Question Paper 2017 Outside Delhi Set-1
CBSE Class XII Mathematics

General Instruction:

- All questions are compulsory.
- The question paper consists of 29 questions divided into four section A, B, C and D. Sections A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section A

1. If for any 2 × 2 square matrix A, A(adj A)= $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, **then write the value of |A|.**

Sol. $A(adj\ A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

By using property

A(adj A) = |A| I_a

$$\Rightarrow |A| I_n = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow |A| I_n = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow |A| = 8$$

2. Determine the value of 'k' for which the following is continuous at x = 3:

$$f(x)=\begin{cases}\dfrac{(x+3)^2-36}{x-3} & ,x\neq 3\\ k & ,x=3\end{cases}$$

Sol. $\lim\limits_{x\to 3} f(x)=\lim\limits_{x\to 3}\dfrac{(x+3)^2-36}{x-3}$

$$=\lim_{x\to 3}\frac{(x+3-6)(x+3+6)}{(x-3)}$$

= 12

given that if f(x) is continuous at x = 3

$$\therefore \lim_{x\to 3} f(x)=f(3)$$

⇒ k = 12

3. Find: $\displaystyle\int \frac{\sin^2 x-\cos^2 x}{\sin x\cos x}dx$

Sol. $\displaystyle\int \frac{\sin^2 x-\cos^2 x}{\sin x\cdot\cos x}dx$

$$=2\int\frac{-\cos 2x}{\sin 2x}dx$$

$$=-2\int\cot 2x\,dx$$

$$=\frac{-2\log|\sin 2x|}{2}+C$$

$$= -\log |\sin 2x| + C$$

4. Find the distance between the planes 2x – y + 2z = 5 and 5x – 2·5y + 5z = 20.

Sol. $2x - y + 2z = 5$ ..... (1)

$5x - 2\cdot5y + 5z = 20$ ..... (2)

or $2x - y + 2z = 8$

Distance between plane (1) & (2)

$$= \left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right| = \left|\frac{3}{\sqrt{9}}\right| = 1$$

Section B

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Sol. Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ be a skew symmetric matrix of order 3

$$\therefore |A| = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$|A| = -a(0 + bc) + b(ac - 0)$

$= -abc + abc = 0$ Proved

6. Find the value of c in Rolle's theorem for the function f(x) = x^3 – 3x in $[-\sqrt{3}, 0]$.

Sol. $f(x) = x^3 - 3x$

(i) f(x) being a polynomial is continuous on $[-\sqrt{3}, 0]$

(ii) $f(-\sqrt{3})$ = f (0) = 0

(iii) f'(x) = $3x^2 - 3$ and this exist uniquely on $[-\sqrt{3}, 0]$

∴ f(x) is derivable on $[-\sqrt{3}, 0]$

∴ f(x) satisfies all condition of Rolle's theorem

∴ There exist atleast one c ∈ (– 3 , 0) where f'(c) = 0

⇒ $3c^2 - 3 = 0$

⇒ c = ± 1 ⇒ c = – 1

7. The volume of a cube is increasing at the rate of 9 cm3/s. How fast is its surface area increasing when the length of an edge is 10 cm?

Sol. Assumed volume of cube = V

Given that, $\frac{dV}{dt} = 9cm^3 / \sec$

$\frac{dA}{dt} = ?$

l = 10 cm

$\frac{dV}{dt} = \frac{d}{dt}(l)^3 = 9 \Rightarrow 3l^2 \frac{dl}{dt} = 9$

$\frac{dl}{dt} = \frac{3}{l^2}$ .... (1)

Now $\frac{dA}{dt} = \frac{d}{dt}(6\,l^2) = 12l\frac{dl}{dt} = 12l \times \frac{3}{l^2}$ (from (1))

$$= \frac{36}{l} = \frac{36}{10} = 3.6\ cm^2 / \text{sec}$$

8. Show that the function f(x) = x3 – 3x2 + 6x – 100 is increasing on R.

Sol. f(x) = $x^3 - 3x^2 + 6x - 100$

f'(x) = $3x^2 - 6x + 6$

f'(x) = $3(x^2 - 2x + 2)$

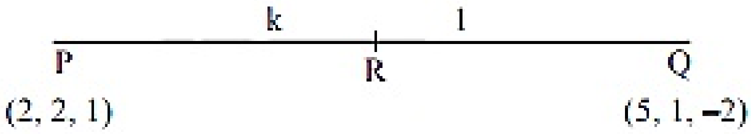
f'(x) = $3[(x - 1)^2 + 1]$

f'(x) > 0 for all $x \in R$

So, f(x) is increasing on R.

9. The x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, –2) is 4. Find its z-coordinate.

Sol.



Let R divides PQ in the ratio k:1

$$R\left(\frac{5k+2}{k+1}, \frac{k+2}{k+1}, \frac{-2k+1}{k+1}\right)$$

Given x co-ordinate of R = 4

$$\therefore \frac{5k+2}{k+1} = 4$$

$\Rightarrow k = 2$

$\therefore$ z co-ordinate$=\dfrac{-2(2)+1}{2+1}=-1$

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

Sol. A = {2, 4, 6} $\quad P(A)=\dfrac{3}{6}=\dfrac{1}{2}$

B = {1, 2, 3} $\quad P(B)=\dfrac{3}{6}=\dfrac{1}{2}$

$A\cap B=\{2\} \quad P(B)=\dfrac{3}{6}=\dfrac{1}{2}$

Here, $P(A)P(B)=\dfrac{1}{2}\times\dfrac{1}{2}=\dfrac{1}{4}$

Since, $P(A\cap B)\neq P(A)P(B)$.

11. Two tailors, A and B, earnj & 300 and j & 400 per day respectively. A can stitch 6 shirts and 4 pairs of trouserswhile B can stitch 10 shirts nd 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

Sol.

	Tailor A	Tailor B	Minimum Total No.
No. of shirts	6	10	60
No. of trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and tailor B works for x days and y days respectively

$\therefore$ x ≥ 0, y ≥ 0

minimum number of shirts = 60

$\therefore$ 6x + 10y ≥ 60

3x + 5y ≥ 30

minimum no. of trousers = 32

$\therefore$ 4x + 4y ≥ 32

⇒ x + y ≥ 8

Let z be the total labour cost

$\therefore$ z = 300 x + 400 y

$\therefore$ The given L.P. Problem reduces to : z = 300x + 400y

x ≥ 0, y ≥ 0, 3x + 5y ≥ 30 and x + y ≥ 8

12. Find: $\int \frac{dx}{5-8x-x^2}$

Sol. $\int \frac{dx}{5-8x-x^2}$

$$= -\int \frac{dx}{\{(x+4)^2 - 21\}}$$

$$= \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$$

$$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21}+(x+4)}{\sqrt{21}-(x+4)} \right| + C$$

Section C

13. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, **then find the value of x.**

Sol. $\tan^{-1}\left[\dfrac{\dfrac{x-3}{x-4}+\dfrac{x+3}{x+4}}{1-\left(\dfrac{x^2-9}{x^2-16}\right)}\right]=\dfrac{\pi}{4}$

$$\frac{(x+4)(x-3)+(x+3)(x-4)}{(x^2-16)-(x^2-9)}=1$$

$2x^2 - 24 = -7$

$2x^2 = -7 + 24$

$$x^2=\frac{17}{2}$$

$$x=\pm\sqrt{\frac{17}{2}}$$

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}=(a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix}A=\begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

Sol. Use $R_1 = R_1 - R_2$; $R_2 = R_2 - R_3$; $R_3 = R_3$

L.H.S.

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a-1)(a+1) & (a-1) & 0 \\ 2(a-1) & (a-1) & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking common $(a - 1)^2$

$$= (a-1)^2 \begin{vmatrix} (a+1) & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$= (a - 1)^2[(a + 1)(1 - 0) - 1(2 - 0)]$

$= (a - 1)^2[(a + 1)-2]$

$= (a - 1)^3$

= R.H.S.

OR

Let matrix A is

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 2a-c & 2b-d \\ a & b \\ -3a+4c & -3b+4d \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Comparing both the sides

2a – c = -1,

2b – d = -8

And a = 1, b = -2

After solving we get

C = 3, d = -4

So, $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

15. if $x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

OR

If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

Sol. We have $x^y + y^x = a^b$.

Differentiating w.r.t. x, we get $\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0$. ...... (1)

Let u = x^y $\therefore \log u = y \log x$

$$\Rightarrow \quad \frac{1}{u}\frac{du}{dx} = y.\frac{1}{x} + \log x.\frac{dy}{dx}; \Rightarrow \quad \frac{du}{dx} = u\left(\frac{y}{x} + \log x \frac{dy}{dx}\right)$$

Or $\frac{d}{dx}(x^y) = x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$ ..... (2)

Let v = y^x $\therefore$ log v = x log y

$$\Rightarrow \frac{1}{v}\frac{dv}{dx} = x.\frac{1}{y}\frac{dy}{dx} + \log y.1; \Rightarrow \frac{dv}{dx} = v\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$$

Or $\frac{d}{dx}(y^x) = y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right)$ ...... (3)

Using (2) and (3) in (1),

We get $x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right) + y^x\left(\frac{x}{y}\frac{dy}{dx} + \log y\right) = 0.$ ...... (4)

$$\Rightarrow (x^y \log x + xy^{x-1})\frac{dy}{dx} = -(y^x \log y + yx^{y-1}) \text{ or } \frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}$$

OR

Let $e^y (x + 1) = 1$

$$e^y(1) + (x+1)e^y\frac{dy}{dx} = 0$$

$\Rightarrow (x+1)\frac{dy}{dx} + 1 = 0$ .... (1)

Again differentiating w.r.t. x

$$\therefore \quad (x+1)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right).1 = 0$$

$$\frac{d^2y}{dx^2} = -\frac{\frac{dy}{dx}}{(x+1)}$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} \cdot \frac{dy}{dx}$$ [equation (1)]

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

16. Find: $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$

Sol. $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$

$$= \int \frac{\cos\theta \, d\theta}{(4+\sin^2\theta)(5-4(1-\sin^2\theta))}$$

$$\int \frac{\cos\theta \, d\theta}{(\sin^2\theta+4)(4\sin^2\theta+1)}$$

Put $\sin\theta = t$

$\text{Cos}\,\theta \, d\theta = dt$

$$\therefore I = \int \frac{1}{(4+t^2)(1+4t^2)} dt$$

Consider

$$\frac{1}{(4+t^2)(1+4t^2)} = \frac{At+B}{4+t^2} + \frac{Ct+D}{1+4t^2}$$

$1 = (At + B)(1 + 4t^2) + (Ct + D)(4 + t^2)$

= At + B + 4At^3 + 4Bt^2 + 4C + Ct^3 + 4D + Dt^2

= (4A + C)t^3 + (4B + D)t^2 + (A + 4C)t + (B + 4D)

4A + C = 0 ⇒ C = -4 A

4B + D = 0 ⇒ D = -4 B

A + 4C = 0 ⇒ A = -4 C

B + 4D = 1

By solving we get A = 0, $B = -\frac{1}{15}, C = 0, D = \frac{4}{15}$

$$\therefore \frac{1}{(4+t^2)(1+4t^2)} = \frac{-1/15}{4+t^2} + \frac{4/15}{1+4t^2}$$

$$\therefore I = -\frac{I}{15}\int \frac{1}{4+t^2}dt + \frac{4}{15} \times \frac{1}{4}\int \frac{1}{\frac{1}{4}+t^2}dt$$

$$= -\frac{1}{15} \times \frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right) + \frac{1}{15} \times \frac{1}{1}\tan^{-1}\left(\frac{t}{1/2}\right) + C$$

$$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right) + \frac{2}{15}\tan^{-1}(2t) + C$$

$$= \frac{2}{15}\tan^{-1}(2\sin\theta) - \frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right) + C$$

17. Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}$

OR

Evaluate: $\int_{1}^{4}\{|x-1|x-2|+|x-4|\}dx$

Sol. $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$ ...... (1)

$$I=\int_0^{\pi}\frac{(\pi-x)(-\tan x)}{-\sec x-\tan x}dx$$

$$I=\int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}dx \quad ...... (2)$$

Adding (1) & (2)

$$2I=\int_0^{\pi}\frac{\pi\tan x}{\sec x+\tan x}dx$$

$$\Rightarrow 2I=2\pi\int_0^{\pi/2}\frac{\tan x}{\sec x+\tan x}dx$$

$$\left\{\because \int_0^{2a}f(x)dx=2\int_0^{a}f(x)dx \text{ whenever } f(2a-x)=f(x)\right\}$$

$$I=\pi\int_0^{\pi/2}\frac{\tan x}{\sec x+\tan x}dx$$

$$I=\pi\int_0^{\pi/2}\frac{\tan x(\sec x-\tan x)}{\sec^2 x+\tan^2 x}dx$$

$$I=\pi\int_0^{\pi/2}(\sec x\tan x-\tan^2 x)dx$$

$$=\pi\int_0^{\pi/2}(\sec x\tan x-\sec^2 x+1)dx$$

$$I=\pi\left[\sec x-\tan x+x\right]_0^{\pi/2}$$

$$= \pi\left[\lim_{x\to\frac{\pi}{2}^-}(\sec x - \tan x) + \frac{\pi}{2} - \sec 0\right]$$

$$= \pi \lim_{x\to\frac{\pi}{2}^-} \frac{1-\sin x}{\cos x} + \frac{\pi^2}{2} - \pi$$

$$= \pi \lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1-\sin^2 x}{\cos x(1+\sin x)} + \frac{\pi^2}{2} - \pi$$

$$= \frac{\pi^2}{2} - \pi$$

OR

Let f (x) = |x – 1| + |x – 2| + |x – 4|

We have three critical points x = 1, 2, 4

(i) when x < 1

(ii) when 1≤ x < 2

(iii) when 2 ≤ x < 4

(iv) when x ≥ 4

f(x) = -(x – 1) – (x – 2) – (x – 4) if x < 1

= (x – 1) – (x – 2) – (x – 4) if 1 ≤ x < 2

= (x – 1) + (x – 2) – (x – 4) if 2 ≤ x < 4

= (x – 1) + (x – 2) + (x – 4) if x ≥ 4

∴ f(x) = -3x + 7 if x < 1

= -x + 5 if $1 \leq x < 2$

= x + 1 if $2 \leq x < 4$

= 3x – 7 if $x \geq 4$

$$\therefore I = \int_1^4 f(x)dx$$

$$\therefore I = \int_1^2 f(x)dx + \int_2^4 f(x)dx$$

$$\therefore I = \int_1^2 (-x+5)dx + \int_2^4 (x+1)dx$$

$$= \left[-\frac{x^2}{2} + 5x\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4$$

$$= \left(-\frac{4}{2} + 10\right) - \left(-\frac{1}{2} + 5\right) + \left(\frac{16}{2} + 4\right) - \left(\frac{4}{2} + 2\right)$$

$$= 8 - \frac{9}{2} + 12 - 4 = \frac{23}{2}$$

18. Solve the differential equation ($\tan^{-1}$ x – y)dx = (1 + x^2) dy.

Sol. We have

$$\frac{dy}{dx} = \frac{\tan^{-1} x - y}{1+x^2}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1} x}{1+x^2}$$

$$I.F = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1} x}$$

$$y.e^{\tan^{-1}x} = \int \frac{\tan^{-1}x}{1+x^2} \times e^{\tan^{-1}x} dx$$

Put t = $\tan^{-1}x$

$$dt = \frac{1.dx}{1+x^2}$$

$$= \int t.e^t \, dt$$

$$= t.e^t - \int 1.e^t dt$$

$$y.e^{\tan^{-1}x} = t\,e^t - e^t + c$$

$$y.e^{\tan^{-1}x} = (\tan^{-1}x - 1)e^{\tan^{-1}x} + c$$

$$y = \tan^{-1}x - 1 + ce^{\tan^{-1}x}$$

19. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ **and** $3\hat{i} - 4\hat{j} - 4\hat{k}$ **respectively, are the vertices of a right-angled triangle, Hence find the area of the triangle.**

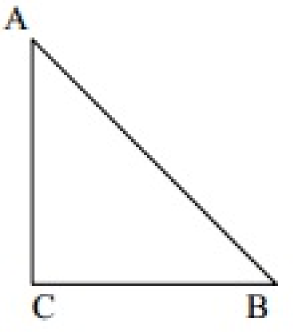
Sol. $\overline{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$

$$\overline{BC} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\overline{CA} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\overline{BC} \cdot \overline{CA} = 0$$

$$\overline{BC} \perp \overline{CA}$$



∴ ΔABC is a right angled triangle

$$\Delta = \frac{1}{2}|\overline{BC}||\overline{AC}|$$

$$\Delta = \frac{1}{2}\sqrt{4+1+1}\sqrt{1+9+25}$$

$$= \frac{1}{2}\sqrt{6}\sqrt{35}$$

$$= \frac{1}{2}\sqrt{210}$$

20. Find the value λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, 2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k},$ are coplanar.

Sol. We have

P.V. of A $= 3\hat{i}+6\hat{j}+9\hat{k}$

P.V. of B $= \hat{i}+2\hat{j}+3\hat{k}$

$$\overline{AB} = -2\hat{i}-4\hat{j}-6\hat{k}$$

$$\overline{AC} = -\hat{i}-3\hat{j}-8\hat{k}$$

$$\overline{AD} = \hat{i}+(\lambda-9)\hat{k}$$

Now $\overline{AB}(\overline{AC}\times\overline{AD}) \Rightarrow \begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & (\lambda-9) \end{vmatrix}$ =0

$$\Rightarrow -2(-3\lambda+27)+4(-\lambda+9+8)-6(0+3)=0$$

$$\Rightarrow 6\lambda-54-4\lambda+68-18=0$$

λ = 2

$\because \overline{AB}, \overline{AC}, \overline{AD}$ are coplanar and so the points A, B, C and D are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Sol. X denote sum of the number so, X can be 4, 6, 8, 10, 12

X	Number on card	P(x)	X p(x)	X^2 P(x)
4	(1, 3)	$\frac{1}{4}\times\frac{1}{3}\times 2=\frac{1}{6}$	2/3	8/3
6	(1, 6)	$\frac{1}{4}\times\frac{1}{3}\times 2=\frac{1}{6}$	1	6
8	(3, 5) or (1, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2+\frac{1}{4}\times\frac{1}{3}\times 2=\frac{1}{3}$	8/3	64/3
10	(3, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2=\frac{1}{6}$	5/3	50/3
12	(5, 7)	$\frac{1}{4}\times\frac{1}{3}\times 2=\frac{1}{6}$	2	24

$$Mean = \sum XP(x) = 8$$

$$Variance = \sum X^2 P(x) - \left(\sum X\,P(x)\right)^2 = \frac{212}{3} - 64 = \frac{20}{3}$$

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

Sol. Let E_1 be students having 100% attendance

E_2 be students having irregular attendance

E be students having A grade

$$P(E_1) = \frac{30}{100} \qquad P(E_2) = \frac{70}{100}$$

$$P\left(\frac{E}{E_1}\right) = \frac{70}{100} \times \frac{30}{100} = 21\%$$

$$P\left(\frac{E}{E_2}\right) = \frac{10}{100} \times \frac{70}{100} = 7\%$$

By Baye's theorem,

So, $$P\left(\frac{E_1}{E}\right) = \frac{P(E_1) P\left(\frac{E}{E_1}\right)}{P(E_1)\,P\left(\frac{E}{E_1}\right) + P(E_2)\,P\left(\frac{E}{E_2}\right)} = \frac{\frac{30}{100} \times \frac{21}{100}}{\frac{30}{100} \times \frac{21}{100} + \frac{70}{100} \times \frac{7}{100}} = \frac{63}{63+49}$$

$= \frac{63}{112}$

23. Maximize Z = x + 2y Subject to the constraints

$X + 2y \geq 100$

$2x - y \leq 0$

$2x + y \leq 200$

$X, y \geq 0$

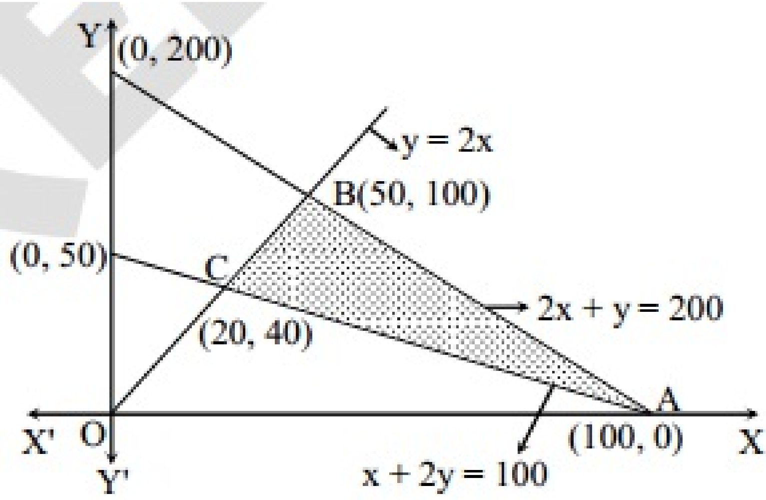
Solve the above LP graphically.

Sol. $x + 2y = 100$

$2x - y = 0$ .... (1)

$2x + y = 200$ ...... (2)

$x = 0, y = 0$ ...... (3)



Corner points are A (100, 0), B(50, 100), C(20, 40)

Corner points	Z = x + 2y	
A(100, 0)	100	← minimum

B(50, 100)	250	← Maximum
C(20, 40)	100	← Minimum

Maximum at point B and maximum value 250

Section D

24. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ **and use it to solve the system of equations x – y + z = 4, x – 2y – 2z = 9, 2x + y +3z = 1.**

Sol. Product of the matrices

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$$

Hence $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$

Now, given system of equation can be written in matrix form, as follows

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$x = \frac{24}{8},\ y = \frac{-16}{8},\ z = \frac{-8}{8}$$

$$x = 3,\ y = -2,\ z = -1$$

25. Consider $f : R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find f–1(0) and x such that f–1(x) = 2.

OR

Let A = Q × Q and let * be a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) $\in A$. Determine, whether * is commutative and associative.Then, with respect to * on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

Sol. $f(x)=\dfrac{4x+3}{3x+4}, x\in R-\left\{-\dfrac{4}{3}\right\}$

<u>f is one – one</u>→

Let x^1, x^2 $\in R-\left\{-\dfrac{4}{3}\right\}$ and $f(x_1) = f(x_2)$

$$\Rightarrow \qquad \frac{4x_1+3}{3x_1+4}=\frac{4x_2+3}{3x_2+4}$$

$$\Rightarrow 12x_1x_2+16x_1+9x_2+12x_1x_2+9x_1+16x_2+12$$

∴ f is one – one

<u>f is onto</u>→

Let k $\in R-\left\{\dfrac{4}{3}\right\}$ be any number

f(x) = k $\Rightarrow \qquad \dfrac{4x+3}{3x+4}$

$$\Rightarrow \qquad 4x+3=3kx+4k$$

$$\Rightarrow \qquad x=\frac{4k-3}{4-3k}$$

Also $\dfrac{4k-3}{4-3k}=-\dfrac{4}{3}$

Implies – 9 = -16 (which is impossible)

$\therefore f\left(\dfrac{4k-3}{4-3k}\right)=k$ i.e. f is onto

∴ The function f is invertible i.e. f^{-1} exist inverse of f

Let f^{-1} (x) = k

f(k) = x

$$\Rightarrow \frac{4k+3}{3k+4} = x$$

$$\Rightarrow k = \frac{4x-3}{4-3x}$$

$$\therefore \qquad f^{-1}(x) = \frac{4x+3}{4-3x}, x \in R - \left\{-\frac{4}{3}\right\}$$

$$f^{-1}(0) = -\frac{3}{4}$$

and when

$$f^{-1}(x) = 2$$

$$\Rightarrow \frac{4x-3}{4-3x} = 2$$

$$\Rightarrow 4x-3 = 8-6x$$

$$\Rightarrow 10x = 11$$

$$\Rightarrow x = \frac{11}{10}$$

OR

(i) Let (e, f) be the identify element for *

$\therefore for\ (a, b) \in Q \times Q,$ we have

(a, b) * (e, f) = (a, b) = (e, f) * (a, b)

⇒ (ae, af + b) = (a, b) = (ea, eb + f)

⇒ ae = a, af + b = b, a = ea, b = eb + f

⇒ e = 1, af = 0, e = 1, b = (1) b + f

(∵ a need not be '0')

⇒ e = 1, f = 0, e = 1, f = 0

∴ (e, f) = (1, 0) $\in Q \times Q$

∴ (1, 0) is the identify element of A

(ii) Let (a, b) $\in Q \times Q$

Let (c, d) $\in Q \times Q$

such that

(a, b) * (c, d) = (1, 0) = (c, d) * (a, b)

⇒ (ac, ad + b) = (1, 0) = (ca, cb + d)

⇒ ac = 1, ad + b = 0, ca = 1, cb + d = 0

$$\Rightarrow c = \frac{1}{a}, d = -\frac{b}{a}, \left(\frac{1}{a}\right)b + d = 0 (a \neq 0)$$

$$\therefore (c, d) = \left(\frac{1}{a}, \frac{-b}{a}\right)(a \neq 0)$$

$$\therefore for\ a \neq 0, (a, b)^{-1} = \left(\frac{1}{a}, \frac{-b}{a}\right)$$

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Sol. If each side of square base is x and height is h then volume

$$V = x^2 h \Rightarrow h = \frac{V}{x^2}$$

S is surface area then

S = 4hx + 2x^2 $= 4\left(\frac{V}{x^2}\right)x + 2x^2$

$$\Rightarrow S = \frac{4V}{x} + 2x^2$$

Diff. w.r.t. to x

$$\frac{dS}{dx} = \frac{4V}{x^2} + 4x \text{ and } \frac{d^2S}{dx^2} = +\frac{8V}{x^3} + 4$$

Now $\frac{dS}{dx} = 0 \Rightarrow 4x = \frac{4V}{x^2}$

$$\Rightarrow x^3 = V \Rightarrow x = V^{1/3}$$

at $x = V^{1/3}, \frac{d^2S}{dx^2} > 0$

$$\Rightarrow S \text{ is minimum when } x = V^{1/3}$$

and $h = \frac{V}{x^2} = \frac{V}{V^{2/3}} = V^{1/3} \Rightarrow x = h$

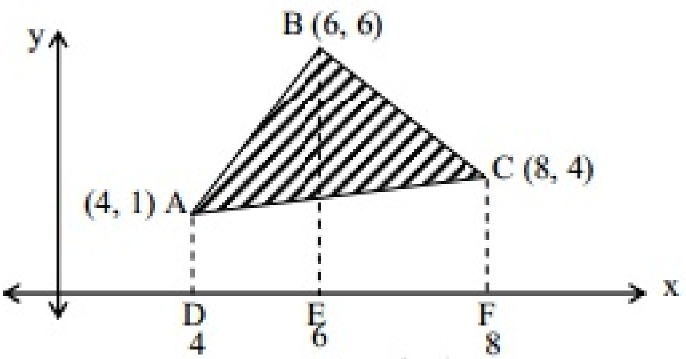
⇒ x = h means it is a cube

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A (4, 1), B (6, 6) and C (8, 4).

OR

Find the area enclosed between the parabola 4y = 3x2 and the straight line 3x – 2y + 12 = 0.

Sol.



Equation of AB is y – 1 $= \frac{6-1}{6-4}(x-4)$

$$\Rightarrow 2y-2=5x-20$$

$$\Rightarrow y=\frac{5x}{2}-9$$

Equation of BC is

$$\Rightarrow y-6=\frac{4-6}{8-6}(x-6)$$

$$\Rightarrow y=-x+12$$

Equation of AC is

$$\Rightarrow y-1=\frac{4-1}{8-4}(x-4)$$

$$\Rightarrow 4y-4=3x-12$$

$$\Rightarrow y=\frac{3x}{4}-2$$

Area of ΔABC = area ABED + area BEFC – area ADFC

$$=\int_4^6\left(\frac{5x}{2}-9\right)dx+\int_6^8(-x+12)\,dx-\int_4^6\left(\frac{3x}{4}-2\right)dx$$

$$=\left|\left(\frac{5x^2}{4}-9x\right)_4^6\right|+\left|\left(\frac{-x^2}{2}+12x\right)_6^8\right|-\left|\left(\frac{3x^2}{8}-2x\right)_4^6\right|=7\,sq\ units$$

OR

Parabola $4y = 3x^2$ ...... (1)

line $3x - 2y + 12 = 0$ ...... (2)

from (2) $y = \dfrac{3x+12}{2}$

putting this value of y is (1) we get

$6x + 24 = 3x^2$

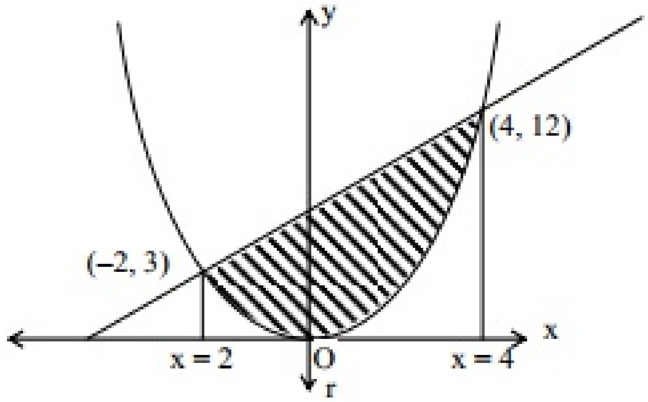
$\Rightarrow x = 4, -2$

when x = 4 then y = 12

x = – 2 then y = 3

Required area

$$= \int_{-2}^{4} (y \text{ of line})dx - \int_{-2}^{4} (y \text{ of parabola})dx$$



$$= \int_{-2}^{4} \left(\frac{3x+12}{2} - \frac{3x^2}{4} \right) dx$$

$$= \frac{3}{4} \int_{-2}^{4} (8 + 2x - x^2) dx$$

$$= \frac{3}{4} \left[8x + x^2 - \frac{x^3}{3} \right]_{-2}^{4} = 27 \, sq.units$$

28. Find the particular solution of the differential equation $(x-y)\dfrac{dy}{dx} = (x+2y)$,

given that y = 0 when x = 1.

Sol. $(x-y)\dfrac{dy}{dx}=(x+2y)$

$$\frac{dy}{dx}=\frac{x+2y}{x-y}$$

Let y = Vx

$$\frac{dy}{dx}=V+x\frac{dV}{dx}$$

$$\Rightarrow V+x\frac{dV}{dx}=\frac{x+2(Vx)}{x-Vx}$$

$$\Rightarrow V+x\frac{dV}{dx}=\frac{1+2V}{1-V}$$

$$\Rightarrow x\frac{dV}{dx}=\frac{1+2V-V+V^2}{1-V}$$

$$\Rightarrow \int\frac{1-V}{1+V+V^2}dV=\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\int\left\{\frac{2V+1-3}{1+V+V^2}\right\}dV=\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\left[\int\frac{2V+1}{1+V+V^2}dV-3\int\frac{dV}{1+V+V^2}\right]=\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{2}\log|1+V+V^2|+\frac{3}{2}\int\frac{dV}{\left(V+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\log|x|+C$$

$$\Rightarrow -\frac{1}{2}\log|1+V+V^2|+\frac{3}{2}\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{V+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=\log|x|+C$$

$$\Rightarrow -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right|+\sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right)=\log|x|+C$$

We have y = 0 when x = 1

$$\Rightarrow 0+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=0+C$$

$$\Rightarrow C=\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=0+C$$

∴ Solution

$$\Rightarrow -\frac{1}{2}\log\left|1+\frac{y}{x}+\frac{y^2}{x^2}\right|+\sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right)=\log|x|+\sqrt{3}\tan^{-1}\frac{1}{\sqrt{3}}$$

29. Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3).

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is 2

Sol. Equation of line passing through

(3, -4, -5) and (2, -3, 1)

$$\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6} \quad \ldots\ldots (1)$$

Equation of plane passing through

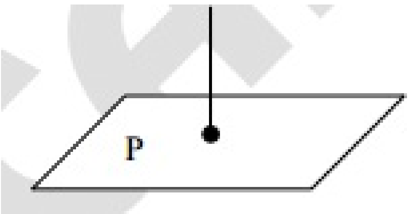
(1, 2, 3) (4, 2, -3) and (0, 4, 3)

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$$

⇒ (x – 1) (12) – (y – 2) (-6) + (z – 3) (6) = 0

⇒ 2x + y + z – 7 = 0 .... (2)

Let any point on line (1)



is P (-k + 3, k – 4, 6k – 5)

it lies on plane

∴ 2(-k + 3) + k – 4 + 6k – 5 – 7 = 0

5k = 10

⇒ k = 2

∴ P (1, -2, 7)

OR

Let the equation of plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \quad ..... (1)$$

It cut the co-ordinate axes at A, B and C

∴A (a, 0, 0), B (0, b, 0), C (0, 0, c)

Let the centroid of ΔABC be (x, y, z)

$$\therefore \left(x = \frac{a}{3}, y = \frac{b}{3}, z = \frac{c}{3} \right) \qquad \text{...... (2)}$$

Given that distance of plane (1) from origin is 3p

$$\therefore \left| \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \right| = 3p$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2}$$

From (2)

$$\Rightarrow \frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2}$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \text{ Proved.}$$