Problem 7 :

a) Given :

Hamstrings hip extension moment arm = 5 cm
Hamstrings knee flexion moment arm = 4 cm
Vasti knee extension moment arm = 4 cm
Soleus ankle extension moment arm = 2.5 cm

Torque = Force x Moment Arm

$\therefore$ Hamstrings hip extension force = $F_{ham-hip-ext}$ =

$$= \frac{-T_3}{5}$$

Hamstrings knee extension moment arm

$$= \frac{-\left\{\left[M_3 g + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)\right] r_3 C\theta_3 - \left[-M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2)\right] r_3 S\theta_3 + I_3 \ddot{\theta}_3\right\}}{5} \rightarrow (1)$$

Hamstrings knee flexion force = $T_2 / 4$

Hamstrings knee flexion moment arm

$$F_{ham-knee-flex} = \frac{\left[T_1 + (F_{gx} r_1 S\theta_1) - (F_{gy} r_1 C\theta_1) + (F_{x_2} d_1 S\theta_1) - (F_{y_2} d_1 C\theta_1) - I_1 \ddot{\theta}_1\right]}{4} \rightarrow (2)$$

where F_{x_2} and F_{y_2} are ~~eqs~~ eq^n (4) and (5) from solved Problem 1.

Hamstrings knee flexion moment arm

$$\boxed{\text{Vasti knee extension force} = \frac{\left[T_2 + (F_{ham-knee-flex} \times 4)\right]}{4}}$$

4 → Vasti knee extension moment arm

where T_2 is from eq^n ⑥ of Prob 1 solved
and $F_{ham-knee-flex}$ is from eq^n ② of this solved problem.

Soleus ankle extension force = $F_{sol-ankle-ext}$ =

$$\boxed{F_{sol-ankle-ext} = \frac{-T_1}{2.5}}$$

2.5 → Soleus ankle extension moment arm

where T_1 is from eq^n ③ of ~~from~~ Prob ① solved

$$T_1 = F_{gx} h - F_{gy} a$$

Which muscle should you beef up for jumping?

→ Quadriceps and hamstrings are primary thrusters when performing a vertical jump. As, hamstrings are acting on two joints i.e. hip and knee, it also becomes reasonable to beef up hamstrings for a vertical leap. The plot shows that hamstring forces are less as compared to quadriceps, therefore if we beef up hamstrings we could perform a better vertical jump. But if you want to jump higher, it's equally important to awaken and strengthen assisting muscles - ~~your~~ calves, the muscles around your hips, and your glutes.

b) :- Static optimization resolves the net joint moments into individual muscle forces at each instant in time. The muscle forces are resolved by minimizing the sum of squared muscle activations. The static optimization method uses the known motion of the model to solve the equations of motion for the unknown generalized forces (e.g. joint torques). The forces could be subjected to one of the following muscle activation-to-force conditions:

Ideal force generators :-

$$\sum_{m=1}^{n} (a_m F_m^o) r_{mj} = \tau_j$$

or, constrained by force-length-velocity properties:

$$\sum_{m=1}^{n} [a_m f(F_m^o, l_m, v_m)] r_{m,j} = \tau_j$$

while we have to minimize the objective function:

$$J = \sum_{m=1}^{n} (a_m)^p \qquad \left(a_m = \frac{f_m}{F_m^{max}} \right)$$

where n is the number of muscles in the body; a_m is the activation level of muscle m at a discrete time step; F_m^o is its maximum isometric force; l_m is it's length; v_m is its shortening velocity; $f(F_m^o, l_m, v_m)$ is its force-length-velocity surface; r_{mj} is its moment arm about the j^{th} joint axis; τ_j is the generalized force acting about the j^{th} joint axis; and p is a user-defined constant.

→

In static optimization, the question arises which muscles should be activated to generate desired net joint moments during the jumping activity. We can find the solution to this problem by assuming no activity for dorsiflexor and also we have to assume that each muscle will generate same amount of force. This is how we can solve for the number of unknowns to get the minimization of cost function. However, this approach is not physiologically reasonable.

The goal of static optimization is to solve for muscle activations that produce the dynamics of an observed motion. Since there are more muscles than the DOF in the human body, the problem of many possible solution exists, hence the need for optimization.