Question Paper 2016 Foreign Set-1

CBSE Class XII Mathematics

General Instruction:

- All question are compulsory.
- Please check that this Question Paper contains 26 Questions.
- Questions 1 to 6 in Section-A are Very Short Answer Type Questions carrying one mark each.
- Questions 7 to 19 in Section-B are Long Answer I Type Questions carrying 4 marks each.
- Questions 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.
- Please write down the serial number of the Question before attempting it.

Section A

1. If $\begin{pmatrix} 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = A,$ **then write the order of matrix A.**

Sol. 1 × 1

2. If $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix} = 8,$ **write the value of x.**

Sol. Expanding we get

$$x^3 = -8 \Rightarrow x = -2$$

3. If $A = \begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix}$ **is written as A = P + Q, where P is a symmetric matrix and Q is skew symmetric matrix, then write the matrix P.**

Sol. $P = \frac{1}{2}(A + A') \therefore P = \begin{bmatrix} 3 & 6 \\ 6 & 9 \end{bmatrix}$

4. If $\bar{a}, \bar{b}, \bar{c}$ **are unit vectors such that** $\bar{a} + \bar{b} + \bar{c} = \bar{0}$, **then write the value of** $\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$.

Sol. $(\bar{a} + \bar{b} + \bar{c}) \cdot (\bar{a} + \bar{b} + \bar{c}) = 0$

$\Rightarrow |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}) = 0$

$\Rightarrow \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = -\frac{3}{2}$

5. If $\left|\bar{a} \times \bar{b}\right|^2 + \left|\bar{a} \cdot \bar{b}\right|^2 = 400$ **and** $\left|\bar{a}\right| = 5$, **then write the value of** $\left|\bar{b}\right|$.

Sol. $a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta = 400$

$\Rightarrow \quad |\bar{b}| = 4$

6. Write the equation of a plane which is at a distance of $5\sqrt{3}$ **units from origin and the normal to which is equally inclined to coordinate axes.**

Sol. $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 5\sqrt{3}$ or x + y + z = 15 $\left[\frac{1}{2} \textit{ mark for dc's of normal}\right]$

Section B

7. Prove that:

$$\cot^{-1}\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}=\frac{x}{2}, 0<x<\frac{\pi}{2}$$

OR

Solve for x:

$$\tan^{-1}\left(\frac{x-2}{x-1}\right)+\tan^{-1}\left(\frac{x+2}{x+1}\right)=\frac{\pi}{4}$$

Sol. LHS=$\cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$

$$=\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$=\frac{x}{2}=RHS$$

OR

$$\tan^{-1}\left[\frac{\frac{x-2}{x-1}+\frac{x+2}{x+1}}{1-\frac{x-2}{x-1}\cdot\frac{x+2}{x+1}}\right]=\frac{\pi}{4}$$

$$\Rightarrow \quad \frac{2x^2-4}{3}=\tan\frac{\pi}{4}$$

$$\Rightarrow \quad x=\pm\sqrt{\frac{7}{2}}$$

8. A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is Rs 9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is Rs 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society?

Sol. Let each poor child pay Rs x per month and each rich child pay Rs y per month.

$\therefore$ 20x + 5y = 9000

5x + 25y = 26000

In matrix form,

$$\begin{bmatrix} 20 & 5 \\ 5 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1} B$

$$A^{-1} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{475}\begin{bmatrix} 25 & -5 \\ -5 & 20 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} = \begin{bmatrix} 200 \\ 1000 \end{bmatrix}$$

$\Rightarrow$ x = 200, y = 1000

Value: Compassion or any relevant value

9. Find the value of a and b, if the function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & x \le 1 \\ bx + 2, & x > 1 \end{cases}$$

Sol. $f_1' = 2x + 3 = 5$

$f'_{1+} = b$

$f'_{1-} = f'_{1+} \Rightarrow \boxed{b = 5}$

$\lim_{x \to 1^-} f(x) = f(1) = \lim_{x \to 1^+} f(x)$

$\Rightarrow 4 + a = b + 2$

$\Rightarrow \boxed{a = 3}$

10. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ **w.r.t.** $\sin^{-1}\frac{2x}{1+x^2}$, **if** $x \in (-1, 1)$

OR

If x = sin t and y = sin pt, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^y y = 0.$

Sol. Let $u = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$\therefore \quad u = \tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$\Rightarrow \quad \frac{du}{dx} = \frac{1}{2(1+x^2)}$

$$v = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$= 2\tan^{-1} x$$

$$\Rightarrow \qquad \frac{dv}{dx} = \frac{2}{1+x^2}$$

$$\therefore \qquad \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{1}{4}$$

OR

$$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$$

$$y = \sin pt \Rightarrow \frac{dy}{dt} = p\cos pt$$

$$\frac{dy}{dx} = \frac{P\cos pt}{\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{\cos t(-p^2\sin pt) - p\cos pt(-\sin t)}{\cos^2 t}\cdot\frac{dt}{dx}$$

$$= \frac{-p^2\sin pt\cos t + p\cos pt\sin t}{\cos^3 t}$$

Now $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$ $\left[\text{Substituting value of } y, \frac{dy}{dx} \,\&\, \frac{d^2y}{dx^2}\right]$

11. Find the angle of intersection of the curves y^2 = 4ax and x^2 = 4by.

Sol. Eqn of given curves

Their point of intersections are (0, 0) and $\left(4a^{1/3}b^{2/3}, 4a^{2/3}b^{1/3}\right)$

$$y^2 = 4ax \Rightarrow \frac{dy}{dx} = \frac{2a}{y},\ \text{slope} = \frac{a^{1/3}}{2b^{1/3}} \quad \text{.... (i)}$$

$$x^2 = 4by \Rightarrow \frac{dy}{dx} = \frac{x}{2b},\ \text{slope} = \frac{2a^{1/3}}{b^{1/3}} \quad \text{.....(ii)}$$

At (0, 0), angle between two curves is $90°$

Or

Acute angle θ between (i) and (ii) is

$$\theta = \tan^{-1}\left\{\frac{3}{2}\left(\frac{a^{1/3}b^{1/3}}{a^{2/3}+b^{2/3}}\right)\right\}$$

12. Evaluate: $\int_0^{\pi} \frac{x}{1+\sin\alpha \sin x}dx$

Sol. $I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin\alpha \sin x}dx$

$$2I = \pi\int_0^{\pi} \frac{dx}{1+\sin\alpha \sin x}$$

$$= 2\pi\int_0^{\pi/2} \frac{dx}{1+\sin\alpha \sin x}$$

$$\Rightarrow \qquad I = \pi\int_0^{\pi/2} \frac{dx}{1+\sin g a\dfrac{2\tan\dfrac{x}{2}}{1+\tan^2\dfrac{x}{2}}}$$

$$I = 2\pi\int_0^{1} \frac{2dt}{1+t^2+2t\sin\alpha} \quad \text{Put } \tan\frac{x}{2} = t$$

$$\Rightarrow I = 2\pi\int_0^1 \frac{dt}{(t+\sin\alpha)^2+\cos^2\alpha}$$

$$= \frac{2\pi}{\cos\alpha}\left[\tan^{-1}\left(\frac{t+\sin\alpha}{\cos\alpha}\right)\right]_0^1$$

$$\Rightarrow I = \frac{\pi}{\cos\alpha}\left(\frac{\pi}{2}-\alpha\right)$$

13. Find: $\int (2x+5)\sqrt{10-4x-3x^2}\,dx$

OR

Find: $\int \frac{(x^2+1)\,(x^2+4)}{(x^2+3)\,(x^2-5)}dx$

Sol. $I = \int (2x+5)\sqrt{10-4x-3x^2}\,dx$

$$= -\frac{1}{3}\int (-4-6x)\sqrt{10-4x-3x^2}\,dx + \frac{11}{3}\int \sqrt{10-4x-3x^2}\,dx$$

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\int\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2-\left(x-\frac{2}{3}\right)^2}\,dx$$

$$= -\frac{2}{9}(10-4x-3x^2)^{3/2} + \frac{11\sqrt{3}}{3}\left[\frac{\left(x-\frac{2}{3}\right)\sqrt{\left(\frac{\sqrt{34}}{3}\right)^2-\left(x-\frac{2}{3}\right)^2}}{2} + \frac{17}{9}\sin^{-1}\frac{3x-2}{\sqrt{34}}\right] + C$$

OR

$x^2 = y$ (say)

$$\frac{(y+1)(y+4)}{(y+3)(y-5)}=1+\frac{A}{y+3}+\frac{B}{y-5}$$

Using partial fraction we get $A=\frac{1}{4}, B=\frac{27}{4}$

$$\int\frac{(x^2+1)(x^2+4)}{(x^2+3)(x^2-5)}dx=\int I.dx+\frac{1}{4}\int\frac{dx}{x^2+3}+\frac{27}{4}\int\frac{dx}{x^2-5}$$

$$=x+\frac{1}{4\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}+\frac{27}{8\sqrt{5}}\log\left|\frac{x-\sqrt{5}}{x+\sqrt{5}}\right|+C$$

14. Find: $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

Sol. $I=\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x=t\Rightarrow\frac{dx}{\sqrt{1-x^2}}=dt$

$$=\int t\cdot\sin t\,dt$$

$$=-t\cos t+\sin t+c$$

$$=-\sqrt{1-x^2}\sin^{-1}x+x+c$$

15. Solve the following differential equation:

$y^2dx+(x^2-xy+y^2)dy=0$

Sol. $y^2dx+(x^2-xy+y^2)\,dy=0$

$$\Rightarrow \quad \frac{dx}{dy} = -\frac{(x^2 - xy + y^2)}{y^2}$$

Put x = vy $\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$$v + y\frac{dv}{dy} = \frac{(v^2y^2 - y^2v + y^2)}{y^2}$$

$$\Rightarrow \quad \frac{dv}{v^2+1} = \frac{dy}{y}$$

Integrating both sides

$$\tan^{-1} v = -\log y + c$$

$$\Rightarrow \quad \tan^{-1}\frac{x}{y} = -\log y + c$$

16. Solve the following differential equation:

($\cot^{-1}$y + *x*) dy = (1 + y^2) d*x*

Sol. $\frac{dx}{dy} - \frac{x}{1+y^2} = \frac{\cot^{-1} y}{1+y^2}$

$$I.F = e^{-\int\frac{1}{1+y^2}} = e^{\cot^{-1} y}$$

$$\Rightarrow \quad \frac{d}{dy}(x \cdot e^{\cot^{-1} y}) = \frac{\cot^{-1} t\, e^{\cot^{-1}} y}{1+y^2}$$

Integrating, we get

$$x \cdot e^{\cot-1y} = \int \frac{\cot^{-1} y\, e^{\cot-1y}}{1+y^2} dy$$

Put $\cot^{-1} y = t$

$$= -\int t\, e^t\, dt$$

$$= (1-t)\, e^t + c$$

$$\Rightarrow \qquad x = (1 - \cot^{-1} y) + c e^{-\cot^{-1} y}$$

17. If $\bar{a} \times \bar{b} = \bar{c} \times \bar{d}$ and $\bar{a} \times \bar{c} = \bar{b} \times \bar{d}$, show that $\bar{a} - \bar{d}$ is parallel to $\bar{b} - \bar{c}$, where $\bar{a} \neq \bar{d}$ and $\bar{b} \neq \bar{c}$.

Sol. $\bar{a} \times \bar{b} = \bar{c} \times \bar{d}$ ....(i)

$\bar{a} \times \bar{c} = \bar{b} \times \bar{d}$ ....(ii)

$$(1) - (2) \Rightarrow \bar{a} \times (\bar{b} - \bar{c}) = \bar{d} \times (\bar{b} - \bar{c})$$

$$\Rightarrow \qquad (\bar{a} - \bar{d}) \times (\bar{b} - \bar{c}) = \bar{0}$$

$$\Rightarrow \qquad (\bar{a} - \bar{d}) \parallel (\bar{b} - \bar{c})$$

18. Prove that the line through A(0, –1, –1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(–4, 4, 4).

Sol. Equation of line $\overline{AB}$

$$\bar{r} = (-\hat{j} - \hat{k}) + \lambda(4\hat{i} + 6\hat{j} + 2\hat{k})$$

Equation of line $\overline{CD}$

$$\bar{r} = (3\hat{i} + 9\hat{j} + 4\hat{k}) + \mu(-7\hat{i} - 5\hat{j})$$

$$\overline{a_2} - \overline{a_1} = 3\hat{i} + 10\hat{j} + 5\hat{k}$$

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 2 \\ -7 & -5 & 0 \end{vmatrix} = 10\hat{i} - 14\hat{j} + 22\hat{k}$$

$$(\overline{a_2} - \overline{a_1}) \cdot (\overline{b_1} \times \overline{b_2}) = 30 - 140 + 110 = 0$$

⇒ Lines intersect

19. A box has 20 pens of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement, at most 2 are defective.

OR

Let, X denote the number of colleges where you will apply after your results and

P (X = *x*) denotes your probability of getting admission in *x* number of colleges. It is given that

$$\mathbf{P(X = x)} = \begin{cases} kx & ; \quad if\ x = 0\ or\ 1 \\ 2\ kx & ; \quad if\ x = 2 \\ k(5-x) & ; \quad if\ x = 3\ or\ 4 \\ 0 & ; \quad if\ x > 4 \end{cases}$$

where k is a positive constant. Find the value of k. Also find the probability that you will get admission in (i) exactly one college (ii) at most 2 colleges (iii) at least 2 colleges.

Sol. Let selection of defective pen be considered success

$$P = \frac{2}{20} = \frac{1}{10}; q = \frac{9}{10}$$

Reqd probability = P(x = 0) + P(x = 1) + P(x = 2)

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 + {}^5C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 + \frac{1}{10} \times \left(\frac{9}{10}\right)^3$$

$$= \left(\frac{9}{10}\right)^3 \times \frac{4}{25}$$

OR

$$\sum_{i=0}^{4} P(x_i) = 1$$

$$\Rightarrow \qquad 8k = 1 \Rightarrow k = \frac{1}{8}$$

(i) P(x=1) $= \frac{1}{8}$

(ii) P(at most 2 colleges) = P(0) + P(1) + P(2)

$$= \frac{5}{8}$$

(iii) P(at least 2 colleges) = 1 – [P(x = 0) + P(x = 1)]

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

Section C

20. If f, $g : R \to R$ be two functions defined as f(x) $= |x| + x$ and $g(x) = |x| - x, \forall x \in R$. Then find fog and gof. Hence find fog(-3), fog(5) and gof (-2).

Sol. f(x) = |x| + x, g(x) = |x| - x $\quad \forall\ x \in R$

(fog)(x) = f(g(x))

= | |x| - 1| + |x| - x

(gof) (x) = g(f(x)

= | |x| + x| - |x| - x

(fog) (-3) = 6

(fog) (5) = 0

(gof) (-2) = 2

21. If a, b and c are all non-zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$, **then prove that**

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1=0$$

OR

If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, **find adj·A and verify that A(adj·A) = (adj·A)A = | A| I_3.**

Sol. $abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} = 0$

$$C_1 \to C_1 + C_2 + C_3$$

$$\Rightarrow abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b}+1 & \frac{1}{c} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}=0$$

$$\Rightarrow abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix}=0$$

$$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$$

$$\Rightarrow \qquad abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=0$$

$$\because \qquad a, b, c, \neq 0$$

$$\therefore \qquad 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$$

OR

$$|A|=1$$

$$adj\,\mathrm{A}=\begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{A}(adj\,\mathrm{A})=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I$$

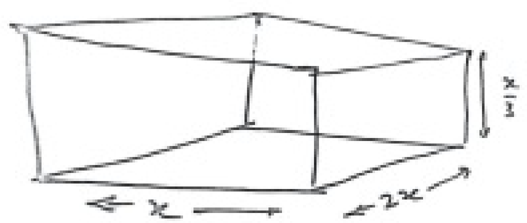
$$|A|I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

22. The sum of the surface areas of a cuboid with sides x, $2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.

OR

Find the equation of tangents to the curve y = cos(x + y), –2π ≤ x ≤ 2π that are parallel to the line x + 2y = 0.

Sol. $S = 6x^2 + 4\pi r^2$



$$\Rightarrow r = \sqrt{\frac{S - 6\pi^2}{4\pi}} \quad \text{...... (i)}$$

$$V = \frac{2x^3}{3} + \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}x^3 + \frac{4}{3}\pi\left(\frac{S - 6x^2}{4\pi}\right)^{3/2}$$

$$= \frac{2x^3}{3} + \frac{(S - 6x^2)^{3/2}}{6\sqrt{x}}$$

$$\frac{dV}{dx} = 2x^2 - \frac{3x}{\sqrt{\pi}}\sqrt{S-6x^2}$$

$$\frac{dV}{dx} = 0 \Rightarrow 2x^2 = \frac{3x\sqrt{S-6x^2}}{\sqrt{\pi}}$$

$$\Rightarrow r = \frac{x}{3}\ [u\sin g\ (i)]$$

$$\frac{d^2V}{dx^2} = 4x\left[\frac{3x}{\sqrt{\pi}}\cdot\frac{(-12x)}{2\sqrt{S-6x^2}} + \frac{3}{\sqrt{\pi}}\sqrt{S-6x^2}\right]$$

$$\left.\frac{d^2V}{dx^2}\right|_{x=\frac{r}{3}} > 0$$

⇒ V is minimum at $x = \frac{r}{3}\ i.e.r = 3x$

Minimum value of sum of volume $= \left(\frac{2x^3}{3} + 36\pi x^3\right)$ cubic units

OR

Equation of given curve

Y = cos (x + y)

$$\Rightarrow \frac{dy}{dx} = -\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$$

Given line x + 2y = 0, it slope $= -\frac{1}{2}$

Condition of || lines

$$\frac{-\sin(x+y)}{1+\sin(x+y)} = -\frac{1}{2}$$

$$\Rightarrow \sin(x+y) = 1$$

⇒ cos (x + y) = 0 y = 0 using (i)

⇒ cos x = 0 $\Rightarrow x = (2n+1)\frac{\pi}{2}, n \in I$

$$\therefore x = \frac{-3\pi}{2}, \frac{\pi}{2} \in [-2\pi, 2\pi]$$

Thus tangents are | | to the line x + 2y = 0

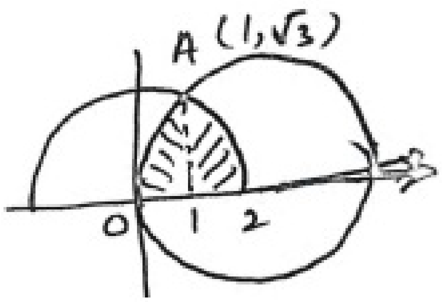
Only at pts $\left(-\frac{3\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$

∴ Required equation of tangents are

$$y - 0 = -\frac{1}{2}\left(x + \frac{3\pi}{2}\right) \Rightarrow 2x + 4y + 3\pi = 0$$

23. Using integration find the area of the region bounded by the curves $y = \sqrt{4 - x^2}$, x^2 + y^2 – 4x = 0 and the x-axis.

Sol. Their point of intersection $\left(1, \sqrt{3}\right)$



Correct Figure

$$Required\ Area = \int_0^1 \sqrt{(2)^2 - (x-2)^2}\,dx + \int_1^2 \sqrt{2^2 - x^2}\,dx$$

$$= \left[\frac{(x-2)\sqrt{4x-x^2}}{2} + 2\sin^{-1}\frac{x-2}{0}\right]_0^1 + \left[\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\frac{x}{2}\right]_1^2$$

$$= \left(\frac{5\pi}{3} - \sqrt{3}\right) Sq.units$$

24. Find the equation of the plane which contains the line of intersection of the planes *x* + 2y + 3z – 4 = 0 and 2*x* + y – z + 5 = 0 and whose *x*-intercept is twice its z-intercept. Hence write the vector equation of a plane passing through the point (2, 3, –1) and parallel to the plane obtained above.

Sol. Equation of family of planes passing through two given planes

(x + 2y + 3z – 4) + k (2x + y – z + 5) = 0

⇒ (1 + 2k) x + (2 + k) y + (3 – k) z = 4 – 5k .....(i)

$$\Rightarrow \qquad \frac{x}{\frac{4-5k}{1+2k}} + \frac{y}{\frac{4-5k}{2+k}} + \frac{z}{\frac{4-5k}{3-k}} = 1$$

As per condition

$$\frac{4-5k}{1+2k} = \frac{2(4-5k)}{(3-k)}$$

$$\Rightarrow \qquad k = \frac{4}{5}\ or\ \frac{1}{5}$$

For $k = \frac{1}{5}$, Eqn. of planes is 7x + 11y + 14z = 15

For $k = \frac{4}{5}$, Eqn. of planes is 13x + 14y + 11z = 0

Equation of plane passing through (2, 3, -1) and parallel to the plane is:

$7(x - 2) + 11(y - 3) + 14(z + 1) = 0$

$\Rightarrow 7x + 11y + 14z = 33$

Vector from: $\bar{r} \cdot (7\hat{i} + 11\hat{j} + 14\hat{k}) = 33$

25. Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from A to B.

Sol. Let H_1 be the event 2 red balls are transferred

H_2 be the event 1 red and 1 balck ball, transferred

H_3 be the event 2 black and 1 balck ball transferred

E be the event that ball drawn from B is red

$$P(H_1) = \frac{^3C_2}{^8C_2} = \frac{3}{28} \quad P(E/H_1) = \frac{6}{10}$$

$$P(H_2) = \frac{^3C_1 \times ^5C_1}{^8C_2} = \frac{15}{28} \quad P(E/H_2) = \frac{5}{10}$$

$$P(H_3) = \frac{^5C_2}{^8C_2} = \frac{10}{28} \quad P(E/H_3) = \frac{4}{10}$$

$$P(H_1/E) = \frac{\frac{3}{28} \times \frac{6}{10}}{\frac{3}{28} \times \frac{6}{10} + \frac{15}{28} \times \frac{5}{10} + \frac{10}{28} \times \frac{4}{10}}$$

$$= \frac{18}{133}$$

26. In order to supplement daily diet, a person wishes to take X and Y tablets. The

contents (in milligrams per tablet) of iron, calcium and vitamins in X and Y are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

The person needs to supplement at least 18 milligrams of iron, 21 milligrams of calcium and 16 milligrams of vitamins. The price of each tablet of X and Y is Rs 2 and Rs 1 respectively. How many tablets of each type should the person take in order to satisfy the above requirement at the minimum cost ? Make an LPP and solve graphically.

Sol. Let x tablets of type X and y tablets of type Y are taken Minimise C = 2x + y

Subjected to

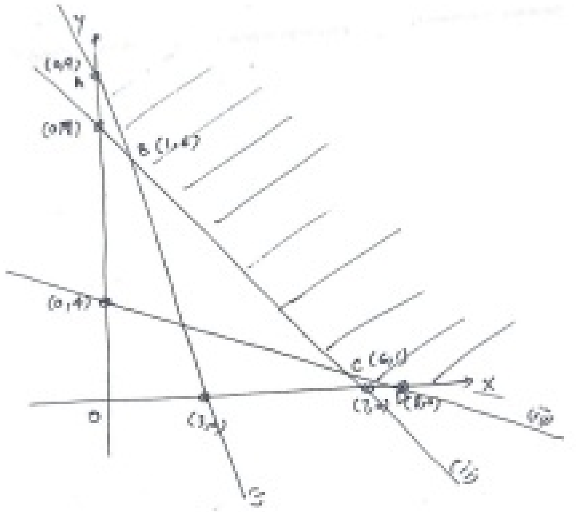
$6x + 2y \geq 18$

$3x + 3y \geq 21$

$2x + 4y \geq 16$

$x, y \geq 0$

correct graph



$$Cl_{A(0,9)} = 9$$

$$Cl_{B(1,6)} = 8 \leftarrow Minimum\ value$$

$$Cl_{C(6,1)} = 13$$

$$Cl_{D(8,0)} = 16$$

2x + y < 8 does not pass through unbounded region

Thus, minimum value of C = 8 at x = 1, y = 6