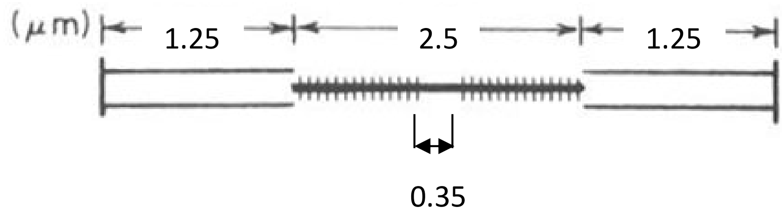
MCEN 4228/5228
Modeling of Human Movement
(K): Knowledge Problem, (C): Challenge Problem, (EX): Extra Credit
HW02

1. **(K)** Due to global warming, there has been a lot of glacial melt. While this is bad for the planet, paleontologists have been discovering fantastically preserved dinosaur specimens. Recently, a T-rex specimen was found that had intact samples of muscle tissue. The sample is so well preserved that it is possible to measure the lengths of the actin and myosin filaments in the sarcomere. You are interested in determining the optimal sarcomere length for T-rex, so you ask the scientists who found the specimen if they can send the dimensions of the filaments. You were sent the drawing below.



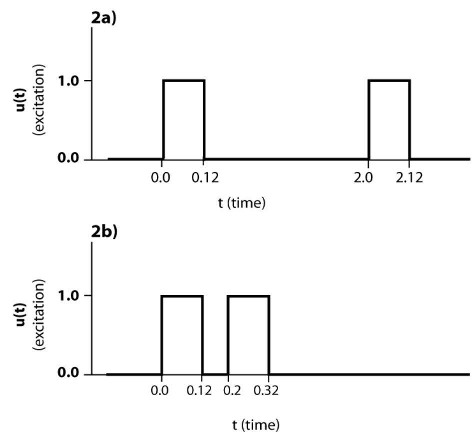
Using this information, draw the active portion of the force-length curve for the T-rex sarcomere, where the x-axis is sarcomere length (in microns) and the y-axis is tension (% maximum). For each of the end points of the plot and for each transition in the slope of the force-length curve that you draw, indicate the length on the x-axis at which this occurs with an actual numerical value. Locations of transitions on the y-axis need only be approximate.

2. **(K)** Given the neural excitation (*u(t)*) profiles shown below (the first plot is part a, the second plot is part b), edit the computer program provided (*hw2q2.m*) to plot the muscle activation (*a(t))* as a function of time for each plot. For both parts, assume that the time constant for activation is **0.12 seconds** and the time constant for deactivation is **0.24 seconds**.

   Is the peak activation higher for part a) or part b)? Why?

   NOTE: use the muscle activation equation provided in the class slides, which is also explained in the 1989 paper by Felix Zajac (posted on Canvas) and also provided here:

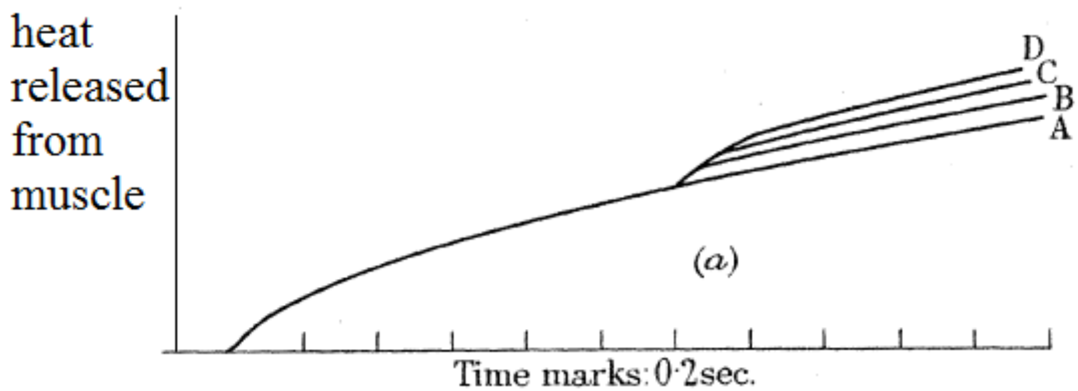
$$\dot{a(t)} = -a(t)\left(\frac{1-u(t)}{\tau_{deact}} + \frac{u(t)}{\tau_{act}}\right) + \left(\frac{1}{\tau_{act}}\right)u(t)$$



3. **(K)** To see the force-length curve in action, make a fist with your wrist straight. Note your grip strength (e.g. by squeezing a plastic water bottle). Now flex your wrist maximally and squeeze. Then extend your wrist maximally and squeeze. Plot your grip strength vs. wrist flexion angle. Explain.

4. **(C)** An elastic tendon can have huge effects on what muscle must do. It can make bouncing possible while keeping the muscle isometric. But there are other advantages as well.
   a) Based on the figures from lecture for COM vertical motion vs. time during running, how much energy would be stored in a mass-spring model during the stance phase with similar motion over the course of 5 minutes? Assume a change in COM height per step of 10 cm and a spring stiffness k = 200 N/mm, and a stride frequency of 1.3 strides/sec. Results can be approximate. List any assumptions that you had to make.
   b) If there was no spring, but muscle performed the same displacements, then how much positive mechanical work must the muscles perform?
   c) Assuming a muscle efficiency of 25%, (100 * mechanical work/metabolic energy = muscle efficiency), calculate how much metabolic energy is required for the same displacements.
   d) How might an elastic tendon help in running?

5. **(EC for all)** In class we talked about the force-velocity relationship of muscle as being the result of experiments where muscles shortened against a series of known loads and had their velocities measured. In reality, A.V. Hill discovered the force-velocity relationship indirectly in 1938 by studying heat released from muscles during a contraction against a load (for your reference please find Hill's paper under Resources in Canvas). In this problem, you will derive the force-velocity relationship of muscle by beginning with Hill's original results.



The figure above shows heat released from muscles as a function of time. The curve labeled A is a muscle that is contracting isometrically against a stop. For subsequent experiments (B-D) an isometrically contracting muscle was released from the first stop at a time of 1.2 seconds and allowed to contract a distance x before reaching another stop, during which time the heat from the muscle was recorded. Trace B corresponds to a shortening distance of 2.0cm. Trace C corresponds to a shortening distance of 3.6 cm. And Trace D corresponds to a shortening distance of 5.2 cm.

a) Write the equation for the work done when a muscle shortens a distance ***x*** under a load F.

b) Hill found in his experiments that the extra heat (Q) liberated when a muscle contracted some distance x (i.e. the distance between A and B, between A and C, and between A and D) was proportional to the distance that the muscle shortened: $Q=ax$ where a is constant. Considering that there is heat released from the system, write an equation for the total energy expended (i.e., sum of work done and heat released) by a muscle shortening a distance x under a constant load F.

c) Write an equation for the **rate** of energy expended by a muscle shortening a distance x under a constant load F.

d) Experimentally, Hill found that the rate of energy expended is linearly proportional to the negative of the load F, and that the rate of energy expended reduced to zero when $F=F_0$ (F_0 is a constant). Write an equation for this using "b" as the proportionality constant.

e) Rewrite your equation from d) in terms of F(v). This is the Force-velocity relationship of muscle, derived from Hill's experiments of heat released from muscle. Plot this relationship using the following values for constants: a=14.35 g.wt., b=1.03 cm/s, $a/F_0=0.22$.

f) Now, from your F(v) relationship, write the equation for muscle power as a function of velocity. Plot this relationship using a=14.35 g.wt., b=1.03 cm/s, $a/F_0=0.22$.

g) At what velocity do the muscles from Hill's experiments display peak power?

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