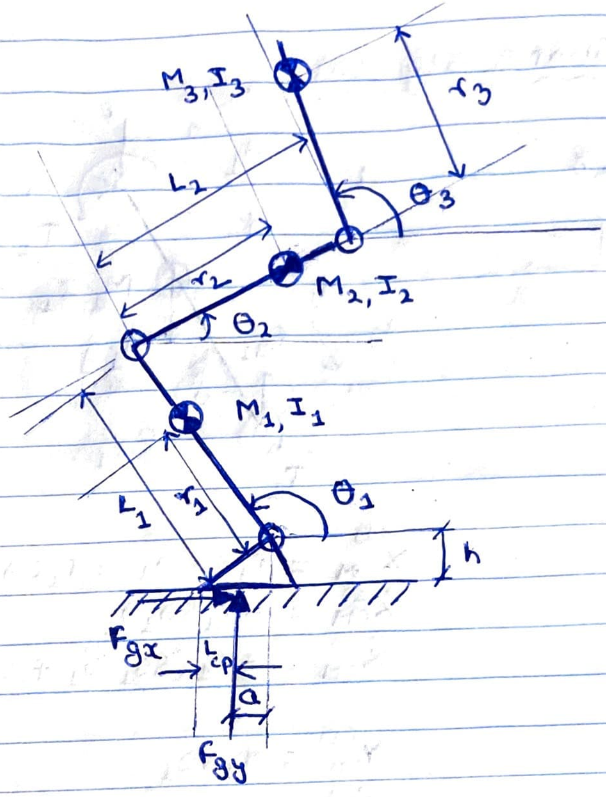
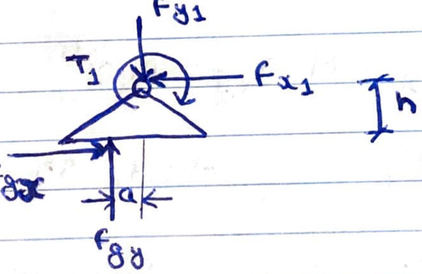
Problem 1 :



Foot segment :

↻ ⇒ ⊖ve

↺ ⇒ ⊕ve



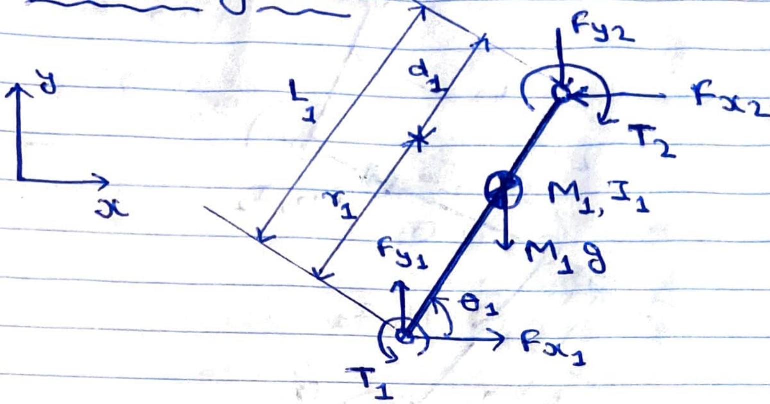
A → Ankle

$$\Sigma F_x = M_0 \ddot{x}_0 \Rightarrow F_{gx} - F_{x_1} = 0 \Rightarrow F_{x_1} = F_{gx} \rightarrow (1)$$

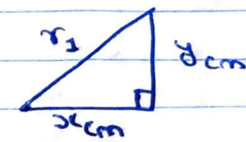
$$\Sigma F_y = M_0 \ddot{y}_0 \Rightarrow F_{gy} - F_{y1} = 0 \Rightarrow F_{y_1} = F_{gy} \rightarrow (2)$$

$$\Sigma M_A = 0 \Rightarrow F_{gx} \cancel{a}^{h} - F_{gy} a - T_1 = 0 \Rightarrow \boxed{T_1 = F_{gx} h - F_{gy} a} \rightarrow (3)$$

Shank segment :



$d_1 = L_1 - r_1$



$$x_{CM}^{(1)} = r_1 \cos\theta_1 = r_1 c\theta_1$$
$$\dot{x}_{CM}^{(1)} = -r_1 s\theta_1 \dot{\theta}_1$$
$$\ddot{x}_{CM}^{(1)} = -r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$y_{CM}^{(1)} = r_1 s\theta_1$$
$$\dot{y}_{CM}^{(1)} = r_1 c\theta_1 \dot{\theta}_1$$
$$\ddot{y}_{CM}^{(1)} = r_1 (c\theta_1 \cdot \ddot{\theta}_1 - s\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Sigma F_x = M_1 a_{x_1} = M_1 \ddot{x}_{CM}^{(1)}$$

$$\Rightarrow \quad F_{x_1} - F_{x_2} = -M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow \quad F_{x_2} = F_{x_1} + M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2)$$

$$\Rightarrow \quad F_{x_2} = F_{gx} + M_1 r_1 (s\theta_1 \cdot \ddot{\theta}_1 + c\theta_1 \cdot \dot{\theta}_1^2) \quad \rightarrow 4$$

$$(\because \; F_{x_1} = F_{gx})$$

$$\Sigma F_y = M_1 a_{y_1} = M_1 \ddot{Y}_{CM}^{(1)}$$

$$\Rightarrow F_{y_1} - F_{y_2} - M_1 g = M_1 r_1 (c\theta_1 . \ddot{\theta}_1 - s\theta_1 . \dot{\theta}_1^2)$$

$$\Rightarrow F_{y_2} = F_{y_1} - M_1 g - M_1 r_1 (c\theta_1 . \ddot{\theta}_1 - s\theta_1 . \dot{\theta}_1^2)$$

$$\Rightarrow F_{y_2} = F_{gy} - M_1 g - M_1 r_1 (c\theta_1 . \ddot{\theta}_1 - s\theta_1 . \dot{\theta}_1^2) \rightarrow 5$$

$$(\because F_{y_1} = F_{gy})$$

S → Shank

$$\Sigma M_{S,CM} = I_1 \ddot{\theta}_1$$

$$\Rightarrow T_1 - T_2 + F_{x_1} r_1 s\theta_1 - F_{y_1} r_1 c\theta_1 + F_{x_2} d_1 s\theta_1 - F_{y_2} d_1 c\theta_1 = I_1 \ddot{\theta}_1$$

$$\Rightarrow \boxed{T_2 = T_1 + F_{x_1} r_1 s\theta_1 - F_{y_1} r_1 c\theta_1 + F_{x_2} d_1 s\theta_1 - F_{y_2} d_1 c\theta_1 - I_1 \ddot{\theta}_1}$$

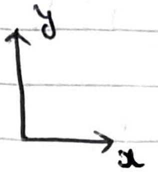
$$\Rightarrow \boxed{\begin{aligned} T_2 = {} & (F_{gx} h - F_{gy} a) + F_{gx} r_1 s\theta_1 - F_{gy} r_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 . \ddot{\theta}_1 + c\theta_1 . \dot{\theta}_1^2)] d_1 s\theta_1 - I_1 \ddot{\theta}_1 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 . \ddot{\theta}_1 - s\theta_1 . \dot{\theta}_1^2)] d_1 c\theta_1 \rightarrow 6 \end{aligned}}$$

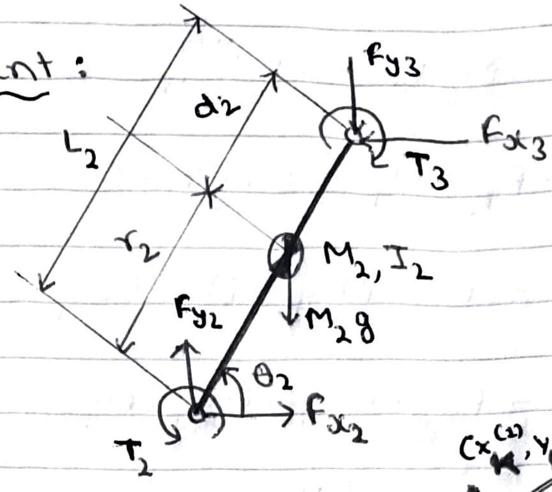
from 1, 2, 3, 4 and 5 eq 7

This is the expression for T_2 in terms of θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, F_H, F_V, along with the masses, inertial properties, and dimensions of the limbs.

→

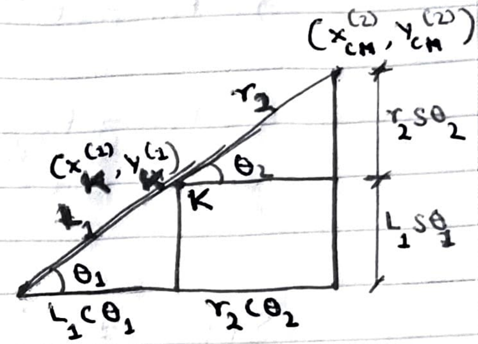
Thigh segment :





$d_2 = L_2 - r_2$

K → Knee



$$x_{CM}^{(2)} = L_1 c\theta_1 + r_2 c\theta_2$$
$$\dot{x}_{CM}^{(2)} = -L_1 s\theta_1 \dot{\theta}_1 - r_2 s\theta_2 \dot{\theta}_2$$
$$\ddot{x}_{CM}^{(2)} = -L_1(s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - r_2(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$y_{CM}^{(2)} = L_1 s\theta_1 + r_2 s\theta_2$$
$$\dot{y}_{CM}^{(2)} = L_1 c\theta_1 \dot{\theta}_1 + r_2 c\theta_2 \dot{\theta}_2$$
$$\ddot{y}_{CM}^{(2)} = L_1(c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + r_2(c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)$$

$$\Sigma F_x = M_2 a_{x_2} = M_2 \ddot{x}_{CM}^{(2)}$$

$$\Rightarrow F_{x_2} - F_{x_3} = -M_2 L_1(s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) - M_2 r_2(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{x_3} = F_{x_2} + M_2 L_1(s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 r_2(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{x_3} = F_{gx} + M_1 r_1 (s\theta_1 . \ddot{\theta}_1 + c\theta_1 . \dot{\theta}_1^2) +$$
$$M_2 L_1(s\theta_1 . \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 r_2(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2) \rightarrow (7)$$

(From eqn (4))

$$\Sigma F_y = M_2 a_{y_2} = M_2 \ddot{Y}_{CM}^{(2)}$$

$$\Rightarrow F_{y_2} - F_{y_3} - M_2 g = M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) + M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)$$

$$\Rightarrow F_{y_3} = F_{y_2} - M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) - M_2 g$$

$$\begin{aligned}\Rightarrow F_{y_3} = {} & F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 g - \\ & M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2) \quad \rightarrow 8\end{aligned}$$

(from eq^n 5)

$T \rightarrow$ Thigh

$$\Sigma M_{T,CM} = I_2 \ddot{\theta}_2$$

$$\Rightarrow T_2 - T_3 + F_{x_2} r_2 s\theta_2 - F_{y_2} r_2 c\theta_2 + F_{x_3} d_2 s\theta_2 - F_{y_3} d_2 c\theta_2 = I_2 \ddot{\theta}_2$$

$$\Rightarrow \boxed{T_3 = T_2 + F_{x_2} r_2 s\theta_2 - F_{y_2} r_2 c\theta_2 + F_{x_3} d_2 s\theta_2 - F_{y_3} d_2 c\theta_2 - I_2 \ddot{\theta}_2}$$

$$\Rightarrow \boxed{\begin{aligned} T_3 = {} & (F_{gx} h - F_{gy} a) + F_{gx} r_1 s\theta_1 - F_{gy} r_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2)] d_1 s\theta_1 - I_1 \ddot{\theta}_1 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2)] d_1 c\theta_1 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2)] r_2 s\theta_2 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2)] r_2 c\theta_2 - I_2 \ddot{\theta}_2 - \\ & [F_{gy} - M_1 g - M_1 r_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) - M_2 L_1 (c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2) \\ & \quad - M_2 g - M_2 r_2 (c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2)] d_2 c\theta_2 + \\ & [F_{gx} + M_1 r_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) + M_2 L_1 (s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2) \\ & \quad + M_2 r_2 (s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2)] d_2 s\theta_2 \quad \rightarrow 9 \end{aligned}}$$

Combining similar terms, we get,

$$\boxed{\begin{aligned}
T_3 = {} & F_{gx}\left(h + r_1 s\theta_1 + d_1 s\theta_1 + r_2 s\theta_2 + d_2 s\theta_2\right) - \\
& F_{gy}\left(a + r_1 c\theta_1 + d_1 c\theta_1 + r_2 c\theta_2 + d_2 c\theta_2\right) + \\
& \left[\left(s\theta_1 \ddot{\theta}_1 + c\theta_1 \dot{\theta}_1^2\right)\left(M_1 r_1 d_1 s\theta_1 + M_1 r_1 r_2 s\theta_2 + M_1 r_1 d_2 s\theta_2 \right.\right. \\
& \left.\left. + M_2 L_1 d_2 s\theta_2\right)\right] + \\
& \left[\left(c\theta_1 \ddot{\theta}_1 - s\theta_1 \dot{\theta}_1^2\right)\left(M_1 r_1 d_1 c\theta_1 + M_1 r_1 r_2 c\theta_2 + M_1 r_1 d_2 c\theta_2 \right.\right. \\
& \left.\left. + M_2 L_1 d_2 c\theta_2\right)\right] - \\
& \left(I_1 \ddot{\theta}_1 + I_2 \ddot{\theta}_2\right) + M_1 g\left(d_1 c\theta_1 + r c\theta_2 + d_2 c\theta_2\right) + \\
& M_2 g\left(d_2 c\theta_2\right) + \left[\left(s\theta_2 \ddot{\theta}_2 + c\theta_2 \dot{\theta}_2^2\right)\left(M_2 r_2 d_2 s\theta_2\right)\right] \\
& + \left[\left(c\theta_2 \ddot{\theta}_2 - s\theta_2 \dot{\theta}_2^2\right)\left(M_2 r_2 d_2 c\theta_2\right)\right] \quad \rightarrow (9)
\end{aligned}}$$