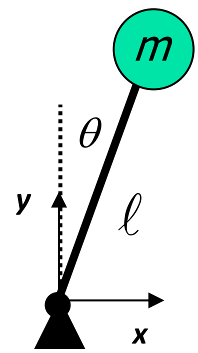
MCEN 4228/5228
Modeling of Human Movement
HW01
(K): Knowledge Problem, (C): Challenge Problem, (EX): Extra Credit (for Undergrads)

1. (K) There are several biomechanists who think that an inverted pendulum alone is not enough to describe the dynamics of walking. Let's explore this "controversy" while analyzing a simple inverted pendulum. Below is a picture of an inverted pendulum of mass "*m*" and length "ℓ" (assuming the leg has no mass and only serves to transmit force). Its angular position with respect to the *y* (vertical) axis is given by θ.



a. Based on the figure above, derive expressions for the *x* and *y* positions of the mass, *m*, as a function of the length of the limb (ℓ) and rotation angle (θ).

b. Derive expressions for the vertical and horizontal ground reaction forces as a function of the length of the limb (ℓ), mass (*m*), angular position (θ), angular velocity, ($\dot{\theta}$), and angular acceleration ($\ddot{\theta}$). Assume that gravity is directed in the -*y* direction.

c. Given the following:

$$\dot{\theta} = \sqrt{\frac{2}{\ell}\left(\frac{C}{m\ell} - g\cos\theta\right)} \qquad \ell = 1\ m \qquad m = 1\ kg \qquad C = 10\ J \qquad g = 9.81 m/s^2$$

Compute and plot the vertical and forward velocity of the mass as θ varies over a range from θ = -30° to θ = 30°. Briefly discuss how these plots relate to the motion paths shown in the determinants of gait discussion from class.

d. For the same information given in part **c**, plot the gravitational potential energy and the forward kinetic energy of the mass as θ varies from θ = -30° to θ = 30°. Briefly discuss your results.

e. What are the assumptions made in modeling gait as an inverted pendulum?

2. (C) The Froude number (sounds like "food" with an "r") is a useful parameter to study how locomotion changes with body size. It is a dimensionless variable that relates a scalar dimension of the body (typically leg length) to the forward velocity of the body. Theoretically, animals of different sizes that rely on pendulum-like mechanics for gait will use the same form of locomotion at a given Froude number (e.g., a T-Rex and a chicken traveling at the same Froude number would look the same). The Froude number is defined as:

$$Fr = \frac{v^2}{g \cdot \ell}$$

where: v = the speed of movement (in m/s), g = the acceleration due to gravity (in m/s^2), and
ℓ = the effective length of the limb (in m)

a. Using your results from problem 2b (and assuming the same pendulum model described in problem 2), show (using equations) why the walk-run transition theoretically occurs at Fr = 1. (hint: for this analysis assume that $\theta = 0$, and assume the walk-run transition happens when the vertical component of the ground reaction force equals zero).

b. Dr. Evil (a somewhat normal adult male with a leg length of 0.9m) and Mini-Me (an exact 1/8 replica of Dr. Evil) need to go to the other side of their secret lair in Nevada to feed their sharks and repair some laser beams. Dr. Evil demands that Mini-Me hold his hand so they can remain side-by-side as they travel to the shark tank. Dr. Evil walks to the shark tank at optimum walking speed (Fr = 0.25). Once arriving at the destination, Mini-Me says that he is too tired from running to keep up with Dr. Evil to feed the sharks. How can you explain this? Use equations in your explanation.

c. Dr. Evil is bored with only having a miniature version of himself and orders the biomechanical engineers on his lunar base to build a lunar robot (named "Moon-Me") that looks, walks, and behaves exactly how Dr. Evil does on Earth. Why is the lunar engineering team doomed to failure? Use equations to make your argument.

To Submit Online:
1. ONE homework document as a .pdf: handwritten, scanned or a combination.