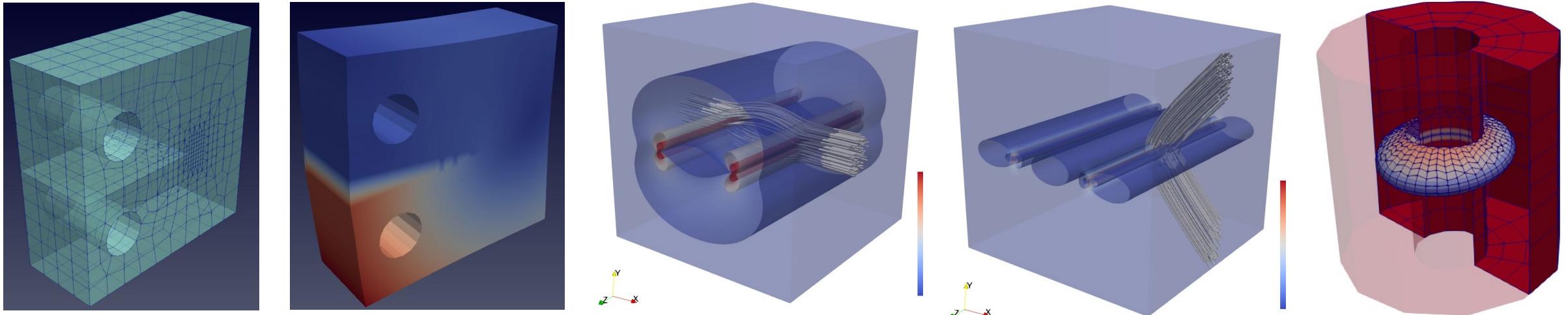


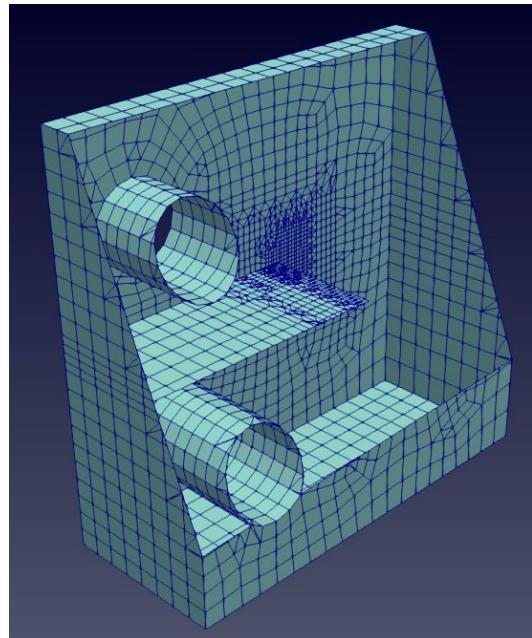
Visualization of crack propagation modeled by boundary element method

Han Tran, Rohit Singh

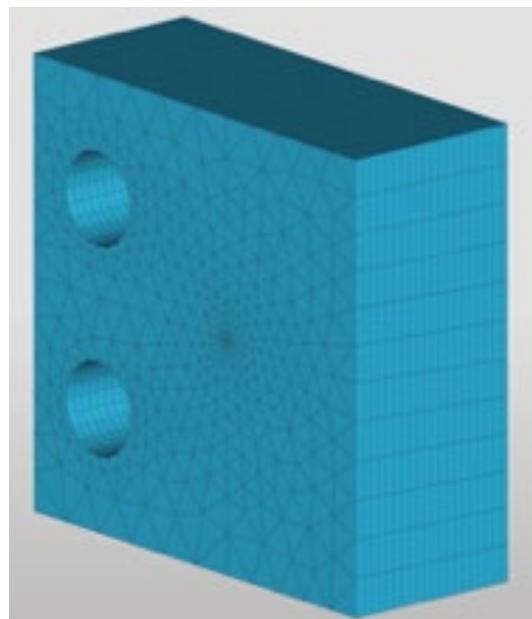


Overview

- Crack modeling is important in order to assess the integrity of structures
 - Symmetric Galerkin boundary element method (SGBEM) is powerful in crack modeling
 - There is no technique to visualize the results analyzed by SGBEM
 - Our goals:
 1. Visualize crack properties
 2. Visualize crack growth
- To enhance the effectiveness of the SGBEM for crack modeling



BEM:
1070 elements



FEM:
14,760 elements

Project description

- **Data:** synthesis data, generated in [1] [2]
- **Specific tasks:**
 1. How do we visualize the geometry of the structures and the topology of cracks modeled by SGBEM?
 2. How do we visualize the plastic zones in front of the crack front ?
 3. How do we visualize the predicted direction if an existing crack is supposed to grow ?
 4. How do we visualize the shape of crack during its propagation modeled by SGBEM ?

[1] S. Li, M.E. Mear, L. Xiao, “Symmetric weak-form integral equation methods for three-dimensional fracture analysis”, Computer Methods in Applied Mechanics and Engineering 151: 435-459 (1998).

[2] Jaroon Rungamornrat, Mark E. Mear, “A weakly-singular SGBEM for analysis of cracks in 3D anisotropic media”, Computer Methods in Applied Mechanics and Engineering 197: 4319-4332 (2012).

1. Visualization of crack modeled by SGBEM

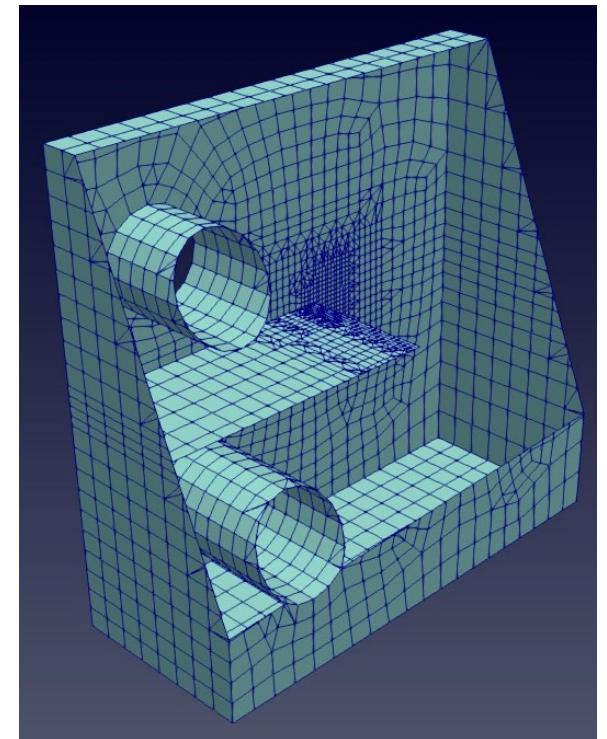
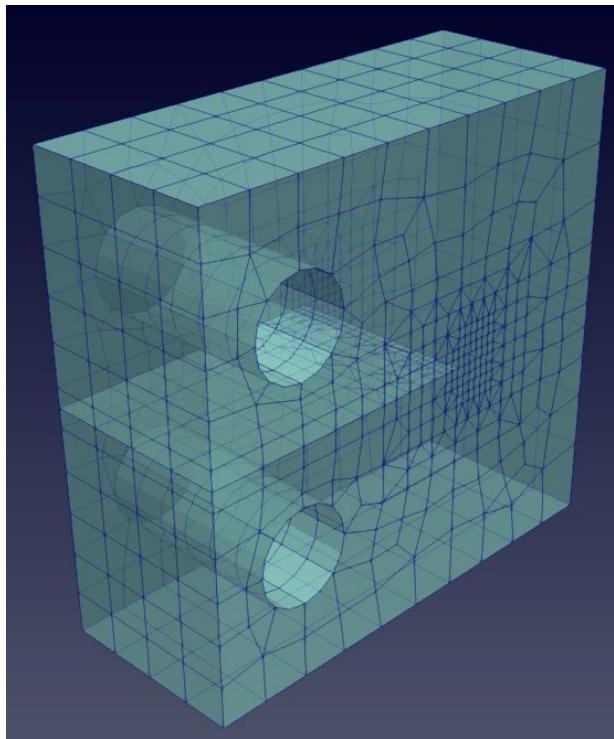
- Mesh (nodal coordinates + element connectivity)
- Displacement vector at nodes
- Traction vector at nodes
- Crack opening at nodes on crack
- Stress intensity factors (K_I , K_{II} , K_{III})



Lagacy VTK-format files



Paraview



Discretization Model

1. Visualization of crack modeled by SGBEM

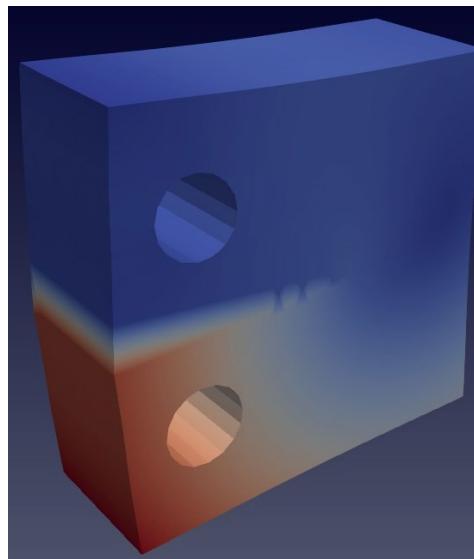
- Mesh (nodal coordinates + element connectivity)
- Displacement vector at nodes
- Traction vector at nodes
- Crack opening at nodes on crack
- Stress intensity factors (K_I , K_{II} , K_{III})



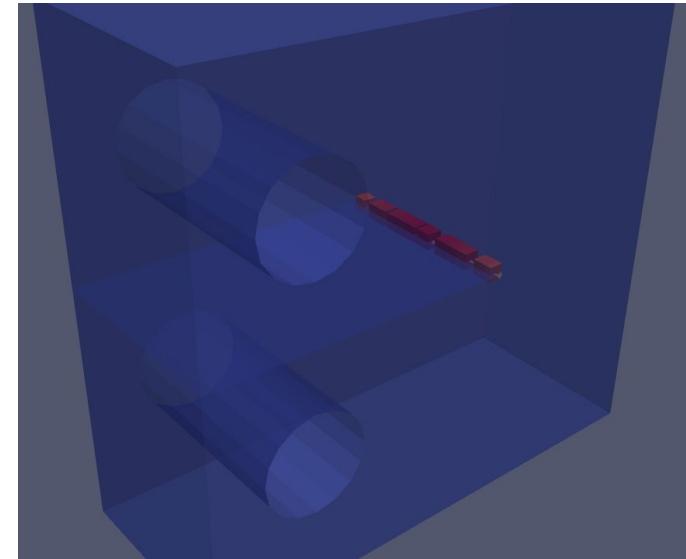
Lagacy VTK-format files



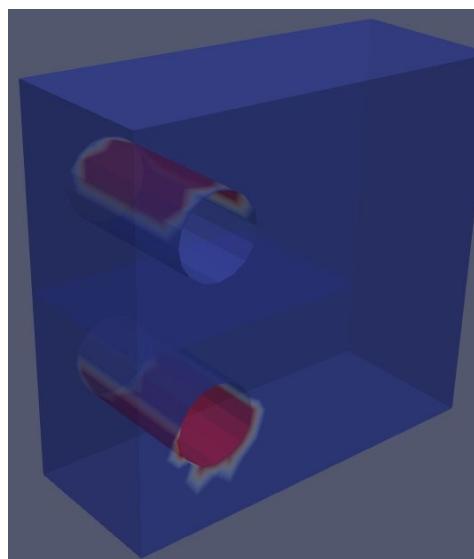
Paraview



Displacement & Traction



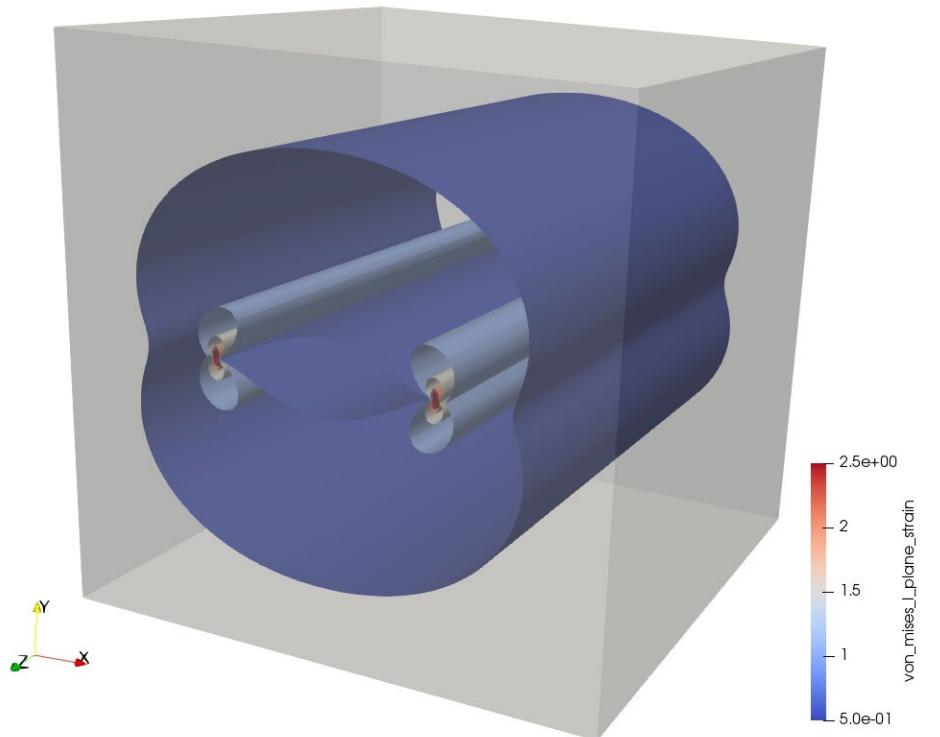
Stress intensity factors



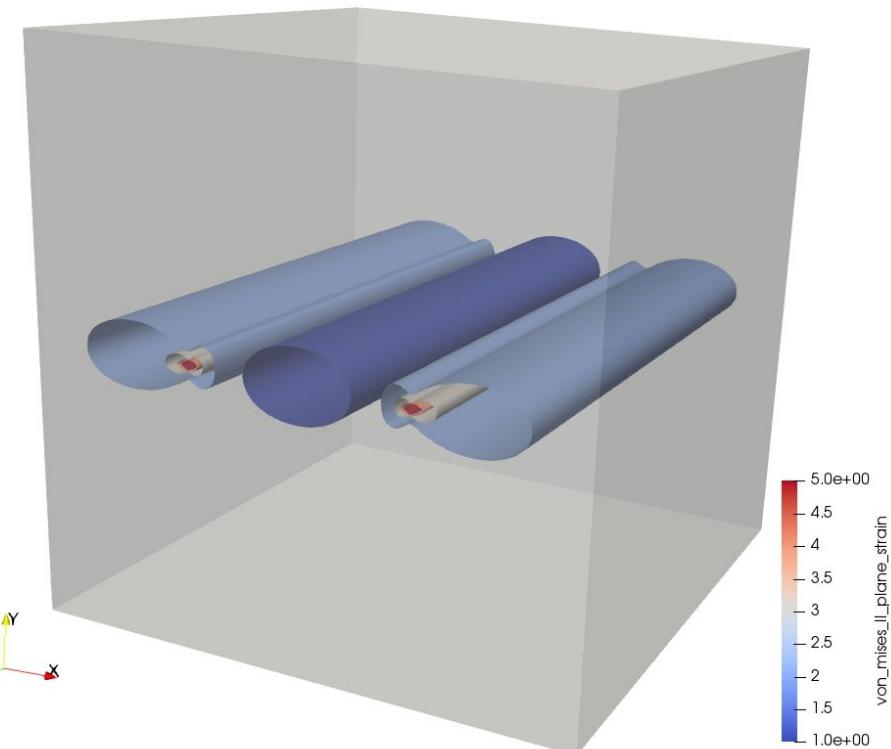
2. Visualization of plastic zones

$$\sigma_e = \frac{1}{\sqrt{2}} \left((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) \right)^{1/2}$$

$\sigma_e = \sigma_{YS}$ Inelastic deformation starts, σ_{YS} is uniaxial yield strength



Plastic zones: crack subjected to far-field normal stress



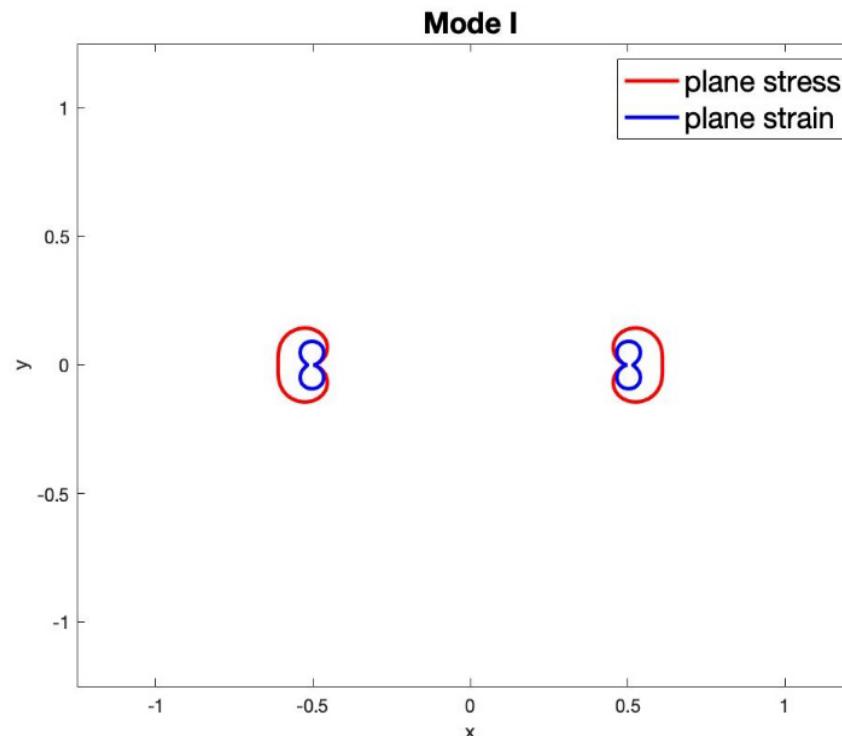
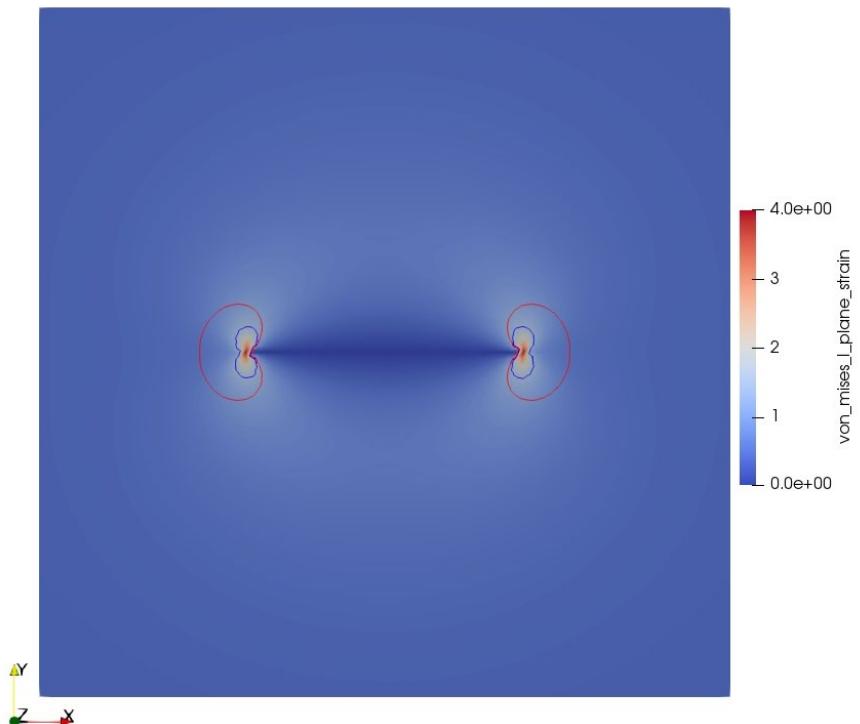
Plastic zones: crack subjected to far-field shear stress

2. Visualization of plastic zones

$$\sigma_e = \frac{1}{\sqrt{2}} \left((\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) \right)^{1/2}$$

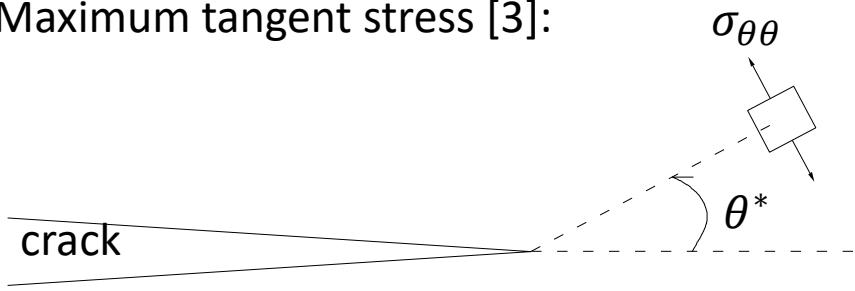
$\sigma_e = \sigma_{YS}$ → Inelastic deformation starts, σ_{YS} is uniaxial yield strength

Method: compute σ_e from stress tensor, generate isosurfaces



3. Visualization of predicted direction of crack growth

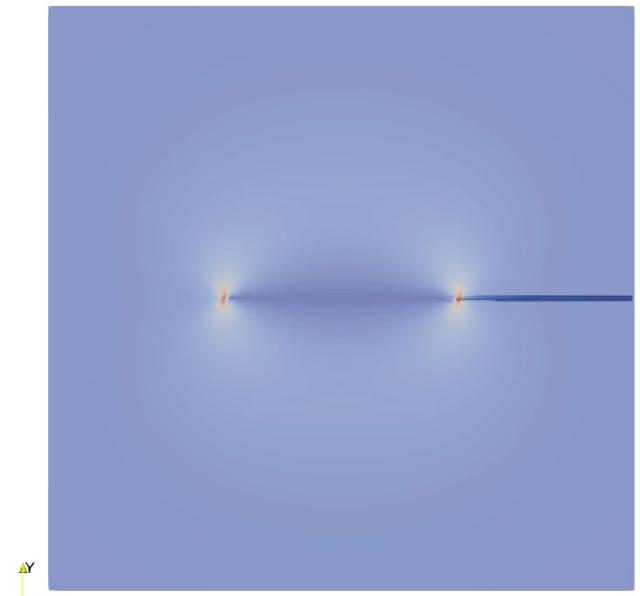
Maximum tangent stress [3]:



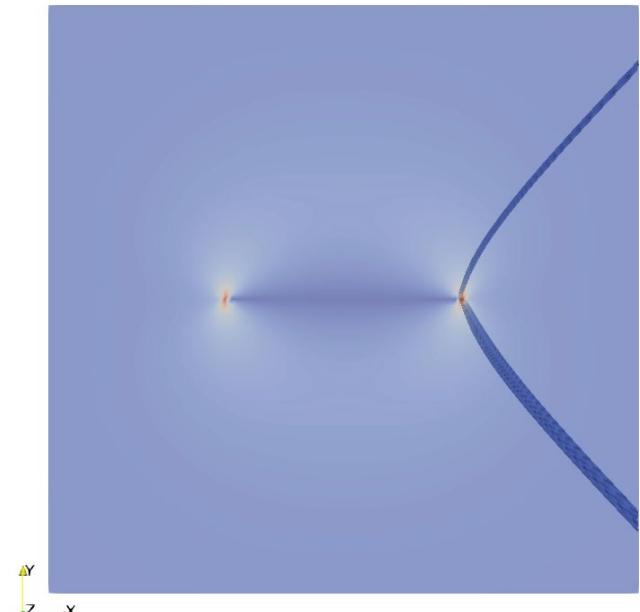
Method:

- Given stress tensor σ_{ij} at every data point, compute eigenvector n_θ that corresponds to the largest eigenvalue $\sigma_{\theta\theta}$
- Determine the direct $n_r \perp n_\theta$
- Generate streamlines that receive n_r as tangent vectors. The streamline going through crack dictate the predicted direction of crack growth.

Mode-I crack;
Analytic solution
 $\theta^* = 0^\circ$

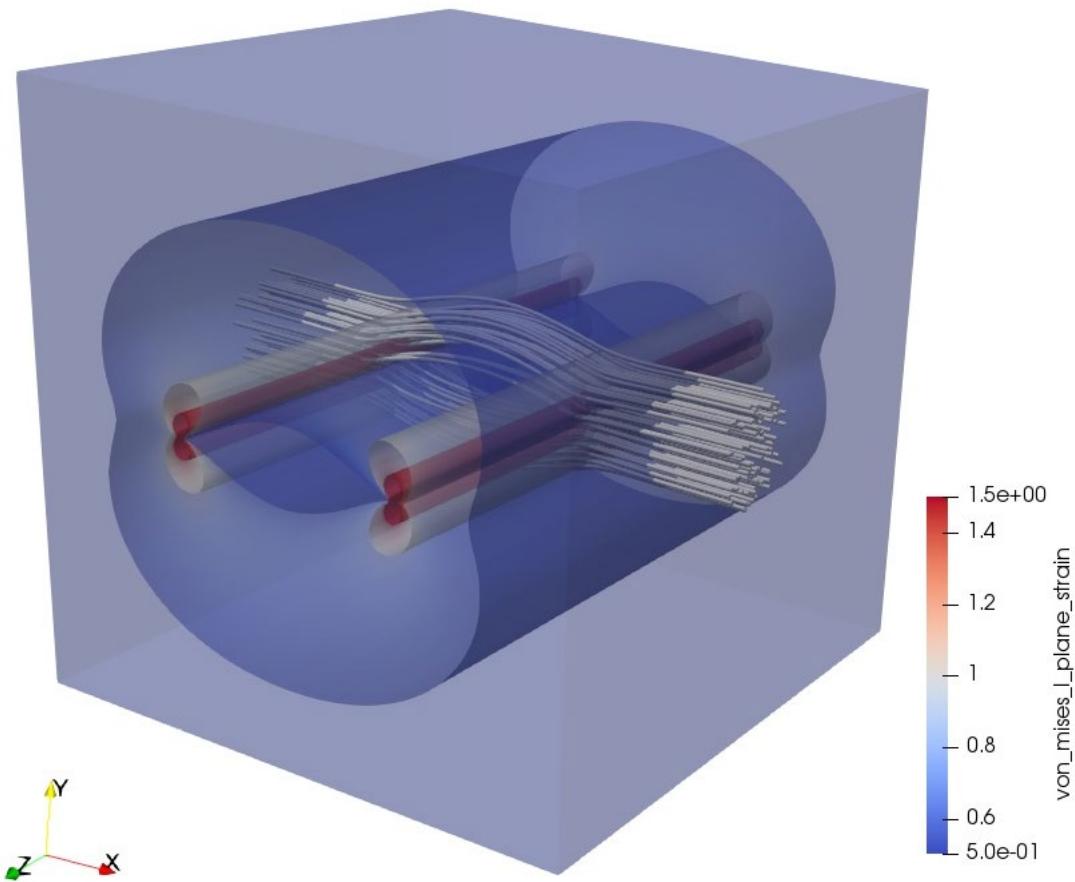


Mode-II crack;
Analytic solution
 $\theta^* = 71^\circ$

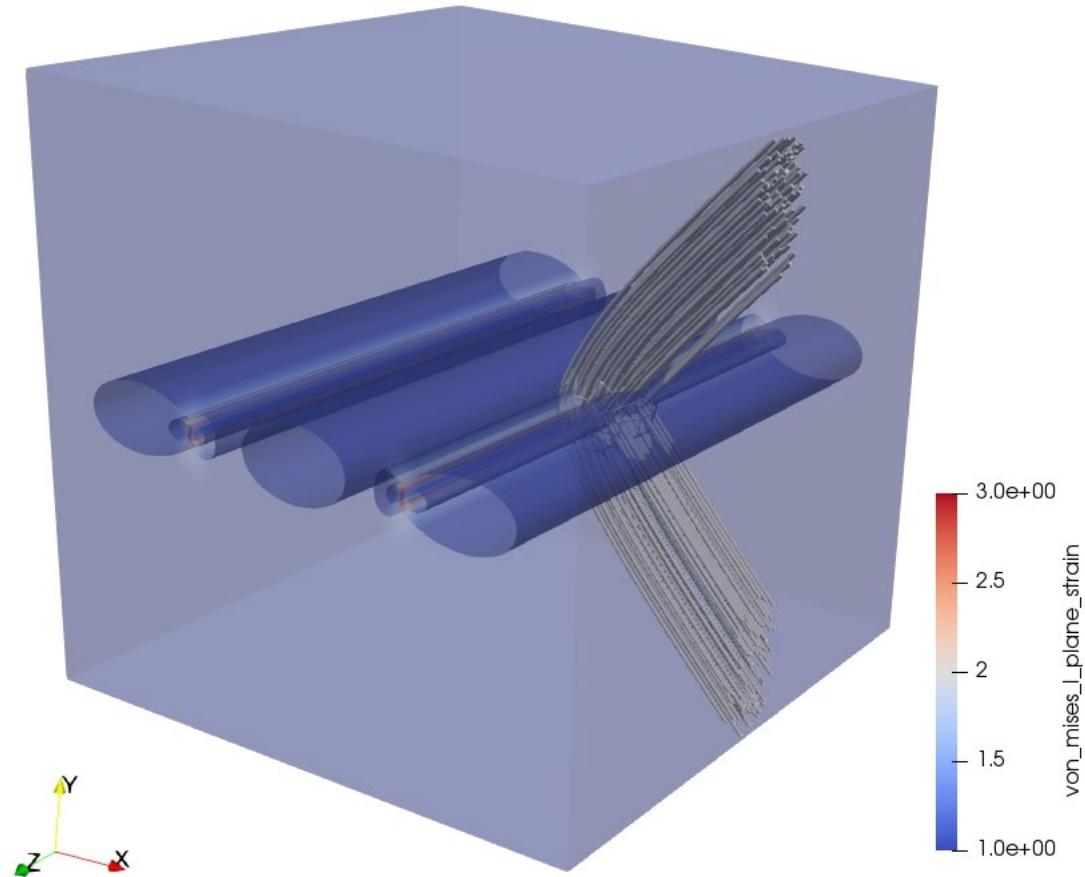


[3] F. Erdogan and G. C. Sih, "On the Crack Extension in Plates Under Plane Loading and Transverse Shear", J. Basic Eng 85(4): 519-525 (1963)

Combine: plastic zones and crack growth direction

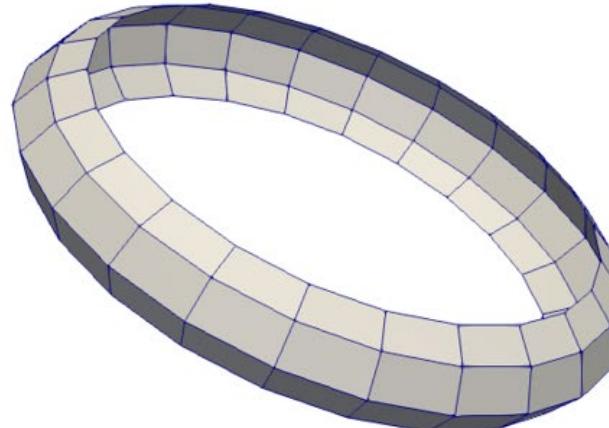
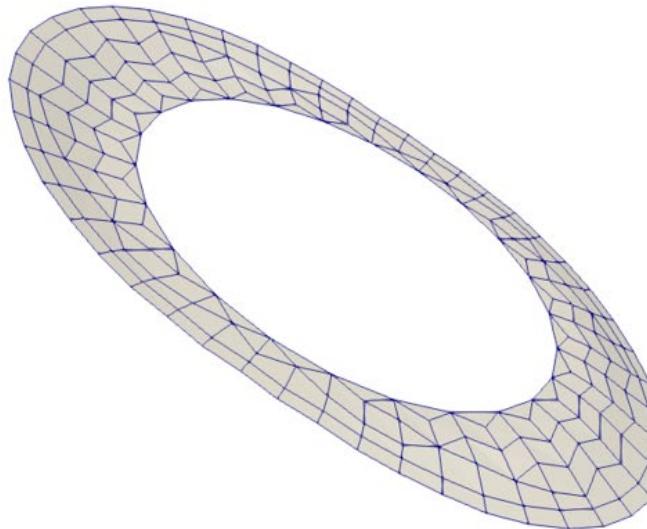
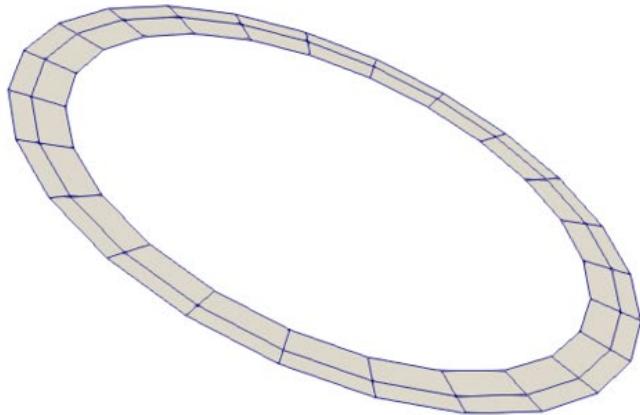


Crack subjected to far-field normal stress



Crack subjected to far-field shear stress

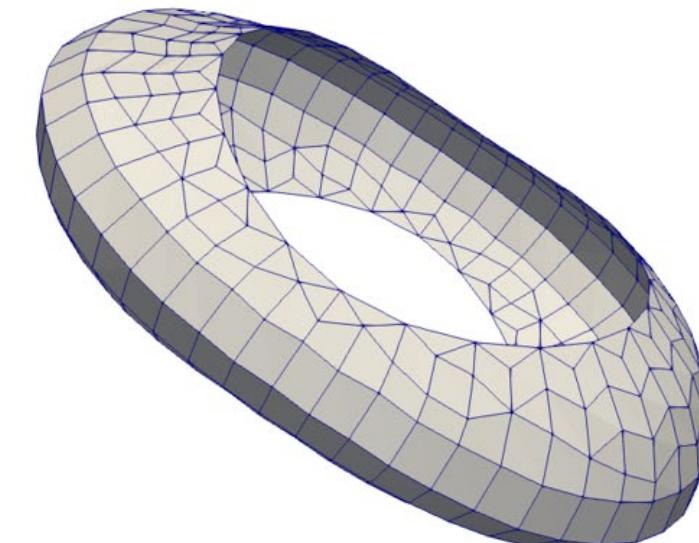
4. Visualization of crack propagation: planar to actual



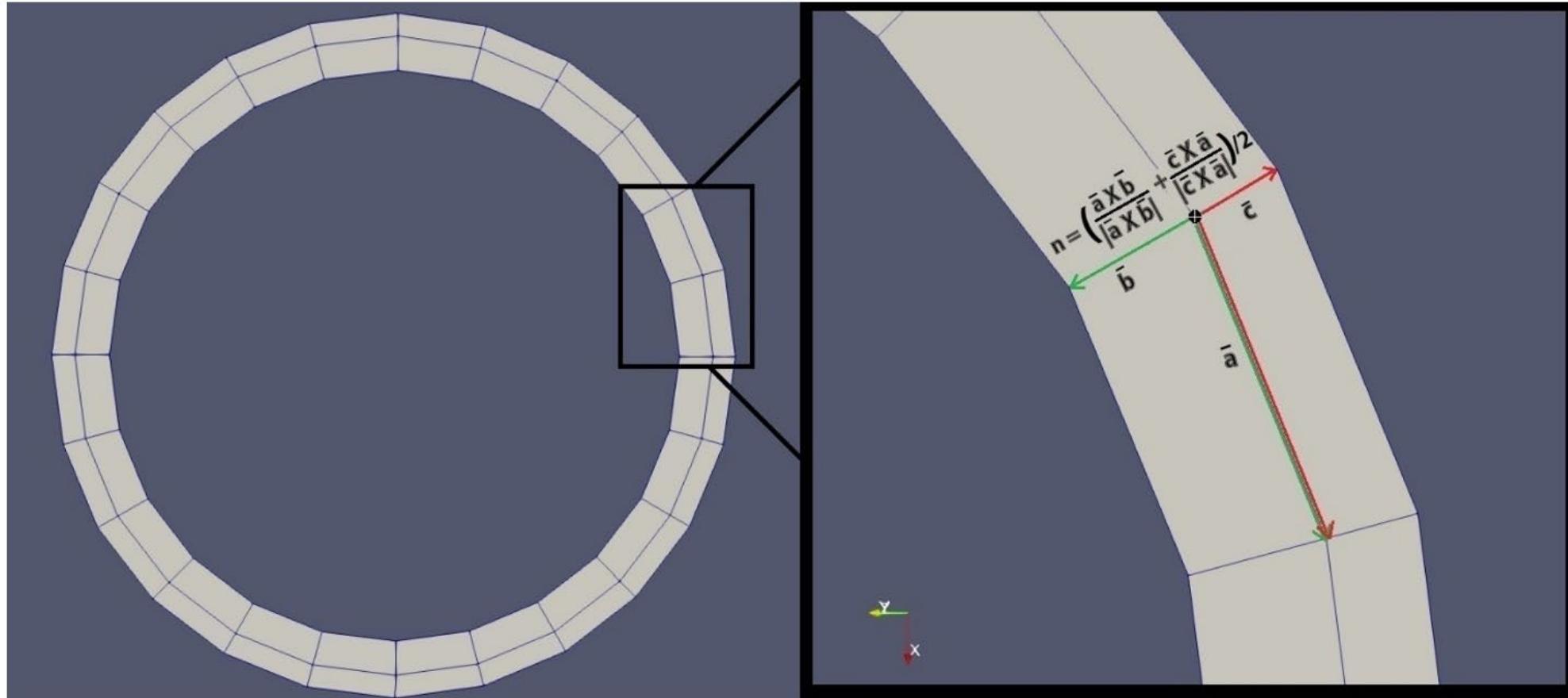
The upper crack plane position v_{up} and lower crack plane position v_{down} for any vertex v can be expressed in terms of the vertex normal n and crack opening's magnitude s as:

$$v_{up} = v + (n \cdot s)/2$$

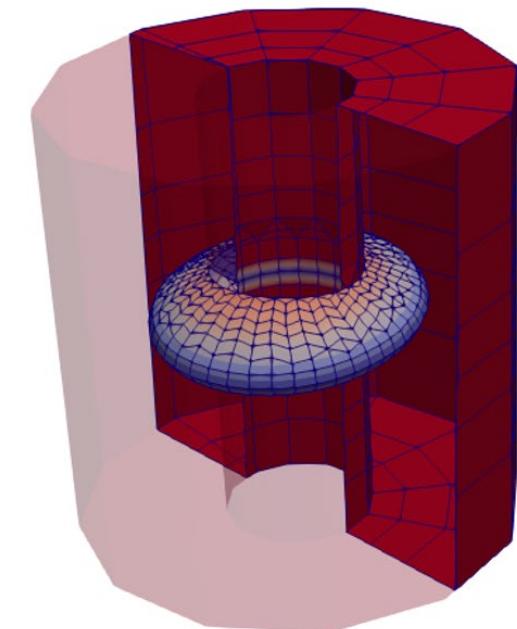
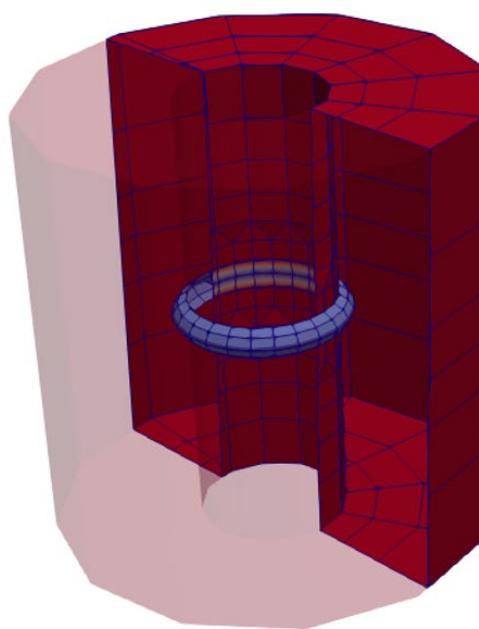
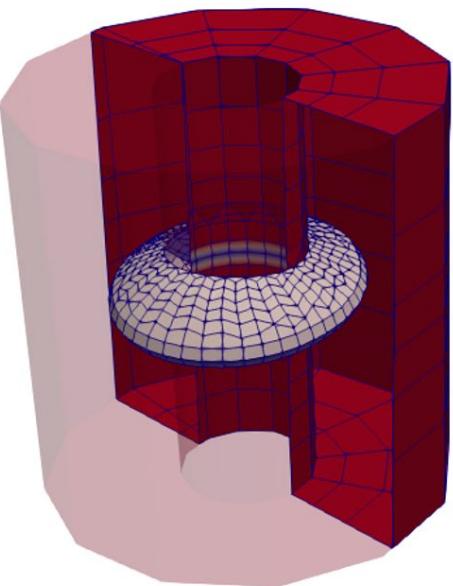
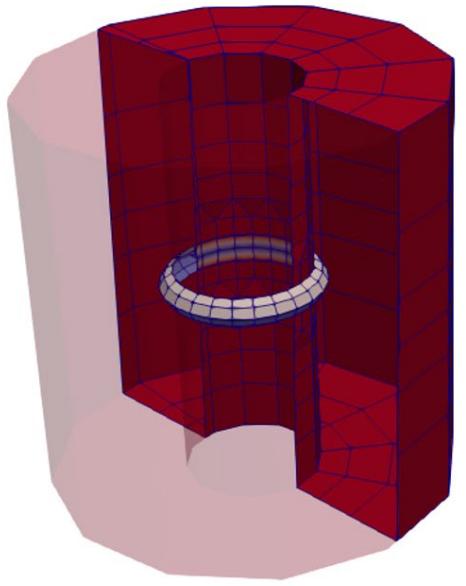
$$v_{down} = v - (n \cdot s)/2$$



4. Calculation of vertex normals on crack plane



4. Mapping color to crack opening magnitude



4. Crack growth up to 99 stages

