Assignment - II

Q.1. Prove that the following De Morgan law $\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$ is true for fuzzy sets $\tilde{A}$ & $\tilde{B}$, when using the Zadeh's operators for union, intersection and complement.

Ans. ⇒ $\mu_{\tilde{A} \cup \tilde{B}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$.

$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$.

$\mu_{\bar{\tilde{A}}}(x) = 1 - \mu_{\tilde{A}}(x)$.

$\overline{(A \cup B)}$ = L.H.S.

$= 1 - \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$.

$\bar{A} \cap \bar{B}$ = R.H.S.

$= \min(1 - \mu_{\tilde{A}}(x), 1 - \mu_{\tilde{B}}(x))$

If we consider $\mu_{\tilde{A}}(x) > \mu_{\tilde{B}}(x)$, then

$\max(\mu_A(x), \mu_B(x)) = \mu_A(x)$ and

$\min(1 - \mu_{\tilde{A}}(x), 1 - \mu_{\tilde{B}}(x)) = 1 - \mu_A(x)$.

∴ L.H.S. = R.H.S.

$\overline{(A \cup B)} = (\bar{A} \cap \bar{B})$.

Q.2. If $\tilde{A}$ and $\tilde{B}$ are two fuzzy sets with membership functions $\mu_A(x) = \{0.2, 0.5, 0.6, 0.1, 0.9\}$ and $\mu_B(x) = \{0.1, 0.5, 0.2, 0.7, 0.8\}$, find the value of $\mu_{A \cap B}$ and $\mu_{A \cup B}$.

Ans. ⇒ $\mu_A(x) = \{0.2, 0.5, 0.6, 0.1, 0.9\}$.

$\mu_B(x) = \{0.1, 0.5, 0.2, 0.7, 0.8\}$.

$\mu_{A \cap B} = \{0.1, 0.5, 0.2, 0.1, 0.8\}$.

$\mu_{A \cup B} = \{0.2, 0.5, 0.6, 0.7, 0.9\}$.

Q.3. Consider a fuzzy set A as defined below

$A = \{(20,0), (30,0.2), (40,0.4), (50,0.6), (60,0.8), (70,1), (80,1)\}$

Find α-cut of A for $\alpha = 0.4$.

Ans.⇒ $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$.

$\alpha = 0.4$.

$\therefore \mu_\alpha = \{(40,0.4), (50,0.6), (60,0.8), (70,1), (80,1)\}$.