# Quantifying Movement

MCEN 4/5228

Modeling of Human Movement

Fall 2021

# Inverse problems

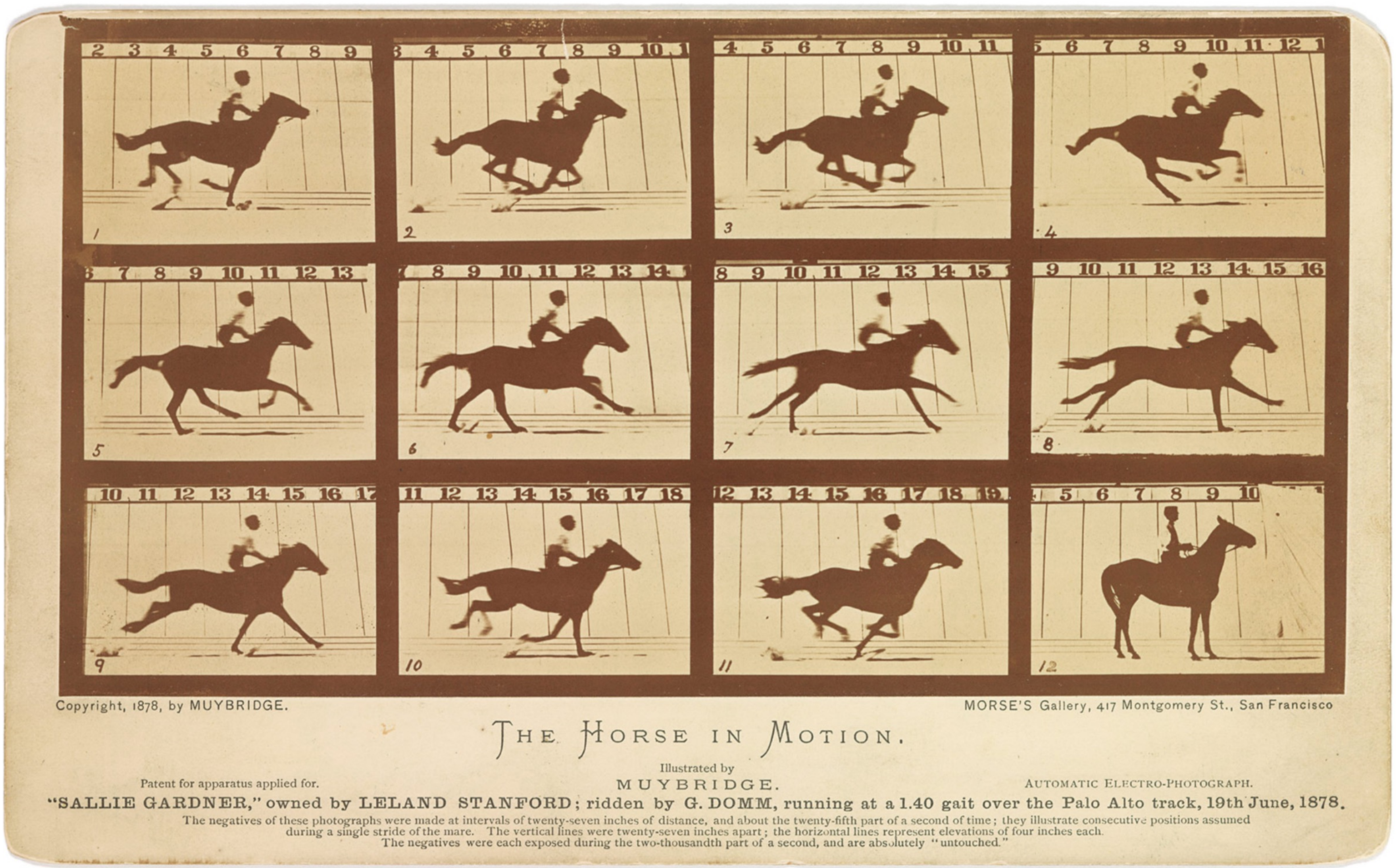
- Involve phenomena that cannot be observed directly, and must use indirect measurements to analyze
- Use observations to understand underlying phenomena responsible
- Most problems in Biomechanics are inverse problems
  - Examples?
- Forward problems: induce a response deliberately and study observed effects

# Quantifying movement

- Measurement techniques
- Unconstrained inverse kinematics
- Constrained inverse kinematics

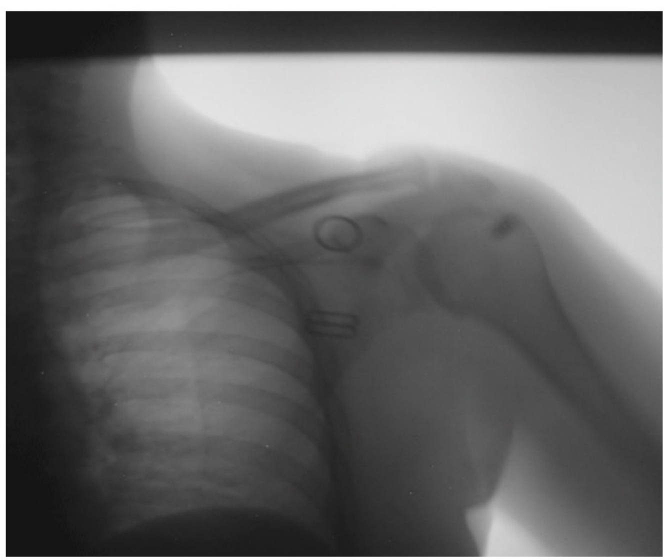
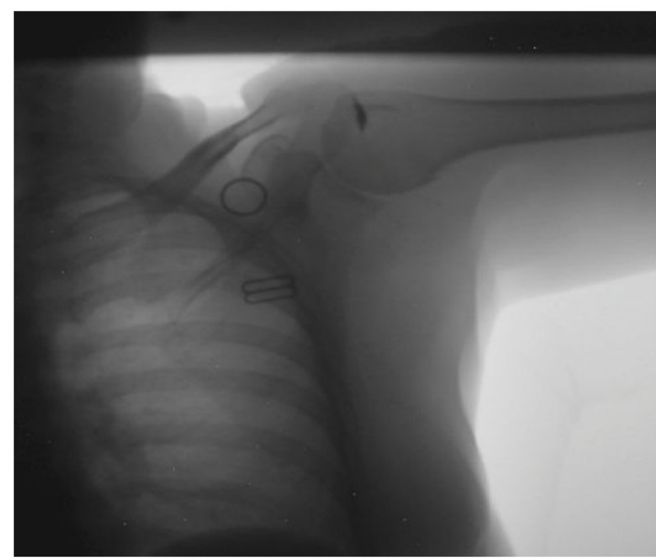
# Measurement techniques

- ”The Horse in Motion”



# Video analysis

- Remains key component of biomechanical analyses
- Other techniques being developed:
  - Cineflouroscopy: x-ray images taken over time

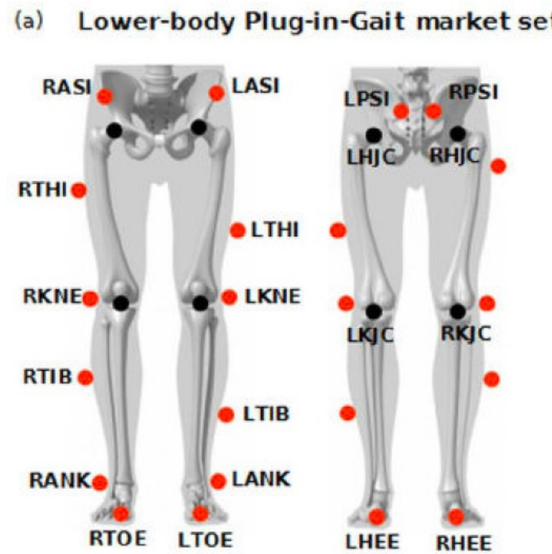



# Video analysis

- Remains key component of biomechanical analyses
- Other techniques being developed:
  - Cineflouroscopy: x-ray images taken over time
  - Implanting bone-anchored pins
  - IMUs

# Motion capture “mocap”

- Current standard
- Use video cameras to track small spherical markers affixed to subject’s skin
- Use inverse kinematics to calculate underlying skeletal joint angles


(a) Lower-body Plug-in-Gait market set
RASI
LASI
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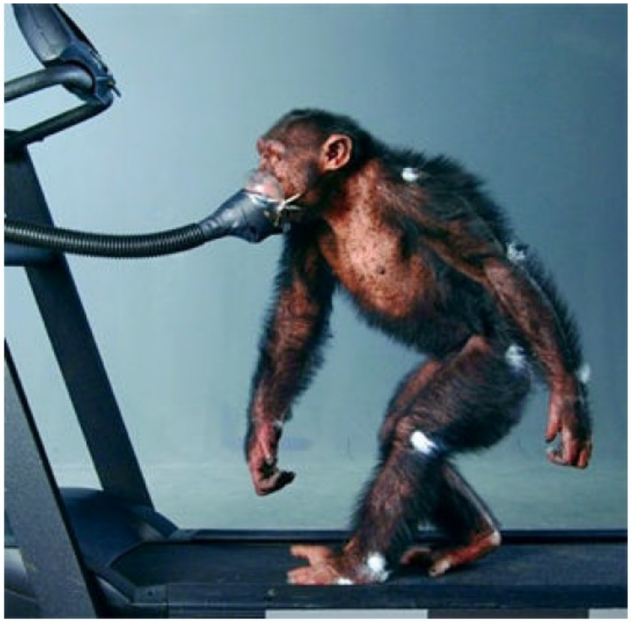
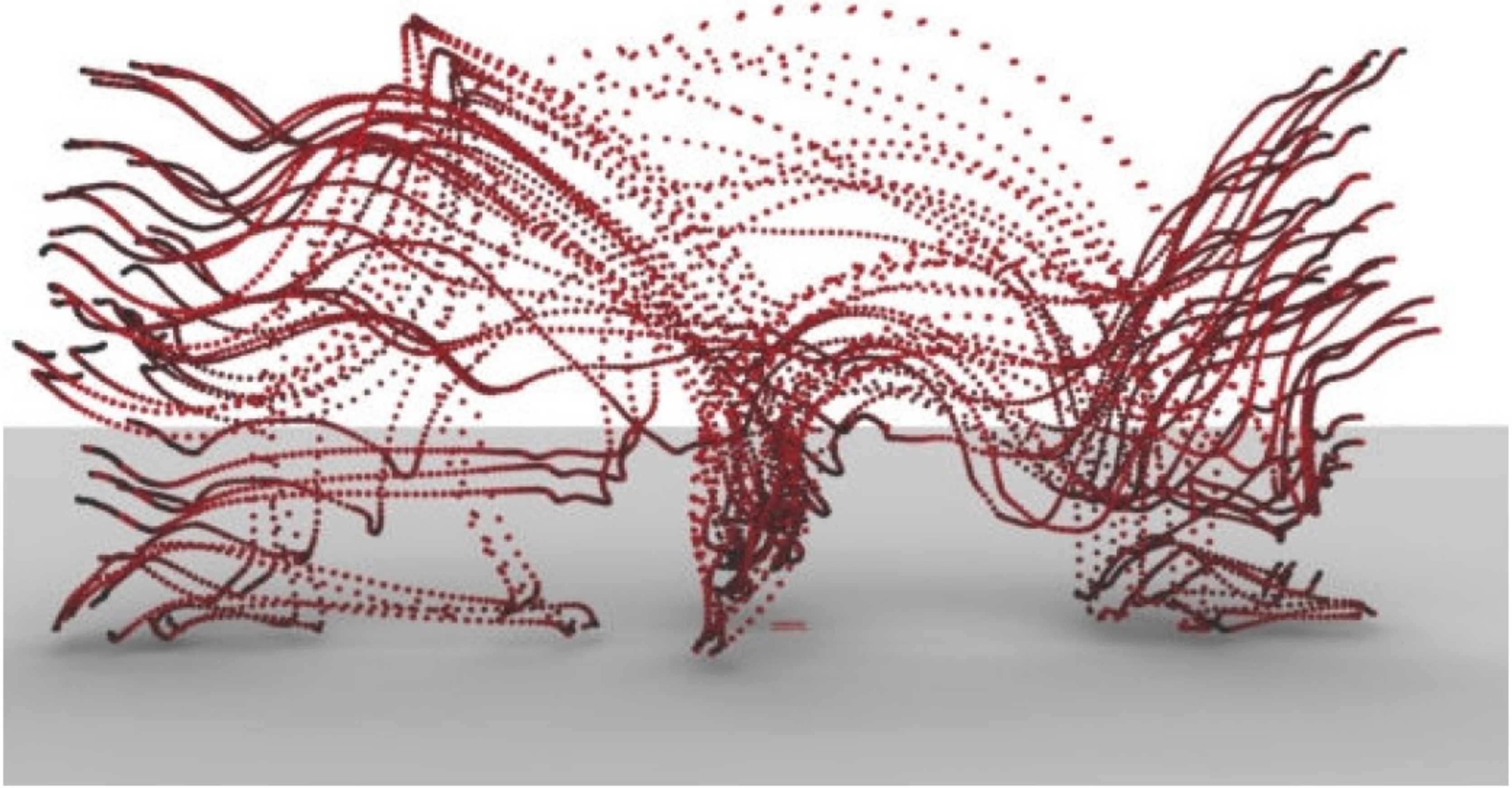
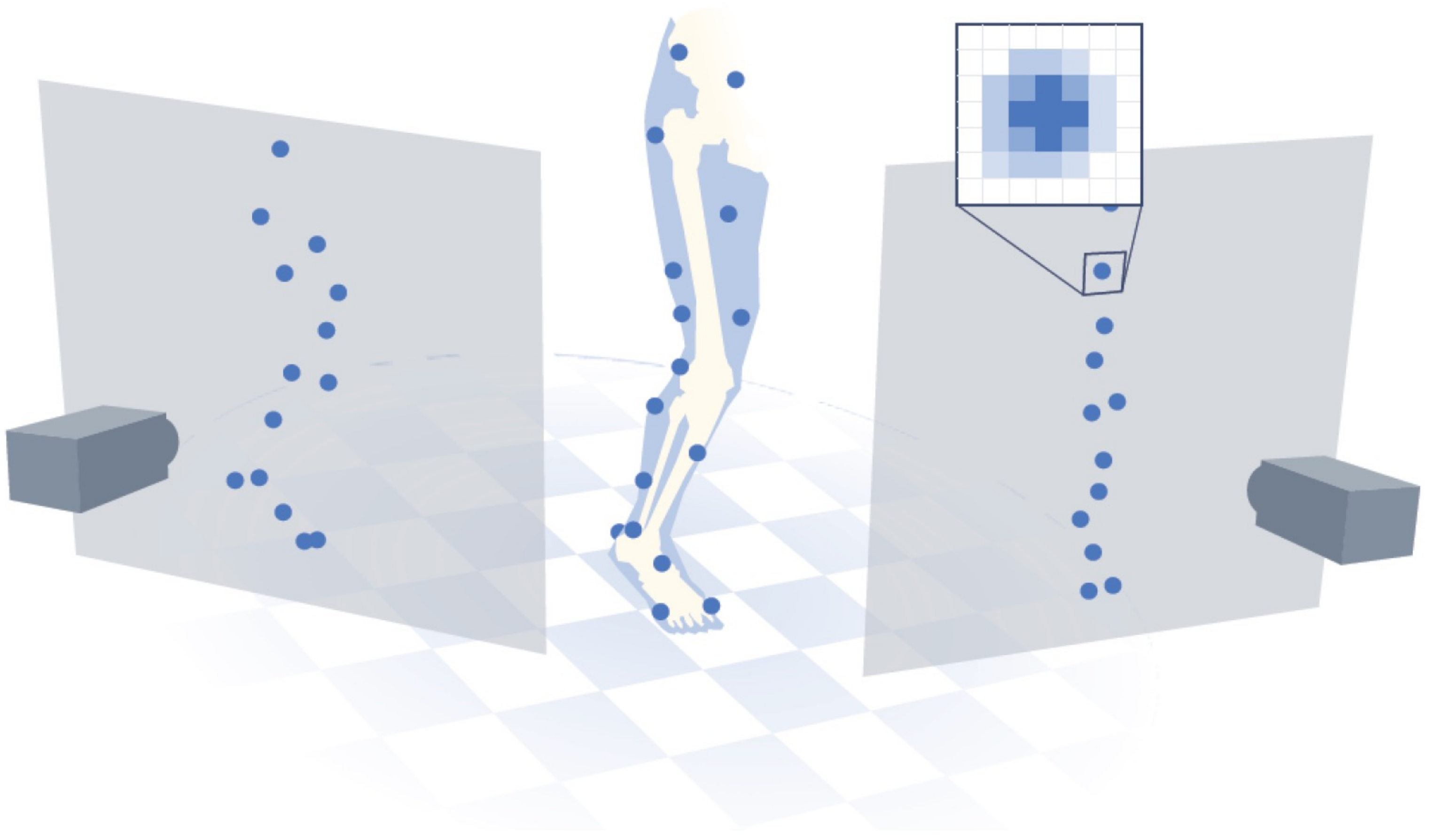




# Mocap challenges

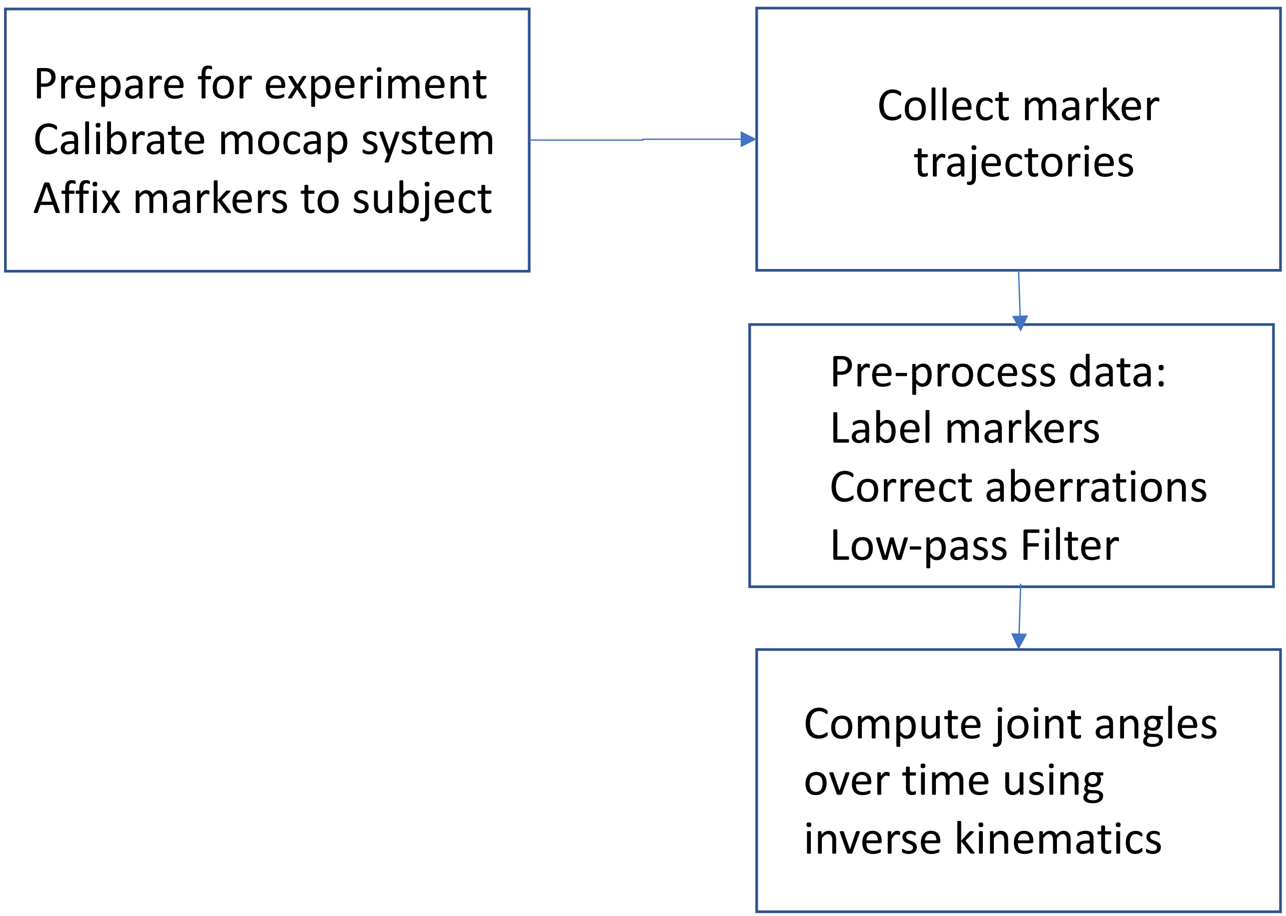


# Mocap Challenges

- Marker occlusion
- Motion artifact
- Marker labelling errors

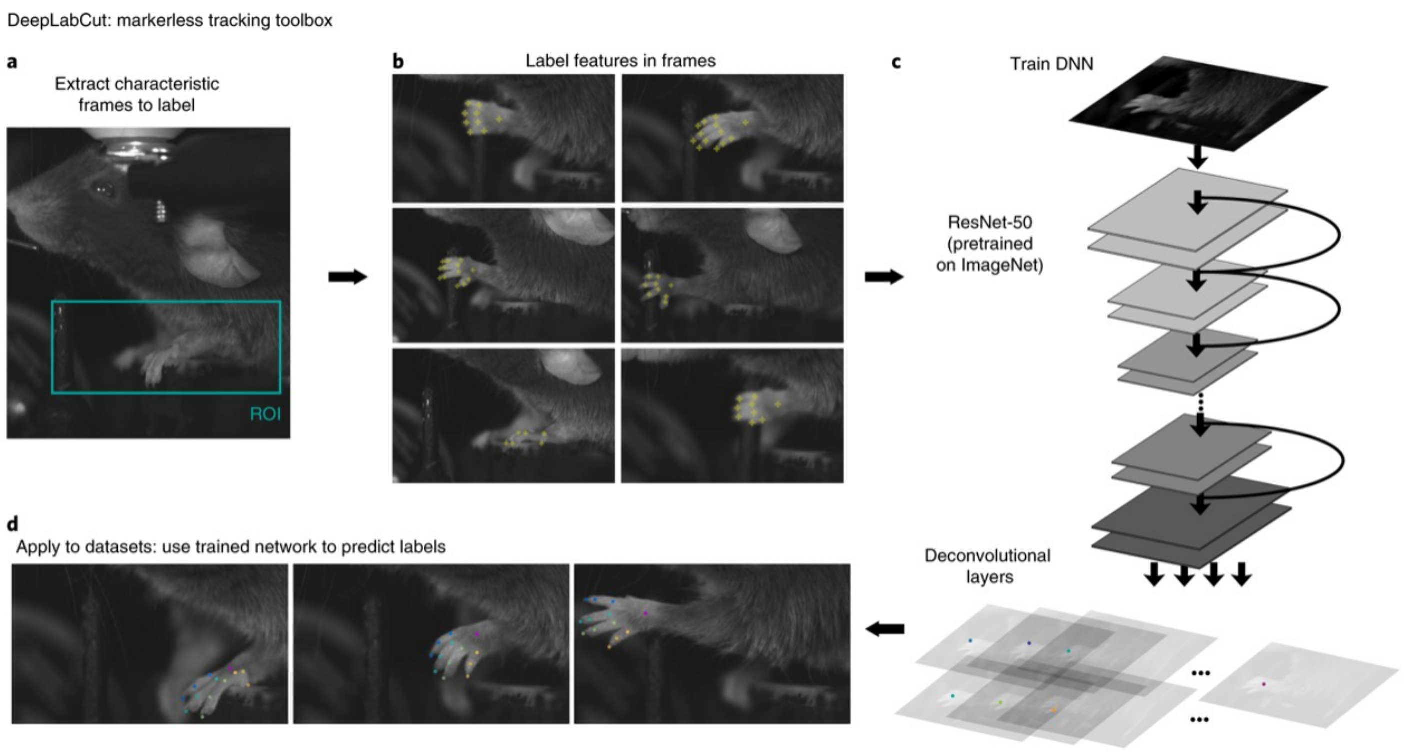
- Solutions:
  - Virtual markers
  - Filtering
  - Interpolation

# Mocap → Joint angles



# The future

- Markerless motion capture
  - DeepLabCut
    - http://www.mousemotorlab.org/deeplabcut



# The future

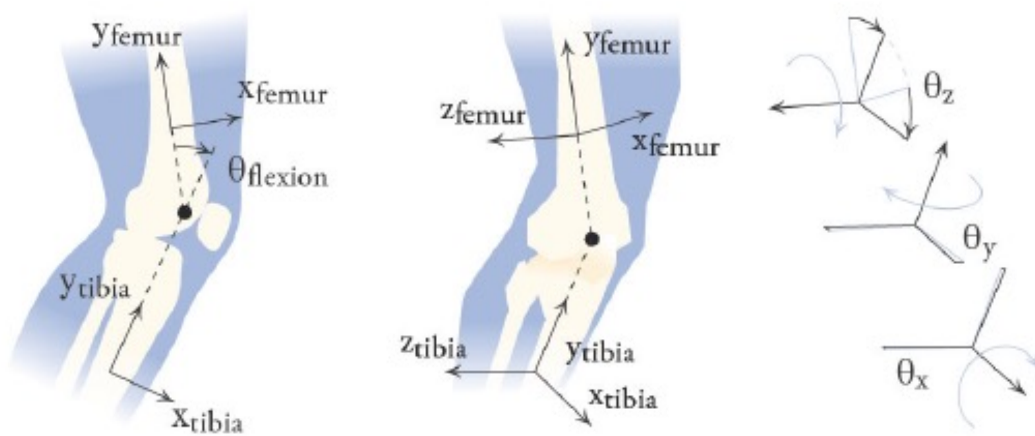
- Markerless motion capture
- IMUs
    - Integrated into clothing
    - Smart phones

# Quantifying movement

- Measurement techniques
- Unconstrained inverse kinematics
- Constrained inverse kinematics

# Unconstrained inverse kinematics

- Goal is to estimate joint angles from marker trajectories
- Must establish a reference frame based on positions of markers on segments
- Compute joint angle over time by comparing orientations of reference frames fixed to adjacent segments



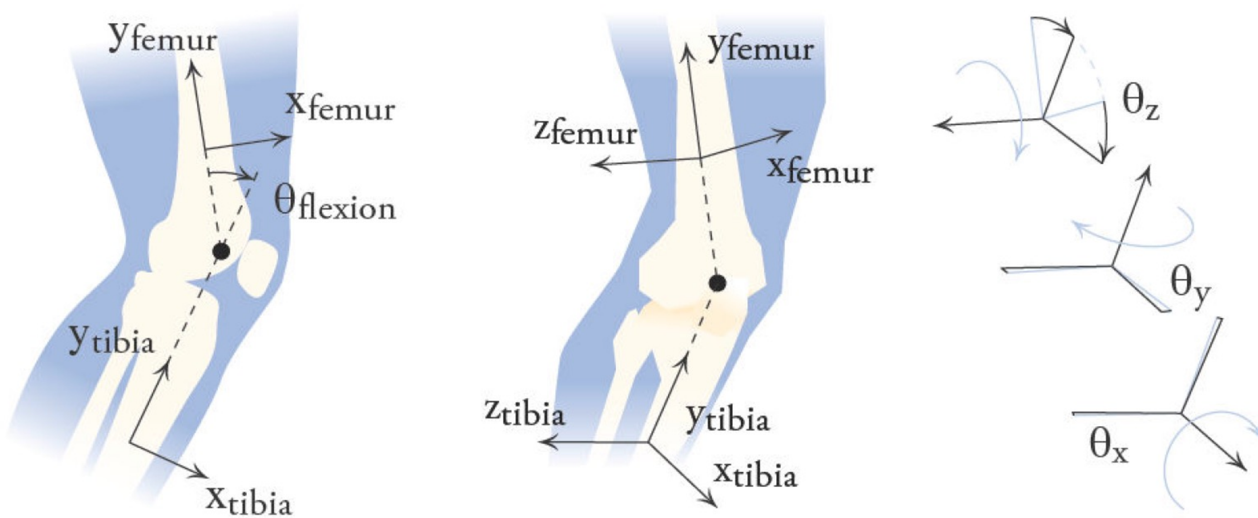
# Unconstrained inverse kinematics

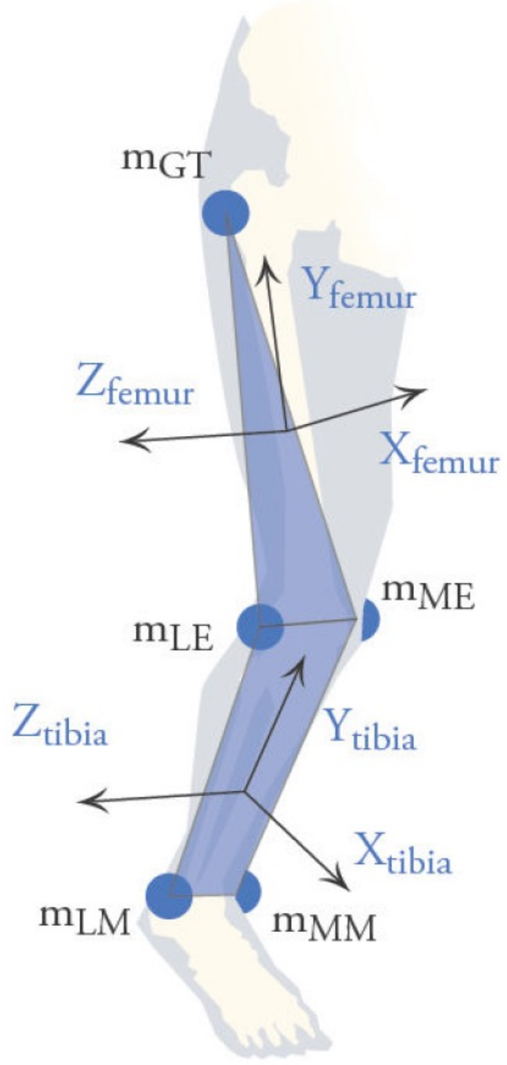
- Goal is to estimate joint angles from marker trajectories
- Must establish a reference frame based on positions of markers on segments
- Compute joint angle over time by comparing orientations of reference frames fixed to adjacent segments
- “unconstrained” because no limits imposed on limb lengths or underlying skeletal model
- What about joint angular velocities and accelerations?

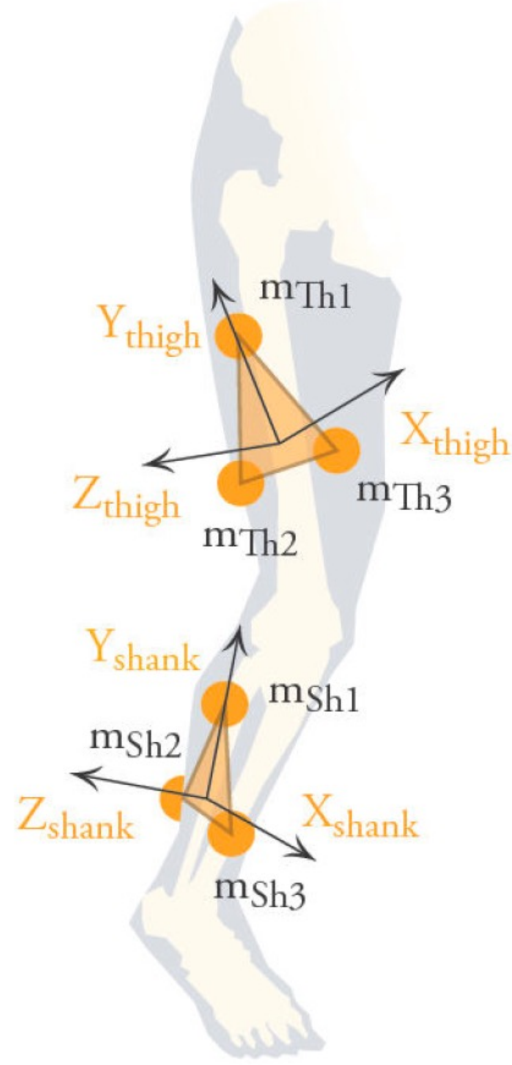
# Joint angle analysis

# Challenges…

- Ideally track reference frames on femur and tibia
- BUT… markers do not track anatomical segments
- Must transform transform marker motion to anatomical segment motion



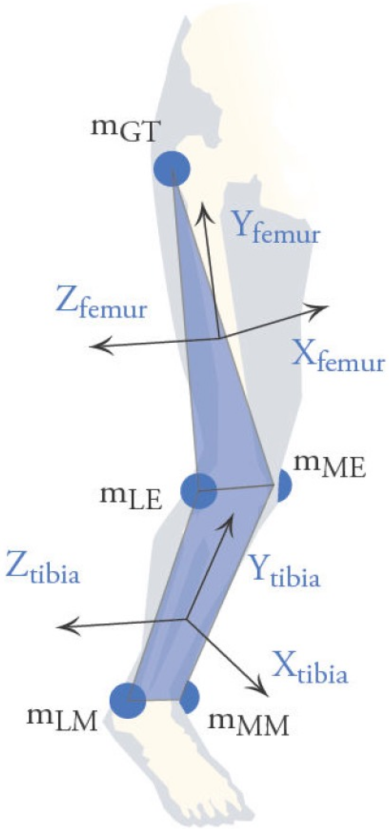



Anatomical Reference Frame

Tracking (marker) Reference Frame

# Anatomical reference frame




Represent underlying skeletal structure

Defined by placing maker on anatomical landmarks (left)

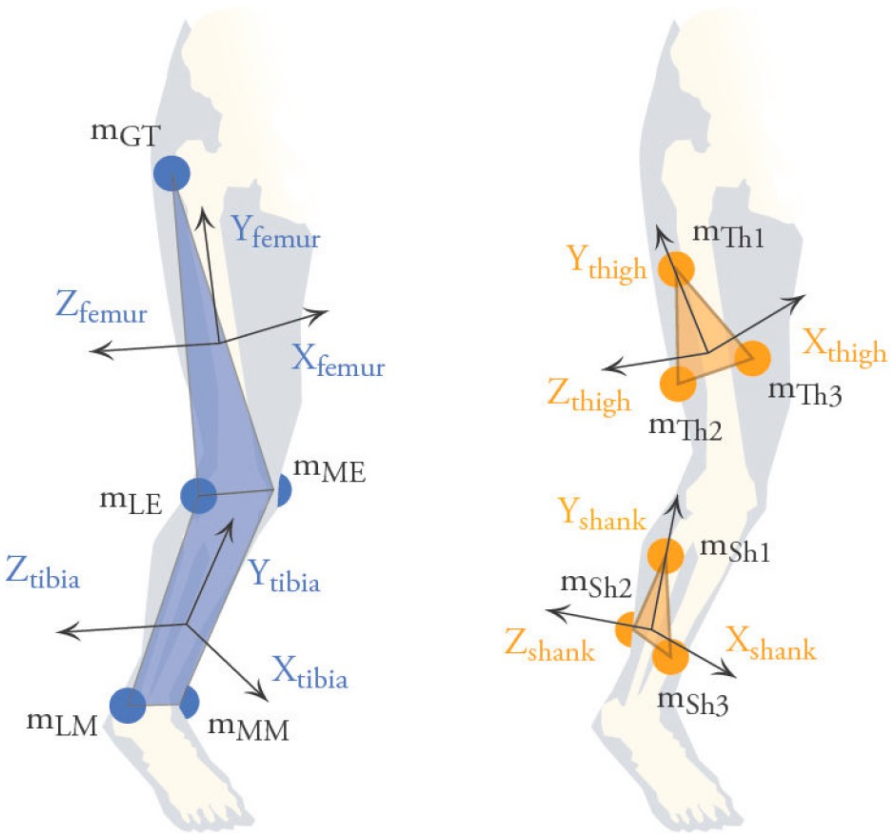
Example (Femur):

- The origin is midway between the greater trochanter marker (mGT) and the knee joint center (midpoint between femoral epicondyle markers, mLE and mME)
- $Z_{femur}$ is parallel to the knee joint axis, which is defined as the vector from the medial to lateral femoral epicondyle markers, normalized to unit length
- $X_{femur}$ is the cross product of $Z_{femur}$ and a vector from the mGT to one of the femoral epicondyle markers, normalized to unit length.
- $Y_{femur}$ is the cross product of $Z_{femur}$ and $X_{femur}$ which completes the right-handed reference frame.

Knee flexion angle: orientation of the tibia reference frame relative to the femur reference frame in the sagittal plane

Anatomical Reference Frame

# Tracking reference frame




Tracking Reference Frame

- Also fixed to body segments
- But may not be aligned with anatomically relevant axes
- Are defined by at least three makers on each segment
- Placed in locations that remain visible with small amounts of soft tissue motion
- Allow more convenient placement of markers

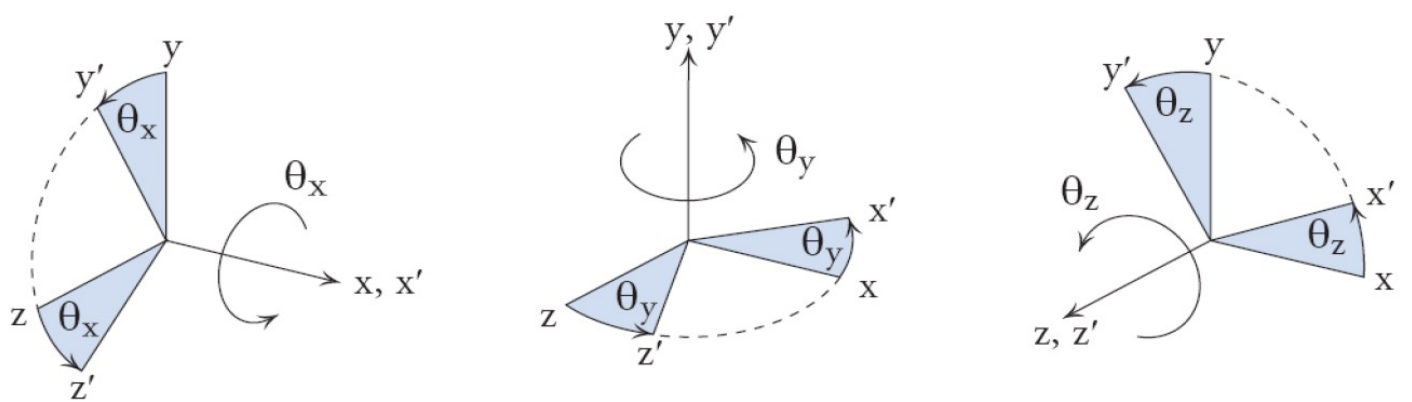
# Transformation matrices

$$R^x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$R^y = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}$$

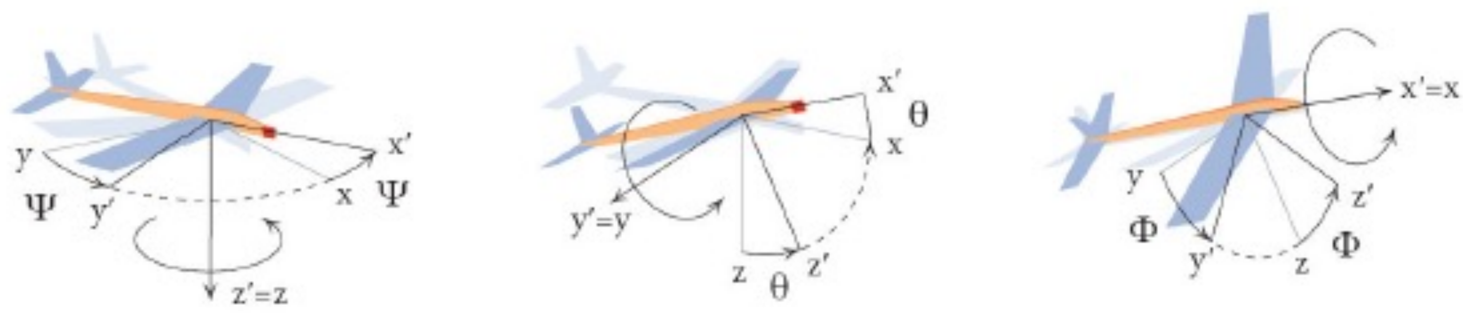
$$R^z = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Use transformation matrices to describe anatomical joint angles from mocap data.
- Any spatial rotation can be expressed as a sequence of elementary rotations.



# Euler angles

- Three angles are required to describe the orientation of one reference frame in space to another
- When these three parameters are the angles of three elementary rotations, they are called Euler angles
- A common sequence of rotations (z-y-x):




Yaw
Rotate Ψ about z-axis

Pitch
Rotate θ about y-axis

Roll
Rotate Φ about x-axis

# Euler angles

- Rotation matrix that related orientations of the original and final frames is:

$$R = R^z\Big|_{\theta_z=\psi} R^y\Big|_{\theta_y=\theta} R^x\Big|_{\theta_x=\phi}$$

$$= \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) \\ \cos(\theta)\sin(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{r}_{xx} & \mathrm{r}_{xy} & \mathrm{r}_{xz} \\ \mathrm{r}_{yx} & \mathrm{r}_{yy} & \mathrm{r}_{yz} \\ \mathrm{r}_{zx} & \mathrm{r}_{zy} & \mathrm{r}_{zz} \end{bmatrix}$$

$$R = R^z\Big|_{\theta_z=\psi} \; R^y\Big|_{\theta_y=\theta} \; R^x\Big|_{\theta_x=\phi}$$

$$= \begin{bmatrix} \cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) \\ \cos(\theta)\sin(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{r}_{xx} & \mathrm{r}_{xy} & \mathrm{r}_{xz} \\ \mathrm{r}_{yx} & \mathrm{r}_{yy} & \mathrm{r}_{yz} \\ \mathrm{r}_{zx} & \mathrm{r}_{zy} & \mathrm{r}_{zz} \end{bmatrix}$$

- Can calculate the z-y-x Euler angles from an arbitrary rotation matrix by equating last two equations.

$$\phi = \operatorname{atan2}(r_{32}, r_{33})$$

$$\theta = \operatorname{atan2}\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$$

$$\psi = \operatorname{atan2}(r_{21}, r_{11})$$

# Transformation matrices

- If reference frames A and B are fixed to adjacent body segments
  - An arbitrary point expressed in frame B ($[p]_B$) can be expressed in frame A ($[p]_A$):

$$[p]_A = {}^{A}R_{B}\,[p]_B$$

  - Where the columns of ${}^{A}R_{B}$ are the coordinates of unit vectors pointed along frame B's xyz axes when expressed in frame A.

# When the origins don't coincide

- Relationship between frames can be described by a 4x4 transformation matrix that captures their relative position and orientation:

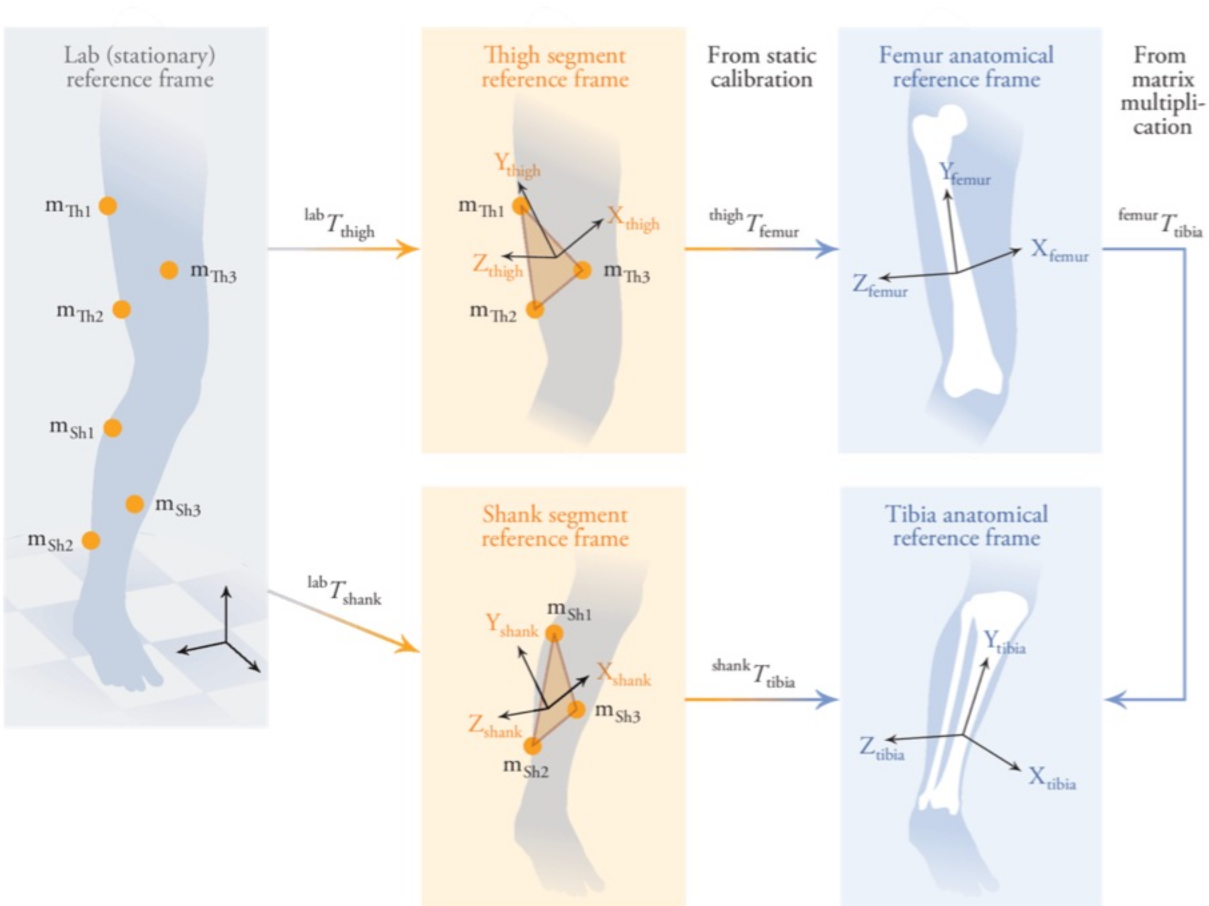
$$ {}^{\mathbf{A}}T_{\mathbf{B}} = \left[\begin{array}{ccc|c} & {}^{\mathbf{A}}R_{\mathbf{B}} & & [p^{\mathbf{A_oB_o}}]_{\mathbf{A}} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

$[p^{AoBo}]_A$ is the position vector from the origin of frame A to the origin of frame B, expressed in frame A.

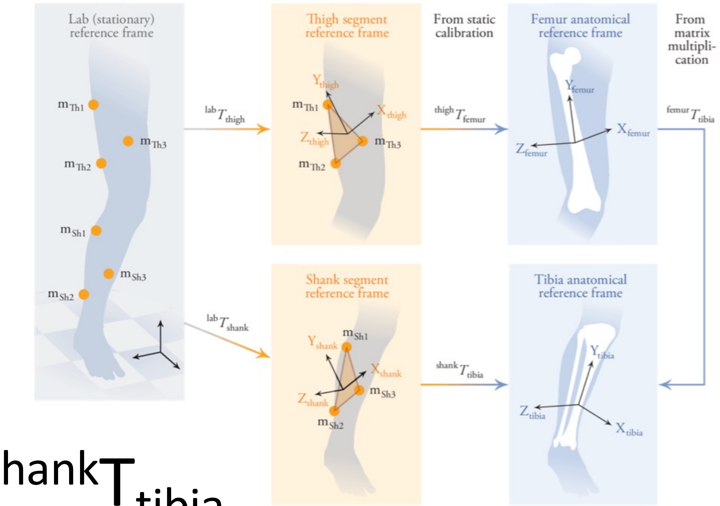
Point expressed in frame B can be expressed in frame A

$$ \begin{Bmatrix} [p]_{\mathbf{A}} \\ 1 \end{Bmatrix} = {}^{\mathbf{A}}T_{\mathbf{B}} \begin{Bmatrix} [p]_{\mathbf{B}} \\ 1 \end{Bmatrix} $$

# Calculating anatomical joint angles



# Calculating anatomical joint angles

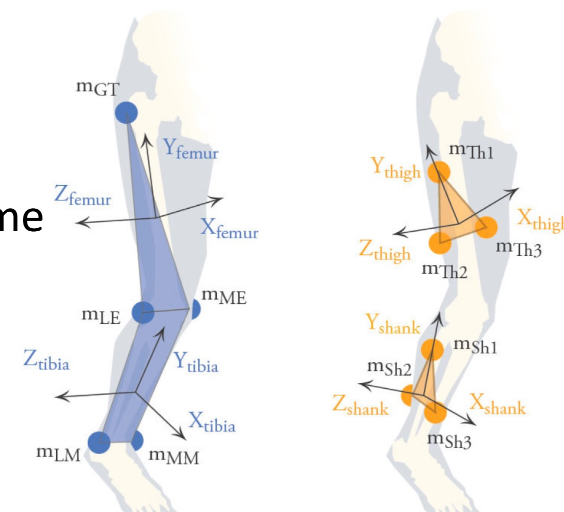



$$^{femur}T_{tibia} = {}^{femur}T_{thigh}\ {}^{thigh}T_{lab}\ {}^{lab}T_{shank}\ {}^{shank}T_{tibia}$$

${}^{lab}T_{shank}$ : Calculated from shank markers in lab reference frame
${}^{lab}T_{thigh}$ : Calculated from thigh markers in lab reference frame

${}^{shank}T_{tibia}$: Calculated from tibia markers in lab reference frame
Next, transformed to shank reference frame

${}^{thigh}T_{femur}$: Calculated from femur markers in lab reference frame
Next, transformed to thigh reference frame



# Quantifying movement

- Measurement techniques
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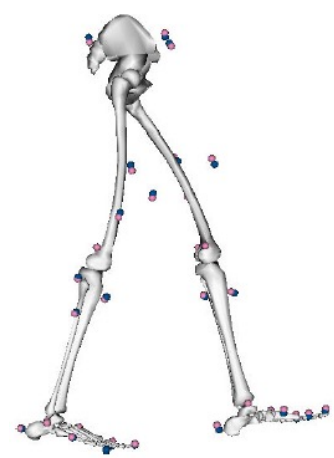
- Constrained inverse kinematics

# Constrained inverse kinematics (IK)

- Unconstrained IK limitations
  - Body segments can change length during movement
  - Leads to error in dynamics calculations

- Constrained IK
  - Uses global optimization to minimize the distance between the locations of experimental markers and analogous markers on underlying skeletal model.
  - Model is composed of rigid bodies, connected by joints that only allow physiological motion.



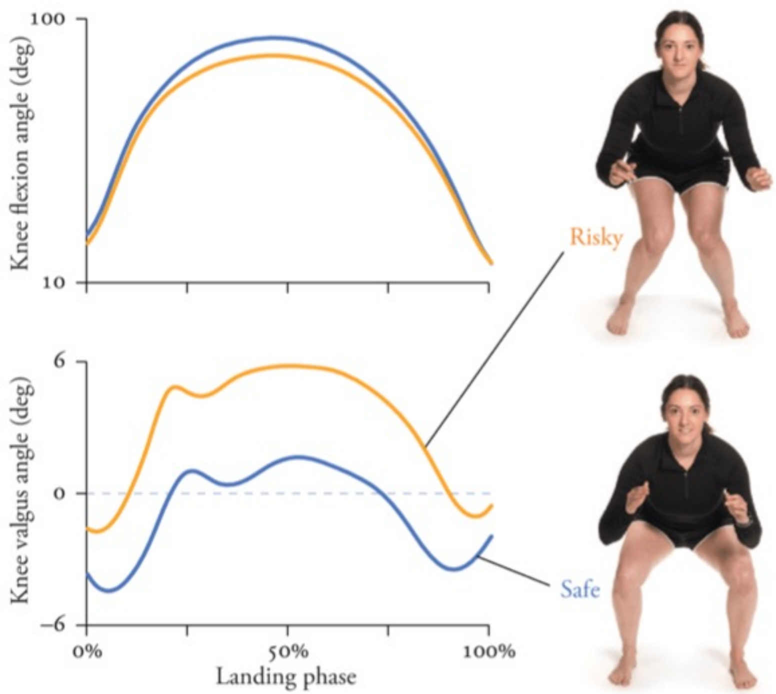
$$J = \min_{\underline{q}} \left\{ \sum_{k \in \text{Markers}} w_k \left\| \underline{x}_k^{\exp} - \underline{x}_k(\underline{q}) \right\|^2 \right\}$$

# Applications

- Joint angles useful for computing joint moments and muscle forces
- Also intrinsically useful
  - distinguish between normal and pathological gait patterns
  - Monitor rehabilitation during stroke recovery
  - Predict injury risk in athletes and prescribing training programs

# ACL injury risk example

- Knee kinematics during landing for a female soccer player
- Increased knee valgus angle→ injury risk



# Quantifying movement

- Measurement techniques
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# For next class

Download OpenSim

https://opensim.stanford.edu

https://simtk.org/frs/?group_id=91