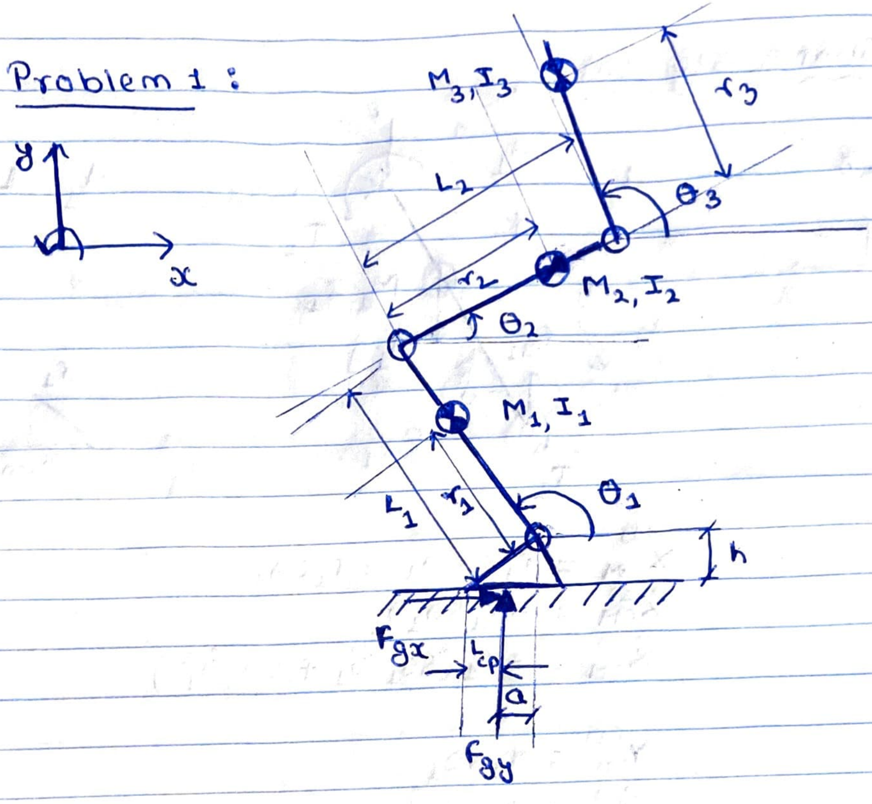
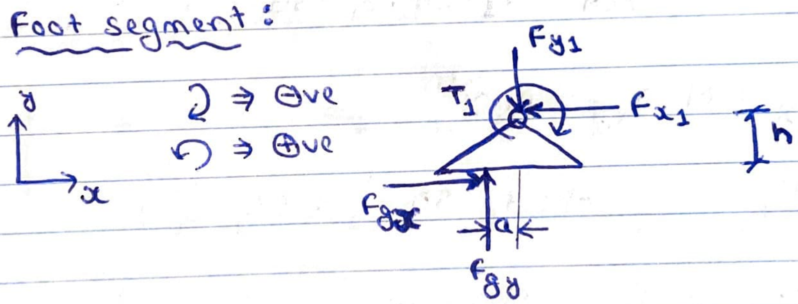
Problem 1 :



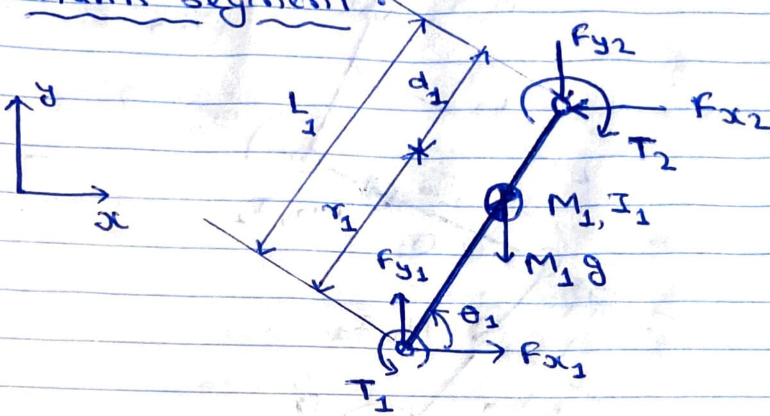
Foot segment :



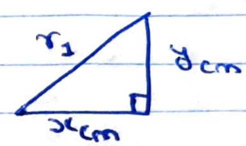
A → Ankle

$\Sigma F_x = M_0 \ddot{x}_0$	$\Sigma F_y = M_0 \ddot{y}_0$	$\Sigma M_A = 0$
$\Rightarrow F_{gx} - F_{x1} = 0$	$\Rightarrow F_{gy} - F_{y1} = 0$	$\Rightarrow F_{gx} \cancel{a}^{h} - F_{gy} a - T_1 = 0$
$\Rightarrow F_{x1} = F_{gx} \rightarrow ①$	$\Rightarrow F_{y1} = F_{gy} \rightarrow ②$	$\Rightarrow \boxed{T_1 = F_{gx} h - F_{gy} a}$ $\rightarrow ③$

Shank segment :



$d_1 = L_1 - r_1$



$$x_{CM}^{(1)} = r_1 \cos\theta_1 = r_1 c\theta_1$$
$$\dot{x}_{CM}^{(1)} = -r_1 s\theta_1 \dot{\theta}_1$$
$$\ddot{x}_{CM}^{(1)} = -r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$y_{CM}^{(1)} = r_1 s\theta_1$$
$$\dot{y}_{CM}^{(1)} = r_1 c\theta_1 \dot{\theta}_1$$
$$\ddot{y}_{CM}^{(1)} = r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Sigma f_x = M_1 a_{x_1} = M_1 \ddot{x}_{CM}^{(1)}$$

$$\Rightarrow f_{x_1} - f_{x_2} = -M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow f_{x_2} = f_{x_1} + M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow f_{x_2} = f_{gx} + M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2) \quad \rightarrow (4)$$

$(\because \; f_{x_1} = f_{gx})$

$$\Sigma F_y = M_1 a_{y_1} = M_1 \ddot{Y}_{CM}^{(1)}$$

$$\Rightarrow F_{y_1} - F_{y_2} - M_1 g = M_1 r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow F_{y_2} = F_{y_1} - M_1 g - M_1 r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow F_{y_2} = F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2) \rightarrow (5)$$

$$(\because F_{y_1} = F_{gy})$$

S→ Shank

$$\Sigma M_{S,CM} = I_1 \ddot{\theta}_1$$

$$\Rightarrow T_1 - T_2 + F_{x_1} r_1 s\theta_1 - F_{y_1} r_1 c\theta_1 + F_{x_2} d_1 s\theta_1 - F_{y_2} d_1 c\theta_1 = I_1 \ddot{\theta}_1$$

$$\Rightarrow \boxed{T_2 = T_1 + F_{x_1} r_1 s\theta_1 - F_{y_1} r_1 c\theta_1 + F_{x_2} d_1 s\theta_1 - F_{y_2} d_1 c\theta_1 - I_1 \ddot{\theta}_1}$$

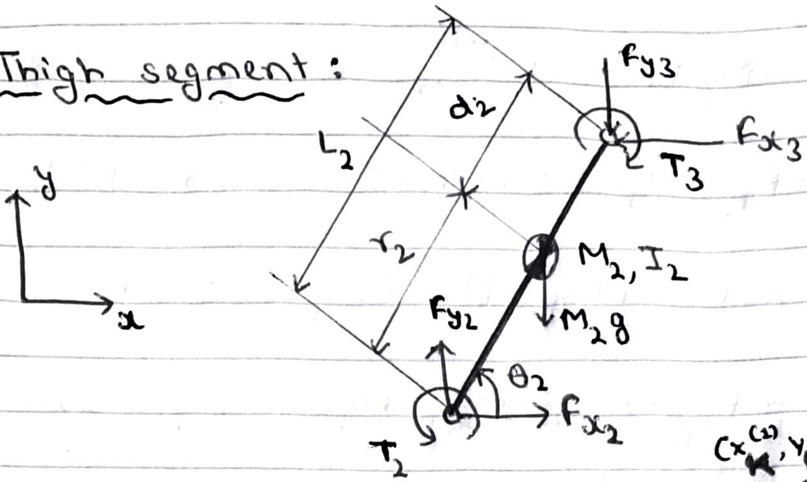
$$\Rightarrow \boxed{\begin{aligned} T_2 = {} & (F_{gx} h - F_{gy} a) + F_{gx} r_1 s\theta_1 - F_{gy} r_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)] d_1 s\theta_1 - I_1 \ddot{\theta}_1 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2)] d_1 c\theta_1 \rightarrow (6) \end{aligned}}$$

from (1), (2), (3), (4) and (5) eq 7

This is the expression for T_2 in terms of θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, F_H, F_V, along with the masses, inertial properties, and dimensions of the limbs.

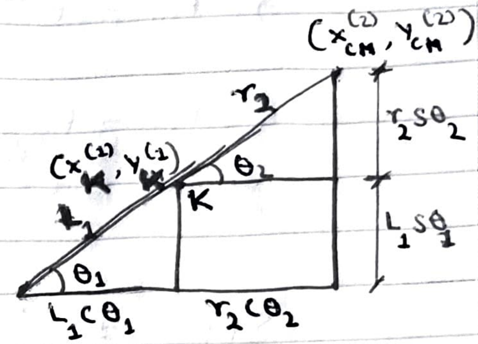
→

Thigh segment:



$d_2 = L_2 - r_2$

$K \rightarrow$ Knee



$$x_{CM}^{(2)} = L_1 c\theta_1 + r_2 c\theta_2$$
$$\dot{x}_{CM}^{(2)} = -L_1 s\theta_1 \dot{\theta}_1 - r_2 s\theta_2 \dot{\theta}_2$$
$$\ddot{x}_{CM}^{(2)} = -L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$y_{CM}^{(2)} = L_1 s\theta_1 + r_2 s\theta_2$$
$$\dot{y}_{CM}^{(2)} = L_1 c\theta_1 \dot{\theta}_1 + r_2 c\theta_2 \dot{\theta}_2$$
$$\ddot{y}_{CM}^{(2)} = L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)$$

$$\Sigma F_x = M_2 a_{x_2} = M_2 \ddot{X}_{CM}^{(2)}$$

$$\Rightarrow F_{x_2} - F_{x_3} = -M_2 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - M_2 r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{x_3} = F_{x_2} + M_2 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{x_3} = F_{gx} + M_1 r_1 (s\theta_1 . \ddot{\theta}_1 + c\theta_1 . \dot{\theta}_1^2) +$$
$$M_2 L_1 (s\theta_1 . \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$\rightarrow$ (7)

(From eqn (4))

$$\Sigma F_y - M_2 g = M_2 a_{y_2} = M_2 \ddot{Y}_{CM}^{(2)}$$

$$\Rightarrow F_{y_2} - F_{y_3} - M_2 g = M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{y_3} = F_{y_2} - M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) - M_2 g$$

$$\Rightarrow F_{y_3} = F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 g - M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) \rightarrow (8)$$

(from eq^n (5))

$T \rightarrow$ Thigh

$$\Sigma M_{T,CM} = I_2 \ddot{\theta}_2$$

$$\Rightarrow T_2 - T_3 + F_{x_2} r_2 s\theta_2 - F_{y_2} r_2 c\theta_2 + F_{x_3} d_2 s\theta_2 - F_{y_3} d_2 c\theta_2 = I_2 \ddot{\theta}_2$$

$$\Rightarrow \boxed{T_3 = T_2 + F_{x_2} r_2 s\theta_2 - F_{y_2} r_2 c\theta_2 + F_{x_3} d_2 s\theta_2 - F_{y_3} d_2 c\theta_2 - I_2 \ddot{\theta}_2}$$

$$\Rightarrow \boxed{\begin{aligned} T_3 = {} & (F_{gx} h - F_{gy} a) + F_{gx} r_1 s\theta_1 - F_{gy} r_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2)] d_1 s\theta_1 - I_1 \ddot{\theta}_1 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2)] d_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2)] r_2 s\theta_2 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2)] r_2 c\theta_2 - I_2 \ddot{\theta}_2 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) \\ & \quad - M_2 g - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)] d_2 c\theta_2 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) \\ & \quad + M_2 r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)] d_2 s\theta_2 \rightarrow (9) \end{aligned}}$$

Combining similar terms, we get,

$$
\boxed{
\begin{aligned}
T_3 = {} & F_{gx}\left(h + r_1 s\theta_1 + d_1 s\theta_1 + r_2 s\theta_2 + d_2 s\theta_2\right) - \\
& F_{gy}\left(a + r_1 c\theta_1 + d_1 c\theta_1 + r_2 c\theta_2 + d_2 c\theta_2\right) + \\
& \left[\left(s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2\right)\left(M_1 r_1 d_1 s\theta_1 + M_1 r_1 r_2 s\theta_2 + M_1 r_1 d_2 s\theta_2 \right.\right. \\
& \qquad \left.\left. + M_2 L_1 d_2 s\theta_2\right)\right] + \\
& \left[\left(c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2\right)\left(M_1 r_1 d_1 c\theta_1 + M_1 r_1 r_2 c\theta_2 + M_1 r_1 d_2 c\theta_2 \right.\right. \\
& \qquad \left.\left. + M_2 L_1 d_2 c\theta_2\right)\right] - \\
& \left(I_1 \ddot{\theta}_1 + I_2 \ddot{\theta}_2\right) + M_1 g\left(a_1 c\theta_1 + r c\theta_2 + d_2 c\theta_2\right) + \\
& M_2 g\left(d_2 c\theta_2\right) + \left[\left(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2\right)\left(M_2 r_2 d_2 s\theta_2\right)\right] \\
& + \left[\left(c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2\right)\left(M_2 r_2 d_2 c\theta_2\right)\right] \rightarrow (9)
\end{aligned}
}
$$

Problem 2: Function

```
function [T1, T2, T3, err_Fx_3, err_Fy_3, err_T3] = HW06_Function(theta1,theta2, ...
    thetadot1, thetadot2,thetadotdot1,thetadotdot2,GRF_hori,GRF_vert)
```

Loading constants and other necessary variables

```
k = matfile('kinematic_data.mat');
c = matfile('constants.mat');
h = c.h;
g = c.g;
I1 = c.I1;
I2 = c.I2;
I3 = c.I3;
L1 = c.L1;
L2 = c.L2;
L3 = c.L3;
m1 = c.m1;
m2 = c.m2;
m3 = c.m3;
r1 = c.r1;
r2 = c.r2;
r3 = c.r3;
a = k.a;
theta3 = k.theta3;
thetadot3 = k.thetadot3;
thetadotdot3 = k.thetadotdot3;
```

Computation of Net Joint Torque at Ankle

```
T1 = (h*GRF_hori) - (GRF_vert.*a);
```

Necessary force and dimension computations

```
Fx_2 = GRF_hori + (m1*r1.*((sin(theta1).*thetadotdot1) + ...
    (cos(theta1).*thetadot1.^2)));
Fy_2 = GRF_vert - (m1*g) - (m1*r1.*((cos(theta1).*thetadotdot1) - ...
    (sin(theta1).*thetadot1.^2)));
d1 = L1 - r1;
```

Computation of Net Joint Torque at Knee

```
T2 = T1 + (GRF_hori.*r1.*sin(theta1)) - (GRF_vert.*r1.*cos(theta1))...
    + (Fx_2*d1.*sin(theta1)) - (Fy_2*d1.*cos(theta1)) - I1.*thetadotdot1;
```

Necessary force and dimension computations

```
Fx_3 = Fx_2 + (m2*r2.*((sin(theta2).*thetadotdot2) + (cos(theta2).*thetadot2.^2))) ...
    + (m2*L1.*((sin(theta1).*thetadotdot1) + (cos(theta1).*thetadot1.^2)));
Fy_3 = Fy_2 - (m2*r2.*((cos(theta2).*thetadotdot2)- (sin(theta2).*thetadot2.^2))) ...
```

```
    - (m2*L1.*((cos(theta1).*thetadotdot1) - (sin(theta1).*thetadot1.^2)));
d2 = L2 - r2;
```

Computation of Net Joint Torque at Hip

```
T3 = T2 + (Fx_2.*r2.*sin(theta2)) - (Fy_2.*r2.*cos(theta2))...
    + (Fx_3.*d2.*sin(theta2)) - (Fy_3.*d2.*cos(theta2)) - I2.*thetadotdot2;
```

Hand of God computations

Necessary force computations

```
%Fx_3_hg = -m3*L1.*((sin(theta1).*thetadotdot1) + (cos(theta1).*thetadot1.^2)) ...
   % -m3*L2.*((sin(theta2).*thetadotdot2) + (cos(theta2).*thetadot2.^2)) ...
    %-m3*r3.*((sin(theta3).*thetadotdot3) + (cos(theta3).*thetadot3.^2));

Fx_3_hg =  -m3*r3*(sin(theta3).*thetadotdot3 + cos(theta3).*thetadot3.^2);

%Fy_3_hg = m3*g + m3*L1.*((cos(theta1).*thetadotdot1) - (sin(theta1).*thetadot1.^2)) .
    %+m3*L2.*((cos(theta2).*thetadotdot2) - (sin(theta2).*thetadot2.^2)) ...
    %+m3*r3.*((cos(theta3).*thetadotdot3) - (sin(theta3).*thetadot3.^2));

 Fy_3_hg = m3*g + m3*r3*(cos(theta3).*thetadotdot3 - sin(theta3).*thetadot3.^2);
```

"Hand of God": Computation of Net Joint Torque at Hip

```
T3_hg = (r3*Fy_3_hg.*cos(theta3)) - (r3*Fx_3_hg.*sin(theta3)) + I3.*thetadotdot3;
```

Error calculations

```
err_Fx_3 = Fx_3_hg - Fx_3;
err_Fy_3 = Fy_3_hg - Fy_3;
err_T3 = T3_hg - T3;

end
```

Problem 3: Script

```
clear;clc;
```

Loading mat files

```
load('constants.mat');
load('kinematic_data.mat')
load('force_data.mat');
```

Calling the function Prob2 to get desired outputs

```
[T1, T2, T3, err_Fx_3, err_Fy_3, err_T3] = HW06_Function(theta1,theta2, ...
    thetadot1,thetadot2,thetadotdot1,thetadotdot2,GRF_hori,GRF_vert);
```

Necessary moment arms of muscles

```
ham_hip_ext_mom_arm = 0.05; % in m
ham_knee_flex_mom_arm = 0.04; % in cm
vasti_knee_ext_mom_arm = 0.04; % in cm
soleus_ankle_ext_mom_arm = 0.025; % in cm
```

Forces calculated in the hamstrings, vasti, and soleus muscles

```
F_ham_hip_ext = -T3/ham_hip_ext_mom_arm;
F_ham_knee_flex = T2./ham_knee_flex_mom_arm;
F_vasti_knee_ext = (T2+F_ham_knee_flex*ham_knee_flex_mom_arm)/...
vasti_knee_ext_mom_arm;
F_soleus_ankle_ext = -T1/soleus_ankle_ext_mom_arm;
```

Accounting for only positive muscle forces

```
F_ham_hip_ext = F_ham_hip_ext(F_ham_hip_ext>=0);
index1 = find(F_ham_hip_ext>=0);

F_ham_knee_flex = F_ham_knee_flex(F_ham_knee_flex>=0);
index2 = find(F_ham_knee_flex>=0);

F_vasti_knee_ext = F_vasti_knee_ext(F_vasti_knee_ext>=0);
index3 = find(F_vasti_knee_ext>=0);

F_soleus_ankle_ext = F_soleus_ankle_ext(F_soleus_ankle_ext>=0);
index4 = find(F_soleus_ankle_ext>=0);
```

Plots of Net Joint Torques at Ankle, Knee and Hip

```
plot(time, T1)
hold on
plot(time, T2)
hold on
plot(time, T3)
xlabel('Time (seconds)')
ylabel('Net Joint Torques (Newton-m)')
h1 = legend('T1: Ankle', 'T2: Knee', 'T3: Hip');
rect = [0.15, 0.60, .30, .30];
```

```
set(h1, 'Position', rect);
title('Net Joint Torque vs Time')
grid on
```

Plot of error in Fx_3 and Fy_3

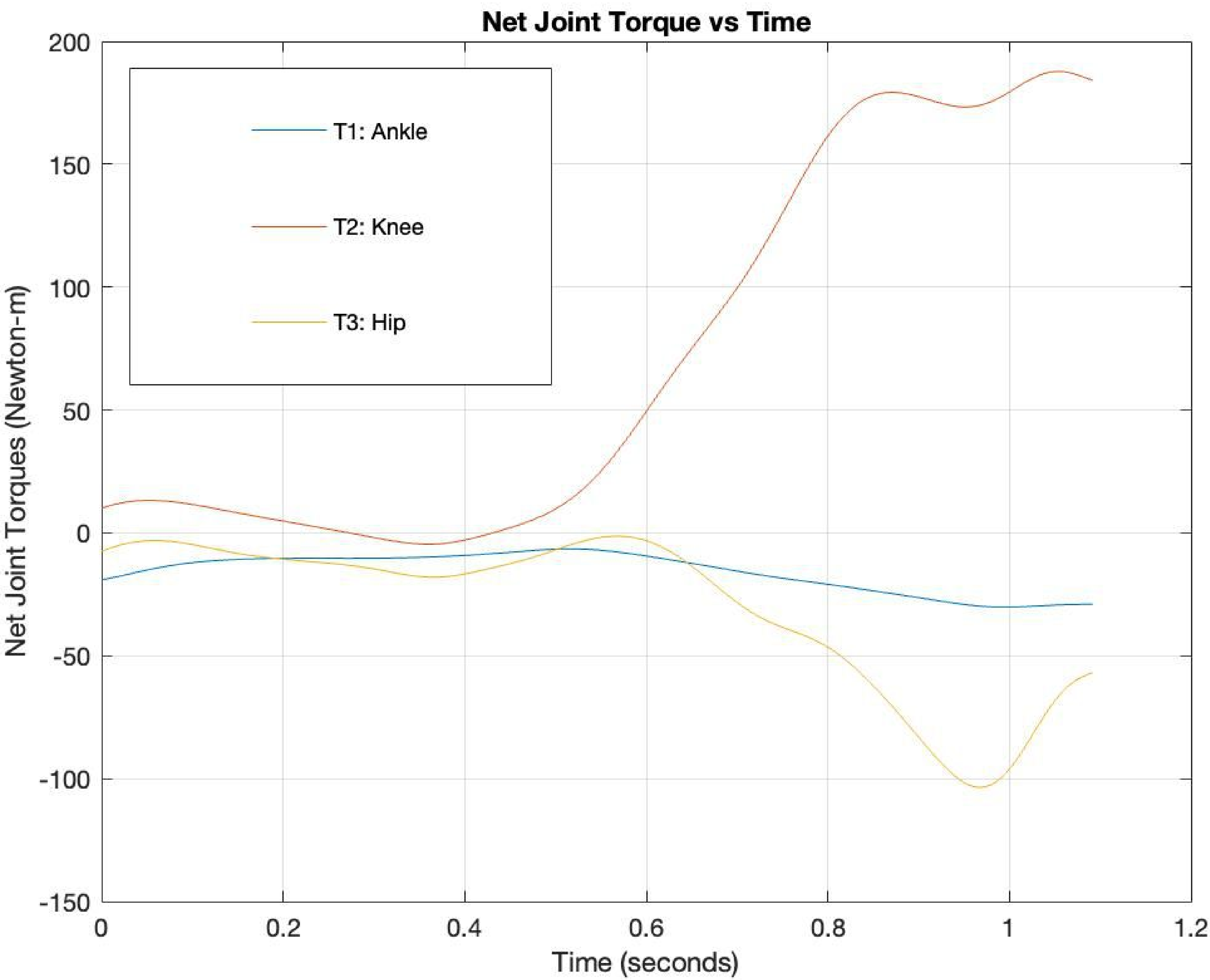
```
figure
plot(time, err_Fx_3)
hold on
plot(time, err_Fy_3)
xlabel('Time (seconds)')
ylabel('Error (Newton)')
title('Error in component of forces vs Time')
legend('Error in Fx_3', 'Error in Fy_3')
grid on
```

Plot of error in Net Joint Torque at Hip

```
figure
plot(time, err_T3)
xlabel('Time (seconds)')
ylabel('Error (Newton-m)')
title('Error in T3 vs Time')
grid on
```

Plot of forces developed in muscles

```
figure
plot(time(index1), F_ham_hip_ext)
hold on
plot(time(index2), F_ham_knee_flex)
hold on
plot(time(index3), F_vasti_knee_ext)
hold on
plot(time(index4), F_soleus_ankle_ext)
xlabel('Time (seconds)')
ylabel('Force (Newton)')
h2 = legend('Hamstring Hip Extension','Hamstring Knee Flexion', ...
    'Vasti Knee Extension','Soleus Ankle Extension');
rect = [0.15, 0.60, .30, .30];
set(h2, 'Position', rect);
title('Muscle Force vs Time')
grid on
```

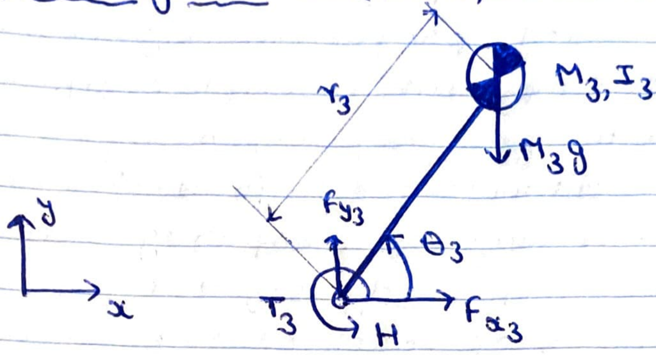


Net Joint Torque vs Time
T1: Ankle
T2: Knee
T3: Hip
Net Joint Torques (Newton-m)
200
150
100
50
0
-50
-100
-150
0
0.2
0.4
0.6
0.8
1
1.2
Time (seconds)

1st method, used in MATLAB script

Problem 4:

HAT Segment: (Head, Arms and Torso)



Taking H as the origin,

$$x_{cm}^{(3)} = r_3 \cos\theta_3 = r_3 C\theta_3$$
$$\dot{x}_{cm}^{(3)} = -r_3 S\theta_3 \cdot \dot{\theta}_3$$
$$\ddot{x}_{cm}^{(3)} = -r_3 C\theta_3 \dot{\theta}_3^2 - r_3 S\theta_3 \ddot{\theta}_3$$

$$y_{cm}^{(3)} = r_3 S\theta_3$$
$$\dot{y}_{cm}^{(3)} = r_3 C\theta_3 \dot{\theta}_3$$
$$\ddot{y}_{cm}^{(3)} = -r_3 S\theta_3 \dot{\theta}_3^2 + r_3 C\theta_3 \ddot{\theta}_3$$

$$\Sigma f_x = M_3 \ddot{x}_{cm}^{(3)}$$
$$\Rightarrow f_{x_3} = -M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2) \rightarrow ①$$

$$\Sigma f_y = M_3 \ddot{y}_{cm}^{(3)}$$
$$\Rightarrow f_{y_3} - M_3 g = M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)$$
$$\Rightarrow f_{y_3} = M_3 g + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2) \rightarrow ②$$

$$\Sigma M_{HAT,CM} = I_3 \ddot{\theta}_3$$

$$\Rightarrow T_3 + F_{x_3} r_3 S\theta_3 - F_{y_3} r_3 C\theta_3 = I_3 \ddot{\theta}_3$$

$$\Rightarrow T_3 = F_{y_3} r_3 C\theta_3 - F_{x_3} r_3 S\theta_3 + I_3 \ddot{\theta}_3$$

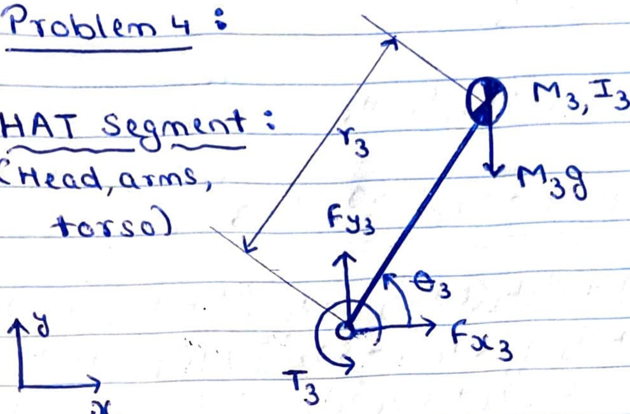
$$\Rightarrow \boxed{T_3 = \left[M_3 g + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)\right] r_3 C\theta_3 - \left[-M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2)\right] r_3 S\theta_3 + I_3 \ddot{\theta}_3}$$

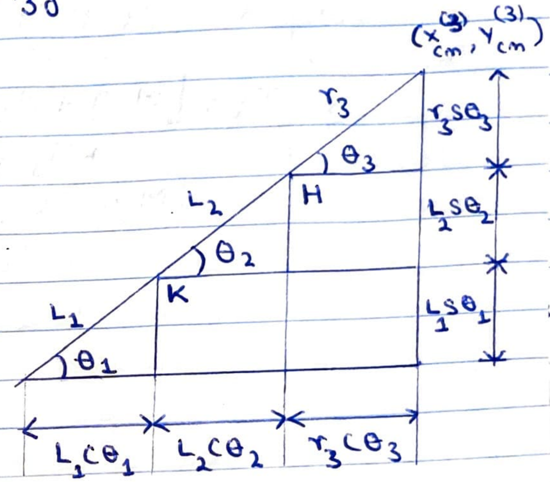
(from ① and ②)

2nd method

Problem 4 :

HAT Segment :
(Head, arms, torso)





Taking ankle as the origin,

$$X_{CM}^{(3)} = L_1 C\theta_1 + L_2 C\theta_2 + r_3 C\theta_3$$

$$\dot{X}_{CM}^{(3)} = -L_1 S\theta_1 \dot{\theta}_1 - L_2 S\theta_2 \dot{\theta}_2 - r_3 S\theta_3 \dot{\theta}_3$$

$$\ddot{X}_{CM}^{(3)} = -L_1 (S\theta_1 \ddot{\theta}_1 + C\theta_1 \dot{\theta}_1^2) - L_2 (S\theta_2 \ddot{\theta}_2 + C\theta_2 \dot{\theta}_2^2) - r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2)$$

$$Y_{CM}^{(3)} = L_1 S\theta_1 + L_2 S\theta_2 + r_3 S\theta_3$$

$$\dot{Y}_{CM}^{(3)} = L_1 C\theta_1 \dot{\theta}_1 + L_2 C\theta_2 \dot{\theta}_2 + r_3 C\theta_3 \dot{\theta}_3$$

$$\ddot{Y}_{CM}^{(3)} = L_1 (C\theta_1 \ddot{\theta}_1 - S\theta_1 \dot{\theta}_1^2) + L_2 (C\theta_2 \ddot{\theta}_2 - S\theta_2 \dot{\theta}_2^2) + r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)$$

$$\Sigma F_x = M_3 \ddot{X}_{CM}^{(3)}$$

$$\Rightarrow F_{x3} = -M_3 L_1 (S\theta_1 \ddot{\theta}_1 + C\theta_1 \dot{\theta}_1^2) - M_3 L_2 (S\theta_2 \ddot{\theta}_2 + C\theta_2 \dot{\theta}_2^2) - M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2) \rightarrow ①$$

$$\Sigma F_y = M_3 \ddot{Y}_{CM}^{(3)}$$

$$\Rightarrow F_{y3} - M_3 g = M_3 L_1 (C\theta_1 \ddot{\theta}_1 - S\theta_1 \dot{\theta}_1^2) + M_3 L_2 (C\theta_2 \ddot{\theta}_2 - S\theta_2 \dot{\theta}_2^2) + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)$$

$$\Rightarrow F_{y3} = M_3 g + M_3 L_1 (C\theta_1 \ddot{\theta}_1 - S\theta_1 \dot{\theta}_1^2) + M_3 L_2 (C\theta_2 \ddot{\theta}_2 - S\theta_2 \dot{\theta}_2^2) + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2) \rightarrow ②$$

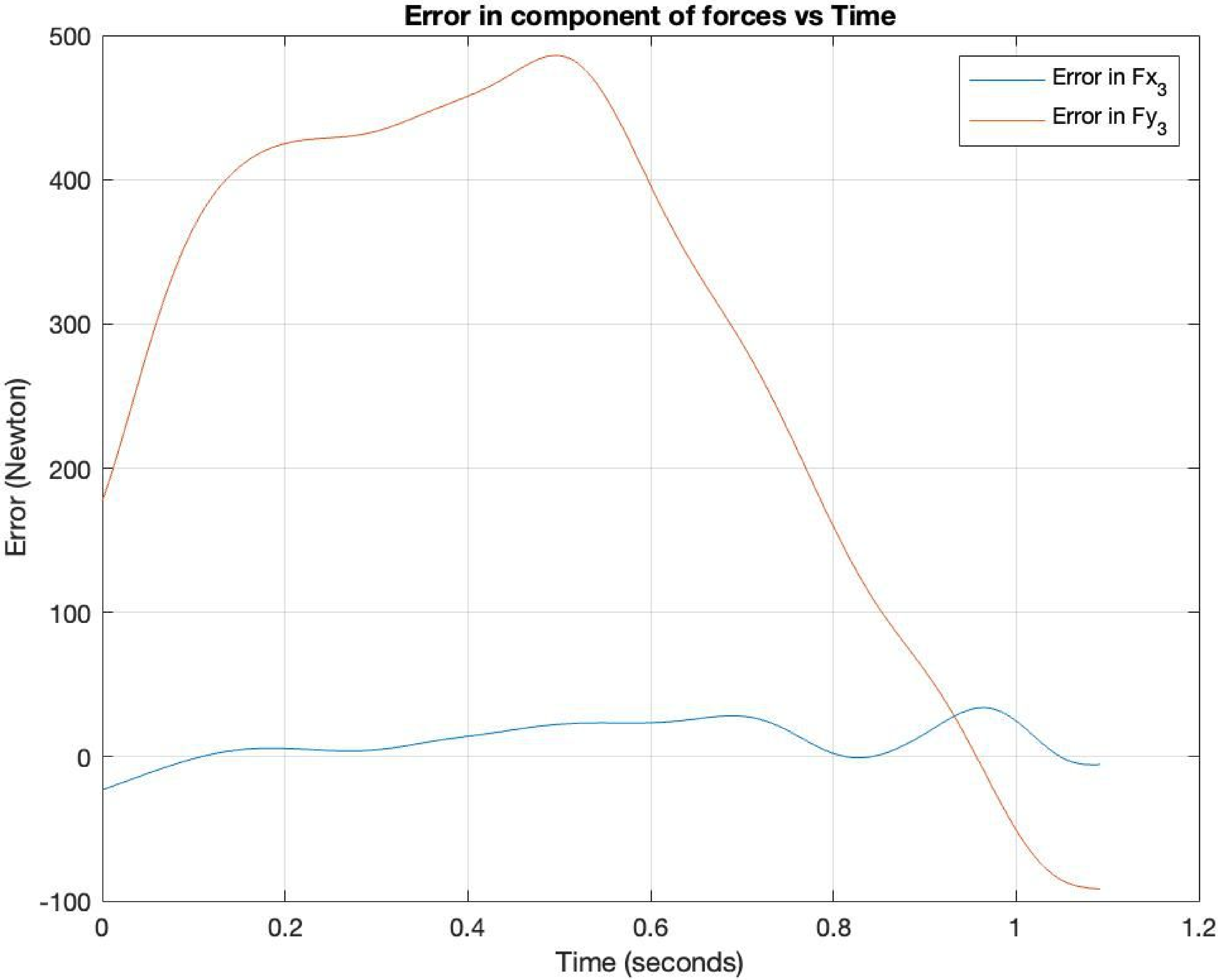
$$\Sigma M_{HAT,CM} = I_3 \ddot{\theta}_3$$

$$\Rightarrow T_3 + f_{x_3} r_3 s\theta_3 - f_{y_3} r_3 c\theta_3 = I_3 \ddot{\theta}_3$$

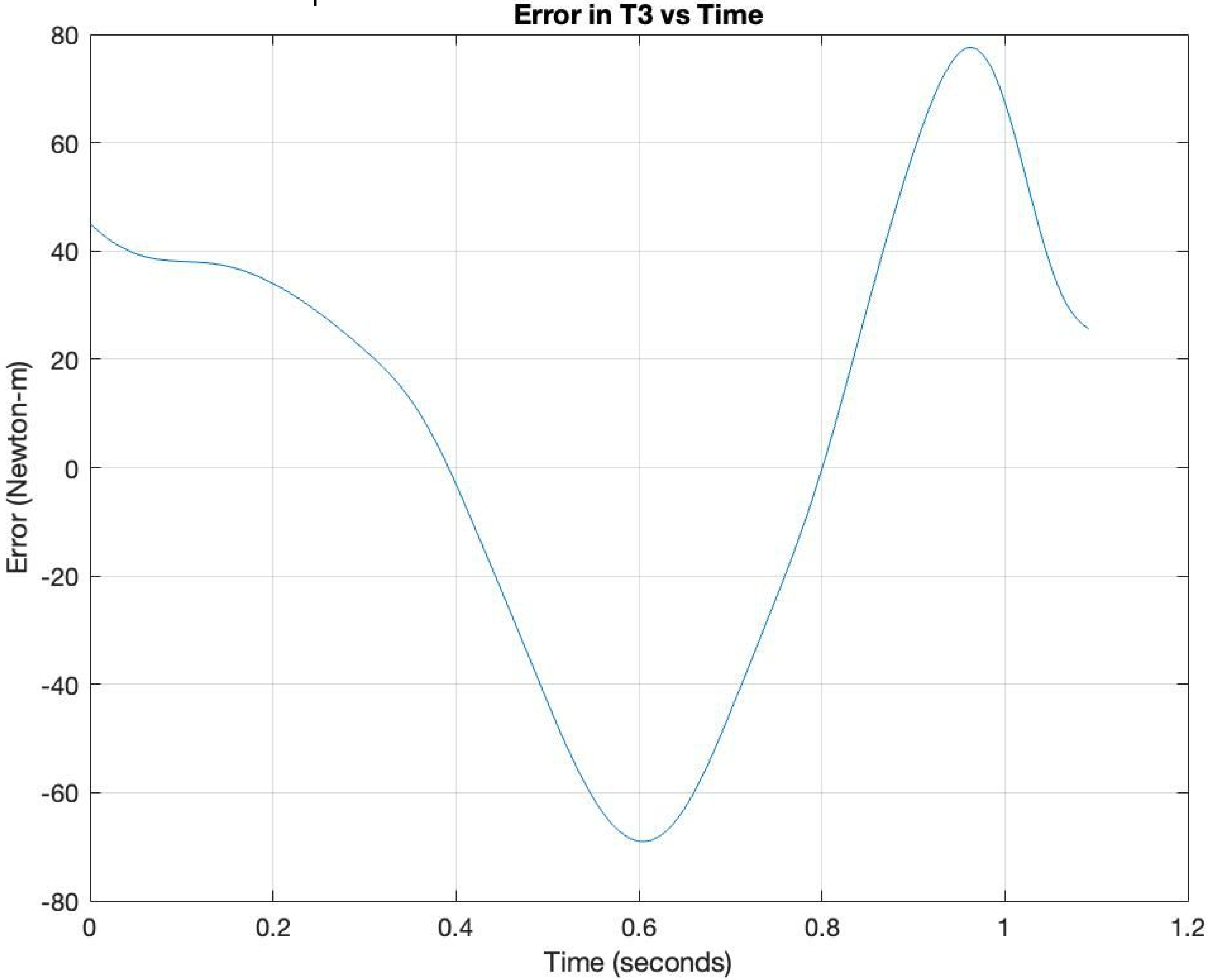
$$\Rightarrow \boxed{T_3 = f_{y_3} r_3 c\theta_3 - f_{x_3} r_3 s\theta_3 + I_3 \ddot{\theta}_3}$$

$$\Rightarrow \boxed{\begin{aligned} T_3 = \Big[& M_3 g + M_3 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + M_3 L_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) \\ & + M_3 r_3 (c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2)\Big](r_3 c\theta_3) + I_3 \ddot{\theta}_3 + \\ \Big[& M_3 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_3 L_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2) \\ & + M_3 r_3 (s\theta_3 \ddot{\theta}_3 + c\theta_3 \dot{\theta}_3^2)\Big](r_3 s\theta_3) \end{aligned}}$$

Hand of God Forces



Hand of God Torque



Problem 6:

Kinematics and forces are usually not consistent due to incorrect modelling assumptions and measurement errors. Also, the balance between external and inertial forces and moments becomes inconsistent due to several sources of modelling and experimental errors. The net joint torques obtained from an inverse dynamic analysis starting at the unconstrained end of a chain of segments and ending at the feet are different from those obtained when the analysis is started at the feet. This problem arises because the system of equations of motion for a complete linked segment model is overdetermined. A system of equations is considered overdetermined if there are more equations than unknowns. The main sources of error in calculations of net joint torques are inaccuracies in segmental motions and estimates of anthropometric body segment parameters. Errors affecting the trajectories of joint centres, the orientation of joint functional axes, the joint angular velocities, the accuracy of inertial parameters and force measurements, can weigh differently in the estimation of joint moments. Errors comprised in joint parameters include the position of the joint centres, and the position and direction of the joint axes of rotations affects both joint kinematics and dynamics. Selection of the coordinate system influences inverse dynamic analysis (IDA) and this influence can arise from the depinition of coordinate system used to

→

describe body segment anatomy. Uncertainty in estimated net joint torques derived from IDA, range from 6% to 232% of the peak net torque and thereby it becomes necessary to address this residual errors ~~forces~~ to ~~be to~~ make the system "dynamically consistent". However, in certain tasks, these residual errors are quite large and therefore strategies based on motion or force variation are applied. In high-speed tasks generally these errors ~~get~~ are high and causes to invalidate the conclusions of the dynamic analysis. Uncertainty in the joint torque could also result because of foot mass and ground reaction forces. This large effect of error arises due to the swing phase while performing a vertical jump.

<u>Bonus question</u>: Ways to minimize the magnitudes of the three components of the "Hand of God"

→ Several ways have been implemented to minimize the errors which leads to dynamical ~~x~~ inconsistency. Adding low-value residual pelvis (force and torque) actuators could deal with such problem. Optimization control algorithm can be used for tracking problem, in which implicit form of dynamics is used. Equations of motion ~~of~~ are introduced as path constraints, as well as residual forces and moments acting can be used as path constraints. These algorithms can deal with dynamic inconsistency in high-speed tasks, obtaining low residual forces and moments while keeping similar kinematics. the constrained optimization algorithm ensures a consistent description of forces and kinematics, thereby improving the validity of calculated net joint torques. Also, another way of computing first approximations of body segment parameters using a series of short calibration motions. These refined optimal body segment parameters derived from combination of motion profiles helps in improving net joint torque calculations and reducing the error of magnitudes of the three components of the "Hand of God".

Problem 7:

a) Given:

Hamstrings hip extension moment arm = 5 cm
Hamstrings knee flexion moment arm = 4 cm
Vasti knee extension moment arm = 4 cm
Soleus ankle extension moment arm = 2.5 cm

Torque = Force x Moment Arm

∴ Hamstrings hip extension force = $F_{ham\_hip\_ext}$ =

$$= \frac{-T_3}{5}$$

Hamstrings knee extension moment arm

$$= \frac{-\left\{\left[M_3 g + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)\right] r_3 C\theta_3 - \left[-M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2)\right] r_3 S\theta_3 + I_3 \ddot{\theta}_3\right\}}{5} \rightarrow (1)$$

Hamstrings knee flexion force = $T_2 / 4$

Hamstrings knee flexion moment arm

$$F_{ham\_knee\_flex} = \frac{\left[T_1 + (F_{gx} r_1 S\theta_1) - (F_{gy} r_1 C\theta_1) + (F_{x_2} d_1 S\theta_1) - (F_{y_2} d_1 C\theta_1) - I_1 \ddot{\theta}_1\right]}{4} \rightarrow (2)$$

where F_{x_2} and F_{y_2} are ~~eq~~ eq^n (4) and (5) from solved Problem 1.

Typed derivation can also be seen in the function and script pages

Hamstrings knee flexion moment arm

$$\boxed{\text{Vasti knee extension force} = \frac{\left[T_2 + \left(f_{ham\text{-}knee\text{-}flex} \times 4\right)\right]}{4}}$$

4 → Vasti knee extension moment arm

where T_2 is from eq^n ⑥ of Prob 1 solved
and $f_{ham\text{-}knee\text{-}flex}$ is from eq^n ② of this solved problem.

Soleus ankle extension force $= f_{sol\text{-}ankle\text{-}ext} =$

$$\boxed{f_{sol\text{-}ankle\text{-}ext} = -\frac{T_1}{2.5}}$$

2.5 → soleus ankle extension moment arm

where T_1 is from eq^n ③ ~~from~~ of Prom ① solved

$$T_1 = f_{gx} h - f_{gy} a$$

Which muscle should you beef up for jumping?

→ Quadriceps and hamstrings are primary thrusters when performing a vertical jump. The vasti muscle group, which is the lump of three heads of the quadriceps, seems to be the most active muscle group of all the three. This is because it develops the maximum amount of force over time as compared to hamstrings and the soleus. It seems that we should beef up vasti muscle group for better vertical leap. But also, as hamstrings are acting on two joints i.e. hip and knee, it also becomes responsible to build up hamstring muscles. But if we want to jump higher, it's equally important to awaken and strengthen assisting muscles - calves, the muscles around hips, and the glutes.

b) :- Static optimization resolves the net joint moments into individual muscle forces at each instant in time. The muscle forces are resolved by minimizing the sum of squared muscle activations. The static optimization method uses the known motion of the model to solve the equations of motion for the unknown generalized forces (e.g. joint torques). The forces could be subjected to one of the following muscle activation-to-force conditions:

Ideal force generators :-

$$\sum_{m=1}^{n} (a_m f_m^{o}) r_{mj} = \tau_j$$

or, constrained by force-length-velocity properties:

$$\sum_{m=1}^{n} [a_m f(f_m^{o}, l_m, v_m)] r_{m,j} = \tau_j$$

while we have to minimize the objective function:

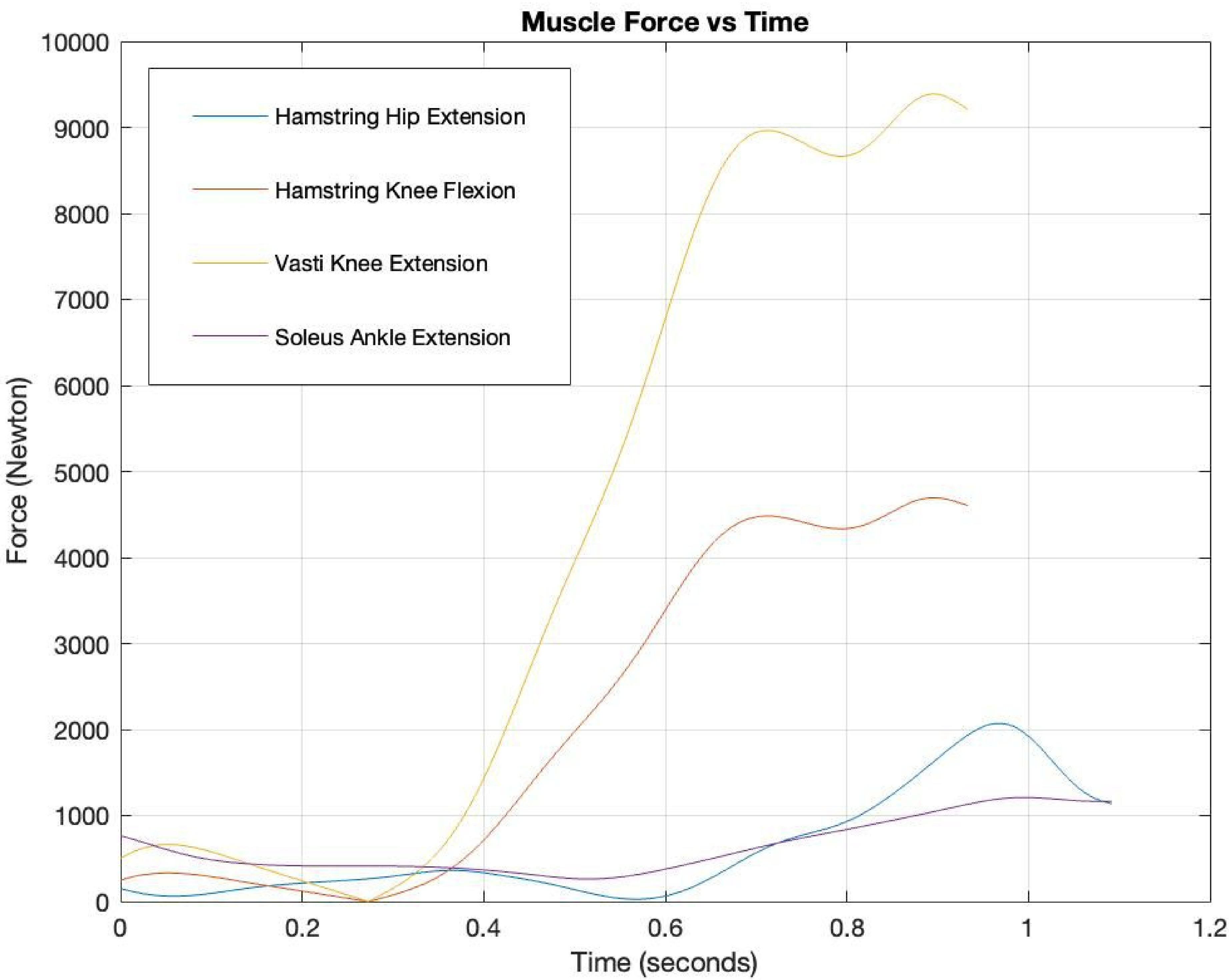
$$J = \sum_{m=1}^{n} (a_m)^p \qquad \left(a_m = \frac{f_m}{f_m^{max}} \right)$$

where n is the number of muscles in the body; a_m is the activation level of muscle m at a discrete time step; f_m^{o} is its maximum isometric force; l_m is it's length; v_m is its shortening velocity; $f(f_m^{o}, l_m, v_m)$ is its force-length-velocity surface; r_{mj} is its moment arm about the j^{th} joint axis; τ_j is the generalized force acting about the j^{th} joint axis; and p is a user-defined constant.

→

In static optimization, the question arises which muscles should be activated to generate desired net joint moments during the jumping activity. We can find the solution to this problem by assuming no activity for dorsiflexor and also we have to assume that each muscle will generate same amount of force. This is how we can solve for the number of unknowns to get the minimization of cost function. However, this approach is not physiologically reasonable.

The goal of static optimization is to solve for muscle activations that produce the dynamics of an observed motion. Since there are more muscles than the DOF in the human body, the problem of many possible solution exists, hence the need for optimization.



Muscle Force vs Time
Hamstring Hip Extension
Hamstring Knee Flexion
Vasti Knee Extension
Soleus Ankle Extension
Force (Newton)
Time (seconds)
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
0.2
0.4
0.6
0.8
1
1.2