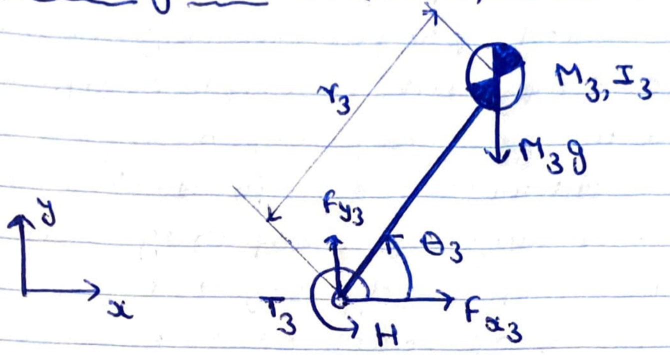
Problem 4:

HAT Segment : (Head, Arms and Torso)



Taking H as the origin,

$$x_{cm}^{(3)} = r_3 \cos\theta_3 = r_3 C\theta_3$$
$$\dot{x}_{cm}^{(3)} = -r_3 S\theta_3 \cdot \dot{\theta}_3$$
$$\ddot{x}_{cm}^{(3)} = -r_3 C\theta_3 \dot{\theta}_3^2 - r_3 S\theta_3 \ddot{\theta}_3$$

$$y_{cm}^{(3)} = r_3 S\theta_3$$
$$\dot{y}_{cm}^{(3)} = r_3 C\theta_3 \dot{\theta}_3$$
$$\ddot{y}_{cm}^{(3)} = -r_3 S\theta_3 \dot{\theta}_3^2 + r_3 C\theta_3 \ddot{\theta}_3$$

$$\Sigma f_x = M_3 \ddot{x}_{cm}^{(3)}$$
$$\Rightarrow f_{x3} = -M_3 r_3 (S\theta_3 \ddot{\theta}_3 + C\theta_3 \dot{\theta}_3^2) \rightarrow (1)$$

$$\Sigma f_y = M_3 \ddot{y}_{cm}^{(3)}$$
$$\Rightarrow f_{y3} - M_3 g = M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2)$$
$$\Rightarrow f_{y3} = M_3 g + M_3 r_3 (C\theta_3 \ddot{\theta}_3 - S\theta_3 \dot{\theta}_3^2) \rightarrow (2)$$

$$\Sigma M_{HAT,CM} = I_3 \ddot{\theta}_3$$

$$\Rightarrow T_3 + f_{x_3} r_3 s\theta_3 - f_{y_3} r_3 c\theta_3 = I_3 \ddot{\theta}_3$$

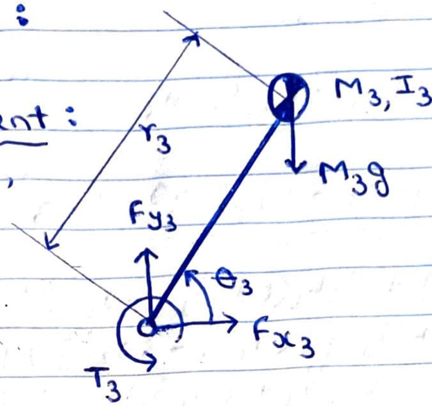
$$\Rightarrow T_3 = f_{y_3} r_3 c\theta_3 - f_{x_3} r_3 s\theta_3 + I_3 \ddot{\theta}_3$$

$$\Rightarrow \boxed{\begin{aligned} T_3 = {} & \left[M_3 g + M_3 r_3 \left(c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2\right)\right] r_3 c\theta_3 \;- \\ & \left[-M_3 r_3 \left(s\theta_3 \ddot{\theta}_3 + c\theta_3 \dot{\theta}_3^2\right)\right] r_3 s\theta_3 + I_3 \ddot{\theta}_3 \end{aligned}}$$

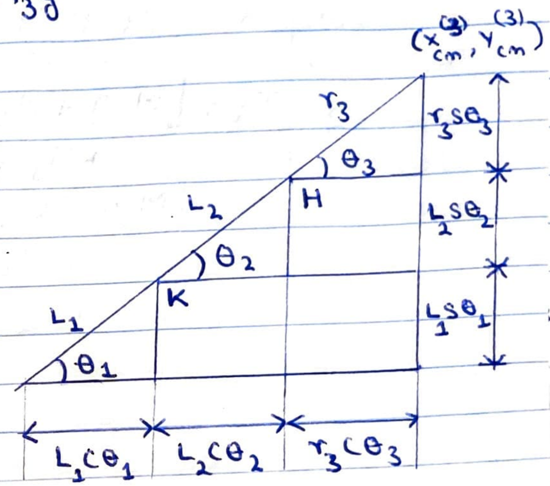
(from 1 and 2)

Problem 4 :

HAT Segment :
(Head, arms, torso)



Taking ankle as the origin,



$$x_{CM}^{(3)} = L_1 c\theta_1 + L_2 c\theta_2 + r_3 c\theta_3$$

$$\dot{x}_{CM}^{(3)} = -L_1 s\theta_1 \dot{\theta}_1 - L_2 s\theta_2 \dot{\theta}_2 - r_3 s\theta_3 \dot{\theta}_3$$

$$\ddot{x}_{CM}^{(3)} = -L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - L_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2) - r_3 (s\theta_3 \ddot{\theta}_3 + c\theta_3 \dot{\theta}_3^2)$$

$$y_{CM}^{(3)} = L_1 s\theta_1 + L_2 s\theta_2 + r_3 s\theta_3$$

$$\dot{y}_{CM}^{(3)} = L_1 c\theta_1 \dot{\theta}_1 + L_2 c\theta_2 \dot{\theta}_2 + r_3 c\theta_3 \dot{\theta}_3$$

$$\ddot{y}_{CM}^{(3)} = L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + L_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) + r_3 (c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2)$$

$$\Sigma F_x = M_3 \ddot{x}_{CM}^{(3)}$$

$$\Rightarrow F_{x3} = -M_3 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - M_3 L_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2) - M_3 r_3 (s\theta_3 \ddot{\theta}_3 + c\theta_3 \dot{\theta}_3^2) \rightarrow ①$$

$$\Sigma F_y = M_3 \ddot{y}_{CM}^{(3)}$$

$$\Rightarrow F_{y3} - M_3 g = M_3 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + M_3 L_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) + M_3 r_3 (c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2)$$

$$\Rightarrow F_{y3} = M_3 g + M_3 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + M_3 L_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) + M_3 r_3 (c\theta_3 \ddot{\theta}_3 - s\theta_3 \dot{\theta}_3^2) \rightarrow ②$$

$$\Sigma M_{HAT,CM} = I_3\ddot{\theta}_3$$

$$\Rightarrow T_3 + f_{x_3} r_3 s\theta_3 - f_{y_3} r_3 c\theta_3 = I_3\ddot{\theta}_3$$

$$\Rightarrow \boxed{T_3 = f_{y_3} r_3 c\theta_3 - f_{x_3} r_3 s\theta_3 + I_3\ddot{\theta}_3}$$

$$\Rightarrow \boxed{\begin{aligned} T_3 = \Big[&M_3 g + M_3 L_1 (c\theta_1\ddot{\theta}_1 - s\theta_1\dot{\theta}_1^2) + M_3 L_2 (c\theta_2\ddot{\theta}_2 - s\theta_2\dot{\theta}_2^2) \\ &+ M_3 r_3 (c\theta_3\ddot{\theta}_3 - s\theta_3\dot{\theta}_3^2)\Big](r_3 c\theta_3) + I_3\ddot{\theta}_3 + \\ \Big[&M_3 L_1 (s\theta_1\ddot{\theta}_1 + c\theta_1\dot{\theta}_1^2) + M_3 L_2 (s\theta_2\ddot{\theta}_2 + c\theta_2\dot{\theta}_2^2) \\ &+ M_3 r_3 (s\theta_3\ddot{\theta}_3 + c\theta_3\dot{\theta}_3^2)\Big](r_3 s\theta_3) \end{aligned}}$$