C_{42} + g(2,3),\ C_{43} + g(3,2)\}$ (5)

Again compute $g(2,\{3\}) = \min\{C_{23} + g(3,\phi)\}$

$= \min\{9+6\}$

$\boxed{g(2,\{3\}) = 15}$

Now compute $g(3,\{2\}) = \min\{c_{32} + g(2,1)\}$

$= \min\{13+5\}$

$\boxed{g(3,\{2\}) = 18}$

Sub in (5)

$g(4,\{2,3\}) = \min\{(8+15), (9+18)\}$

$= \min\{23, 26\}$

$\boxed{g(4,\{2,3\}) = 23}$

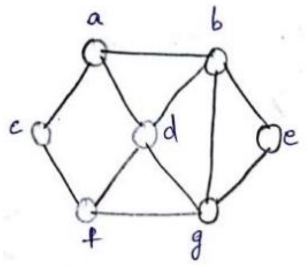
Sub $g(2,(3,4))$, $g(3,(2,4))$ and $g(4,(2,3))$ in eq (2).

$g(1,\{2,3,4\}) = \min\{(10+25), (15+25), (20+23)\}$

$= \min\{35, 40, 43\}$

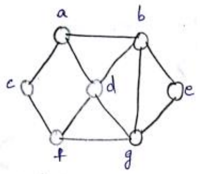
$\boxed{g(1,\{2,3,4\}) = 35}$

5) What is a Hamiltanian Cycle? Explain how to find Hamiltanian path and cycle using backtracking algorithm.



Ans:-

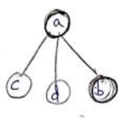
→ Apply backtracking to find hamiltonian cycle in the following graph



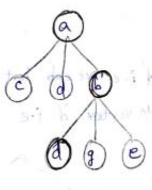
Sol :-

STEP 1 :- (a) ← root.

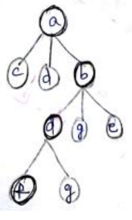
STEP 2 :-



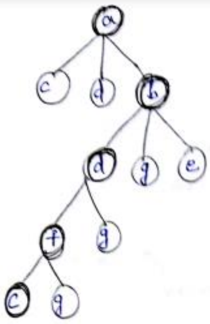
STEP 3 :-



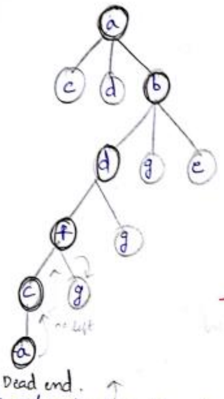
STEP 4 :-



STEP 5 :-

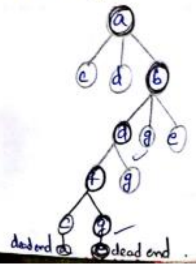


STEP 6 :-

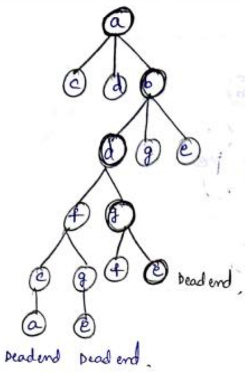


⇒ Here we got only 5 vertices but there are only 7 vertices so Dead end. & back track.

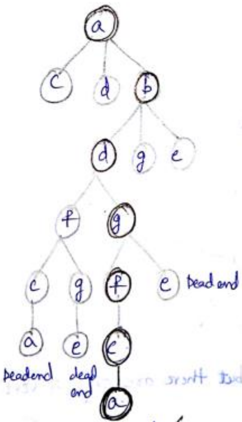
⇒ STEP 7 :-



STEP 8 :-

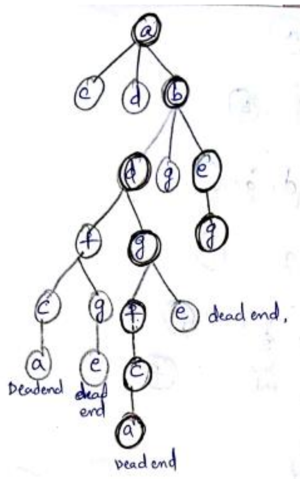


STEP 9 :-

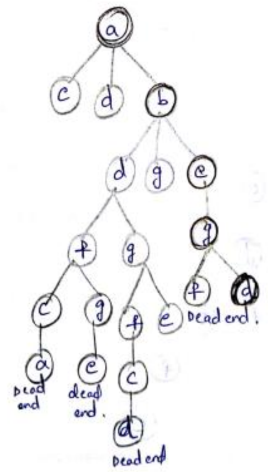


Dead end. (6 vertices but we have 7 vertices) back track.

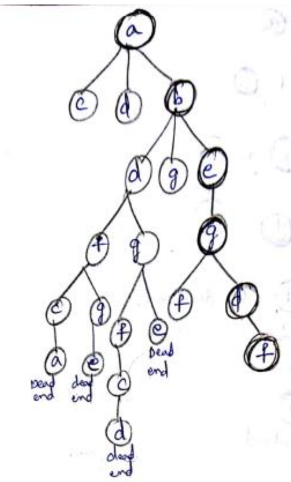
STEP 10 :-



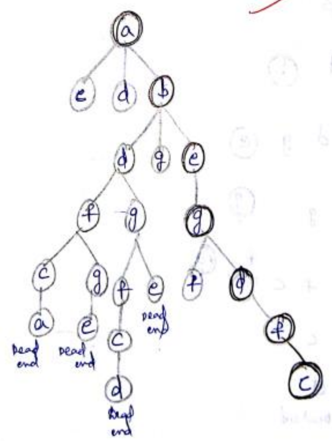
STEP 11 :-

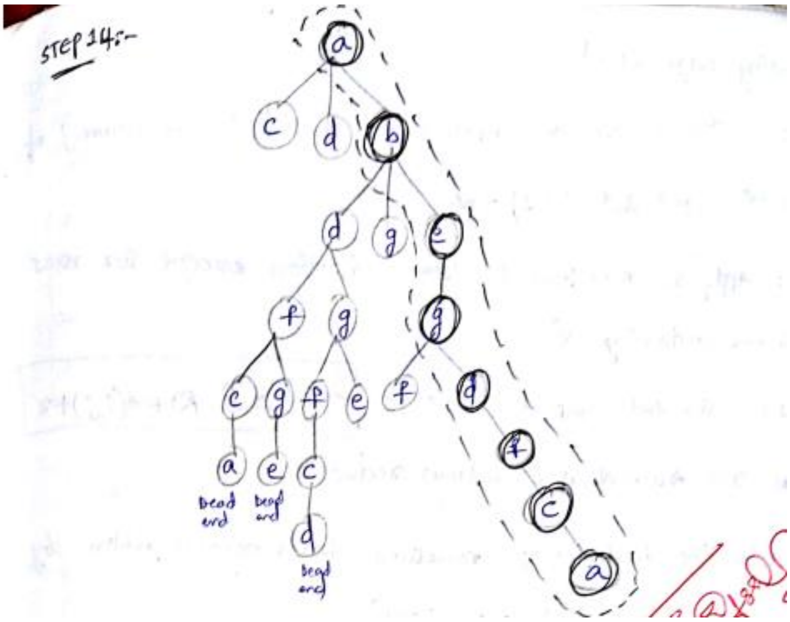


STEP 12 :-



STEP 13 :-





6) What is meant by non-deterministic algorithm? Write a non-deterministic searching algorithm.

Ans:-

Non-Deterministic Algorithm:-

The algorithm in which every operation is uniquely defined is called Deterministic Algorithm

The algorithm in which every operation may not have unique result, such type of algorithms is called NON-Deterministic algorithm.

→ NON-Deterministic means that no particular rule is followed to make the guess.

→ The NON-Deterministic algorithm is "two stage" algorithm.

i) NON-Deterministic (Guessing) stage: It generate an arbitary string that can be though of as a candidate solution.

ii) Deterministic (verification) stage: In this stage, It takes as Input the candidate solution and the Instance to the problem and return "yes" if the candidate solution represents Actual solution.

Algorithm:

```
Algorithm NON-Determin()
{
 for i:=1 to n do
   A[i]:=choose(i);          } // Guessing stage.
   if (A[i]=x) then
   {
     write (i);
     success();               } // verification stage
   }
   write(0);
 fail()
}
```

In the above program given non-Deterministic algorithm there are three functions used.

1. **choose** :- Arbitrarily choose one of the element from given input set

2. Fail :- Indicates the unsuccessful completion.

3. Success :- Indicates successful completion.

7) Write the N-Queen Problem algorithm using backtracking technique? And draw the permutation tree for 4-Queen Problem?

Ans:-

10m

* ⇒ N-Queens problem :- Here we have to discuss 4-Queen & 8 Queens problem.

(i) 4 Queen problem :- Consider a 4x4 chess board and it contains 4 Queens Q_1, Q_2, Q_3 & Q_4. The objective is to place the 4 Queens on chess board in such a way that no two queens should be placed in the same row, same column and same diagonal position.

⇒ The explicity constraints are 4 queens are to be placed on 4x4 chess board in 4x4 ways. The implicity constraints are no two queens are in the same row, same column and same diagonal position. Let (x_1, x_2, x_3, x_4) are the solution vectors

STEP 1:- Consider empty 4x4 chess board as shown in below

STEP 2 :- place the queen Q_1 in 1st row & 1st column i.e

Q_1			

STEP 3 :- Now place the queen 'Q_2' in 2nd row and 3rd column, 4th column. Now choose 2nd row 3rd column i.e.

Q_1			
		Q_2	

STEP 4 :- We are unable to place 'Q_3' in 3rd row. Apply the backtracking method we go back to 'Q_2' and place it somewhere else i.e 2nd row & 4th column.

Q_1			
			Q_2

STEP 5 :- Now place Queen Q_3 in 3rd row, 2nd column.

Q_1			
			Q_2
	Q_3		

STEP 6 :- The fourth Queen, should be placed in 4th row but, Q_1, Q_2, Q_3 attack to Q_4, so apply the back-tracking method, we go back to Q_3 and remove it. Again it is not possible, go back to Q_2 and remove it. Again it is not possible, go back to Q_1 move it to next column as shown below.

	Q_1		

STEP :- 7 :- Now place Queen Q_2 in 2nd row, 4th column

	Q_1		
			Q_2

Step 8 :- Now place Queen Q_3 in 3rd, 1 column, because no one attack to Q_3

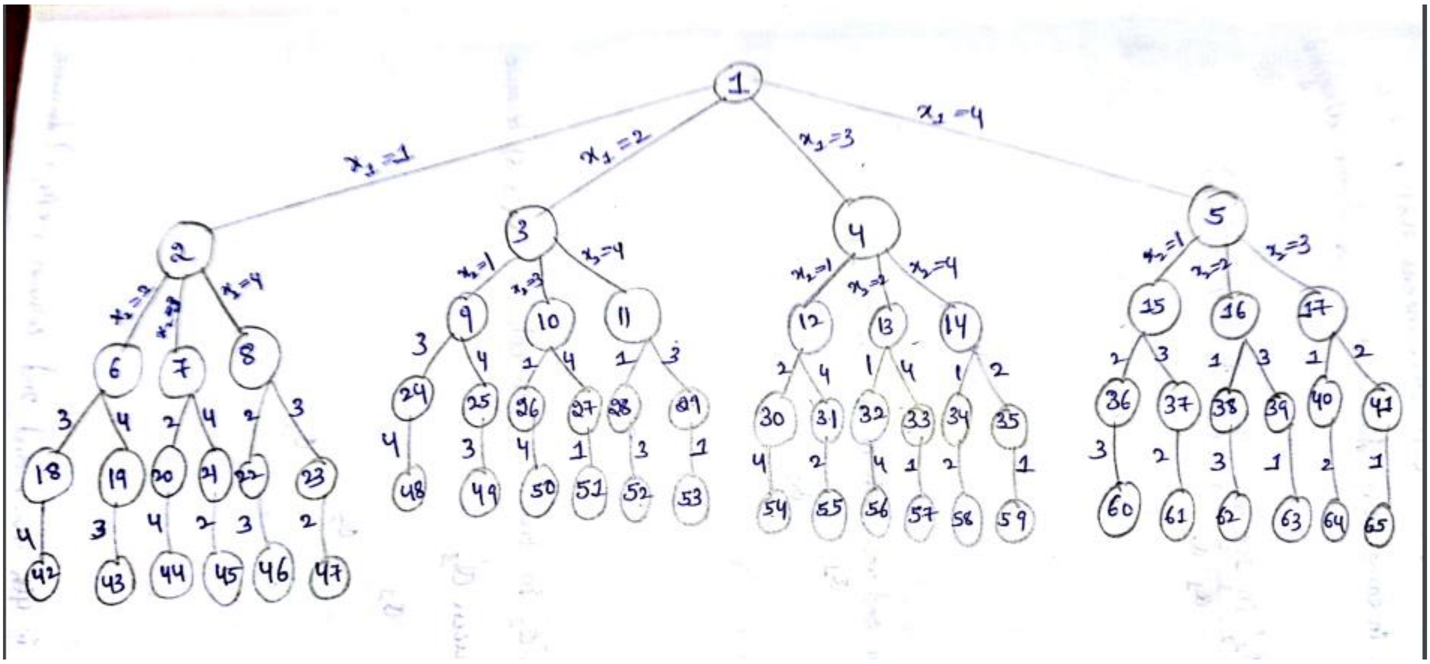
	Q_1		
			Q_2
Q_3			

STEP 9 :- Place Queen Q_4 in 4th row, 3rd column because no one attack to Q_4

	Q_1		
			Q_2
Q_3			
		Q_4	

$$\therefore (x_1, x_2, x_3, x_4) = (2, 4, 1, 3)$$

⇒ Draw the state space tree for 4-Queen's problem :-



8) a) Explain 3CNF Satisfiability problem

Ans:-

(ii) 3SAT Problem:

A 3SAT problem is a problem which takes a boolean formula "s" in CNF form with each clause having exactly three literals and check whether "s" is satisfied or not. CNF can represent as (+) * (+) * (+) (or) (v) ∧ (v) ∧ (v) ... etc..

ex: $(\bar{a}+b+\bar{g})(c+\bar{c}+f)(\bar{b}+d+\bar{f})(a+c+\bar{h})$

(or)

$(\bar{a} \vee b \vee \bar{g}) \wedge (c \vee \bar{c} \vee f) \wedge (\bar{b} \vee d \vee \bar{f}) \wedge (a \vee c \vee \bar{h})$

Here "+" nothing but "v" symbol

" * '(.)' nothing but "∧" symbol

Theorem: 3SAT is in NP complete

Proof: let "s" be the boolean formula having 3 literals in each clause for which we can construct a simple non-Deterministic Algorithm which can guess an Assignment of boolean values to "s". if the "s" is evaluated as '1' then "s" is satisfied. then we can prove that 3SAT is in NP-complete

b) Write a short note on NP Hard and NP Complete Classes

Ans:-

NP-hard and NP-complete classes :-

To measuring the complexity of an algorithm, we use the input length as the parameter. An algorithm 'A' is of polynomial complexity if there exists a polynomial p( ) such that the computing time of A is O(p(n)) for every input of size 'n'

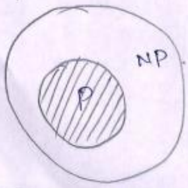
→ P is the set of all decision problem solvable by Deterministic Algorithms in polynomial time.

→ NP is the set of all decision problems solvable by NON-deterministic algorithms in polynomial time

→ Since the Deterministic Algorithms are just a special case of non-deterministic algorithm, so we conclude that $\boxed{P \subseteq NP}$

→ what we do not know, and what has become Perhaps (maybe), the most famous unsolved problem in computer Science, is whether $\boxed{P = NP \text{ or } P \neq NP}$

→ Is it possible that for all the problems in NP



∴ All 'p' problems are solve in NP but not viceversa

Fig:- relation b/w P & NP.

9) a) Distinguish between Dynamic Programming and Greedy method.

Ans:-

Feature	Greedy method	Dynamic programming
Feasibility	In a greedy Algorithm, we make whatever choice seems best at the moment in the hope that it will lead to global optimal solution.	In Dynamic Programming we make decision at each step considering current problem and solution to previously solved sub problem to calculate optimal solution .
Optimality	In Greedy Method, sometimes there is no such guarantee of getting Optimal Solution.	It is guaranteed that Dynamic Programming will generate an optimal solution as it generally considers all possible cases and then choose the best.
Recursion	A greedy method follows the problem solving heuristic of making the locally optimal choice at each stage.	A Dynamic programming is an algorithmic technique which is usually based on a recurrent formula that uses some previously calculated states.
Memoization	It is more efficient in terms of memory as it never look back or revise previous choices	It requires dp table for memoization and it increases it's memory complexity.
Time complexity	Greedy methods are generally faster. For example, Dijkstra's shortest path algorithm takes O(ELogV + VLogV) time.	Dynamic Programming is generally slower. For example, Bellman Ford algorithm takes O(VE) time.
Fashion	The greedy method computes its solution by making its choices in a serial forward fashion, never looking back or revising previous choices.	Dynamic programming computes its solution bottom up or top down by synthesizing them from smaller optimal sub solutions.
Example	Fractional knapsack .	0/1 knapsack problem

b) Find an optimal sequence to the n=5 jobs where profits (P1,P2,P3,P4,P5) = (20,15,10,5,1) and deadlines (d1,d2,d3,d4,d5) = (2,2,1,3,3)

Ans:-

Q) b) Find an optimal sequence to the n=5, jobs where profits $(P_1, P_2, P_3, P_4, P_5) = (20, 15, 10, 15, 1)$ & deadlines $(D_1, D_2, D_3, D_4, D_5) = (2, 2, 1, 3, 3)$

Sol:- Given:-

$n = 5$

Profits $(P_1, P_2, P_3, P_4, P_5) = (20, 15, 10, 15, 1)$

Deadlines $(D_1, D_2, D_3, D_4, D_5) = (2, 2, 1, 3, 3)$

In the above problem maximum deadline is 3 units

∴ The feasible solution set must have less than (or) equal to 3 elements (jobs)

Now arrange the Job profits in decreasing order, we get

~~$(P_1, P_2, P_3,$~~

$(P_1, P_2, P_4, P_3, P_5) = (20, 15, 15, 10, 1)$

$(D_1, D_2, D_4, D_3, D_5) = (2, 2, 3, 1, 3)$

S.No	Feasible solution	Processing sequence	Profits
1)	{1}	1	20
2)	{1, 2}	1,2 | 2,1	20+15 = 35
3)	{1, 2, 4}	1, 2,4 | 2,1,4	20+15+15 = 50
4)	{1, 2, 3}	1,2,3 | 1,3,2	20+15+10 = 45
5)	{1, 2, 5}	1, 2,5 | 2,1,5	20+15+1 = 36

→ Solution 3 is optimal solution, the Job must be processed in the order (1, 2, 4) (or) (2, 1, 4)

∴ The value of optimal solution is 50

Here, we can take only 3 elements because the max deadline is 3 units.

10) Solve Travelling Salesman Problem using Branch and Bound Algorithm for the following adjacency matrix.

$$\begin{bmatrix} \infty & 10 & 12 & 3 \\ 4 & \infty & 6 & 12 \\ 13 & 5 & \infty & 7 \\ 7 & 9 & 6 & \infty \end{bmatrix}$$

Ans:-

10) Solve Travelling Salesman Problem using ~~Backtracking~~ Branch & Bound Algorithm for the following adjacency matrix.

$$\begin{bmatrix} \infty & 10 & 12 & 3 \\ 4 & \infty & 6 & 12 \\ 13 & 5 & \infty & 7 \\ 7 & 9 & 6 & \infty \end{bmatrix}$$

Sol:-

Row Reduction:-

Least value in Row

$$\begin{array}{c|c} \begin{bmatrix} \infty & 10 & 12 & 3 \\ 4 & \infty & 6 & 12 \\ 13 & 5 & \infty & 7 \\ 7 & 9 & 6 & \infty \end{bmatrix} & \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array} \end{array} \Longrightarrow \begin{bmatrix} \infty & 7 & 9 & 0 \\ 0 & \infty & 2 & 8 \\ 8 & 0 & \infty & 2 \\ 1 & 3 & 0 & \infty \end{bmatrix}$$

$r = 18$

Column Reduction:-

$$\begin{bmatrix} \infty & 7 & 9 & 0 \\ 0 & \infty & 2 & 8 \\ 8 & 0 & \infty & 2 \\ 1 & 3 & 0 & \infty \end{bmatrix}$$

$0 \quad 0 \quad 0 \quad 0 \; (r=0)$

$\therefore$ No column Reduction

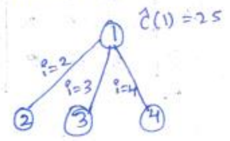
1st Reduced Matrix

$$A = \begin{bmatrix} \infty & 7 & 9 & 0 \\ 0 & \infty & 2 & 8 \\ 8 & 0 & \infty & 2 \\ 1 & 3 & 0 & \infty \end{bmatrix}$$

$\therefore r = 18 + 0$

$\boxed{\hat{C}(1) = r = 25}$

$\therefore$ State space Tree is



→ Consider path (1, 2):–

It means that set '∞' to 1st row & 2nd column and set (2, 1) = ∞ of reduced matrix 'A' i.e

$$A = \begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & 8 \\ 8 & \infty & \infty & 2 \\ 1 & \infty & 0 & \infty \end{bmatrix} & \begin{matrix} - \\ 2 \\ 2 \\ 0 \end{matrix} \\ & r = 4 \end{array}$$

Row Reduction:–

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 6 \\ 6 & \infty & \infty & 0 \\ 1 & \infty & 0 & \infty \end{bmatrix}$$
$$\begin{matrix} 1 & \infty & 0 & 0 \end{matrix} \quad (r = 1)$$

Column Reduction:–

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 6 \\ 5 & \infty & \infty & 0 \\ 0 & \infty & 0 & \infty \end{bmatrix}$$

$\therefore r = 4 + 1 = 5$

$\hat{C}(2) = \hat{C}(1) + A(1, 2) + r$

$= 25 + 7 + 5$

$\boxed{\hat{C}(2) = 37}$

~~∴ State space tree is~~

→ Consider path (1, 3):–

It means that set '∞' to 1st row & 3rd column & set (3, 1) = ∞ of reduced matrix 'A' i.e

$$A = \begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 8 \\ \infty & 0 & \infty & 2 \\ 1 & 3 & \infty & \infty \end{bmatrix} & \begin{matrix} - \\ 0 \\ 0 \\ 1 \end{matrix} \\ & r = 1 \end{array}$$

Row Reduction:-

$$A = \begin{array}{c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 8 \\ \infty & 0 & \infty & 2 \\ 0 & 3 & \infty & \infty \end{bmatrix} \\ \begin{array}{cccc} 0 & 0 & \infty & 2 \end{array} \ (r=2) \end{array}$$

Column Reduction:-

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & 6 \\ \infty & 0 & \infty & 0 \\ 0 & 3 & \infty & \infty \end{bmatrix}$$

$\therefore r = 1 + 2 = 3$

$\hat{c}(3) = \hat{c}(1) + A(1,3) + r$

$= 25 + 8 + 3$

$\boxed{\hat{c}(3) = 36}$

→ Consider path (1,4):-

It means that we set '∞' to 1st row & 4th column & set (4,1) = ∞ of reduced matrix (A) i.e

$$A = \begin{array}{c} \begin{array}{c|c} \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty \\ 8 & 0 & \infty & \infty \\ \infty & 3 & 0 & \infty \end{bmatrix} & \begin{array}{c} \infty \\ 0 \\ 0 \\ 0 \end{array} \end{array} \\ \begin{array}{cccc} 0 & 0 & 0 & \infty \end{array} \ \boxed{r=0} \end{array}$$

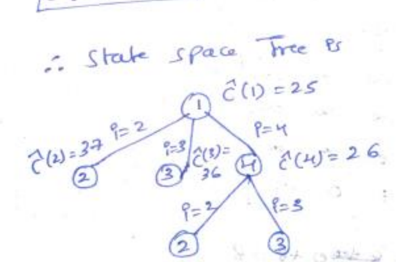
∴ No row reduction & Column Reduction

$\hat{c}(4) = \hat{c}(1) + A(1,4) + r$

$= 25 + 1 + 0$

$\boxed{\hat{c}(4) = 26}$

∴ State space Tree is



Compare (1,2), (1,3), (1,4). Here (1,4) has minimum cost then consider 2nd reduced matrix for path (1,4) i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & 2 & \infty \\ 8 & 0 & \infty & \infty \\ \infty & 3 & 0 & \infty \end{bmatrix}$$

→ Consider path (4,2):–

It means set 4th row & 2nd column & $A(2,1)$ to '∞' of 2nd reduced matrix 'A' i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 2 & \infty \\ 8 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \infty \\ 2 \\ 8 \\ \infty \end{matrix}$$

$\gamma = 10$

→ Row Reduction:–

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty \\ 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

$$\begin{matrix} 0 & \infty & 0 & \infty \end{matrix} \quad (\gamma = 0)$$

→ Column Reduction:–

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty \\ 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

$\therefore \gamma = 10 + 0 = 10$

$\hat{C}(2) = \hat{C}(4) + A(4,2) + \gamma$

$= 26 + 3 + 10$

$\boxed{\hat{C}(2) = 39}$

→ Consider Path (4,3):–

It means set 4th row & 3rd column & $A(3,1)$ to '∞' of 2nd reduced matrix 'A' i.e,

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \infty \\ 0 \\ 0 \\ \infty \end{matrix}$$

$$\begin{matrix} 0 & 0 & \infty & \infty \end{matrix} \quad (\gamma = 0)$$

∴ No Row Reduction & Column Reduction

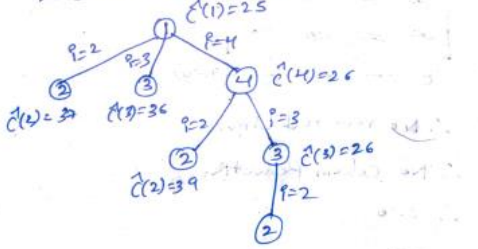
$\boxed{\therefore \gamma = 0}$

$\hat{C}(3) = \hat{C}(4) + A(4,3) + \gamma$

$= 26 + 0 + 0$

$\boxed{\hat{C}(3) = 26}$

∴ State space tree is



→ ~~Consider path (3,2)~~

Compare (4,2) & (4,3). Here (4,3) has minimum cost then consider 3rd reduced matrix for path (4,3) i.e

$$A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \\ \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix}$$

→ Consider path (3,2):-

It means that set 3rd row & 2nd column & A(2,1) as '∞' of 3rd reduced matrix 'A' i.e

$$\begin{array}{cc} A = \begin{bmatrix} \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty \end{bmatrix} & \begin{matrix} - \\ 0 \\ - \\ - \end{matrix} \\ \begin{matrix} 0 & - & - & - \end{matrix} & (r=0) \end{array}$$

∴ No row reduction

∴ No column reduction

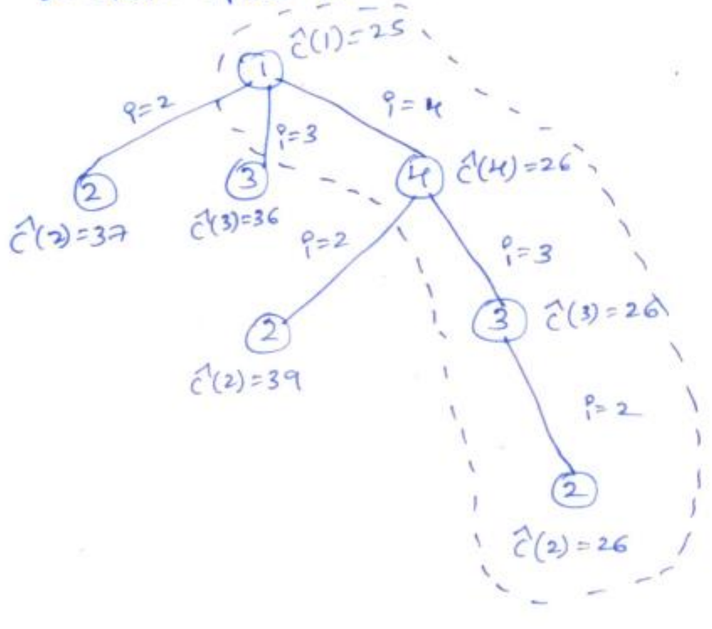
∴ $r=0$

$\hat{C}(2) = \hat{C}(3) + A(3,2) + r$

$= 26 + 0 + 0$

$\boxed{\hat{C}(2) = 26}$

∴ State space tree is



<u>All the best</u>
<u>Now do the</u>
<u>Rest…</u>