CBSE Class 12th Mathematics

Compartment Delhi SET-1 2017

General Instructions:

1. All questions are compulsory.
2. This question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

Question numbers 1 to 4 carry 1 mark each.

1. Let A and B are matrices of order 3 × 2 and 2 × 4 respectively. Write order of matrix (AB).

2. Write the equation of tangent drawn to the curve y = sin x at the point (0, 0).

3. Find : $\int \frac{1}{x(1+\log x)} dx$

4. Write the angle between the vectors $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.

SECTION – B

Question numbers 5 to 12 carry 2 marks each.

5. In the following matrix equation use elementary operation $R_2 \to R_2 + R_1$ and write the equation thus obtained.

$$\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 8 & -3 \\ 9 & -4 \end{pmatrix}$$

6. Find the value of k for which the function

$$f(x)\begin{cases}\dfrac{x^2+3x-10}{x-2}, & x \neq 2\\ k, & x=2\end{cases}$$

is continuous at x = 2.

7. The radius r of a right circular cone is decreasing at the rate of 3 cm/minute and the height h is increasing at the rate of 2 cm/minute. When r = 9 cm and h = 6 cm, find the rate of change of its volume.

8. Find: $\int \sqrt{x^2 - 2x}dx$

9. Find the differential equation of the family of curves $y^2 = 4$ ax.

10. Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y = e^{3x}$$

11. If the points with position vectors $10\hat{i} + 3\hat{j}, 12\hat{i} - 5\hat{j}$ and $\lambda\hat{i} + 11\hat{j}$ are collinear, find the value of λ.

12. A firm has to transport at least 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ` 400 and each small van is Rs.200. Not more than Rs.3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

SECTION – C

Question numbers 13 to 23 carry 4 marks each.

13. Prove that : $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x, -\dfrac{1}{\sqrt{2}} \leq x \leq 1$

14. $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$ then using properties of determinants, find the value of $\dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z}$, where x, y, z ≠ 0.

OR Using elementary operations, find the inverse of the following matrix A

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}.$$

15. If x = a (cos θ + θ sin θ) and y = a (sin θ – θ cos θ), then find $\frac{d^2y}{dx^2}$.

16. Find the equation of tangent to the curve y = cos (x + y), $2\pi \le x \le 0$, that is parallel to the line x + 2y = 0.

17. Find: $\int \frac{x+5}{3x^2+13x-10}dx$

OR Evaluate: $\int_0^{\pi/4} \frac{1}{\cos^2 x + 4\sin^2 x}$

18. Find: $\int \frac{x^2 dx}{(x-1)(x^2+1)}$

19. Find the general solution of the following differential equation:

$$x\cos\left(\frac{x}{y}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$$

20. If four points A, B, C and D with position vectors $4\hat{i}+3\hat{j}+3\hat{k}, 5\hat{i}+x\hat{j}+7\hat{k}, 5\hat{i}+3\hat{j}$ and $7\hat{i}+6\hat{j}+\hat{k}$ respectively are coplanar, then find the value of x.

21. Find the value of p so that the lines

$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{1}$ and $\frac{7-7x}{3p} = \frac{5-y}{1} = \frac{11-z}{7}$ are at right angles

OR Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 and twice of its y-intercept is equal to three times its z-intercept.

22. Solve the following Linear Programming problem graphically:

Minimize: z = 6x + 3y

Subject to the constraints: $\begin{cases} 4x+y \ge 80 \\ x+5y \ge 115 \\ 3x+2y \le 150 \\ x \ge 0, y \ge 0 \end{cases}$

23. There are three categories of students in a class of 60 students :

A : Very hard working students B : Regular but not so hard working

C : Careless and irregular

10 students are in category A, 30 in category B and rest in category C. It is found that probability of students of category A, unable to get good marks in the final year examination is, 0.002, of category B it is 0.02 and of category C, this probability is 0.20. A student selected at random was found to be the one who could not get good marks in the examination. Find

the probability that this student is of category C. What values need to be developed in students of category C ?

SECTION – D

Question numbers 24 to 29 carry 6 marks each.

24. Let f: $\mathbb{R} - \left\{ -\frac{4}{3} \right\} \to \mathbb{R}$ be a function is $f(x) = \frac{4x}{3x+4}$. Show that, in $f : \left\{ -\frac{4}{3} \right\} \to$ Range of f, f is one-one and onto. Hence find f^{-1}. Range $f \to \mathbb{R} - \left\{ -\frac{4}{3} \right\}$.

OR Let $A = \mathbb{R} \times \mathbb{R}$ and * be the binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on A, if any.

25. Find matrix A, if $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

26. Find the intervals in which the function f given by
$f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.
OR Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α, is one-third that of the cone. Hence find the greatest volume of the cylinder.

27. Using integration find the area of the region $\{(x, y): y^2 \le 4x,\ 4x^2 + 4y^2 \le 9\}$.

28. Find the equation of plane containing the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-38}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$.

29. A fair coin is tossed 8 times, find the probability of
(i) exactly 5 heads (ii) at least six heads (iii) at most six heads
OR Three cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.