Quantifying Movement Example

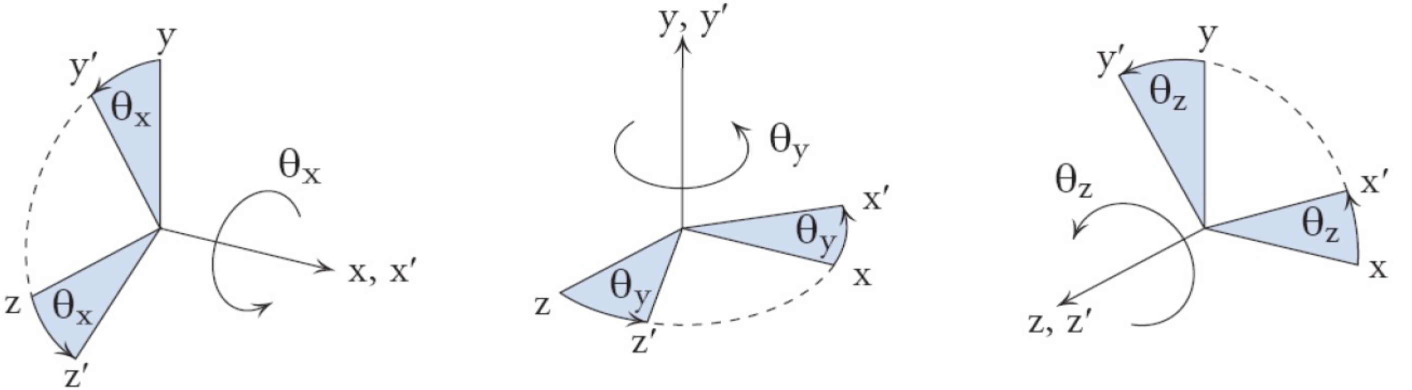
Transformation matrices

$$R^x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$R^y = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}$$

$$R^z = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Use transformation matrices to describe anatomical joint angles from mocap data.
- Any spatial rotation can be expressed as a sequence of elementary rotations.



Euler angles

- Rotation matrix that related orientations of the original and final frames is:

$$R = R^z\Big|_{\theta_z=\psi} R^y\Big|_{\theta_y=\theta} R^x\Big|_{\theta_x=\phi}$$

$$= \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) \\ \cos(\theta)\sin(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{r}_{xx} & \mathrm{r}_{xy} & \mathrm{r}_{xz} \\ \mathrm{r}_{yx} & \mathrm{r}_{yy} & \mathrm{r}_{yz} \\ \mathrm{r}_{zx} & \mathrm{r}_{zy} & \mathrm{r}_{zz} \end{bmatrix}$$

$$R = R^z\Big|_{\theta_z=\psi} R^y\Big|_{\theta_y=\theta} R^x\Big|_{\theta_x=\phi}$$

$$= \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) \\ \cos(\theta)\sin(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{r}_{xx} & \mathrm{r}_{xy} & \mathrm{r}_{xz} \\ \mathrm{r}_{yx} & \mathrm{r}_{yy} & \mathrm{r}_{yz} \\ \mathrm{r}_{zx} & \mathrm{r}_{zy} & \mathrm{r}_{zz} \end{bmatrix}$$

- Can calculate the z-y-x Euler angles from an arbitrary rotation matrix by equating last two equations.

$$\phi = \operatorname{atan2}(r_{32}, r_{33})$$ Flexion/extension

$$\theta = \operatorname{atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$$ Varus/valgus

$$\psi = \operatorname{atan2}(r_{21}, r_{11})$$ Internal/external rotation

XYZ Euler angles

$$ {}^{A}R^{B}_{X'Y'Z'}(\alpha,\beta,\gamma) = \begin{bmatrix} c\beta c\gamma & -c\beta s\gamma & s\beta \\ s\alpha s\beta c\gamma + c\alpha s\gamma & -s\alpha s\beta s\gamma + c\alpha c\gamma & -s\alpha c\beta \\ -c\alpha s\beta c\gamma + s\alpha s\gamma & c\alpha s\beta s\gamma + s\alpha c\gamma & c\alpha c\beta \end{bmatrix} $$

$$ \beta = \operatorname{atan2}(r_{13}, \sqrt{r_{11}^2 + r_{12}^2}) $$

Varus/valgus

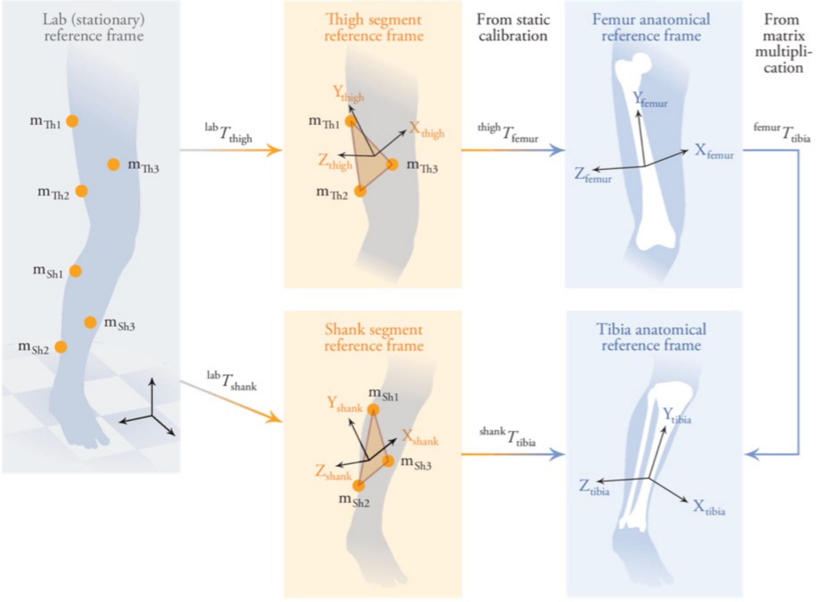
$$ \alpha = \operatorname{atan2}\left(-\frac{r_{23}}{c\beta}, \frac{r_{33}}{c\beta} \right) $$

Flexion/extension

$$ \gamma = \operatorname{atan2}\left(-\frac{r_{12}}{c\beta}, \frac{r_{11}}{c\beta} \right) $$

Internal/external rotation

Calculating anatomical joint angles

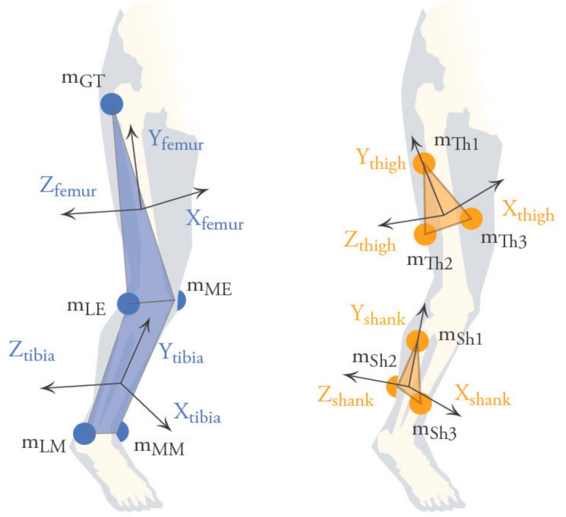


$$ {}^{acs.thigh}T_{acs.shank} = {}^{acs.thigh}T_{lcs.thigh}\ {}^{lcs.thigh}T_{gcs}\ {}^{gcs}T_{lcs.shank}\ {}^{lcs.shank}T_{acs.tibia} $$

${}^{gcs}T_{lcs.shank}$: Calculated from shank markers in lab reference frame (LCS to GCS)
${}^{gcs}T_{lcs.thigh}$: Calculated from thigh markers in lab reference frame (LCS to GCS)

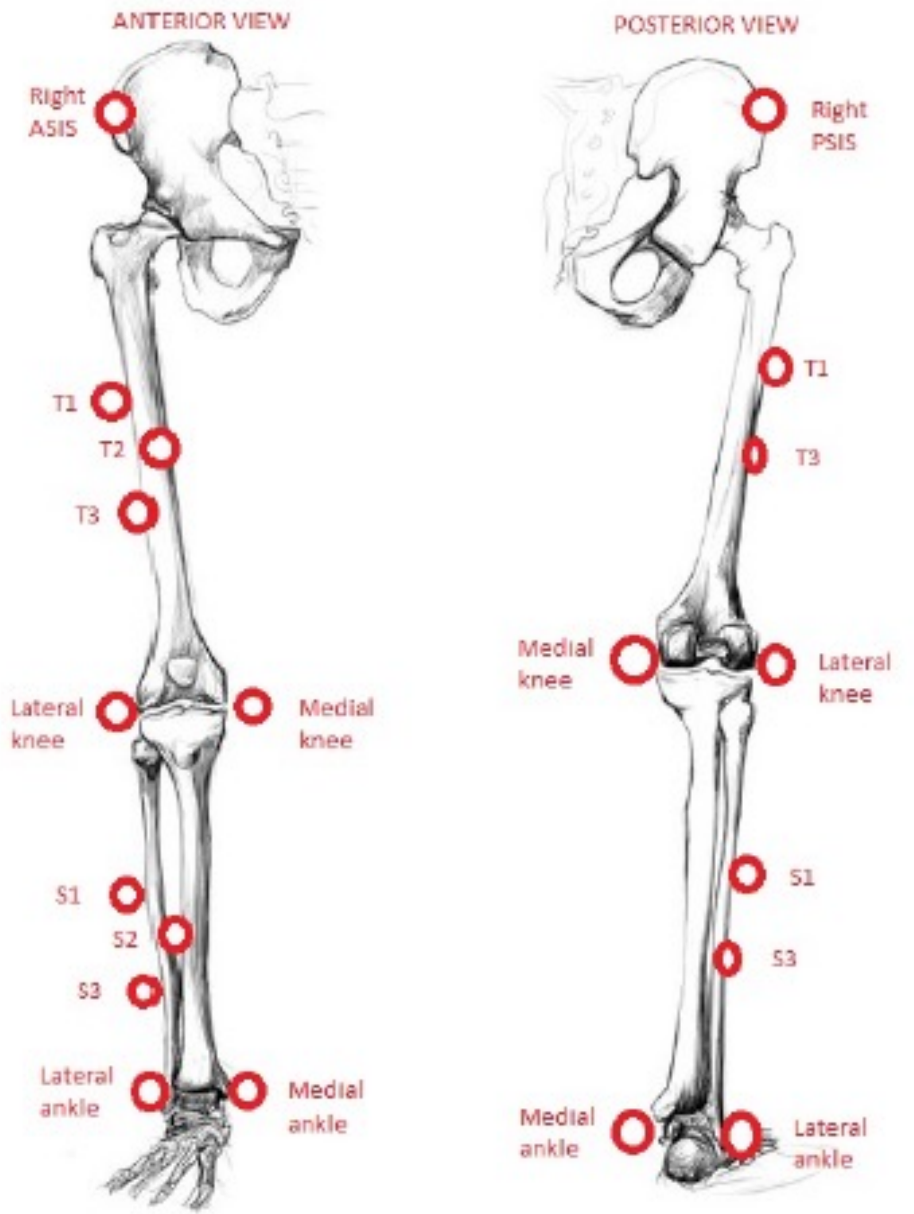
${}^{lcs.shank}T_{acs.shank}$: Calculated from tibia markers in lab reference frame (ACS to GCS)
Next, transformed to shank reference frame (ACS to LCS for shank)

${}^{lcs.thigh}T_{acs.thigh}$: Calculated from femur markers in lab reference frame (ACS to GCS)
Next, transformed to thigh reference frame (ACS to LCS for thigh)



In the second part of this homework assignment you will be guided through the matrix calculations and analyses necessary to perform inverse kinematics and obtain knee joint angles from motion capture data.

Data has been collected on a subject performing a vertical jump, starting from a standing position and going down into a deep knee bend before leaping up. The marker positions on the right leg are approximately represented in the following diagram, where T1, T2, and T3 are markers for defining the local coordinate system (LCS) of the thigh and S1, S2, and S3 are markers for defining the shank LCS. All other markers are important for finding joint axes of rotation and anatomical coordinate systems (ACS), which are defined on an initial static trial. The static trial data for T1-3, S1-3, and the anatomical landmarks can be found in the **static_trial.mat** file, and the data giving the motion of T1-3 and S1-3 during the dynamic trial can be found in the **dynamic_trial.mat** file (the video for this trial is on Canvas to help you visualize what's going on). Both files are on Canvas (download the "HW5_data" zipped folder). When you load the ".mat" files, you will see a list of defined come up in your workspace. Each marker has three columns of data that correspond to the global X, Y, and Z position of that marker at each time point. Note that the static trial only has one time point, and the dynamic trial has many time points.



Static Trial (static_trial.mat):
T1-3
S1-3
Anatomical Landmarks

Dynamic Trial (dynamic_trial.mat):
T1-3
S1-3

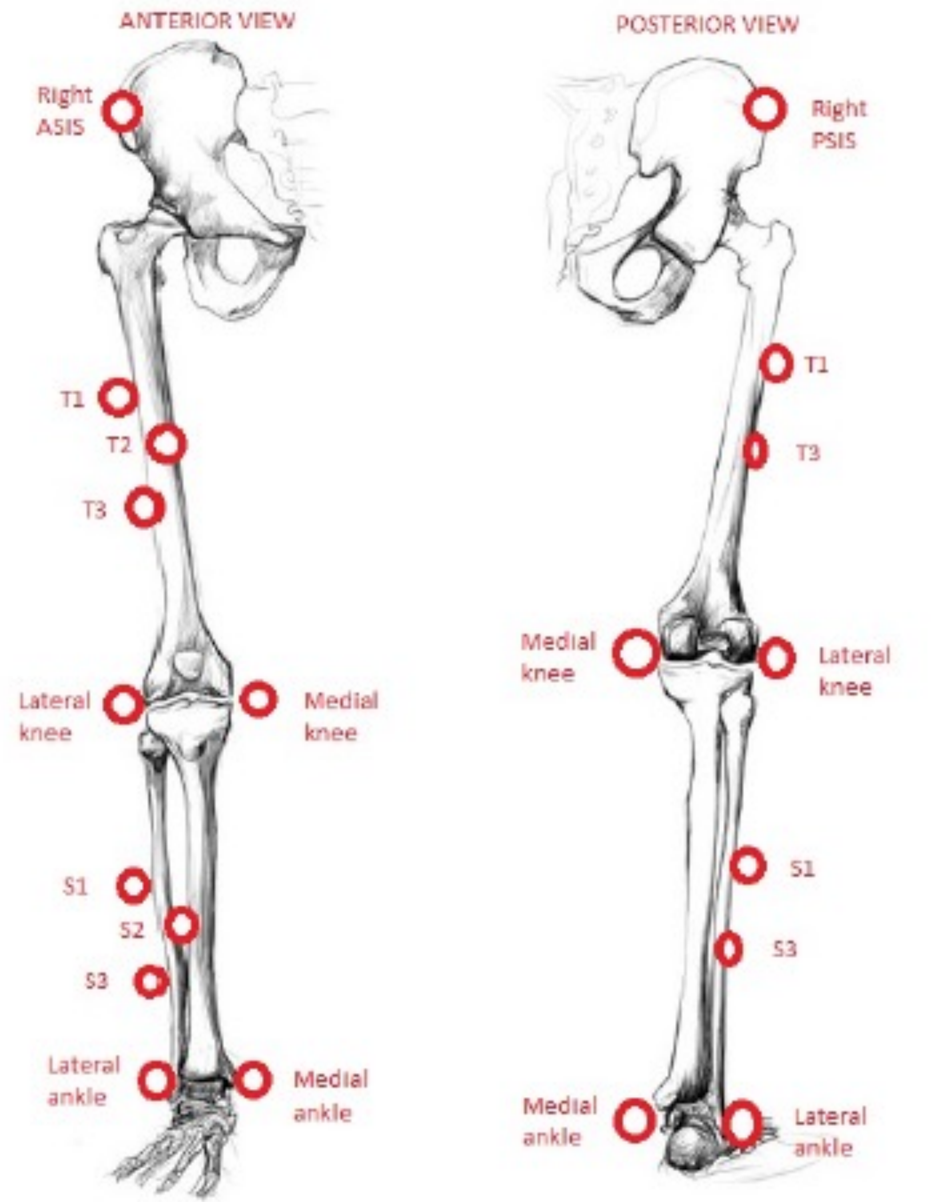
Static Trial Contents

- Load('static_trial.mat')

  T1-3
  S1-3
  Anatomical Landmarks

Workspace

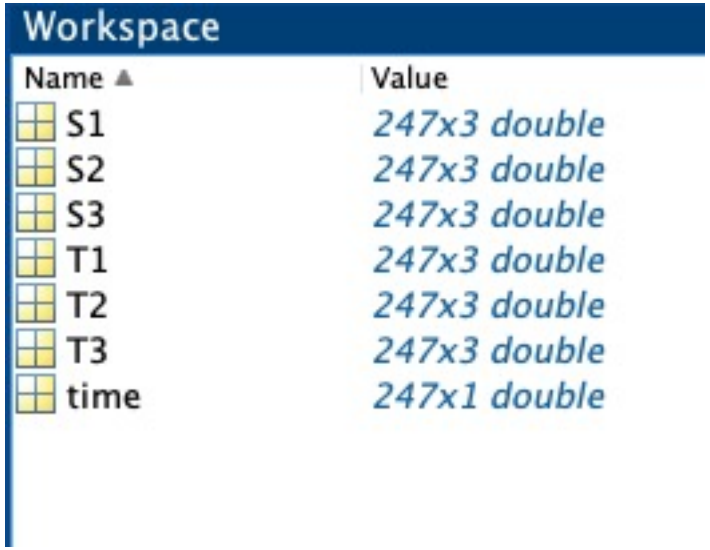
Name ▲	Value
ASIS	[325.0862,522.0791,1.0271e+03]
LAT_ANKLE	[315.9144,544.0333,78.1969]
LAT_KNEE	[293.3926,495.8734,499.9116]
MED_ANKLE	[402.7979,526.1310,103.6250]
MED_KNEE	[406.9175,495.8000,486.6986]
PSIS	[337.5655,695.8406,1.0527e+03]
S1	[270.4434,522.7805,322.9194]
S2	[331.8686,472.3892,283.1944]
S3	[300.1669,508.5782,219.0040]
T1	[263.9885,520.7175,708.1314]
T2	[328.6312,452.4911,675.8262]
T3	[292.6235,470.0835,622.2356]
time	0



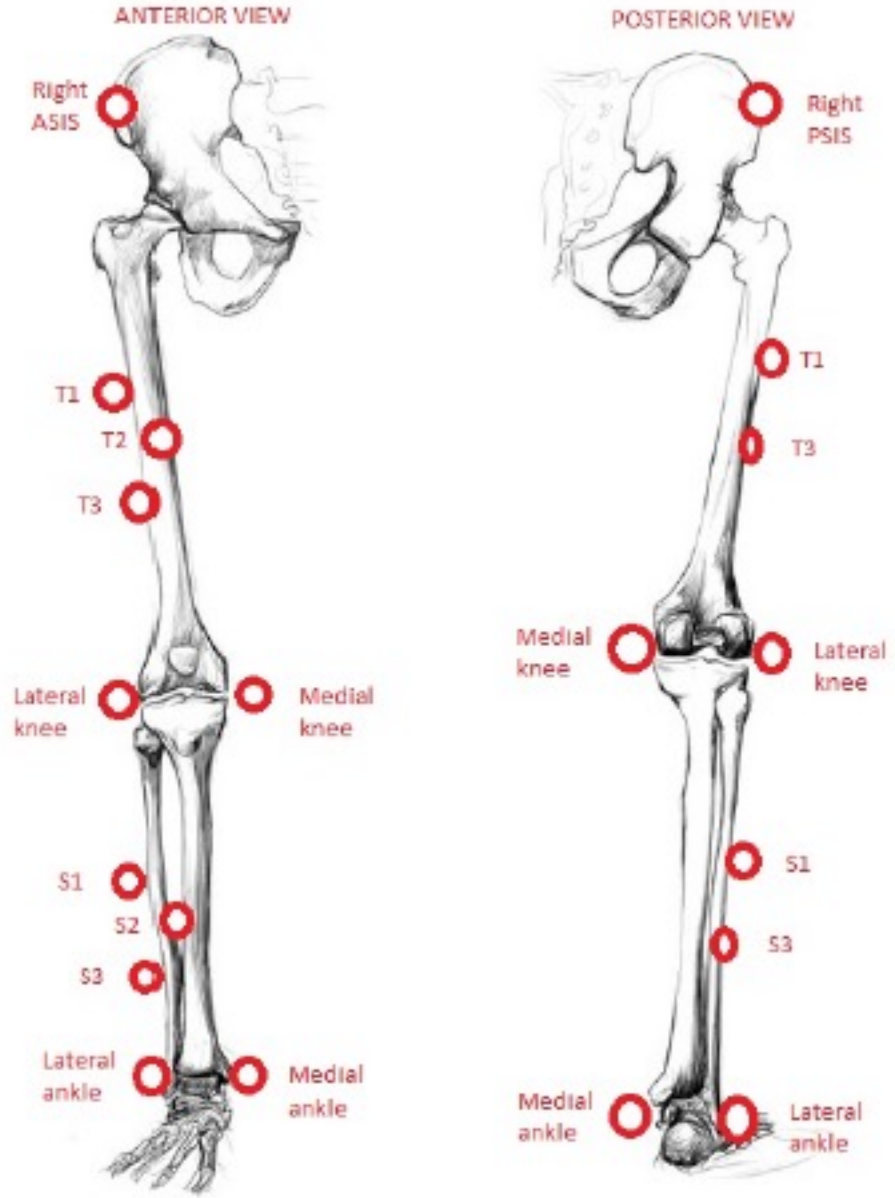
Dynamic Trial Contents

- Load(‘dynamic_trial.mat’)

Dynamic Trial (dynamic_trial.mat):
T1-3
S1-3


Workspace

Name	Value
S1	247x3 double
S2	247x3 double
S3	247x3 double
T1	247x3 double
T2	247x3 double
T3	247x3 double
time	247x1 double



Creating a coordinate system

Given three points in global coordinate system create system of the thigh with respect to the Global Coordinate System.

T1	[263.9885,520.7175,708.1314]
T2	[328.6312,452.4911,675.8262]
T3	[292.6235,470.0835,622.2356]

- Marker T2 is used as the LCS origin.
- The x-axis is directed from marker T2 to marker T1.
- The y-axis is normal to the x-axis and the vector from marker T2 to marker T3.
- The z-axis is normal to both the x-axis and the y-axis.

```
o_lcsthigh=T2;
x_lcsthigh=(T1-T2)/norm(T1-T2);
y_lcsthigh=cross((T3-T2),x_lcsthigh)/norm(cross((T3-T2),x_lcsthigh));
z_lcsthigh=cross(x_lcsthigh,y_lcsthigh)/norm(cross(x_lcsthigh,y_lcsthigh));
```

Creating a rotation matrix

Unit vectors of local CS in global CS are columns of the rotation matrix that transorms a point in local CS coordinates to global CS coordinates.

T1	[263.9885,520.7175,708.1314]
T2	[328.6312,452.4911,675.8262]
T3	[292.6235,470.0835,622.2356]

```
o_lcsthigh=T2;
x_lcsthigh=(T1-T2)/norm(T1-T2);
y_lcsthigh=cross((T3-T2),x_lcsthigh)/norm(cross((T3-T2),x_lcsthigh));
z_lcsthigh=cross(x_lcsthigh,y_lcsthigh)/norm(cross(x_lcsthigh,y_lcsthigh));
```

$$R_{lcs}^{gcs} = \left[x_{lcsthigh} \;\; y_{lcsthigh} \;\; z_{lcsthigh} \right]$$

Creating a transformation matrix

Unit vectors of local CS in global CS are columns of the rotation matrix that transforms a point in local CS coordinates to global CS coordinates.

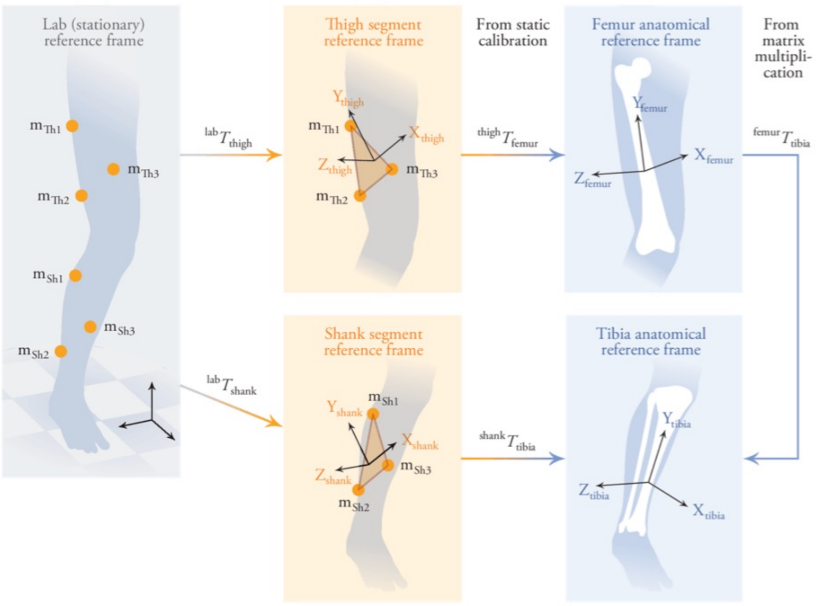
T1	[263.9885,520.7175,708.1314]
T2	[328.6312,452.4911,675.8262]
T3	[292.6235,470.0835,622.2356]

```
o_lcsthigh=T2;
x_lcsthigh=(T1-T2)/norm(T1-T2);
y_lcsthigh=cross((T3-T2),x_lcsthigh)/norm(cross((T3-T2),x_lcsthigh));
z_lcsthigh=cross(x_lcsthigh,y_lcsthigh)/norm(cross(x_lcsthigh,y_lcsthigh));
```

$$R_{lcs}^{gcs} = \left[x'_{lcsthigh} \;\; y'_{lcsthigh} \;\; z'_{lcsthigh} \right]$$

$$T_{lcs}^{gcs} = \begin{bmatrix} R_{lcs}^{gcs} & o_{lcsthigh} \\ [0\,0\,0] & 1 \end{bmatrix}$$

Calculating anatomical joint angles

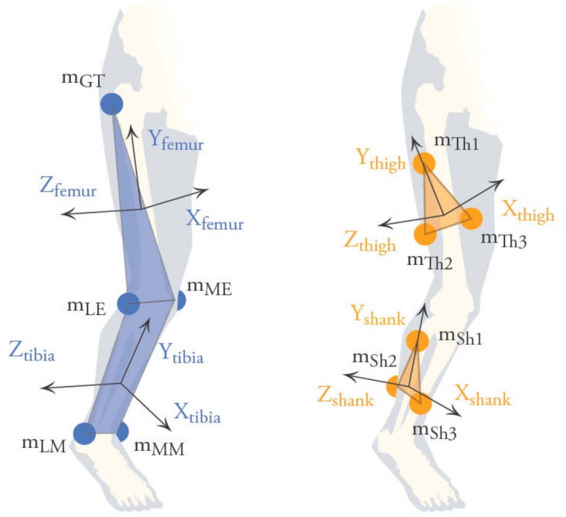


$${}^{acs.thigh}T_{acs.shank} = {}^{acs.thigh}T_{lcs.thigh}\ {}^{lcs.thigh}T_{gcs}\ {}^{gcs}T_{lcs.shank}\ {}^{lcs.shank}T_{acs.tibia}$$

${}^{gcs}T_{lcs.shank}$: Calculated from shank markers in lab reference frame (LCS to GCS)
${}^{gcs}T_{lcs.thigh}$: Calculated from thigh markers in lab reference frame (LCS to GCS)

${}^{lcs.shank}T_{acs.shank}$: Calculated from tibia markers in lab reference frame (ACS to GCS)
Next, transformed to shank reference frame (ACS to LCS for shank)

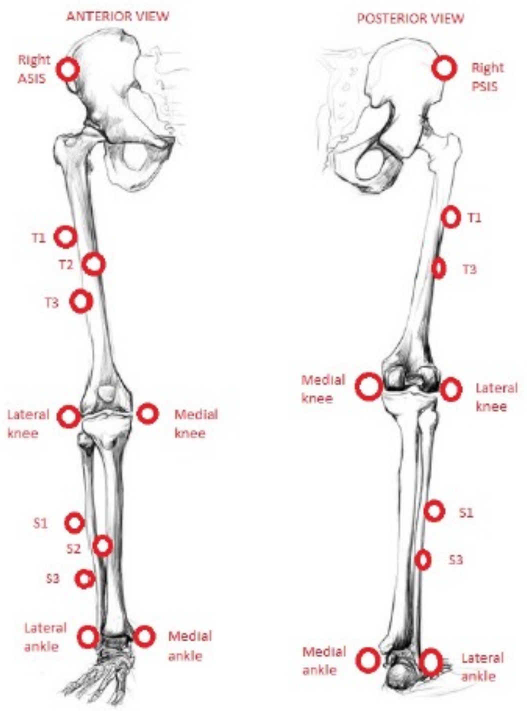
${}^{lcs.thigh}T_{acs.thigh}$: Calculated from femur markers in lab reference frame (ACS to GCS)
Next, transformed to thigh reference frame (ACS to LCS for thigh)



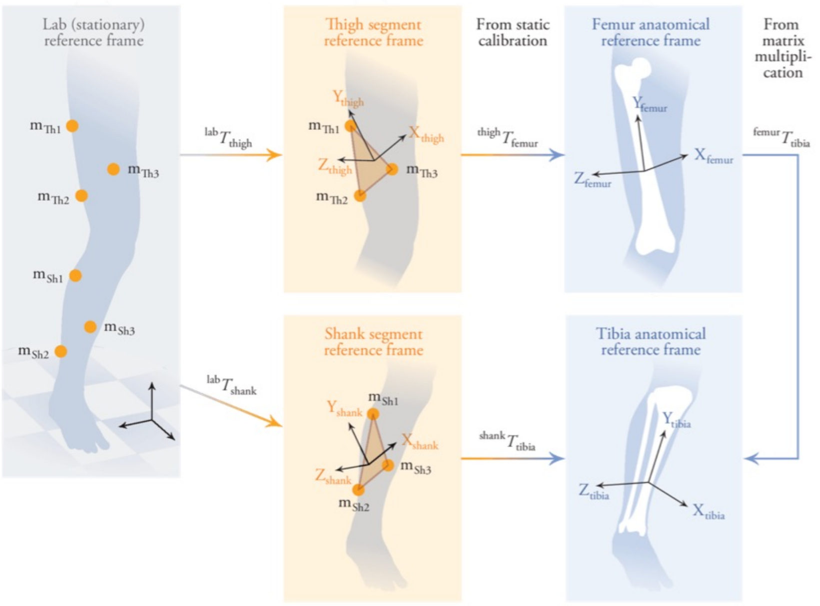
Steps

1. Segment 1 (Thigh)
    1. Create Transformation matrix for LCS to GCS (using T1-T3) T_{lcs}^{gcs}

2. Static Trial: Segment 2 (shank)
    1. Create Transformation matrix for LCS to GCS (using S1-S3) T_{lcs}^{gcs}



Calculating anatomical joint angles



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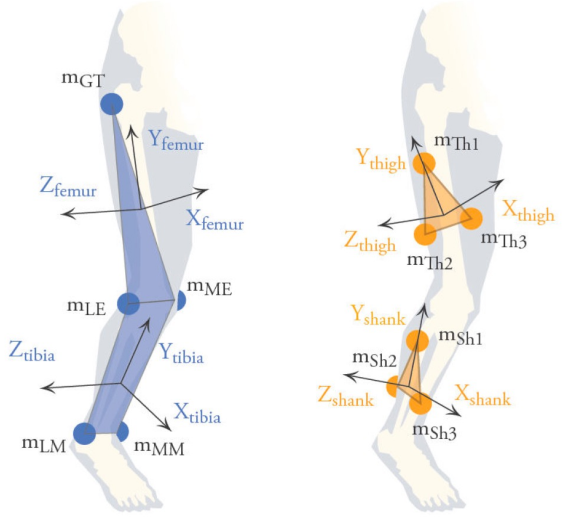
Next, transformed to shank reference frame (ACS to LCS for shank)

$^{lcs.thigh}T_{acs.thigh}$: Calculated from femur markers in lab reference frame (ACS to GCS)
Next, transformed to thigh reference frame (ACS to LCS for thigh)



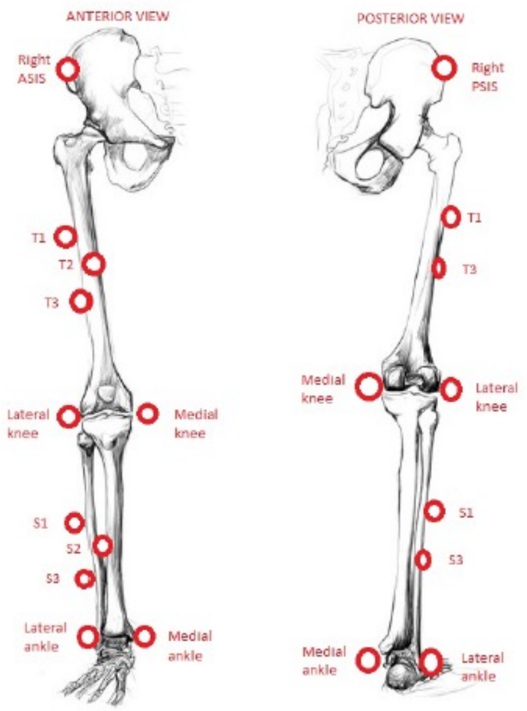
Steps

1. Segment 1 (Thigh)
   1. Create Transformation matrix for LCS to GCS (using T1-T3) T_{lcs}^{gcs}
   2. Create Transformation matrix for ACS to GCS

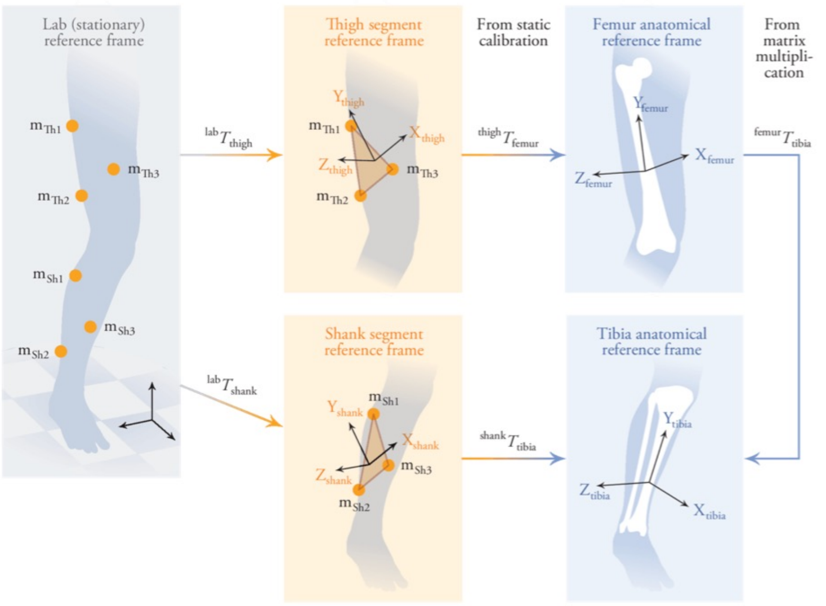
2. Static Trial: Segment 2 (shank)
   1. Create Transformation matrix for LCS to GCS (using S1-S3) T_{lcs}^{gcs}
   2. Create Transformation matrix for ACS to GCS

3. Derive the 4x4 transformation matrix that expresses the ACS with respect to the LCS for that segment (does not change):

$$T_{acs}^{lcs} = \left[T_{lcs}^{gcs}\right]^{-1}\left[T_{acs}^{gcs}\right]$$



Calculating anatomical joint angles



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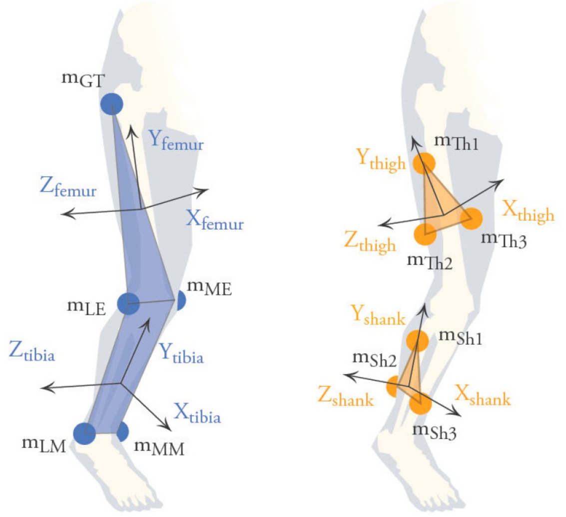
Next, transformed to thigh reference frame (ACS to LCS for thigh)



Steps

1. Segment 1 (Thigh)
   1. Create Transformation matrix for LCS to GCS (using T1-T3)
   2. Create Transformation matrix for ACS to GCS

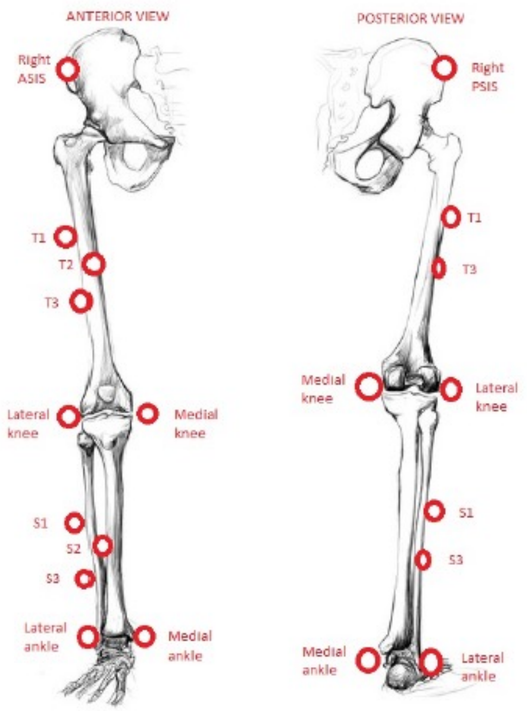
2. Static Trial: Segment 2 (shank)
   1. Create Transformation matrix for LCS to GCS (using S1-S3)
   2. Create Transformation matrix for ACS to GCS

3. Derive the 4x4 transformation matrix that expresses the ACS with respect to the LCS for that segment (does not change):

$$T_{acs}^{lcs} = \left[T_{lcs}^{gcs}\right]^{-1}\left[T_{acs}^{gcs}\right]$$

4. Derive the 4x4 transformation matrix the ACS of one segment to the other:

$$T_{acs.shank}^{acs.thigh} = \left[T_{acs.thigh}^{lcs.thigh}\right]^{-1}\left[T_{lcs.thigh}^{gcs}\right]^{-1}\left[T_{lcs.shank}^{gcs}\right]\left[T_{acs.shank}^{lcs.shank}\right]$$



Steps

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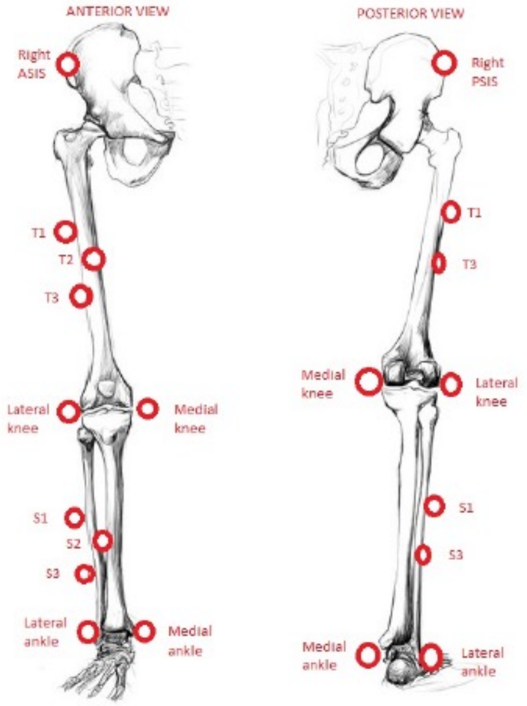

In Dynamic trial, repeat steps 1.1, 2.1 and 4 at every time step.

Joint Angles

$$T_{acs.shank}^{acs.thigh} = \left[T_{acs.thigh}^{lcs.thigh}\right]^{-1}\left[T_{lcs.thigh}^{gcs}\right]^{-1}\left[T_{lcs.shank}^{gcs}\right]\left[T_{acs.shank}^{lcs.shank}\right]$$

$$= \begin{bmatrix} r_{xx} & r_{xy} & r_{xz} \\ r_{yx} & r_{yy} & r_{yz} \\ r_{zx} & r_{zy} & r_{zz} \end{bmatrix}$$

$$\phi = \operatorname{atan2}(r_{32}, r_{33})$$

Flexion/extension

$$\theta = \operatorname{atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$$

Varus/valgus

$$\psi = \operatorname{atan2}(r_{21}, r_{11})$$

Internal/external rotation