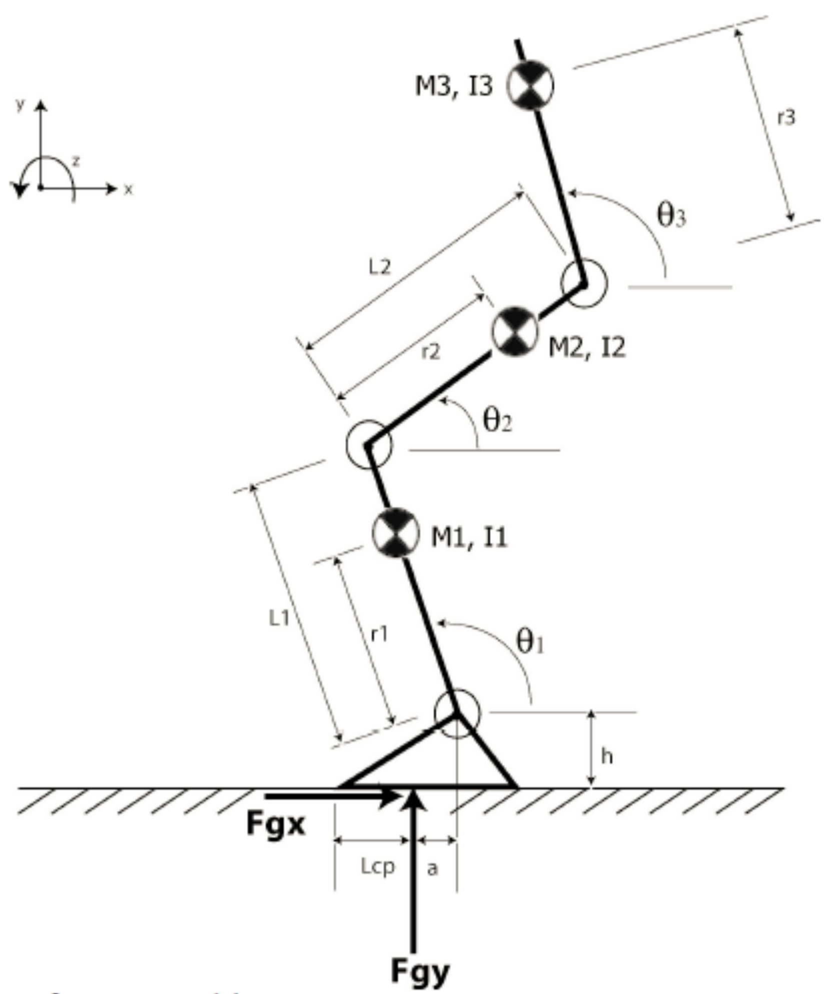
MCEN 4228/5228
Modeling of Human Movement
HW06

This homework assignment will guide you through the process of inverse dynamics. You will derive equations based on kinematics and force-plate data, then put them in action in MATLAB in order to analyze real data (posted in the zipped folder labeled “HW6_data” [constants.mat, kinematic_data.mat,force_data.mat]– note the video for this motion is also posted there – different subject this time).

Once again we have decided to study the motion of a person doing a vertical jump. This time we want to know what the internal joint forces and torques are. From the motion capture data we’ve obtained angular displacement for the shank, thigh, and trunk. By performing numerical differentiation, we’ve also calculated absolute angular velocities and accelerations of each body segment. A simple three-segment, planar linkage is assumed and utilized to model the human skeleton in the figure below.



First, derive an analytical expression for the equations of motion using the kinematics and force plate data.

1. Using both kinematic and force data, derive complete analytical expressions for the net joint torques exerted at the ankle (T1), knee (T2), and hip (T3) during the jumping motion. The equations should be in terms of θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, F_H, F_V, along with the masses, inertial properties, and dimensions of the limbs (all assumed to be inputs). Assume the foot remains flat on the floor.

2. Create a MATLAB function in which you write the analytical expressions from Problem 1 for T1, T2, and T3 (as well as all other necessary calculations and constants to solve for them). Write it so that you can input θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, $\ddot{\theta}_2$, F_H, F_V, and output T1, T2, and T3. Be sure to identify your calculations (e.g., segment 1 kinematics) using comments within the code.

3. In a MATLAB script, load the *kinematic_data.mat*, *constants.mat*, and *force_data.mat* files provided on Collab. Note that the angular positions are in units of radians, angular velocities in units radians/s, and angular accelerations in units of radians/s^2. In a new loop, call your function from Problem 2 and input the appropriate data from the .mat files so that you obtain arrays of T1, T2, and T3 values for each time point. Plot and label all three curves.

You'll notice that you (hopefully) did not have to use expressions for θ_3, $\dot{\theta}_3$, $\ddot{\theta}_3$. However, we do measure those motions. Let's explore what happens if you use that data.

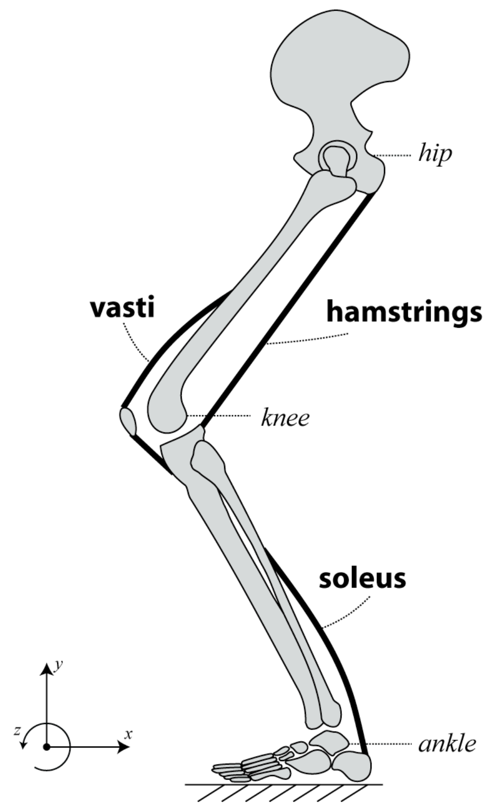
4. Derive a complete expression for the F_{3x}, F_{3y}, and T3, as a function of θ_3, $\dot{\theta}_3$, $\ddot{\theta}_3$, along with the mass and inertial properties of segment 3.

5. Add the expressions from problem 4 to your MATLAB script in problem 2. Once you have done that add lines to make three "error" calculations: one for F_{3x}, one for F_{3y}, and one for T3 (which is the difference between your calculation for these variables in part 4 and your calculation from part 2). Plot and label these three curves. In inverse dynamics, we call these error forces the "Hand of God." These additional forces (coming from thin air, hence the term "Hand of God") are generally applied to the top segment in order to make the inverse dynamics model "dynamically consistent."

6. Explain where you think the errors you calculated in Problem 5 come from, why do you think we need to account for them, and what it means if the errors are high. (BONUS question: what might be ways to minimize the magnitudes of the three components of the "Hand of God"?)

7. Now you will explore how you might estimate the muscle forces involved in the jumping task. Let's say that your EMG measurements showed that only three muscles were active during this task – the hamstrings (lumped as one muscle), the vasti (three heads of the quadriceps that are all lumped as one muscle), and the soleus. The figure below shows the lower limb geometry including these muscles.

7a. Use the time-varying values for T1, T2, and T3 that you found in problem 3, combined the moment arms for each muscle listed, **calculate the time-varying forces being developed in the hamstrings, vasti, and soleus muscles for this task.** (NOTE: Remember that muscle forces should only be *positive*.) Based on your answer, which muscle should you beef up for jumping?



Hamstrings hip extension moment arm = 5.0 *cm*
Hamstrings knee flexion moment arm = 4.0 *cm*
Vasti knee extension moment arm = 4.0 *cm*
Soleus ankle extension moment arm = 2.5 *cm*
(you can assume for this problem that moment arms are constant..)

7b. Lets say you did the same experiment on a new subject, but now detected significant activity in a total of six muscles. Briefly describe how you could use static optimization to estimate the muscle forces. (**BONUS**: take your proposed approach and implement it in MATLAB and come up with a solution. You can choose whichever six muscles you want in the leg to do this, and use the literature to find the parameters (such as moment arms, PCSAs, etc) needed.)

To Submit Online:

1. ONE homework document as a .pdf that includes:

- Handwritten or typed derivations for Problems 1 and 4.
- Labeled plots of T1, T2, and T3 values vs. time (Problem 3)
- Labeled plots of "Hand of God" forces and torques vs. time (Problem 5)
- Your written/typed answers to the questions posted in Problem 6.
- Handwritten or typed derivation for Problem 7a & answer to 7b.
- Labeled plots of hamstrings, vasti, and soleus muscle forces (Problem 7)
- All code, printed out– *be neat and fastidious*!