Inverse Dynamics

MCEN 4/5228

Modeling of Human Movement

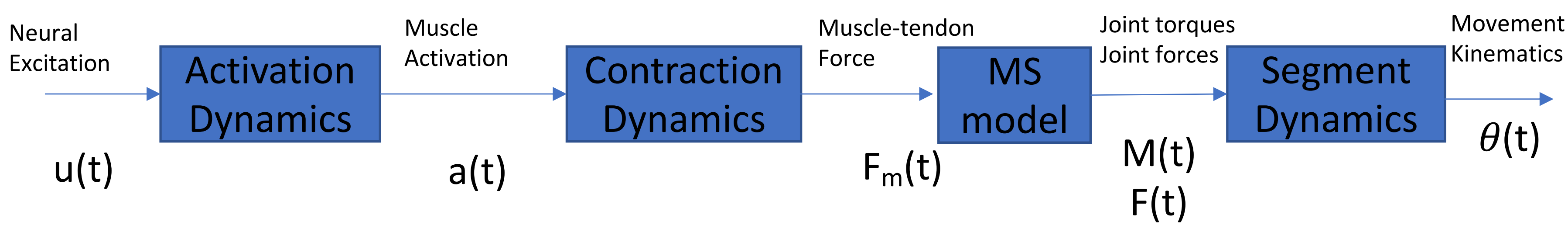
Fall 2021

Inverse Dynamics

- Forward vs Inverse Dynamics
- Measuring ground reaction forces
- Center of pressure
- Inverse dynamics example
- Inverse dynamics without ground reaction forces
- Clinical application

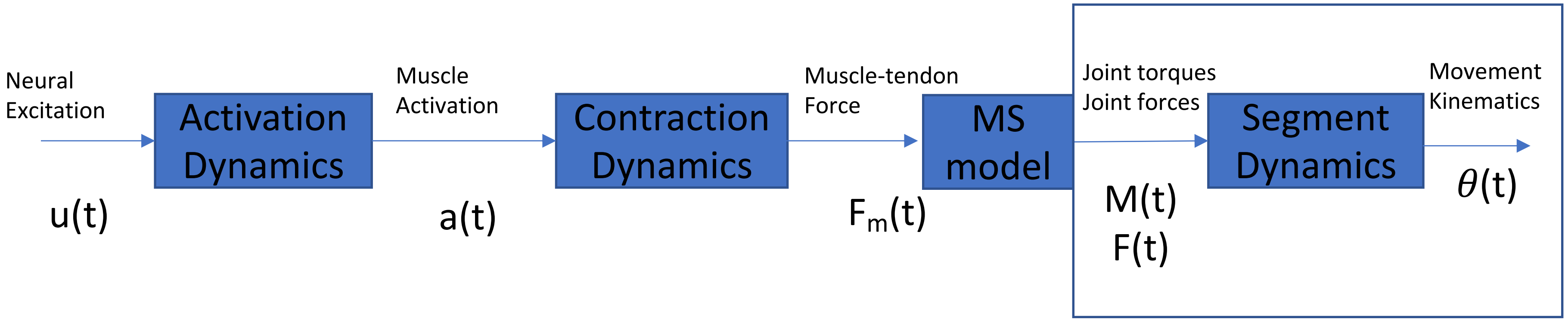
Modeling pipeline

- Activation Dynamics
- Contraction Dynamics
  - F-L
  - F-V
  - Tendon F-L
- MS model
  - Muscle moment arms
- Segment Dynamics
  - Equations of motion



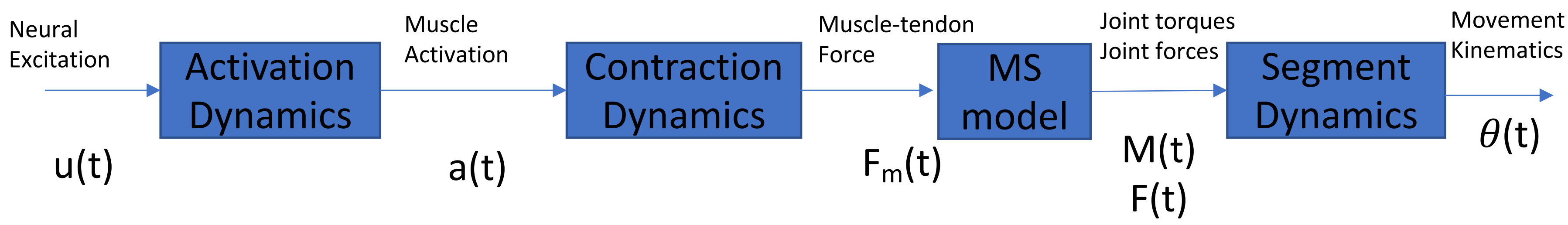
Forward and inverse dynamics

- Forward dynamics: process of predicting motion that would result from applying a given set of forces and torques on a system (→)
- Inverse dynamics: process of determining joint forces and torques given kinematics (←)



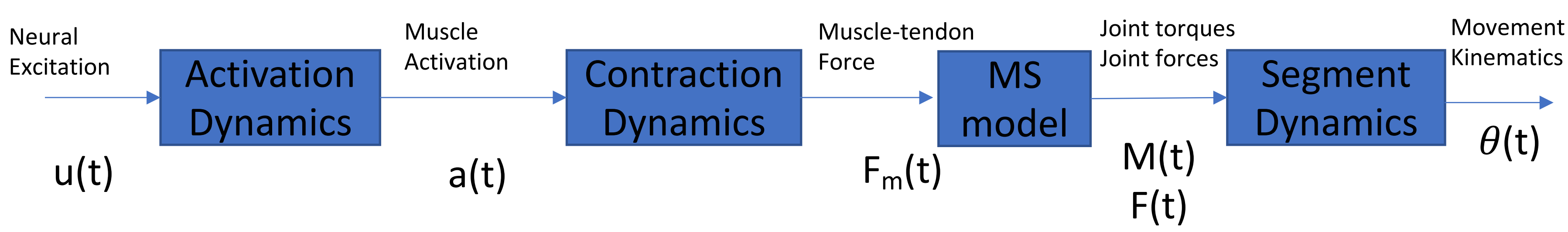
Applications of Inverse Dynamics

- Estimation of joint loads for injury prevention and rehabilitation
- Estimation of muscle forces (with an additional step)



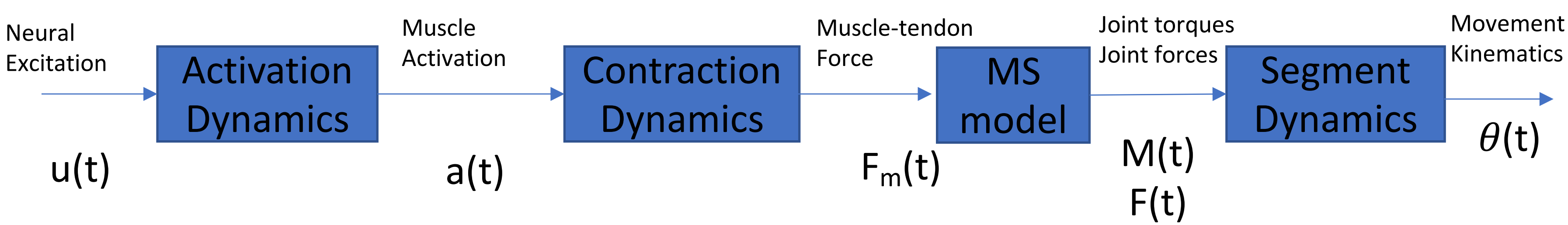
Limitations of Inverse Dynamics

- Model assumptions
- Noisy experimental data
- Does not provide information about individual muscles



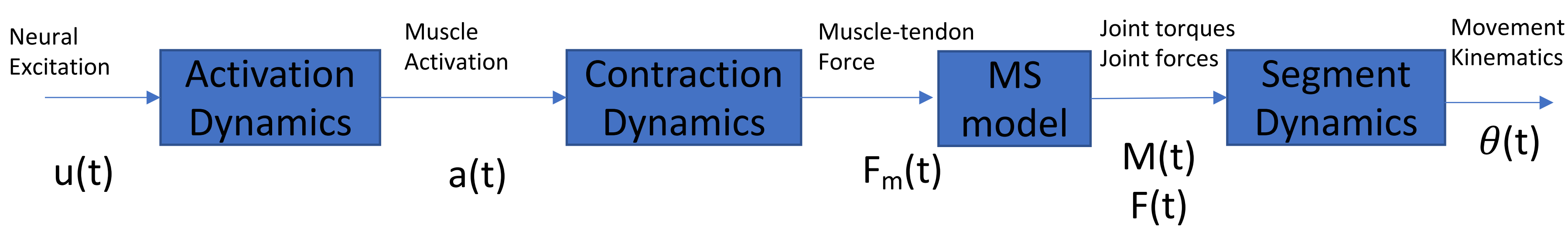
Applications of forward dynamics

- Understanding muscle function
- Determining individual muscle contributions to:
  - Segment accelerations
  - Joint loads
  - Metabolic cost
  - Performance
- Assessing performance
  - Max jump height



Limitations of forward dynamics

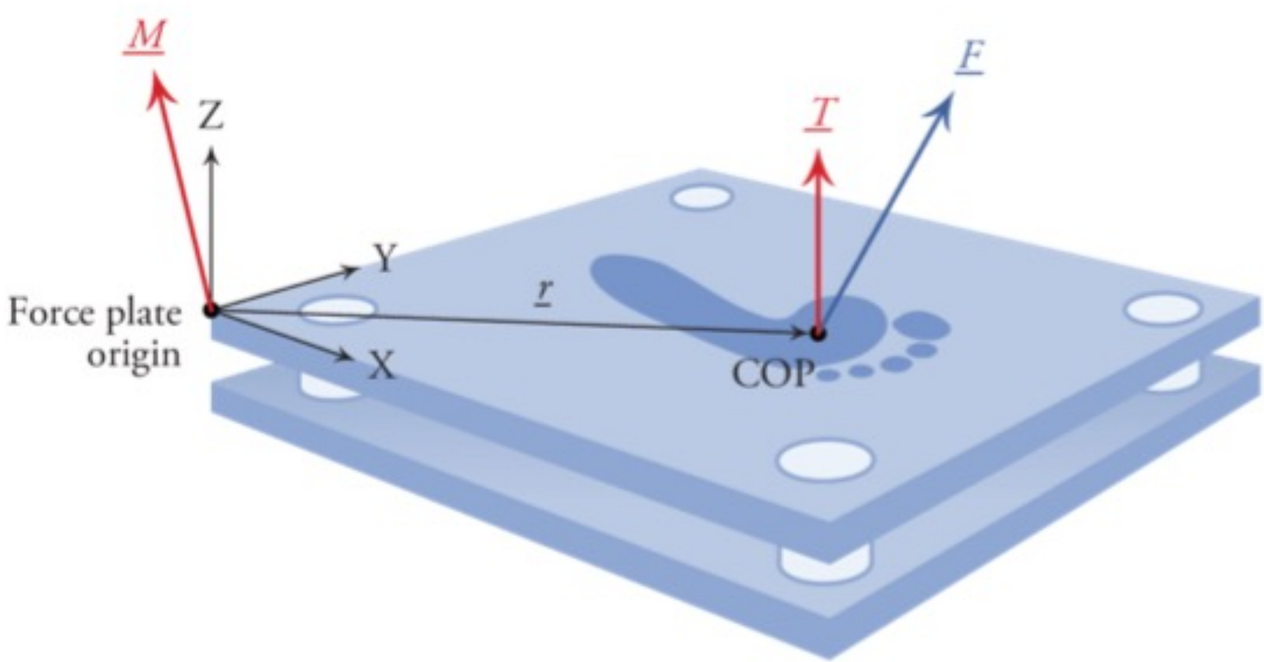
- Difficult to produce a well-coordinated movement
  - Optimal tracking
    - Minimize RMS error between experiment and model
  - Hypothesize goal of the motor task
  - There is no unique solution
    - Muscle recruitment is not trivial
  - Validation can be difficult



Solving inverse dynamics problems

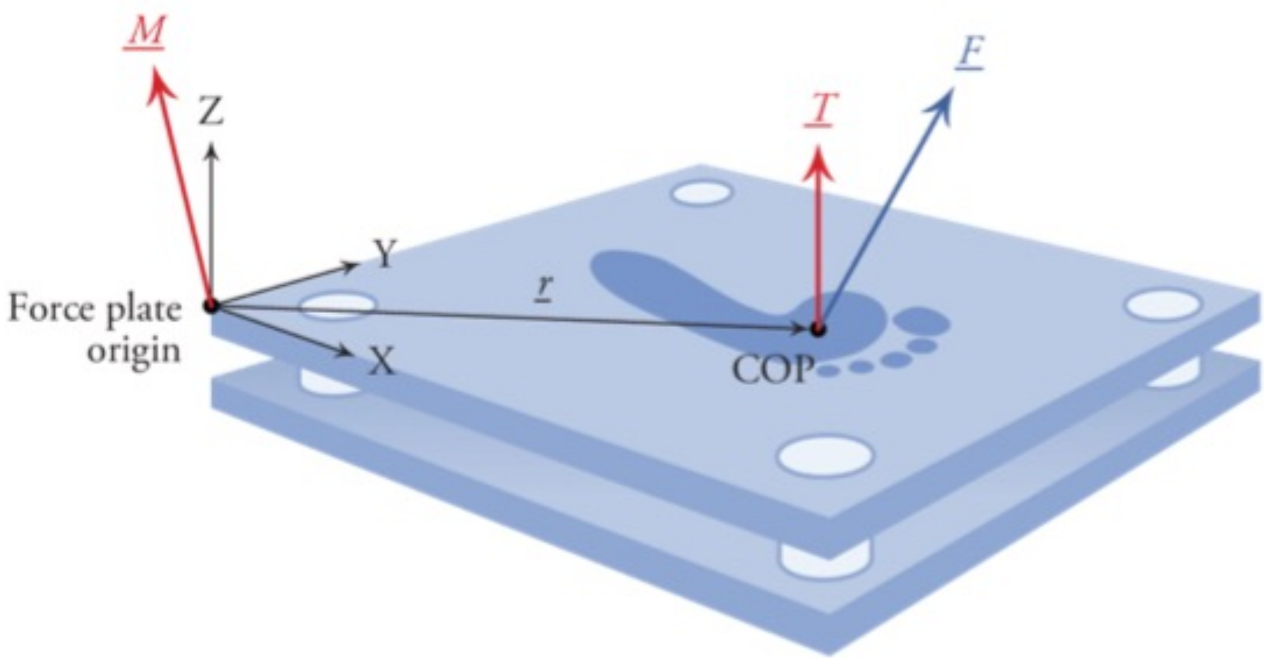
- Many different approaches
- Typical approach described here
  - Use ground reaction forces if they are available
  - Derive equations of motion
  - Describe algorithm to determine net joint forces and torques

Measuring ground reaction forces

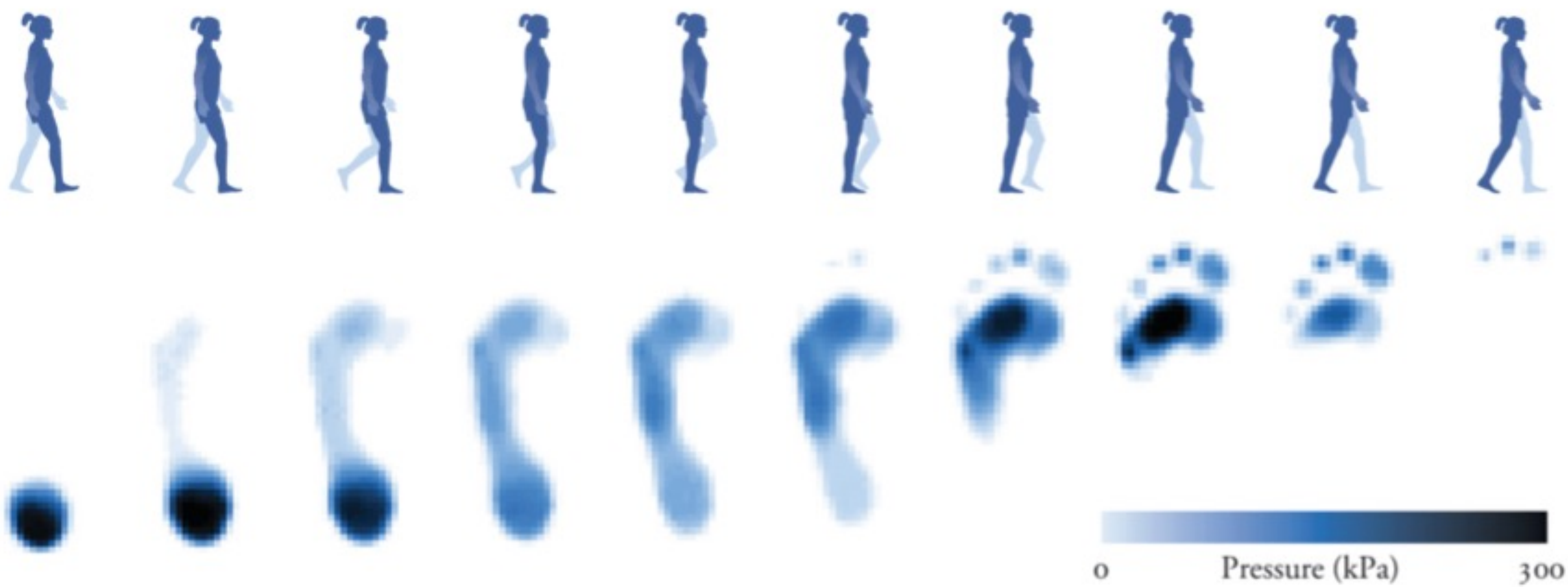


Force plate

- Measures ground reaction forces (GRFs)
  - Resultant force, **F** and total moment **M**

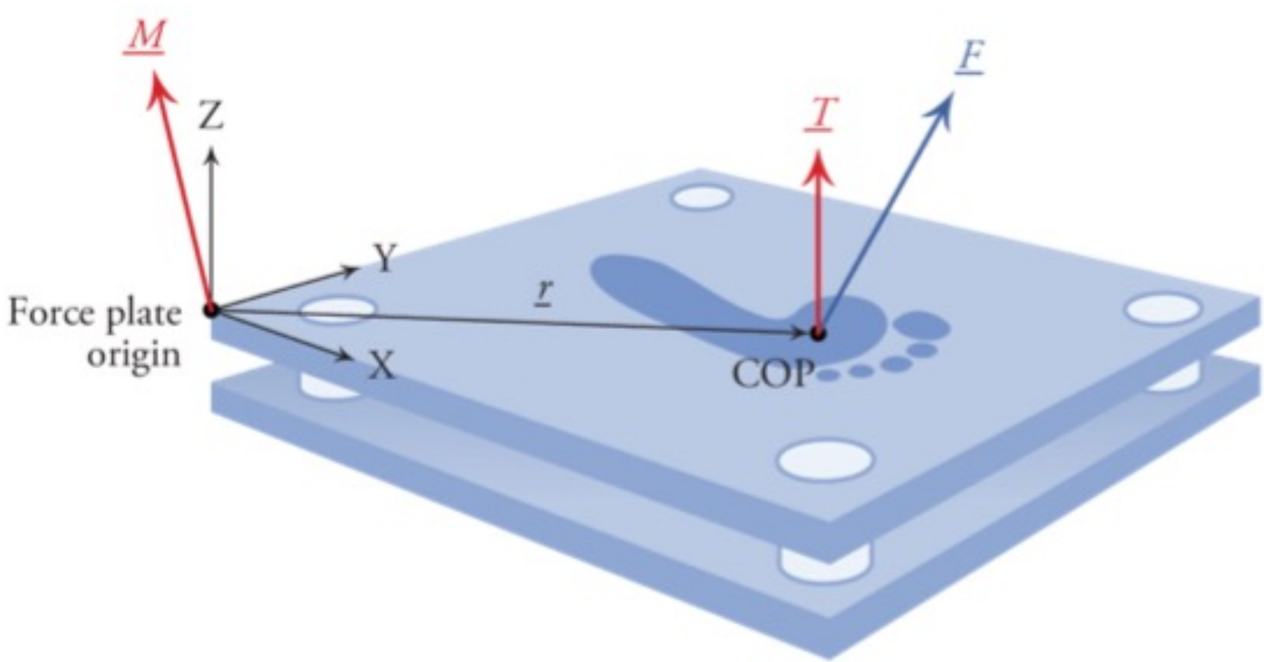


Center of pressure



Force plate

- Measures ground reaction forces (GRFs)
  - Resultant force, **F** and total moment **M**
  - Used to calculate center of pressure, COP, and free moment, **T**



Center of pressure

- Center of pressure (COP)
  - Location at which the GRF would be applied if the pressure distributed over the sole of the foot were concentrated at a point

$$\underline{M} = \underline{r} \times \underline{F} + \underline{T}$$

  - **M**: moment at FP origin
  - **r**: vector from the origin to the COP
  - **F**: resultant ground reaction force
  - **T** : equivalent torque applied at the COP

Center of pressure equations

$$\underline{M} = \underline{r} \times \underline{F} + \underline{T}$$

- 3 equations ($M_{x,y,z}$) but 6 unknowns ($r_{x,y,z}$, $T_{x,y,z}$)
- $r_z = 0$: COP remains on the force plate
- T_x and $T_y = 0$: surface friction can only generate moment about the vertical axis

$$\underline{r} = \begin{Bmatrix} -M_y / F_z \\ M_x / F_z \\ 0 \end{Bmatrix}$$

$$T_z = M_z - r_x F_y + r_y F_x$$

Inverse dynamics algorithm

- Given:
  - a representative computational model of a subject
  - The subject’s joint kinematics over time
  - measurements of the external forces applied to the subject
- Find:
  - the net joint forces and torques that must have been present to produce the given motion

Inverse dynamics (ID) algorithm

- Given:
  - a representative computational model of a subject
  - the subject's joint kinematics over time
  - measurements of the external forces applied to the subject

- Find:
  - the net joint forces and torques that must have been present to produce the given motion

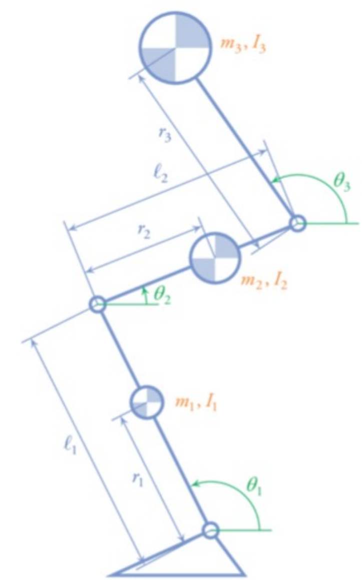
Apply laws of mechanics to each body segment in the model and compute the internal forces and torques acting at each joint.

Comments:

- Joint angles often estimated from optical marker trajectories using an IK algorithm, then smoothed and differentiated to obtain angular velocities and accelerations.
- Possible to perform ID analysis without measurements of external forces.

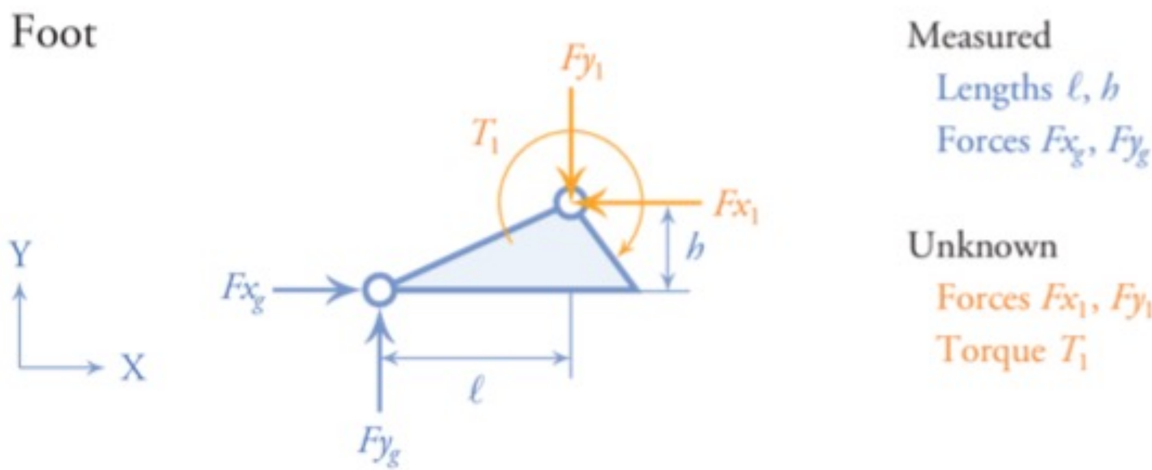
Example: ID during the squat

- Experimental setup and approximate sagittal plane model
- Planar model of a two-legged squat:
  - 4 rigid bodies connected by three pin joints (ankle,knee,hip)
  - symmetric
  - foot has negligible mass
  - lump left and right segments
  - lump head arms torso: HAT
  - model is scaled to subject



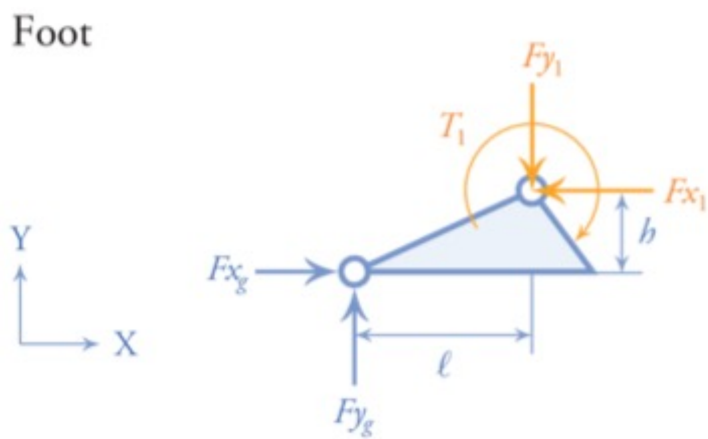
- Approach:
  - begin at feet and use a sequence of free body diagrams to compute net forces and torques applied at Each joint

Foot segment



- Newton's third law of motion
  - Convention: draw vectors in the positive direction on the proximal body and in the negative direction on the distal body.

Foot segment



Measured
Lengths ℓ, h
Forces Fx_g, Fy_g

Unknown
Forces Fx_1, Fy_1
Torque T_1

$$\sum M^k = I_b \ddot{\theta}_b + \underline{r}^k \times m_b \underline{a}_b$$

$$\sum F_X = m_0 \ddot{x}_0$$

$$Fx_g - Fx_1 = 0$$

$$\boxed{Fx_1 = Fx_g}$$

$$\sum F_Y = m_0 \ddot{y}_0$$

$$Fy_g - Fy_1 = 0$$

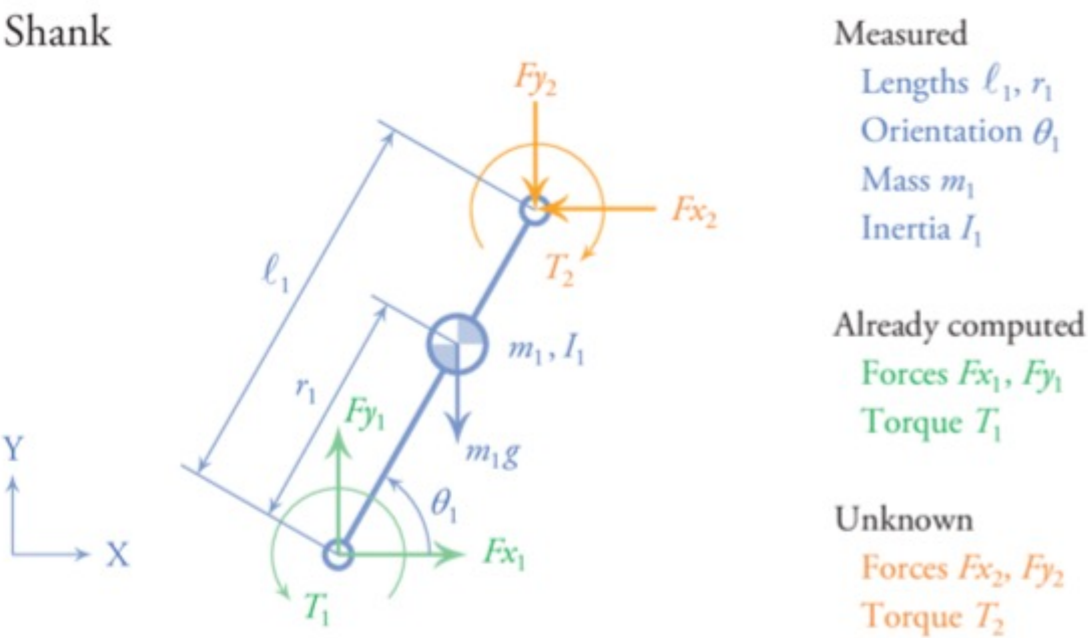
$$\boxed{Fy_1 = Fy_g}$$

$$\sum M^A = 0$$

$$Fx_g h - Fy_g l - T_1 = 0$$

$$\boxed{T_1 = Fx_g h - Fy_g l}$$

Shank segment



$$x_1 = r_1 c\theta_1$$

$$\dot{x}_1 = -r_1 s\theta_1 \dot{\theta}_1$$

$$\ddot{x}_1 = -r_1\left(s\theta_1\ddot{\theta}_1 + c\theta_1\dot{\theta}_1^2\right)$$

$$y_1 = r_1 s\theta_1$$

$$\dot{y}_1 = r_1 c\theta_1 \dot{\theta}_1$$

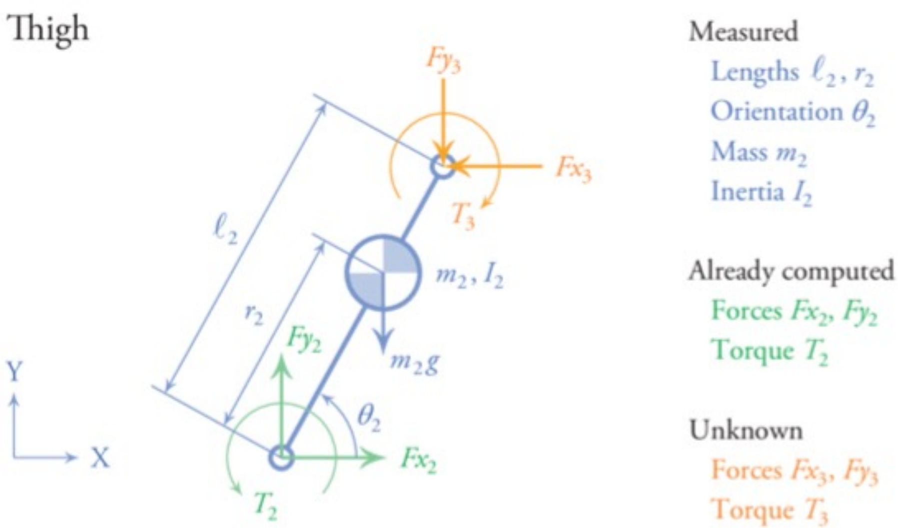
$$\ddot{y}_1 = r_1\left(c\theta_1\ddot{\theta}_1 - s\theta_1\dot{\theta}_1^2\right)$$

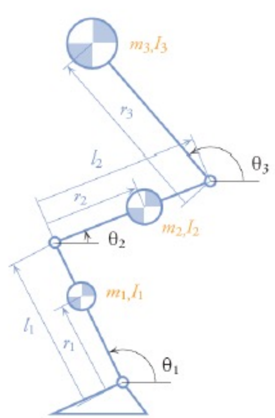
$$Fx_1 - Fx_2 = -m_1 r_1\left(s\theta_1\ddot{\theta}_1 + c\theta_1\dot{\theta}_1^2\right)$$

$$Fy_1 - Fy_2 - m_1 g = m_1 r_1\left(c\theta_1\ddot{\theta}_1 - s\theta_1\dot{\theta}_1^2\right)$$

$$T_1 - T_2 + Fx_1 r_1 s\theta_1 - Fy_1 r_1 c\theta_1 + Fx_2 d_1 s\theta_1 - Fy_2 d_1 c\theta_1 = I_1\ddot{\theta}_1 \qquad d_1 \equiv l_1 - r_1$$

Thigh segment



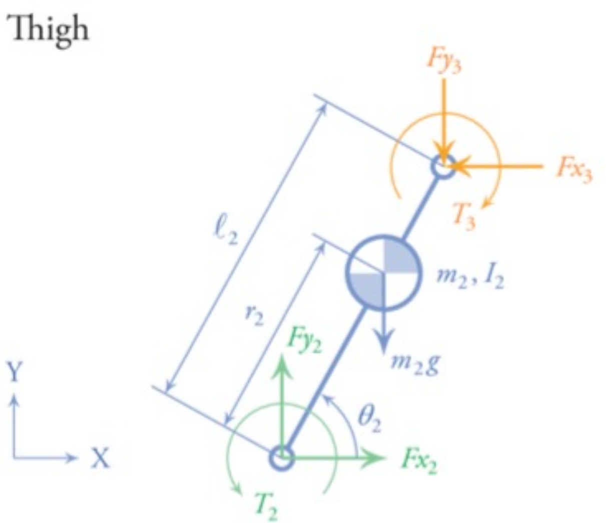


Need:

$$x_2, \dot{x}_2, \ddot{x}_2$$

$$y_2, \dot{y}_2, \ddot{y}_2$$

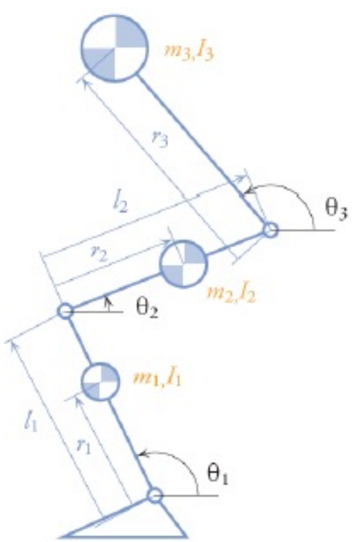
Thigh segment



Measured
- Lengths ℓ_2, r_2
- Orientation θ_2
- Mass m_2
- Inertia I_2

Already computed
- Forces Fx_2, Fy_2
- Torque T_2

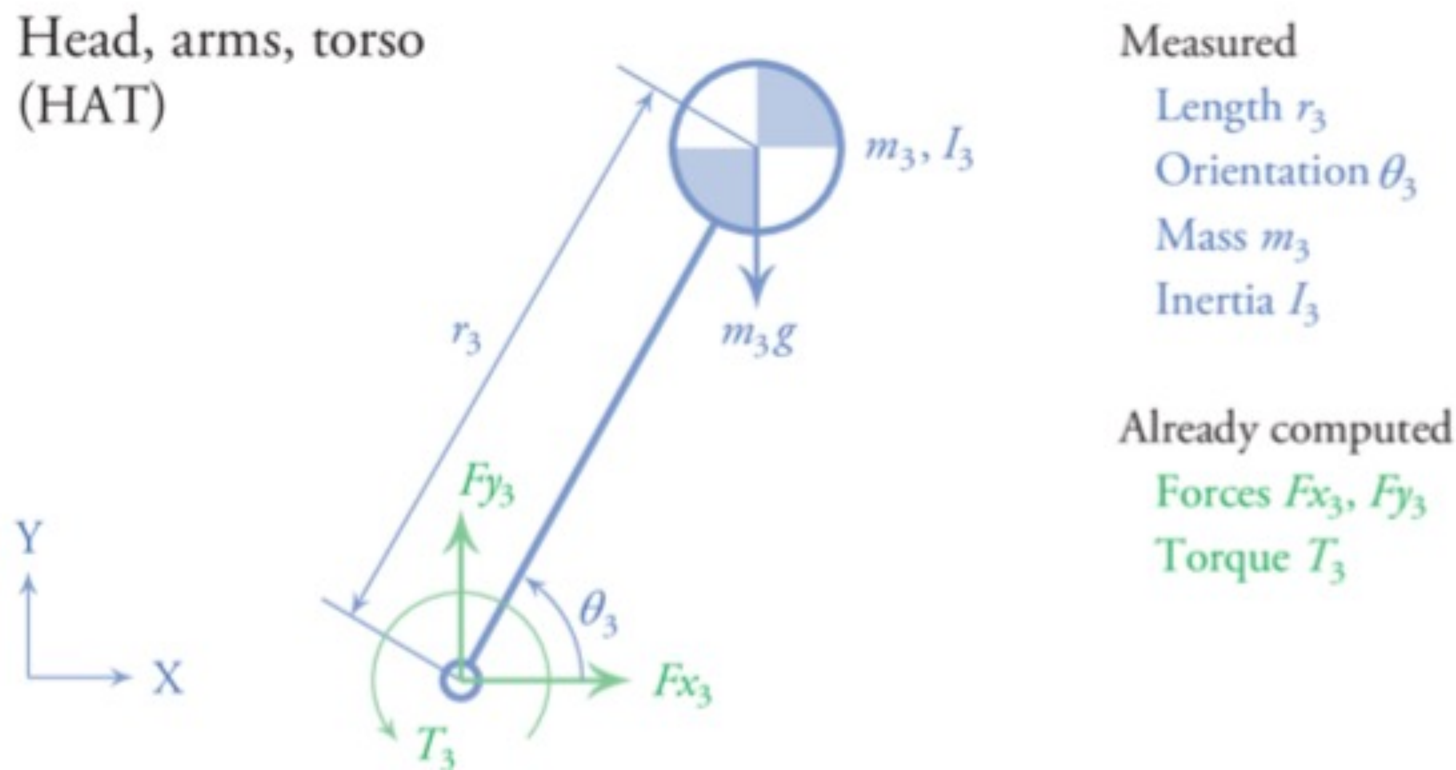
Unknown
- Forces Fx_3, Fy_3
- Torque T_3



$$x_2 = l_1 \mathrm{c}\theta_1 + r_2 \mathrm{c}\theta_2$$
$$\dot{x}_2 = -l_1 \mathrm{s}\theta_1 \dot{\theta}_1 - r_2 \mathrm{s}\theta_2 \dot{\theta}_2$$
$$\ddot{x}_2 = -l_1\left(\mathrm{s}\theta_1 \ddot{\theta}_1 + \mathrm{c}\theta_1 \dot{\theta}_1^2\right) - r_2\left(\mathrm{s}\theta_2 \ddot{\theta}_2 + \mathrm{c}\theta_2 \dot{\theta}_2^2\right)$$

$$y_2 = l_1 \mathrm{s}\theta_1 + r_2 \mathrm{s}\theta_2$$
$$\dot{y}_2 = l_1 \mathrm{c}\theta_1 \dot{\theta}_1 + r_2 \mathrm{c}\theta_2 \dot{\theta}_2$$
$$\ddot{y}_2 = l_1\left(\mathrm{c}\theta_1 \ddot{\theta}_1 - \mathrm{s}\theta_1 \dot{\theta}_1^2\right) + r_2\left(\mathrm{c}\theta_2 \ddot{\theta}_2 - \mathrm{s}\theta_2 \dot{\theta}_2^2\right)$$

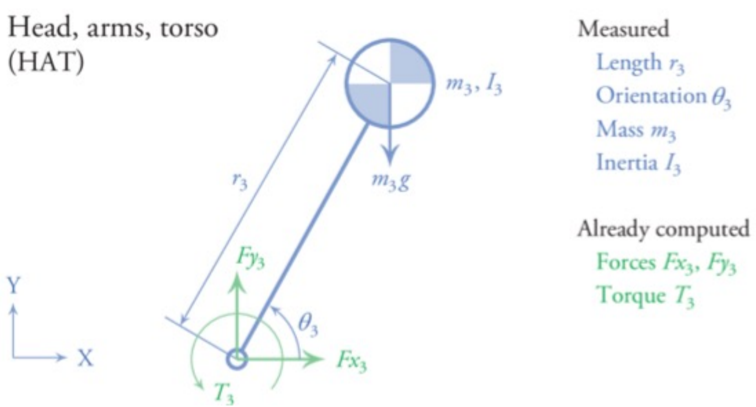
HAT segment



Head, arms, torso
(HAT)
m_3, I_3
r_3
$m_3 g$
Fy_3
Y
X
θ_3
Fx_3
T_3
Measured
Length r_3
Orientation θ_3
Mass m_3
Inertia I_3
Already computed
Forces Fx_3, Fy_3
Torque T_3

ID without GRFs

- If GRFs are unknown, we have 5 unknowns in the dynamic equations from the foot: $Fx_1, Fy_1, T_1, Fx_g, Fy_g$
- Undetermined system and need additional info
- One solution: replace foot dynamic equations with dynamic equations from HAT



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ r_3 s\theta_3 & -r_3 c\theta_3 & 1 \end{bmatrix} \begin{Bmatrix} Fx_3 \\ Fy_3 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} m_3 \ddot{x}_3 \\ m_3 \ddot{y}_3 + m_3 g \\ I_3 \ddot{\theta}_3 \end{Bmatrix}$$

Verifying dynamic consistency

- What if we were equally confident in both the GRFs and the HAT segment kinematics?

  →overdetermined system (more equations than unknowns)

  →use extra information to improve the model parameters

  →Can begin at foot segment and use the equations to obtain hip forces and torques, and then apply those forces to the HAT to predict linear and angular accelerations and compare to measured kinematics

$$\hat{\ddot{x}}_3 = \tfrac{1}{m_3} Fx_3$$

$$\hat{\ddot{y}}_3 = \tfrac{1}{m_3} Fy_3 - g$$

$$\hat{\ddot{\theta}}_3 = \tfrac{1}{I_3}\left(Fx_3 r_3 \mathrm{s}\theta_3 - Fy_3 r_3 \mathrm{c}\theta_3 + T_3\right)$$

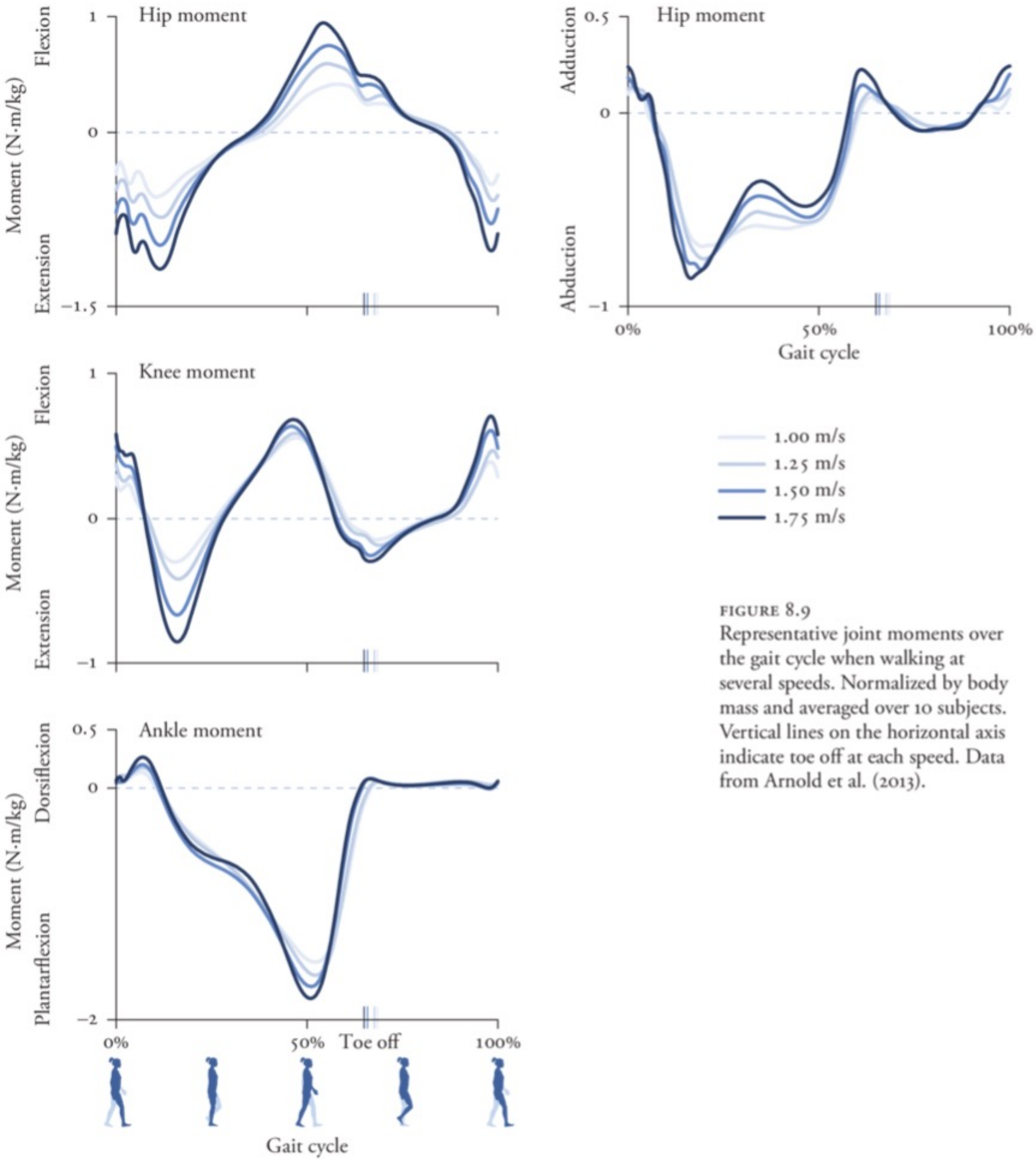
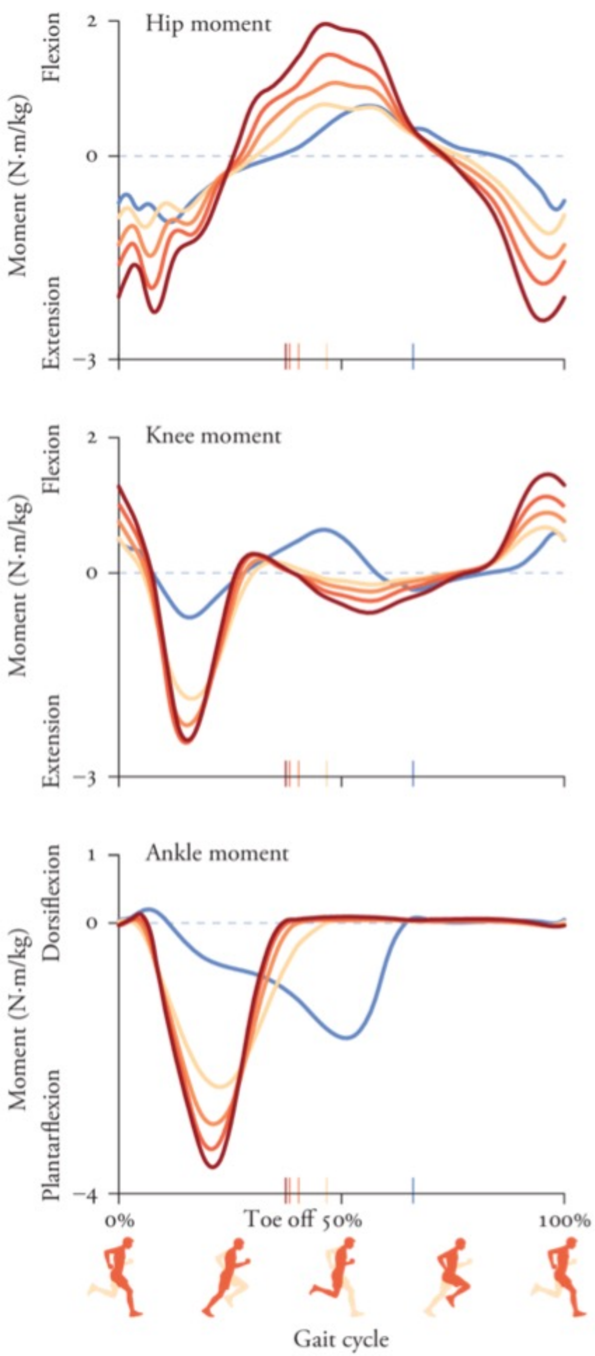


FIGURE 8.9
Representative joint moments over the gait cycle when walking at several speeds. Normalized by body mass and averaged over 10 subjects. Vertical lines on the horizontal axis indicate toe off at each speed. Data from Arnold et al. (2013).



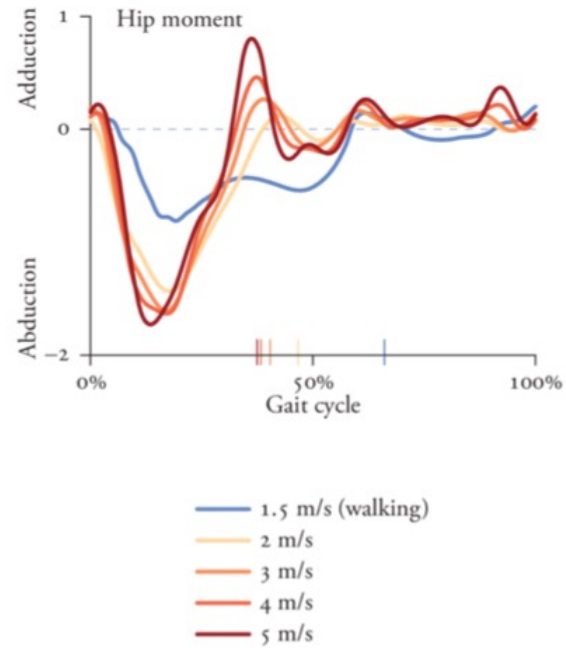
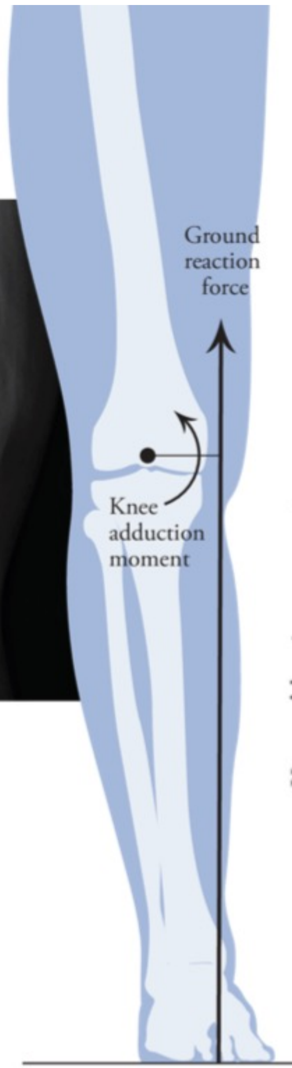


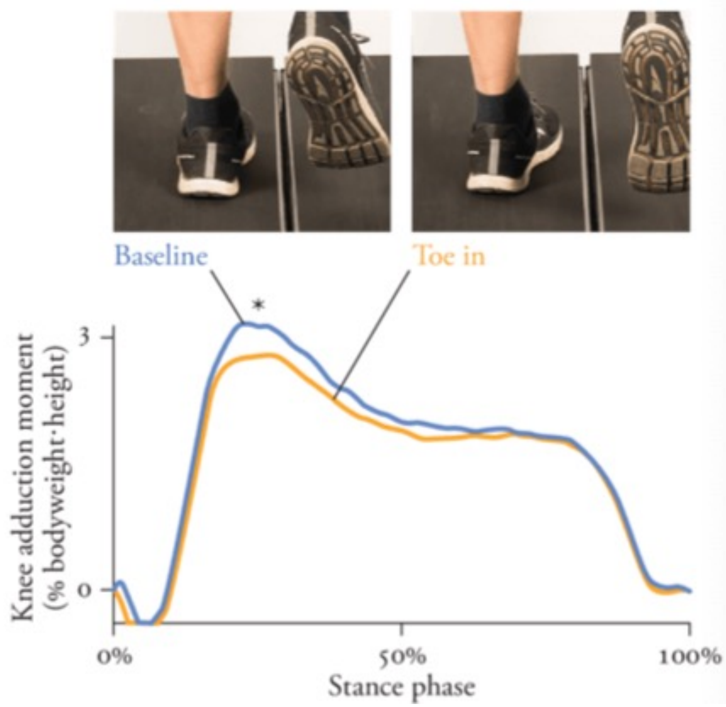
FIGURE 8.10
Representative joint moments over the gait cycle when running at several speeds (walking at 1.5 m/s from Figure 8.9 is shown for reference). Normalized by body mass and averaged over 10 subjects. Vertical lines on the horizontal axis indicate toe off at each speed. Data from Hamner and Delp (2013).

Application: gait retraining to reduce knee pain

- Knee osteoarthritis (OA) affects about 20% of adults over the age of 45
- Analysis of knee dynamics can help us understand how OA develops and how best to treat it

Application: gait retraining to reduce knee pain





GRF generates an external knee adduction moment during stance

Moment loads medial compartment of the knee, leading to it supporting 2-3x more weight than the lateral compartment

Peak occurs in early stance and is linked to presence, severity and progression of medial compartment OA

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