Board Paper 2016

SUMMATIVE ASSESSMENT- II Set-1

CBSE Class 10 Mathematics

General Instructions :

(i) All questions are compulsory.

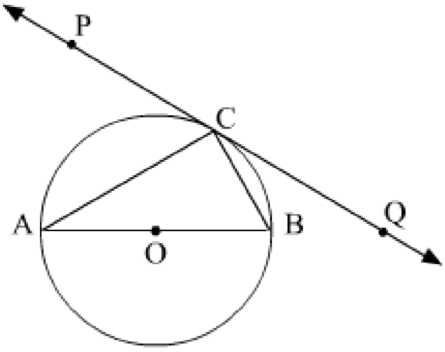
*(ii) The question paper consists of **31** questions divided into four sections – **A, B, C** and **D**.*

*(iii) Section A contains **4** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section **D** contains **11** questions of **4** marks each.*

(iv) Use of calculated is not permitted.

Section A

Q1 In fig. 1, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$.

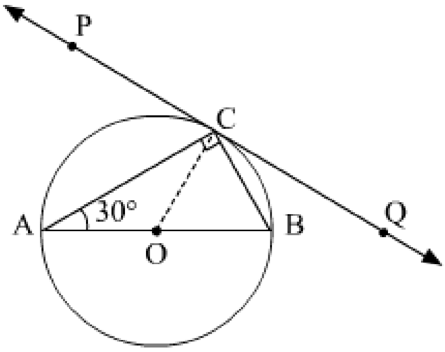


Ans:- Given: $\angle CAB = 30°$

AB is a diameter to the circle with centre O.

$\therefore \angle ACB = 90°$

Join OC.



$\therefore$ OC = OA (Radii of the circle)

∠CAB = ∠ACO = 30°

We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ ∠OCP = 90°

∠PCA + ∠OCA = 90°

∠PCA = 90°−30°=60°

∴ ∠PCA = 60°

Q2 For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P.?

Ans:- If a, b and c are in AP, then $2b = a + c$.

It is given that $k + 9$, $2k - 1$ and $2k + 7$ are in AP.

∴ $2(2k - 1) = (k + 9) + (2k + 7)$

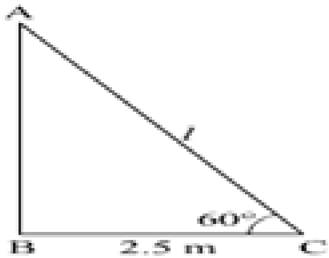
⇒ $4k - 2 = 3k + 16$

⇒ $k = 18$

Thus, the value of k is 18.

Q3 A ladder, leaning against a wall, makes an angle of 60° with the horitonzal. If the foot of the ladder is 2.5 m amay from the wall, find the length of the ladder.

Ans:- The given information is represented in the figure shown below:



Here, AC is the ladder with length l and AB is the wall.

In ΔABC,

$$\cos 60^\circ = \frac{BC}{AC}$$

$$(\because \cos\theta = \frac{Base}{Hypotenuse})$$

$$\Rightarrow 12 = \frac{2.5}{l}$$

$$\Rightarrow l = 2.5 \times 2 = 5 metres$$

Thus, the length of the ladder is 5 metres.

Q4 A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither red card nor a queen.

Ans:- Out of 52 cards, one card can be drawn in 52 ways.

∴ Total number of events = 52

In a pack of 52 playing cards, there are 26 red cards and 26 black cards that include 2 red queens and 2 black queens, respectively.

∴ Number of cards that are neither red nor queen = 52 − (26 + 2) = 24

⇒ Favourable number of events = 24

∴ Required probability $= \frac{24}{52} = \frac{6}{13}$

Section B

Q5 If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p\left(x^2 + x\right) + k = 0$ has equal roots, find the value of *k*.

Ans:- −5 is a root of the quadratic equation $2x^2 + px - 15 = 0$.

$$\therefore 2(-5)^2 + p(-5) - 15 = 0$$

$$\Rightarrow 50 - 5p - 15 = 0$$

$$\Rightarrow 35 = 5p$$

$$\Rightarrow p = 7$$

$$It\ is\ given\ that p\left(x^2 + x\right) + k$$

$$= 0.7\left(x^2 + x\right) + k = 0$$

$$\Rightarrow 7x^2 + 7x + k = 0$$

If the roots are equal, then D = 0.

$$D = b^2 - 4ac = 0$$
$$\Rightarrow 49 - 28k = 0$$
$$\Rightarrow 49 = 28k$$
$$\Rightarrow k = \frac{7}{4}$$

Q6 Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

Ans:- It is given that P and Q are the points of trisection of the line segment joining points A(2, –2) and B(–7, 4) such that P is nearer to A. Therefore, P divides the line segment AB internally in the ratio 1 : 2 and Q divides AB internally in the ratio 2 : 1.

Using section formula, we have

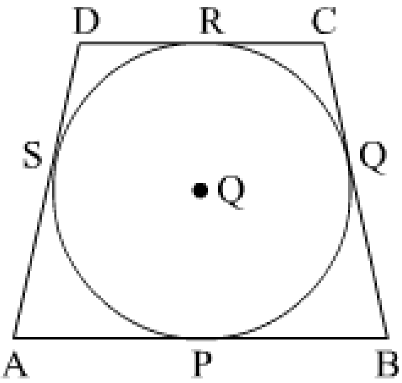
Coordinates of P

$$= \left(\frac{1\times(-7)+2\times 2}{1+2}, \frac{1\times 4+2\times(-2)}{1+2} \right)$$
$$= \left(\frac{-7+4}{3}, \frac{4-4}{3} \right)$$
$$= (-1, 0)$$

Coordinates of Q

$$= \left(\frac{2\times(-7)+1\times 2}{2+1}, \frac{2\times 4+1(2)}{2+1} \right)$$
$$= \left(\frac{-14+2}{3}, \frac{8-2}{3} \right) = (-4, 2)$$

Q7 In Fig. 2, a quadrilateral ABCD is drawn to circumscribe a circle with centre O in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB+ CD = BC + DA.



Ans:- We know that the tangents drawn from the exterior point to a circle are equal in length.

So,

From point D, DR = DS .....(i)

From point A, AP = AS .....(ii)

From point B, BP = BQ .....(iii)

From point C, CR = CQ .....(iv)

Adding (i), (ii), (iii) and (iv), we get

DR + AP + BP + CR = DS + AS + BQ + CQ

(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)

CD + AB = DA + BC

AB + CD = BC + DA

Hence proved.

Q8 Prove that the points (3, 0), (6, 4) and (−1, 3) are the vertices of a right angled isosceles triangle.

Ans:- Let A(3, 0), B(6, 4) and C(−1, 3) be the vertices of the given triangle.

Using distance formula, we have

$$AB = \sqrt{(4-0)^2 + (6-3)^2} = \sqrt{25} = 5\,units$$

$$BC = \sqrt{(3-4)^2 + (-1-6)^2} = \sqrt{50} = 5\sqrt{2}\,units$$

$$CA = \sqrt{(0-3)^2 + (3+1)^2} = \sqrt{25} = 5\,units$$

Now,

$$(5)^2+(5)^2=\left(5\sqrt{2}\right)^2$$

$$\Rightarrow AB^2+CA^2=BC^2$$

$\Rightarrow \Delta ABC$ is a right-angled triangle, right angled at A.

Also,

AB = CA = 5 units

Therefore, ΔABC is a right-angled isosceles triangle.

Hence, the given points are the vertices of a right-angled isosceles triangle.

Q9 The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

Ans:- Let a and d be the first term and the common difference of the AP, respectively.

It is given that $a_4 = 0$.

$\therefore a + 3d = 0$

$\Rightarrow a = -3d$

Now,

$a_{25} = a + 24d$

$\Rightarrow a_{25} = -3d + 24d = 21d$ .....(1)

Also,

$a_{11} = a + 10d$

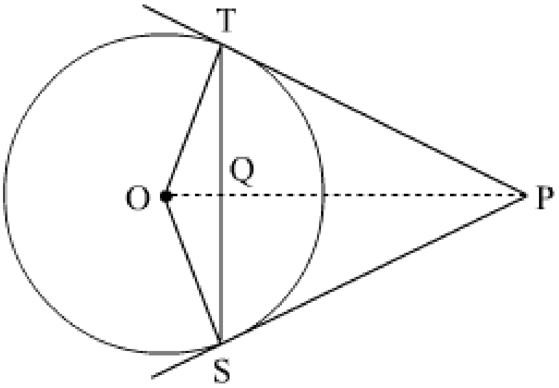
$\Rightarrow a_{11} = -3d + 10d = 7d$ .....(2)

From (1) and (2), we have

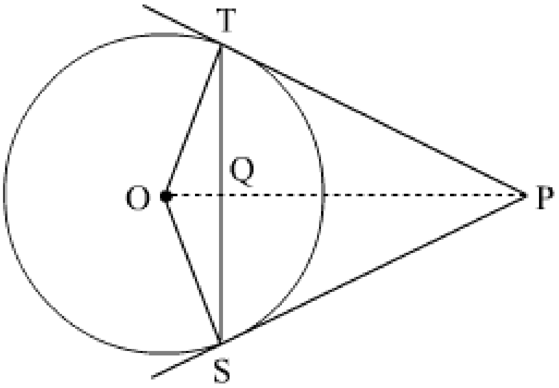
$$a_{25} = 3 \times a_{11}$$

Q10 In Fig. 3, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius *r*.

If OP = 2*r*, show that ∠OTS = ∠OST = 30°.



Ans:-



Given:

O is the centre and r is the radius of the circle.

PT and PS are tangents to the circle.

OP = 2r

To prove: ∠OTS=∠OST=30°

Proof:

PT and PS are tangents drawn to the circle.

∴ ∠OTP=∠OSP=90° (Tangent to a circle is perpendicular to the radius through the point of contact.)

$In\ \Delta OTP,$

$$\sin\angle OPT = \frac{OT}{OP} = \frac{r}{2r}$$

$$\Rightarrow \sin\angle OPT = \frac{1}{2}$$

$$\Rightarrow \angle OPT = 30°$$

Now,

∠OTP+∠OPT+∠TOP=180° (Angle sum property)

$\Rightarrow 90° + 30° + \angle TOP = 180°$

$\Rightarrow \angle TOP = 180° - 120° = 60°$

ΔPTS is an isosceles triangle and OP is the angle bisector of $\angle TPS$.

$\Rightarrow TS \perp OP$

$\therefore \angle OQT = \angle OQS = 90°$

In ΔOTQ,

$\angle OQT + \angle OTQ + \angle TOQ = 180°$ (Angle sum property)

$\Rightarrow 90° + \angle OTQ + 60° = 180°$

$\Rightarrow \angle OTQ = 180° - 150° = 30°$

Similarly,

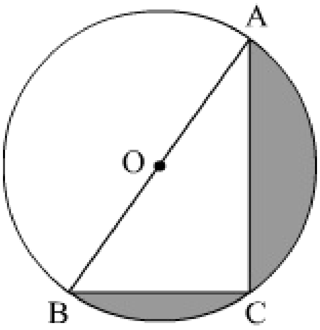
$\angle OSQ = 30°$

$\therefore \angle OTS = \angle OST = 30°$

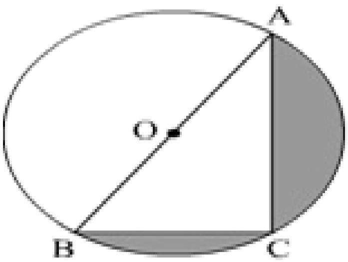
Section C

Q11 In fig. 4, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region.

(Take π = 3.14)



Ans:-



It is given that AB = 13 cm and AC = 12 cm.

We know that angle inscribed in a semicircle is 90°.

$\therefore \angle ACB = 90°$

So, ΔABC is a right-angled triangle.

In ΔABC,

$AC^2 + BC^2 = AB^2$

$\Rightarrow 144 + BC^2 = 169$

$\Rightarrow BC^2 = 25$

$\Rightarrow BC = 5$ cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AC$

$= \frac{1}{2} \times 5 \times 12 = 30 cm2$

Radius of the circle,

$r = \frac{AB}{2} = \frac{13}{2}\ cm$

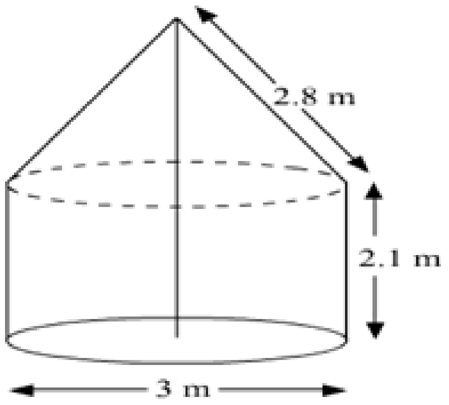
Area of semicircle

$ACB = \frac{\pi r^2}{2} = \frac{3.14}{2} \times \frac{3.14}{2} \times \frac{13}{2}$

$= 66.33 cm^2 \left(Approx\right)$

∴ Area of the shaded region = Area of semicircle ACB − Area of ΔACB = 66.33 − 30 = 36.33 cm^2 (Approx.)

Q12 In the figure, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m, respectively, and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of Rs 500/sq. metre. $\left(Use\ \pi = \frac{22}{7}\right)$



Ans:- Canvas needed to make the tent = Curved surface area of the conical part + Curved surface area of the cylindrical part

Radius of the conical part = Radius of the cylindrical part

$= r = \frac{3}{2}\,m$

Slant height of the conical part = l = 2.8 m

Height of the cylindrical part = h = 2.1 m

Curved surface area of the conical part

$= \pi r l = \frac{22}{7} \times \frac{3}{2} \times 2.8 m^2$

Curved surface area of the cylindrical part

$= 2\pi r h = 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1 m^2$

∴ Total area of the canvas needed to make the tent

$$= \frac{22}{7} \times \frac{3}{2} \times 2.8 + 2 \times \frac{22}{7} \times \frac{3}{2} \times 2.1$$

$$= \frac{22}{7} \times \frac{3}{2} \times (2.8 + 4.2)$$

$$= \frac{22}{7} \times \frac{3}{2} \times 7 = 33 m^2$$

Cost of the canvas $=$ rS. $500 / m^2$

∴ Total cost of the canvas needed to make the tent = 500 × 33 = ?16,500

Q13 If the point P(x, y) is equidistant from the points A(a + b, b − a) and B(a − b, a + b). Prove that bx = ay.

Ans:- It is given that the point P(x, y) is equidistant from the points A(a + b, b − a) and B(a − b, a + b).

$\therefore PA = PB$

$$\Rightarrow \sqrt{(a+b-x^2)+(b-a-y^2)}$$

$$= \sqrt{(a-b-x^2)+(a+b-y^2)}$$

$$\Rightarrow (a+b-x^2)(b-a-y^2)$$

$$= (a-b-x^2)+(a+b-y^2)$$

$$\Rightarrow (a+b-x^2)-(a-b-x^2)$$

$$= (a+b-y^2)-(b-a-y^2)$$

$$\Rightarrow (a+b-x+a-b-x)(a+b-x-a+b+x)$$

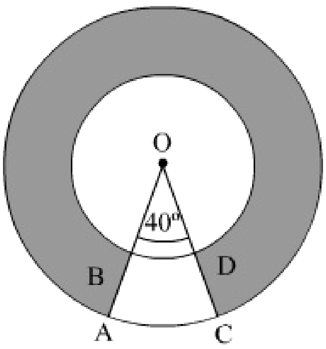
$$= (a+b-y+b-a-y)(a+b-y-b+a+y)$$

$$\Rightarrow (2a-2x)(2b) = (2b-2y)(2a)$$

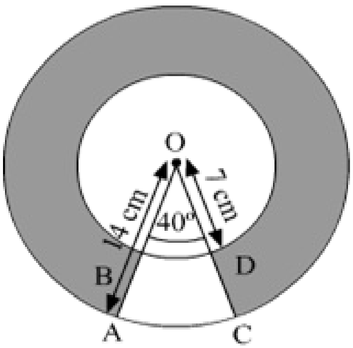
$$\Rightarrow (a-x)b = (b-y)a$$

$$\Rightarrow ab - bx = ab - ay \Rightarrow bx = ay$$

Q14 In fig. 6, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm, where ∠AOC=40°. $\left(Use\, \pi = \frac{22}{7} \right)$



Ans:- We have



Area of region ABDC = Area of sector AOC – Area of sector BOD

$$=\left(\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times14\times14-\frac{40^\circ}{360^\circ}\times\frac{22}{7}\times7\times7\right)cm^2$$

$$=\left(\frac{1}{9}\times22\times14\times2-\frac{1}{9}\times22\times7\right)cm^2$$

$$=\frac{22}{9}(28-7)cm^2$$

$$=\frac{154}{3}cm^2$$

Area of the circular ring

$$=\left(\frac{22}{7}\times14\times14-\frac{22}{7}\times7\times7\right)cm^2$$

$$=(22\times14\times2-22\times7)cm^2$$

$$=22(28-7)cm^2=22\times21cm^2=462\,cm^2$$

∴ Area of shaded region = Area of the circular ring − Area of region ABDC

$$=\left(462-\frac{154}{3}\right)cm^2$$

$$=\frac{1232}{3}cm^2$$

$$=410.67\,cm^2 \quad (Approx)$$

Q15 If the ratio of the sum of first n terms of two A.P's is (7n + 1) : (4n + 27), find the of their mth terms.

Ans:- Let a and a' be the first terms and d and d' be the common differences of the two AP's.

Now, the sum of their n terms is given by

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]and$$

$$S'n=\frac{n}{2}\left[2a'+(n-1)d'\right]$$

$$\therefore SnS'n=\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2a'+(n-1)d'\right]}$$

$$=\frac{2a+(n-1)d}{2a'+(n-1)d'}$$

It is given that

$$\frac{Sn}{S'n}=\frac{7n+1}{4n+27}$$

$$\Rightarrow \frac{2a+(n-1)d}{2a'+(n-1)d'}=\frac{7n+1}{4n+27} \quad .....(1)$$

Now, in order to find the ratio of the m^{th} terms of the two given AP's, we replace n by $(2m - 1)$ in (1).

$$\therefore \frac{2a+(2m-2)d}{2a'+(2m-2)d'}$$

$$=\frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\Rightarrow \frac{a+(m-1)d}{a'+(m-1)d'}$$

$$=\frac{14m-6}{8m+23}$$

Hence, the ratio of the m^{th} terms of the two AP's is $(14m - 6) : (8m + 23)$.

Q16 Solve for X:

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}$$

$$=\frac{2}{3}, x \neq 1, 2, 3$$

Ans:-

$$\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}$$

$$=\frac{2}{3}, x \neq 1, 2, 3$$

$$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\Rightarrow \frac{2x-4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$$

$$\Rightarrow \frac{x-2}{(x-1)(x-2)(x-3)}=\frac{1}{3}$$

$$\Rightarrow 3=(x-1)(x-3)$$

$$\Rightarrow 3 = x^2 - 4x + 3$$
$$\Rightarrow x^2 - 4x = 0$$
$$\Rightarrow x(x-4) = 0$$
$$\Rightarrow x=0, x=4$$

Q17 A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. $\left(Use\, \pi = \frac{22}{7} \right)$

Ans:- Let:

r_1 = Radius of the conical vessel = 5 cm

h_1 = Height of the conical vessel = 24 cm

r_2 = Radius of the cylindrical vessel = 10 cm

Suppose water rises up to a height h_2 cm in the cylindrical vessel.

Now,

Volume of the water in cylindrical vessel = Volume of water in conical vessel

$$\Rightarrow \pi r_2^2 h_2 = \frac{1}{3} \pi r_1^2 h_1$$
$$\Rightarrow 3 r_2^2 h_2 = r_1^2 h_1$$
$$\Rightarrow 3 \times 10 \times 10 \times h_2 = 5 \times 5 \times 24$$
$$\Rightarrow h_2 = \frac{5 \times 5 \times 24}{3 \times 10 \times 10}$$
$$\Rightarrow h_2 = 2\, cm$$

Thus, the rise in the water level in the cylindrical vessel is 2 cm.

Q18 A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by 359 cm. Find the diameter of the cylindrical vessel.

Ans:- Increase in the height of water level in the cylindrical vessel due to sphere (h) = 329 cm

Radius of the sphere (R) = 6 cm

Let radius of the cylindrical vessel be *r*.

Rise in volume of water in cylinder = Volume of sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3$$

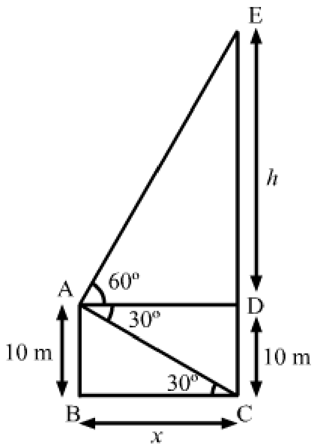
$$\Rightarrow r^2 = \frac{4\times 6^3 \times 9}{3\times 32}$$

Hence, diameter of the cylindrical vessel is 18 cm (2 × 9).

$$\Rightarrow r = \sqrt{27\times 3} = 9\,cm$$

Q19 A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill.

Ans:-



Suppose the man is standing on the deck of the ship at point A.

Let CE be the hill with base at C.

It is given that the angle of elevation of point E from A is 60° and the angle of depression of point C from A is 30°.

So,

∠DAE = 60°

∠CAD = 30°

Now,

∠CAD = ∠ACB = 30° (Alternate angles)

AB = 10 m

Suppose ED = *h* m and BC = *x* m.

In ?EAD, we have

$$tan 60° = \frac{ED}{AD}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$(AD = BC = x)$$

$$\Rightarrow h = x\sqrt{3} \quad ....(1)$$

$In \Delta ABC,$ we have

$$tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$\Rightarrow x = 10\sqrt{3} \quad ....(2)$$

∴ Distance of the hill from the ship $= 10\sqrt{3} m$

From (1) and (2), we have

$$h = 10\sqrt{3} \times \sqrt{3} \quad = 30 m$$

∴ Height of the hill = h + 10 = 30 + 10 = 40 m

Q20 Three different coins are tossed together. Find the probability of getting

(i) exactly two heads

(ii) at least two heads

(iii) at least two tails.

Ans:- Following are the possible outcomes of tossing three coins together:

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

⇒ Total number of outcomes = 8

(i) Outcomes of getting exactly two heads are HHT, HTH and THH.

⇒ Favourable number of outcomes = 3

Probability of getting exactly two heads = $\frac{3}{8}$

(ii) Outcomes of getting at least two heads are HHH, HHT, HTH and THH.

⇒ Favourable number of outcomes = 4

Probability of getting atleast two heads $= \frac{4}{8} = \frac{1}{2}$

(iii) Outcomes of getting at least two tails are HTT, THT, TTH and TTT.

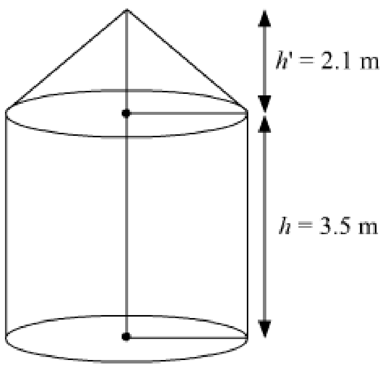
⇒ Favourable number of outcomes = 4

Probability of getting atleast two tails $= \frac{4}{8} = \frac{1}{2}$

<u>Section D</u>

Q21 Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem? $\left(Use\ \pi = \frac{22}{7}\right)$

Ans:- Consider the following figure:



Area of the tent = CSA of cylindrical base + CSA of the conical top

$$= 2\pi rh + \pi rl$$

$$l = \sqrt{(h')^2 + r^2}$$

$$= \sqrt{2.1^2 + 2.8^2}$$

$$= \sqrt{4.41 + 7.84}$$

$$= \sqrt{12.25}$$

$$= 3.5m$$

Area of the tent is given by

$= \pi r l + 2\pi r h$

$= \pi r (l + 2h)$

$= \frac{22}{7} \times 2.8(3.5 + 3.5 \times 2)$

$= \frac{22}{7} \times 2.8(3.5 + 7)$

$= \frac{22}{7} \times 2.8 \times 10.5$

$= 92.4 / m^2$

Total area of the canvas required to make 1 tent = 92.4/m^2.

Cost of the canvas used to make 1 tent = 92.4 × 120 = ?11,088

Cost of canvas to make 1500 tents = ?1,66,32,000

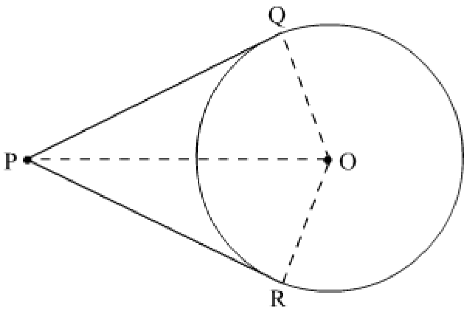
∴ Share of one school $= \frac{1,66,32,000}{50} = \text{Rs. } 3,32,640$

Thus, the amount shared by each school is ?3,32,640.

The given problem shows the kind behaviour of the school authorities.

Q22 Prove that the lengths of the tangents drawn from an external point to a circle are equal.

Ans:- Let us consider a circle with centre O, a point P lying outside the circle and two tangents PQ and PR on the circle from P.



To prove: PQ = PR

Proof: Join OP, OQ and OR.

Then, ∠OQP and ∠ ORP are right angles, as these are the angles between the radii and tangents.

Now,

∠OQP = ∠ORP = 90° (Tangent at any point of a circle is perpendicular to the radius through the point of contact.)

$In \Delta OQP$ and ΔORP,

OQ = OR (Radii of the same circle)

OP = OP (Common)

∠OQP = ∠ORP (Each 90°)

∴ ΔOQP ≅ ΔORP (RHS)

and

PQ = PR (CPCT)

Hence proved.

Q23 Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

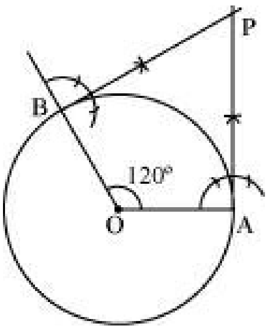
Ans:- Follow the given steps to construct the tangents.

Step 1: Draw a circle of radius 4 cm, with O as centre.

Step 2: Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at A.

Step 3: Draw a radius OB, making an angle of 120° (180° − 60°) with OA.

Step 4: Draw a perpendicular to OB at B. Let these perpendiculars intersect at P.



Here, PA and PB are two tangents drawn to the circle inclined at an angle of 60° to each other.

Justification

The construction can be justified by proving that ∠APB = 60°.

∠OAP = 90° (Construction)

∠OBP = 90° (Construction)

∠AOB = 120° (Construction)

We know that the sum of all interior angles of a quadrilateral is 360°.

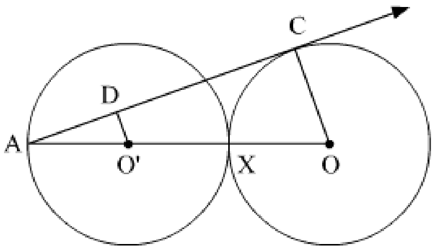
∠OAP + ∠AOB + ∠OBP + ∠APB = 360°

⇒ 90° + 120° + 90° + ∠APB = 360°

⇒ ∠APB = 60°

This justifies the construction.

Q24 In the figure, two equal circles, with centres O and O', touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$



Ans:- Given:

O'X = OX (Radius of two equal circles)

O'D⊥AC

It is known that radius is perpendicular to the tangent.

⇒ OC⊥AC

∠O'DA=∠ACO=90°

Therefore, line O'D is parallel to line OC. (Corresponding angles)

In ?ADO' and ?ACO',

∠O'*DA*=∠ACO=90°

∠DAO'=∠CAO (Common)

So, by AA similarity,

?ADO'~?ACO

Therefore, corresponding sides will be in the same ratio.

$$\frac{AD}{AC} = \frac{AO'}{AO} = \frac{DO'}{CO} \quad ...(1)$$

And

AO' = O'X = XO (Radius)

So, AO = AO' + O'X + XO = 3AO'

Now,

$$\frac{AO'}{AO} = \frac{AO'}{3AO'} = \frac{1}{3} \quad ...(2)$$

From (1) and (2), we have

$$\frac{DO'}{CO} = 13$$

Q25 Solve for $x : \frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, x \neq -1, -2, -4$

Ans:- $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}, x \neq -1, -2, -4$

$$\Rightarrow \frac{(x+2)+2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\Rightarrow \frac{3x+4}{(x+1)(x+2)} = \frac{4}{x+4}$$

$$\Rightarrow (3x+4)(x+4) = 4(x+1)(x+2)$$

$$\Rightarrow 3x^2 + 16x + 16 = 4x^2 + 12x + 8$$

$$\Rightarrow x^2 - 4x - 8 = 0$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-8)}}{2}$$

$$\left(\because x = \frac{-b \pm \sqrt{D}}{2a} \right)$$

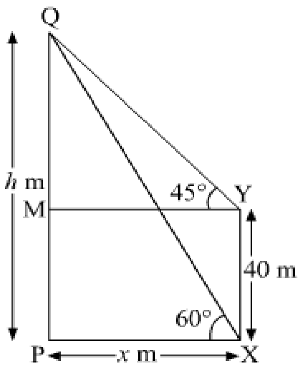
$$\Rightarrow x = \frac{4 \pm \sqrt{48}}{2}$$

$$\Rightarrow x = \frac{4 \pm 4\sqrt{3}}{2}$$

$$\Rightarrow x = 2 \pm 2\sqrt{3}$$

Q26 The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. $\left(Use \sqrt{3} = 1.73 \right)$

Ans:- Let the height of the tower PQ be *h* m and the distance PX be *x* m.



As YX = 40 m,

QM = (h − 40) m

$In\ \Delta QPX,$

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = \sqrt{3} \quad ....(1)$$

$In\ \Delta QMY,$

$$\tan 45° = \frac{h-40}{x}$$

$$(\because MY = PX = xm)$$

$$\Rightarrow 1 = \frac{h-40}{x}$$

$$\Rightarrow x = h - 40$$

$$\Rightarrow h = x + 40 \quad ...(2)$$

From (1) and (2), we get

$$\sqrt{3}x = x + 40$$

$$\Rightarrow x\left(\sqrt{3} - 1\right) = 40$$

$$\Rightarrow x(1.73 - 1) = 40$$

$$\Rightarrow 0.73x = 40$$

$$\Rightarrow x = \frac{40}{0.73} = 54.79 \text{ m}$$

$\therefore h = 54.79 + 40 = 94.79m$ [From (2)]

Hence, the height of the tower is 94.79 m and the distance PX is 54.79 m.

Q27 The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X.

Ans:- Sum of the numbers of houses preceding the house numbered X = 1 + 2 + 3 + ... + (X − 1)

$$=\frac{(X-1)}{2}\left[2\times 1+(X-1-1)\right]$$

$$=\frac{X(X-1)}{2}$$

Sum of the numbers of houses following the house numbered X = (X + 1) + (X + 2) + (X + 3) + ... + 49

Using the formula, we get

Sum of n terms, $S_n=\frac{n}{2}(a+l)$ (Here, a is the first term and l is the last term of an AP.)

$$=\frac{(49-X)}{2}\left[49+(X+1)\right]$$

$$=\frac{(49-X)(50+X)}{2}$$

Sum of the numbers of houses following the house numbered X = Sum of the numbers of houses preceding the house numbered X

$$\Rightarrow \frac{(49-X)(50+X)}{2}=\frac{X(X-1)}{2}$$

$$\Rightarrow (49-X)(50+X)=X(X-1)$$

$$\Rightarrow 2450+49X-50X-X^2=X^2-X$$

$$\Rightarrow 2X^2=2,450$$

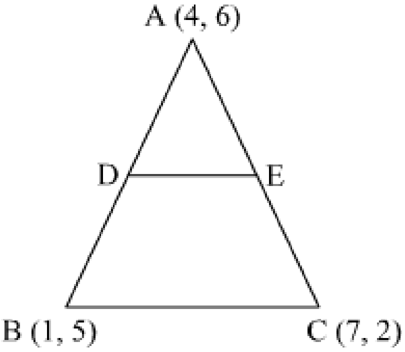
$$\Rightarrow X^2=1,225$$

$$\Rightarrow X=35$$

Hence, at X = 35, the sum of numbers of houses preceding the house numbered X is equal to sum of the numbers of houses following X.

Q28 In fig. 8, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E, respectively, such that

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. **Calculate the area of ΔADE and compare it with area of** ΔABC.



Ans:- In ?ADE and ?ABC,

∠DAE = ?∠BAC (Common)

$\frac{AD}{AB} = \frac{AE}{AC}$ (Given)

?ADE ~ ?ABC (SAS similarity)

Now,

$$Area\,of\,\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|4(5-2) + 1(2-6) + 7(6-5)|$$

$$= 12 \times |15| = \frac{15}{2}\,sq.units$$

Since ?ADE ~ ?ABC,

$$\Rightarrow \frac{Ar\,\Delta ADE}{Ar\,\Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\Rightarrow Area\,of\,\Delta ADE = \frac{1}{9}\,Area\,of\;ABC$$

$$= \frac{1}{9} \times \frac{15}{2} = \frac{5}{6}$$

$$\Rightarrow Area\,of\,\Delta ADE = \frac{5}{6}\,sq.units$$

Also,

Area of?ADE : Area of?ABC=1 : 9

Q29 A number *x* is selected at random from the numbers 1, 2, 3 and 4. Another number *y* is selected at random from the numbers 1, 4, 9 and 16. Find the probability that

product of x and y is less than 16.

Ans:- Number x can be selected in four ways. Corresponding to each such way are four ways of selecting number y.

Therefore, the two numbers can be in selected in 16 ways as follows:

(1, 1), (1, 4), (1, 9), (1, 16),

(2, 1), (2, 4), (2, 9), (2, 16),

(3, 1), (3, 4), (3, 9), (3, 16),

(4, 1), (4, 4), (4, 9), (4, 16)

∴ Total number of possible outcomes = 16

The product xy will be less than 16 if x and y are chosen in one of the following ways:

(1, 1), (1, 4), (1, 9),

(2, 1), (2, 4),

(3, 1), (3, 4),

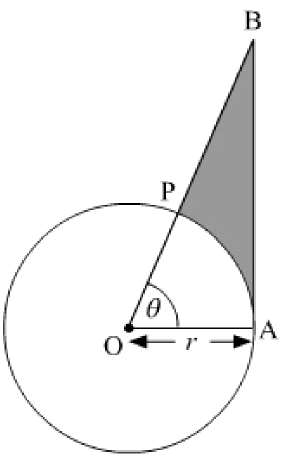
(4, 1)

∴ Favourable number of outcomes = 8

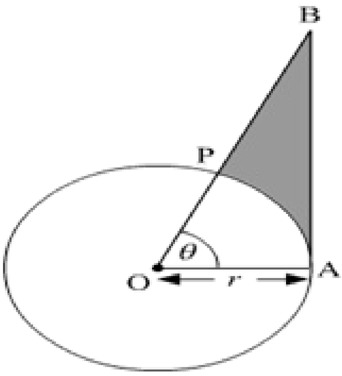
∴ Probability that the product of x and y is less than $16 = \frac{8}{16} = \frac{1}{2}$

Hence, the required probability is $\frac{1}{2}$

Q30 In Fig. 9, is shown a sector OAP of a circle with centre O, containing ∠θ. AB is perpendicular the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[tan\theta + sec\theta + \frac{\pi\theta}{180} - 1\right]$.



Ans:- It is given that the radius of the circle is r and $\angle AOP = \theta$.



In ?AOB,

$$\cos\theta = \frac{OA}{OB}$$

$$\Rightarrow OB = \frac{OA}{\cos\theta}$$

$$\Rightarrow OB = r\sec\theta$$

$$\therefore PB = OB - OP = r\sec\theta - r$$

Also,

$$\tan\theta = \frac{AB}{OA}$$

$$\Rightarrow AB = OA\tan\theta$$

$$\Rightarrow AB = r\tan\theta$$

$$Length\ of\ the\ arc\ AP = \frac{\theta}{360^\circ}\times 2\pi r = \frac{\theta\pi}{180^\circ}r$$

∴ Perimeter of the shaded region = PB + AB + Length of the arc AP

$$= r\sec\theta - r + r\tan\theta + \frac{\pi\theta}{180^\circ}r$$

$$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180^\circ} - 1\right)$$

Q31 A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream.

Ans:- Given: Speed of the boat in still water = 24 km/h

Let the speed of the stream be *x* km/h.

Now,

Speed of the boat upstream = Speed of the boat in still water − Speed of the stream

∴ Speed of the boat upstream = $(24 - x)$ km/h

Speed of the boat downstream = Speed of the boat in still water + Speed of the stream

∴ Speed of the boat downstream = $(24 + x)$ km/h

Time taken in the upstream journey = Time taken in the downstream journey + 1 h

$$\therefore \frac{Distance\,covered\,upstream}{Speed\,of\,the\,boat\,upstream}$$

$$= \frac{Distance\,covered\,downstream}{Speed\,of\,the\,boat\,downstream} + 1\,h$$

$$\Rightarrow \frac{32}{(24-x)} = \frac{32}{(24+x)} + 1$$

$$\Rightarrow \frac{32}{(24-x)} - \frac{32}{(24+x)} = 1$$

$$\Rightarrow \frac{32(24+x-24+x)}{(24-x)(24+x)} = 1$$

$$\Rightarrow \frac{64x}{576-x^2} = 1$$

$$\Rightarrow 64x = 576 - x^2$$

$$\Rightarrow x^2 + 64x - 576 = 0$$

$$\Rightarrow x^2 + 72x - 8x - 576 = 0$$

$$\Rightarrow x(x+72) - 8(x+72) = 0$$

$$\Rightarrow (x-8)(x+72) = 0$$

$$\Rightarrow x = 8\,or\,x = -72$$

$$\therefore x = 8$$

$$[x \neq -72\,as\,the\,speed\,cannot\,be\,negative.]$$

Hence, the speed of the stream is 8 km/h.