Exercise 5.1.1 : Design context free grammars for the following languages.

Ⓐ The set $\{0^n 1^n \mid n \geq 1\}$, that is the set of all strings of one or more 0's followed by an equal number of 1's.

$L = \{01, 0011, 000111, \ldots\}$

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$S \to 0S1 \mid 01$

$G = (V, T, P, S)$

$V = \{S\}$

$T = \{0, 1\}$

$P = \quad S \to 0S1 \mid 01$

$S \to S$

Ⓑ The set $\{a^i b^j c^k \mid i \neq j \text{ or } j \neq k\}$, that is, the set of strings of a's followed by b's followed by c's, such that there are either a different number of a's and b's or a different number of b's and c's, or both.

$S \to AB \mid CD$

$A \to aA \mid \varepsilon$

$B \to bBc \mid E \mid cD$

$C \to aCb \mid E \mid aA$

$D \to cD \mid \varepsilon$

$E \to bE \mid b$

The grammar works as

- A generates zero or more a's
- D generates zero or more c's
- E generates one or more b's
- B generates an equal number of b's and c's, then produces either one or more b's (via E) or one or more c's (via CD)

That is, B generates strings in $b^* c^*$ with an unequal number of b's and c's

- Similarly, C generates unequal number of a's then b's
- Thus, AB generates strings in $a^* b^* c^*$ with an unequal number of b's and c's while CD generates strings in $a^* b^* c^*$ with an unequal number of a's and b's

Context free grammar

$G = (V, T, P, S)$

$V = \{S, A, B, C, D, E\}$

$T = \{a, b, c\}$

P = Productions as listed above

$S = \{S\}$

(c) The set of all strings of a's and b's that are not of the form ww, that is not equal to any string repeated.

$G = (V, T, P, S)$

$V = \{S, A, B\}$

$T = \{a, b\}$

$P = \quad S \rightarrow AB \mid BA \mid A \mid B$

$A \rightarrow aAa \mid aAb \mid bAa \mid bAb \mid a$

$B \rightarrow aBa \mid aBb \mid bBa \mid bBb \mid b$

$S \rightarrow \{S\}$

(d) The set of all strings with twice as many 0's as 1's

$L = \{\varepsilon, 0011, 001111, 00011111, \ldots\}$

$S \rightarrow SS \mid 00S1 \mid 1S00 \mid 0S1S0 \mid \varepsilon$

context free grammar

$G = (V, T, P, S)$

$V = \{S\}$

$T = \{0, 1\}$

Productions (P) $= S \rightarrow SS \mid 00S1 \mid 1S00 \mid 0S1S0 \mid \varepsilon$

$\{S \rightarrow S1S0S0S \mid S0S1S0S \mid S0S0S1S \mid \varepsilon\}$

$S = \{S\}$

Ex:- 5.1-2

The following grammar generates the language of regular expression $0^* 1 (0+1)^*$:

$S \rightarrow A1B \rightarrow 1$

$A \rightarrow \underset{2}{0A} \mid \varepsilon - 3$

$B \rightarrow \underset{4}{0B} \mid \underset{5}{1B} \mid \underset{6}{\varepsilon}$

Give leftmost and rightmost derivations of the following strings

00101

Leftmost derivation $S \xRightarrow{1} A1B \xRightarrow{2} 0A1B \xRightarrow{2} 00A1B \xRightarrow{3} 001B \xRightarrow{4} 0010B$
$\xRightarrow{5} 00101B \xRightarrow{6} 00101$

Rightmost: $S \xRightarrow{1} A1B \xRightarrow{4} A10B \xRightarrow{5} A101B \xRightarrow{6} A101 \xRightarrow{2} 0A101 \Rightarrow$
$\xRightarrow{2} 00A101 \xRightarrow{3} 00101$

(b) 1001

Leftmost : ~~B → 1B ⇒~~

$S \xrightarrow{1} A1B \xRightarrow{3} 1B \xRightarrow{4} 10B \xRightarrow{4} 100B \xRightarrow{5} 1001B \xRightarrow{6} 1001$

Rightmost : $S \xrightarrow{1} A1B \xRightarrow{4} A10B \xRightarrow{4} A100B \xRightarrow{5} A1001B \xRightarrow{6} A1001 \xRightarrow{3} 1001$

(c) 00011

Leftmost : $S \xrightarrow{1} A1B \xRightarrow{2} 0A1B \xrightarrow{2} 00A1B \xRightarrow{2} 000A1B \xRightarrow{3} 0001B \Rightarrow$
$\xRightarrow{5} 00011B \xRightarrow{6} 00011$

Rightmost: $S \xrightarrow{1} A1B \xRightarrow{5} A11B \xRightarrow{6} A11 \xRightarrow{2} 0A11 \xRightarrow{2} 00A11 \Rightarrow$
$\xRightarrow{2} 000A11 \xRightarrow{3} 00011$

Ex:-5.4.5: This question concerns the grammar from Exercise 5.1.2 which we reproduce here:

$S \rightarrow A1B$

$A \rightarrow 0A \mid \varepsilon$

$B \rightarrow 0B \mid 1B \mid \varepsilon$

(a) Show that this grammar is unambiguous

Ans:- The grammar is said to be unambiguous when the given grammar can be derived from ~~th~~ both the right and leftmost derivation.

let us example ~~for~~ to prove that the grammar is unambiguous

Ex:-00101

leftmost: $S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 0010B \rightarrow 00101B \rightarrow 00101$

Rightmost: $S \rightarrow A1B \rightarrow A10B \rightarrow A101B \rightarrow 0A101 \rightarrow 00A101 \rightarrow 00101$

So the grammar is unambiguous

(b) The grammar that is ~~un~~ambiguous for the language is 00101.

$S \rightarrow A1B$

$A \rightarrow 0A \mid \varepsilon$

$B \rightarrow 1B \mid \varepsilon$

neglecting 0B

Left derivation

~~Derivation~~

$S \rightarrow A1B \rightarrow 0A1B \rightarrow 00A1B \rightarrow 0011$

Right derivation

$S \rightarrow A1B \rightarrow A11B \rightarrow 0111$

Hence, 00101 is ~~un~~ambiguous as we cannot derive from both the derivation from the language