Session Ending Examination (2015-2016) Set - 4

Class XI (Mathematics)

Time: 3 Hrs M.M: 100

General Instructions:

a) All the questions are compulsory.

b) The Question Paper consists of 26 Questions divided into three sections A, B and C

c) Section-A comprises of 6 questions of one mark each.

d) Section-B consists of 13 questions of four marks each.

e) Section-C comprises of 7 questions of Six marks each.

f) There is no overall choice. However, an internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

g) Use of calculator, is not permitted.

Section A

1. If (x – 1, y + 3) = (2, x + 4), then find the values of x and y.

2. Rephrase the following sentence in conditional form 'Working hard ensures that you will pass the examination'.

3. How many three digit numbers are divisible by 7?

4. Find the length of latusrectum of the parabola $y^2 = -8x$.

5. Write the contrapositive of the following statement 'If a triangle is equilateral, then it is isosceles'.

6. Write the negation of the following statement, 'All Mathematicians are men'.

Section B

7. If f is a real function defined by $f(x) = \frac{x-1}{x+1}$, then prove that $f(2x) = \frac{3f(x)+1}{f(x)+3}$.

8. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

9. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, then find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$

Or

Prove that $\cot\frac{\pi}{24} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

10. If α and β are different complex numbers with $|\beta|=1$, then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

11. To pass in a subject, one must obtain an average of 33 out of 100 or higher to pass in the subject in five examinations. If a student's marks in the four examinations are 28, 31, 40 and 37, then find the minimum marks a student must obtain to pass in the subject.
A student obtained 42 marks in the fifth exam. Do you think student has passed in the subject? What value system does he possess?

12. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected, if the team has

(i) no girl?

(ii) at least one boy and one girl?

13. Find the equation of a line which passes through the point of intersection of lines 3x + y - 7 = 0 and x + 2y + 5 = 0 and is perpendicular to the line 5x - 2y + 6 = 0.

Or

If P and P' are perpendiculars from the origin on the straight lines whose equations are $x\sec\theta + y\operatorname{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$, then prove that $4P^2 + (P')^2 = a^2$.

14. Prove that $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$

15. Four students in traditional dresses represent four states of India, standing at the points represented by O(0, 0, 0), A(a, 0, 0), B(0, b, 0) and C(0, 0, c). Find the place, in terms of coordinates, where a girl representing 'BHARATMATA be placed so that girl is equidistant from the four students. What message does it convey?

16. Find the equation of circle which passes through (3, -2), (-2, 0) and has its centre on the line 2x - y = 3.

Or

Find the equation of the ellipse whose axes are along the coordinate axes, vertices are

$(\pm 5, 0)$ and foci at $(\pm 4, 0)$.

17. The mean and variance of 7 observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12 and 14, then find the remaining 2 observations.

18. Find the mean deviation from mean of the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of students	6	8	14	16	4	2

Or

Find the mean deviation from median of the following distribution.

Marks	0-10	10-20	20-30	30-40	40-50
Numbers of students	5	8	15	16	6

19. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue?

(ii) at least one will be green?

Section C

20. If f be the exponential function and g be the logarithmic function, then find

(i) (f + g) (1)

(ii) (fg) (1)

(iii) (3f) (1)

(iv) (5g) (1)

21. Solve the following equation $\cot^2\theta + \frac{3}{\sin\theta} + 3 = 0$.

22. Prove by Principle of Mathematical Induction

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \ldots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4},\ n \in N.$$

Or

Prove by Principle of Mathematical Induction n(n +1) (n + 5) is a multiple of 3 for all $n \in N$.

23. Solve the following system of in-equations graphically $12x+12y \leq 840$,

$3x+6y \leq 300$, $8x+4y \leq 480$, $x \geq 0$, $y \geq 0$

24. Using binomial theorem, expand $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$, $x \neq 0$.

Or

Show that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{n!} 2^n \cdot x^n$

25. Find the sum to n terms of the series $\frac{1}{1 \cdot 2}+\frac{1}{2 \cdot 3}+\frac{1}{3 \cdot 4}+\ldots+\frac{1}{n \cdot (n+1)}$.

26. (i) Differentiate $\frac{e^x}{1+\sin x}$ with respect to x.

(ii) Evaluate $\lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x-\frac{\pi}{6}}$