Muscle force optimization

MCEN 4/5228

Modeling of Human Movement

Fall 2021

Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

Biological and numerical optimizers

- Humans are optimizers
  - Movement is inherently an under-determined problem
  - Through trial and error settle on movement solution that optimizes a subconscious cost function

Biological and numerical optimizers

- Numerical optimization
  - Use similar exploratory strategies to find optimal solutions for under-determined problems
  - Guesses: *candidate solutions,* each of which proposes a numerical value for all unknowns or *design variables* (for example: muscle excitations)
  - Suitability of each solution determined by evaluating the *objective function* or *cost function*, an expression that quantifies the goodness or favorability of the solution
  - The candidate solution that provides the best objective function value is the optimal solution

Biological and numerical optimizers

- Constrained optimization
  - Nature of solution is also determined by *constraints*, leading to a *feasible* set of candidate solutions.
  - Muscle redundancy example:
    - All muscles must generate tensile forces
    - Moments must sum to the previously calculate net joint moments
  - Constraints can only reduce the size of the *solution space.*

Generic Optimization Problem

minimize	$J(\underline{x})$	Adjust design variables $\underline{x}$ to minimize objective function $J(\underline{x})$
subject to	$g_i(\underline{x}) \leq 0, \quad i = 1, \ldots, n^i$	while satisfying n^i inequality constraints,
	$h_j(\underline{x}) = 0, \quad j = 1, \ldots, n^j$	satisfying n^j equality constraints,
	$\underline{x}^{\text{lower}} \leq \underline{x} \leq \underline{x}^{\text{upper}}$	and respecting bounds on the design variables.

Biological and numerical optimizers

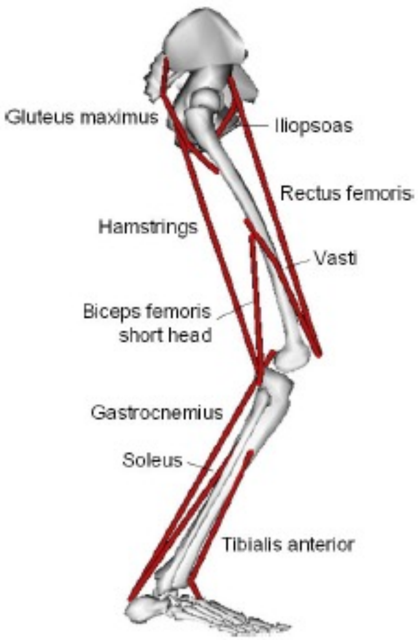
- Subconsciously, the brain selects movements using these same principles.
- *Consider reaching for the pen on your desk…*
- *Movement modelers use numerical optimization to solve the muscle redundancy problem*
    - *Many different combinations of muscle activations can lead to the same net joint torques*

Muscle force optimization

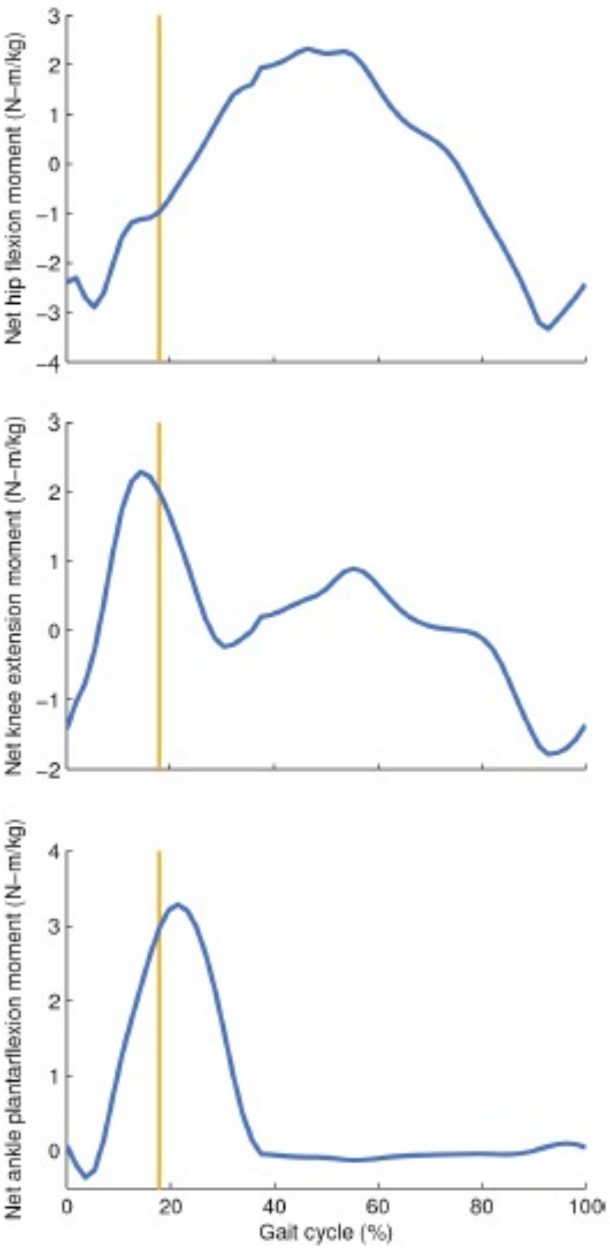
- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

Static optimization

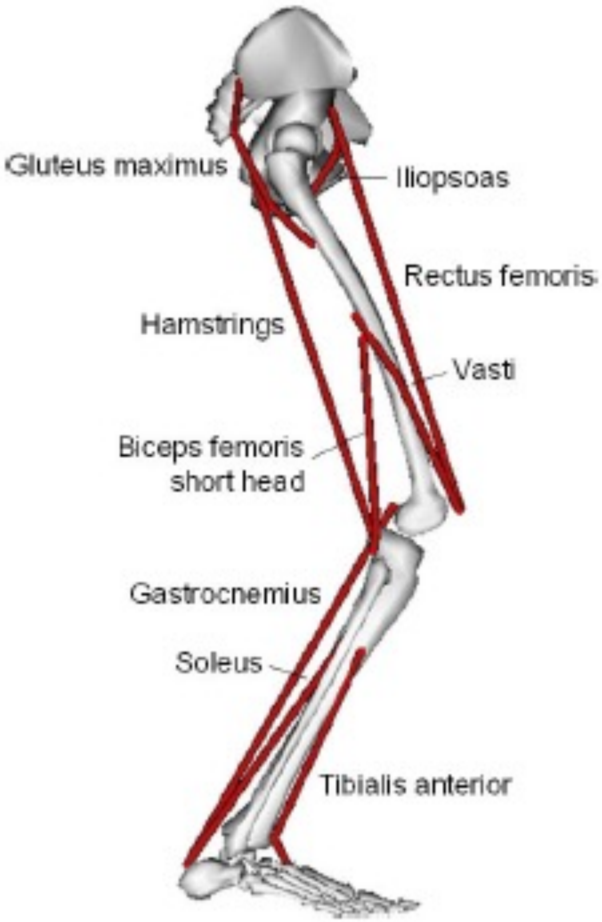
- Net joint moments during running with time of peak vertical GRF (m = 69.4kg)
- How do we activate our muscles to generate these moments?



Muscle or group	Instantaneous force-generating capacity (N)	Instantaneous moment arms (mm)			PCSA (cm²)
		Hip flexion	Knee flexion	Ankle dorsi.	
Gluteus maximus	3316	–55.8			70.2
Iliopsoas	4237	42.8			73.7
Hamstrings	3744	–54.4	42.1		94.8
Rectus femoris	2328	48.6	–48.5		39.0
Biceps femoris short head	1369		35.3		26.8
Vasti	8655		–43.6		150.9
Gastroc-nemius	4097		19.3	–39.0	75.0
Soleus	6435			–36.4	118.3
Tibialis anterior	1207			45.1	30.2



Simplifying assumptions



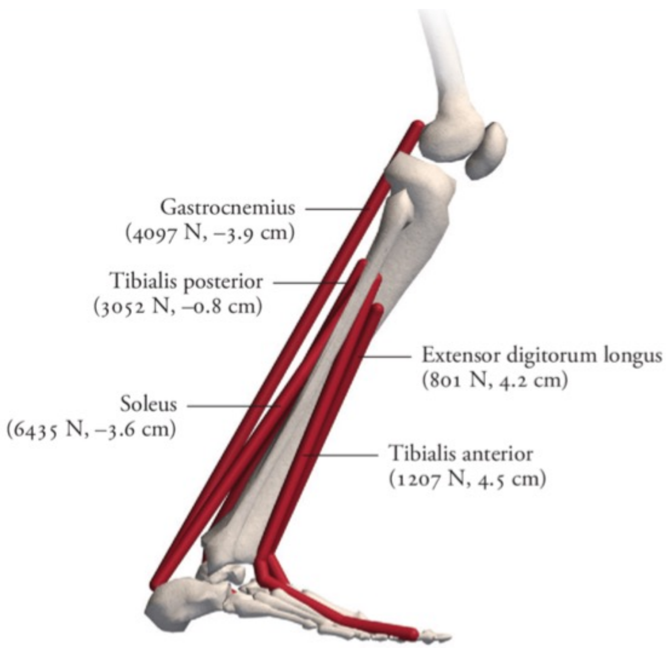
- Muscle act in 3D
- Joint stability
- Minimize total muscle force?
- Muscle mechanics

- Not so trivial to predict muscle activations!

Let’s start with a simpler problem

Goal: generate a net ankle plantarflexion moment of 100Nm

Q: which muscles should be activated?

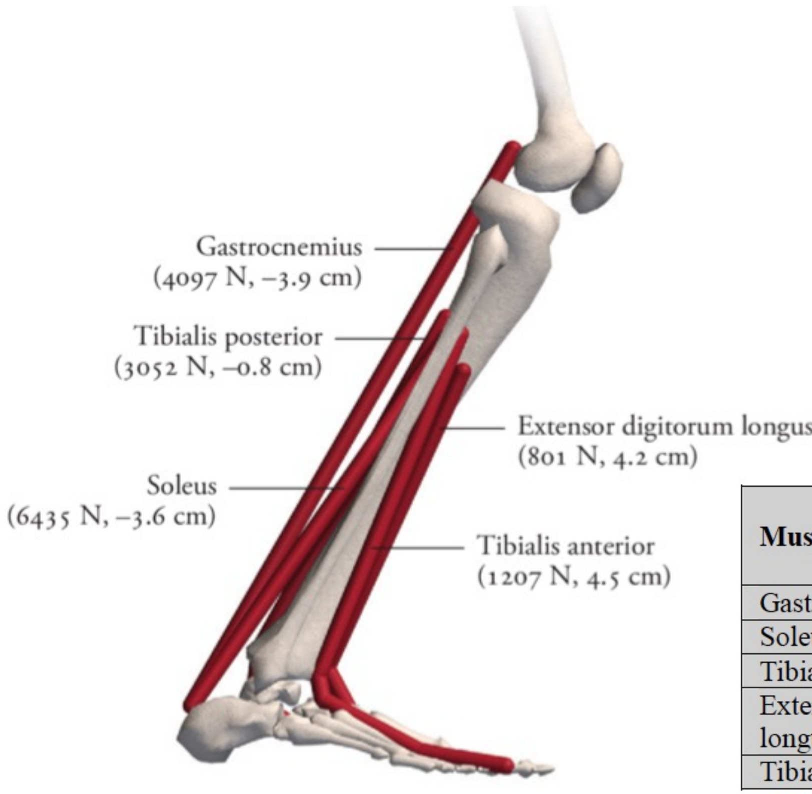


Muscle	**Instantaneous force-generating capacity (N)**	**Instantaneous moment arm (mm)**	**PCSA (cm^2)**
Gastrocnemius	4097	–39.0	75.0
Soleus	6435	–36.4	118.3
Tibialis posterior	3052	–8.4	52.9
Extensor digitorum longus	801	42.3	17.1
Tibialis anterior	1207	45.1	30.2

Sol1: Reduce number of unknowns

Goal: generate a net ankle plantarflexion moment of 100Nm

Q: which muscles should be activated?



Solution: reduce number of unknowns
→ No dorsiflexor activity
→ Still have one equation / 3unknowns

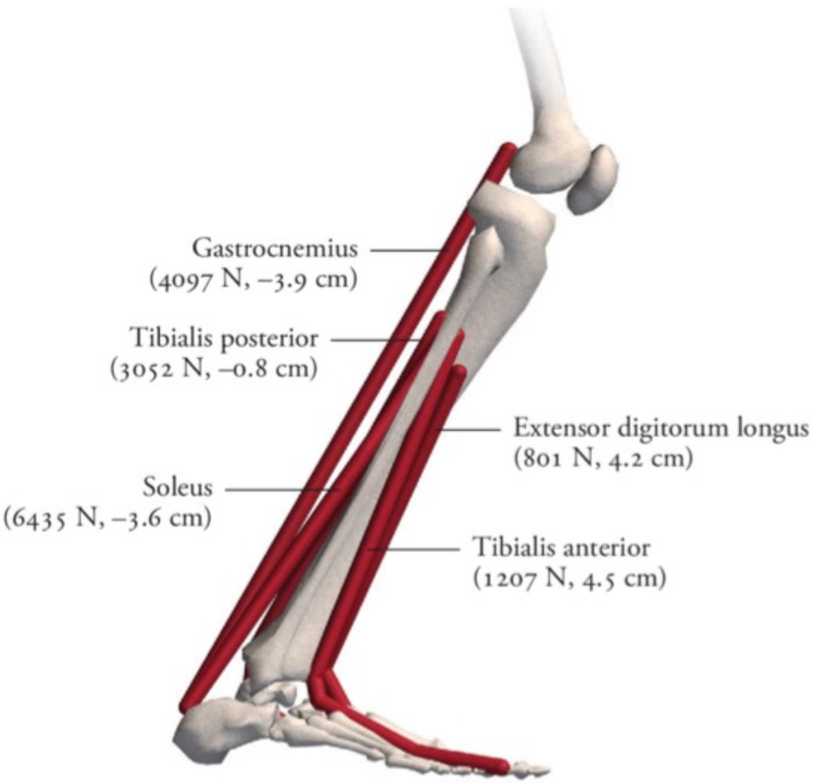
Solution: Assume each muscle will generate same amount of force
→ Two additional equations
→ Not physiologically reasonable

Muscle	Instantaneous force-generating capacity (N)	Instantaneous moment arm (mm)	PCSA (cm^2)
Gastrocnemius	4097	–39.0	75.0
Soleus	6435	–36.4	118.3
Tibialis posterior	3052	–8.4	52.9
Extensor digitorum longus	801	42.3	17.1
Tibialis anterior	1207	45.1	30.2

Optimization approach

Goal: generate a net ankle plantarflexion moment of 100Nm

Q: which muscles should be activated?



Optimization Problem 1: Find the ankle plantarflexor muscle forces that produce a desired joint moment

minimize	$J(\underline{F})$	Objective function where smaller values are favored
subject to	$0.039F^{GAS} + 0.036F^{SOL} + 0.008F^{TP} = 100$	Muscles must produce the desired net ankle moment
	$0 \leq F^{GAS} \leq 4097$ $0 \leq F^{SOL} \leq 6435$ $0 \leq F^{TP} \leq 3052$	Muscle forces must lie within physiological ranges

$$J = F_1 + F_2 + F_3 \equiv \sum_{i=1}^{3} F_i$$

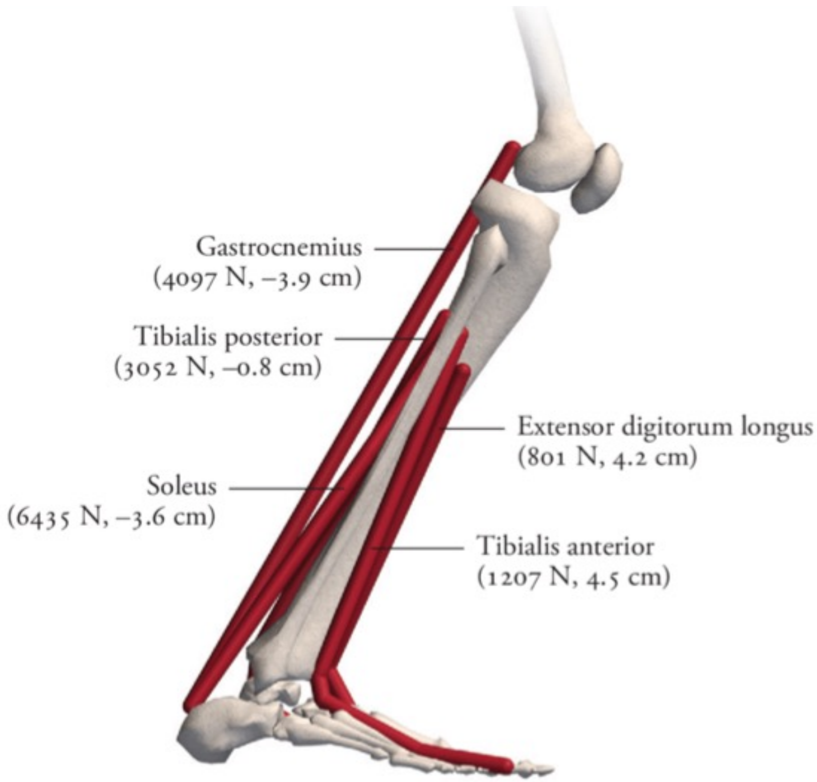
$$F \approx [2564, 0, 0]^T$$

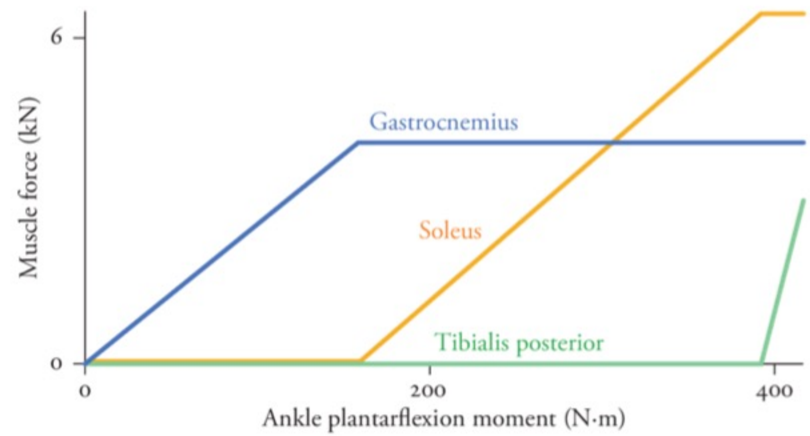
Below max gastroc force

Optimization approach

Goal: generate a net ankle plantarflexion moment of 100Nm

Q: which muscles should be activated?





$$J = F_1 + F_2 + F_3 \equiv \sum_{i=1}^{3} F_i$$

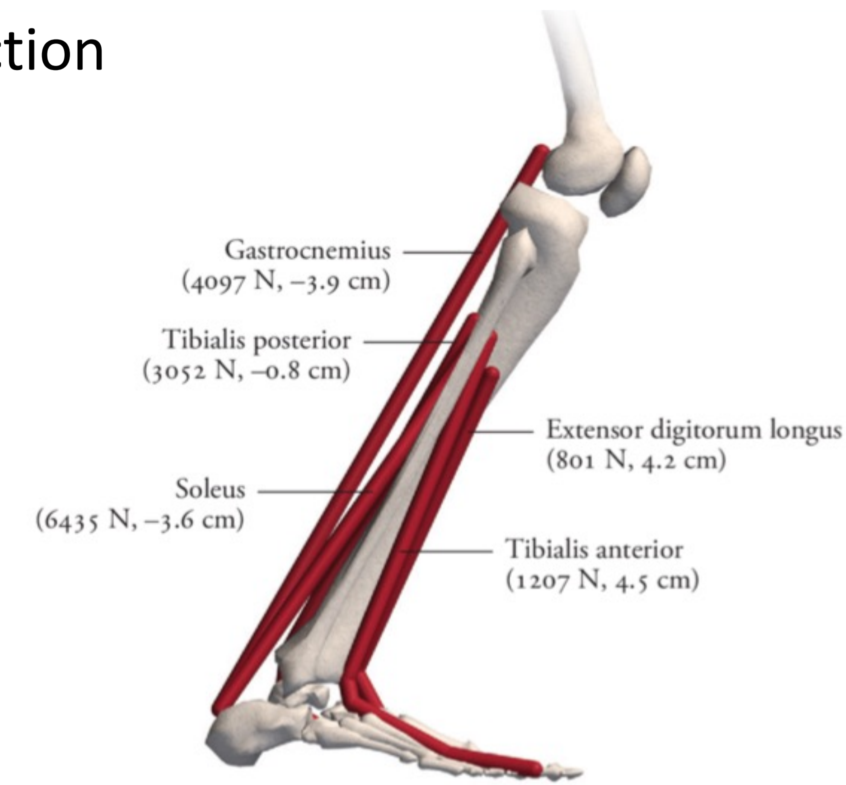
Not realistic to have each muscle reach its max force before the next one is recruited

Optimization approach

Goal: generate a net ankle plantarflexion moment of 100Nm
Q: which muscles should be activated?

Need more complex objective function

Difficult to solve via inspection



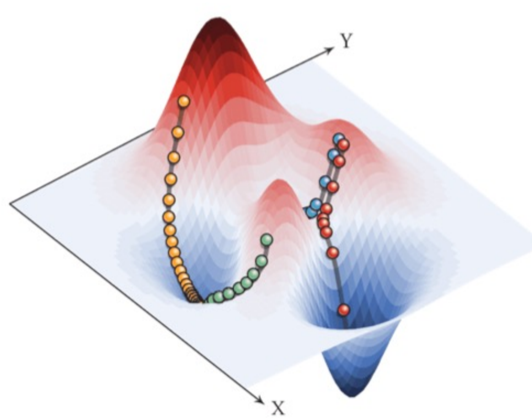
Local optimization algorithms

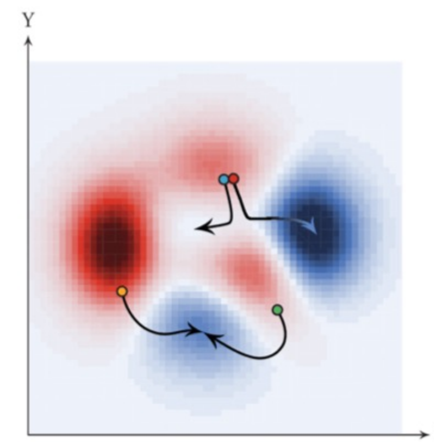
- Start from an initial guess (candidate solution)
- Use steepest descent approach to find minimum
- Algorithm terminates when it reaches a local optimum
- Solution depends on initial guess and may not be globally optimal

Example: objective function in two variables, J(x,y)

Gradient descent:
Starts at a point in the solution space
Takes steps downhill in direction of steepest local slope
Stops when movement in any direction would increase J(x,y)

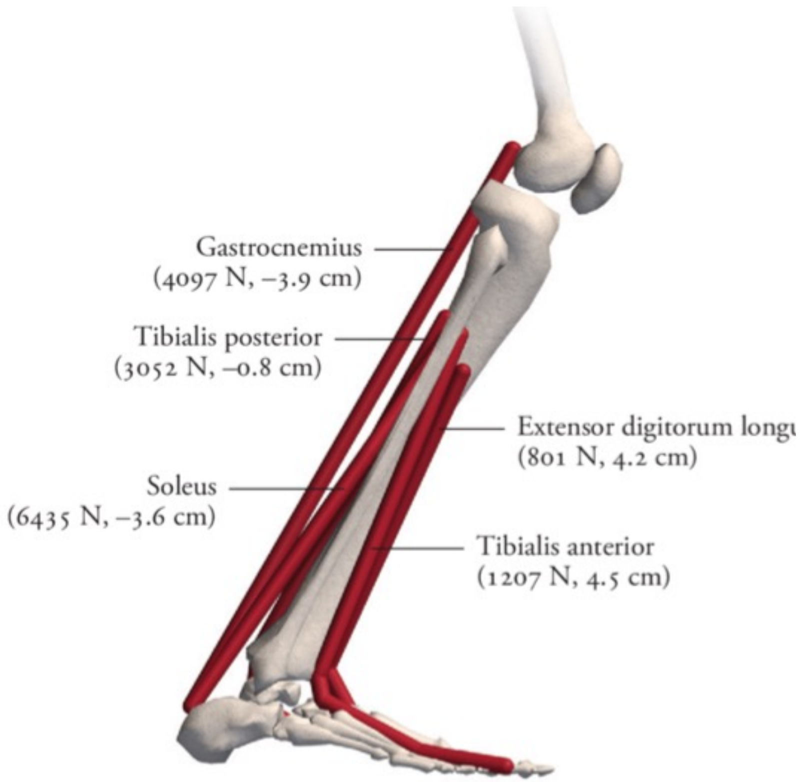
MATLAB: *fminsearch, fmincon*





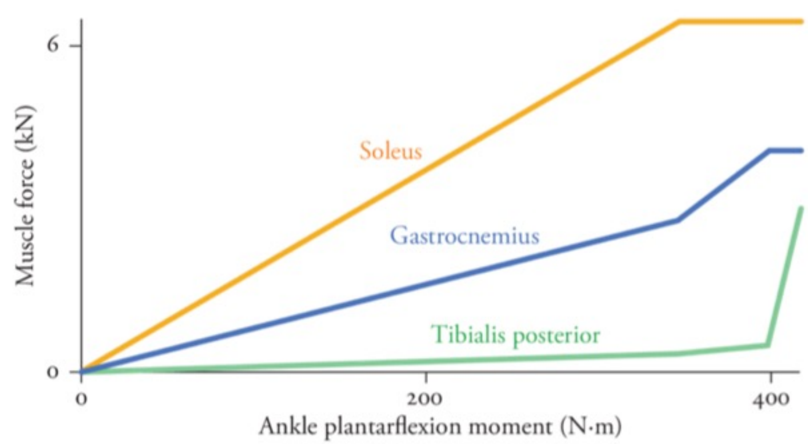
Optimization approach

Goal: generate a net ankle plantarflexion moment of 100Nm
Q: which muscles should be activated?



Minimize squared muscle activations:

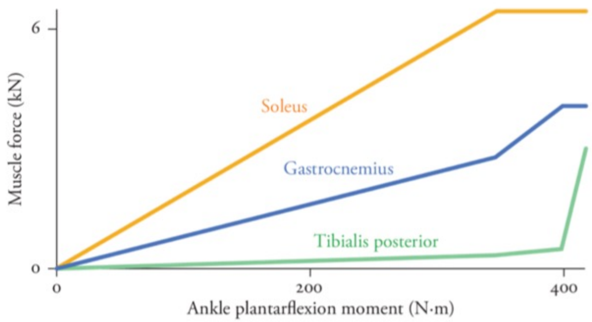
$$J = \sum_{i=1}^{3} \left(\frac{F_i}{F_i^{\max}} \right)^2 \equiv \sum_{i=1}^{3} a_i^2$$

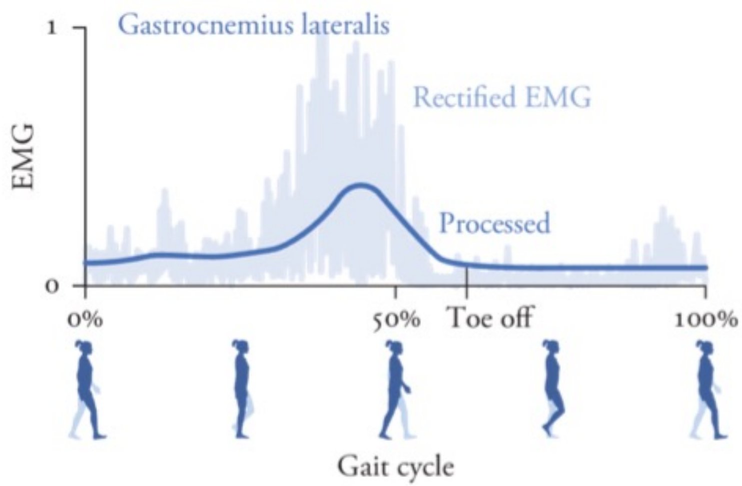


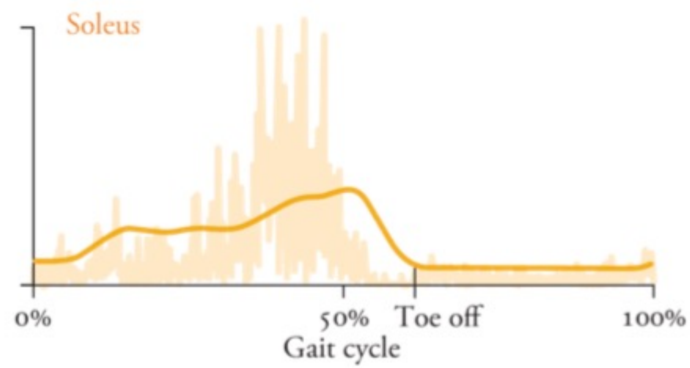
Optimization approach

Minimizing sum of muscle activations squared matches well with experimental observations

In general, all muscles are recruited to some degree regardless of the required plantarflexor moment



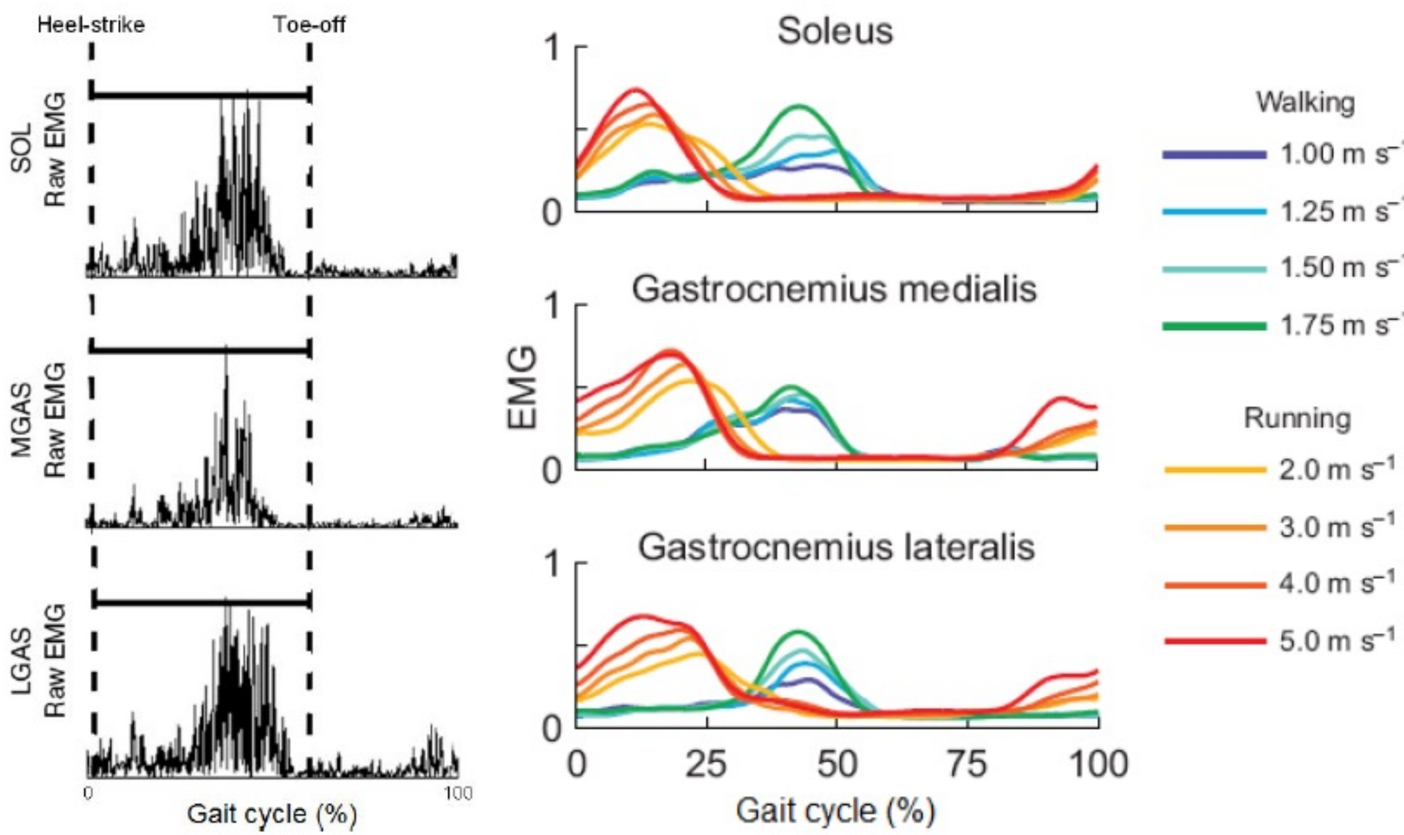




Optimization approach

Minimizing sum of muscle activations squared matches well with experimental observations

In general, all muscles are recruited to some degree regardless of the required plantarflexor moment

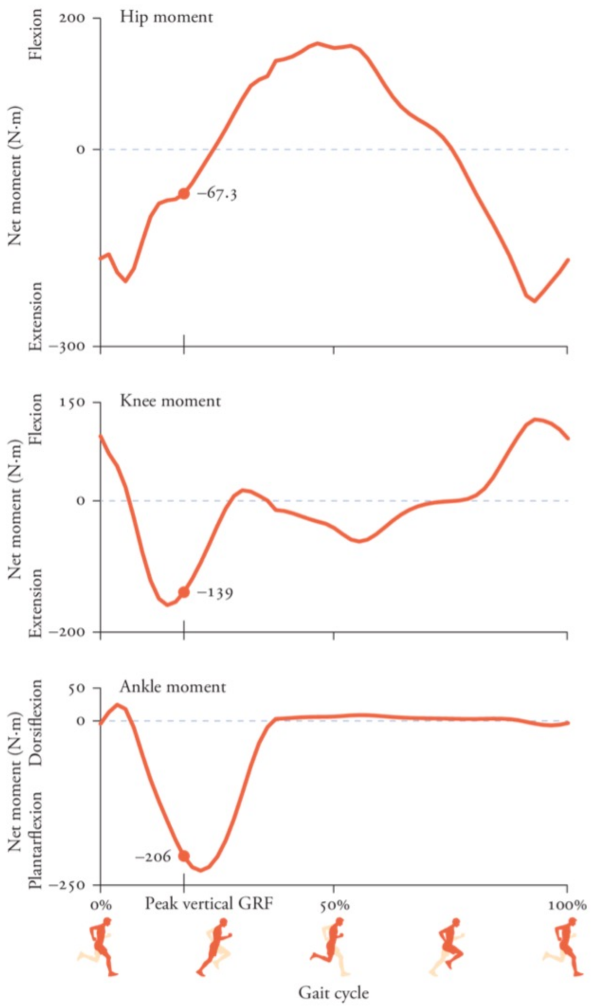


Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

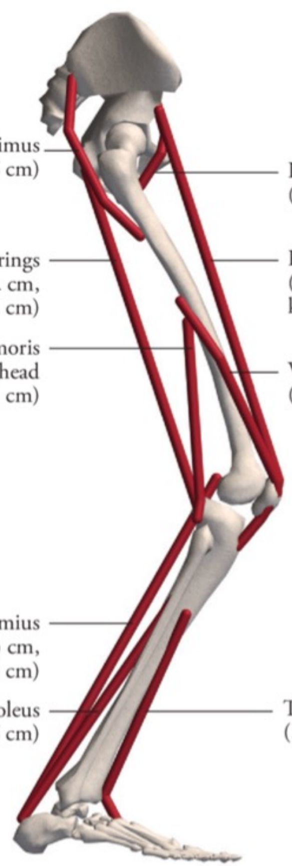
Muscle forces during walking and running

- Goal: muscle forces at a given instant during stance phase of running (instant of peak vertical GRF)



Muscle forces during walking and running

Goal: muscle forces at a given instant during stance phase of running (instant of peak vertical GRF)



Optimization Problem 2: Find muscle activations at the instant of peak vertical ground reaction force during running

minimize $$J(\underline{a}) = \sum_{i=1}^{9} a_i^2$$

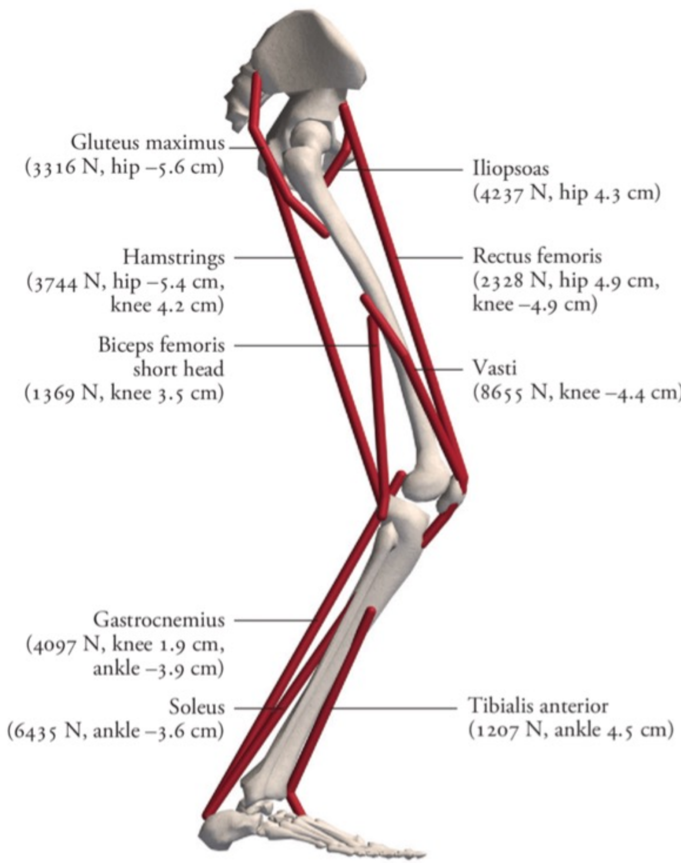
subject to
$$\sum_{i=1}^{9} a_i \left(r_i^{\text{hip}} F_i^{\text{max}}\right) = -67.3$$
$$\sum_{i=1}^{9} a_i \left(r_i^{\text{knee}} F_i^{\text{max}}\right) = -139$$
$$\sum_{i=1}^{9} a_i \left(r_i^{\text{ankle}} F_i^{\text{max}}\right) = -206$$
$$0 \le a_i \le 1 \quad \text{for } i = 1, \ldots, 9$$

Muscle or group	Force, F_i (N)
Gluteus maximus	875
Iliopsoas	0
Hamstrings	340
Rectus femoris	0
Biceps femoris short head	0
Vasti	4134
Gastrocnemius	1396
Soleus	4167
Tibialis anterior	0

Muscle forces during walking and running

Goal: muscle forces during gait cycle

Repeat analysis at evenly-spaced instants over gait cycle



Optimization Problem 2: Find muscle activations at the instant of peak vertical ground reaction force during running

minimize $$J(\underline{a}) = \sum_{i=1}^{9} a_i^2$$

subject to $$\sum_{i=1}^{9} a_i \left(r_i^{\text{hip}} F_i^{\max}\right) = -67.3$$

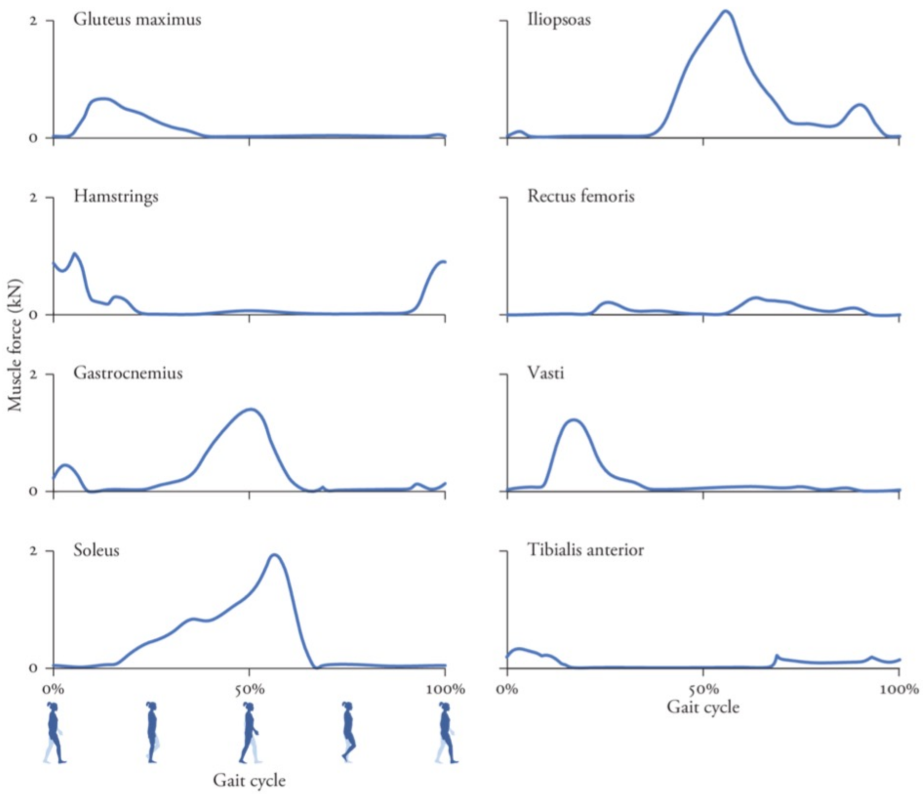
$$\sum_{i=1}^{9} a_i \left(r_i^{\text{knee}} F_i^{\max}\right) = -139$$

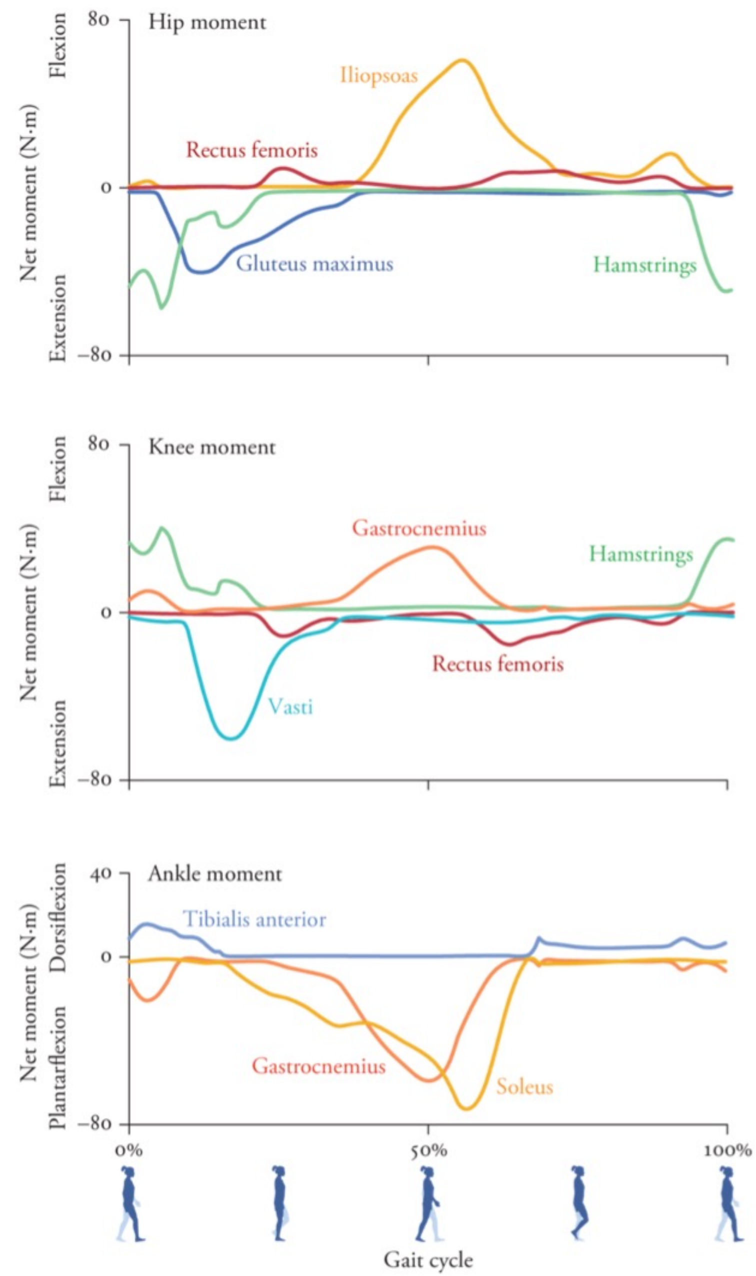
$$\sum_{i=1}^{9} a_i \left(r_i^{\text{ankle}} F_i^{\max}\right) = -206$$

$$0 \le a_i \le 1 \quad \text{for } i = 1, \ldots, 9$$

Walking

Walking at 1.67 m/s; m=67.1kg

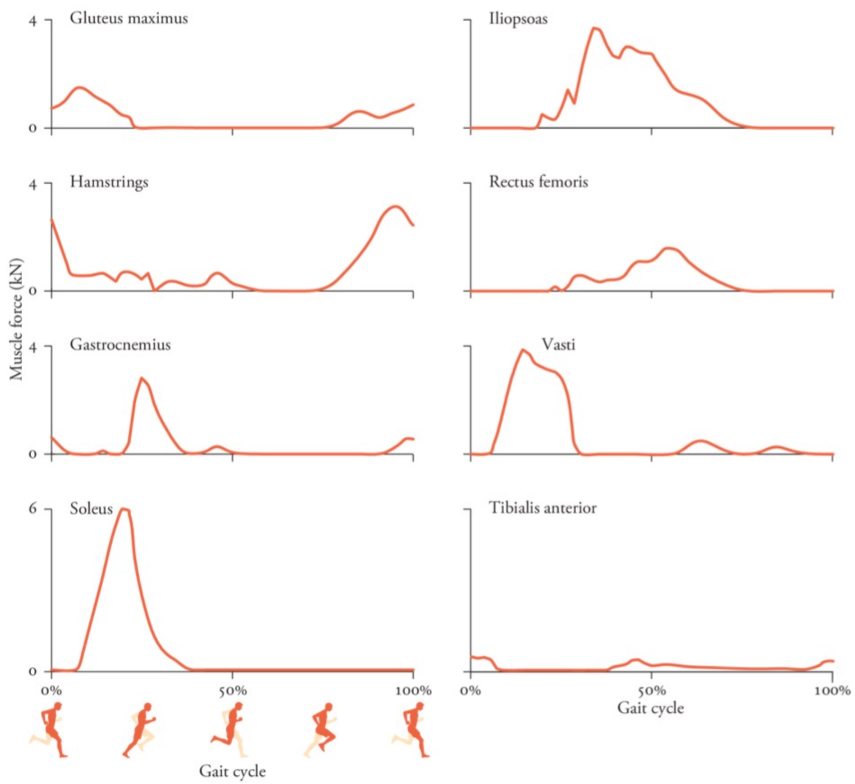


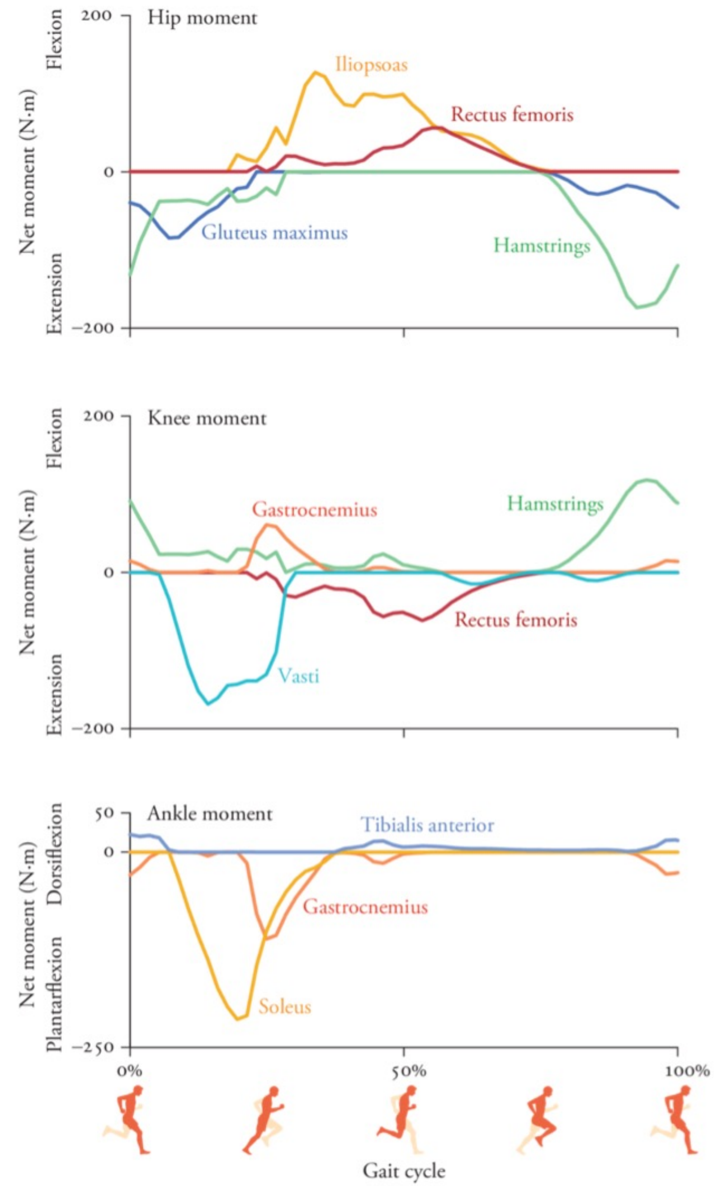


Dembia et al. 2017

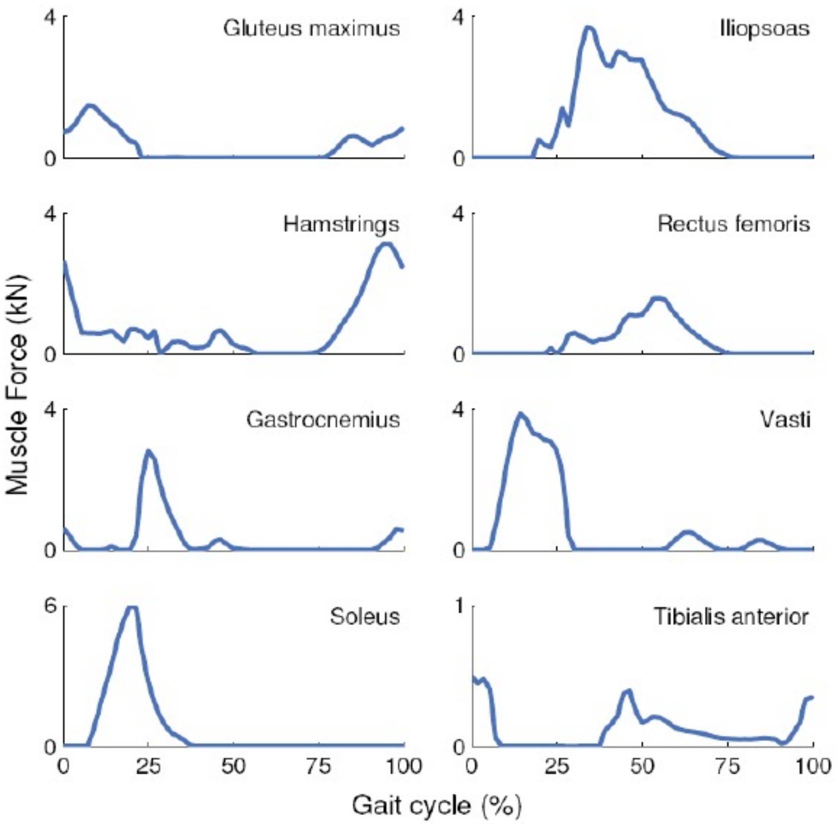
Running

Running at 5 m/s; m=69.4kg

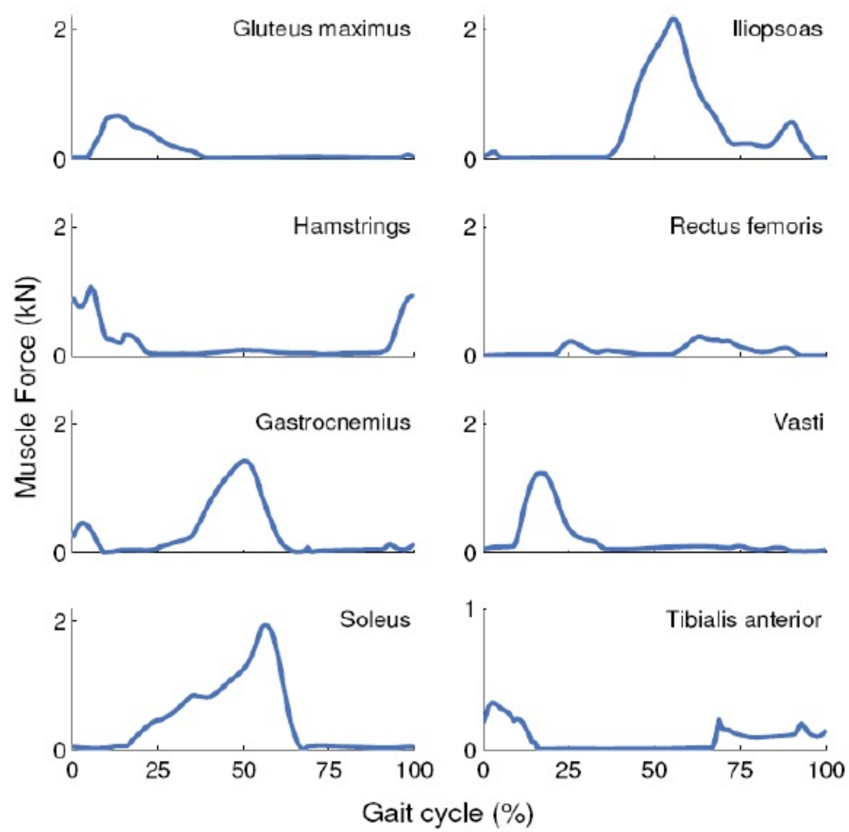




Hamner and Delp, 2013



Gluteus maximus
Iliopsoas
Hamstrings
Rectus femoris
Gastrocnemius
Vasti
Soleus
Tibialis anterior
Muscle Force (kN)
Gait cycle (%)



Gluteus maximus
Iliopsoas
Hamstrings
Rectus femoris
Gastrocnemius
Vasti
Soleus
Tibialis anterior
Muscle Force (kN)
Gait cycle (%)

Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

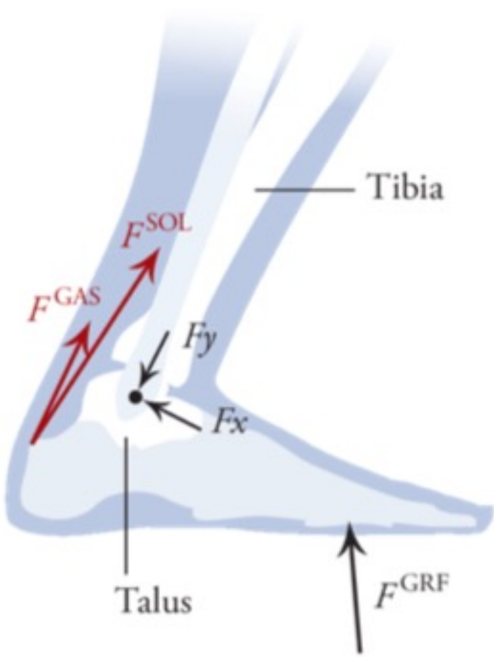
Estimating joint loads

- Can only estimate joint loads when muscle forces are known
- Distinguish:
  - “joint reaction forces” obtained via inverse dynamics
    - Would be actual joint loads only if our muscle applied pure joint torques like rotational motors
  - “bone-on-bone” forces that would be measured in vivo

  - Our muscles do not generate torques directly but generate pulling force on the skeleton. These forces produce moments as well as compressive and shearing forces in the joint. The contribution of muscle forces to joint loads is substantial.

Example

- Look at instant of peak vertical GRF during running



$$
\begin{aligned}
Fx &= F_{\mathrm{GAS},x} + F_{\mathrm{SOL},x} + F_{\mathrm{GRF},x} \\
&= 44\ \mathrm{N} + 495\ \mathrm{N} - 838\ \mathrm{N} \\
&= -299\ \mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
Fy &= F_{\mathrm{GAS},y} + F_{\mathrm{SOL},y} + F_{\mathrm{GRF},y} \\
&= 1357\ \mathrm{N} + 4088\ \mathrm{N} + 1379\ \mathrm{N} \\
&= 6824\ \mathrm{N}
\end{aligned}
$$

1. Shear force in the ankle caused by the GRF is opposed by muscle forces leading to a 50% reduction
2. Muscle forces contribute 80% of total compressive force → 10x body weight!

Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

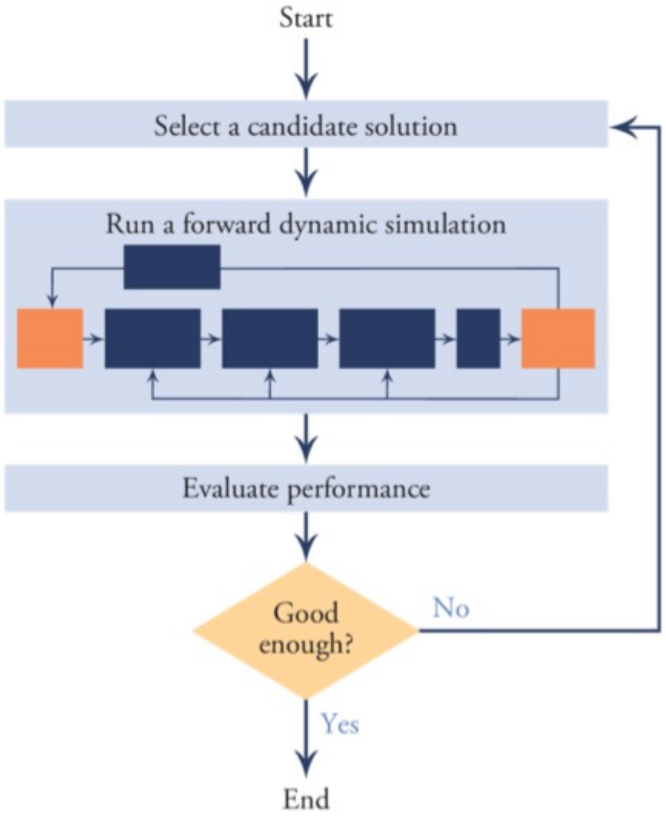
Dynamic optimization

- Limitations of static optimization
  - Objective function depends only on instantaneous quantities (but we may be anticipating and preparing for the future)
    - May be insufficient to fully understand muscle coordination during movements such as throwing, jumping, and sprinting where one needs to consider the entire movement to predict muscle activations at a given instant
  - Kinematics and net joint moments are known a priori
    - Does not allow generation of de novo movement (and corresponding muscle coordination patterns). For example, how will the body adapt to an exoskeleton or a surgery, or how do we maximize athletic performance.

- Dynamic optimization uses a model of the system dynamics to determine the muscle coordination and motion that optimize a mathematical description of a motor task.

Dynamic optimization

Dynamic optimization uses a model of the system dynamics to determine the muscle coordination and motion that optimize a mathematical description of a motor task.



Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump

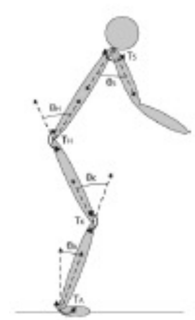
Muscle coordination during a standing long jump

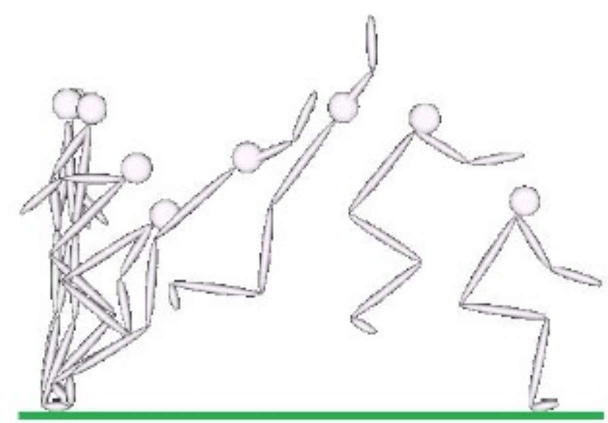
- Modeled the long jump using a planar 5-segment model.
- Physiological torque actuators at the ankle, knee and hip
- Activity of each torque actuator modeled with a piecewise linear function
- Nodes of torque function are design variables in the optimization problem
- Objective function maximizes distance while avoiding bad things

Simulation-Based Design for Wearable Robotic Systems: An Optimization Framework for Enhancing a Standing Long Jump

Carmichael F. Ong, Jennifer L. Hicks, and Scott L. Delp*

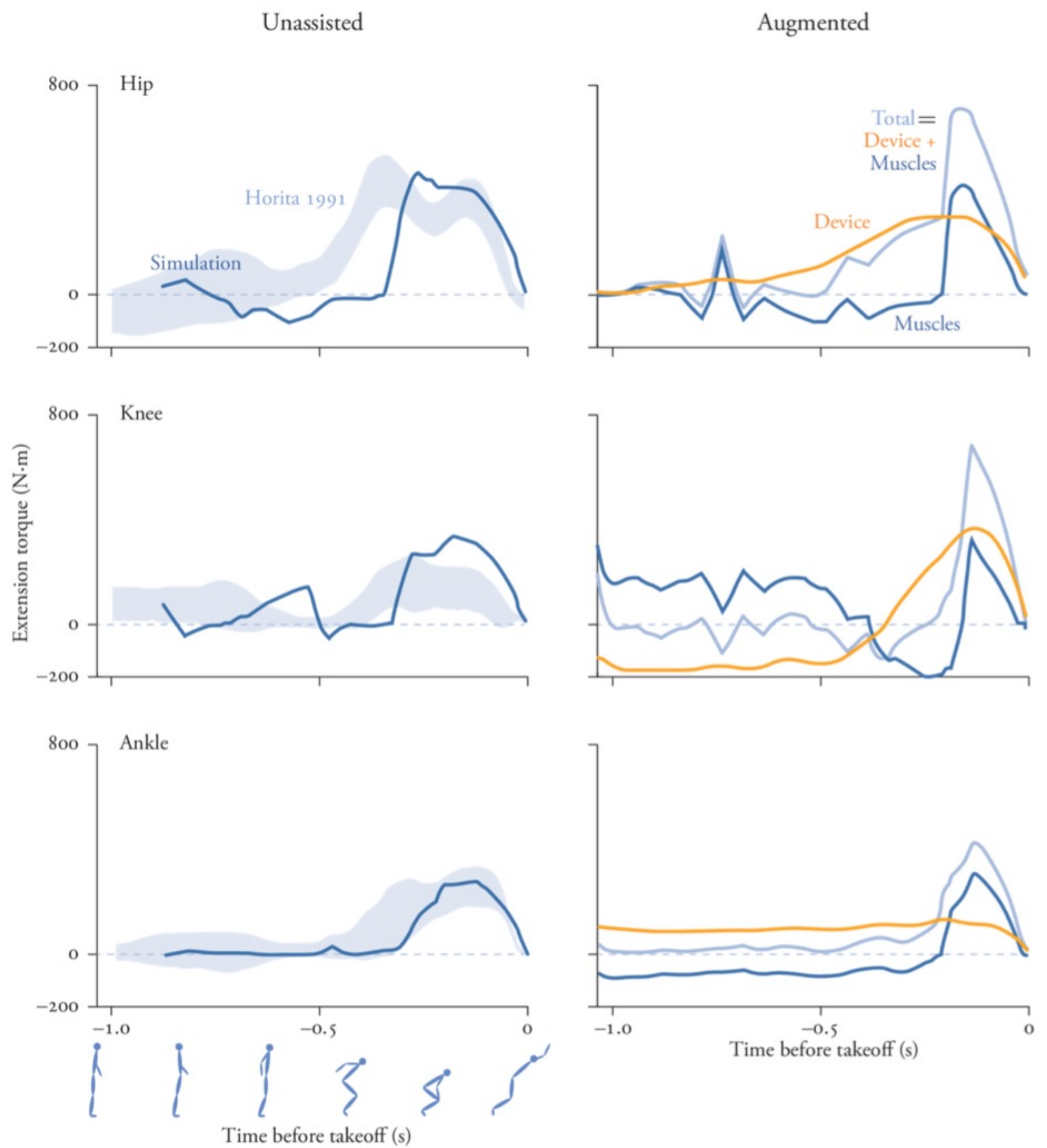
$$\begin{aligned}
\text{minimize} \quad J = &- d && \text{Reward longer distances} \\
&+ w_1\,(K_{CMx} + K_{CMy}) && \text{Penalize falls at landing} \\
&+ w_2\,(K_{injury}) && \text{Discourage use of ligaments} \\
&+ w_3\,(K_{slip}) && \text{Penalize slipping at takeoff} \\
&+ w_4\,(K_{time}) && \text{Reward counter-movements}
\end{aligned}$$





Ong et al., 2016

- Optimized unassisted model
- Optimized assisted model
    - Augmented model with massless rotational springs at the hip, knee and ankle
- Assisted model increased jump distance
    - 2.27m to 3.32m



Ong et al., 2016

Muscle force optimization

- Biological and numerical optimizers
- Static optimization
- Muscle forces during walking and running
- Estimating joint loads
- Dynamic optimization
- Muscle coordination during the long-jump