HOMEWORK #1

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Course: MCEN 5228 – Modeling of Human Movement

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Q1)	a) Biceps Brachii: elbow flexion b) Triceps (all heads): elbow extension c) Anterior Deltoid: shoulder flexion d) Posterior Deltoid: shoulder extension e) Pectoralis Major: glenohumeral adduction f) Rectus Femoris: hip flexion g) Vastus Lateralis: knee extension h) Vastus Medialis: knee extension i) Vastus Intermedius: knee extension j) Semitendinosus: hip extension k) Semimembranosus: hip extension l) Biceps Femoris: knee flexion m) Gastrocnemius: ankle plantarflexion n) Soleus: ankle plantarflexion o) Tibialis Anterior: ankle dorsiflexion p) Gluteus Maximus: hip extension q) Gluteus Medius: hip abduction
Q2)	a) Movement in Sagittal plane: Walking b) Movement in Transverse plane: Twisting lunges c) Movement in Frontal plane: Jumping jack exercises

Q3) :→ $\ddot{x}_m + \frac{b}{m}\dot{x}_m + \frac{k_m}{m}x_m = 0$

This is a homogeneous linear second-order differential equation. The general solution of the differential equation is

$$x(t) = C\exp(\lambda_1 t) + D\exp(\lambda_2 t)$$

where C and D are arbitrary constants.

Let $p = b/m$ and $q = k_m/m$

$$\Rightarrow \frac{d^2x_m}{dt^2} + p\frac{dx_m}{dt} + \frac{k_m}{m}x_m = 0$$

$$\Rightarrow \frac{d^2x_m}{dt^2} + p\frac{dx_m}{dt} + q x_m = 0$$

Auxillary equation for this differential equation is,

$$\Rightarrow \lambda^2 + p\lambda + q = 0$$

The roots for the auxillary equation are

$$\lambda_1 = \frac{-p + \sqrt{p^2 - 4q}}{2} \quad \text{and} \quad \lambda_2 = \frac{-p - \sqrt{p^2 - 4q}}{2}$$

$$\Rightarrow \lambda_1 = \frac{-(b/m) + \sqrt{(b^2/m^2) - (4k_m/m)}}{2} \quad \text{and}$$

$$\lambda_2 = \frac{-(b/m) - \sqrt{(b^2/m^2) - (4k_m/m)}}{2}$$

$$\therefore x_m(t) = C.\exp\left[-t\left(\sqrt{b^2 - 4k_m.m} + b\right)/2m\right] + D.\exp\left[t\left(\sqrt{b^2 - 4k_m.m} - b\right)/2m\right]$$

Q4) :→ $\frac{dx(t)}{dt} = 5x(t) + 3$, where $x(0) = 0$

$$\Rightarrow \frac{dx(t)}{5x(t)+3} = d(t)$$

$$\Rightarrow \int \frac{dx(t)}{5x(t)+3} = \int d(t) + c_1$$

Let, $5x(t) + 3 = u$

Then, $5\,dx(t) = du \quad \Rightarrow dx(t) = du/5$

Substituting back in our equation, we get,

$$\Rightarrow \int \frac{du/5}{u} = \int d(t) + c_1$$

$$\Rightarrow \frac{1}{5} \ln u = t + c_1$$

$$\Rightarrow u = e^{5t} . e^{c_1} = e^{5t} . c_2 \quad (\text{where, } c_2 = e^{c_1})$$

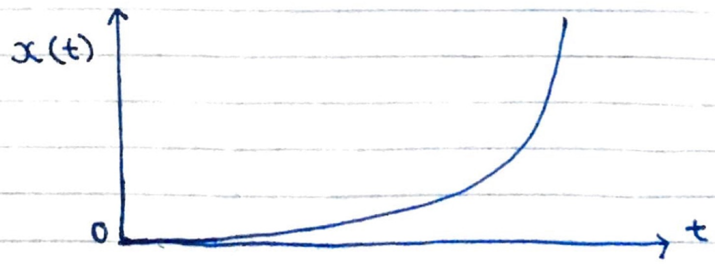
$$\Rightarrow 5x(t) + 3 = c_2 . e^{5t}$$

$$\Rightarrow x(t) = (c_2 e^{5t} - 3)/5$$

$$\Rightarrow x(t) = \frac{c_2}{5} e^{5t} - \frac{3}{5} = c_3 e^{5t} - \frac{3}{5} \quad (\text{where, } c_3 = \frac{c_2}{5})$$

Also, $x(0) = 0 = c_3 e^{0} - \frac{3}{5} \quad \Rightarrow c_3 = 3/5$

$$\therefore \boxed{x(t) = \frac{3}{5}\left(e^{5t} - 1\right)}$$



Q5) :→ Given two vectors $\vec{x}$ and $\vec{y}$, $\vec{x} \times \vec{y}$ is a vector that is perpendicular to both $\vec{x}$ and $\vec{y}$.

$\therefore$ Let, $\vec{w} = \vec{x} \times \vec{y}$

$$\Rightarrow \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix}$$

$$\Rightarrow \vec{w} = \hat{i}\left(9-(-25)\right) - \hat{j}\left(12-(-5)\right) + \hat{k}\left(-20-(-3)\right)$$

$$\Rightarrow \vec{w} = 34\hat{i} - 17\hat{j} - 17\hat{k}$$

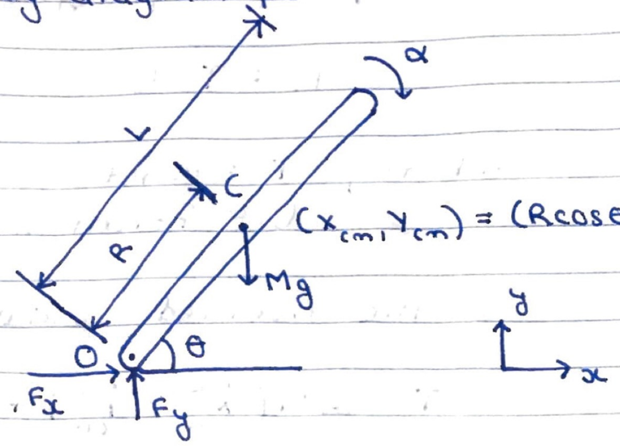
Unit vector, $\hat{w}$ is given by, $\frac{\vec{w}}{||\vec{w}||}$

$$\Rightarrow ||\vec{w}|| = \sqrt{(34)^2 + (-17)^2 + (-17)^2} = 17\sqrt{6}$$

$$\Rightarrow \hat{w} = \frac{34\hat{i} - 17\hat{j} - 17\hat{k}}{17\sqrt{6}} = \frac{\sqrt{6}}{3}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$$

Hence, required vector, $\hat{w} = \left(\frac{\sqrt{6}}{3}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

Q6) :→ Free body diagram for the inverted pendulum :-



Q7) :→ Method 1) :→ Using Newtonian Mechanics :

We know that, $M_\theta = I_\theta \alpha$

(where, M_o is moment about point O)

$\Rightarrow \quad M_o = Mg(R\cos\theta)$

Also, $I_o = I_{cm} + MR^2$

$\therefore \quad M_o = MgR\cos\theta = (I_{cm} + MR^2)\alpha$

$$\Rightarrow \quad \boxed{\alpha = \frac{MgR\cos\theta}{(I_{cm} + MR^2)}}$$

$\rightarrow$

Method 2) :→ Using Lagrangian Mechanics:

$$L = T - U$$

where, T = Kinetic energy
U = Potential energy

For the inverted pendulum problem,

$$T = \frac{1}{2}M\left(\dot{x}_{CM}^2 + \dot{y}_{CM}^2\right) + \frac{1}{2}I_{CM}\dot{\theta}^2$$

$$\Rightarrow T = \frac{1}{2}M\left(R^2\dot{\theta}^2\cos^2\theta + R^2\dot{\theta}^2\sin^2\theta\right) + \frac{1}{2}I_{CM}\dot{\theta}^2$$

$$\Rightarrow T = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$U = Mg(R\sin\theta) = MgR\sin\theta$$

So, $L = T - U$

$$\Rightarrow L = \frac{1}{2}MR^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2 - MgR\sin\theta$$

According to Euler-Lagrange equations, or Lagrange's equations of the second kind,

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0 \quad — (A)$$

So, $$\frac{\partial L}{\partial \theta} = -MgR\cos\theta \quad — (i)$$

→

Also, $\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} MR^2 2\dot{\theta} + \frac{1}{2} I 2\dot{\theta} = \dot{\theta}(MR^2 + I)$

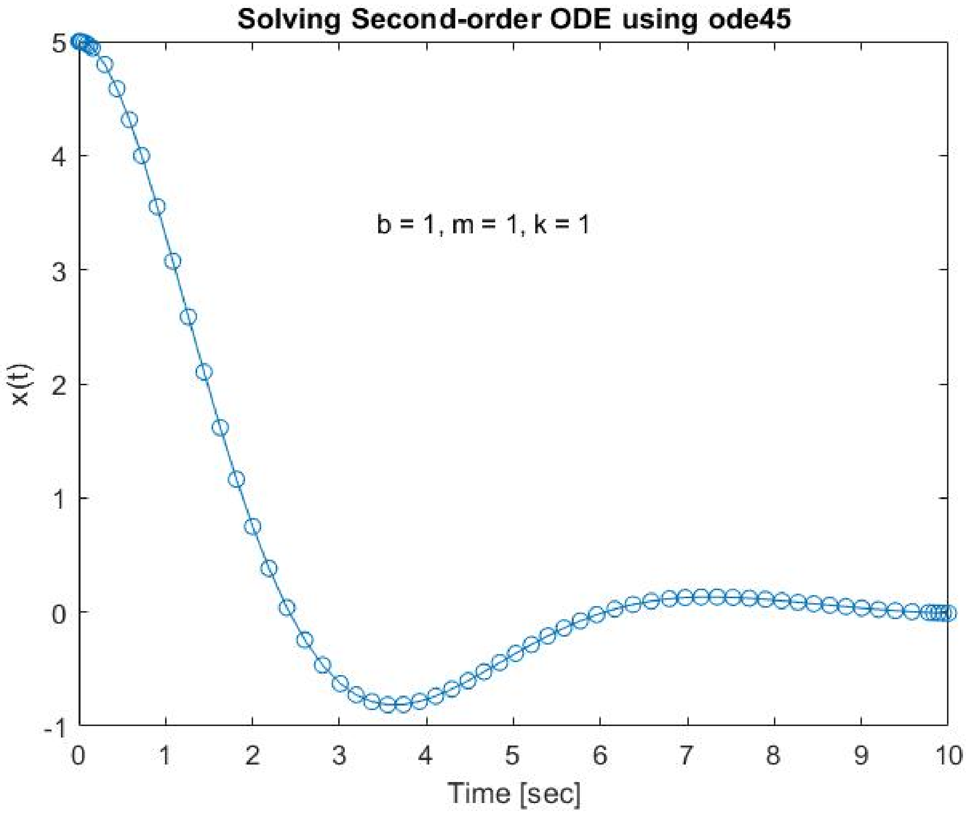
$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \ddot{\theta}(MR^2 + I)$ — (ii)

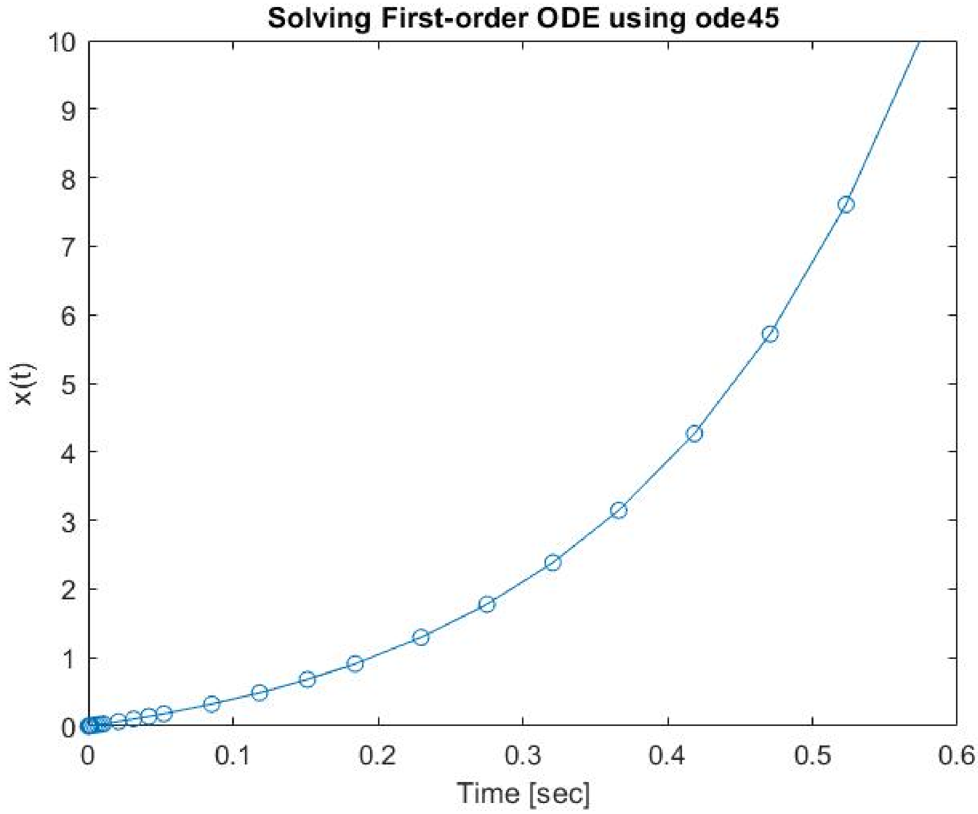
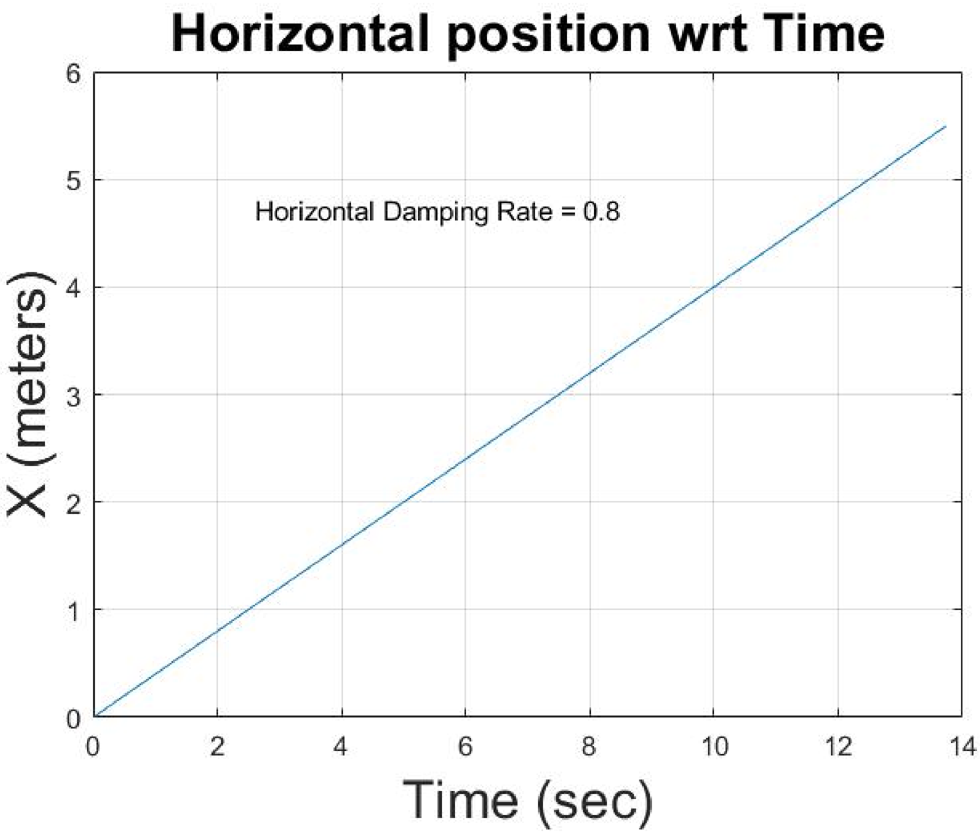
Substituting (i) and (ii) in (A), we get,

$\Rightarrow \quad -MgR\cos\theta - \ddot{\theta}(MR^2 + I) = 0$

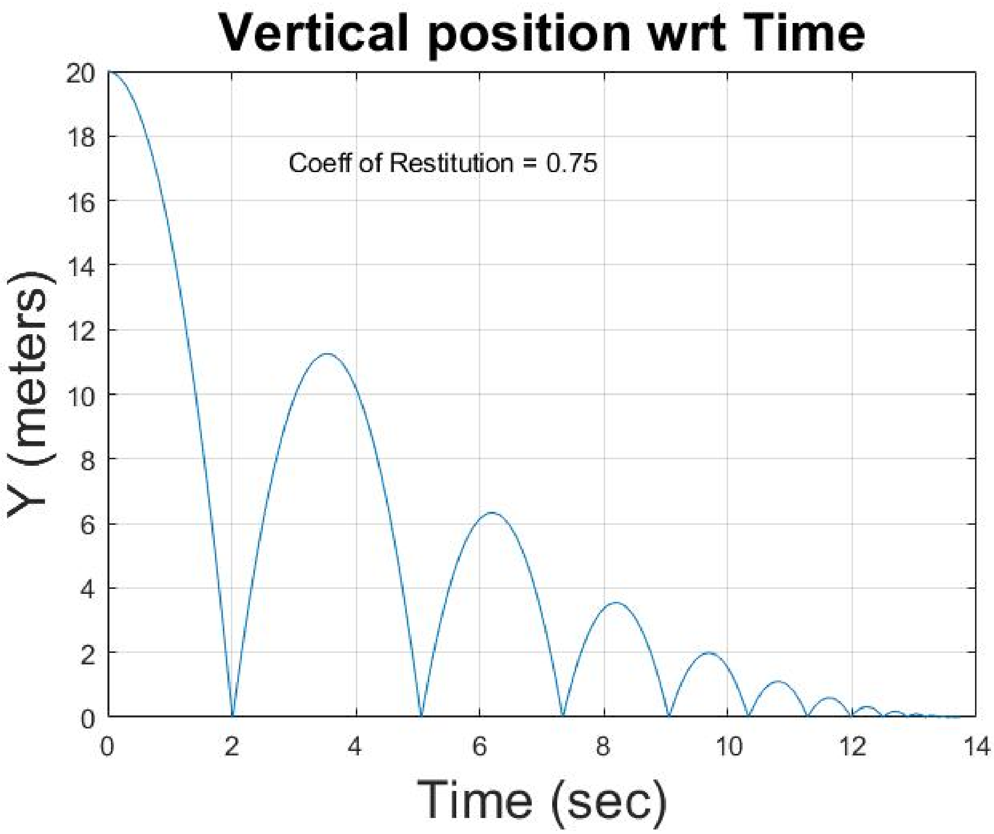
$\Rightarrow \quad \ddot{\theta}(MR^2 + I) = -MgR\cos\theta$

$\Rightarrow \quad \boxed{\ddot{\theta} = \frac{-MgR\cos\theta}{(I + MR^2)}}$

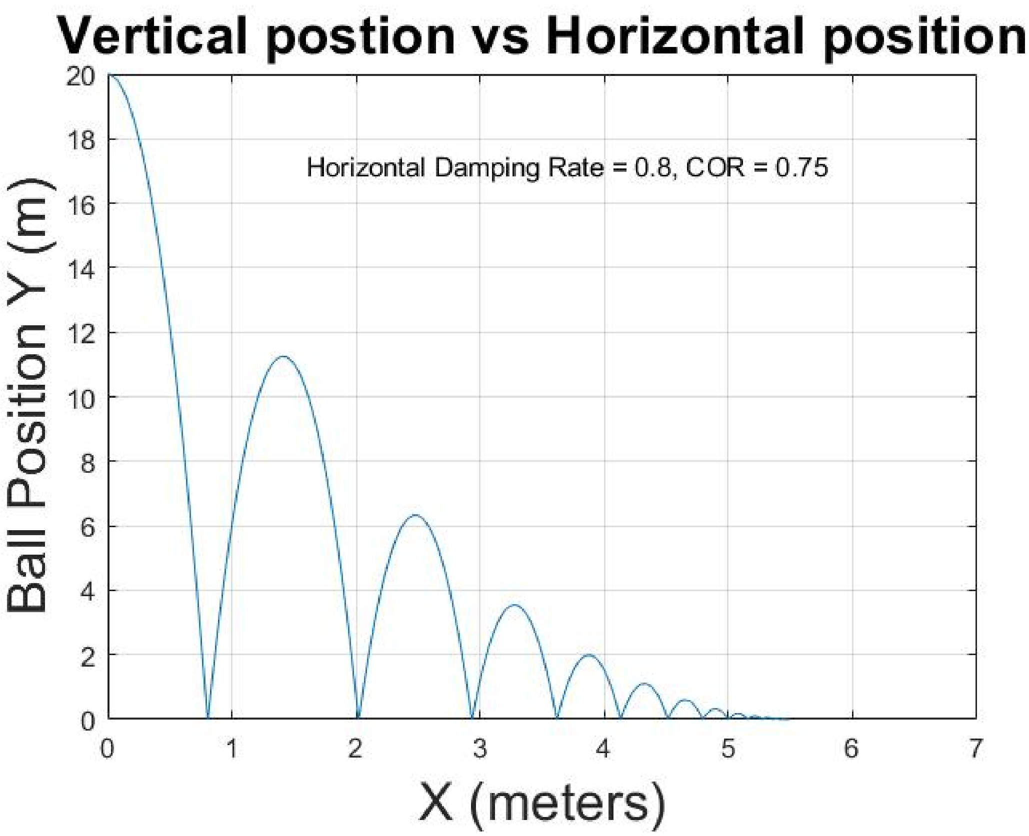
Q8)	According to Hooke's law, the displacement or size of deformation is directly proportional to the deforming force or load for relatively small deformations of an object. The equation for Hooke's law is, F = k*x, where F is the force, x is the length of extension/compression and k is a constant of proportionality known as the spring constant. Hooke's law can represent muscles and tendons (the tissue connecting muscle to bone). It is because of the elasticity of biological substances and muscle fibers. Some tendons have a high collagen content so there is relatively little strain, on the other hand, support tendons can have strain up to 10%.
Q9)	a) 

	b)  Solving First-order ODE using ode45 x(t) Time [sec] 0 1 2 3 4 5 6 7 8 9 10 0 0.1 0.2 0.3 0.4 0.5 0.6
Q10)	a)  Horizontal position wrt Time Horizontal Damping Rate = 0.8 X (meters) Time (sec) 0 1 2 3 4 5 6 0 2 4 6 8 10 12 14

b)


Vertical position wrt Time
Coeff of Restitution = 0.75
Y (meters)
Time (sec)
0
2
4
6
8
10
12
14
16
18
20
0
2
4
6
8
10
12
14

c)


Vertical postion vs Horizontal position
Horizontal Damping Rate = 0.8, COR = 0.75
Ball Position Y (m)
X (meters)
0
2
4
6
8
10
12
14
16
18
20
0
1
2
3
4
5
6
7