25/11/19

SEMESTER - II [CALCULUS]

PRACTICAL No. 1

TOPIC :- Limits and continuity.

Q1.) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3}x}{\sqrt{3a+x} - 2\sqrt{x}} \right]$.

Q2.) $\lim_{y \to a} \left[\dfrac{\sqrt{a+x} - \sqrt{a}}{y\sqrt{a+y}} \right]$

Q3) $\lim_{x \to \pi/6} \left[\dfrac{\cos 5x - \sqrt{3}\sin x}{\pi - 6x} \right]$

Q4.) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Q5) Examine the continuity of the following function.

1.) $$\left.\begin{aligned} f(x) &= \frac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 < x \leq \pi/2 \\ &= \frac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{aligned}\right\} \text{ at } x = \pi/2$$

2.) $$\left.\begin{aligned} f(x) &= \frac{x^2-9}{x-3}, & 0 < x < 3 \\ &= x+3, & 3 \leq x < 6 \\ &= \frac{x^2-9}{x+3}, & 6 \leq x \leq 9 \end{aligned}\right\} \text{ at } x = 3 \text{ and } x = 6.$$

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es

Q6) Find value of K so that the function $f(x)$ is continuous at the indicated point.

1.) $f(x) = \frac{1-\cos 4x}{x^2}, \; x<0$
$= K, \; x=0$ } at $x=0$

2.) $f(x) = (\sec^2 x)^{\cot^2 x}$
$= K$ } at $x=0$

3.) $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}, \; x \neq \pi/3$
$= K, \; x = \pi/3$ } at $x = \pi/3$

Q7.) Discuss the continuity of following functions which of these function have removable discontinuity ? Redefine function. so as to remove the discontinuity ?

1.) $f(x) = \frac{1-\cos 3x}{x \tan x}, \; x \neq 0$
$= 9, \; x=0$ } at $x=0$.

2.) $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}, \; x \neq 0$
$= \pi/60, \; x=0$ } at $x=0$

Q8.) If $f(x) = \frac{(e^{x^2} - \cos x)}{x^2}$

for $x \neq 0$ is continous at $x=0$ find $f(0)$

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3.) $\lim_{x\to\pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}.$

$x - \pi/6 = h \quad \cdot x = h + \pi/6 \quad h \to 0$

$= \lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\,(h+\pi/6)}{\pi - 6\,(h+\pi/6)}$

$= \lim_{h\to 0} \frac{(\cos h.\ \cos\pi/6 - \sin h.\sin\pi/6) - \sqrt{3}\,(\sin h \cos\pi/6 + \cos h\ \sin\pi/6)}{-6h}$

$= \lim_{h\to 0} \frac{\left[\left(\cos h.\frac{\sqrt{3}}{2}\right) - \left(\sin h.\frac{1}{2}\right)\right] - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2} + \cos h\,\frac{1}{2}\right)\Big]}{-6h}$

$= \lim_{h\to 0} \frac{\left(\cos\frac{\sqrt{3}h}{2} - \sin\frac{h}{2}\right) - \left(\sin\frac{3h}{2} + \cos\frac{\sqrt{3}}{2}h\right)}{-6h.}$

$= \lim_{h\to 0} \frac{\cancel{-}\frac{\sin 4h}{2}}{\cancel{-}6h.}$

$= \lim_{h\to 0} \frac{\sin 2h}{6h}$

$= \frac{1}{3} \cdot \lim_{h\to 0} \frac{\sin h}{h}$

$= \underline{\underline{\frac{1}{3}.}}$

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$$\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$= \lim_{x\to\pi/2^-} \frac{2\sin x\cos x}{\sqrt{2\sin^2 x}} = \lim_{x\to\pi/2^-} \frac{2.\sin x\cos x}{\sqrt{2}\ \sin x}.$$

$$= \frac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x$$

$= 0.$

$\therefore$ L.H.L $\neq$ R.H.L.

$\therefore$ f is not continous at $x = \pi/2$.

5.) 2.) $$f(x) = \begin{cases} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \le x \le 6 \\ = \frac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$$ at $x=3$ & $x=6$.

1) at $x=3$

$$f(3) = \frac{x^2-9}{x-3} = 0.$$

f at $x=3$ define

~~$\lim_{x\to3^+} f(x) = \lim_{x\to3^+}$~~

~~$f(3) = x+3 = 3+3 = 6.$~~

~~f is define at $x=3$.~~

2) $$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (x+3) = 6.$$

$$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 6.$$

$\therefore$ L.H.L = R.H.L.

$\therefore$ f is continous at $x=3$

for $x=6$.

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9}$$

$$\lim_{x\to6^+} = \frac{x^2-9}{x+3}$$

$$\lim_{x\to6^+} \frac{(x-3)(x+3)}{(x+3)} = \lim_{x\to6^+} (x-3) = 6-3 = 3.$$

$$\lim_{x\to6^-} x+3 = 3+6 = 9.$$

L.H.L $\neq$ R.H.L

function is not continous.

Q6.) 1.) $$f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x<0 \\ = k & x=0 \end{cases}$$ at $x=0$

→ f is continous at $x=0$

$$\lim_{x\to0} f(x) = f(0)$$

$$\lim_{x\to0} \frac{1-\cos 4x}{x^2} = k$$

$$2\lim_{x\to0} \frac{2\sin^2 2x}{x^2} = k \therefore 2\lim_{x\to0}\left(\frac{\sin^2 2x}{x}\right)^2 = k.$$

$2(2)^2 = k$

$\therefore k = 8.$

6.ii) $$f(x) = \begin{cases} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x=0 \end{cases}$$ at $x=0$.

$= 4/3.$

6 iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x} \quad x = \pi/3$

$\quad = k \qquad\qquad x = \pi/3 \Big\}$ at $x = \pi/3$

$\rightarrow x - \pi/3 = h \qquad\qquad \therefore \lim\limits_{x \to \pi/3} f(x) = f(\pi/3)$

$x = h + \pi/3 \quad h \to 0$

$f(\pi/3 + h) - \sqrt{3} \qquad\qquad \therefore \lim\limits_{x \to \pi/3} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} = k.$

$$\lim_{h \to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3}\left(\dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}\right)}{\pi - \pi - 3h.}$$

$$\lim_{h \to 0} \frac{\sqrt{3}(1 - \tan \pi/3 . \tan h) - \tan \pi/3 - \tan h}{(-3h)(1 - \tan \pi/3 \cdot \tan h).}$$

$$\lim_{h \to 0} \frac{\sqrt{3}(1 - \sqrt{3} \tan h) - \sqrt{3} - \tan h}{-3h(1 - \sqrt{3} \tan h)}$$

$$\lim_{h \to 0} \frac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{-3h(1 - \sqrt{3} \tan h)}$$

$$\lim_{h \to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3} \tan h)} = \frac{4}{3} \lim_{h \to 0} \left(\frac{\tan h}{h}\right)\left(\frac{1}{1 - \sqrt{3} \tan h}\right)$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}(0)} = \underline{\underline{\frac{-4}{3}.}}$$

7.2) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2} \quad x \neq 0$
$= \pi/6 \quad x=0$ } at $x=0$

at $x=0$

$\lim_{x\to 0} (e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)/x^2$

$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin \pi x/180}{x}$

$3 \log e \frac{1}{180} = \frac{\pi}{60} \therefore = f(0)$

f is continous at $x=0$

8.) $f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x=0$

is continous at $x=0$

$\therefore$ f is continous at $x=0$.

$\lim_{x\to 0} f(x) = f(0)$

$= \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \cancel{(e^{x^2}-1)} + (1-\cos x)/x^2$

$= \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} 2 \frac{\sin^2 x/2}{x^2}$

$= \log e + 2\left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 on Numerator & Denominator.

$= 1 + 2 \times \frac{1}{9} = \frac{3}{2} = f(0)$.

9.) $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \quad x \neq \pi/2$

f (0) is continous at $x = \pi/2$

$= \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$.

$= \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1+\sin x}{1-\sin^2 x(\sqrt{2}+\sqrt{1+\sin x}}$

$= \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \frac{1}{2\times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$

$f(\pi/2) = \frac{1}{4\sqrt{2}}$

6 (ii) $f(x) = (\sec^2 x)^{\cot^2 x}, \quad x \neq 0$
$= k \quad$ ~~$x=0$~~ } at $x=0$

$f(x) = \cancel{(\sec^2 x)}^{\cot^2 x}$.

using

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$.

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and,
$\cot^2 x = 1/\tan^2 x$.

$\therefore \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$.

$\therefore \lim_{x\to 0} (1+\tan^2 x)^{1/\tan^2 x}$.

we know that,

$\lim_{x\to 0} (1+px)^{1/px} = e$.

$K = e$

$\boxed{\therefore K = e}$

PRACTICAL No. 2

DERIVATIVE

Q1) Show that the following function defined from $R \to R$ are differentiable.

1) $\cot x$

$f(x) = \cot x$

$Df(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$.

put $x-a=h$

$x = a+h$; as $x\to a$, $h\to 0$.

$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(\cancel{a}+h-\cancel{a})\tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$.

$= \lim_{h\to 0} \frac{\tan(a-a-h) - 1+\tan a\tan(a+h)}{h \times \tan(a+h)\tan a}$

$\therefore \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$.

$\therefore \tan A - \tan B = \tan(A-B)(1+\tan A\tan B)$

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$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}.$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= -\frac{1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a}.$$

$D\,f(a) = -\operatorname{cosec}^2 a$

$\therefore$ f is differentiable $\forall a \in R$.

2.) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$.

$$D\,f(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}.$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}.$$

$$= \lim_{x\to a} \frac{\sin a - \sin x.}{(x-a)\sin a.\sin x.}$$

put $x-a=h$.

$x = a+h$.

as $x\to a$, $h\to 0$.

$$D\,f(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

Formula :- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right).\sin\left(\frac{a-a-h}{2}\right)}{h\times \sin a.\sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times 1/2 \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= \lim_{h\to 0} \cancel{\sin h} - 1/\cancel{2} \times \frac{\cancel{2}\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a.\operatorname{cosec} a.$$

3.) $\sec x$

$f(x) = \sec x$.

$$D\,f(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a.}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a.}$$

$$= \lim_{x\to a} \frac{1/\cos x - 1/\cos a}{x-a.}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x.}$$

put $x-a=h$.

$x = a+h$.

as $x\to a$, $h\to 0$.

$$\therefore D\,f(h) = \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{h\times \cos a \cos(a+h)}$$

Formula :- $-2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right).\sin\left(\frac{\cancel{a}-\cancel{a}-h}{2}\right)}{h\times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right).\sin(-h/2)}{\cos a \cos(a+h) \times -h/2} \times -1/2$$

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$= -\frac{1}{2} \times -2 \sin\left(\frac{2a+0}{2}\right) \over \cos a \cos(a+0)$

~~$= -\frac{1}{2} \times -2 \sin 2a+0$~~

$= \cancel{-}\frac{1}{2} \times \cancel{+2} \frac{\sin a}{\cos a \times \cos a}$

$= \tan a \times \sec a.$

Q2) If $f(x) = 4x+1$, $x \leq 2$

$= x^2+5$, $x > 0$, at $x=2$, then.

find function is differentiable or not.

Soln:- L.H.D $= Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-8}{x-2}.$

$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{x-2}}$

$Df(2^-) = 4.$

R.H.D:

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= 2+2$

$Df(2^+) = 4$

R.H.D = L.H.D.

f is differentiable at $x=2$.

Q3) If $f(x) = 4x+7$, $x<3$

$= x^2+3x+1$, $x \geq 3$ at $x=3$, then

find f is differentiable or not ?

Soln :- R.H.D.

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}.$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3.}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3.}$

$= \lim_{x \to 3^+} \frac{\cancel{x(x+} x^2+6x-3x-18}{x-3.}$

$= \lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3.}$

$= \lim_{x \to 3^+} \frac{\cancel{(x-3)}(x+6)}{\cancel{(x-3)}}$

$= 3+6$

$Df(3^+) = 9$

L.H.D $= Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3.}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3.}$

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$$= \lim_{x \to 3^-} \frac{4x - 12}{x - 3}.$$

$$= \lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$$

$Df(3^-) = 4.$

$\because$ R.H.D $\neq$ L.H.D.

$\therefore$ f is not differentiable at $x = 3$.

Q4.) If $f(x) = 8x - 5 \quad , x \leq 2$

$= 3x^2 - 4x + 7 \quad , x > 2$ at $x = 2$, then.

Find f is differentiable or not.

Solⁿ:- $f(2) = 8 \times 2 - 5 = 16 - 5 = 11.$

R.H.D. :

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}.$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}.$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x - 2}.$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 3 \times 2 + 2$

$Df(2^+) = 8$

L.H.D:-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}.$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}.$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{(x-2)}}$$

$Df(2^-) = 8.$

$\because$ L.H.D = R.H.D.

$\therefore$ f is differentiable at $x = 3$.

AB
20/01/2020

PRACTICAL NO. 3.

APPLICATION OF DERIVATIVE

Q1.) Find the intervals in which function is increasing or ~~deserea~~ decreasing.

1.) $f(x) = x^3 - 5x - 11$

2.) $f(x) = x^2 - 4x.$

3.) $f(x) = 2x^3 + x^2 - 20x + 4.$

4.) $f(x) = x^3 - 27x + 5.$

5.) $f(x) = 69 - 24x - 9x^2 + 2x^3.$

Q2.) Find the intervals in which function is concave upwards.

1.) $y = 3x^2 - 2x^3$

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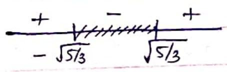
and f is decreasing if $f'(x) < 0$.

$\therefore 3x^2 - 5 < 0$.

$\therefore 3(x^2 - 5/3) < 0$.

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$.



$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2.) $f(x) = x^2 - 4x$.

$f'(x) = 2x - 4$.

$\because$ f is increasing if $f'(x) > 0$.

$\therefore 2x - 4 > 0$.

$2(x-2) > 0$.

$x - 2 > 0$.

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 < 0$.

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$.

$\therefore x \in (-\infty, 2)$

3.) $f(x) = 2x^3 + x^2 - 20x + 4$.

$\therefore f'(x) = 6x^2 + 2x - 20$.

$\because$ f is increasing if $f'(x) > 0$.

$\therefore 6x^2 + 2x - 20 > 0$.

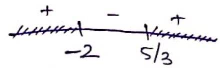
$\therefore 2(3x^2 + x - 10) > 0$.

$\therefore 3x^2 + x - 10 > 10$.

$\therefore 3x^2 + 6x - 5x - 10 > 0$.

$\therefore 3x(x+2) - 5(x+2) > 0$

$\therefore (x+2)(3x-5) > 0$.



$x \in (-\infty, -2) \cup (5/3, \infty)$

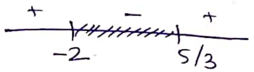
and f is decreasing if $f'(x) < 0$.

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$.

$\therefore 3x^2 + x - 10 < 0$.

$\therefore (x+2)(3x-5) < 0$



$x \in (-2, 5/3)$

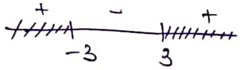
4.) $f(x) = x^3 - 27x + 5$.

$f'(x) = 3x^2 - 27$.

$\because$ f is increasing if $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$.

$(x-3)(x+3) > 0$.



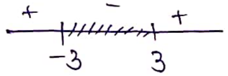
$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing if $f'(x) < 0$.

$\therefore 3x^2 - 27 < 0$.

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$.



$\therefore x \in (-3, 3)$

5.) $f(x) = 2x^3 - 9x^2 - 24x + 69$.

$f'(x) = 6x^2 - 18x - 24$.

$\because$ f is increasing if $f'(x) > 0$.

$\therefore 6x^2 - 18x - 24 > 0$.

$\therefore 6(x^2 - 3x - 4) > 0$.

$x^2 - 4x + x - 4 > 0$.

$x(x-4) + 1(x-4) > 0$.

$(x-4)(x+1) > 0$.

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

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and f is decreasing if $f'(x) < 0$.

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$.

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x-4)(x+1) < 0$.

+ − +
-1 4

$\therefore x \in (-1, 4)$

Q.2)

1.) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$.

$\therefore f''(x) = 6 - 12x$.

f is concave upward if $f''(x) > 0$

$\therefore f(6 - 12x) > 0$.

$12(1/2 - x) > 0$.

$x - 1/2 > 0$.

~~$x > 1/2$~~ $\therefore x \in (1/2, \infty)$

2.) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

$f(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$

+ − +
1 2

$x \in (-\infty, 1) \cup (2, \infty)$

3.) $y = x^3 - 27x + 5$.

$f'(x) = 3x^2 - 27$.

$f''(x) = 6x$.

f is concave upward if $f''(x) > 0$.

$\therefore 6x > 0$.

$x > 0$.

$\therefore x \in (0, \infty)$

4.) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$.

f is concave upward if $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$.

$\therefore x - 3/2 > 0$

$\therefore x \in (3/2, \infty)$

5.) $y = 2x^3 + x^2 - 20x + 4$.

$f(x) = 2x^3 + x^2 - 20x + 4$.

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$.

f is concave upward if $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$.

$\therefore (x + 1/6) > 0$.

$\therefore$ ~~$x \in$~~ ($\therefore x \le -1/6$

$\because f''(x) \not> 0$

$\therefore$ There exist no interval.

PRACTICAL No. 4.

Topic :- Application of derivative & Newton's Method.

Q1) Find maximum & minimum value of following

1) $f(x) = x^2 + \frac{16}{x^2}$

2) $f(x) = 3 - 5x^3 + 3x^5$

3) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1 \; [-2, 3]$

Q2) Find the root of the following equation by Newton's method (Take 4 iteration only) correct upto 4 decimal.

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

2) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Q1)

1) $f(x) = x^2 + 16/x2$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/\cancel{2}16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + 16/4$

$= 4 + 4$

$= 8$

$\therefore f''(2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$.

$\therefore$ Function reaches minimum value at $x = 2$ and $x = -2$.

2) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

Consider, $f'(x) = 0$.

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0 \quad \therefore$ f has minimum value at $x = 1$

$f(1) = 3-5(1)^3+3(1)^5$
$= 6-5$
$= 1$

$\therefore f''(-1) = -30(-1)+60(-1)^3$
$= 30-60$
$= -30<0$

$\therefore$ f has maximum value at $x=-1$

$\therefore f(-1) = 3-5(-1)^3+3(-1)^5$
$= 3+5-3$
$= 5$

$\therefore$ f has maximum value 5 at $x=-1$ and has the minimum value 1 at $x=1$

3.) $f(x) = x^3-3x^2+1$

$\therefore f'(x) = 3x^2-6x$

Consider, $f'(x)=0$

$\therefore 3x^2-6x=0$

$\therefore 3x(x-2)=0$

$\therefore 3x=0$ or $x-2=0$

$\therefore x=0$ or $x=2$.

$\therefore f''(x) = 6x-6$

$\therefore f''(0) = 6(0)-6$
$= -6<0$

$\therefore$ f has maximum value at $x=0$

$\therefore f(0) = (0)^3-3(0)^2+1 = \underline{\underline{1}}$.

$\therefore f''(2) = 6(2)-6$
$= 12-6$
$= 6>0$

$\therefore$ f has minimum ~~at~~ value at $x=2$.

$\therefore f(2) = 2^3-3(2)^2+1$
$= 8-12+1$
$= -3$

$\therefore$ f has maximum value 1 at $x=0$ and has minimum value -3 at $\underline{x=2}$.

4.) $f(x) = 2x^3-3x^2-12x+1$

$\therefore f'(x) = 6x^2-6x-12$

Consider, $f'(x)=0$

$\therefore 6x^2-6x-12=0$

$\therefore 6(x^2-x-2)=0$

$x^2-x-2=0$

$\therefore x^2+x-2x-2=0$

$\therefore x(x+1)-2(x+1)=0$

$\therefore (x-2)(x+1)=0$

$\therefore x=2$ or $x=-1$

$\therefore f''(x) = 12x-6$

$\therefore f''(2) = 12(2)-6$
$= 24-6$
$= 18>0$

$\therefore$ f has minimum value at $x=2$

$\therefore f(2) = 2(2)^3-3(2)^2-12(2)+1$
$= 2(8)-3(4)-24+1$
$= -19$.

$\therefore f''(-1) = 12(-1)-6$
$= -12-6$
$= -18<0$

$\therefore$ f has maximum value at $x=-1$

$\therefore f(-1) = 2(-1)^3-3(-1)^2-12(1)+1$.
$= -2-3+12+1$
$= 8$

$\therefore$ f has maximum value 8 at $x=-1$ and has minimum value -19 at $x=2$.

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Q2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5 \quad x_0 = 0 \rightarrow$ Given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$\therefore x_1 = x_0 - f(x_0)/f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= 0.1712$

∴ The root of the equation is $\underline{0.1712}$.

2.) $f(x) = x^3 - 4x - 9 \quad [2, 3]$

$f'(x) = 3x^2 - 4$

$f(2) = \cancel{2^3} 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

∴ By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= \underline{17.9851}$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 2.7071 - 0.0102/17.9851$

$= 2.7071 - 0.0056$

$= 2.7015$.

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$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.0901$.

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= \underline{2.7065}$.

3.) $f(x) = x^3 - 1.8x^2 - 10x + 17$ [1,2]

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$f(2) = 2^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation.
By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$= 2 - 2.2/5.2$

$= 2 - 0.4230 = \underline{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 1.577 + 0.6755/8.2164$

$= 1.577 + 0.0822$

$= \underline{1.6592}$

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PRACTICAL No. 5

TOPIC :- Integration

Q1.) Solve the following integration.

1.) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

2.) $\int (4e^{3x}+1)\,dx.$

3.) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$

4.) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx.$

5.) $\int t^7.\sin(2t^4)\,dt.$

6.) $\int \sqrt{x}\,(x^2-1)\,dx$

7.) $\int 1/x^3 . \sin(1/x^2)\,dx$

8.) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

~~9.) $\int e\cos^3 x$~~ 9.) $\int e^{\cos^2 x} . \sin 2x\,dx.$

10.) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

Q.1)

1.) $I=\int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

Comparing with $\int \frac{dx}{\sqrt{x^2-a^2}} = \cancel{x}^2 = (x+1)^2$

$\therefore I = \log|x+\sqrt{x^2+a^2}| + c.$

$= \log|x+1+\sqrt{(x+1)^2-(2)^2}| + c$

2.) $I = \int (4e^{3x}+1)\,dx.$

$= \int 4e^{3x}\,dx + \int 1\,dx.$

$= \frac{4e^{3x}}{3} + x + c$

3.) $I = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx.$

$= 2\int x^2\,dx - 3\int \sin x\,dx + 5\int \sqrt{x}\,dx$

$= \frac{2}{3}x^3 \cancel{dx} + 3\cos x + \frac{5x^2}{3}x^{3/2} + c.$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + c$

4) $\int \frac{x^3+3x+4}{\sqrt{x}} dx.$

$\Rightarrow I = \int \frac{x^3+3x+4}{\sqrt{x}} dx.$

$= \int \frac{x^3+3x+4dx}{\sqrt{x}}$

$= \int \left(\frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\right) dx.$

$= \int x^{5/2} dx + 3\int x^{-1/2} dx + 4\int x^{-1/2} dx.$

$= \frac{2}{7} x^{7/2} + 2x^{3/2} + 8\sqrt{x} + c$

5) $I = \int t^7 \sin(2t^4)\, dx.$

let $t^4 = x.$

$4t^3\, dt = dx.$

$\therefore I = \frac{1}{4}\int 4t^3 . t^4 \sin(2t^4)\, dt.$

$= \frac{1}{4}\int x . \sin(2x)\, dx$

$= \frac{1}{4}\left[x \int \sin 2x - \int [\int \sin 2x . dx/dx\right]$

$= \frac{1}{4}\left[-\frac{x \cos 2x}{2} + \frac{1}{2}\int \cos 2x . 1\right]$

$= \frac{1}{4}\left[\frac{-x\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + c.$

$= -\frac{1}{8} x . \cos 2x + \frac{1}{16} \sin 2x + c.$

$= -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16} \sin(2t^4) + c.$

6) $\int \sqrt{x}\,(x^2-1)\, dx.$

$\Rightarrow I = \int \sqrt{x}\,(x^2-1)\, dx.$

$= \int (\sqrt{x} . x^2 - \sqrt{x})\, dx.$

$= \int (x^{5/2} - \sqrt{x})\, dx.$

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c.$

7) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx.$

let $\frac{1}{x^2} = t$

$x^{-2} = t.$

$-\frac{2}{x^3}\, dx = dt.$

$I = -\frac{1}{2}\int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= -\frac{1}{2}\int \sin t$

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$= \frac{-1}{2}(-\cos t) + c$

$= \frac{1}{2} \cos t + c.$

Resubstitution $t = 1/x^2$

$\therefore I = \frac{1}{2} \cos\left(1/x^2\right) + c$

8.) $I = \int \frac{\cos x}{\sqrt[3]{\sin 2x}}.$

let $\sin x = t.$

$\therefore \cos x \, dx = dt.$

$\therefore I = \int \frac{dt}{\sqrt[3]{t^2}}$

$= \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3} \, dt.$

$= -3 \, t^{1/3} + c.$

$= 3 (\sin x)^{1/3} + c.$

$= 3. \sqrt[3]{\sin x} + c.$

9.) $I = \int e^{\cos^2 x} . \sin^2 x \, dx.$

let $\cos^2 x = t.$

$-2 \cos x . \sin x \, dx = dt.$

$-2 \sin 2x \, dx = dt.$

$\therefore I =$ ~~[illegible]~~ $\int -2 \sin 2x . e^{\cos^2 x} \, dx.$

$= -\int e^t \, dt.$

$= -e^t + c.$

Resubstituting $t = \cos^2 x$

$\therefore I = -e^{\cos^2 x} + c$

10.) $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

let $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x) \, dx = dt.$

$(x^2 - 2x) \, dx = dt/3$

$\therefore I = \int 1/3t \, dt.$

$= 1/3 \int dt/t$

$= \frac{1}{3} \log t + c.$

$= \frac{1}{3} \log\left(x^3 - 3x^2 + 1\right) + c$

PRACTICAL No. 6

Topic :- Application of Integration & Numerical integration.

Q1) Find the length of the following curve.

1) $x = t\sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$
for t belong to $[0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$x = t - \sin t$, $y = 1 - \cos t$

$\frac{dx}{dt} = 1 - \cos t$, $\frac{dy}{dt} = 0 - (\sin t)$

$\frac{dy}{dt} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2}\, dt$$

$$= \left[-4\cos\left(\frac{t}{2}\right)\right]_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4$$

$$= 8.$$

2) $y = \sqrt{4-x^2}$ $x \in [-2,2]$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\, dx.$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx.$$

$$= 4\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$= \underline{2\pi}.$$

3) $y = x^{3/2}$ in $[0,4]$

$\rightarrow$ $f'(x) = \frac{3}{2} x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$L = \int_a^b \sqrt{1+[f'(x)]^2}\, dx.$$

$$= \int_0^x \sqrt{1+\frac{9}{4}x}\, dx$$

put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$.

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\, dx = \left[\frac{4}{9}\cdot\frac{2}{3}\left(u^{3/2}\right)\right]_1^{1+\frac{9}{4}x}.$$

$$L = \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$$

4) $x = 3\sin t$, $y = 3\cos t$

$\rightarrow$ $\frac{dx}{dt} = 3\cos t$

$$\frac{dy}{dt} = -3\cos t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt.$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt.$$

$$= \int_0^{2\pi} 3\sqrt{\cancel{x}1}\, dt.$$

$$= 3\int_0^{2\pi} \cancel{9}\, 1.\, dt.$$

$$= 3\left[t\right]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$$= 3(2\pi)$$

$$\boxed{L = 6\pi}$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1,2]$

→ $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy.$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy.$

$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy.$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$

Q2) 1) $\int_0^2 e^{x^2} dx$ with $n=4$

→ In each case the width of the sub internal be,

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

and so the sub intervals will be $[0, 0.5]$, $[0.5, 1]$ $[1, 1.5]$, $[1.5, 2]$

By simpson rule,

$\int e^{x^2} dx = \frac{1/2}{3} \cdot (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$

$\approx \frac{1/2}{3}(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2})$

≈ 17.3536

2) $\int_1^4 x^2 dx$, $n=4$.

→ $\Delta x = \frac{4-0}{4} = 1$

$\therefore \int_a^b f(x)\, dx = \frac{\Delta x}{3}[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$

$= \frac{1}{3}[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$

$= \frac{1}{3}[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2].$

$= \frac{64}{3}$

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PRACTICAL No. 7

Topic : Differential Equations.

Q1) Solve the following differential equations :-

1) $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$p(x) = \frac{1}{x}$ $\quad \phi(x) = \frac{e^x}{x}$

$IF\ e^{\int 1/x\, dx}$

x

$y(IF) = \int \phi(x)(IF)\, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x\, dx + c.$

$xy = e^x + c$

2) $e^x\frac{dy}{dx} + 2e^{x}y = 1$

$\rightarrow \frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad , \phi(x) = e^{-x}$

$\int P(x) = dx.$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int \phi(x)(IF)\, dx + c$

$= \int e^x\, dx + c.$

$= y.\ e^{2x}$

$= e^x + c$

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3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\rightarrow \cancel{x}\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad , \quad \phi(x) = \frac{\cos x}{x^2}$

$IF = e^{\int P(x)dx}$
$= e^{\int 2x\,dx}$.

$y(IF) = \int \phi(x)\,(IF)\,dx + c$
$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c$
$= \int \cos x + c$
$\therefore x^2 y = \sin x + c$

4) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\rightarrow \frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x \qquad \phi(x) = \sin x^{\cancel{x}}/x^3$

$P(x) = \int 3/x\,dx$.
$= x^3$

$IF = e^{\int P(x)\,dx}$

$y(IF) = \int \phi(x)\,(IF)\,dx + c$
$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$.
$= \int \sin x + c$

$x^3 y = -\cos x + c$

7.) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

$x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1-dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \cancel{\frac{dy}{dx}} \sin^2 v.$

$\frac{dv}{dx} = 1 - \sin^2 v.$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v \, dv = \int dx.$

$\tan v = x + c.$

$\tan(x-y+1) = x + c.$

8.) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y = 0$

$2 + \frac{3dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\therefore \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$= \frac{v-1+2v+v}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{(v+2)}$

$\therefore \int \frac{v+2}{v+1} dv = 3dx.$

$\therefore \int \frac{v+1}{v} dx + \int \frac{1}{v+1} dv = \int 3dx.$

$\therefore v \log|v| = 3x + c$

$\therefore 2x + 3y + \log|2x+3y+1| = 3x + c$

$\therefore 3y = x - \log|2x+3y+1| + c$

PRACTICAL No. 8

TOPIC :- Euler's Method

Q1) $\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2) = ?$

→ $f(x) = y + e^x - 2 \quad x_0 = 0 \; , \; y(0) = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$$\therefore \boxed{y(2) = 9.8215}$$

Q2) $\frac{dy}{dx} = 1 + y^2 , \quad y(0) = 1 \quad h = 0.2$ find $y(1) = ?$

→ $y_0 = 0 \; , \; y_0 = 0 \; , \; h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6418	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$$\boxed{y(1) = 1.2939}$$

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Q3.) $\frac{dy}{dx} = \sqrt{\frac{x}{4}}$ $y(0) = 1$, $h = 0.2$, find $y(1) = ?$

$x_0 = 0$, $y(0) = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$\therefore y(1) = 1.5051$

Q4.) 1.) $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$, find $y(2)$, $h = 0.5$

$h = 0.25$

→ $y_0 = 2$, $x_0 = 1$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

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PRACTICAL No. 9

TOPIC :- LIMITS & PARTIAL ORDER DERIVATIVES.

Q1.) Evaluate the following limits :-

1.) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$= \dfrac{-64+3+1-1}{9}$

$= \dfrac{-64+3}{9}$

$= \dfrac{-61}{9}$

2.) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$= \dfrac{1(4-8)}{2}$

$= \dfrac{-4}{2}$

$= -2.$

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3.) $L = \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^2(x-yz)}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x)^2-(yz)^2}{x^2(x-yz)}$

$\because (x)^2-(yz)^2 = (x-yz)(x+yz)$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$

$= \frac{1+1}{(1)^2} = \frac{2}{1}$

$= \underline{\underline{2}}$ ....

Q2.) Find f_x, f_y for each of the following f.

1.) $f(x,y) = xy.e^{x^2+y^2}$

→ $f(x) = y.e^{x^2+y^2}.(2x)$

$= \underline{\underline{2xy.e^{x^2+y^2}}}$

$f(y) = x.e^{x^2+y^2}.(2y)$

$= \underline{\underline{2xy.e^{x^2+y^2}}}$

2.) $f(x,y) = e^x \cos y$

→ $f(x) = e^x(\cos y)$, $f(y) = e^x(-\sin y)$

$= \underline{\underline{e^x.\cos y}}$ $\quad = \underline{\underline{-e^x \sin y}}$ ...

3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

→ $f(x) = 3x^2y^2 - 6xy + 0 + 0$, $f(y) = x^3.(2y) + 3x^2(1) + 0 + 0$

$= 3x^2y^2 - 6xy$ $\quad = \underline{\underline{2x^3y + 3x^2}}$ ...

Q3.) Using definition find values of f_x, f_y at (0,0) for $f(x,y) = \frac{2x}{1+y^2}$

→ $f(x,y) = \frac{2x}{1+y^2}$, $(a,b) = (0,0)$

$f(x)(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$f(x)(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h.}$

$= \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}}$

$\underline{\underline{f(x)(0,0) = 2.}}$ ...

$f(y)(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

$f(y)(0,0) = \lim_{h\to 0} \frac{f(0,0+h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

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$$f(y)(0,0) = \lim_{h \to 0} \frac{0-0}{h}$$

$$= \underline{\underline{0}}.$$

$\therefore$ $\underline{f(x)\ (0,0)} = 2.$

$\underline{\underline{f(y)(0,0) = 0.}}$

Q4.) Find all Second Order Partial derivatives of f. Also verify whether, $f(xy) = f(yx)$

1.) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\rightarrow f(x) = 3x^2 + 6xy^2 - \frac{1}{x^2+1} \cdot (2x)$

$= \underline{\underline{3x^2 + 6xy^2 - \frac{2x}{x^2+1}}}$

$f(y) = 6x^2y - \frac{1}{x^2+1} \cdot (0)$

$= \underline{\underline{6x^2y}}.$

$f(xx) = 6x + 6y^2 - \frac{\cancel{2}}{\cancel{2}x}$

$= \underline{\underline{6x + 6y^2 - \frac{1}{x}}}.$

$f(yy) = \underline{\underline{6x^2}}$

$f(xy) = 0 + 12xy - 0$, $f(yx) = \underline{\underline{12xy}}$

$= \underline{\underline{12xy}}$

$\because$ $f(xy) = f(yx) = 12xy.$

Hence $\underline{\underline{\text{Proved (verified)}}}$

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Q5) Find the linearization of $f(x,y)$ at given point.

1.) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{1+1} = \sqrt{2}$.

$f(x) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot (\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}}$

$f(x)(1,1) = \frac{1}{\sqrt{2}}$

$f(y) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot (\cancel{2}y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

$f(y)(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(1,1) + f(x)(1,1)(x-1) + f(y)(1,1)(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$.

$= \sqrt{2} + \frac{2}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{2}{\sqrt{2}}(x+y-2)$.

2.) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

$\rightarrow f(\pi/2, 0) = 1-\pi/2+0$

$= 1-\pi/2$

$f(x) = -1+y\cos x$, $f(y) = -x+\sin x(1)$

$f(x)(\pi/2,0) = -1+0$, $= -x+\sin x$

$= -1$, $f(y)(\pi/2,0) = -\pi/2+1$

$\therefore L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f(y)(a,b)(y-b)$

$= (1-\pi/2) + (-1)(x-\pi/2) + (-\pi/2+1)(y-0)$

$= 1-\cancel{\pi/2}-x+\cancel{\pi/2}-\pi/2\,y+y$

$= 1-x-\pi/2\,y+y$.

$= 1-x+y-\frac{\pi}{2}y$

3.) $f(x,y) = \log x+\log y$ at $(1,1)$

$\rightarrow f(1,1) = \log(1)+\log(1)$

$= \log(2)$

$f(x)(1,1) = \frac{1}{x}$, $f(y)(1,1) = \frac{1}{y}$

$\therefore L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f(y)(a,b)(y-b)$

$= f(1,1) + f(x)(1,1)(x-1) + f(y)(1,1)(y-1)$

$= \log(2) + \frac{1}{x}(x-1) + \frac{1}{y}(y-1)$

$= \log(2) + 1-\frac{1}{x} + \cancel{1-\frac{1}{y}}$

$= \log(2) + 2-\frac{1}{x}-\frac{1}{y}$

$= \cancel{\log}\ 2+\log(2)-\left(\frac{1}{x}+\frac{1}{y}\right)$

PRACTICAL No. 10

DIRECTIONAL DERIVATIVE, GRADIENT VECTOR AND MAXIMA, MINIMA, TANGENT AND NORMAL VECTORS.

Q1) Find the directional derivative of the following function at given points & in the direction of given vector.

1) $f(x,y) = x+2y-3$, $a=(1,-1)$, $u = 3i-j$.

→ $u = 3i-j$ is not a unit vector

$\therefore |u| = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $= \frac{u}{|u|} = \frac{3i-j}{\sqrt{10}}$

$$u = \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$$

$f(a) = f(1,-1) = 1-2-3 = 4.$

$$f(a+hu) = f\left((1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$$

$$= f\left(1+\frac{3h}{\sqrt{10}}, -1-\frac{h}{\sqrt{10}}\right)$$

$$= 1+\frac{3h}{\sqrt{10}} + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3.$$

$$= 1+\frac{3h}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3.$$

$$= -4+\frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-4} + \cancel{k}/\sqrt{10} \cancel{+4}}{\cancel{k}}$$

$$= \frac{1}{\sqrt{10}}$$

2.) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

→ $u = i + 5j$ is not a unit vector.

$\therefore$ $|u| = \sqrt{1+25} = \sqrt{26}$

Unit vector along u is $= \frac{u}{|u|} = \frac{i+5j}{\sqrt{26}}$

$$u = \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1$$

$$= 16 - 12 + 1 = 4 + 1 = 5$$

$$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1.$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1.$$

$$= 5 + \frac{36h}{\sqrt{26}} + \frac{25h^2}{26.} = \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5.$$

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$$D_u.f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$$
$$= \lim_{h\to 0} \left(\frac{\frac{18\cancel{h}}{5} + \cancel{8} - \cancel{8}}{\cancel{h}}\right)$$
$$= \frac{18}{5}$$

Q2.) Find Gradient vector for the following function at given point.

1.) $f(x,y) = x^y + y^x, \quad a = (1,1)$

$f(x) = y.\, x^{y-1} + y^x.\log y$

$f(y) = x^y.\log x + x.y^{x-1}$

$$\nabla f(x,y) = (f(x), f(y))$$
$$= (y.x^{y-1} + y^x.\log y,\ x^y.\log x + x.y^{x-1})$$
$$= (1+0,\ 1+0)$$
$$= (1,1)$$

2.) $f(x,y) = (\tan^{-1}x).y^2, \quad a = (1,-1)$

→ $f(x) = \frac{1}{1+x^2}\cdot y^2 = \frac{y^2}{1+x^2}$

$f(y) = \tan^{-1}x.(2y) = (2y)(\tan^{-1}x)$

$\nabla f \cancel{(x,y)} = (f_x, f_y)$

$$\nabla f(1,-1) = \left(\frac{y^2}{1+x^2}, (2y)(\tan^{-1}x)\right)$$
$$= \left(\frac{1}{1+1}, (2).\tan^{-1}(1)\right)$$
$$= \left(\frac{1}{2}, \cancel{2}.\frac{\pi}{\cancel{4}_2}\right)$$

$$\therefore \nabla f(x,y) = \left(\frac{1}{2}, \frac{\pi}{2}\right)$$

3.) $f(x,y,z) = xyz - e^{x+y+z}, \quad a = (1,-1,0)$

→ $f(x) = yz\cancel{(1)} - e^{x+y+z}$.

$f(y) = xz - e^{x+y+z}$.

$f(z) = xy - e^{x+y+z}$.

$$\nabla f(x,y,z) = (f_x, f_y, f_z)$$
$$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$$
$$= (0 - e^0, 0 - e^0, -1 - e^0)$$
$$= (0-1, 0-1, -1-1)$$
$$= (-1, -1, -2)$$

Q3.) Find the equation of tangent & normal to each of the following curves using at given points.

1.) $x^2\cos y + e^{xy} = 2$ ~~at~~ $(1,0)$

→ Here, $f(x,y) = x^2\cos y + e^{xy} - 2 = 0$.

$f(x) = 2x(\cos y) + e^{xy}(1) \quad , \quad f(y) = -x^2(\sin y) + e^{xy}$

$= 2x(\cos y) + e^{xy}$

point is $(x_0, y_0) = (1,0)$

$$\therefore f(x)(x_0,y_0) = 2(1)(\cos(0)) + e^0$$
$$= 2(1) + 1 = 2+1 = 3$$

$f_y(x_0,y_0) = -(1)^2(\sin 0) + e^0$
$= -1(0) + 1$
$= 0 + 1$
$= \underline{1}$..

Equation of tangent is
$f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) = 0.$
$3(x-1) + 1(y-0) = 0.$
$3(x-1) + y = 0$
$3x - 3 + y = 0.$
$\underline{3x + y - 3 = 0.}$

$ax + by + c = 0.$
Let $bx - ay + d = 0.$

Eqn of normal is,
$(1)x - 3y + d = 0.$
$x - 3y + d = 0.$

But normal is passing through $(1,0)$
$\therefore 1 - 0 + d = 0.$
$1 + d = 0$
$\boxed{d = -1}$

$\therefore$ Eqn of normal is $\underline{x - 3y - 1 = 0}$..

2.) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$
→ Here, $f(x,y) = x^2+y^2-2x+3y+2=0$
$\therefore f(x) = 2x-2.$, $f(y) = 2y+3.$

$\because$ point is $(x_0,y_0) = (2,-2)$
$x_0 = 2,\ y_0 = -2.$

$f(x) = 2(2)-4$, $f(y) = 2(-2)+3$
$= 4-2 = \underline{2}.$, $= -4+3 = \underline{-1}$..

Eqn of tangent is,
$f(x)_{(x_0,y_0)}(x-x_0) + f(y)_{(x_0,y_0)}(y-y_0) = 0.$
$2(x-2) + (-1)(y+2) = 0.$
$2x - 4 - y - 2 = 0$
$\underline{2x - y - 6 = 0.}$...

$ax + by + c = 0$
Let $bx - ay + d = 0$

Eqn of normal is
$-1x - 2y + d = 0.$

But normal is passing thr' $(2,-2)$
$\therefore -1(2) - 2(-2) + d = 0.$
$-2 + 4 + d = 0.$
$2 + d = 0.$
$\boxed{d = -2}$

$\therefore$ Eqn of normal is $-1x - 2y - 2 = 0$
$\underline{-x - 2y - 2 = 0.}$...

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Q4) Find the eq^n of tangent & normal line to each of the following surface. 33

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x) = 2x + z.$

$f(y) = 2x + 3$

$f(z) = -2y + x.$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore$ $x_0 = 2,\ y_0 = 1,\ z_0 = 0.$

$f(x)\ (x_0, y_0, z_0) = 2(2) + 0 = 4.$

$f(y)\ (x_0, y_0, z_0) = 2(0) + 3 = 3$

$f(z)\ (x_0, y_0, z_0) = -2(1) + 2 = 0$

Eq^n of tangent,

$f(x)\ (x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$\therefore\ 4(x-2) + 3(y-1) + 0(z-0) = 0$

$\therefore\ 4x - 8 + 3y - 3 = 0.$

$\therefore\ \underline{4x + 3y - 11 = 0}$ → Eq^n of tangent.

Eq^n of normal at $(4,3,-11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$\therefore\ \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

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Q5. Find the local maxima and minima for the following function.

1.) $f(x,y) = 3x^2 + y^2 - 3xy - 3xy + 6x - 4y$.

$f(x) = 6x - 3y + 6$.
$f(y) = 2y - 3x - 4$.

$f(x) = 0$

$$6x - 3y + 6 = 0$$
$$3(2x - y + 2) = 0$$
$$2x - y + 2 = 0$$
$$\underline{2x - y = -2} \quad \ldots\ (1)$$

$f(y) = 0$

$$2y - 3x - 4 = 0$$
$$2y - 3x = 4 \quad \ldots\ (2)$$

Multiply eqⁿ (1) by 2,

$$\therefore\ 4x - \cancel{2y} = -4.$$
$$\cancel{2y} - 3x = 4$$
$$x = 0.$$

Substitute value of x in eqⁿ (1)

$$2(0) - y = -2.$$
$$\cancel{-}y = \cancel{-}2 \quad \therefore \boxed{y = 2}$$

$\therefore$ Critical points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here, $r > 0$
$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$\therefore\ 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$0 + 4 - 0 + 0 - 8 = \underline{\underline{-4}}.$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f(x) = 8x^3 + 6xy$
$f(y) = 3x^2 - 2y$

$f_x = 0$

$$\therefore\ 8x^3 + 6xy = 0$$
$$2x(4x^2 + 3y) = 0$$
$$4x^2 + 3y = 0 \quad \ldots\ (1)$$

$f(y) = 0$

$$3x^2 - 2y = 0 \quad \ldots\ (2)$$

Multiply eqⁿ (1) by 3 & eqⁿ (2) by 4.

$$\cancel{12x^2} + 9y = 0$$
$$\underline{-\cancel{12x^2} \overset{+}{-} 8y = 0}$$
$$y = 0$$

Substitute value of y in eqⁿ (1)

$$\therefore\ 4x^2 + 3(0) = 0$$
$$4x^2 = 0.$$
$$\underline{x = 0}$$

Critical point is $(0,0)$

$r = f(xx) = 24x^2 + 6x$

$t = f(yy) = 0 - 2 = -2$

$s = f(xy) = 6x - 0 = 6x = 6(0) = 0$

r at (0,0)

$= 24(0) + 6(0) = 0$

$\therefore r = 0.$

$rt - s^2 = 0(-2) - 0^2$

$= 0 - 0 = \underline{0}.$

$r = 0$ & $rt - s^2 = 0.$

(noting to say)

$\therefore f(xy)$ at (0,0)

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0.$

$= \underline{\underline{0}}$

3.) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} = -1 \quad \underline{\underline{\therefore x = -1}}$

$f_y = 0 \quad \therefore -2y + 8 = 0$

$y = \frac{-8}{-2} = 4$

$\underline{\underline{\therefore y = 4}}$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

Scanned with CamScanner