CS 6635 - Scientific Visualization - Spring 2019 **Final Project Report**

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Project Title: **Visualization of crack propagation modeled by boundary element method**

1. Overview and goals of the project

Numerical modeling of cracks and prediction of crack growth in solids are important tasks for engineers to assess the integrity of structures under real working conditions. Symmetric Galerkin boundary element method (SGBEM) [1] [2] [3] possesses advantages when modeling cracks that other domain-based numerical methods (e.g. finite element, finite difference methods) do not have: no need to discretize the interior of structure which is particularly beneficial for crack modeling (to avoid discretization of the domain in front of crack front which requires very fine mesh); highly accurate results are obtained even when using relatively coarse meshes.

Although SGBEM is proved to be powerful in crack modeling, there is no method/technique to visualize the analysis results. This could be one of the reasons why the application of SGBEM is very limited in the reality of engineering industries.

This project is to implement the visualization of crack modeling and simulation of crack propagation done by SGBEM. The visualization mainly includes two parts:

1. Visualizing crack properties including the geometry, crack opening, plastic zone, predicted direction of crack growth.
2. Visualizing crack growth including the topology of crack propagation starting from initial configuration till the end of the simulation.

The ultimate purpose of this project is to enhance the effectiveness of the SGBEM for crack modeling. Besides, the visualizing methods proposed in this project could be applied to other numerical methods as well.

2. Background and related work

The background of this project is the SGBEM developed in [1] [2] [3] for modeling of cracks in general anisotropic elastic media and multi-field media.

3. Project description

The data used for this project is synthesis data which is generated from the code developed in [1] and [2]. Besides, for visualizing crack properties, we also develop a small source code (written in Fortran 90) to generate data for a simple crack problem using analytic solution so that we can compare the visualized results with the exact solution.

To implement the two goals of the project mentioned above, we need to find solutions to the following questions:

1. How do we visualize the crack modeling generated by SGBEM developed in [1] [2]? Specifically, how do we visualize the geometry of the structures and the topology of cracks modeled by SGBEM.

2. How do we visualize the plastic zones in front of the crack front where the material is supposed to be plastic due to high concentration of stress?
3. How do we visualize the predicted direction if an existing crack is supposed to grow under a critical condition of loading?
4. How do we visualize the shape of crack during its propagation modeled by SGBEM?

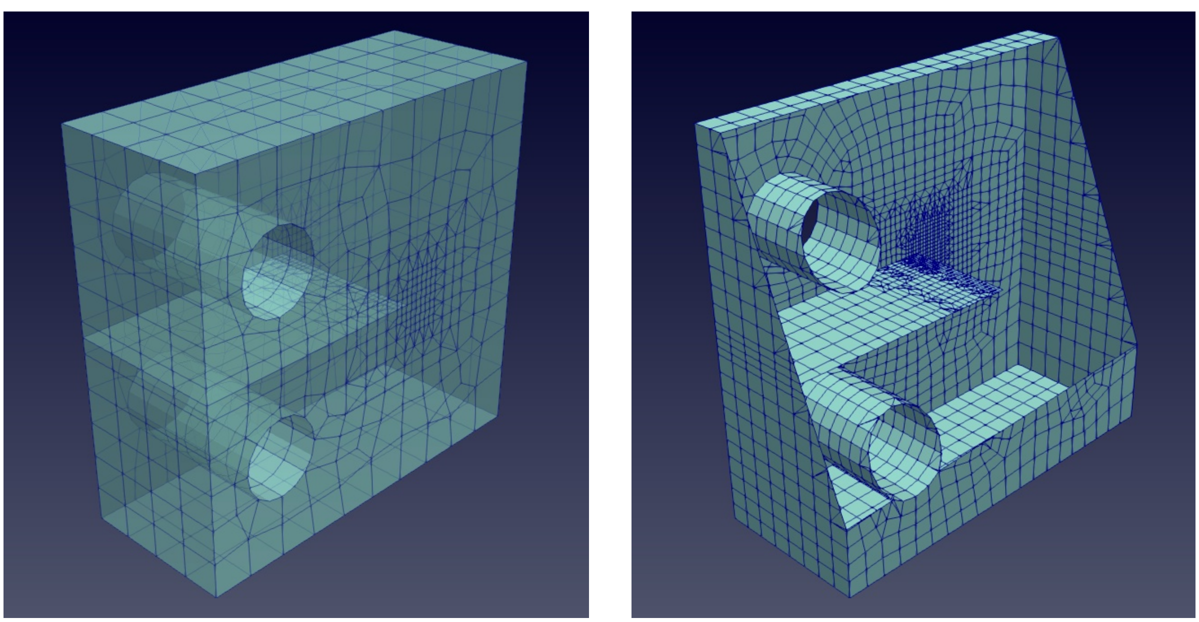
The answers for those questions are presented in the next section.

4. Implementation

In this section, we will describe in details what we have done to answer the four questions mentioned above.

4.1. Visualization of crack model generated by SGBEM

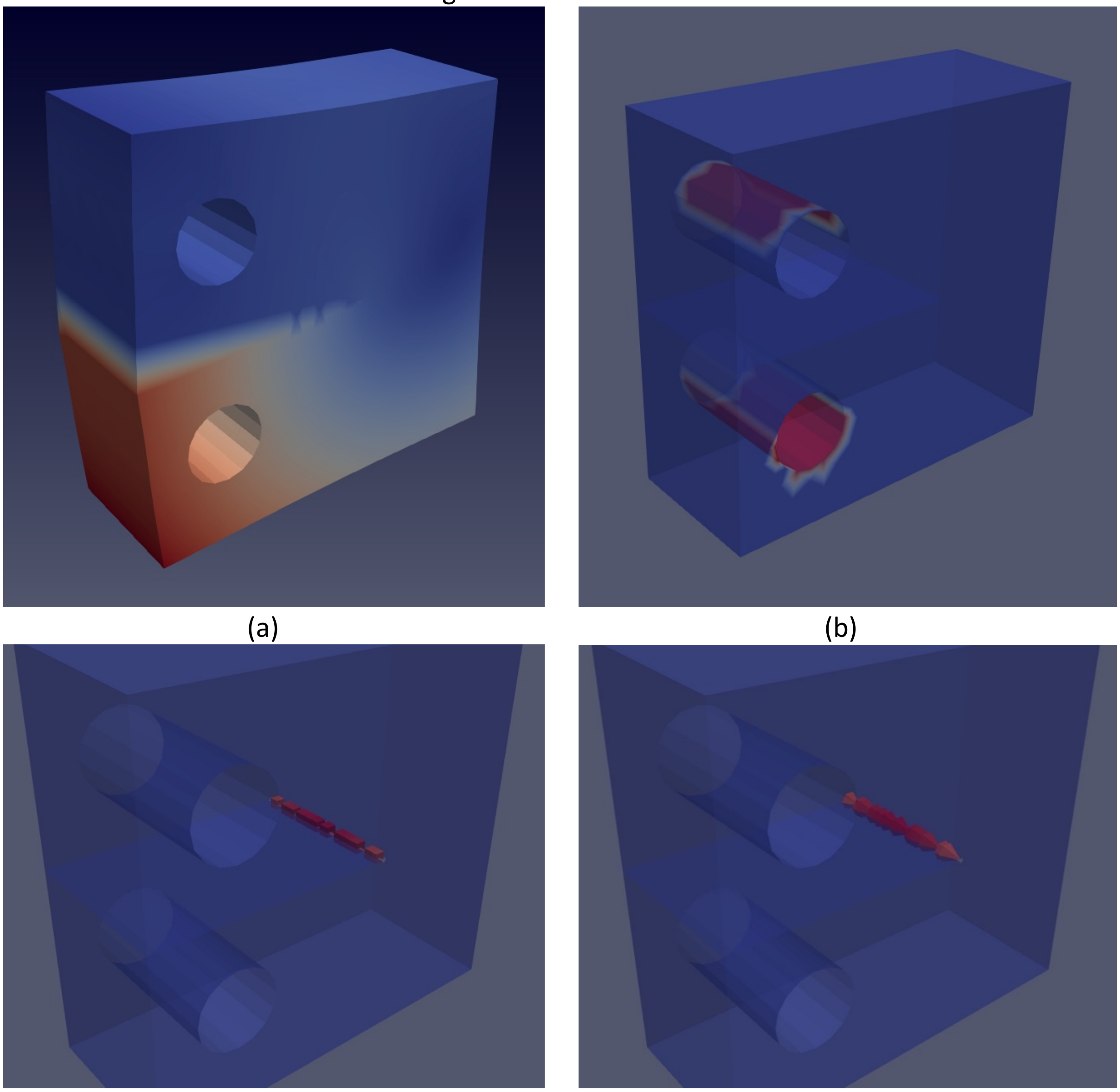
Data generated by SGBEM [1] [2] are ASCII files that include the discretization (coordinates of nodes and element connectivity) and the analysis results (displacements and tractions at nodes on boundary, crack opening at nodes on crack, stress intensity factors at nodes along crack front). Based on the data, we generate legacy VTK file format [5] so that we can use Paraview [4] to visualize the data. We used Fortran 90 to write a small code to read the ASCII files and generate VTK files. Figure 1 describes the modelling of a compact tension specimen (CTS) that include a plane crack. As mentioned in the Overview, the SGBEM does not discretize the interior of the structure, and the crack is modeled as a single surface (even though there are 2 separated surfaces in reality). Visualizing improvement of this special feature of crack representation will be the task 4 of this project which will be discussed at the end of this section.



(a) (b)

Figure 1: Visualization of the discretized model of compact tension specimen: (a) entire specimen; (b) Clip filter for interior view

Other vector fields of crack model such as deformed shape (i.e. displacement vector), surface load (i.e. traction vector), and stress intensity factors (vector of {K_I, K_{II}, K_{III}}) at nodes along crack front are also visualized as shown in Figure 2.



(a) (b) (c) (d)

Figure 2: Visualization of mechanical properties of crack model:
(a) Deformed shape with color displaying magnitude of displacement vector; (b) Surface load with color displaying magnitude of traction vector; (c) Stress intensity factors along crack front using box glyphs; (d) Stress intensity factors along crack front using cone glyphs

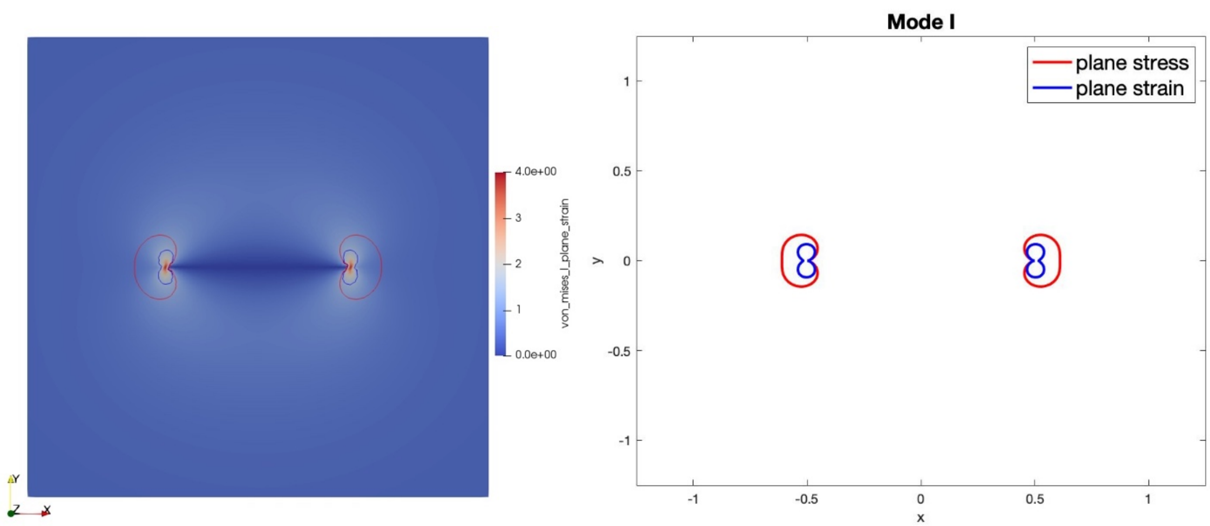
4.2. Visualization of plastic zones

Linear elastic fracture mechanics predicts infinite stresses at the crack front. However, in real material, stresses at crack front are finite due to the inelastic material deformation (such as plasticity in metals) in regions closed to crack front [6]. The elastic stress analysis becomes increasingly inaccurate as the plastic region at the crack tip grows. Therefore, an accurate prediction of the plastic zone is essential to crack modeling.

In this project, we want to visualize the size and shape of plastic zones in front of crack tip using isosurfaces. According to Von-Mises criterion [6], the effective stress is defined as

$$\sigma_e = \frac{1}{\sqrt{2}}\left((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\right)^{1/2}. \quad (1)$$

Plastic yielding occurs when $\sigma_e = \sigma_{YS}$, where σ_{YS} is the uniaxial yield strength. The plastic zone is then visualized by computing the isosurface of the scalar value σ_e. To do that, we compute σ_e according (1), then using Isocontour filter of Paraview to plot isosurfaces of σ_e. Figure 3 shows an example of plastic zone for the problem of crack in infinite domain subjected to far-field normal stress σ_{22}. For comparison, we also plot the analytic solution [6] in Figure 3. It is seen that our method of using isosurface agrees well with the analytic solution.



(a) (b)

Figure 3: Visualization of plastic zone in front of crack tip for a crack in infinite domain subjected to uniform far-field stress σ_{22} (mode I crack): (a) Using Paraview by generating isosurfaces of effective stress σ_e computed from stress tensor; (b) Analytic solution computed from asymptotic stress field [6]

Figure 4 shows another example of plastic zones for both mode I (opening mode) and mode II (shearing mode) of a crack in infinite domain subjected to far-field normal stress σ_{22} and σ_{12} respectively. By setting different isovalues for effective stress σ_e, we can see a spectrum of plastic deformation around crack.

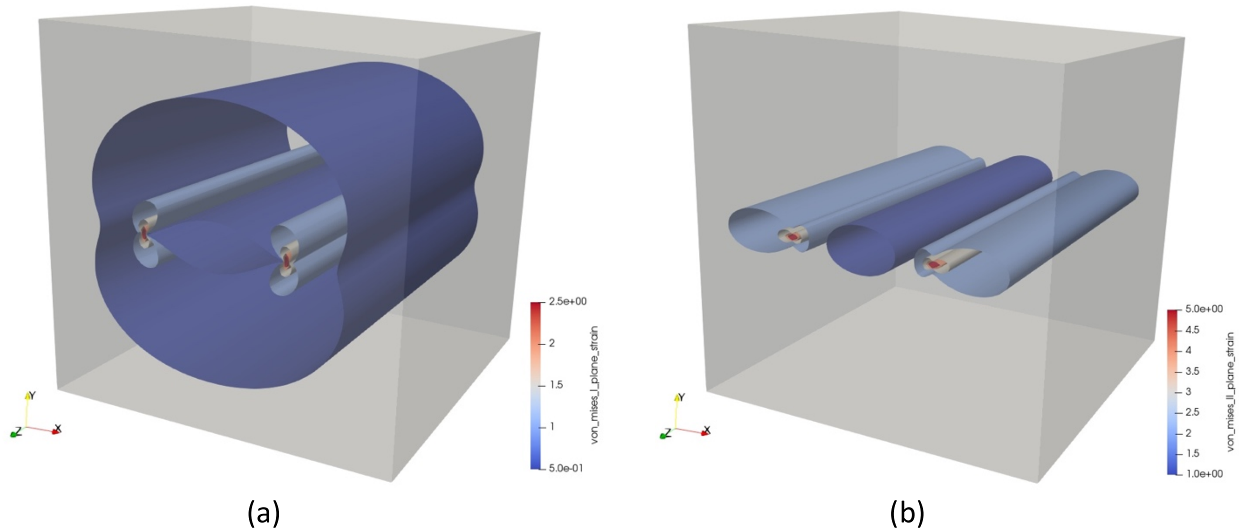


Figure 4: Isosurfaces of effective stress σ_e to visualize the spectrum of plastic deformation: (a) crack subjected to far-field normal stress (mode I), plane strain condition; (b): crack subjected to far-field shear stress (mode II)

4.3. Visualization of predicted direction of crack growth

Under some critical loading, crack starts propagating throughout structures. The growth direction depends on how the stresses in front of crack tip distributed. One of the most common criteria used to predict growth direction is the maximum tangential stress (MTS) [7] which assumes that crack will grow in the direction θ^* that gives maximum tangential stress $\sigma_{\theta\theta}$ as shown schematically in Figure 5 below.

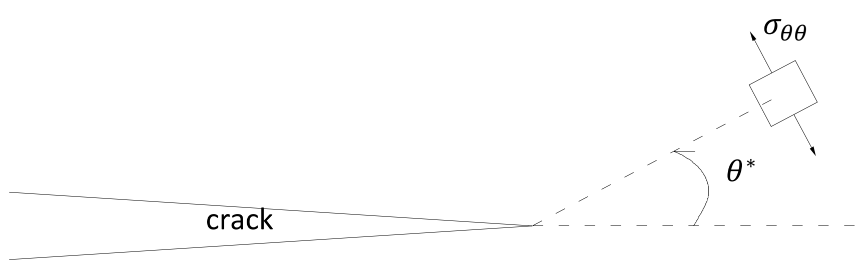


Figure 5: Crack growth direction according to MST criterion [7]

The analytic solution to growth direction θ^* is given in [7] and [8] as

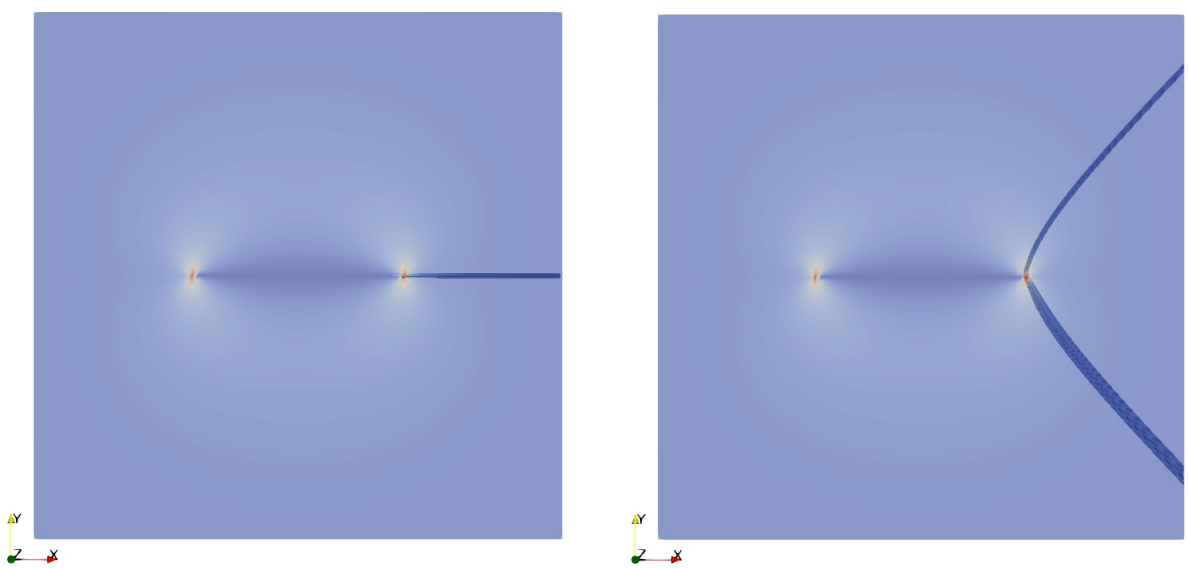
$$\theta^* = 2\tan^{-1}\left(\frac{-2K_{II}}{K_I + \sqrt{K_I^2 + 8K_{II}^2}}\right), \tag{2}$$

where K_I and K_{II} are mode-I and mode-II stress intensity factors respectively. It is clear that for pure mode-I problem (i.e. $K_{II} = 0$) the growth angle is $\theta^* = 0$, and for pure mode-II problem (i.e. $K_I = 0$) the growth angle is $\theta^* \approx 71^o$.

Using MST criterion, we propose a method to visualize the predicted direction of crack growth using streamlines as follows:

- Given stress tensor σ_{ij} at every data point, compute eigenvector n_θ that corresponds to the largest eigenvalue $\sigma_{\theta\theta}$
- Determine the direct $n_r \perp n_\theta$
- Generate streamlines that receive n_r as tangent vectors. The streamline going through crack dictate the predicted direction of crack growth.

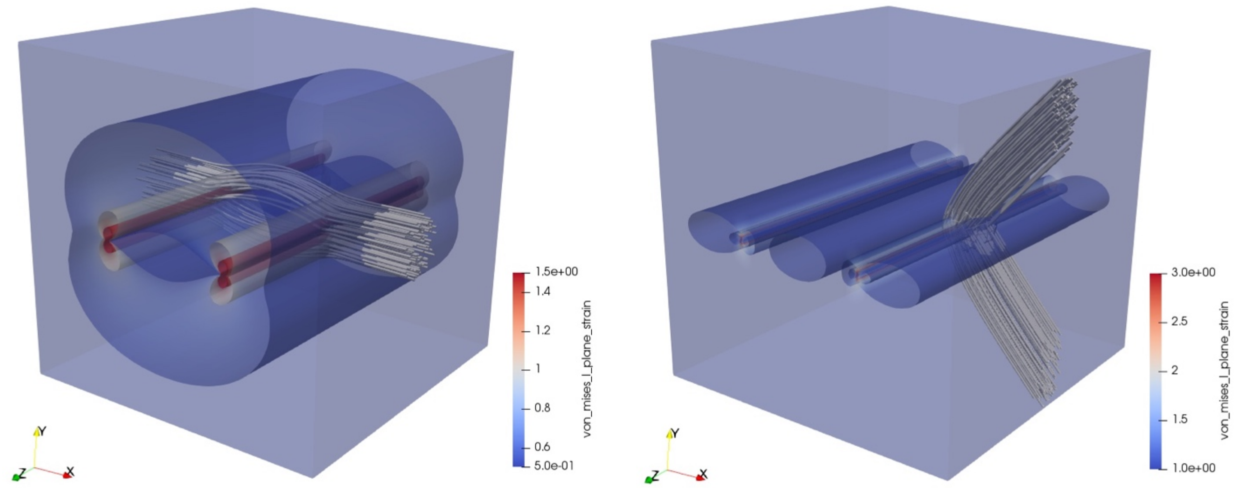
To verify our proposed method, we visualize the growth direction in pure mode-I and mode-II problems and compare with the analytic solution (2). Figure 6 shows the results of visualization of predicted direction of crack growth. We can see that for mode-I crack (Figure 5a), the streamline going through crack tip has angle $\theta^* = 0$, and for mode-II crack (Figure 5b), the streamline going through crack tip has angle $\theta^* \approx 71^o$. Note that for mode-II crack, there are 2 values for θ^*: one for maximum (positive value) $\sigma_{\theta\theta}$ and one for minimum (negative value) $\sigma_{\theta\theta}$. That explains why we see the streamline as shown in Figure 5b.



(a) (b)

Figure 5: Visualization of predicted direction of crack growth using MTS criterion [7]:
(a) pure mode-I crack; (b) pure mode-II crack

By combining two visualization methods described above, we can get an overall view about plastic deformation and predicted direction of crack growth. Figure 6 presents two examples for this combination: one for mode-I crack and the other for mode-II crack.

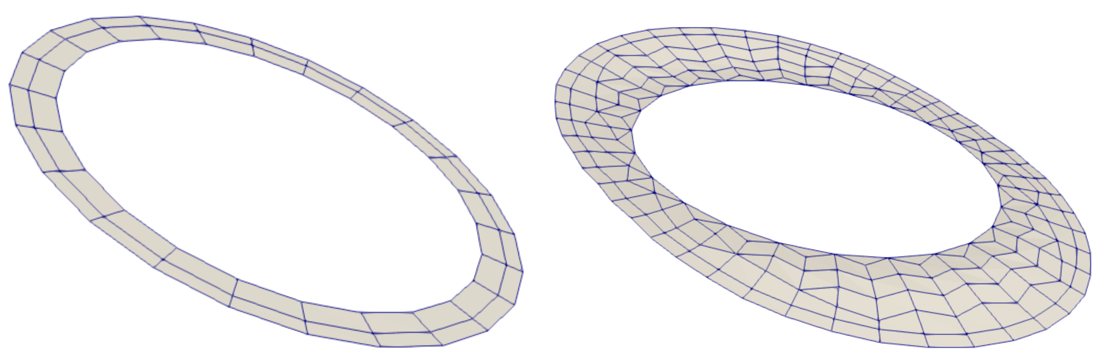


(a) (b)

Figure 6: Combination of visualization of plastic zones and predicted direction of crack growth: (a) pure mode-I crack problem; (b) pure mode-II crack problem

4.4. Visualization of crack propagation

To be able to visualize the variation in shape and magnitude of a crack with time can be a useful tool in understanding the underlying dynamics. We wrote a python script to read the data from the ASCII files generated from SBGEM and generate a stream of VTK-format files- one for file each stage of crack growth. The VTK files when visualized in sequence show growth of the crack with time.



(a) (b)

Figure 7: Crack visualization as per data generated by SBGEM:
(a) Initial shape of crack; (b) Shape of crack at step 50

As shown in Figure 7, the SGBEM data does not discretize the interior of the structure, and the crack is modeled as a single surface (even though there are 2 separated surfaces in reality). In order to get a more realistic visualization of the crack it is necessary to represent the crack as an opening with the upper plane and the lower plane calculated from the available planar data. We applied simple concepts of 3D computer graphics and vector calculus to achieve this. For this we calculated the normal at each vertex of the grid using vector product as shown in Figure 8 below.

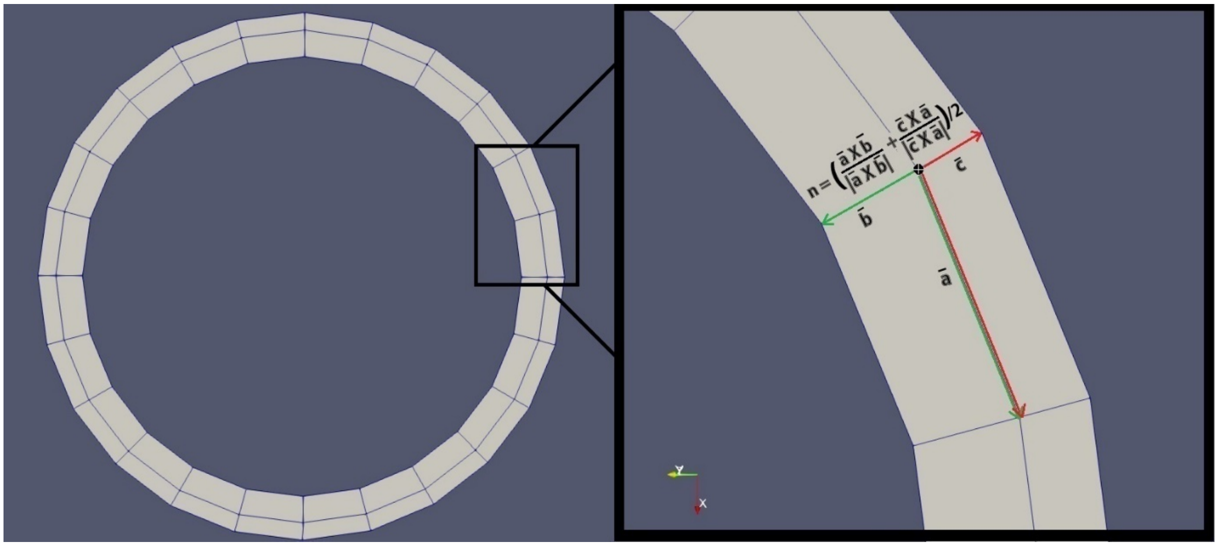
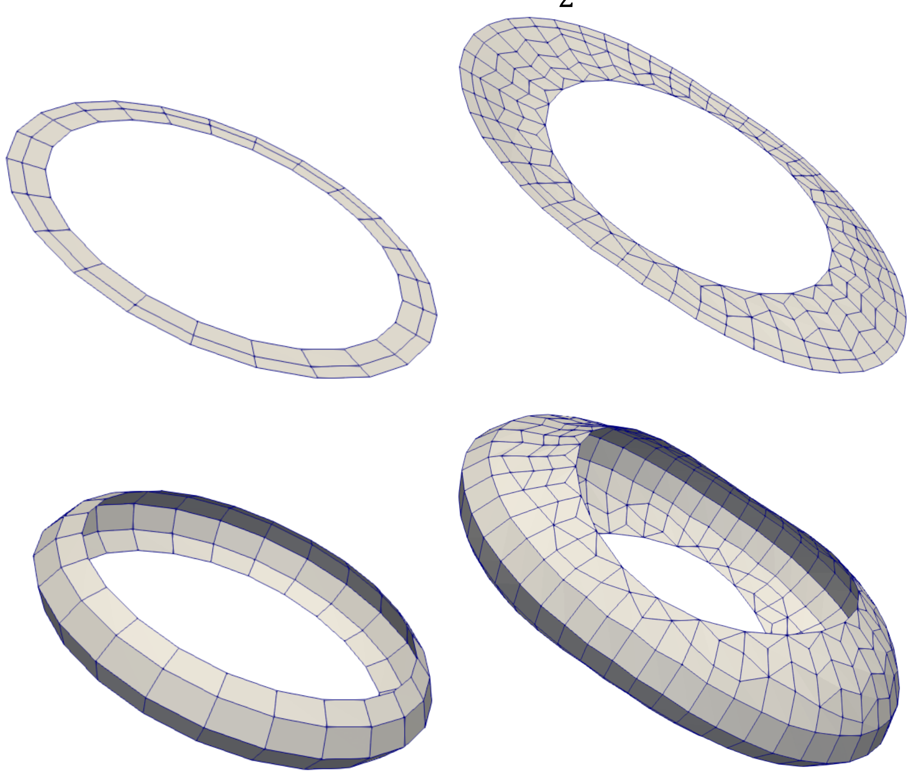


Figure 8: Vertex normal as average of cross product of vectors with neighboring vertices

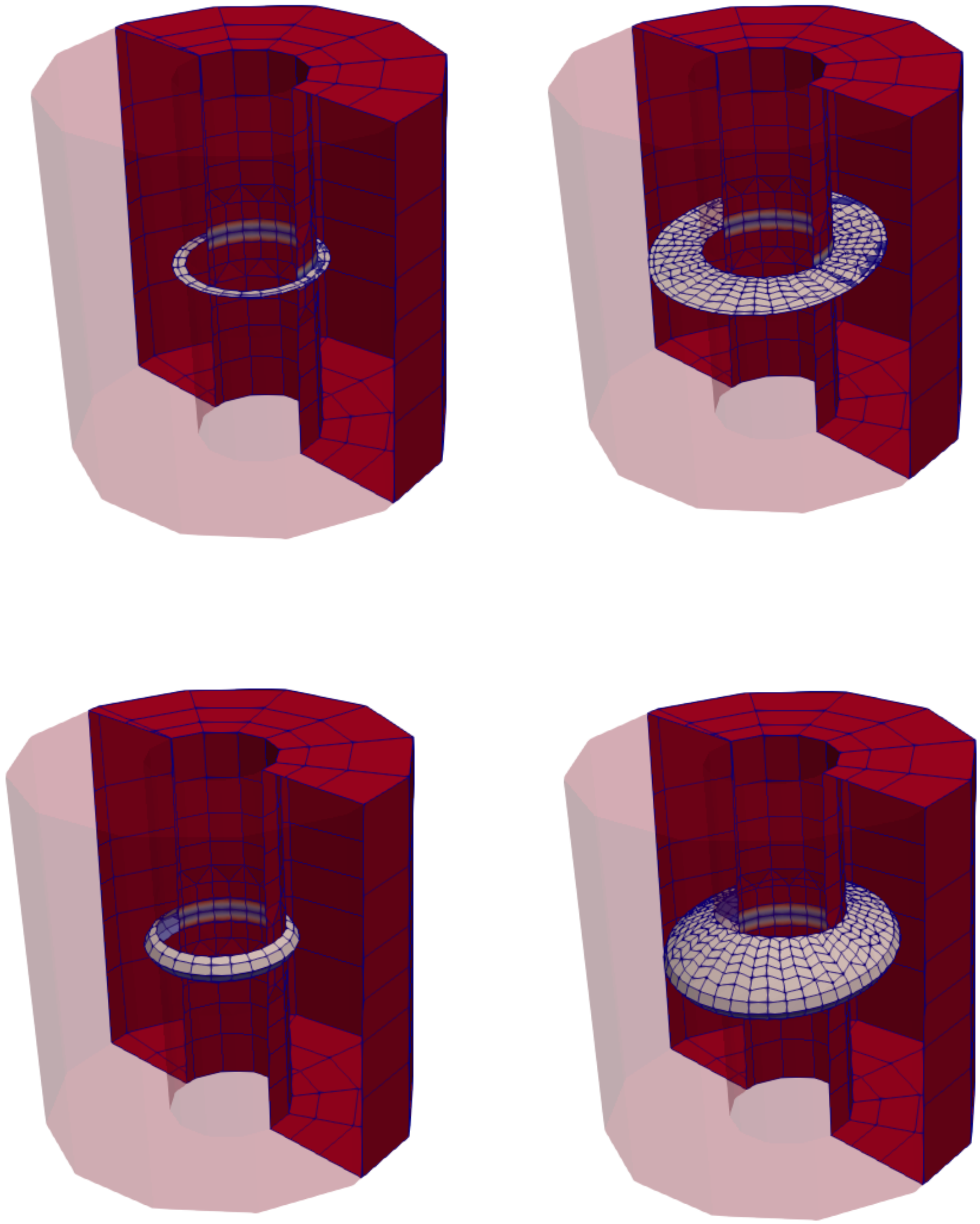
The upper crack plane position v_{up} and lower crack plane position v_{down} for any vertex v can be determined in terms of the normal n and crack opening's magnitude s as:

$$v_{up} = v + n(\frac{s}{2})$$
$$v_{down} = v - n(\frac{s}{2})$$



(a) (b)

Figure 9: Planar view of crack and corresponding 3D view:
(a) Crack at step 1; (b) Crack at step 50



(a) (b)

Figure 10: Planar view of crack in cylinder and corresponding 3D view
(a) Crack at step 1; (b) Crack at step 98

One aspect missing from our crack representation till now is color-mapping. We can improve perception by using appropriate colors to represent crack magnitude at a vertex. Of course, ParaView can do the mapping for us but we felt it would be a good to have a pre-calculated colormap specific to our data set included in the VTK files as a look-up table. Therefore, we configured our python script to do the necessary interpolation at vertices and scale our crack magnitude range to fit into a 1024 cool to warm color map. This improves the visualization quality drastically.

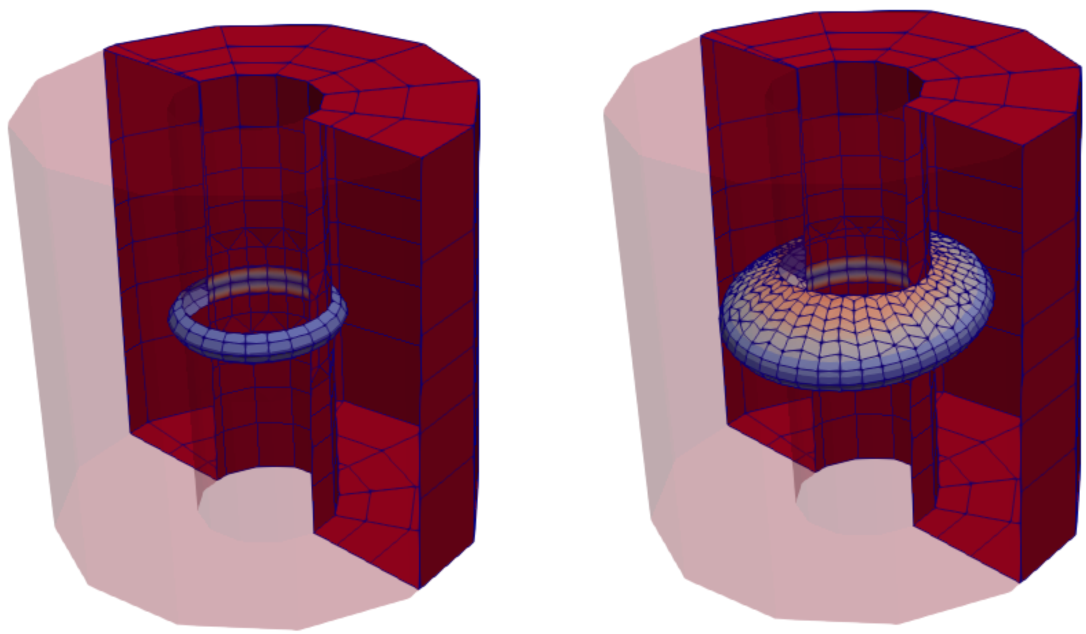


Figure 11: Planar view of crack in cylinder and corresponding 3D view with color mapping
(a) Crack at step 1; (b) Crack at step 98

In the final step we tried to check the correctness of this 3D crack model calculated based on our approach. We used the ruler filter on ParaView to give us the distance between two corresponding points at step 98- $Point1$ picked from the 3D upper plane and $Point\ 2$ picked from the 3D lower plane of the data generated. This distance should correspond to the crack magnitude for that vertex- $Point\ 0$ at step 98 in the original SBGEM data.

Point	X	Y	Z	Crack Magnitude
Plane-Point 0	0	-1.4142	0.7071	0.6182
Upper-Point 1	4.40E-05	-1.636373084	0.929131315	0.6182
Lower-Point 2	-4.40E-05	-1.192026916	0.485068685	0.6182

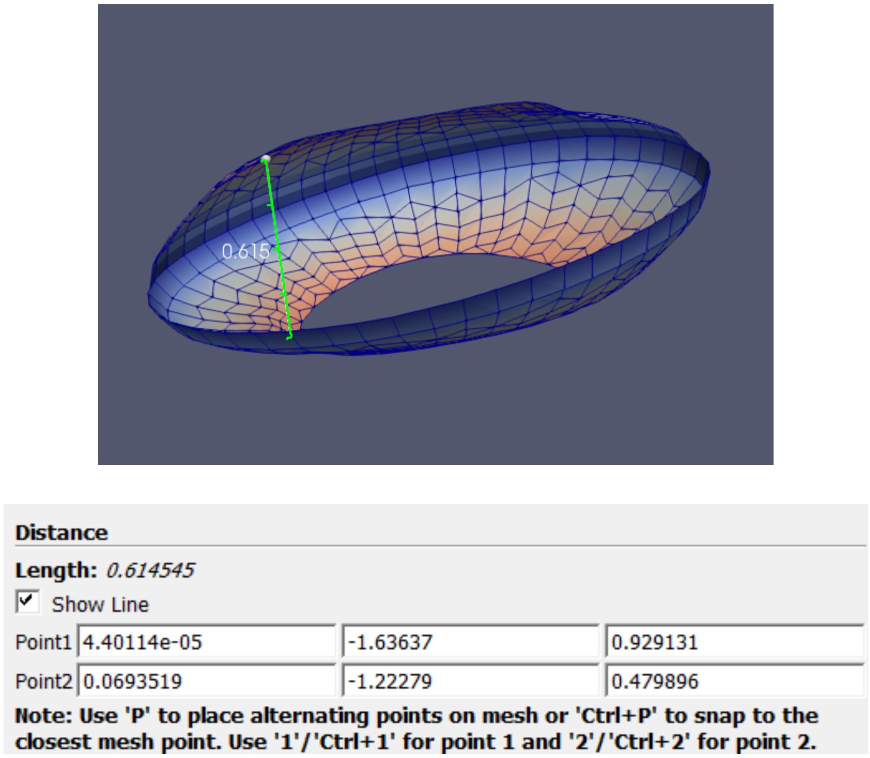


Figure 12: Distance between corresponding upper and lower-plane points i.e. crack opening.

The distance measured by the ruler in ParaView deviates from the crack data value by just 0.4%. Correctness of our approach is verified.

5. Learning outcomes

By implementing this project, we obtained the following learning outcomes.

- Understanding the basic theory of linear elastic fracture mechanics: stress concentration in front of crack tip, stress intensity factors, plastic zones, crack growth direction, crack opening and propagation/development of crack in structures
- Knowing how to generate VTK files based on the data from other sources, so that data can be visualized using available tools in Paraview.
- Knowing how to apply visualization techniques that we learned in class (isosurfaces of scalar fields, streamlines of vector fields, generate colormap for point data to improve perception) to a practical problem in engineering.
- Knowing how to write small scripts (in Fortran and Python) to implement the above-mentioned ideas

6. Evaluation

We believe that the project is successful. We obtained the two main objectives that were set in in the project proposal:

- Visualization of fracture properties of a crack
- Visualization of crack propagation

We also obtain some of the additional objectives:

- Visualize the deformed shape of the structure based on displacement vector at nodes on boundary.
- Generate colormap for point data to improve the perception.

7. Additional comments

Although what have been done in the project is far away from reaching a research project, we learned the fundamental steps of doing research: specify problem – define questions need to answers – seek methods to solve – implement – verify and validate the obtained result – evaluate the outcomes. Throughout this process, we understand more about the methods and techniques that we learned in class. Most importantly, we know how to apply what we learned in class to solve a practical problem. Last but not least, each team member learned and practiced how to work in group where discussion and helping each other are very helpful to obtain the goals.

References

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