Question Paper 2016 DelhiSet-2
CBSE Class XII Mathematics

General Instruction:

- All question are compulsory.
- Please check that this Question Paper contains 26 Questions.
- Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying one mark each.
- Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.
- Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.
- Please write down the serial number of the Question before attempting it.

Section A

1. Matrix $A = \begin{vmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{vmatrix}$ is given to be symmetric, find value of a and b.

2. Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2:1.

3. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - j + 4\hat{k}$ represent the two sides AB and AC, respectively of a. Find the length of the median through A.

4. Find the vector equation of plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{i} + 6\hat{k}$.

5. Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

6. If A is a square matrix such that $A^2 = I$, then find the simplified value of

$$(A-I)^3 + (A+I)^3 - 7A$$

Section B

7. Show that the equation of normal at any point t on the curve x = 3 cost t – $\cos^3$t and

y = 3 sin t – $\sin^3$t is 4(y $\cos^3$t – $\sin^3$t) = 3 sin 4t.

8. Find $\int \dfrac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\, \theta.$

OR

Evaluate

$$\int_0^\pi e^{2x} \cdot \sin\left(\frac{\pi}{4} + x\right) dx$$

9. Find $\int \dfrac{\sqrt{x}}{\sqrt{a^3 - x^3}}\, dx.$

10. Evaluate $\int_{-1}^{2} \left| x^3 - x \right| dx.$

11. Find the particular solution of the differential equation

$$(1 - y^2)(1 + \log x)\, dx + 2xy\, dy = 0$$

Given that y = 0 when x = 1.

12. Find the general solution of the following differential equation:

$$\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

13. Show that the vector $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar if $\bar{a}, \bar{b}, \bar{b}+\bar{c}$ and $\bar{c}+\bar{a}$ are coplanar.

14. Find the vector and Cartesian equations of the line through the point (1, 2, -4) perpendicular to the two lines.

$$\bar{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right) \text{ and}$$

$$\bar{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}-8\hat{j}-5\hat{k}\right).$$

15. Three persons A, B and C apply for a job of manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

16. Prove that: $\tan^{-1}\dfrac{1}{5}+\tan^{-7}\dfrac{1}{7}+\tan^{-3}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}=\dfrac{\pi}{4}$

OR

Solve for x: $2\tan^{-1}(\cos x)=\tan^{-1}(2\cos ec\ x)$

17. The monthly incomes of Aryan and Babban are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value?

18. If x = a sin 2t(1 + cos 2t) and y = b cos 2t (1 – cos 2t), find the values of $\dfrac{dy}{dx}$ at $t=\dfrac{\pi}{4}$ and

$t=\dfrac{\pi}{3}.$

If y = X^X, prove that $\dfrac{d^2y}{dx^2} - \dfrac{1}{y}\left(\dfrac{dy}{dx}\right)^2 - \dfrac{y}{x} = 0.$

19. Find the values of p and q for which

$$f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \dfrac{x}{2} \\ P, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{q(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$$

Is continuous at $x = \dfrac{\pi}{2}$

Section C

20. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\dfrac{4r}{3}$. Also find maximum volume in terms of volume of the sphere.

Find the intervals in which f(x) = sin 3x – cos 3x, 0 < x < π, is strictly increasing or strictly decreasing.

21. Using integration find the area of the region $\{(x, y): x^2 + y^2 \le 2x, y^2 \ge ax, x, y \ge 0\}$.

22. Find the coordinate of the point P where the line through A (3, -4, -5) and B(2, -3, 1) crosses the plane passing through three points L(2, 2, 1), M(3, 0, 1) and N(4, -1, 0). Also, find the ratio in which P divides the line segment AB.

23. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

24. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs 7 profit and b at a profit of Rs 4. Find the production level per day for maximum profit graphically.

25. Let $f: N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(4)$ and $f^{-1}(163)$.

26. Prove that $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by (x + y + z) and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix $A = \begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use

it to solve the following system of linear equations:

$$8x + 7y + 3z = 19$$
$$2x + y + z = 5$$
$$x + 2y + 2 = 7$$