Finite State Machines With Output

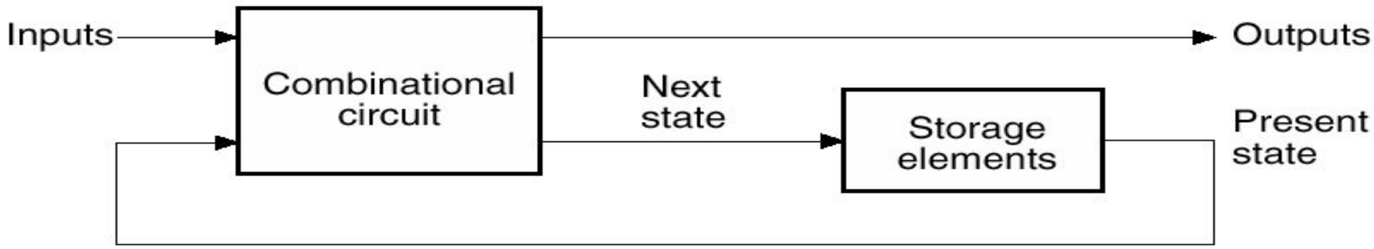
CSE 322

Formal Language and Automata Theory

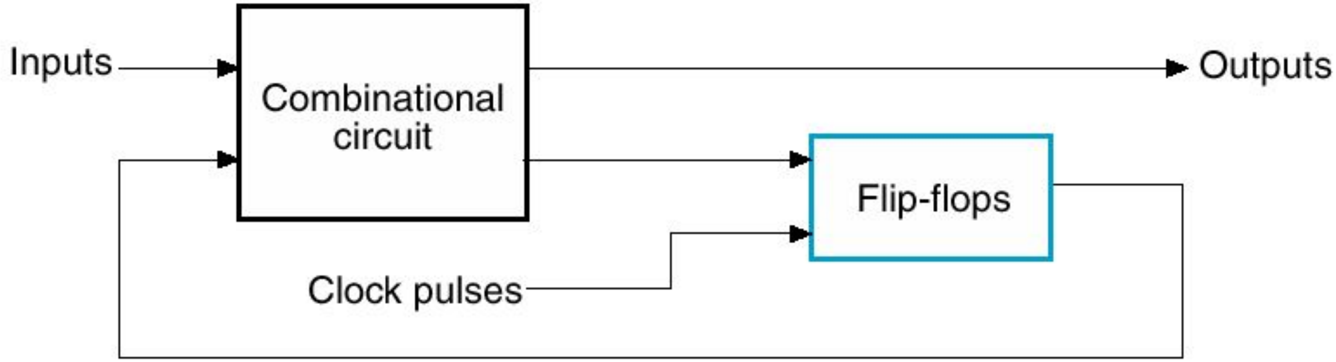
Today's Topics

- State Machines
  - How to design machines that go through a <u>sequence</u> of events
    - "sequential machines"

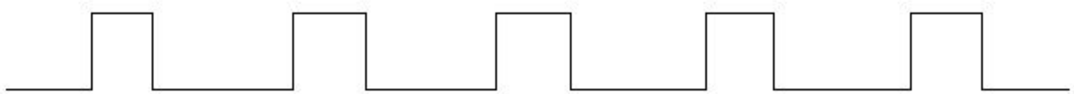
  - Basically close the feedback loop in this picture:



Synchronous Sequential Logic



(a) Block diagram



(b) Timing diagram of clock pulses

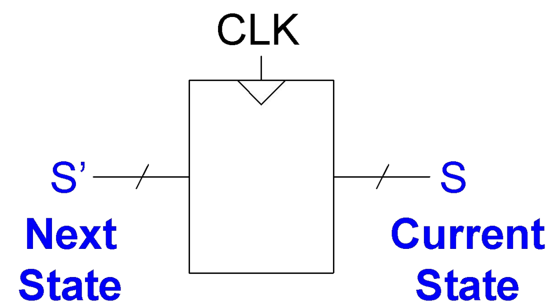
- Flip-flops/registers contain the system's <u>state</u>
  - state changes only at clock edge
    - so system is *synchronized* to the clock
  - all flip-flops receive the same clock signal (important!)

Two common types

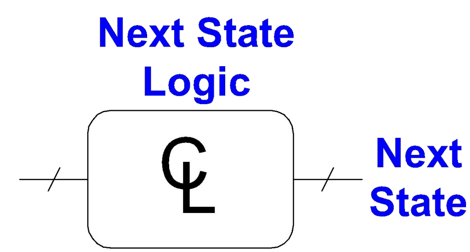
- Two common synchronous sequential circuits:
  - **Finite State Machines (FSMs)**
  - Pipelines

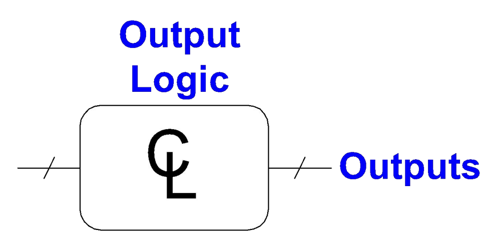
Finite State Machine (FSM)

- Consists of:
  - State register that
    - holds the current state
    - updates it to the "next state" at clock edge



  - Combinational logic (CL) that
    - computes the next state
      - using current state and inputs
    - computes the outputs
      - using current state (and maybe inputs)



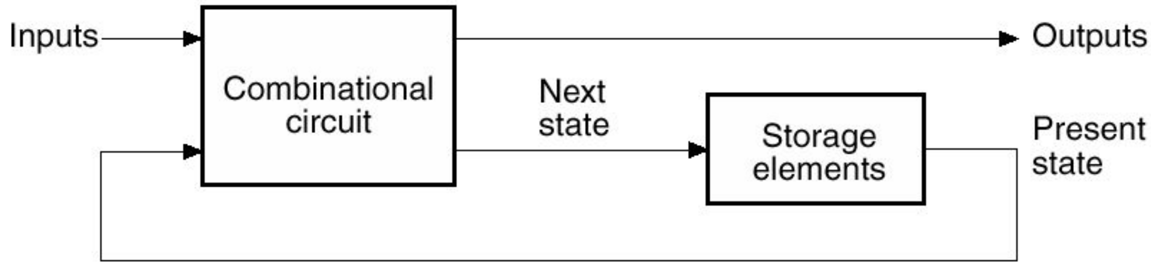


More and Mealy FSMs

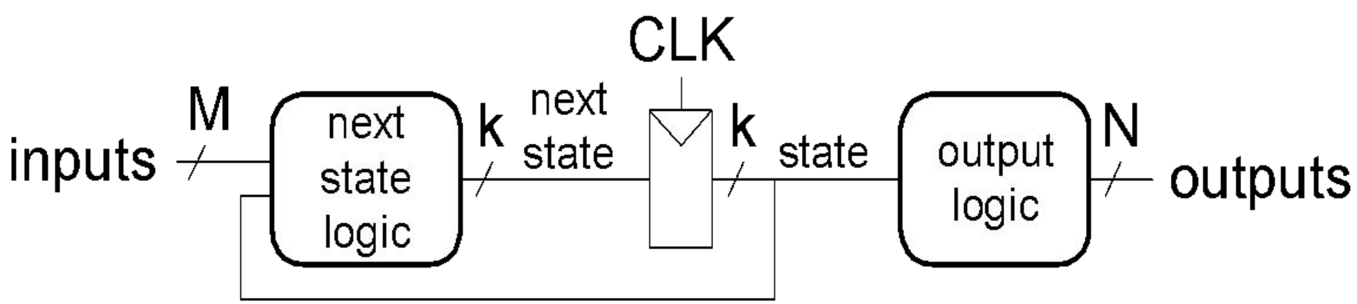
- Two types of finite state machines differ in the output logic:
  - **<u>Moore FSM</u>**:
    - outputs depend only on the current state
  - **<u>Mealy FSM</u>**:
    - outputs depend on the current state <u>and the inputs</u>
  - can convert from one form to the other
    - Mealy is more general

- In Both:
  - Next state is determined by current state and inputs

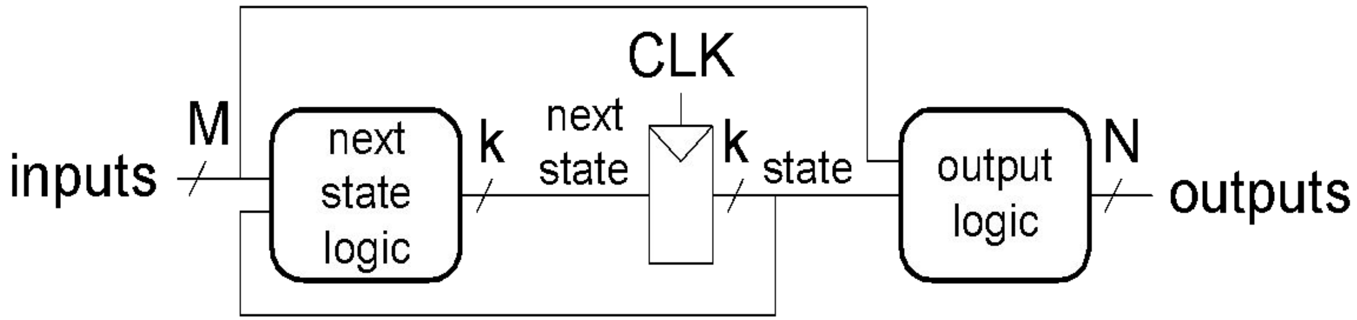
Moore and Mealy FSMs



Moore FSM



Mealy FSM



Formal Definition of Moore Machine

Moore machines are finite state machines with outp ut value and its output depends only on present state. It can be defined as $(Q, q0, \sum, O, \delta, \lambda)$ where:

Q is finite set of states.

q0 is the initial state.

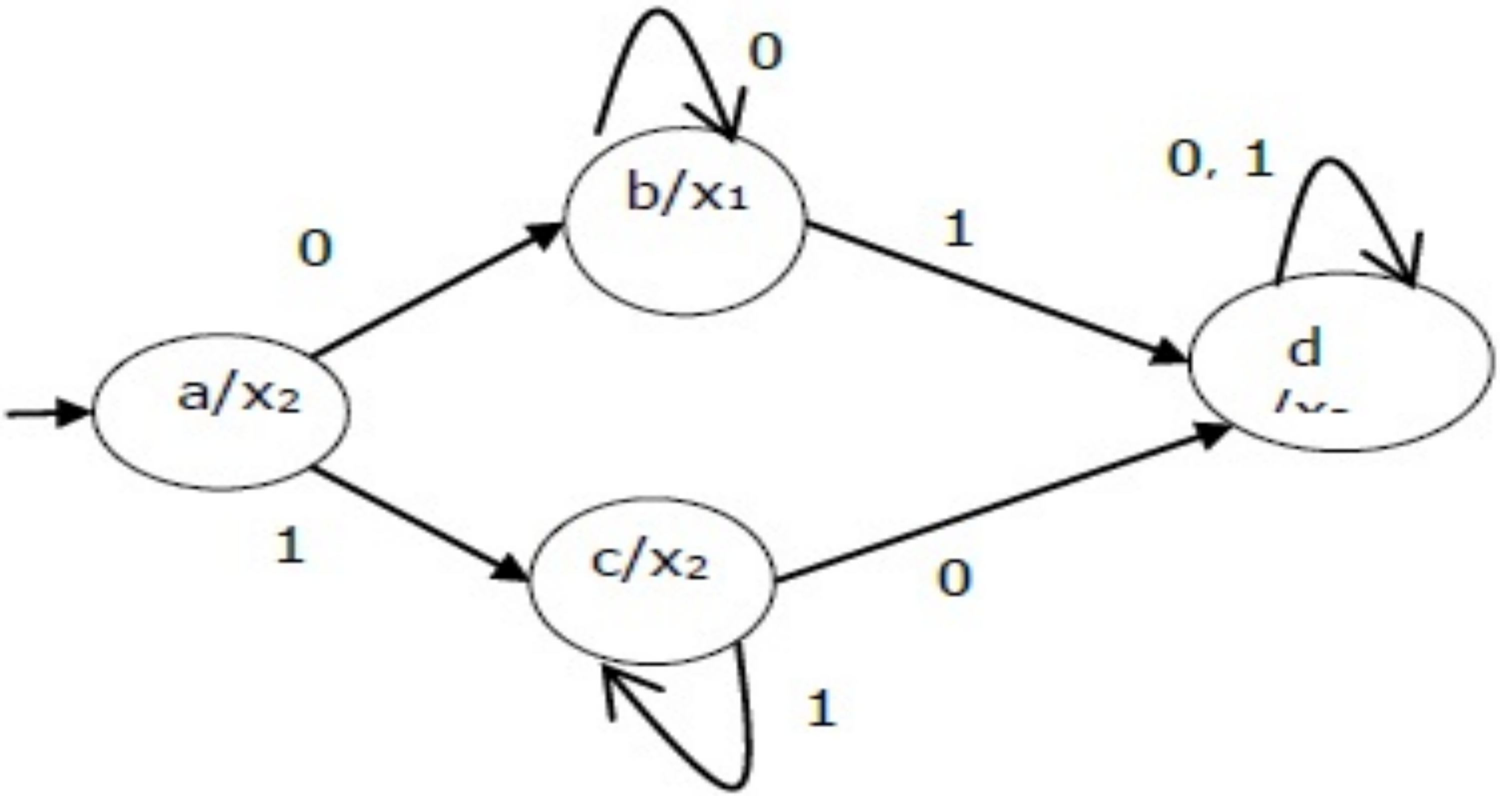
$\sum$ is the input alphabet.

O is the output alphabet.

δ is transition function which maps $Q \times \sum \rightarrow Q$.

λ is the output function which maps $Q \rightarrow O$.

Representation method of Moore Machine



The state table of a Moore Machine is shown below –

Present state	Next State		Output
	Input = 0	Input = 1	
→ a	b	c	x_2
b	b	d	x_1
c	c	d	x_2
d	d	d	x_3

Formal Definition of Mealy Machine

A Mealy Machine is an FSM whose output depends on the present state as well as the present input.

It can be described by a 6 tuple $(Q, \sum, O, \delta, X, q_0)$ where −

Q is a finite set of states.

$\sum$ is a finite set of symbols called the input alphabet.

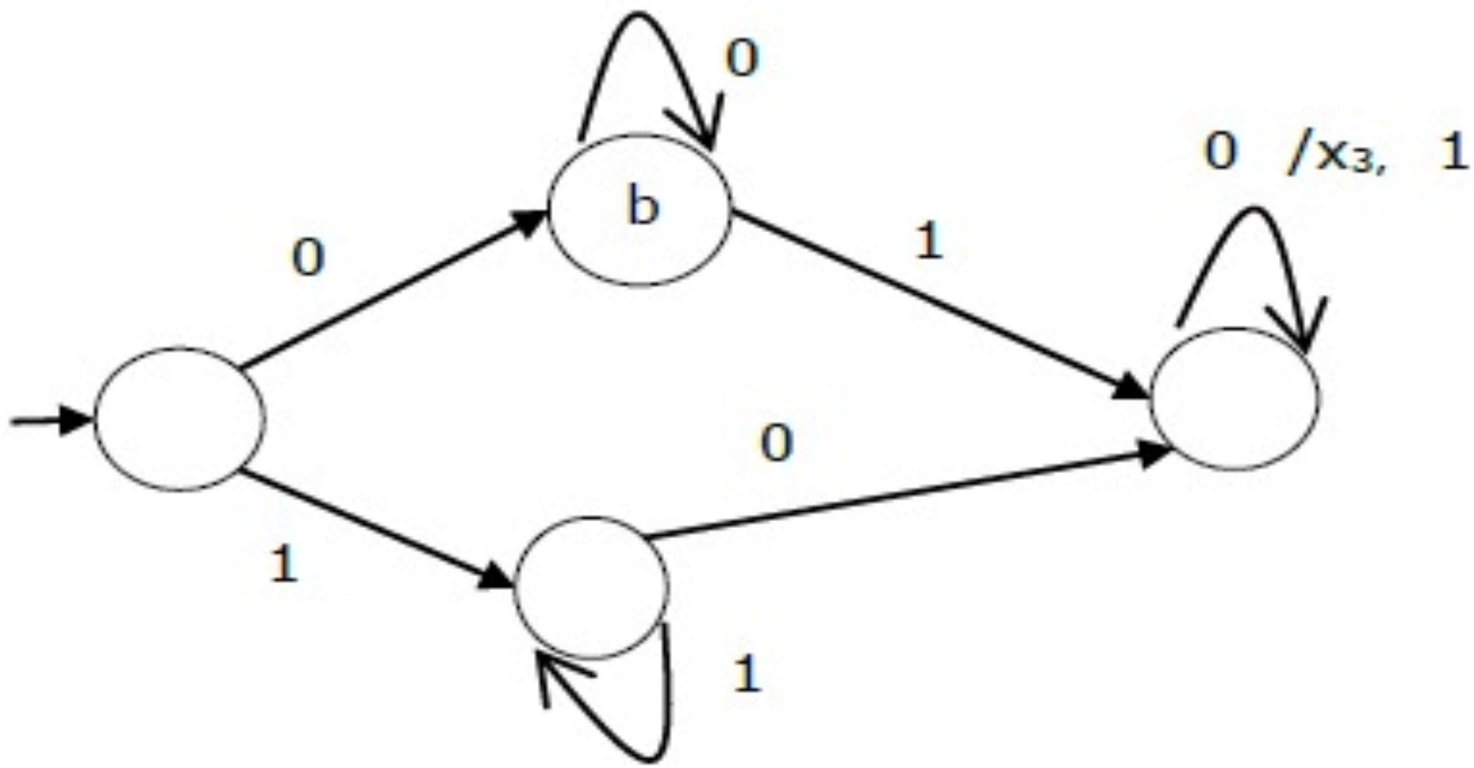
O is a finite set of symbols called the output alphabet.

δ is the input transition function where $\delta: Q \times \sum \rightarrow Q$

X is the output transition function where $X: Q \times \sum \rightarrow O$

$\mathbf{q_0}$ is the initial state from where any input is processed ($q_0 \in Q$).

Representation method of Mealy Machine

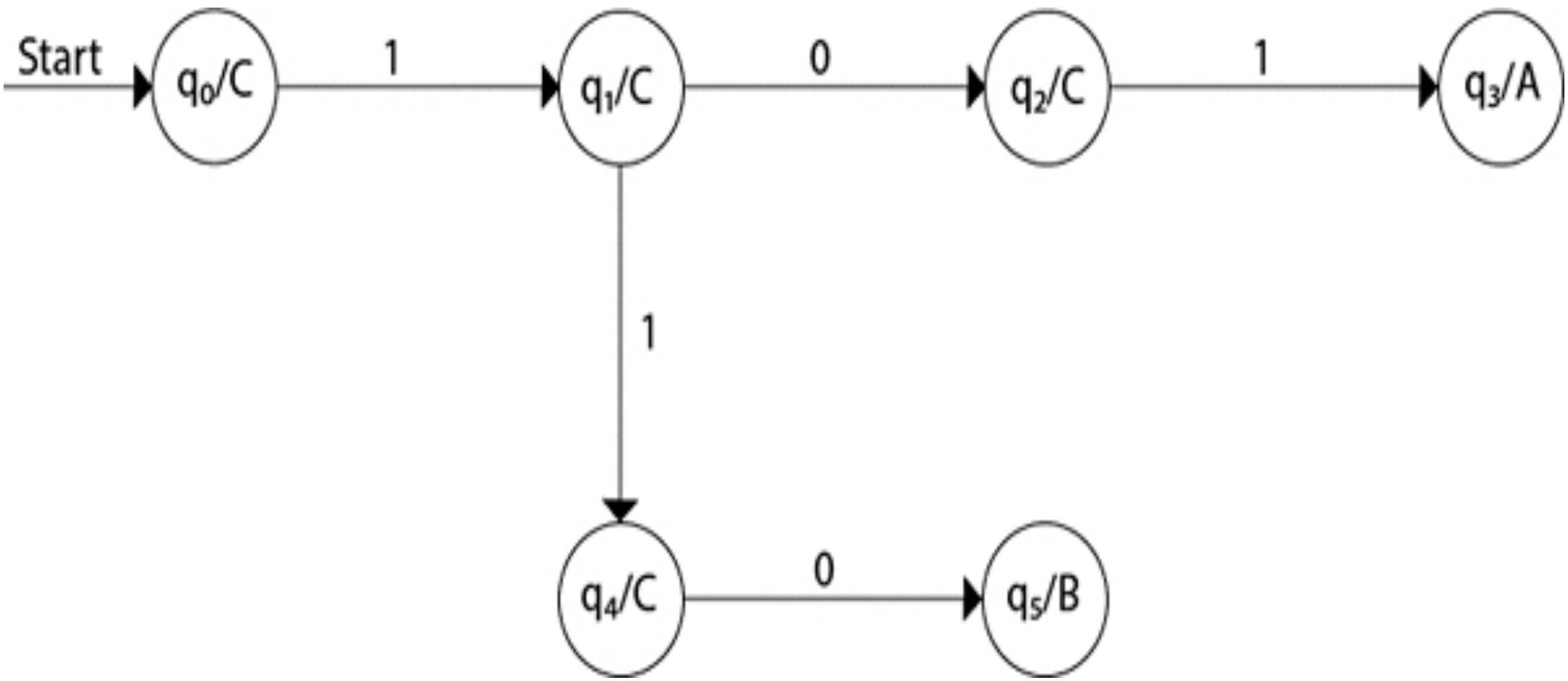


The state table of a Mealy Machine is shown below –

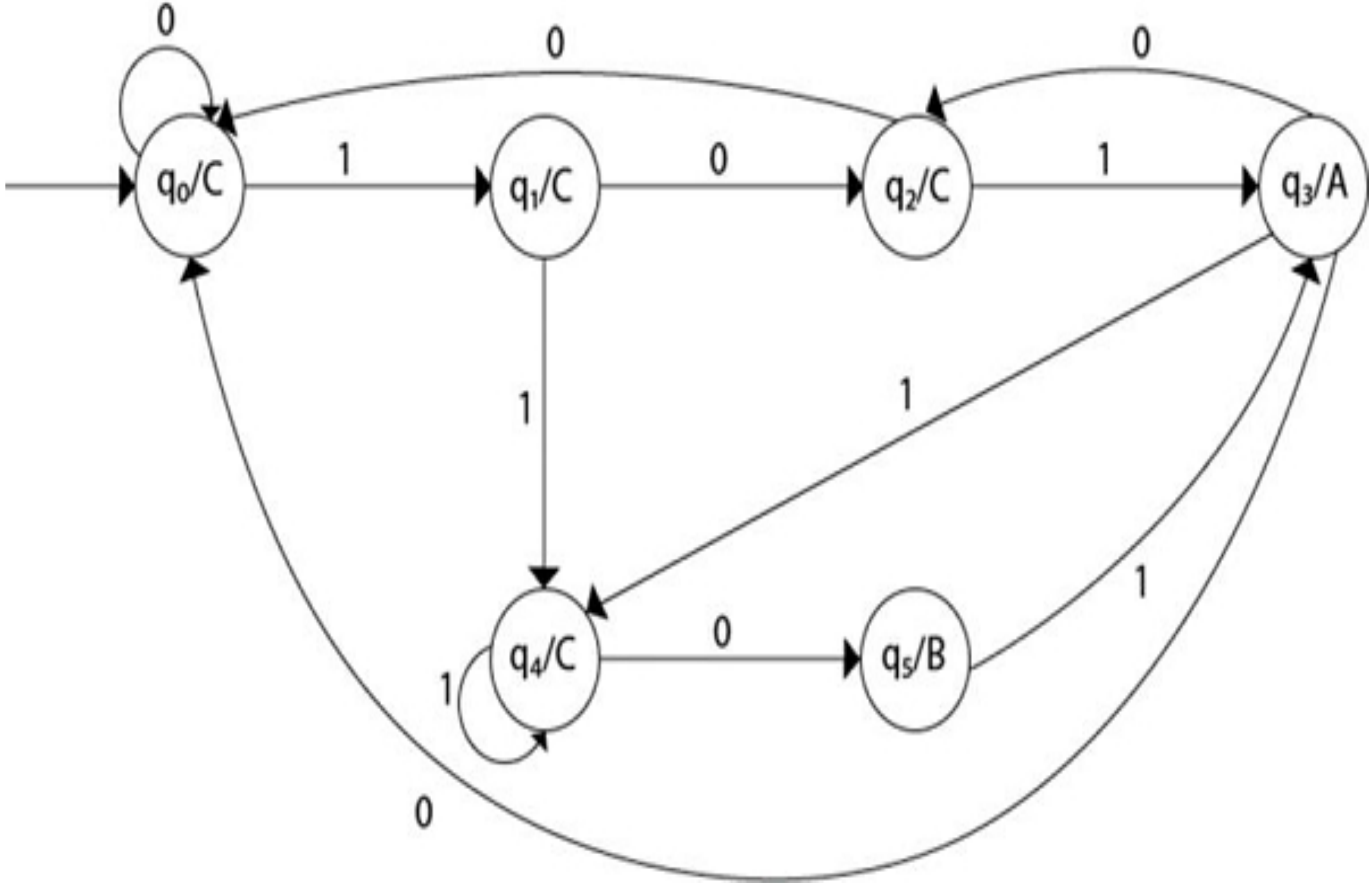
Present state	Next state			
	input = 0		input = 1	
	State	Output	State	Output
→ a	b	x_1	c	x_1
b	b	x_2	d	x_3
c	d	x_3	c	x_1
d	d	x_3	d	x_2

Designing of Moore Machine

Ex#1 :Design a Moore machine for a binary input sequence such that if it has a substring 101, the machine outputs A, if the input has substring 110, it outputs B otherwise it outputs C



Ex#1



EX#2: Design a Moore machine to generate 1's complement of a binary number.

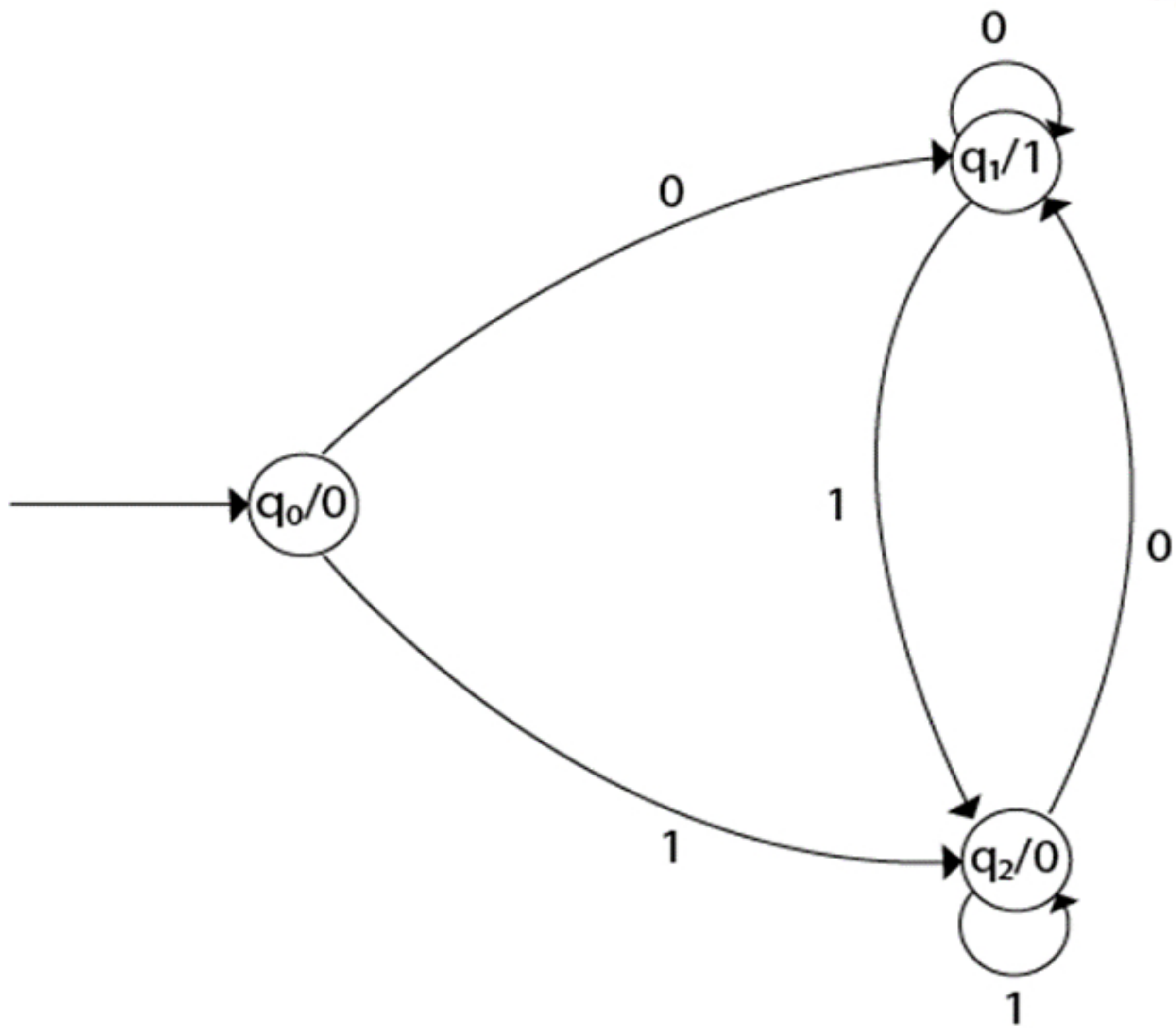
Explanation

Step 1 − q0 is the start state on input ‘0’ goes to q1 state and on ‘1’ goes to state q2 generating output 0.

Step 2 − q1 on input ‘0’ goes to q1 itself and on ‘1’ goes to q2 generating output ‘1’.

Step 3 − q2 on input ‘0’ goes to q1 and on ‘1’ goes to q2 generating output ‘0’.

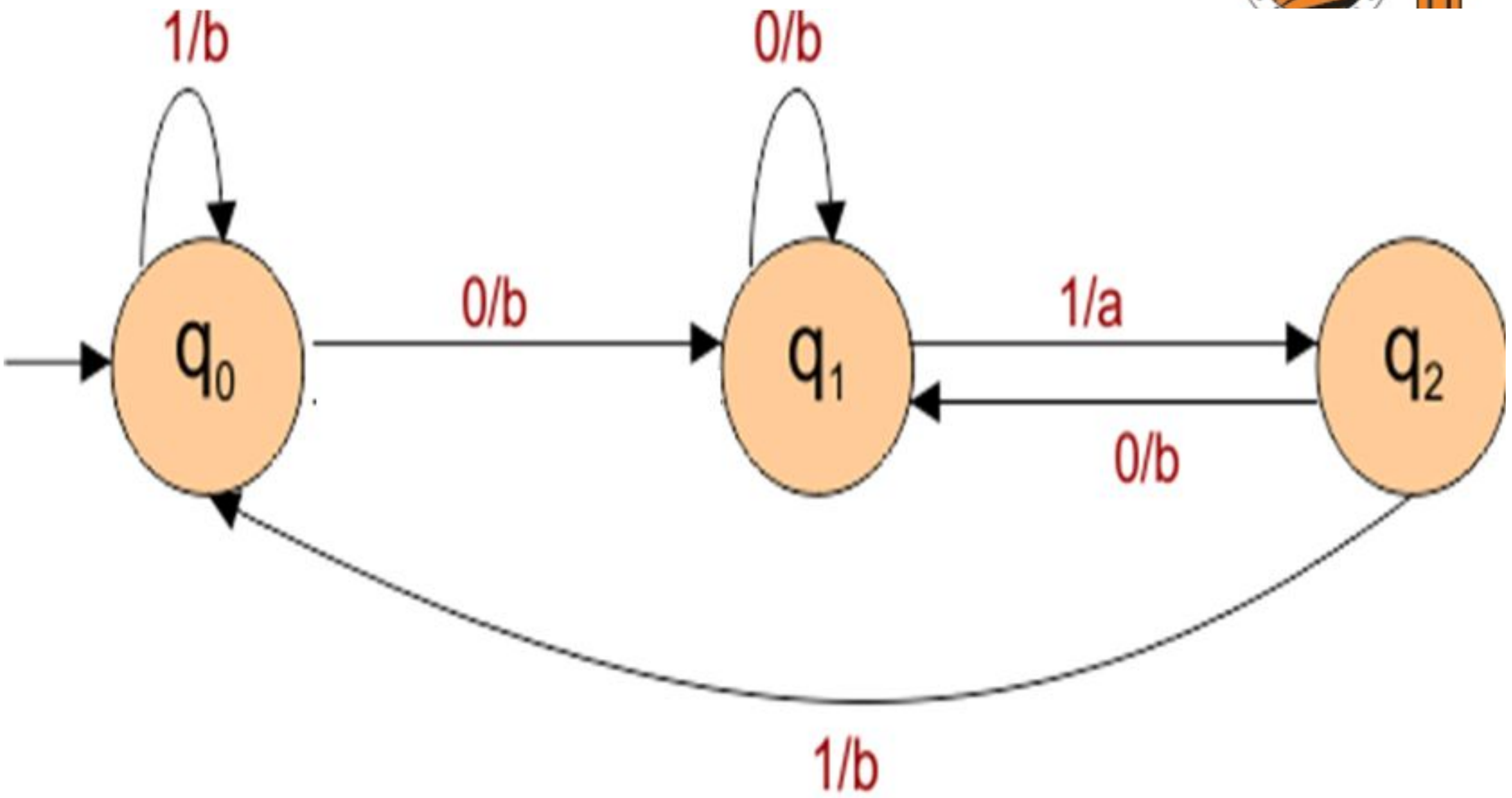
Ex#2:



Designing of Mealy Machine

Ex # 1 :Design a Mealy Machine that prints “a” whenever the sequence “01” is encountered in any input binary string.

Solution of Ex#1



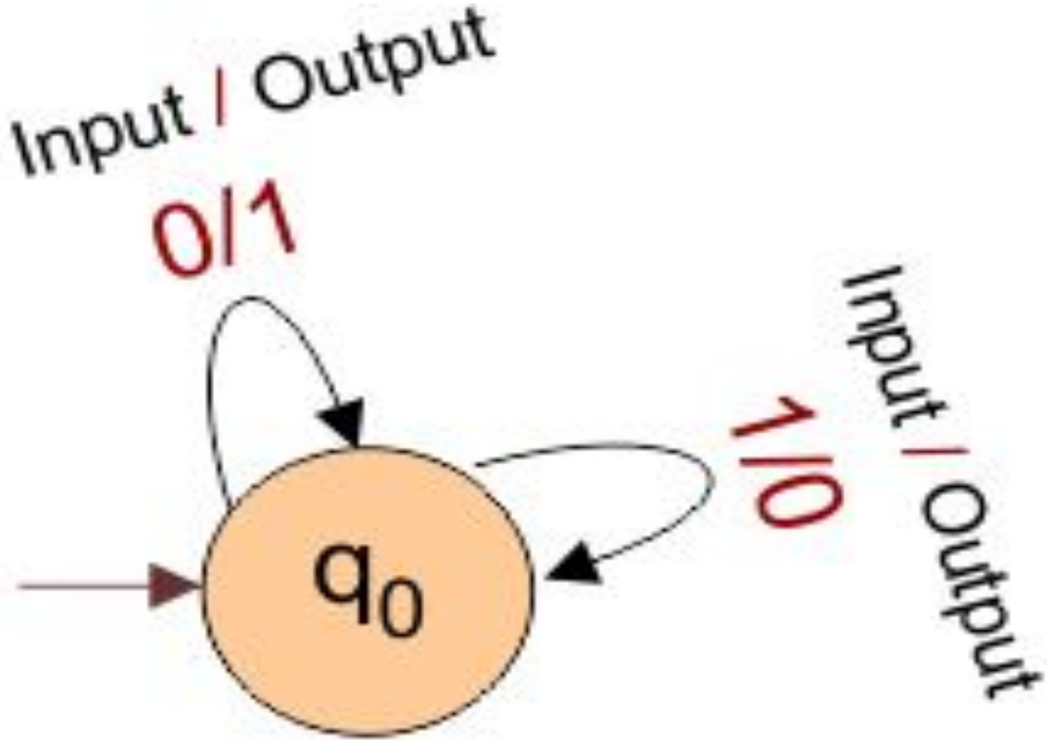
Mealy Machine

For Example : Take one binary number 10111001 then look at thetake one binary number 10111001 then look at the following table following table

Input	1	0	1	1	1	0	0	1
State	q0	q1	q2	q0	q0	q1	q1	q2
Output	b	b	a	'b	b	b	b	a

Ex#2: **Design a Moore machine to generate 1's complement of a any given binary input.**

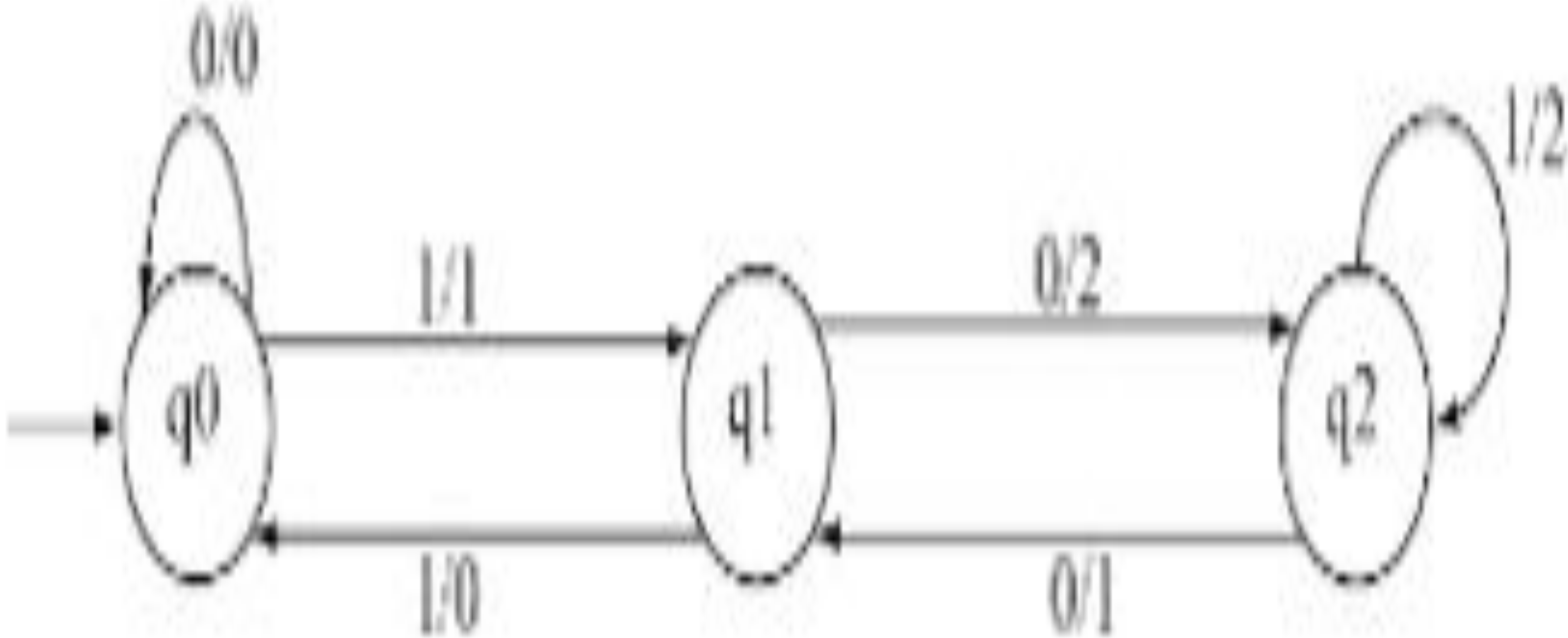
Solution of Ex#2:



1st Complement in Mealy Machine

Ex#3: Design a mealy machine to determine the residue mod 3 of a binary number.

Solution of EX#3



Equivalence in Moore and Mealy Machine

1. Conversion from Mealy to Moore
2. Conversion from Moore to Mealy

1. Conversion from mealy to moore

Mealy Machine to Moore Machine

Algorithm

Input: Mealy Machine
Output: Moore Machine
Step 1 Calculate the number of different outputs for each state (Qi) that are available in the state table of the Mealy machine.

Step 2 If all the outputs of Qi are same, copy state Qi . If it has n distinct outputs, break Qi into n states as Qin where n = 0, 1, 2.......

Example Let us consider the following Mealy Machine

Present State	Next State			
	a=0		a=1	
	Next State	Output	Next State	Output
→a	d	0	b	1
b	a	1	d	0
c	c	1	c	0
d	b	0	a	1

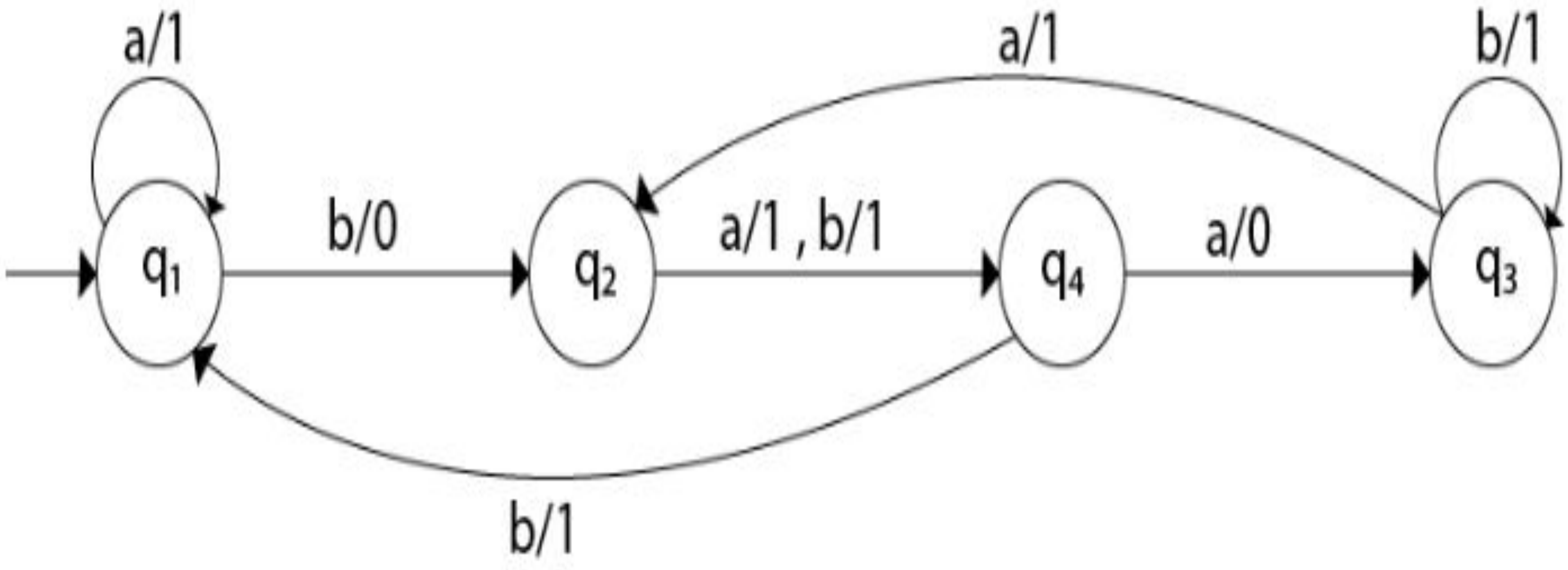
Here, states ‘a’ and ‘d’ give only 1 and 0 outputs respectively, so we retain states ‘a’ and ‘d’. But states ‘b’ and ‘c’ produce different outputs 1and0. So, we divide b into b0, b1 and c into c0, c1.

Present State	Next State		Output
	a=0	a=1	
→a	d	b_1	1
b_0	a	d	0
b_1	a	d	1
c_0	c_1	C_0	0
c_1	c_1	C_0	1
d	b_0	a	0

Now Draw the Transition Diagram of equivalent Moore Machine

Ex#2:

Convert the following Mealy machine into equivalent Moore machine.

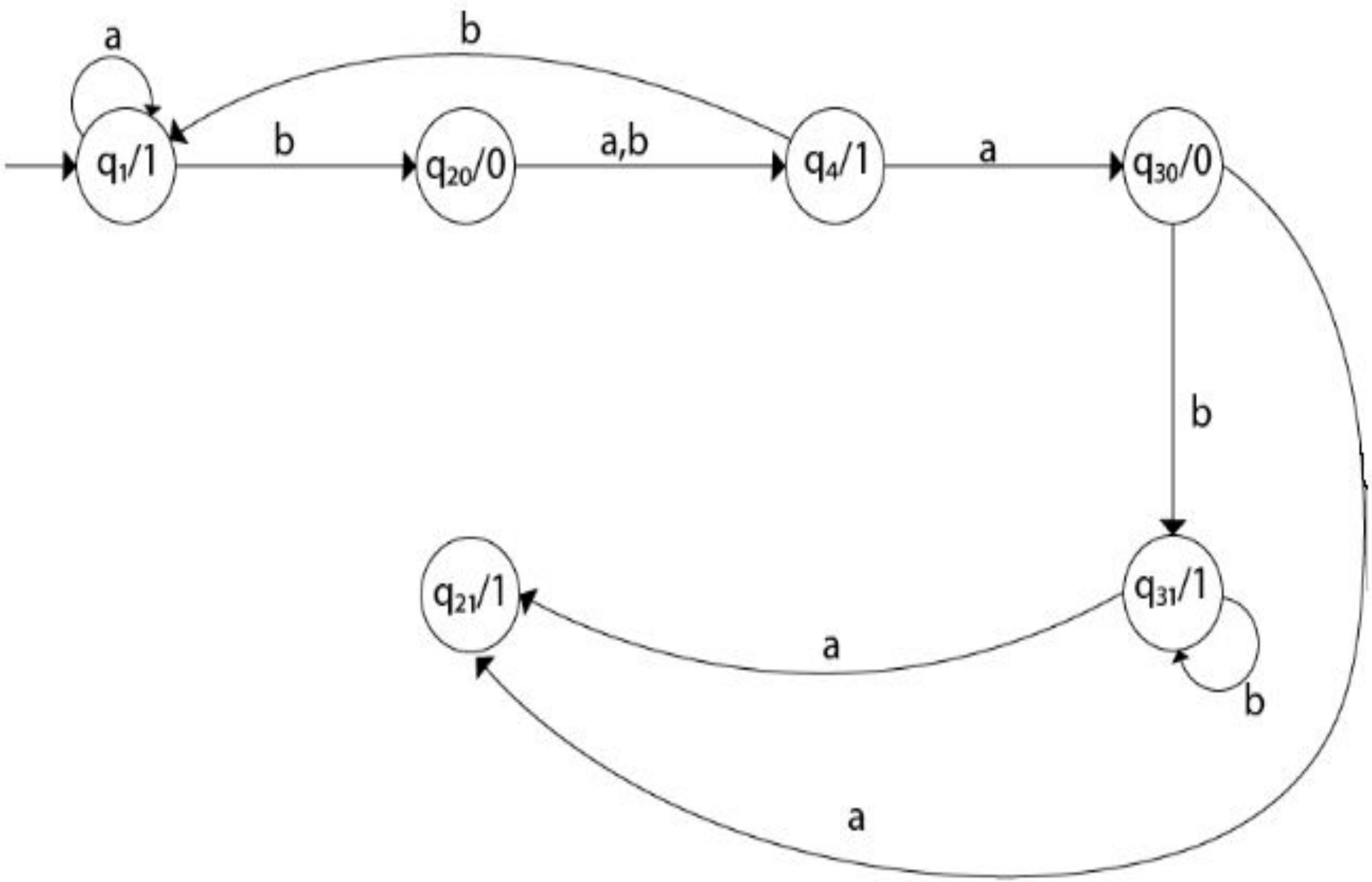


Transition table for above Mealy machine is as follows:

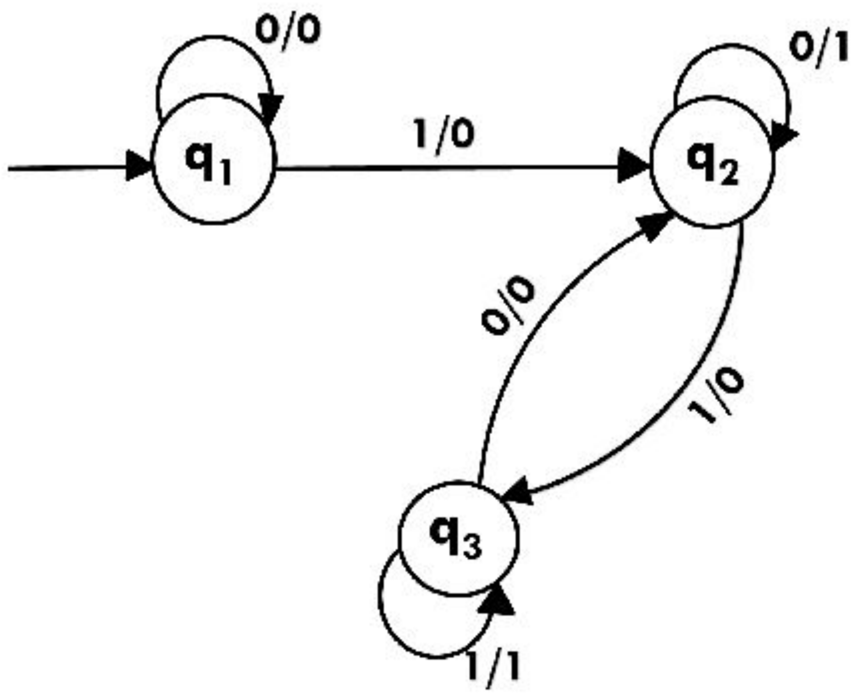
Present State	Next State			
	a		b	
	State	O/P	State	O/P
q_1	q_1	1	q_2	0
q_2	q_4	1	q_4	1
q_3	q_2	1	q_3	1
q_4	q_3	0	q_1	1

1. For state q1, there is only one incident edge with output 0. So, we don't need to split this state in Moore machine.
2. For state q2, there is 2 incident edge with output 0 and 1. So, we will split this state into two states q20(state with output 0) and q21(with output 1).
3. For state q3, there is 2 incident edge with output 0 and 1. So, we will split this state into two states q30(state with output 0) and q31(state with output 1).
4. For state q4, there is only one incident edge with output 0. So, we don't need to split this state in Moore machine.

Transition diagram for Moore machine will be:

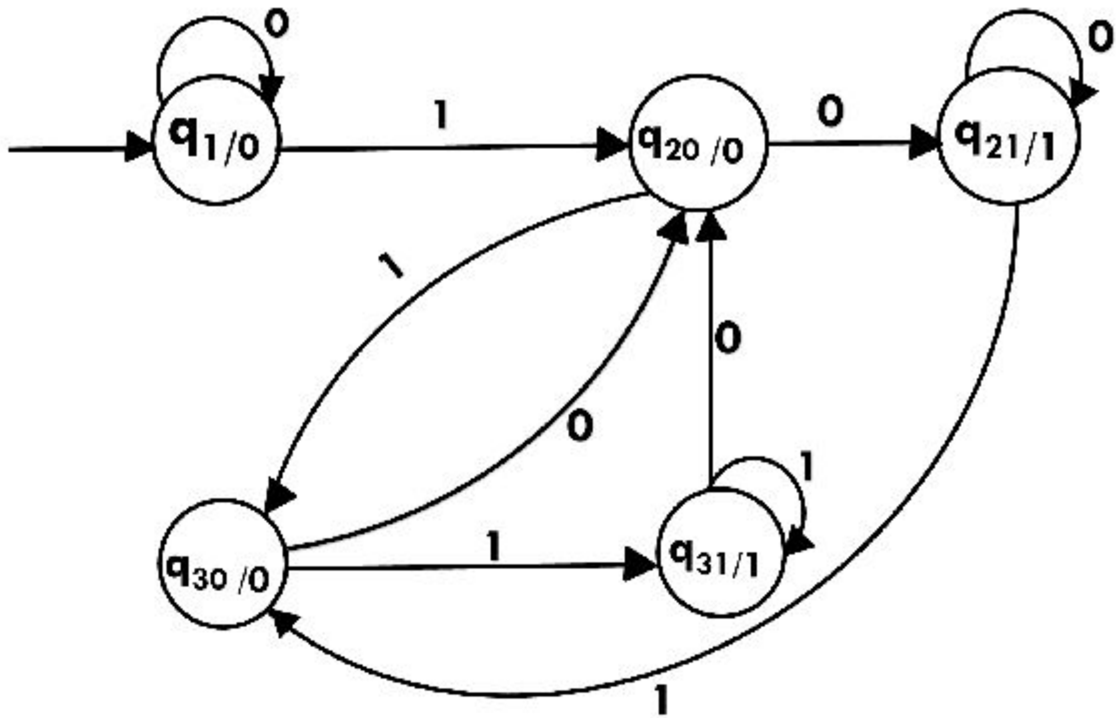


Ex# 3:



Solution

Transition diagram for Moore machine will be:

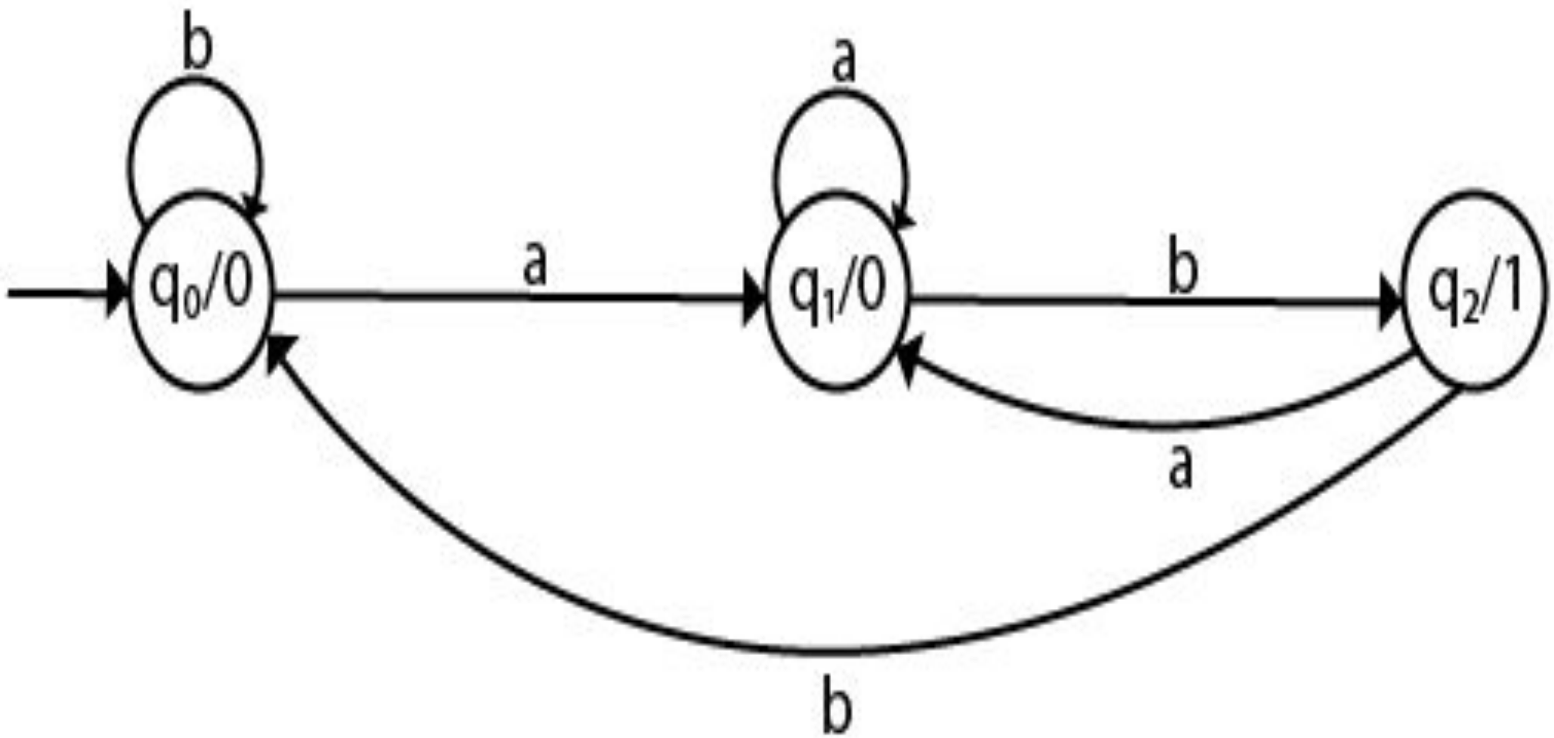


2. Conversion from Moore To Mealy

1. In the Moore machine, the output is associated with every state, and in the mealy machine, the output is given along the edge with input symbol. The equivalence of the Moore machine and Mealy machine means both the machines generate the same output string for same input string.

2. We cannot directly convert Moore machine to its equivalent Mealy machine because the length of the Moore machine is one longer than the Mealy machine for the given input. To convert Moore machine to Mealy machine, state output symbols are distributed into input symbol paths. We are going to use the following method to convert the Moore machine to Mealy machine.

Convert the given Moore machine into its equivalent Mealy machine.



The equivalent Mealy machine can be obtained as follows:

$\lambda'(q0, a) = \lambda(\delta(q0, a))$
$= \lambda(q1)$
$= 0$

$\lambda'(q0, b) = \lambda(\delta(q0, b))$
$= \lambda(q0)$
$= 0$

The λ for state q1 is as follows:

$\lambda'(q1, a) = \lambda(\delta(q1, a))$
$= \lambda(q1)$
$= 0$

$\lambda'(q1, b) = \lambda(\delta(q1, b))$
$= \lambda(q2)$
$= 1$

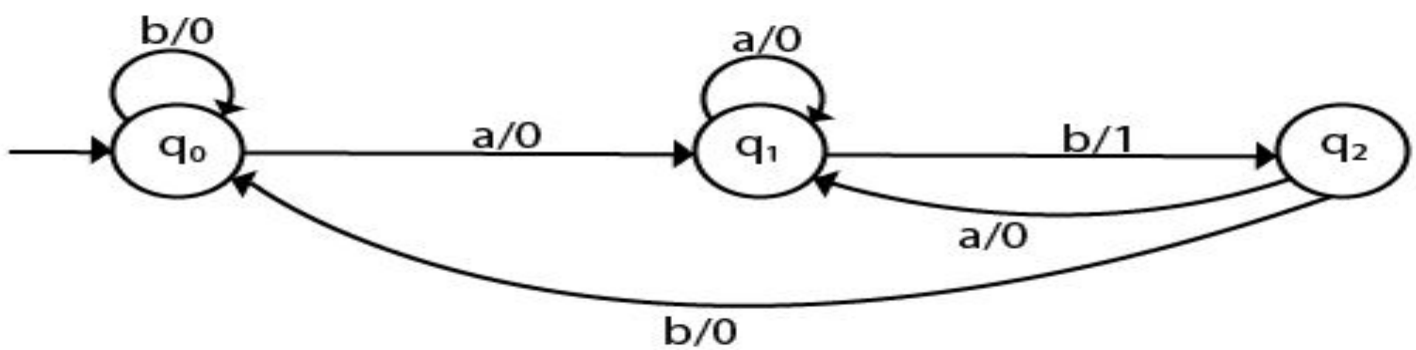
The λ for state q2 is as follows:

$\lambda'(q2, a) = \lambda(\delta(q2, a))$
$= \lambda(q1)$
$= 0$

$\lambda'(q2, b) = \lambda(\delta(q2, b))$
$= \lambda(q0)$
$= 0$

Q \ Σ	Input a		Input b	
	State	Output	State	Output
q_0	q_1	0	q_0	0
q_1	q_1	0	q_2	1
q_2	q_1	0	q_0	0

The equivalent Mealy machine will be,



Ex#2:

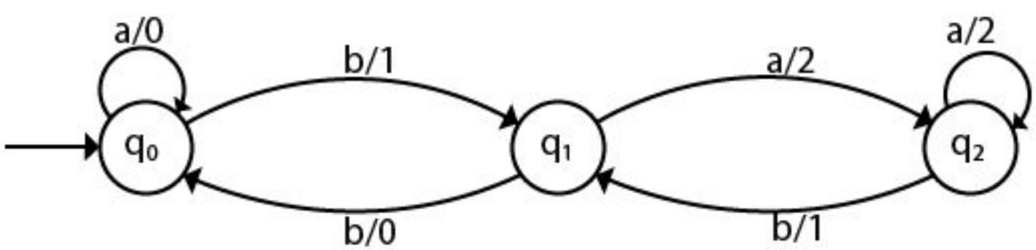
Convert the given Moore machine into its equivalent Mealy machine.

Q	a	b	Output(λ)
q0	q0	q1	0
q1	q2	q0	1
q2	q1	q2	2

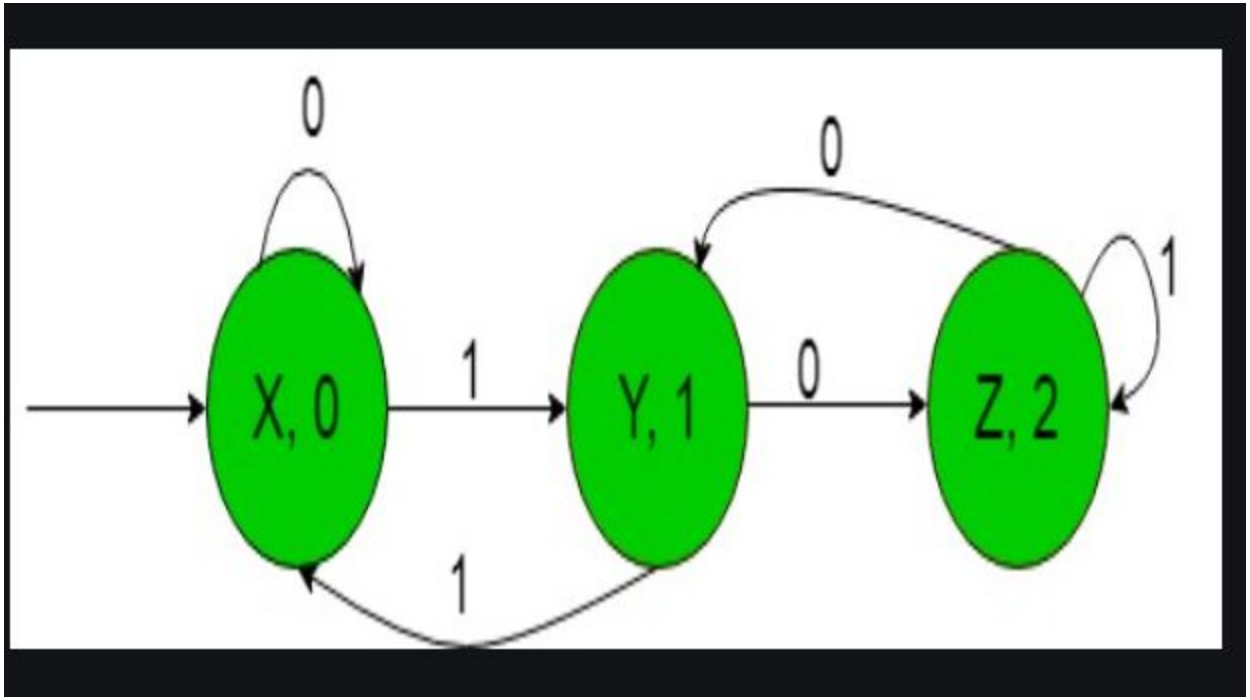
Solution of Ex#2

Q \ Σ	Input a		Input b	
	State	O/P	State	O/P
q_0	q_0	0	q_1	1
q_1	q_2	2	q_0	0
q_2	q_1	1	q_2	2

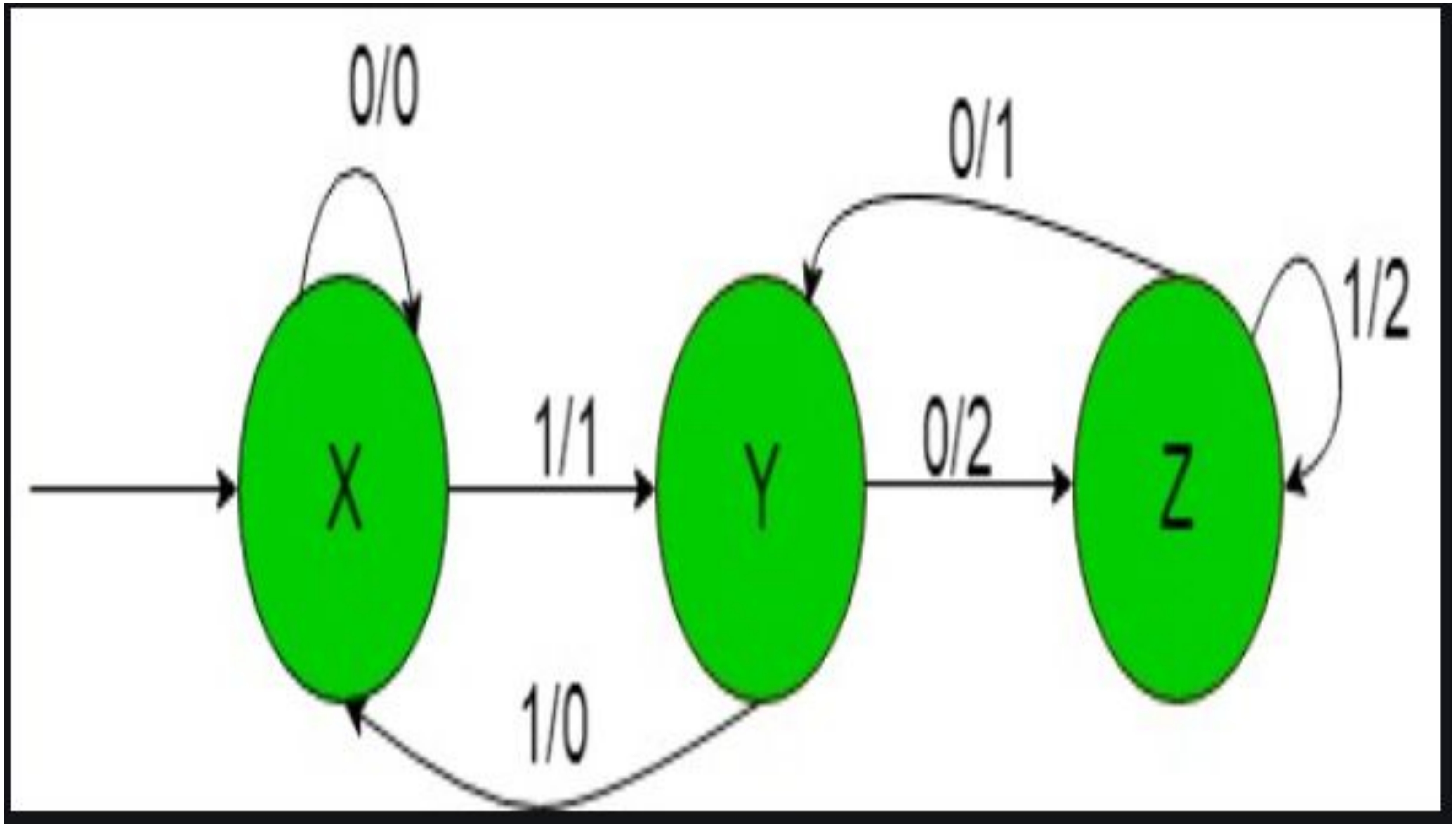
The equivalent Mealy machine will be,



Ex#3:



Solution of Ex#3



END of TOPIC

Thanks to All

Any Query ????