<u>HOMEWORK #01</u>

Course: **<u>MCEN 5228 – Modeling of Human Movement</u>**

Professor: **<u>Alaa Ahmed</u>**

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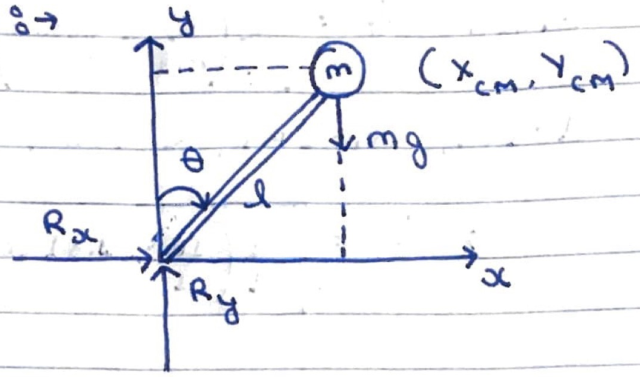
Problem 1) :→



a) Using Right angle triangle properties,

$$Y_{CM} = l\cos\theta \quad -①$$

$$X_{CM} = l\sin\theta \quad -②$$

Taking derivatives of both ① and ②, we get,

$$\dot{Y}_{CM} = -l(\sin\theta)\dot{\theta}$$

$$\dot{X}_{CM} = l(\cos\theta)\dot{\theta}$$

Taking derivatives again, we get,

$$\ddot{Y}_{CM} = -l\left[\sin\theta.\ddot{\theta} + \cos\theta.\dot{\theta}^2\right]$$

$$\ddot{X}_{CM} = l\left[\cos\theta.\ddot{\theta} - \sin\theta.\dot{\theta}^2\right]$$

b) Taking forces along X-direction,

$$R_x + m\ddot{X}_{CM} = 0$$

$$\Rightarrow R_x = -m\ddot{X}_{CM}$$

$$\therefore R_x = -ml\left[(\cos\theta)\ddot{\theta} - (\sin\theta)\dot{\theta}^2\right]$$

$$\boxed{R_x = -ml\cos\theta\,\ddot{\theta} + ml\sin\theta\,\dot{\theta}^2}$$

Taking forces along Y-direction,

$$R_y + m\ddot{Y}_{CM} = mg$$

$$\Rightarrow R_y = mg - m\ddot{Y}_{CM}$$

$$\Rightarrow \boxed{R_y = mg + ml\sin\theta\,\ddot{\theta} + ml\cos\theta\,\dot{\theta}^2}$$

c) Given, $\dot{\theta} = \sqrt{\frac{2}{l}\left(\frac{C}{ml} - g\cos\theta\right)}$

where, $l = 1m$, $m = 1kg$, $C = 10J$, $g = 9.81\ m/s^2$

$$\Rightarrow \quad \dot{\theta} = \sqrt{\frac{2}{1}\left(\frac{10}{1} - 9.81\cos\theta\right)}$$

$$\Rightarrow \quad \dot{\theta} = \sqrt{20 - 19.62\cos\theta}$$

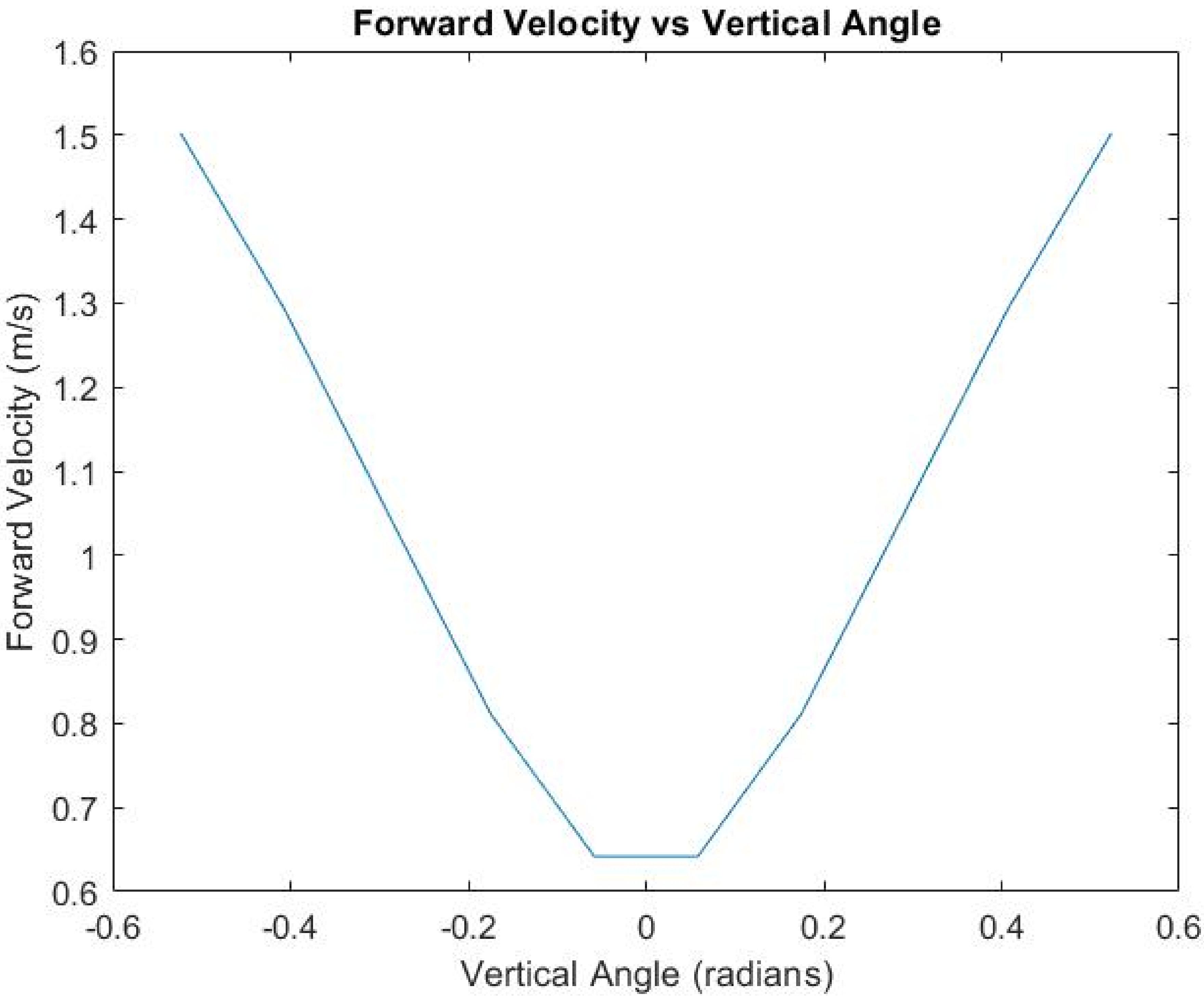
As, from the part a), we got,

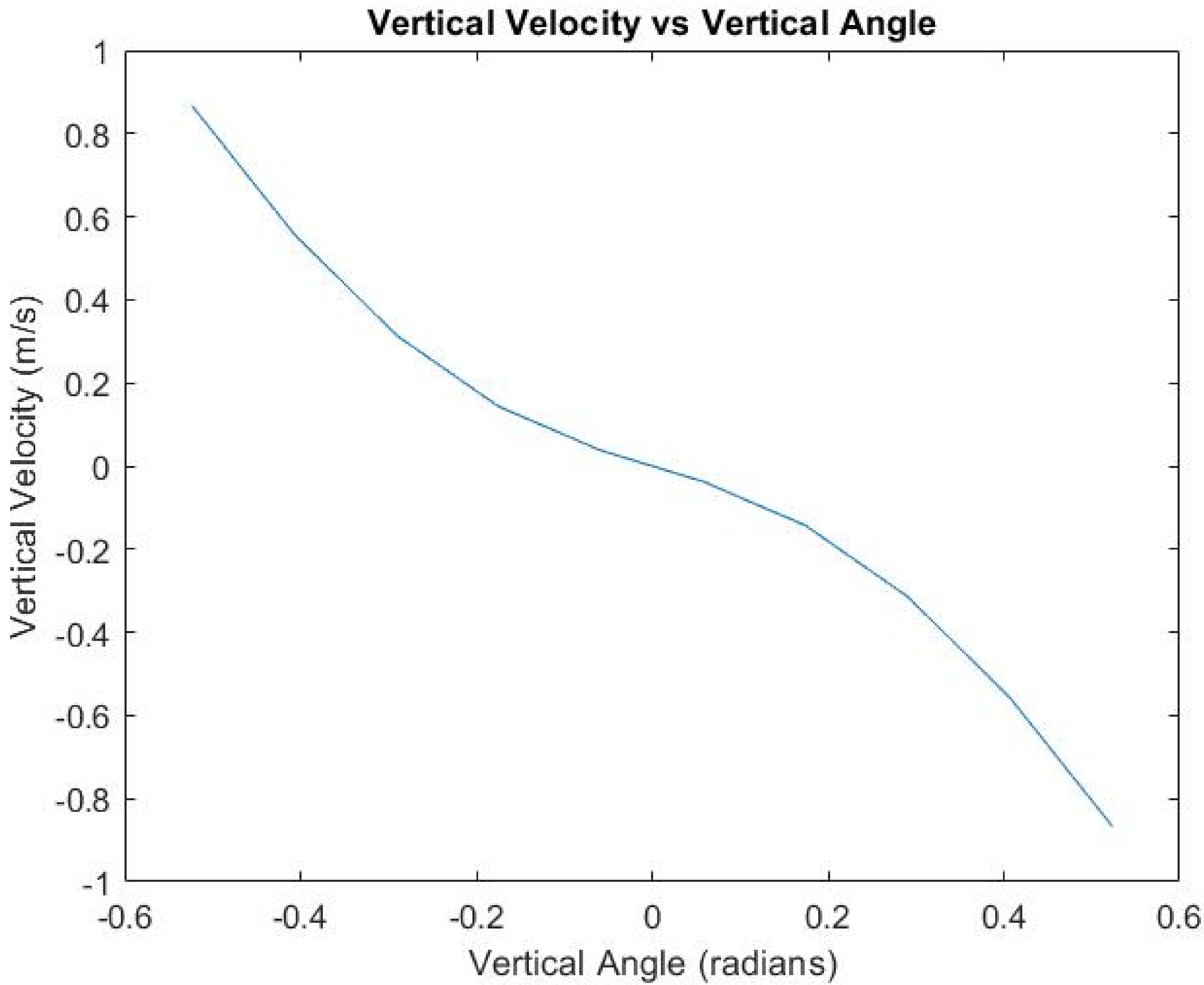
$$\dot{Y}_{CM} = -l\sin\theta.\dot{\theta}$$
$$\dot{X}_{CM} = l\cos\theta.\dot{\theta}$$

$$\therefore \quad \dot{Y}_{CM} = -(\sin\theta)\sqrt{20 - 19.62\cos\theta} \quad — (3)$$
$$\dot{X}_{CM} = (\cos\theta)\sqrt{20 - 19.62\cos\theta} \quad — (4)$$

Discussion:

Forward velocity as well as vertical velocity of Centre of Mass (COM) of the body while walking fluctuates with each stance. In the 1st half of stance, forward velocity ($\dot{X}_{CM}$) decreases and ~~the~~ in the second half of stance $\dot{X}_{CM}$ increases. ~~Also~~ However, the height of the COM from level ground increases in 1st half of stance and decreases in the 2nd half of stance. This phenomenon can be clearly seen in the plots. The vertical velocity ($\dot{Y}_{CM}$) undergoes change in sign with keeping the magnitude values same over the range of angle as it varies from -30° to 30°. Thus, inverted pendulum to an extent can be used to model level walking but for capturing more of the dynamical aspects we have to use Double Pendulum.


Forward Velocity vs Vertical Angle
Forward Velocity (m/s)
Vertical Angle (radians)
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


Vertical Velocity vs Vertical Angle
Vertical Velocity (m/s)
Vertical Angle (radians)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

d) Potential energy due to gravity = $mg\,Y_{CM}$

$$\Rightarrow \text{P.E.} = mg\,Y_{CM} = mg\,l\cos\theta$$

$$\Rightarrow \text{P.E.} = 9.81\cos\theta \quad — (5)$$

Forward kinetic energy $= \frac{1}{2} \cdot m \cdot (\dot{x}_{CM})^2$

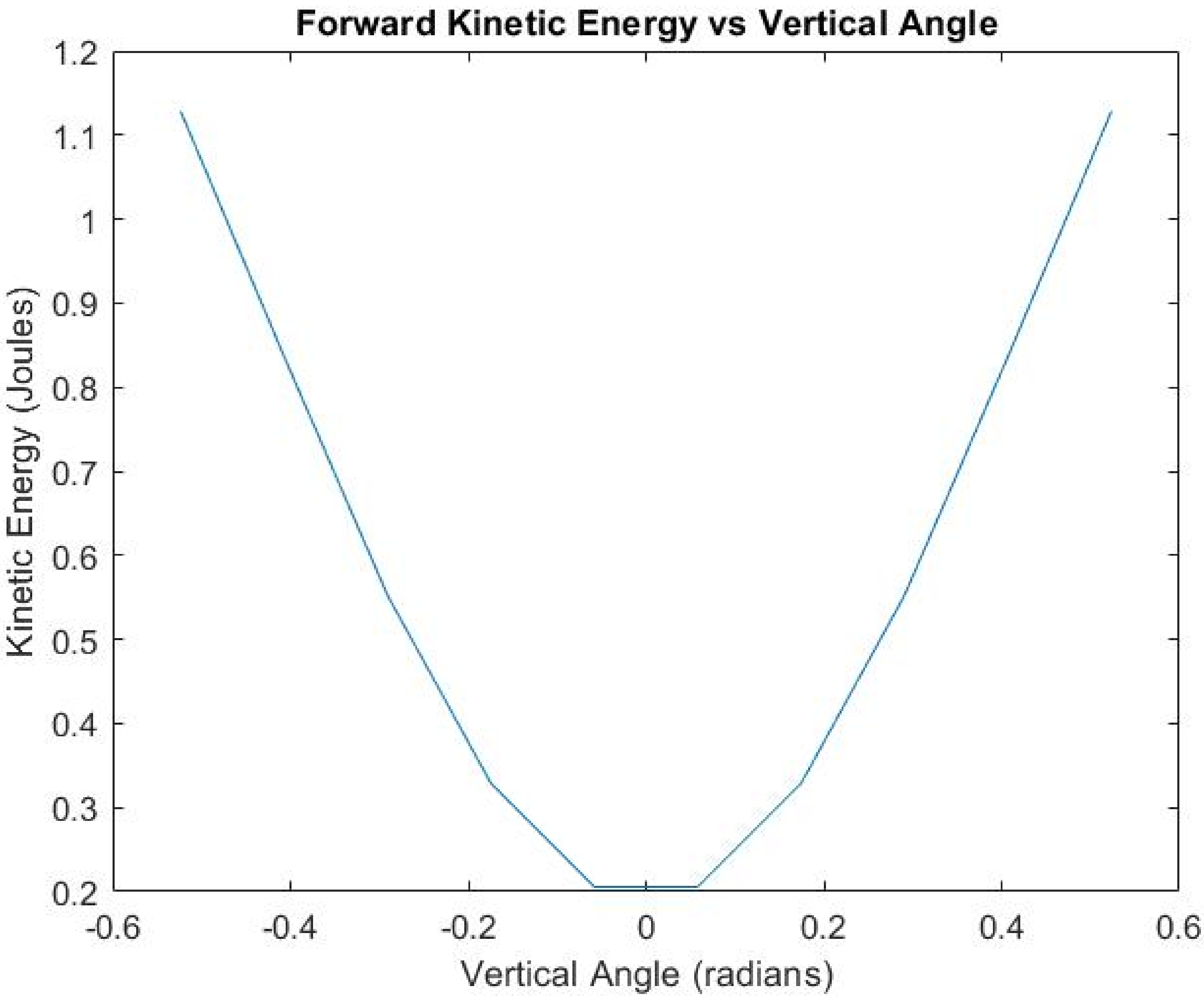
$$\Rightarrow KE = \frac{1}{2} m \left(\dot{x}_{CM}\right)^2 = \frac{1}{2} m \left(l\cos\theta\,\dot{\theta}\right)^2$$

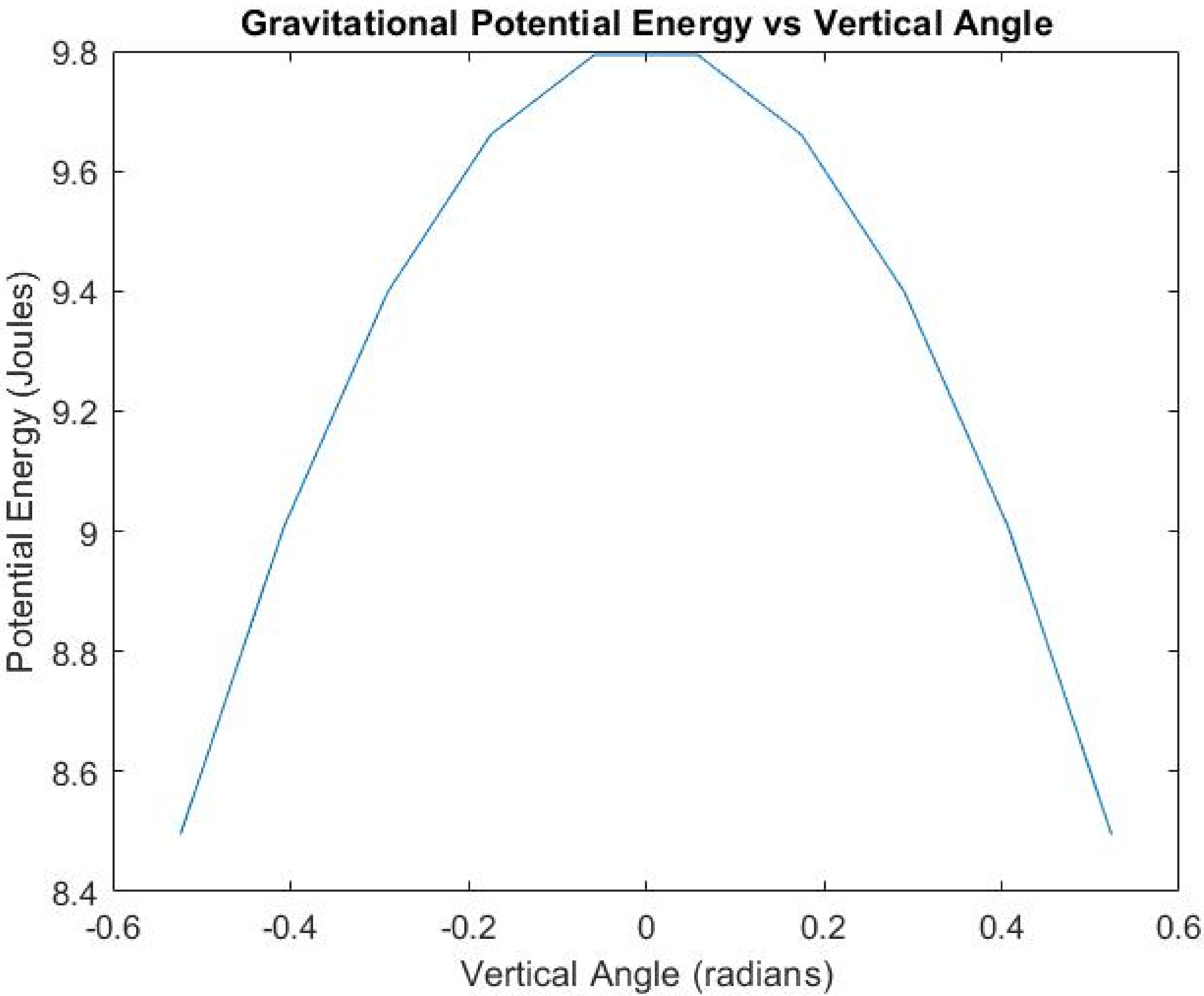
$$\Rightarrow KE = \frac{\cos^2\theta\,(20 - 19.62\cos\theta)}{2} \quad \text{— (from (4) eq}^n\text{)}$$

— (5)

<u>Discussion</u>:

Kinetic energy and Gravitational potential energy fluctuate within each stance of walking. In the first half of stance, $\dot{x}_{CM}$ decreases and height of centre of mass of body (COM) increases. Therefore, kinetic energy gets converted to potential energy. Although, in the second half of stance, $\dot{x}_{CM}$ increases and height of COM decreases. So, potential energy gets converted to kinetic energy. These two phenomenon can be clearly visualized by the plots for KE and GPE of a inverted pendulum. The plots shows that as vertical angle moves from -30° to 30°, KE and GPE follows the ~~same~~ similar energetics as of the energetics of level walking.


Forward Kinetic Energy vs Vertical Angle
Kinetic Energy (Joules)
Vertical Angle (radians)
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


Gravitational Potential Energy vs Vertical Angle
Potential Energy (Joules)
Vertical Angle (radians)
9.8
9.6
9.4
9.2
9
8.8
8.6
8.4
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

e) Following assumptions were made in modeling gait as an inverted pendulum:-

(i) Mass is assumed to be concentrated at a single point.

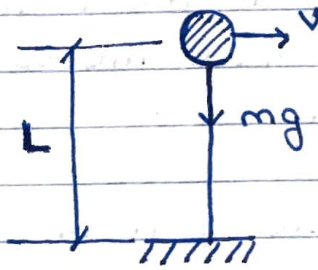
(ii) The link is considered massless.

(iii) We consider motion in a single plane, sideways motion is neglected.

Problem 2 :-

a) For this analysis it is given that to assume the same inverted pendulum with vertical angle $\theta = 0^\circ$, and that walk-run transition happens when vertical component of the ground reaction force equals zero.

$\therefore$



In this position, centripetal force to the inverted pendulum is provided by the weight of the pendulum.

$$\Rightarrow \quad mg = \frac{mv^2}{L}$$

$$\Rightarrow \quad g = \frac{v^2}{L} \qquad \Rightarrow v \leq \sqrt{gL}$$

$$\Rightarrow \quad \boxed{v_{max} = \sqrt{gL}} \quad — ①$$

Equating these forces, we find the max^m walking speed (v_{max}) for an inverted pendulum. At speeds above v_{max}, the foot will leave the ground and $\therefore$ walk-run transition occurs at $Fr = 1$.

Since, $Fr = \frac{v^2}{gL}$

from ① $\Rightarrow$ $Fr = \frac{v^2}{gL} = \frac{v_{max}^2}{gL} = \frac{gL}{gL} = 1$

b) Given that, Dr. Evil has a leg length of 0.9 m and Mini-Me is an exact 1/8 replica of Dr. Evil. Therefore, leg length of Mini-Me will be 0.9/8 = 0.1125 m.

From our equation (1), we know that,

$$v = \sqrt{gL}$$

Therefore,

$$\frac{v_{Dr.Evil}}{v_{Mini-Me}} = \sqrt{\frac{L_{Dr.Evil}}{L_{Mini-Me}}} \quad (\because v \propto \sqrt{L})$$

$$\Rightarrow \frac{v_{Dr.Evil}}{v_{Mini-Me}} = \sqrt{\frac{0.9}{(0.9/8)}} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow v_{Dr.Evil} = 2\sqrt{2}\,(v_{Mini-Me})$$

Walking speed of Dr. Evil is $2\sqrt{2}$ (or 2.8284) times more than that of Mini-Me because of shorter legs of Mini-Me. Therefore, Mini-Me is tired as he has to run to keep up with Dr. Evil who is walking at a comfortable pace to feed the sharks.

c) Given that, Moon-Me, a lunar robot looks, walks, and behaves exactly as Dr. Evil on Earth. Therefore, the leg-length of both Dr. Evil and Moon-Me will be same.

From our equation ①, we know that,

$$v = \sqrt{gL}$$

Therefore,

$$\frac{v_{Dr.Evil}}{v_{Moon\text{-}Me}} = \sqrt{\frac{g_{Earth}}{g_{Moon}}} \quad (\because v \propto \sqrt{g})$$

$$\Rightarrow \frac{v_{Dr.Evil}}{v_{Moon\text{-}Me}} = \cancel{\sqrt{\frac{1}{6}}} \sqrt{\frac{6}{1}}$$

$$(\because g_{Earth} / g_{Moon} = 6)$$

$$\Rightarrow v_{Dr.Evil} = \sqrt{6}\,(v_{Moon\text{-}Me}) \approx 2.45\, v_{Moon\text{-}Me}$$

Therefore, a normal walking speed on Earth will cause Moon-me to lift off the surface of moon. The testing done on moon by the lunar engineering team will yield different results. To compensate for this, the leg length of the robot has to be made 6 times that of Dr. Evil and that will cause failure in design and manufacturing aspects of the robot.