CHAPTER

3 Trigonometric Ratios and Identities

INTRODUCTION

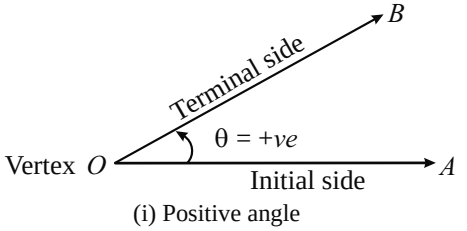
The word trigonometry is derived from three greek words.

Tri	Gon	Metron
↓	↓	↓
three	sides	Measure

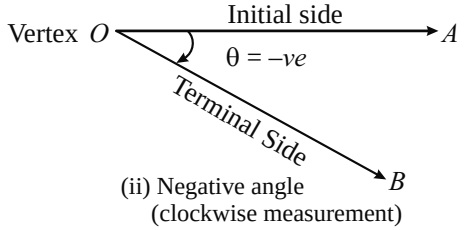
In the ancient time trigonometry defines relations between elements of a triangle. In a triangle there are six basic elements, three sides and three angles. Any three line segments will form a triangle if they satisfy three triangular inequalities i.e. the sum of any two lines segment is greater than third side. In Euclidean geometry the sum of three angles of a triangle is 180º. These requirements impose limitations on the manner in which the relations between the elements are defined.

ANGLE

Angle is a measure of rotation of a given ray about its initial point. The original ray is called the initial side and the final position of the ray after rotation is called the terminal side of the angle. The point of rotation is called the vertex. If the direction of rotation is anticlockwise, the angle is said to be positive and if the direction of rotation is clockwise, then the angle is negative.



(i) Positive angle
(anticlockwise measurement)



(ii) Negative angle
(clockwise measurement)

SYSTEMS FOR MEASUREMENT OF ANGLES

An angle can be measured in the following systems.

1. **Sexagesimal System (British System):** In this system $\frac{1}{360}$ of a complete circular turn is called a degree (°), $\frac{1}{60}$ of a degree is called a minute (′) and $\frac{1}{60}$ of a minute is called a second (″).

   One right angle = 90°, 1° = 60′, 1′ = 60″

ADVANCED LEARNING

2. **Centesimal System (French System):** In this system $\frac{1}{400}$ of a complete circular turn is called a grade (g), $\frac{1}{100}$ of a grade is called a minute (') and $\frac{1}{100}$ of a minute is called a second (").

   ∴ One right angle = 100^g; $1^g = 100'$; $1' = 100''$

   Note: The minutes and seconds in the Sexagesimal system are different with the minutes and seconds respectively in the Centesimal System. Symbols in both systems are also different.

3. **Circular System (Radian Measurement):** The angle subtended by an arc of a circle whose length is equal to the radius of the circle at the centre of the circle is called a radian. In this system the unit of measurement is radian (^{c}) As the circumference of a circle of radius 1 unit is 2π, therefore one complete revolution of the initial side subtends an angle of 2π radian.

   More generally, in a circle of radius r, an arc of length r will subtend an angle of 1 radian. It is well-known that equal arcs of a circle subtend equal angle at the centre. Since in a circle of radius r, an arc of length r subtends an angle whose measure is 1 radian, an arc of length ℓ will subtend an angle whose measure is $\frac{\ell}{r}$ radian. Thus, if in a circle of radius r, arc of length ℓ subtends an angle θ radian at the centre, we have $\theta = \frac{\ell}{r}$ or $\ell = r\theta$.

Train Your Brain

Example 7: Find cos (315°).

Sol. cos (315°) = cos (360° – 45°) = cos (–45°)

$= \cos(45°) = \frac{1}{\sqrt{2}}$

Example 8: Find tan (330°).

Sol. tan (330°) = tan (360° – 30°) = tan (–30°)

$= -\tan 30° = \frac{-1}{\sqrt{3}}$

Example 9: Prove that sin420° cos390° + cos(–300°) sin(–330°) = 1.

Sol. sin (360º+ 60º) cos (360º + 30º) + cos (–360º + 60º) sin (–360º + 30º) = sin 60º cos 30º + cos 60º sin 30º

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$

Example 10: Prove that sin 240° sin 510° – sin330° cos 390° = 0

Sol. sin (270º – 30º) sin (540º – 30º) – sin (360°– 30°) cos (360º + 30º)

= – cos 30º sin30º + sin30º cos30º = 0

Example 11: Prove that $\cos A + \sin(270° + A) - \sin(270° - A) + \cos(180° + A) = 0$

Sol. $\cos A + \sin(270° + A) - \sin(270° - A) + \cos(180° + A)$

$= \cos A - \cos A - (-\cos A) + (-\cos A) = 0$

Example 12: Find the value of tan 35º.tan 40º.tan 45º. tan 50º.tan 55º.

Sol. We have,

tan 35º.tan 40º.tan 45º.tan 50º.tan 55º

$= \{\tan 35° \times \tan 55°\}\{\tan 40° \times \tan 50°\} \times \tan 45°$

$= \{\tan 35° \times \cot 35°\}.\{\tan 40° \times \cot 40°\} \times \tan 45° = 1$

Example 13: Find the value of tan(1º).tan(2º).tan(3º)..... tan(89º).

Sol. We have,

tan(1º).tan(2º).tan(3º).....tan(89º)

= tan(1º).tan(2º).tan(3º).....tan(44º)

tan(45º) tan(46º).....tan(87º) tan(88º) tan(89º)

= {tan(1º) × tan(89º)}.{ tan(2º) × tan(88º)}.....

{tan(44º) × tan(46º)}. tan(45º) = 1

Example 14: Find the value of cos(1º).cos(2º).cos(3º)..... cos(189º).

Sol. We have,

cos(1º).cos(2º).cos(3º).....cos(189º)

= cos(1º).cos(2º).cos(3º).....cos(89º)

cos(90º) cos(91º).....cos(189º)

= cos(1º).cos(2º).cos(3º)....cos(89º) × 0 × cos(91º) .... cos(189º) = 0

Concept Application

6. If $5\cos^2\alpha - 2\sin\alpha - 2 = 0$, $(5\pi/4 < \alpha < 7\pi/4)$, then find the value of cot $\alpha/2$.

7. Let $a = \tan\left(\frac{-2\pi}{3}\right)$, $b = \sin\left(\frac{-17\pi}{4}\right)$, c = cot (– 300°) d = cos (– 315°), then

(*a*) $a < b < c < d$

(*b*) $a < c < b < d$

(*c*) $b < c < d < a$

(*d*) $b < c < a < d$

8. Find the value of

$$\cos^2\frac{\pi}{16} + \cos^2\frac{3\pi}{16} + \cos^2\frac{5\pi}{16} + \cos^2\frac{7\pi}{16}$$

9. Match the columns:

Column-I		Column-II	
A.	sin (1050°)	p.	$-1/\sqrt{3}$
B.	sin (120°)	q.	3/4
C.	$\cos^2$ (120°)	r.	$-\sqrt{3}/2$
D.	tan (120°)	s.	1/2
E.	sin (135°)	t.	$1/\sqrt{2}$
F.	$\sin^2 9\pi/4$	u.	$-\sqrt{3}$
G.	sin $11\pi/3$	v.	1/4
H.	$\cos^2 31\pi/6$	w.	$\sqrt{3}/2$
I.	tan $41\pi/6$	x.	–1/2

1. If $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$ then, $\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}=$

(*a*) $\frac{1}{(a+b)^3}$ (*b*) $\frac{1}{(a-b)^3}$

(*c*) $(a+b)^3$ (*d*) $(a-b)^3$

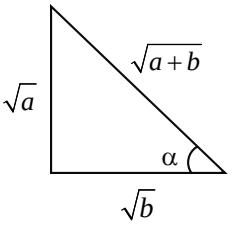
Sol. $\frac{\sin^4\alpha}{a}+\frac{\cos^4\alpha}{b}=\frac{1}{a+b}$

$\frac{(a+b)}{a}\sin^4\alpha+\frac{(a+b)}{b}\cos^4\alpha=1$

$\Rightarrow \sin^4\alpha+\frac{b}{a}\sin^4\alpha+\frac{a}{b}\cos^4\alpha+\cos^4\alpha=1$

$\Rightarrow 1-2\sin^2\alpha\cos^2\alpha+\frac{b}{a}\sin^4\alpha+\frac{a}{b}\cos^4\alpha=1$

$\Rightarrow \left(\sqrt{\frac{b}{a}}\sin^2\alpha-\sqrt{\frac{a}{b}}\cos^2\alpha\right)^2=0$



$\Rightarrow \tan^2\alpha=\frac{a}{b}$

$\Rightarrow$ Now, $\frac{\sin^8\alpha}{a^3}+\frac{\cos^8\alpha}{b^3}$

$=\frac{a^4}{(a+b)^4}\frac{1}{a^3}+\frac{b^4}{(a+b)^4}\frac{1}{b^3}=\frac{1}{(a+b)^3}$

Therefore, option (*a*) is the correct answer.

2. If $X=\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$,

$Y=\cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$, then

$\frac{X}{Y}-\frac{Y}{X}=$

(*a*) $2\tan2\theta$ (*b*) $2\tan^2\theta$

(*c*) $2\cot^2\theta$ (*d*) None of these

Sol. $X=\sin\left(\theta+\frac{7\pi}{12}\right)+\sin\left(\theta-\frac{\pi}{12}\right)+\sin\left(\theta+\frac{3\pi}{12}\right)$

$=2\sin\left(\frac{\theta+\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(\theta+\frac{\pi}{4}\right)$

$=2\sin\left(\theta+\frac{\pi}{4}\right)$

$Y=\cos\left(\theta+\frac{7\pi}{12}\right)+\cos\left(\theta-\frac{\pi}{12}\right)+\cos\left(\theta+\frac{3\pi}{12}\right)$

$=2\cos\left(\theta+\frac{\pi}{4}\right)\cdot\frac{1}{2}+\cos\left(\theta+\frac{\pi}{4}\right)$

$=2\cos\left(\theta+\frac{\pi}{4}\right)$

Now $\frac{X}{Y}-\frac{Y}{X}=\frac{2\sin(\theta+\pi/4)}{2\cos(\theta+\pi/4)}-\frac{2\cos(\theta+\pi/4)}{2\sin(\theta+\pi/4)}$

$=\left[\frac{-2\cos(2\theta+\pi/2)}{\sin(2\theta+\pi/2)}\right]=2\tan2\theta$

Therefore, option (*a*) is the correct answer.

3. Evaluate: $\cos^6\frac{\pi}{16}+\cos^6\frac{3\pi}{16}+\cos^6\frac{5\pi}{16}+\cos^6\frac{7\pi}{16}$

(*a*) 5/8 (*b*) 5/4 (*c*) 7/16 (*d*) 5/32

Sol. $\left(\cos^6\frac{\pi}{16}+\sin^6\frac{\pi}{16}\right)+\left(\cos^6\frac{3\pi}{16}+\sin^6\frac{3\pi}{16}\right)$

$=2-3\cos^2\frac{\pi}{16}\sin^2\frac{\pi}{16}-3\cos^2\frac{3\pi}{16}\sin^2\frac{3\pi}{16}$

$=2-\frac{3}{4}\left[\sin^2\frac{\pi}{8}+\sin^2\frac{3\pi}{8}\right]$

$=2-\frac{3}{4}\left[\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}\right]=\frac{5}{4}$

Therefore, option (*b*) is the correct answer.

4. If $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta+\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta-\frac{2\pi}{3}\right)}$, then $x + y + z$ is equal to:

(*a*) −1 (*b*) 1 (*c*) 0 (*d*) 2

Sol. Given $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta+\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\lambda$ (say)

$\Rightarrow x+y+z=\lambda\left\{\cos\theta+\cos\left(\theta+\frac{2\pi}{3}\right)+\cos\left(\theta-\frac{2\pi}{3}\right)\right\}$

$=\lambda\left\{\cos\theta+2\cos\theta\cos\frac{2\pi}{3}\right\}=0$

Therefore, option (*c*) is the correct answer.

5. If $\alpha+\beta+\gamma+\delta=2\pi$. Prove that $\cos\alpha+\cos\beta+\cos\gamma+\cos\delta+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

(*a*) 3/2 (*b*) 5/4 (*c*) 3 (*d*) 0

Sol. Given $\alpha+\beta+\gamma+\delta=2\pi$

Tanking L.H.S. we have

$\cos\alpha+\cos\beta+\cos\gamma+\cos\delta+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\gamma+\delta}{2}\cos\frac{\gamma-\delta}{2}+4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\alpha+\delta}{2}\right)$

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

(*a*) $\frac{-4}{5}$ but not $\frac{4}{5}$ (*b*) $\frac{-4}{5}$ or $\frac{4}{5}$

(*c*) $\frac{4}{5}$ but not $-\frac{4}{5}$ (*d*) None of these

2. The value of $\sin20° \sin40° \sin 60° \sin80°$ is

(*a*) $\frac{-3}{16}$ (*b*) $\frac{5}{16}$

(*c*) $\frac{3}{16}$ (*d*) $\frac{1}{16}$

3. If $\sin\theta + \cos\theta = 1$, then the value of $\sin2\theta$ is equal to

(*a*) 1 (*b*) $\frac{1}{2}$

(*c*) 0 (*d*) –1

4. The value of $\sin\left(45° + \theta\right) - \cos\left(45° - \theta\right)$ is

(*a*) $2\cos\theta$ (*b*) $2\sin\theta$

(*c*) 1 (*d*) 0

5. The value of $\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)$ is

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) Not defined

6. If $\tan A = \frac{1}{2}, \tan B = \frac{1}{3}$, then $\tan\left(2A + B\right)$ is equal to

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

7. The value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is

(*a*) 1 (*b*) $\sqrt{3}$

(*c*) $\frac{\sqrt{3}}{2}$ (*d*) 2

8. Which of the following is correct?

(*a*) $\sin1° > \sin1$ (*b*) $\sin1° < \sin1$

(*c*) $\sin1° = \sin1$ (*d*) $\sin1° = \frac{\pi}{18°}\sin1$

9. The value of $\tan75° - \cot75°$ is equal to

(*a*) $2\sqrt{3}$ (*b*) $2 + \sqrt{3}$

(*c*) $2 - \sqrt{3}$ (*d*) 1

VERY SHORT ANSWER TYPE QUESTIONS

10. Find the value of $\sqrt{3}\text{cosec}20° - \sec20°$

11. If $\frac{\sin\left(x+y\right)}{\sin\left(x-y\right)} = \frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y} = \frac{a}{b}$

12. If θ lies in the second quadrant, then show that

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = -2\sec\theta$$

13. If $\cot\theta + \tan\theta = 2\text{cosec}\theta$, then find the general value of θ.

14. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$

15. If $\cos\left(\alpha + \beta\right) = \frac{4}{5}$ and $\sin\left(\alpha - \beta\right) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, find the value of $\tan2\alpha$.

16. Prove that $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$

LONG ANSWER TYPE QUESTIONS

17. Find the value of

$$\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right)$$

18. Show that

$$2\sin^2\beta + 4\cos\left(\alpha + \beta\right)\sin\alpha\sin\beta + \cos2\left(\alpha + \beta\right) = \cos2\alpha$$

19. Find the value of the expression

$$\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$$

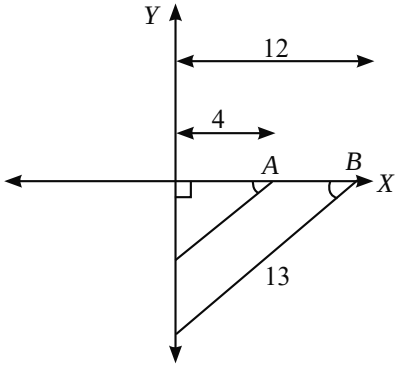
20. If $\sin\left(\theta + \alpha\right) = a$ and $\sin\left(\theta + \beta\right) = b$, then prove that $\cos2\left(\alpha - \beta\right) - 4ab\cos\left(\alpha - \beta\right) = 1 - 2a^2 - 2b^2$

21. If $a\cos2\theta + b\sin2\theta = c$ has α and β as its roots, then prove that $\tan\alpha + \tan\beta = \frac{2b}{a + c}$.

CASE STUDY BASED QUESTIONS

Case Study-I

Rajiv constructs two right angled triangles in the fourth quadrant in such a way that the measure of triangle gives $\cos A = \frac{4}{5}$ and $\cos B = \frac{12}{13}$, where $\frac{3\pi}{2} < A$ and $B < 2\pi$.



Based on the above information, answer the following questions.

22. Find the value of $\cos(A + B)$

(*a*) $\frac{13}{16}$ (*b*) $\frac{13}{65}$ (*c*) $\frac{33}{65}$ (*d*) $\frac{1}{16}$

23. Find the value of $\tan(A + B)$

(*a*) $\frac{36}{33}$ (*b*) $\frac{56}{65}$

(*c*) $-\frac{56}{33}$ (*d*) $\frac{56}{33}$

Case Study-II

In a class test of class XI, a teacher asked to students to consider $A + B = \frac{\pi}{4}$, where A and B are acute angles.

Based on the above information, answer the following questions.

24. Find the value of $(1 + \tan A)(1 + \tan B)$?

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

25. Find the value of $\sin(A + B) - \cos(A + B) + \tan(A + B)$.

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

PRARAMBH (TOPICWISE)

ANGLE AND ITS MEASUREMENT

1. An equilateral triangle has side length 8. The area of the region containing all points outside the triangle but not more than 3 units from a point on the triangle is

(*a*) $9(8 + \pi)$ (*b*) $8(9 + \pi)$

(*c*) $9\left(8 + \frac{\pi}{2}\right)$ (*d*) $8\left(9 + \frac{\pi}{2}\right)$

2. The perimeter of a certain sector of a circle is equal to half that of the circle of which it is a sector. The circular measure of one angle of the sector is

(*a*) $(\pi - 2)$ radian (*b*) $(\pi + 2)$ radian

(*c*) π radian (*d*) $(\pi - 3)$ radian

BASIC TRIGONOMETRIC IDENTITIES

3. Number of values of θ for which $\cos\theta = 0.707$ $(\theta \in (0, 2\pi))$

(*a*) 1 (*b*) 3

(*c*) 4 (*d*) 2

4. If $\frac{2\sin\alpha}{1 + \sin\alpha + \cos\alpha} = \lambda$ then $\frac{1 + \sin\alpha - \cos\alpha}{1 + \sin\alpha}$ is equal to

(*a*) $\frac{1}{\lambda}$ (*b*) λ

(*c*) $1 - \lambda$ (*d*) $1 + \lambda$

5. If $\sin A \tan A = \cos^2 A$ then $\cos^3 A + \cos^2 A$ is equal to

(*a*) 1 (*b*) 2

(*c*) 4 (*d*) None of these

6. If $\operatorname{cosec} A + \cot A = \frac{11}{2}$, then $\tan A$ is

(*a*) $\frac{21}{22}$ (*b*) $\frac{15}{16}$

(*c*) $\frac{44}{117}$ (*d*) $\frac{117}{43}$

7. Which of the following statement is incorrect?

(*a*) Tangent of odd integral multiple of $\frac{\pi}{2}$ is not defined.

(*b*) Cotangent of integral multiple of π is not defined.

(*c*) Tangent of odd integral multiple of π is 0.

(*d*) Cotangent of integral multiple of $\frac{\pi}{2}$ is not defined.

8. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1 + \cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if

(*a*) $\theta \in \left(0, \frac{\pi}{2}\right)$ (*b*) $\theta \in \left(\frac{\pi}{2}, \pi\right)$

(*c*) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (*d*) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

1. The value of $\tan 7\frac{1}{2}^\circ$ is equal to

(*a*) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}+1}$ (*b*) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}-1}$

(*c*) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}+1}$ (*d*) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{2}-1}$

2. If $a \sin x + b \cos(c + x) + b \cos(c - x) = \alpha$, $a < \alpha$, then the minimum value of $|\cos c|$ is:

(*a*) $\sqrt{\frac{\alpha^2 - a^2}{b^2}}$ (*b*) $\sqrt{\frac{\alpha^2 - a^2}{2b^2}}$

(*c*) $\sqrt{\frac{\alpha^2 - a^2}{3b^2}}$ (*d*) $\sqrt{\frac{\alpha^2 - a^2}{4b^2}}$

3. If $\frac{\sin A}{\sin B} = \frac{\sqrt{3}}{2}$ and, $\frac{\cos A}{\cos B} = \frac{\sqrt{5}}{2}$ $0 < A, B < \frac{\pi}{2}$ then $\tan A + \tan B$ is equal to:

(*a*) $\sqrt{\frac{3}{5}}$ (*b*) $\sqrt{\frac{5}{3}}$

(*c*) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ (*d*) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}}$

4. If $\alpha + \gamma = 2\beta$ then the expression $\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$ simplifies to:

(*a*) $\tan\beta$ (*b*) $-\tan\beta$

(*c*) $\cot\beta$ (*d*) $-\cot\beta$

5. If $\log_3 \sin x - \log_3 \cos x - \log_3(1 - \tan x) - \log_3(1 + \tan x) = -1$, then $\tan 2x$ is equal to (wherever defined)

(*a*) –2 (*b*) 3/2

(*c*) 2/3 (*d*) 6

6. $\sum_{r=1}^{n-1} \cos^2 \frac{r\pi}{n}$ is equal to

(*a*) $\frac{n}{2}$ (*b*) $\frac{n-1}{2}$

(*c*) $\frac{n}{2} - 1$ (*d*) $\frac{n+1}{2}$

7. The expression $2 \sin 2^\circ + 4 \sin 4^\circ + 6 \sin 6^\circ + \ldots + 180 \sin 180^\circ$ equals

(*a*) $\cot 1^\circ$ (*b*) $90 \cot 1^\circ$

(*c*) $\sin 1^\circ$ (*d*) $90 \cos 1^\circ$

8. The value of $\tan\frac{\theta}{2}(1 + \sec\theta)(1 + \sec 2\theta)(1 + \sec 4\theta) \ldots (1 + \sec 2^n\theta)$ is

(*a*) $\tan 2^n\theta$ (*b*) $\tan 2^{n-1}\theta$

(*c*) $\tan 2^{n+1}\theta$ (*d*) $\tan 2^{n-2}\theta$

9. The minimum value of $27^{\cos 3x} \cdot 81^{\sin 3x}$ is

(*a*) 1 (*b*) $\frac{1}{81}$ (*c*) $\frac{1}{243}$ (*d*) $\frac{1}{27}$

10. The value of '*a*' for which the equation $\sin x(\sin x + \cos x) = a$ has a real solution are

(*a*) $1 - \sqrt{2} \le a \le 1 + \sqrt{2}$ (*b*) $2 - \sqrt{3} \le a \le 2 + \sqrt{3}$

(*c*) $0 \le a \le 2 + \sqrt{3}$ (*d*) $\frac{1-\sqrt{2}}{2} \le a \le \frac{1+\sqrt{2}}{2}$

11. For $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$ lies in the interval

(*a*) $(-\infty, \infty)$ (*b*) $(-2, 2)$

(*c*) $(0, \infty)$ (*d*) $(-1, 1)$

12. If $f(\alpha, \beta) = \cos^2\alpha + \sin^2\alpha \cdot \cos 2\beta$, then which of the following is incorrect?

(*a*) $f\left(\frac{\pi}{5}, \frac{2\pi}{5}\right) \ne f\left(\frac{2\pi}{5}, \frac{\pi}{5}\right)$

(*b*) $f\left(\frac{\pi}{12}, \frac{\pi}{3}\right) = f\left(\frac{\pi}{3}, \frac{\pi}{12}\right)$

(*c*) $3f\left(\frac{\pi}{5}, \frac{\pi}{3}\right) \ne f\left(\frac{\pi}{3}, \frac{\pi}{5}\right)$

(*d*) $f\left(\frac{\pi}{4}, \frac{\pi}{18}\right) \ne 3f\left(\frac{\pi}{18}, \frac{\pi}{4}\right)$

13. Let $f(x) = \cos 10x + \cos 8x + 3 \cos 4x + 3 \cos 2x$ and $g(x) = 8 \cos x \cdot \cos^3 3x$, then for all x we have

(*a*) $f(x) = g(x)$ (*b*) $2f(x) = 3g(x)$

(*c*) $f(x) = 2g(x)$ (*d*) $2f(x) = g(x)$

14. The value of $\cot 70^\circ + 4 \cos 70^\circ$ is

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\sqrt{3}$

(*c*) $2\sqrt{3}$ (*d*) $\frac{1}{2}$

15. If $\cos^2 A + \cos^2 B + \cos^2 C = 1$, then ΔABC is

(*a*) Equilateral (*b*) Isosceles

(*c*) Right angled (*d*) None of these

34. $\sin\alpha + \sin 2\alpha + \sin 4\alpha + \sin 5\alpha =$

(*a*) $4\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}\sin 3\alpha$ (*b*) 0

(*c*) 1 (*d*) $2\cos\frac{\alpha}{2}\cos\frac{3\alpha}{2}\sin 3\alpha$

35. $\cos(36° - A)\cos(36° + A) + \cos(54° + A)\cos(54° - A) =$

(*a*) $\cos 2A$ (*b*) 1

(*c*) 0 (*d*) $\sin 2A$

36. $\operatorname{cosec}\theta + \operatorname{cosec} 2\theta + \operatorname{cosec} 2^2\theta + \ldots + \operatorname{cosec} 2^{n-1}\theta =$

(*a*) $\cot(\theta/2) - \cot 2^{n-1}\theta$ (*b*) $\cot(\theta/2) + \cot 2^{n-1}\theta$

(*c*) 0 (*d*) $\frac{\tan 2^n\theta}{\tan\theta}$

37. If $A + B + C = \pi$, then $\tan^2\frac{A}{2} + \tan^2\frac{B}{2} + \tan^2\frac{C}{2}$

(*a*) ≤ 1 (*b*) always 0

(*c*) ≤ 2 (*d*) ≥ 1

38. Let $P = \cos(A + B + C) + \cos(-A + B + C) + \cos(A - B + C) + \cos(A + B - C)$ and $Q = \sin(A + B + C) + \sin(-A + B + C) - \sin(A - B + C) + \sin(A + B - C)$ then $\frac{P}{Q} =$

(*a*) $\cos 2B$ (*b*) $\cot B$ (*c*) 0 (*d*) $\sin B$

39. Distance between orthocentre and circumcentre in a triangle with sides 17, 15, 8.

(*a*) 9 (*b*) $\frac{17}{2}$ (*c*) 15/2 (*d*) 4

40. Value of the expression $\log_{1/2}(\sin 6° \cdot \sin 42° \cdot \sin 45° \cdot \sin 66° \cdot \sin 78°)$

(*a*) Lies between 3 and 5

(*b*) Is rational which is not integral

(*c*) Is irrational which is a simple surd

(*d*) Is irrational which is a mixed surd

INTEGER TYPE QUESTIONS

41. If $\sin x + \sin^2 x = 1$, then the value of $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x - 1$ is equal to

42. If $\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$, then the expression

$\cos 2\alpha + (2 + \sqrt{3})\sin 2\alpha$ is

43. The maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$ for real values of θ is

44. If $\cos(\alpha + \beta) + \sin(\alpha - \beta) = 0$ and $\tan\beta = \frac{1}{2009}$; then $\tan 3\alpha$ is:

45. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is

46. If $\frac{\sin\theta}{\cos(3\theta)} + \frac{\sin(3\theta)}{\cos(9\theta)} + \frac{\sin(9\theta)}{\cos(27\theta)} + \frac{\sin(27\theta)}{\cos(81\theta)} = \frac{\sin(k\theta)}{2\cos\theta\cos 81\theta}$, then k equals to

47. $\cot 16° \cdot \cot 44° + \cot 44° \cdot \cot 76° - \cot 76° \cdot \cot 16° =$

48. The number of all possible 5-tuples $(a_1, a_2, a_3, a_4, a_5)$ such that $a_1 + a_2\sin x + a_3\cos x + a_4\sin 2x + a_5\cos 2x = 0$ holds for all x is

49. If m and n are positive integers satisfying $1 + \cos 2\theta + \cos 4\theta + \cos 6\theta + \cos 8\theta + \cos 10\theta = \frac{\cos m\theta \cdot \sin n\theta}{\sin\theta}$ then $m + n$ is equal to

50. $\log[\sec(2022\,\pi) + \sec(2023\,\pi) + \cos(12345678\,\pi)] =$

51. If cosine of odd integral multiple of $\frac{\pi}{2}$ is a, tangent of integral multiple of π is b, sine of integral multiple of π is c, cosine of odd integral multiple of π is d and even integral multiple of e and let $\sin\left(2n\pi + \frac{\pi}{2}\right) = f, \sin\left(2n\pi - \frac{\pi}{2}\right) = g, (n \in I)$ and then $a + b + c + d + e + f + g =$

PARIKSHIT (JEE ADVANCED LEVEL)

SINGLE CORRECT TYPE QUESTIONS

1. If $\tan\frac{\pi}{9}, x$ and $\tan\frac{5\pi}{18}$ are in A.P. and $\tan\frac{\pi}{9}, y$ and $\tan\frac{7\pi}{18}$ are also in A.P., then

(*a*) $2x = y$ (*b*) $x > 2$

(*c*) $x = y$ (*d*) None of these

2. It is known that $\sin\beta = \frac{4}{5}$ and $0 < \beta < \pi$ then the value of $\frac{\sqrt{3}\sin(\alpha+\beta) - \frac{2}{\cos(\pi/6)}\cos(\alpha+\beta)}{\sin\alpha}$ is

(*a*) Independent of α for all β in $(0, \pi)$

(*b*) $\frac{5}{13}$ for $\tan\beta < 0$

MATCH THE COLUMN TYPE QUESTIONS

16. Match the column:

	Column-I		Column-II
A.	The value of $\frac{\sin 22^\circ}{\sin 56^\circ \sin 34^\circ \cot 68^\circ}$ equals to	p.	5
B.	The value of $\left(\cos 65^\circ + \sqrt{3}\sin 5^\circ + \sin 85^\circ\right)^2$ = μ $\cos^2 25^\circ$, then value of μ be	q.	2
C.	If $f(\theta) = 2\sin 2\theta - \sin^2\theta + 3\cos^2\theta$, then number of intergers in the range of $f(\theta)$ equals to	r.	3
D.	If $f(\theta) = \cos(\sin\theta) + \sin(\sin\theta)$, then number of integers in the Range of $f(\theta)$ equals to	s.	1

(*a*) A → r; B → s; C → p; D → q

(*b*) A → q; B → r; C → p; D → q

(*c*) A → q; B → r; C → q; D → p

(*d*) A → q; B → r; C → s; D → p

17. If α and β are the roots of the equation, $a\cos\theta + b\sin\theta = c$ then match the entries of Column-I with the entries of Column-II.

	Column-I		Column-II
A.	sin α + sin β	p.	$\frac{2b}{a+c}$
B.	sin α · sin β	q.	$\frac{c-a}{c+a}$
C.	$\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$	r.	$\frac{2bc}{a^2+b^2}$
D.	$\tan\frac{\alpha}{2}\cdot\tan\frac{\beta}{2} =$	s.	$\frac{c^2-a^2}{a^2+b^2}$

(*a*) A → r; B → s; C → p; D → q

(*b*) A → q; B → r; C → p; D → q

(*c*) A → q; B → r; C → q; D → p

(*d*) A → q; B → r; C → s; D → p

18. Match the columns:

	Column-I		Column-II
A.	cot 35° + cot 145° + 3 is equal to	p.	1
B.	$\frac{1}{\sec^2 120^\circ} + \frac{1}{\text{cosec}^2 120^\circ}$ is equal to	q.	3
C.	If $(\sin 2)^x + (\cos 2)^x = 1$ then x is	r.	0
D.	$\tan\frac{\pi}{11} + \tan\frac{2\pi}{11} + \tan\frac{4\pi}{11} + \tan\frac{7\pi}{11} \tan\frac{9\pi}{11} + \tan\frac{10\pi}{11}$	s.	2
E.	cos 10° + cos 20° + cos 30° + ... + cos 80° + cos 100° + cos 150° + cos 160° + cos 170° + 4	t.	4

(*a*) A → q; B → p; C → s; D → r; E → t

(*b*) A → r; B → s; C → p; D → q; E → q

(*c*) A → r; B → s; C → p; D → t; E → q

(*d*) A → r; B → s; C → p; D → q; E → t

NUMERICAL TYPE QUESTIONS

19. If $\text{cosec}\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, then $a^2 b^2 (a^2 + b^2) =$

20. If the expression tan(55°)tan(65°)tan(75°) simplifies to cot(x°) and m is the numerical value of the expression tan (27°) + tan (18°) + tan (27°) tan (18°), then find the value of $(m + x + 1)$.

21. Given that for $a, b, c, d \in R$. if a sec (190°) – c tan (190°) = d and b sec(190°) + d tan (190°) = c, then find the value of $\left(\frac{a^2+b^2+c^2+d^2}{bd-ac}\right)\sin 20^\circ$.

22. If $P = \sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta$, $Q = \text{cosec}^6\theta - \cot^6\theta - 3\,\text{cosec}^2\theta\cot^2\theta$ and $R = \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$, then find the value of $(P + Q + R)^{(P+Q+R)}$

23. If the angles α & β satisfy the relation $\frac{\sin\beta}{\sin(2\alpha+\beta)} = \frac{n}{m}(|m| > |n|) \Rightarrow \frac{a+\frac{\tan\beta}{\tan\alpha}}{m+n} = \frac{a-\tan\alpha\tan\beta}{m-n}$, then $a =$

24. Let $k = 1°$, then and $\sum_{n=0}^{88}\sec(nk)\sec(n+\alpha)k = \frac{\cos k}{\sin^2 k}$, then α =

25. If α + β + γ = π and $\tan\left(\frac{\beta+\gamma-\alpha}{4}\right)\cdot\tan\left(\frac{\gamma+\alpha-\beta}{4}\right)\cdot\tan\left(\frac{\alpha+\beta-\gamma}{4}\right) = 1$, and a + cos α + cos β + cos γ = 0, then $a =$

26. If $A + B + C = 2S$ and

$\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S = \alpha \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$, then $\alpha =$

27. If $A + B + C = 2S$, and

$\cos^2 A + \cos^2 B + \cos^2 C + 2\cos A \cos B \cos C = 1 + \alpha \cos S \cos(S-A)\cos(S-B)\cos(S-C)$, then $\alpha =$

28. If $\cos(-A+B+C) + \cos(A-B+C) + \cos(A+B-C) + \cos(A+B+C) = \lambda \cos A \cos B \cos C$. Then find λ.

29. If $x + y + z = xyz$, then

$$\frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \frac{\lambda x - x^3}{1-3x^2} \cdot \frac{\lambda y - y^3}{1-3y^2} \cdot \frac{\lambda z - z^3}{1-3z^2}.$$

Find λ.

30. For ΔABC

$$\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1 = \lambda \sin\frac{\pi - A}{4}\sin\frac{\pi - B}{4}\sin\frac{\pi - C}{4}.$$

Then $\lambda =$

PYQ'S (PAST YEAR QUESTIONS)

BASED ON TRIGONOMETRIC FORMULAE

1. If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$, then the value of $\left(a + \frac{1}{a}\right)$ is: **[30 Jan, 2023 (Shift-I)]**

(*a*) 4 (*b*) $4 - 2\sqrt{3}$

(*c*) 2 (*d*) $5 - \frac{3}{2}\sqrt{3}$

2. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to **[10 April, 2023 (Shift-I)]**

(*a*) 3 (*b*) 2

(*c*) 4 (*d*) 1

3. $16\sin(20°)\sin(40°)\sin(80°)$ is equal to: **[26 June, 2022 (Shift-II)]**

(*a*) $\sqrt{3}$ (*b*) $2\sqrt{3}$

(*c*) 3 (*d*) $4\sqrt{3}$

4. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to: **[27 June, 2022 (Shift-I)]**

(*a*) –1 (*b*) $-\frac{1}{2}$ (*c*) $-\frac{1}{3}$ (*d*) $-\frac{1}{4}$

5. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to **[25 July, 2022 (Shift-II)]**

(*a*) $\frac{3}{16}$ (*b*) $\frac{1}{16}$ (*c*) $\frac{1}{32}$ (*d*) $\frac{9}{32}$

6. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are **[28 June, 2022 (Shift-II)]**

(*a*) $-\frac{1}{7}$ and IVth quadrant (*b*) 7 and Ist quadrant

(*c*) –7 and IVth quadrant (*d*) $\frac{1}{7}$ and Ist quadrant

7. The value of $2\sin(12°) - \sin(72°)$ is: **[25 June, 2022 (Shift-II)]**

(*a*) $\frac{\sqrt{5}(1-\sqrt{3})}{4}$ (*b*) $\frac{1-\sqrt{5}}{8}$

(*c*) $\frac{\sqrt{3}(1-\sqrt{5})}{2}$ (*d*) $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

8. If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$ then $16 + \alpha^{-1}$ is equal to..... **[26 June, 2022 (Shift-I)]**

9. Let α and β be real numbers such that $-\frac{\pi}{4} < \beta < 0 < \alpha < \frac{\pi}{4}$. If $\sin(\alpha+\beta) = \frac{1}{3}$ and $\cos(\alpha-\beta) = \frac{2}{3}$ then the greatest integer less than or equal to

$\left(\frac{\sin\alpha}{\cos\beta} + \frac{\cos\beta}{\sin\alpha} + \frac{\cos\alpha}{\sin\beta} + \frac{\sin\beta}{\cos\alpha}\right)^2$ is ____. **[JEE Adv, 2022]**

10. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x+y) = 3/2$, Then, $\sin x + \cos y$ is equal to: **[25 Feb, 2021 (Shift-II)]**

(*a*) $\frac{1+\sqrt{3}}{2}$ (*b*) $\frac{1-\sqrt{3}}{2}$

(*c*) $\frac{\sqrt{3}}{2}$ (*d*) $\frac{1}{2}$

11. The value of $\cot\left(\frac{\pi}{24}\right)$ is: **[25 July, 2021 (Shift-II)]**

(*a*) $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

(*b*) $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

(*c*) $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

(*d*) $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

25. The number of elements in the set

$$S=\left\{x\in\mathbb{R}:2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$$

[29 July, 2022 (Shift-II)]

(*a*) 1 (*b*) 3

(*c*) 0 (*d*) Infinite

26. The maximum value of $3\cos\theta + 5\sin\left(\theta-\frac{\pi}{6}\right)$ for any real value of θ is: **[12 Jan, 2019 (Shift-I)]**

(*a*) $\sqrt{19}$ (*b*) $\frac{\sqrt{79}}{2}$

(*c*) $\sqrt{34}$ (*d*) $\sqrt{31}$

PW CHALLENGERS

1. $\sqrt[3]{\cos\frac{2\pi}{7}}+\sqrt[3]{\cos\frac{4\pi}{7}}+\sqrt[3]{\cos\frac{8\pi}{7}}=\sqrt[3]{\frac{1}{2}(a-3\sqrt[3]{b})}$, then find $a+b$.

2. For every positive integer *n*,

$\tan\frac{\pi}{2n+1}.\tan\frac{2\pi}{2n+1}...\tan\frac{n\pi}{2n+1}=\sqrt{\lambda n+\mu}$. Find $\lambda+\mu$.

3. $\cos\theta=\frac{a}{b+c},\cos\phi=\frac{b}{a+c},\cos\psi=\frac{c}{a+c}$. where θ, ϕ, ψ ∈ (0, π) and *a*, *b*, *c* are sides of triangle *ABC* then $\tan^2\frac{\theta}{2}+\tan^2\frac{\phi}{2}+\tan^2\frac{\psi}{2}$ is equal to _____.

4. If $a=\frac{2\pi}{2023}$ and $\cos a\cos 2a\cos 3a...\cos 1011a=\frac{1}{k}$, then $\frac{\log_2 k}{1011}$ is equal to _____

5. For $0\le\theta<2\pi$, let

$$P=\frac{1}{2}\cos\theta-\frac{1}{4}\sin 2\theta-\frac{1}{8}\cos 3\theta+\frac{1}{16}\sin 4\theta$$
$$+\frac{1}{32}\cos 5\theta-\frac{1}{64}\sin 6\theta-\frac{1}{128}\cos 7\theta+...\text{ and}$$
$$Q=1-\frac{1}{2}\sin\theta-\frac{1}{4}\cos 2\theta-\frac{1}{8}\sin 3\theta+\frac{1}{16}\cos 4\theta$$
$$-\frac{1}{32}\sin 5\theta-\frac{1}{64}\cos 6\theta+\frac{1}{128}\sin 7\theta+...$$

so that $\frac{P}{Q}=\frac{2\sqrt{2}}{7}$. Then $\sin\theta=-\frac{m}{n}$, where *m* and *n* are relatively prime positive integer, then $\left(\frac{m+n}{9}\right)$ is equal to

6. The value the expression

$$\left(\tan^2\frac{\pi}{7}+\tan^2\frac{2\pi}{7}+\tan^2\frac{3\pi}{7}\right)\left(\cot^2\frac{\pi}{7}+\cot^2\frac{2\pi}{7}+\cot^2\frac{3\pi}{7}\right)$$

is equal to.

7. The value of $\left[\tan\frac{3\pi}{11}+4\sin\frac{2\pi}{11}\right]$, (where [·] represents G.I.F) is equal to.

8. Sum of integral value(s) in the range of $f(x) = \cos x$

$\left\{\sin x+\sqrt{\sin^2 x+\sin^2\alpha}\right\}$ $\{x\in R\}$ is equal to

9. If $x\sin a + y\sin 2a + z\sin 3a = \sin 4a$

$x\sin b + y\sin 2b + z\sin 3b = \sin 4b$

$x\sin c + y\sin 2c + z\sin 3c = \sin 4c$

then roots of equation $t^3-\left(\frac{z}{2}\right)t^2-\left(\frac{y+2}{4}\right)t+\left(\frac{z-x}{8}\right)$ = 0, $a,b,c\ne n\pi$, are

(*a*) sin *a*, sin *b*, sin *c* (*b*) cos *a*, cos *b*, cos *c*

(*c*) sin 2*a*, sin 2*b*, sin 2*c* (*d*) cos 2*a*, cos 2*b*, cos 2*c*

10. Let *x*, *y*, *z* be real numbers with $x\ge y\ge z\ge\frac{\pi}{12}$ such that $x+y+z=\frac{\pi}{2}$

(*a*) Maximum $(\cos x\cdot\sin y\cdot\cos z)=\frac{2+\sqrt{3}}{4}$

(*b*) Minimum $(\cos x\cdot\sin y\cdot\cos z)=\frac{1}{8}$

(*c*) Minimum value occurs when $x=y=\frac{5\pi}{24}$ *and* $z=\frac{\pi}{12}$

(*d*) Maximum $(\cos x\cdot\sin y\cdot\cos z)=\frac{2+\sqrt{3}}{8}$

Answer Key

CONCEPT APPLICATION

1. [20π/3] **2.** [90,60,30] **3.** [1.7] **4.** [1] **5.** [8/15] **6.** [–1] **7.** (*c*) **8.** [2]

9. A → x; B → w; C → v; D → u; E → t; F → s; G → r; H → q; I → p **12.** [17] **13.** (*b*)

15. $\left[\sin\alpha=\frac{\sqrt{2+\sqrt{2}}}{2},\cos\alpha=-\frac{\sqrt{2-\sqrt{2}}}{2}\right]$ **16.** [–1/2] **17.** [–2/3] **18.** (*d*) **21.** $\left(-\infty,-\frac{1}{3}\right]\cup\left[\frac{1}{7},\infty\right)$

22. $\left(4-\sqrt{10},4+\sqrt{10}\right)$ **23.** (*b*) **24.** (*d*) **25.** (*b*) **26.** [1/2]

SCHOOL LEVEL PROBLEMS

1. (*b*) **2.** (*c*) **3.** (*c*) **4.** (*d*) **5.** (*c*) **6.** (*c*) **7.** (*c*) **8.** (*b*) **9.** (*a*) **22.** (c)

23. (*c*) **24.** (*d*) **25.** (*c*)

PRARAMBH (TOPICWISE)

1. (*a*) **2.** (*a*) **3.** (*d*) **4.** (*b*) **5.** (*a*) **6.** (*c*) **7.** (*d*) **8.** (*b*) **9.** (*c*) **10.** (*b*)

11. (*a*) **12.** (*a*) **13.** (*d*) **14.** (*a*) **15.** (*b*) **16.** (*c*) **17.** (*d*) **18.** (*d*) **19.** (*b*) **20.** (*d*)

21. (*a*) **22.** (*b*) **23.** (*a*) **24.** (*b*) **25.** (*a*) **26.** (*d*) **27.** (*a*) **28.** (*b*) **29.** (*c*) **30.** (*a*)

31. (*d*) **32.** (*d*) **33.** (*b*) **34.** (*b*) **35.** (*c*) **36.** (*b*) **37.** (*a*) **38.** (*d*) **39.** (*c*) **40.** (*a*)

41. (*a*) **42.** (*c*) **43.** (*c*) **44.** (*c*) **45.** (*d*) **46.** (*a*) **47.** (*c*) **48.** (*c*) **49.** (*b*) **50.** (*a*)

51. (*a*) **52.** (*c*) **53.** (*a*) **54.** (*d*) **55.** (*c*) **56.** (*c*) **57.** (*d*) **58.** (*b*) **59.** (*a*) **60.** (*c*)

61. (*c*) **62.** (*b*) **63.** (*b*) **64.** (*b*) **65.** (*a*)

PRABAL (JEE MAIN LEVEL)

1. (*a*) **2.** (*d*) **3.** (*c*) **4.** (*c*) **5.** (*c*) **6.** (*c*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*d*)

11. (*a*) **12.** (*a*) **13.** (*a*) **14.** (*b*) **15.** (*c*) **16.** (*a*) **17.** (*b*) **18.** (*a*) **19.** (*b*) **20.** (*b*)

21. (*b*) **22.** (*b*) **23.** (*c*) **24.** (*b*) **25.** (*b*) **26.** (*d*) **27.** (*a*) **28.** (*a*) **29.** (*a*) **30.** (*c*)

31. (*b*) **32.** (*c*) **33.** (*a*) **34.** (*a*) **35.** (*a*) **36.** (*a*) **37.** (*d*) **38.** (*b*) **39.** (*b*) **40.** (*b*)

41. [0] **42.** [1] **43.** [4] **44.** [1] **45.** [4] **46.** [80] **47.** [3] **48.** [0] **49.** [11] **50.** [0]

51. [0]

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*a*) **2.** (*d*) **3.** (*c*) **4.** (*c*) **5.** (*d*) **6.** (*d*) **7.** (*a*,*b*) **8.** (*b*,*d*) **9.** (*a*,*b*,*c*,*d*) **10.** (*b*,*c*)

11. (*b*,*c*) **12.** (*a*,*b*) **13.** (*c*) **14.** (*a*) **15.** (*b*) **16.** (*b*) **17.** (*a*) **18.** (*a*) **19.** [1] **20.** [7]

21. [2] **22.** [27] **23.** [1] **24.** [1] **25.** [1] **26.** [4] **27.** [4] **28.** [4] **29.** [3] **30.** [4]

PYQ's (PAST YEAR QUESTIONS)

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*b*) **5.** (*b*) **6.** (*a*) **7.** (*d*) **8.** [80] **9.** [1] **10.** (*a*)

11. (*b*) **12.** (*b*) **13.** (*d*) **14.** (*b*) **15.** [1] **16.** (*a*) **17.** (*d*) **18.** (*d*) **19.** (*b*) **20.** (*b*)

21. (*b*) **22.** (*a*) **23.** (*a*,*b*,*d*) **24.** (*d*) **25.** (*a*) **26.** (*a*)

PW CHALLENGERS

1. [12] **2.** [3] **3.** [1] **4.** [1] **5.** [4] **6.** [105] **7.** [3] **8.** [0] **9.** (*d*) **10.** (*b*,*d*)

CHAPTER 4

Motion in a Plane

SCALAR AND VECTOR

(*i*) Scalar: A **scalar** quantity requires only a number for its complete description. Mass, volume, density, pressure and temperature are all examples of scalar quantities. The mathematics of scalar quantities is the ordinary algebra of numbers.

(*ii*) Vector: A **vector** quantity requires both magnitude and direction for its complete description. Velocity, acceleration, force and momentum are examples of vector quantities.

A vector quantity is usually represented by an arrow. The arrow points in the direction of vector quantity and length of the arrow is proportional to magnitude of the vector quantity. For example if a displacement of 10 m due east is represented by an arrow of length 1 cm pointing east then displacement of 50 m due north will be represented by an arrow of length 5 cm pointing due north.

A vector quantity is usually denoted by an arrow over the corresponding letter such as $\vec{A}$ or $\vec{a}$ and magnitude of vector by $|\vec{A}|$ or $|\vec{a}|$.

Types of Vector

(*i*) Equal vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(*ii*) Parallel vector: Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(*a*) Both have same direction.

(*b*) One vector is scalar (positive) non-zero multiple of another vector.

(*iii*) Anti-parallel vectors: Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when

(*a*) Both have opposite direction.

(*b*) One vector is scalar non-zero negative multiple of another vector.

(*iv*) Zero vector $(\vec{0})$**:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

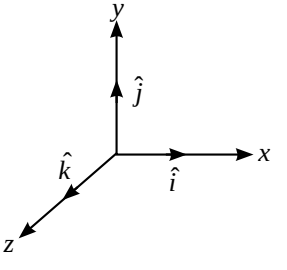
(*v*) Unit vector: A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap/A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A}$

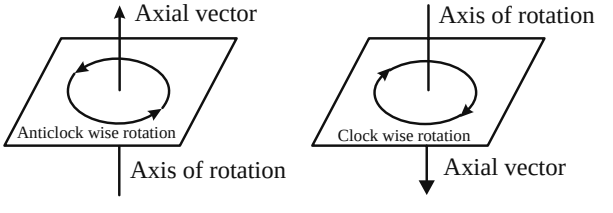
$\Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

(*vi*) Orthogonal unit vectors: If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z* axes respectively then $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we curl the fingers of right hand from *x* to *y* then we must get the direction of *z* along thumb).



(*vii*) Axial Vectors: These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are examples of axial vectors.



(*viii*) Coplanar vectors: Three (or more) vectors are called coplanar vectors if they lie in the same plane. Two (free) vectors are always coplanar.

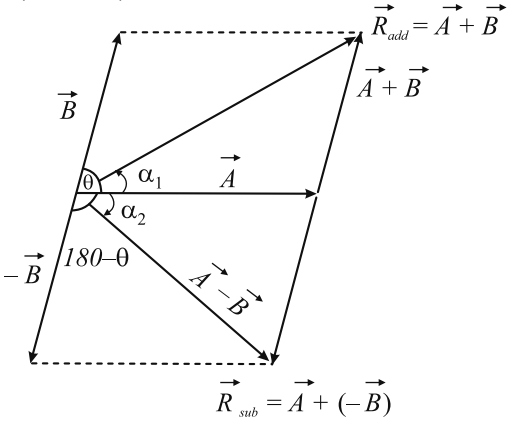
(*ix*) Collinear Vectors: Those vectors which are expressed along same straight line and are parallel or anti-parallel are called collinear vectors.

SUBTRACTION OF VECTORS

Since $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$ and $|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$

$\Rightarrow |\vec{A}-\vec{B}|=\sqrt{A^2+B^2+2AB\cos(180^o-\theta)}$

Since, $\cos(180°-\theta)=-\cos\theta$



$\Rightarrow |\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2AB\cos\theta}$

$$\tan\alpha_1=\frac{B\sin\theta}{A+B\cos\theta} \Rightarrow \tan\alpha_2=\frac{B\sin(180-\theta)}{A+B\cos(180-\theta)}$$

But $\sin(180°-\theta)=\sin\theta$ and $\cos(180°-\theta)=-\cos\theta$

$$\Rightarrow \tan\alpha_2=\frac{B\sin\theta}{A-B\cos\theta}$$

Train Your Brain

Example 1: There are two force vectors, one of 5 N and other of 12 N. At what angle should the two vectors be added to get resultant vector of 17 N, 7 N and 13 N respectively?

(*a*) 0°, 180° and 90° (*b*) 0°, 90° and 180°

(*c*) 0°, 90° and 90° (*d*) 180°, 0° and 90°

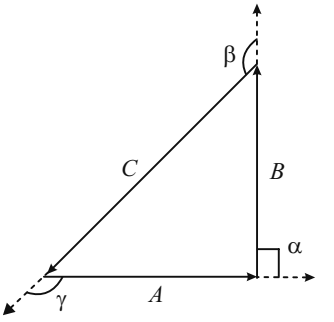
Sol. For 17 N, both the vectors should be parallel i.e. angle between them should be zero. For 7 N, both the vectors should be antiparallel i.e. angle between them should be 180°. For 13 N both the vectors should be perpendicular to each other i.e. angle between them should be 90°.

Example 2: Given that $\vec{A}+\vec{B}+\vec{C}=0$. Out of three vectors two are equal in magnitude and the magnitude of third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. Then the angles between vectors are given by

(*a*) 30°, 60°, 90° (*b*) 45°, 45°, 90°

(*c*) 45°, 60°, 90° (*d*) 90°, 135°, 135°

Sol. From polygon law, three vectors having summation zero should form a closed polygon. (Triangle) since the two vectors are having same magnitude and the third vector is $\sqrt{2}$ times that of either of two having equal magnitude. i.e. the triangle should be right angled triangle



Angle between *A* and *B*, $\alpha = 90^o$

Angle between *B* and *C*, $\beta = 135^o$

Angle between *A* and *C*, $\gamma = 135^o$

Example 3: Two vectors of 10 units and 5 units make an angle of 120° with each other. Find the magnitude and angle of resultant with the vector of 10 unit magnitude.

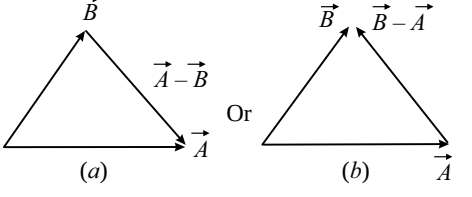
Sol. $|\vec{a}+\vec{b}|=\sqrt{a^2+b^2+2ab\cos\theta}$

$$=\sqrt{100+25+2\times10\times5(-1/2)}\ =\ 5\sqrt{3}$$

$$\tan\alpha=\frac{5\sin120°}{10+5\cos120°}=\frac{5\sqrt{3}}{20-5}=\frac{5\sqrt{3}}{5\times3}=\frac{1}{\sqrt{3}}$$

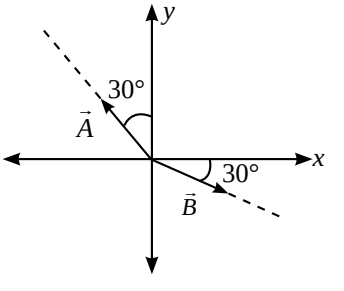
$\Rightarrow \alpha = 30°$

Note: $\vec{A}-\vec{B}$ or $\vec{B}-\vec{A}$ can also be found by making triangles as shown in figure. (*a*) and (*b*)



Concept Application

1. A vector $\vec{A}$ makes an angle 30° with the *y*-axis in anticlockwise direction. Another vector $\vec{B}$ makes an angle 30° with the *x*-axis in clockwise direction. Find angle between vectors $\vec{A}$ and $\vec{B}$.



at $t = 0$, $u = 0$, $\omega_0 = 0$

$\alpha = \frac{a_t}{R} = \frac{\pi}{2}$ rad/s^2

(*a*) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \pi = 0 + \frac{1}{2}\frac{\pi}{2}t^2 \Rightarrow t = 2$ sec

(*b*) $v = u + a_t t = 0 + \frac{\pi}{2} \times 2 = \pi$ m/s

(*c*) $a_t = \frac{\pi}{2}$ m/s^2, $a_c = \frac{v^2}{r} = \pi^2$ m/s^2

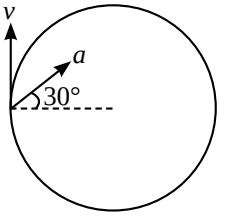
$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{\frac{\pi^2}{4} + \pi^4} = \frac{\pi}{2}\sqrt{1 + 4\pi^2} \text{ m/s}^2$$

25. Figure shows the direction of total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this instant if magnitude of net acceleration is 25 m/sec^2, find:

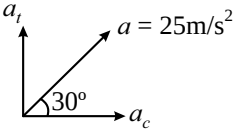
(*i*) The radial acceleration,

(*ii*) The speed of the particle and

(*iii*) Its tangential acceleration



Sol.



(i) $a_c = a\cos 30^\circ = 25\frac{\sqrt{3}}{2}$ m/s^2

(ii) $a_c = \frac{v^2}{R} \Rightarrow v^2 = a_c R = 25\frac{\sqrt{3}}{2} \times 2.5$

$$v = \left(125\frac{\sqrt{3}}{4}\right)^{1/2} \text{ m/s}$$

(iii) $a_t = a\sin 30^\circ = \frac{25}{2}$ m/s^2

26. Two particles *A* and *B* move anticlockwise with the same speed v in a circle of radius *R* and are diametrically opposite to each other. At $t = 0$, A is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. Calculate the time in which *A* collides with *B*, the angle traced by *A* during this time, its angular velocity and radial acceleration of *A* at the time of collision.

Sol. $\omega_{0(rel)} = 0$, $\theta_{rel} = \pi$, $\alpha_{rel} = \frac{72v^2}{25\pi R^2}$

$\theta_{rel} = \omega_{0(rel)}\, t + \frac{1}{2}\alpha_{rel}\, t^2$

$\pi = 0 + \frac{1}{2}\frac{72v^2}{25\pi R^2}t^2 \Rightarrow t = \frac{5\pi R}{6v}$

Angle traced by *A*, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = \frac{v}{R}\cdot\frac{5\pi R}{6v} + \frac{1}{2}\frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)^2$

$= \frac{5\pi}{6} + \pi = \frac{11}{6}\pi$

Angular velocity $\omega = \omega_0 + \alpha t = \frac{v}{R} + \frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)$

$= \frac{v}{R} + \frac{12v}{5R} = \frac{17v}{5R}$

$a_c = \omega^2 R = \left(\frac{17v}{5R}\right)^2 R = \frac{289v^2}{25R}$

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first?

(*a*) Slower one

(*b*) Faster one

(*c*) Both will reach simultaneously

(*d*) Cannot be predicted

2. A projectile can have same range *R* for two angles of projection. If t_1 and t_2 are the times of flight in the two cases, then what is the product of two times of flight?

(*a*) $t_1 t_2 \propto R^2$ (*b*) $t_1 t_2 \propto R$

(*c*) $t_1 t_2 \propto \frac{1}{R}$ (*d*) $t_1 t_2 \propto \frac{1}{R^2}$

3. The range *R* of projectile is same when its maximum heights are h_1 and h_2. What is the relation between *R*, h_1, and h_2?

(*a*) $R = \sqrt{h_1 h_2}$ (*b*) $R = \sqrt{2h_1 h_2}$

(*c*) $R = 2\sqrt{h_1 h_2}$ (*d*) $R = 4\sqrt{h_1 h_2}$

23. Given two vectors. $\vec{A} = 3\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$. Find the

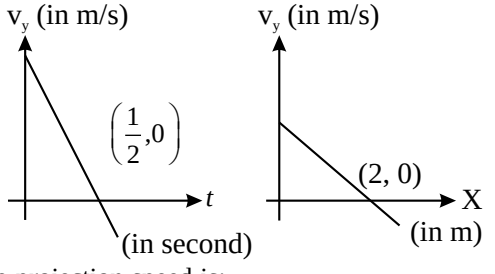
(*a*) area of the triangle whose two sides are represented by the vectors $\vec{A}$ and $\vec{B}$.

(*b*) area of the parallelogram whose two adjacent sides are represented by the vectors $\vec{A}$ and $\vec{B}$.

(*c*) area of the parallelogram whose diagonals are represented by the vectors $\vec{A}$ and $\vec{B}$.

CASE STUDY BASED QUESTIONS

24. Two graphs of the same projectile motion (in the *xy*-plane) projected from origin are shown. *x*-axis is along horizontal direction and *y*-axis is vertically upwards. Take $g = 10\ ms^{-2}$.



(*i*) The projection speed is:

(*a*) $\sqrt{37}\ ms^{-1}$ (*b*) $\sqrt{41}\ ms^{-1}$

(*c*) $\sqrt{14}\ ms^{-1}$ (*d*) $\sqrt{40}\ ms^{-1}$

(*ii*) Projection angle with the horizontal is:

(*a*) $\tan^{-1}\left(\frac{4}{5}\right)$ (*b*) $\tan^{-1}\left(\frac{2}{3}\right)$

(*c*) $\tan^{-1}\left(\frac{5}{4}\right)$ (*d*) $\tan^{-1}\left(\frac{1}{2}\right)$

(*iii*) Maximum height attained from the point of projection is

(*a*) 1.25 m (*b*) 12.5 m

(*c*) 2.25 m (*d*) None of these

25. A uniform circular motion is the motion of a particle travelling at a constant (uniform) speed along circular path and hence its kinetic energy remains same everywhere. But when a particle moves in a vertical circle completing the loop, then its speed goes on changing at every point & hence, its kinetic energy goes on changing but total mechanical energy remains constant.

(*i*) Uniform circular motion is a example of

(*a*) accelerated motion

(*b*) uniform motion

(*c*) non-accelerated motion

(*d*) None of the above

(*ii*) An insect trapped in a circular groove of radius 14 cm moves along the groove steadily and completes 10 revolutions in 100 s. The linear speed of the insect is

(*a*) 4.3 cm s^{-1} (*b*) 8.8 cm s^{-1}

(*c*) 6.3 cm s^{-1} (*d*) 7.3 cm s^{-1}

(*iii*) What is the magnitude of average velocity after half rotation, if a particle is moving with constant speed *v* in a circle?

(*a*) $2v$ (*b*) $\frac{2v}{\pi}$ (*c*) $\frac{v}{\pi}$ (*d*) $\frac{v}{2\pi}$

(*iv*) A body moving along a circular path of radius *R* with velocity *v*, has centripetal acceleration *a*. If its velocity is made equal to 2*v*. What will be the centripetal acceleration?

(*a*) $4a$ (*b*) $2a$ (*c*) $\frac{a}{4}$ (*d*) $\frac{a}{2}$

(*v*) A point moves along a circle with a velocity v = at, where $a = 0.50\ m/s^2$. Find the total acceleration of the point at the moment when it has covered the n^{th} (n = 0.10) fraction of the circle after beginning of the motion.

(*a*) $0.8 ms^{-2}$ (*b*) $0.6 ms^{-2}$

(*c*) $0.7 ms^{-2}$ (*d*) $0.9 ms^{-2}$

PRARAMBH (TOPICWISE)

RESULTANT OF TWO VECTORS

1. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(*a*) Can be zero

(*b*) Cannot be zero

(*c*) Lies in the plane of $\vec{A}$ and $\vec{B}$

(*d*) Lies in the plane of $\vec{A}$ and $\vec{A} + \vec{B}$

2. The vector sum of the forces of 10 N and 6 N can be

(*a*) 2 N (*b*) 8 N (*c*) 18 N (*d*) 20 N

3. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a

(*a*) Scalar quantity (*b*) Pseudo vector

(*c*) Unit vector (*d*) Null vector.

4. A car drives 6.0 km east, then 8 km north, and then 21 km west. The magnitude of the resulting displacement from origin is

(*a*) 35 km (*b*) 23 km

(*c*) 21 km (*d*) 17 km

DIFFERENCE OF TWO VECTORS

5. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(*a*) Magnitude

(*b*) Direction

(*c*) Magnitude as well as direction

(*d*) Neither magnitude nor direction

102. Two bodies of mass 10 kg and 5 kg are moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is
(*a*) R/r (*b*) r/R (*c*) R^2/r^2 (*d*) r^2/R^2

103. A stone is tied to one end of a string 50 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 10 revolutions in 20 s, what is the magnitude of acceleration of the stone?
(*a*) 493 cm/s^2 (*b*) 720 cm/s^2
(*c*) 860 cm/s^2 (*d*) 990 cm/s^2

104. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$, then
(*a*) Both P and Q have the same acceleration
(*b*) Both P and Q do not have any acceleration
(*c*) P has greater acceleration than Q
(*d*) Q has greater acceleration than P

105. The formula for centripetal acceleration in a circular motion is
(*a*) $\vec{\alpha}\times\vec{r}$ (*b*) $\vec{\omega}\times\vec{v}$ (*c*) $\vec{\alpha}\times\vec{v}$ (*d*) $\vec{\omega}\times\vec{r}$

106. A particle is moving in a horizontal circle with constant speed. It has constant
(*a*) Velocity (*b*) Acceleration
(*c*) Kinetic energy (*d*) Displacement

107. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω?
(*a*) The centripetal acceleration remains unchanged.
(*b*) The centripetal acceleration is halved.
(*c*) The centripetal acceleration is doubled.
(*d*) The centripetal acceleration is quadrupled.

108. A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega},\vec{v},\vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is not correct?
(*a*) $\vec{\omega}\perp\vec{v}$ (*b*) $\vec{\omega}\perp\vec{\alpha}$ (*c*) $\vec{\omega}\perp\vec{a}_c$ (*d*) $\vec{v}\perp\vec{a}_c$

109. A particle is moving in circular path with constant tangential acceleration. Time t after the beginning of motion the direction of net acceleration is at 45° to radius vector at the instant. The angular acceleration of the particle at time 't' is proportional to:
(*a*) $\frac{1}{t}$ (*b*) $\frac{1}{t^2}$ (*c*) $\frac{3}{t}$ (*d*) t^0

110. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing its speed at the rate of 'a' m/s^2, then the resultant acceleration will be:
(*a*) $\sqrt{\left(\frac{v^2}{r^2}-a^2\right)}$ (*b*) $\sqrt{\left(\frac{v^4}{r^2}+a^2\right)}$
(*c*) $\sqrt{\left(\frac{v^4}{r^2}-a^2\right)}$ (*d*) $\sqrt{\left(\frac{v^2}{r^2}+a^2\right)}$

PRABAL (JEE MAIN LEVEL)

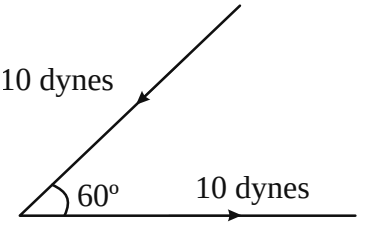
1. If the angle between two forces increases, the magnitude of their resultant
(*a*) decreases
(*b*) increases
(*c*) remains unchanged
(*d*) first decreases and then increases

2. Which of the following sets of displacements might be capable of bringing a car to its initial point
(*a*) 5, 10, 30 and 50 unit
(*b*) 5, 9, 9 and 16 unit
(*c*) 40, 40, 90 and 200 unit
(*d*) 10, 20, 40 and 90 unit

3. If the resultant of two forces of magnitudes P and Q acting at a point at an angle of 60º is $\sqrt{7}\,Q$, then P/Q is
(*a*) 1 (*b*) 3/2 (*c*) 2 (*d*) 4

4. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces. The angle between the two forces is
(*a*) 150º (*b*) 90º (*c*) 60º (*d*) 120º

5. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then their resultant is:



(*a*) 10 dynes (*b*) 20 dynes
(*c*) $10\sqrt{3}$ dynes (*d*) 5 dynes

6. The vector $\vec{P}$ makes 120° with the x-axis and vector $\vec{Q}$ makes 30° with the y-axis. What is their resultant? (taking anticlockwise as positive)
(*a*) $P + Q$ (*b*) $P - Q$
(*c*) $\sqrt{P^2+Q^2}$ (*d*) $\sqrt{P^2-Q^2}$

7. A vector $\vec{A}$ is directed along 30° west of north direction and another vector $\vec{B}$ along 15° south of east. Their resultant cannot be in _____ direction.
(*a*) North (*b*) East
(*c*) North-East (*d*) South

MULTIPLE CORRECT TYPE QUESTIONS

1. Which of the following is a true statement?

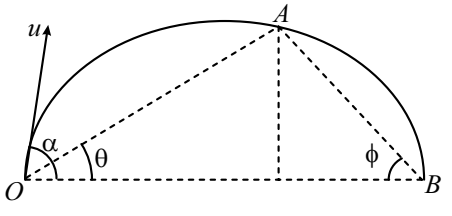
(*a*) A vector cannot be divided by another vector

(*b*) Angular displacement can either be a scalar or a vector

(*c*) Since addition of vectors is commutative therefore vector subtraction is also commutative

(*d*) The resultant of two equal forces of magnitude *F* acting at a point is F if the angle between the two forces is 120°

2. A football is kicked as shown in figure. When it is at point A, OA and OB make angles θ and ϕ with the horizontal. Find the relation between α, θ and ϕ.



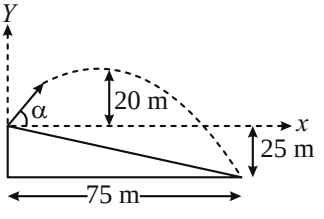
(*a*) $\cot\alpha = \cot\theta + \cot\phi$

(*b*) $\tan\alpha = \tan\theta - \tan\phi$

(*c*) $\cot\alpha = \cot\theta - \cot\phi$

(*d*) $\tan\alpha = \tan\theta + \tan\phi$

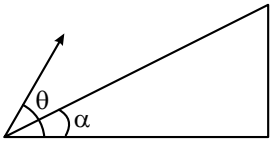
3. A ball thrown down the incline strikes at a point on the incline 25 m below the horizontal as shown in the figure. If the ball rises to a maximum height of 20 m above the point of projection, the angle of projection α (with horizontal *x*-axis) is



(*a*) $\tan^{-1}\frac{4}{3}$ (*b*) $\tan^{-1}\frac{3}{4}$

(*c*) $\tan^{-1}\frac{3}{2}$ (*d*) $\tan^{-1}\frac{2}{3}$

4. A projectile is fired at an angle θ with the horizontal. Find the condition under which it lands perpendicular on an inclined plane of inclination α as shown in figure.



(*a*) $\sin\alpha = \cos(\theta - \alpha)$

(*b*) $\cos\alpha = \sin(\theta - \alpha)$

(*c*) $\tan\theta = \cot(\theta - \alpha)$

(*d*) $\cot(\theta - \alpha) = 2\tan\alpha$

5. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4s. Which of the following are correct?

(*a*) The height of the table is 0.8 m

(*b*) It hits the ground at an angle of 60° with the vertical

(*c*) It covers a horizontal distance 1.6 m from the table

(*d*) It hits the ground with vertical velocity 4 m/s

6. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself

(*a*) The ball will always return to him.

(*b*) The ball will never return to him.

(*c*) The ball will return to him if the cart moves with constant velocity.

(*d*) The ball will fall behind him if the cart moves with some positive acceleration.

7. State which of the following statements is/are false.

(*a*) If two particles are neither approaching nor separating from each other, then their relative velocity is zero.

(*b*) If relative velocity of particle *B* with respect to *A* is $\vec{v}_1$, relative velocity of particle *C* with respect to *B* is $\vec{v}_2$ and particle *A* moves with velocity $\vec{v}_0$ with respect to ground, then the velocity of *C* with respect to ground cannot be zero. (assuming $\vec{v}_1$, $\vec{v}_2$ & $\vec{v}_0$ to be non zero)

(*c*) Four dogs are running along a line in the same direction, such that each is running relative to the dog in front of him with equal speed. Then the rate of separation between the third and the first dog is same as that of the fourth and the second dog. (where first, second, third and fourth are taken in order)

(*d*) At some instant of time at a place, two particles are observed and it is found that their relative velocity is zero. Then they will remain stationary with respect to each other.

8. A river is flowing with a velocity of 2 m/s. A boat is moving downstream along the river. Velocity of the boat in still water is 3 m/s. A person standing on the boat throws a ball (w.r.t. himself) in a plane perpendicular to the direction of motion of the boat with 10 m/s at 60° with the horizontal. When the ball reaches highest point of its path.

(*a*) The speed of ball w.r.t. man standing on boat is 5 m/s

(*b*) The speed of ball w.r.t. river is 3 m/s

(*c*) The speed of ball w.r.t. river is 0 m/s

(*d*) The speed of ball w.r.t. ground is $5\sqrt{2}$ m/s

20. Find the equation of trajectory of the boat.

(*a*) $x-1=\left(\frac{3y}{5}\right)^{1/3}$ (*b*) $x=u\left(\frac{6y}{5}\right)^{1/3}$

(*c*) $x-1=u\left(\frac{6y}{5}\right)^{1/3}$ (*d*) None of these

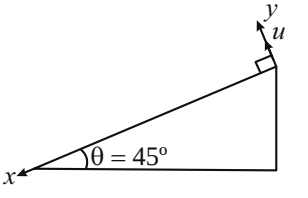
21. Find the drift of the boat when it is in the middle of the river.

(*a*) $u\left(\frac{3d}{5}\right)^{1/3}$ (*b*) $u\left(\frac{3d}{5}\right)^{1/3}+1$

(*c*) $u\left(\frac{6d}{5}\right)^{1/3}$ (*d*) None of these

MATCH THE COLUMN TYPE QUESTIONS

22. An inclined plane makes an angle θ = 45° with horizontal. A stone is projected normally from the inclined plane, with speed *u* m/s at *t* = 0 sec. *x* and *y* axes are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in Column-I with the results in Column-II. (*g* in Column-II is acceleration due to gravity.)



Column-I		Column-II	
A.	The instant of time at which velocity of stone is parallel to *x*-axis	p.	$\frac{2\sqrt{2}u}{g}$
B.	The instant of time at which velocity of stone makes an angle θ = 45° with positive *x*-axis in clockwise direction.	q.	$\frac{2u}{g}$
C.	The instant of time at which (starting from *t* = 0) component of displacement along *x*-axis become half the range on inclined plane is	r.	$\frac{\sqrt{2}u}{g}$
D.	Time of flight on inclined plane is	s.	$\frac{u}{\sqrt{2}g}$

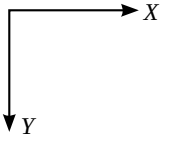
(*a*) A-(r); B-(q); C-(s); D-(p)
(*b*) A-(r); B-(s); C-(q); D-(p)
(*c*) A-(r); B-(s); C-(p); D-(q)
(*d*) A-(p); B-(q); C-(r); D-(s)

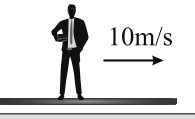
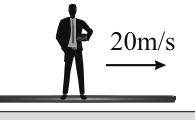
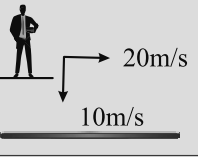
23. A particle is projected from level ground. Assuming projection point as origin, *x*-axis along horizontal and *y*-axis along vertically upwards. If particle moves in *x*-*y* plane and its path is given by $y = ax - bx^2$ where *a*, *b* are positive constants. Then match the physical quantities given in Column-I with the values given in Column-II. (*g* in Column II is acceleration due to gravity.)

Column-I		Column-II	
A.	Horizontal component of velocity	p.	$\frac{a}{b}$
B.	Time of flight	q.	$\frac{a^2}{4b}$
C.	Maximum height	r.	$\sqrt{\frac{g}{2b}}$
D.	Horizontal range	s.	$\sqrt{\frac{2a^2}{bg}}$

(*a*) A-(q); B-(s); C-(r); D-(p)
(*b*) A-(s); B-(r); C-(p); D-(q)
(*c*) A-(r); B-(s); C-(q); D-(p)
(*d*) A-(q); B-(s); C-(r); D-(p)

24. Rain is falling at velocity $10\hat{i}+10\hat{j}$ m/s. Then match the direction of velocity of rain w.r.t. man in Column-II from Column-I.



Column-I		Column-II	
A.		p.	
B.		q.	
C.		r.	
D.		s.	
		t.	

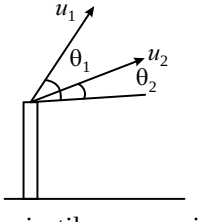
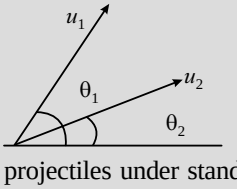
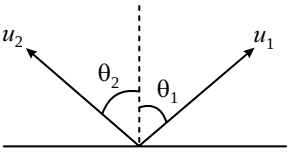
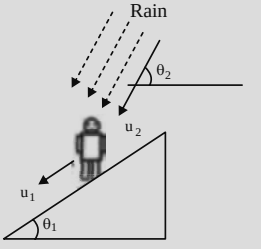
(*a*) A-(q); B-(r); C-(s); D-(t)
(*b*) A-(p); B-(s); C-(t); D-(r)
(*c*) A-(s); B-(t); C-(s); D-(r)
(*d*) A-(q); B-(s); C-(r); D-(t)

25. Match Column-I with Column-II and select the correct answer using the codes given below the lists.

	Column-I		Column-II
A.	If swimmer can swim at 5m/sec in still water and if velocity of water flow is 4m/sec then angle between direction of swimming and direction of river flow to minimize drift.	p.	53°
B.	If swimmer can swim at 5 m/sec in still water and velocity of flow is 3 m/sec then angle between direction of velocity of swimmer with respect to river and the direction of river flow if swimmer crosses the river in minimum time.	q.	127°
C.	If swimmer can swim at 4 m/sec and velocity of flow is 3m/sec then angle of resultant velocity (w.r.t. ground) with the direction of river flow if swimmer swims perpendicular to flow of river.	r.	143°
D.	Angle between direction of fluttering of flag and north if wind blows towards south west direction with a velocity $3\sqrt{2}$ m/sec. Man moves with a velocity 7 m/sec along west, holding flag in his hand.	s.	90°

(*a*) A-(q); B-(r); C-(s); D-(p)
(*b*) A-(r); B-(s); C-(p); D-(q)
(*c*) A-(s); B-(q); C-(s); D-(r)
(*d*) A-(q); B-(s); C-(r); D-(q)

26. Column-I shows certain situations with certain conditions and Column-II shows the parameters in which situations of Column-I match. Which can be possible combination.

	Column-I		Column-II
A.	$u_1 = u_2$; $\theta_1 = \theta_2$	p.	 Two projectiles are projected from a height such that they strike ground at the same time.
B.	$u_1 > u_2$; $\theta_1 > \theta_2$	q.	 Two projectiles under standard ground to ground projection such that horizontal range is same.
C.	$u_1 < u_2$; $\theta_2 > \theta_1$	r.	 Two swimmer starting from same point on a river bank such that time of crossing is same. u_1 and u_2 are velocities relative to river.
		s.	 Person moving downward along slope in rain such that he observes rain vertically

(*a*) A-(p,s,r); B-(q,s,p); C-(r,s)
(*b*) A-(p,r); B-(q,p,s); C-(q,r)
(*c*) A-(p,s,r); B-(q,r,p); C-(q,p,s)
(*d*) A-(p,q,r); B-(q,r); C-(q,r)

27. A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius *R*. Match the quantities given in column-I with corresponding results in column-II

	Column-I		Column-II
A.	Magnitude of tangential acceleration of particle	p.	Decreases with time.
B.	Magnitude of centripetal acceleration of particle	q.	Increases with time
C.	Magnitude of angular speed of particle with respect to centre of circle	r.	Remains constant
D.	Angle between the total acceleration vector and centripetal acceleration vector of particle	s.	Depends on the value of radius *R*

(*a*) A-(q,r); B-(q,s); C-(q,s); D-(p)
(*b*) A-(q,r); B-(q,p); C-(q); D-(p,s)
(*c*) A-(q); B-(q,s); C-(q,s); D-(p,s)
(*d*) A-(s); B-(q,s); C-(q,p); D-(p,r)

NUMERICAL TYPE QUESTIONS

Answer should be rounded of upto two decimal places

28. The ratio of the distance carried away by the water current, downstream, in crossing a river, by a person, making same angle with downstream and upstream is 2: 1. The ratio of the speed of person to the water current cannot be less than_____.

"PROJECTILE MOTION: GROUND TO GROUND PROJECTION"

1. The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be: **[10 April, 2023 (Shift-I)]**

(*a*) 50 *m* (*b*) $50\sqrt{2}\,m$

(*c*) 100 *m* (*d*) $100\sqrt{2}\,m$

2. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Two identical balls *A* and *B* thrown with same velocity 'u' at two different angles with horizontal attained the same range *R*. If *A* and *B* reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1 h_2}$

Reason R: Product of said heights.

$$h_1 h_2 = \left(\frac{u^2 \sin^2\theta}{2g}\right)\cdot\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the correct answer: **[25 June, 2022 (Shift-II)]**

(*a*) Both *A* and *R* are true and *R* is the correct explanation of *A*.

(*b*) Both *A* and *R* are true but *R* is NOT the correct explanation of *A*.

(*c*) *A* is true but *R* is false.

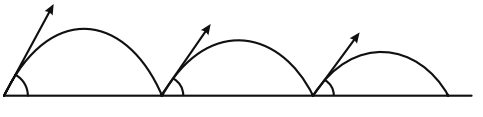
(*d*) *A* is false but *R* is true.

3. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1 - x)$. The *y* component vector of the initial velocity is _____ $\hat{j}$

(Take $g = 10\ m/s^2$) **[JEE Adv, 2022]**

4. A ball is thrown from ground at an angle θ with horizontal and with an initial speed u_0. For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is V_1

After hitting the ground, ball rebounds at the same angle θ but with a reduced speed of u_0/α. Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is $0.8\ V_1$, the value of α is_____ **[JEE Adv, 2019]**



PROJECTILE MOTION ON AN INCLINED PLANE, KINETIC ENERGY OF A PROJECTILE

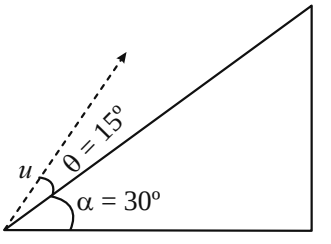
5. A stone is projected at angle 30° to the horizontal. The ratio of kinetic energy of the stone at point of projection to its kinetic energy at the highest point of flight will be **[29 Jan, 2023 (Shift-I)]**

(*a*) 1 : 2 (*b*) 1 : 4

(*c*) 4 : 1 (*d*) 4 : 3

6. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to: **[10 April, 2019 (Shift-II)]**

(Take $g = 10\ ms^{-2}$)



(*a*) 14 cm (*b*) 20 cm

(*c*) 18 cm (*d*) 26 cm

TWO DIMENSIONAL RELATIVE MOTION

7. The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is _____ km h^{-1}.

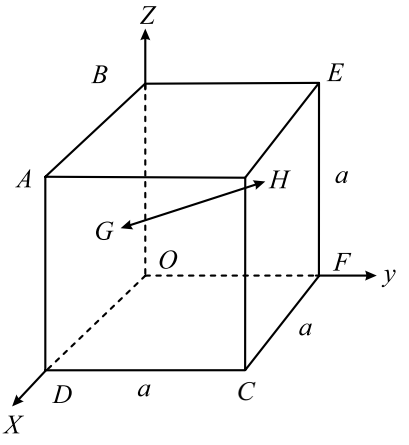
[31 Jan, 2023 (Shift-I)]

8. A fighter jet is flying horizontally at a certain with a speed of 200 ms^{-1}.When it passes directly overhead ad anti-aircraft gun, a bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be _____.

[26 June, 2022 (Shift-I)]

9. A particle is moving along the *x*-axis with its coordinate with time '*t*' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the *y*-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1s$, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then *v*(in m/s) is

[8 Jan, 2020 (Shift-I)]



(*a*) $\frac{1}{2}a(\hat{k}-\hat{i})$ (*b*) $\frac{1}{2}a(\hat{i}-\hat{k})$

(*c*) $\frac{1}{2}a(\hat{j}-\hat{i})$ (*d*) $\frac{1}{2}a(\hat{j}-\hat{k})$

DOT PRODUCT

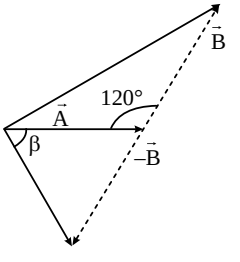
29. If two vectors $\vec{P}=\hat{i}+2m\hat{j}+m\hat{k}$ and $\vec{Q}=4\hat{i}-2\hat{j}+m\hat{k}$ are perpendicular to each other. Then, the value of *m* will be: **[24 Jan, 2023 (Shift-II)]**

(*a*) 1 (*b*) –1

(*c*) –3 (*d*) 2

30. The angle between vector $\left(\vec{A}\right)$ and $\left(\vec{A}-\vec{B}\right)$ is:

[26 Aug, 2021 (Shift-II)]



(*a*) $\tan^{-1}\left(\frac{\sqrt{3}B}{2A-B}\right)$

(*b*) $\tan^{-1}\left(\frac{B\cos\theta}{A-B\sin\theta}\right)$

(*c*) $\tan^{-1}\left(\frac{A}{0.7\,B}\right)$

(*d*) $\tan^{-1}\left(\frac{\frac{-B}{2}}{A-B\frac{\sqrt{3}}{2}}\right)$

31. Let $\left|\vec{A}_1\right|=3,\left|\vec{A}_2\right|=5$ and $\left|\vec{A}_1+\vec{A}_2\right|=5$. The value of $(2\vec{A}_1+3\vec{A}_2).(3\vec{A}_1-2\vec{A}_2)$ is: **[8 April, 2019 (Shift-II)]**

(*a*) –112.5 (*b*) –106.5

(*c*) –118.5 (*d*) –99.5

CROSS PRODUCT

32. If $\vec{P}=3\hat{i}+\sqrt{3}\hat{j}+2\hat{k}$ and $\vec{Q}=4\hat{i}+\sqrt{3}\hat{j}+2.5\hat{k}$ then, the unit vector in the direction of $\vec{P}\times\vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i}+\hat{j}-2\sqrt{3}\hat{k}\right)$. The value of *x* is **[25 Jan, 2023 (Shift-I)]**

33. $\vec{A}$ is a vector quantity such that $|\vec{A}|$ = non-zero constant. Which of the following expression is true for $\vec{A}$?

[25 June, 2022 (Shift-I)]

(*a*) $\vec{A}.\vec{A}=0$ (*b*) $\vec{A}\times\vec{A}<0$ (*c*) $\vec{A}\times\vec{A}=0$ (*d*) $\vec{A}\times\vec{A}>0$

34. If $\vec{P}\times\vec{Q}=\vec{Q}\times\vec{P}$ the angle between $\vec{P}$ and $\vec{Q}$ is θ ($0°<\theta<360°$). The value of 'θ' will be _____ **[25 Feb, 2021 (Shift-II)]**

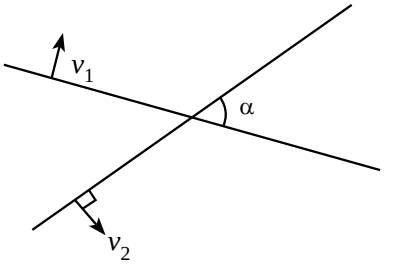
PW CHALLENGERS

SINGLE CORRECT TYPE QUESTIONS

1. Water flows out in all directions with the same speed from a sprinkler consisting of a perforated spherical shell fixed at the end of a hose. When the sprinkler is fixed at the ground, maximum height attained by a water stream is *h*. If the sprinkler is shifted to height *h* above the ground, by what factor will the watered area on the ground change? Neglect diameter of the spherical shell as compared to the height *h*.

(*a*) 8 (*b*) 6 (*c*) 4 (*d*) 2

2. A plane contains two straight lines intersecting each other at an angle α. The lines start moving perpendicular to themselves in the same plane as shown in the figure with velocities v_1 and v_2 respectively. Find the velocity of the point of intersection of the two lines.



(*a*) $\sqrt{v_1^2+v_2^2+2v_1v_2\cos\alpha}$ (*b*) $\frac{\sqrt{v_1^2+v_2^2+2v_1v_2\sin\alpha}}{\sin\alpha}$

(*c*) $\frac{\sqrt{v_1^2+v_2^2+2v_1v_2\sin\alpha}}{\cos\alpha}$ (*d*) $\frac{\sqrt{v_1^2+v_2^2+2v_1v_2\cos\alpha}}{\sin\alpha}$

Answer Key

CONCEPT APPLICATION

1. [150°] **2.** (*a*) 60°, (*b*) 15°, (*c*) 75° **3.** (*a*) 120°, (*b*) 120°, (*c*) 120° **4.** F_1 = 6 N, F_2 = 10 N,

5. $R = 7\sqrt{2}$ *cm* at 45° from *x*-axis **6.** (*b*) **7.** (*d*) **8.** $(\pm\sqrt{12})$ **9.** (*c*) **10.** (*d*) **11.** (*b*) **12.** (*c*)

13. (*a*) **14.** (*a*) **15.** (*d*) **16.** $R = \sqrt{2}\, r + r$ along *OB* **17.** 2 cm along *x*-axis **18.** $(15i + 20j)$

19. [150°] **20.** (*b*) **21.** (*b*) **22.** (*a*) **23.** (*c*) **24.** (*c*) **25.** (*d*) **26.** (*b*) **27.** (*c*)

28. (*b*) **29.** (*a*) **30.** (*a*) **31.** $\pm\dfrac{\hat{i}-4\hat{j}-5\hat{k}}{\sqrt{42}}$ **32.** $\hat{i}+7\hat{j}+5\hat{k}$ **33.** (*c*) **34.** (*a*)

35. (*c*) **36.** (*d*) **37.** (*d*) **38.** (*d*) **39.** (*c*) **40.** (*b*) **41.** (*d*) **42.** (*c*) **43.** (*d*) **44.** (*a*)

45. (*c*) **46.** (*a*) **47.** (*a*) **48.** (*a*) **49.** (*b*) **50.** (i) 10 sec (ii) 980 m (iii) $98\sqrt{2}$ m/s **51.** (*c*) **52.** (*a*)

53. (*c*) **54.** [3 m] **55.** (*b*) **56.** (*b*) **57.** (*d*) **58.** (*a*) **59.** (*b*) **60.** (*b*) **61.** [40 m] **62.** (*c*)

63. (*c*) **64.** (*a*) **65.** (*a*) **66.** (*a*) **67.** 14.29s, 0.45 m/s^2 **68.** $\dfrac{u^2\sin^2\theta}{g}$

SCHOOL LEVEL PROBLEMS

1. (*c*) **2.** (*b*) **3.** (*d*) **4.** (*c*) **5.** (*c*) **6.** (*b*) **7.** (*a*) **8.** (*a*) **9.** (*d*) **10.** (*c*)

11. (*d*) **12.** (*a*) **25.** (i)-(*b*), (ii)-(*b*), (iii)-(*b*), (iv)-(*a*), (v)-(*a*) **26.** (i)-(*a*), (ii)-(*c*), (iii)-(*a*), (iv)-(*d*), (v)-(*c*)

PRARAMBH (TOPICWISE)

1. (*b*) **2.** (*b*) **3.** (*d*) **4.** (*d*) **5.** (*a*) **6.** (*d*) **7.** (*b*) **8.** (*a*) **9.** (*c*) **10.** (*a*)

11. (*a*) **12.** (*d*) **13.** (*b*) **14.** (*b*) **15.** (*a*) **16.** (*b*) **17.** (*d*) **18.** (*b*) **19.** (*d*) **20.** (*b*)

21. (*a*) **22.** (*a*) **23.** (*a*) **24.** (*b*) **25.** (*d*) **26.** (*c*) **27.** (*b*) **28.** (*d*) **29.** (*d*) **30.** (*c*)

31. (*a*) **32.** (*b*) **33.** (*a*) **34.** (*b*) **35.** (*b*) **36.** (*d*) **37.** (*d*) **38.** (*c*) **39.** (*b*) **40.** (*b*)

41. (*c*) **42.** (*d*) **43.** (*a*) **44.** (*a*) **45.** (*d*) **46.** (*d*) **47.** (*c*) **48.** (*b*) **49.** (*b*) **50.** (*b*)

51. (*d*) **52.** (*b*) **53.** (*c*) **54.** (*d*) **55.** (*b*) **56.** (*c*) **57.** (*d*) **58.** (*c*) **59.** (*a*) **60.** (*b*)

61. (*c*) **62.** (*d*) **63.** (*d*) **64.** (*a*) **65.** (*c*) **66.** (*b*) **67.** (*b*) **68.** (*d*) **69.** (*b*) **70.** (*c*)

71. (*a*) **72.** (*b*) **73.** (*a*) **74.** (*c*) **75.** (*c*) **76.** (*b*) **77.** (*c*) **78.** (*b*) **79.** (*c*) **80.** (*a*)

81. (*b*) **82.** (*b*) **83.** (*c*) **84.** (*b*) **85.** (*c*) **86.** (*b*) **87.** (*c*) **88.** (*a*) **89.** (*c*) **90.** (*b*)

91. (*b*) **92.** (*a*) **93.** (*d*) **94.** (*d*) **95.** (*c*) **96.** (*a*) **97.** (*d*) **98.** (*b*) **99.** (*a*) **100.** (*c*)

101. (*c*) **102.** (*a*) **103.** (*a*) **104.** (*c*) **105.** (*b*) **106.** (*c*) **107.** (*a*) **108.** (*b*) **109.** (*b*) **110.** (*b*)

PRABAL (JEE MAIN LEVEL)

1. (*a*) **2.** (*b*) **3.** (*c*) **4.** (*d*) **5.** (*a*) **6.** (*a*) **7.** (*d*) **8.** (*d*) **9.** (*d*) **10.** (*b*)

11. (*d*) **12.** (*b*) **13.** (*b*) **14.** (*d*) **15.** (*d*) **16.** (*d*) **17.** (*d*) **18.** (*a*) **19.** (*b*) **20.** (*c*)

21. (*d*) **22.** (*d*) **23.** (*b*) **24.** (*b*) **25.** (*c*) **26.** (*b*) **27.** (*b*) **28.** (*d*) **29.** (*b*) **30.** (*a*)

31. (*a*) **32.** (*b*) **33.** (*d*) **34.** (*a*) **35.** (*b*) **36.** (*b*) **37.** (*c*) **38.** (*d*) **39.** (*a*) **40.** (*c*)

41. (*a*) **42.** (*b*) **43.** (*a*) **44.** (*a*) **45.** (*b*) **46.** (*b*) **47.** (*d*) **48.** (*a*) **49.** (*d*) **50.** (*a*)

51. (*b*) **52.** (*d*) **53.** (*c*) **54.** (*d*) **55.** (*b*) **56.** (*c*) **57.** (*a*) **58.** (*a*) **59.** (*a*) **60.** [25]

61. [15] **62.** [13] **63.** [25] **64.** [20] **65.** [2] **66.** [5] **67.** [7] **68.** [625] **69.** [9]

CHAPTER

4

Chemical Bonding and Molecular Structure

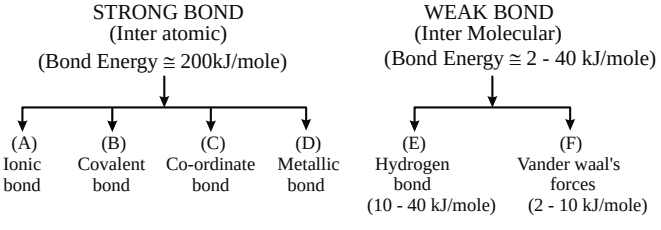
INTRODUCTION

Atoms combine with one another in different ways to form a large number of molecules. The attractive force which holds the constituents (atoms, molecules or ions) together in different chemical species, is called a **chemical bond.**

As the atomic state is regarded as a state of higher energy therefore the atoms combine with one another in a number of ways.

Key Note

- A molecule will only be formed if it is more stable and has a lower energy than the individual atoms.



Cause of Chemical Combination

(I) Atom combines to acquire noble gas configuration.

(II) Outermost electrons participate in bond formation.

TYPES OF BONDS

1. **Electrovalent or ionic bond** (by transfer of e^- from one atom to another)
2. **Covalent bond** (by sharing of a pair of e^- between two atoms)
3. **Coordinate or dative bond** (by donation of a pair of e^- from one atom to another)

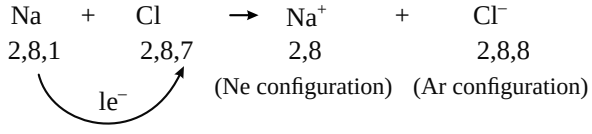
ELECTROVALENT OR IONIC BOND

The chemical bond formed between two or more atoms as a result of the complete transfer of one or more electrons from one atom to another is called **Ionic or electrovalent bond.**

(*i*) Electro +ve atom loses electron (group IA to IIIA)

(*ii*) Electro –ve atom gains electron (group VA to VIIA)

- Electrostatic force of attraction between cation and anion is called ionic bond or electrovalent bond.
- Strength of ionic bond ∝ Electronegativity difference
- Gr. 1 and Gr. 17 group elements form maximum ionic compounds.



Electrovalency: It is defined as number of electrons lost or gained by an atom e.g. Na has electrovalency 1, Mg has 2.

Example:

(*a*) $Mg + O \rightarrow Mg^{2+} + O^{2-}$

2,8,2 2, 6

$2e^-$

electrovalency of Mg = 2
electrovalency of O = 2

(*b*) $Ca + Cl \rightarrow Ca^{2+} + Cl^-$

2,8,8,2 2,8,7

One e^-, Cl 2,8,7, One e^-

electrovalency of Ca = 2
electrovalency of Cl = 1

(*c*) $Ca + O \rightarrow Ca^{2+} + O^{2-}$

2,8,8,2 2,6

$2e^-$

electrovalency of Ca = 2
electrovalency of O = 2

- The force of attraction is equal in all direction so ionic bond is non-directional.
- Ionic compounds do not have molecular formula. It has only empirical formula e.g. NaCl is empirical formula of sodium chloride.

CONDITIONS FOR FORMING IONIC BOND

Formation of Ionic bond depends upon these three factors

(*a*) Ionisation energy: Minimum amount of energy required to remove an electron from the outermost orbit of an isolated gaseous atom to form the +ve ion or cation.

Lesser Ionisation energy → Greater tendency to form cation.

e.g. $Na^+ > Mg^{2+} > Al^{3+}$
$Cs^+ > Rb^+ > K^+ > Na^+ > Li^+$ } ionic bond formation tendency

(*b*) Electron affinity: Amount of energy released when an electron is added to an isolated gaseous atom to form –ve ion (anion).

Higher electron affinity → Greater tendency to form anion

e.g. $F^- > O^{2-} > N^{3-}$ } tendency to form ionic bond

Train Your Brain

Example 1: Calculate the lattice energy of solid KF with the help of following data:

$\Delta H^0_f(KF) = -562.6$ kJ mol^{-1}

$\Delta H_{sub}(K) = 89.6$ kJ mol^{-1}

$\Delta H_{IE}(K) = 419.0$ kJ mol^{-1}

$\Delta H_{diss}(F_2) = 158.2$ kJ mol^{-1}

$\Delta H_{EA}(F) = -332.6$ kJ mol^{-1}

Sol. $K(s) + \frac{1}{2}F_2(g) \longrightarrow KF(s)$; -562.6 kJ mol^{-1}

$K(s) \longrightarrow K(g)$; 89.6 kJ mol^{-1}

$K(g) \longrightarrow K^+(g) + e^-$; 419.0 kJ mol^{-1}

$\frac{1}{2}F_2(g) \longrightarrow F(g)$; $\frac{1}{2}(158.2) = 79.1$ kJ mol^{-1}

$F(g) + e^- \longrightarrow F^{(-)}(g)$; -332.6 kJ mol^{-1}

Let $\Delta H_{lattice}(KF)$ be U, then according to Hess's law of constant heat summation.

$-562.6 = 89.6 + 419.0 + 79.1 - 332.6 + U$

So, U = -817.7 kJ mol^{-1}

Lattice energy of KF = -817.7 kJ mol^{-1}

Example 2: Lattice energy of $BeCO_3$ (I), $MgCO_3$ (II) and $CaCO_3$ (III) are in the order -

(*a*) I > II > III (*b*) I < II < III

(*c*) I < III < II (*d*) II < I < III

Sol. Lattice energy $\propto \frac{1}{\text{Cation size}}$

$BeCO_3 > MgCO_3 > CaCO_3$ (L.E)

L.E. decrease from top to bottom.

Example 3: Which of the following statements is wrong regarding ionic compounds?

(*a*) These are generally in solid state at room temperature.

(*b*) The force of attraction between ions is non directional.

(*c*) Ionic compounds are soluble in all solvents.

(*d*) They conduct electricity in molten and solution state.

Sol. Ionic compound are soluble only in polar solvents.

Example 4: When two atoms combine to form a molecule

(*a*) Energy is released.

(*b*) Energy is absorbed.

(*c*) Energy is neither released nor absorbed.

(*d*) Energy may either released or absorbed.

Sol. To attain stability energy is released.

Concept Application

1. Which of the following factors is dependent upon lattlice energy?

(*a*) Stability of ionic compound

(*b*) Melting point of ionic compound

(*c*) Boiling point of ionic compound

(*d*) All of the above

2. The electrovalency of the element is equal to the:

(*a*) Number of electrons lost

(*b*) Number of electrons gained

(*c*) Number of electrons transferred

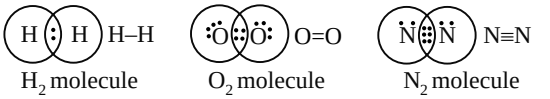
(*d*) Number of electrons lost or gained by the atom of the element during the formation of ions of ionic compound

3. The compound which has the highest lattice energy is

(*a*) LiF (*b*) LiCl (*c*) NaCl (*d*) MgO

COVALENT BOND

(I) A covalent bond is formed by the mutual sharing of electrons between two atoms of electronegative elements to complete their octet (Except H which completes its duplet).



H_2 molecule O_2 molecule N_2 molecule

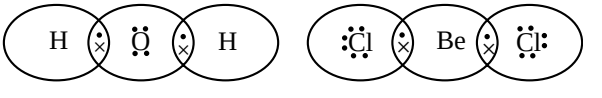
(II) The shared pair of electrons should have opposite spins, and are localised between two atoms concerned.

(III) Sharing of electrons may occur in three ways –

No. of electron shared between two atoms	Electron pair	Bond
2	1	Single bond (–)
4	2	Double bond (=)
6	3	Triple bond (≡)

OCTET RULE

Lewis postulated that atoms achieve the stable octet (i.e. 8 electrons in their outer most shells), when they are linked by chemical bonds.



Obeys octet rule Doesn't obey octet rule

LEWIS SYMBOLS

In the formation of a molecule, only the outer shell electrons take part in chemical combination and they are known as valence electrons. The inner shell electrons are well protected

O_2^+ has bond order 2.5 as one electron is removed from the antibonding molecular orbital.

O_2^- has bond order 1.5 as an additional electron is added to the antibonding molecular orbital.

O_2^{2-} has bond order 1.0 as two electrons are added to the two antibonding molecular orbitals.

Stability order (and bond dissociation energy) of the four species are as under.

$O_2^+ > O_2 > O_2^- > O_2^{2-}$

Bond distances are in the reverse order; $O_2^+ < O_2 < O_2^- < O_2^{2-}$

Out of these four species O_2, O_2^+ and O_2^- are paramagnetic due to the presence of 2, 1, 1 unpaired electrons respectively while O_2^{2-} is diamagnetic as it has no unpaired electron.

(*e*) Fluorine

Total number of electrons in F_2 molecule is 18 and so it resembles the configuration of O_2^{2-}

$F_2 \rightarrow (\sigma 1s)^2, (\sigma^*1s)^2, (\sigma 2s)^2, (\sigma^*2s)^2, (\sigma 2p_z)^2, (\pi 2p_x)^2 = (\pi 2p_y)^2, (\pi^*2p_x)^2 = (\pi^*2p_y)^2$

Its bond order n is 1 and it is diamagnetic.

(*f*) Neon

Ne_2 does not exist as it's bond order comes to be zero.

Key Note

HOMO: Highest Occupied Molecular Orbital

LUMO: Lowest Unoccupied Molecular Orbital

Train Your Brain

Example 32: Though O_2 molecule is paramagnetic yet it is a colourless gas. Why?

Sol. It is because the energy gap between HOMO and LUMO levels in O_2 molecule is so large that radiations of visible light cannot excite an e^- from HOMO to LUMO. In fact O_2 gas shows absorption in UV zone. So it is colourless.

Example 33: Correct order of bond energy is:

(*a*) $N_2 > N_2^+ > N_2^- > N_2^{2-}$ (*b*) $N_2^+ > N_2^- > N_2^{2-} > N_2$

(*c*) $N_2 > N_2^- = N_2^+ > N_2^{2-}$ (*d*) $N_2^- > N_2 = N_2^+ > N_2^{2-}$

Sol. Bond enregy is directly proportional to the bond order.

Bond order of $N_2 = 3$; N_2^+ & $N_2^- = 2.5$

$N_2^{2-} = 2$

But N_2^- has more electrons in antibonding MO's and thus N_2^+ is more stable than N_2^-. So correct order of bond energy will be $N_2 > N_2^+ > N_2^- > N_2^{2-}$

Example 34: In an antibonding molecular orbital, electron density is minimum-

(*a*) Around one atom of the molecule

(*b*) Between the two nuclei of the molecule

(*c*) At the region away from the nuclei of the molecule

(*d*) at no place

Sol. In an antibonding M.O., electron density is minimum between the two nuclei of the molecule.

Example 35: Which of the following are diamagnetic?

(*a*) C_2 (*b*) O_2^{2-} (*c*) Li_2 (*d*) N_2^+

Sol. Species C_2, O_2^{2-}, Li_2 have all the electrons paired but N_2^+ has one unpaired electron in bonding molecular orbital so it is paramagnetic.

Example 36: Which of the following has maximum bond strength?

(*a*) O_2 (*b*) O_2^+ (*c*) O_2^- (*d*) O_2^{2-}

Sol. Bond order of O_2^+ is 2.5 which is maximum among all the species hence its bond strength will also be the maximum.

Example 37: Which have non-integral bond order?

(*a*) O_2^+ (*b*) O_2^-

(*c*) NO (*d*) All of these

Sol. (*a*) $\therefore$ $O_2^+ = 2.5$

(*b*) $O_2^- = 1.5$

(*c*) NO = 2.5

Concept Application

31. In which of the following pairs of molecules/ions, both the species are not likely to exist?

(*a*) H_2^+, He_2^{2-} (*b*) H_2^-, He_2^{2-}

(*c*) H_2^{2+}, He_2 (*d*) H_2^-, He_2^{2+}

32. When N_2 goes to N_2^+, the N–N bond distance.... and when O_2 goes to O_2^+, the O–O bond distance....

(*a*) Decrease, Increases (*b*) Increases ,Decrease

(*c*) Increases, Increases (*d*) None of these

33. In which molecule, last electron is filled in $\sigma 2p_z$?

(*a*) C_2 (*b*) N_2 (*c*) B_2 (*d*) O_2

34. In which of the following set, the value of bond order will be 2.5?

(*a*) O_2^+, NO, NO^{2+}, CN (*b*) CN, NO^{2+}, CN^-, F_2

(*c*) O_2^+, NO^{2+}, O_2^{2+},CN^- (*d*) O_2^{2-}, O_2^-, O_2^+, O_2

35. Among the following which one will have the largest O–O bond length?

(*a*) KO_2 (*b*) O_2

(*c*) $O_2^+ [AsF_6]^-$ (*d*) K_2O_2

HYDROGEN BOND

Nitrogen, oxygen and fluorine are highly electronegative elements. When they are bonded to a hydrogen atom to form covalent bond, the electrons of the covalent bond are shifted towards the more electronegative atom. This partially positively charged hydrogen atom attracts nearby any other more electronegative atom having lone pair. This electrostatic force of attraction is called as hydrogen bond and is weaker than covalent bond. For example, in HF molecule, the

1. The type of hybrid orbitals used by the chlorine atom in ClO_2^- is:

(*a*) sp^3 (*b*) sp^2

(*c*) sp (*d*) None of these

Sol. $^-O–\ddot{\underset{\ddot{}}{Cl}}=O$

Bond pair = 2, Lone Pair = 2

Total Electronic pairs = 4 $\Rightarrow$ Steric no. = 4,

Hybridisation at Cl = sp^3

Therefore, option (*a*) is the correct answer.

2. Specify the coordination geometry around and hybridisation of N and B atoms in a 1 : 1 complex of BF_3 and NH_3.

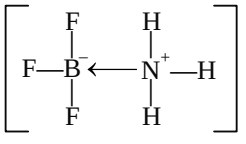
(*a*) N : tetrahedral, sp^3 ; B: tetrahedral, sp^3

(*b*) N : pyramidal, sp^3; B: pyramidal, sp^3

(*c*) N: pyramidal, sp^3; B: planar, sp^2

(*d*) N: pyramidal, sp^3; B: tetrahedral, sp^3

Sol.



Both 'B' and 'N' has sp^3 hybridisation and tetrahedral geometry.

Therefore, option (*a*) is the correct answer.

3. The cyanide ion CN^- and N_2 are isoelectronic, but in contrast to CN^-, N_2 is chemically inert because of

(*a*) Low bond energy

(*b*) Absence of bond polarity

(*c*) Unsymmetrical electron distribution

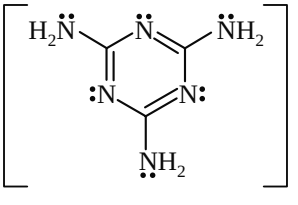
(*d*) Presence of more number of electron in bonding orbitals

Sol. N_2 is a neutral, non-polar, inert molecule while CN^- is a highly polar, highly reactive ion.

Therefore, option (*b*) is the correct answer.

4. The total number of lone-pair of electrons in melamine is:

Sol. Melamine is a heterocyclic compound.



Six lone pairs are present.

Therefore, [6] is the correct answer.

5. Among the triatomic molecules/ions $BeCl_2$, N_3^- , N_2O, NO_2^+, O_3, SCl_2, ICl_2^-, I_3^- and XeF_2, the total number of linear molecules(s)/ion(s) where the hybridisation of the central atom does not have contribution from the d-orbital(s) is [atomic number of S = 16, Cl = 17, I = 53 and Xe = 54].

Sol. $Cl\underset{\uparrow}{\underset{sp}{Be}}Cl$, $N\equiv\underset{\uparrow}{\underset{sp}{N}}\longrightarrow\ddot{\ddot{N}}:$

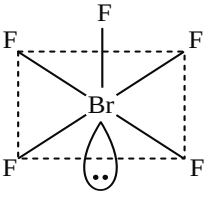
$N\equiv\underset{\uparrow}{\underset{sp}{N}}\rightarrow\ddot{\ddot{O}}:$, $O=\overset{+}{\underset{\uparrow}{\underset{sp}{N}}}=O$

All the above mentioned molecules/ions have sp-hybridised central atom and no Lone pair at central atom, hence linear also.

Therefore, [4] is the correct answer.

6. Based on VSEPR theory, the number of 90° F—Br—F angles in BrF_5 is.

Sol.



To minimize the repulsion, Lone pair would push the Br—F bond pairs in upward direction and all Br—F bond angles will contract.

Therefore, [0] is the correct answer.

7. Stability of the species Li_2 , Li_2^- and Li_2^+ increases in the order of

(*a*) $Li_2 < Li_2^+ < Li_2^-$ (*b*) $Li_2^- < Li_2^+ < Li_2$

(*c*) $Li_2 < Li_2^- < Li_2^+$ (*d*) $Li_2^- < Li_2 < Li_2^+$

Sol. $Li_2 = \sigma1s^2, \sigma^*1s^2, \sigma2s^2$

$$\text{Bond order} = \frac{4-2}{2} = 1$$

$Li_2^+ = \sigma1s^2, \sigma^*1s^2, \sigma2s^1$

$$\text{Bond order} = \frac{3-2}{2} = 0.5$$

$Li_2^- = \sigma1s^2, \sigma^*1s^2, \sigma2s^2, \sigma^*2s^1$

$$\text{Bond order} = \frac{4-3}{2} = 0.5$$

Stability order is $Li_2^- < Li_2^+ < Li_2$

(because Li_2^- has more number of electrons in antibonding orbitals which destabilises the species).

Therefore, option (*b*) is the correct answer.

8. Consider the following molecules/anions

(x) HPO_2F_2 (y) $PO_2F_2^-$

(z) HSO_3F (t) SO_3F^-

Identify the correct statement(s).

(*a*) Both (x) and (z) have one –OH group each

(*b*) Hybridisation of phosphorus in (x) and (y) is same

(*c*) Hybridisation of phosphorus in (y) and sulphur in (z) is same

(*d*) Shapes of (x) and (y) are different

SCHOOL LEVEL PROBLEMS

SINGLE CORRECT TYPE QUESTIONS

1. Which of the following has highest ionic character?

(*a*) $MgCl_2$ (*b*) $CaCl_2$

(*c*) $BaCl_2$ (*d*) $BeCl_2$

2. The molecule that deviates from octet rule is

(*a*) NaCl (*b*) $BeCl_2$

(*c*) MgO (*d*) NH_3

3. Molecule having maximum number of covalent bonds is:

(*a*) NH_4OH (*b*) NH_4Cl

(*c*) $CO(NH_2)_2$ (*d*) CH_3OH

4. The correct sequence of decrease in the bond angles of the following hydrides is

(*a*) $NH_3 > PH_3 > AsH_3 > SbH_3$

(*b*) $NH_3 > AsH_3 > PH_3 > SbH_3$

(*c*) $SbH_3 > AsH_3 > PH_3 > NH_3$

(*d*) $PH_3 > NH_3 > AsH_3 > SbH_3$

5. The correct order towards bond angle is

(*a*) Bond angle does not depend on hybridisation

(*b*) $sp < sp^2 < sp^3$

(*c*) $sp^2 < sp < sp^3$

(*d*) $sp^3 < sp^2 < sp$

6. The shape of covalent molecule AX_3 is.

(*a*) Triangular (*b*) T-shape

(*c*) Pyramidal (*d*) Any of the above

7. Which of the following species given below have shape similar to $XeOF_4$?

(*a*) XeO_3 (*b*) IOF_4^+

(*c*) PCl_5 (*d*) XeF_5^+

8. The hybrid state of C in CS_2 should be:

(*a*) sp^2 (*b*) sp

(*c*) sp^3 (*d*) Not specific

Directions: These questions consist of two statements each, printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.

(*a*) Both Assertion and Reason are True and the Reason is a correct explanation of the Assertion.

(*b*) Both Assertion and Reason are True but Reason is not a correct explanation of the Assertion.

(*c*) Assertion is True but the Reason is False.

(*d*) Assertion is False but Reason is True.

9. **Assertion:** Geometry of NH_3 molecule is tetrahedral.
Reason: Nitrogen is sp^3 hybridized in NH_3

10. **Assertion:** The value of dipole moment helps to predict whether a molecule is polar or nonpolar.
Reason: The dipole moment also helps to predict the geometry of the molecules.

MATCH THE COLUMN TYPE QUESTIONS

11. Match the species in Column-I with the geometry/shape in Column-II.

Column-I		Column-II	
A.	H_3O^+	(p)	Linear
B.	HC ≡ CH	(q)	Angular
C.	ClO_2^-	(r)	Tetrahedral
D.	NH_4^+	(s)	Triogonal bipyramidal
		(t)	Pyramidal

(*a*) A-(p), B-(q), C-(t), D-(r)

(*b*) A-(t), B-(p), C-(q), D-(r)

(*c*) A-(s), B-(r), C-(t), D-(p)

(*d*) A-(r), B-(p), C-(t), D-(s)

12. Match the species in Column-I with the bond order in Column-II.

Column-I		Column-II	
A.	NO	(p)	1.5
B.	CO	(q)	2.0
C.	O_2^-	(r)	2.5
D.	O_2	(s)	3.0

(*a*) A-(r), B-(s), C-(p), D-(q)

(*b*) A-(s), B-(p), C-(q), D-(r)

(*c*) A-(s), B-(r), C-(q), D-(p)

(*d*) A-(r), B-(p), C-(q), D-(s)

SHORT ANSWER TYPE QUESTIONS

13. Using the valence shell electron pair repulsion theory, predict the shapes of

(*i*) H_2S (*ii*) PCl_3

14. (*a*) What characterizes a nonpolar covalent bond, and can you give an example of a substance that exhibits this type of bonding?

(*b*) Calculate the formal charge of each O in ozone.

15. (*a*) "Why does hydrogen sulfide (H_2S) exist as a gas, whereas water (H_2O) is in a liquid state at room temperature?"

(*b*) Explain s-s and p-p overlapping.

16. Determine the structures and hybridizations of

(*a*) $BeCl_2$ (*b*) SF_6

(*c*) BrF_5

17. (*a*) Draw and explain the Lewis structure of nitrite ion.

(*b*) Compare the dipole moments of NH_3 and NF_3.

18. (*a*) Explain the reason behind the smaller bond angle in water as compared to ammonia, despite both molecules having distorted tetrahedral geometries.

(*b*) Based on lattice enthalpy and other considerations, arrange the given below species in increasing order of melting point.

RbCl, LiCl, KCl, NaCl

LONG ANSWER TYPE QUESTIONS

19. (*a*) In addition to having a tetrahedral structure, CH_4 might also have a square planar structure, with the C atom in the middle and the four H atoms arranged at the corners. Explain why CH_4 is not square planar?

(*b*) Why does the BeH_2 molecule exhibit a dipole moment of zero, even though its Be—H bonds are polar in nature?

20. (*a*) Define hydrogen bonding and explain why compounds like carboxylic acids, HF, and NH_3, which exhibit hydrogen bonding, have significantly higher melting and boiling points compared to similar compounds without hydrogen bonds. Use examples to illustrate.

(*b*) Show intramolecular H-bonding in o-nitrophenol.

21. (*a*) Explain the hybridization process in PCl_5 and the reason for the longer length of axial bonds as compared to equatorial bonds.

(*b*) Determine which of the following compounds, C_2H_2 or C_2H_4, has the highest total number of sigma and pi bonds.

22. (*a*) Despite the fact that both CO_2 and H_2O are triatomic molecules, the H_2O molecule is bent whereas the CO_2 molecule is linear. On the basis of the dipole moment, explain this.

(*b*) Among water and dimethyl ether (CH_3–O–CH_3), which molecule has a larger bond angle despite having the same central oxygen atom with the same hybridization? Provide a rationale for your answer.

23. (*a*) Draw the Lewis structures for HNO_2, NO_2, and H_2SO_4, indicating the formal charges on each atom in the molecules.

(*b*) Using molecular orbital theory, explain the differences in bond strength and magnetic properties between O_2^+, O_2^- and O_2^{2-} ions.

CASE STUDY BASED QUESTIONS

24. Chemical bonding is a fundamental concept in chemistry that underpins the formation of molecules and compounds. It explains how atoms interact with one another to achieve stability and attain a lower energy state. There are primarily three types of chemical bonds: covalent, ionic, and metallic. In covalent bonds, atoms share electrons to complete their outer electron shells, forming molecules with distinct properties. Ionic bonds involve the transfer of electrons from one atom to another, creating oppositely charged ions that attract each other. Metallic bonding, on the other hand, is characterized by a "sea of electrons" that move freely among metal ions, giving metals their unique properties such as malleability and electrical conductivity. Understanding chemical bonding is essential for predicting chemical reactions, designing materials, and comprehending the structure of matter.

(*i*) What is the defining characteristic of metallic bonding?

(*a*) Transfer of electrons between atoms

(*b*) Sharing of electrons to complete outer shells

(*c*) A "sea of electrons" moving among metal ions

(*d*) Formation of oppositely charged ions

(*ii*) Which type of chemical bond involves the transfer of electrons from one atom to another?

(*a*) Covalent (*b*) Ionic

(*c*) Metallic (*d*) Hydrogen

(*iii*) What unique properties do metals exhibit due to metallic bonding?

(*a*) High melting points and brittleness

(*b*) Low electrical conductivity and solubility

(*c*) Malleability and electrical conductivity

(*d*) Strong covalent bonds and low density

(*iv*) How do covalent bonds differ from ionic bonds in terms of electron interaction?

(*a*) Covalent bonds involve the transfer of electrons, while ionic bonds involve electron sharing.

(*b*) Covalent bonds involve electron sharing, while ionic bonds involve the transfer of electrons.

(*c*) Covalent bonds and ionic bonds both involve electron transfer.

(*d*) Covalent bonds and ionic bonds are identical in electron behavior.

PRARAMBH (TOPICWISE)

INTRODUCTION, VALENCY

1. The octet rule is not obeyed in-

(*a*) CO_2 (*b*) BCl_3

(*c*) PCl_5 (*d*) Both (*b*) and (*c*)

2. Weakest bond is

(*a*) Ionic bond (*b*) Covalent bond

(*c*) Coordinate bond (*d*) Hydrogen bond

3. Which condition favours the bond formation:

(*a*) Maximum attraction and maximum potential energy

(*b*) Minimum attraction and minimum potential energy

(*c*) Minimum potential energy and maximum attraction

(*d*) None of the above

44. Which one is paramagnetic from the following?

(*a*) O_2^- (*b*) NO

(*c*) Both (*a*) and (*b*) (*d*) CN^-

45. Which of the following represents correctly the order of filling of the molecular orbitals of B_2, C_2, N_2?

(*a*) $\pi_{2p_x} = \pi_{2p_y} < \sigma_{2p_z} < \pi^*_{2p_y} = \pi^*_{2p_x} < \sigma^*_{2p_z}$

(*b*) $\sigma_{2p_z} < \pi_{2p_x} = \pi_{2p_y} < \pi^*_{2p_x} = < \pi^*_{2p_y} < \sigma^*_{2p_z}$

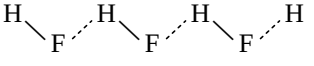
(*c*) $\sigma_{2p_z} = \pi_{2p_x} = \pi_{2p_y} < \pi^*_{2p_x} = < \pi^*_{2p_y} < \sigma^*_{2p_z}$

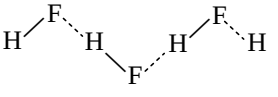
(*d*) $\pi_{2p_x} < \pi_{2p_y} < \sigma_{2p_z} > \pi^*_{2p_y} < \pi^*_{2p_z} = \sigma^*_{2p_z}$

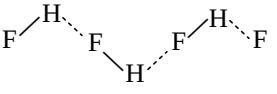
Hydrogen Bonding

46. The H bond in solid HF can be best represented as :

(*a*) H – F....H – F....H – F

(*b*) 

(*c*) 

(*d*) 

47. Contrary to other hydrogen halides, hydrogen fluoride is a liquid at room temperature because

(*a*) Size of F atom is small

(*b*) HF is a weak acid

(*c*) HF molecule are hydrogen bonded

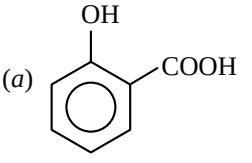
(*d*) Fluorine is highly reactive

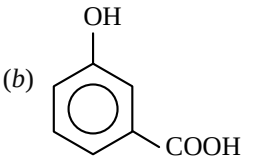
48. The hydrogen bond is strongest in:

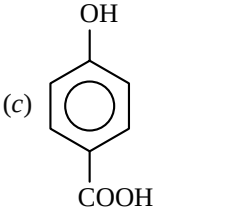
(*a*) O – H - - - S (*b*) S – H - - - O

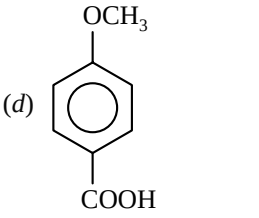
(*c*) F – H - - - F (*d*) O – H - - - O

49. Which of the following has strongest intramolecular hydrogen bonding:

(*a*) 

(*b*) 

(*c*) 

(*d*) 

50. The boiling point of p-nitrophenol is higher than that of o-nitrophenol because:

(*a*) NO_2 group at p-position behaves in a different way from that at o-position

(*b*) Intramolecular hydrogen bonding exists in p-nitrophenol

(*c*) There is intermolecular hydrogen bonding in p-nitrophenol

(*d*) p-nitrophenol has a higher molecular weight than o-nitrophenol

51. Of the following hydrides which has the lowest boiling point?

(*a*) NH_3 (*b*) PH_3 (*c*) SbH_3 (*d*) AsH_3

52. Water has high heat of vaporisation due to

(*a*) Covalent bonding (*b*) H–bonding

(*c*) Ionic bonding (*d*) None of the above

Back Bonding, Bridge Bonding and Odd Electron Molecules

53. In BF_3:

(*a*) B-F bond has some double bond character

(*b*) All the B–F bonds are single covalent in nature

(*c*) Bond energy and bond-length of B–F bond indicate its single bond character

(*d*) All of the above bonds are ionic

PRABAL (JEE MAIN LEVEL)

1. The maximum covalency for representative elements is equal to (excluding 1st and 2nd period):

(*a*) The number of unpaired p-electrons

(*b*) The number of paired d-electrons

(*c*) The number of unpaired s and p-electrons

(*d*) The actual number of s and p-electrons in the outermost shell.

2. The types of bonds present in N_2O_5 are:

(*a*) Only covalent (*b*) Only ionic

(*c*) Ionic and covalent (*d*) Covalent & coordinate

3. Which of the following models best describes the bonding within a layer of the graphite structure?

(*a*) Metallic bonding

(*b*) Ionic bonding

(*c*) Non-metallic covalent bonding

(*d*) Vander Waals force

4. Example of super octet molecule is:

(*a*) SF_6 (*b*) PCl_5 (*c*) IF_7 (*d*) All of these

5. Which of the following species does not contain N – N covalent bond?

(*a*) N_2O_3 (*b*) $N_2O_2^{2-}$ (*c*) N_2O_5 (*d*) N_2O_4

6. Which molecule is T shaped?

(*a*) BeF_2 (*b*) BCl_3 (*c*) NH_3 (*d*) ClF_3

7. The correct order of hybridization of the central atom in the following species NH_3, $PtCl_4^{-2}$, PCl_5 and BCl_3 is

(*a*) dsp^2, sp^3d, sp^2 and sp^3

(*b*) sp^3, dsp^2, sp^3d, sp^2

(*c*) dsp^2, sp^2, sp^3, sp^3d

(*d*) dsp^2, sp^3, sp^2, sp^3d

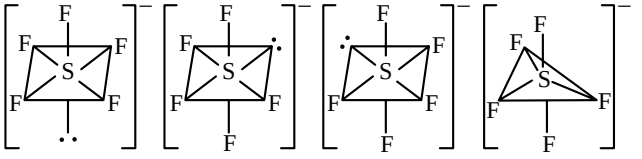
SINGLE CORRECT TYPE QUESTIONS

1. What is the state of hybridisation of Xe in cationic part of solid XeF_6?
(*a*) sp^3d^3 (*b*) sp^3d^2 (*c*) sp^3d (*d*) sp^3

2. KF combines with HF to form KHF_2. The compound contains the species:
(*a*) K^+, F^- and H^+ (*b*) K^+, F^- and HF
(*c*) K^+ and $[HF_2]^-$ (*d*) $[KHF]^+$ and F_2

3. Geometry (i.e. arrangement of electron pairs around central atom) of $ClOF_3$ is similar to the:
(*a*) XeF_4 (*b*) $SOCl_2$ (*c*) I_3^- (*d*) ClO_4^-

4. The shape of SF_5^- can be:



(*a*) I only (*b*) I and II only
(*c*) IV only (*d*) I, II, & III

5. Which of the following statement(s) is/are incorrect?
(*a*) Ethyne gas is more soluble in acetone than in water.
(*b*) CH_3F is more polar than CD_3F due to deuterium (D) being less electronegative than hydrogen (H).
(*c*) Silyl isocyanate (SiH_3NCO) is linear in shape around N while methyl isocyanate (CH_3NCO) is bent in shape around N.
(*d*) In $CH_2=C=C=C=CH_2$, the 2H–atoms on one C–atom lie in a plane perpendicular to the plane in which 2H–atoms on other C–atom lie.

6. Choose the incorrect statements.
(*a*) CH_3NCS molecule has linear arrangement around N.
(*b*) SiH_3NCS molecule has linear arrangement around N.
(*c*) GeH_3NCS molecule has bent arrangement around N.
(*d*) $[C(SiH_3)_3]^-$ molecule has pyramidal arrangement around N.

7. Which of the following molecule exist?
(*a*) SH_6 (*b*) HFO_4 (*c*) FeI_3 (*d*) $HClO_3$

8. Select correct option about $C_3N_3Cl_3$ molecule?
(*a*) It is cyclic molecule
(*b*) Geometry at carbon is triangular planar with sp^2 hybridisation of carbon
(*c*) Geometry at nitrogen is triangular planar with sp^2 hybridisation of N
(*d*) Ratio of $\sigma : \pi$: L.P. is 3 : 1 : 4

9. Which of the following statements is incorrect?
(*a*) In ClF_3, the axial Cl—F bond length is longer than equatorial Cl—F bond length.
(*b*) In SF_4, F–S–F equatorial bond angle is not 120° but 104° due to lp-bp repulsions.
(*c*) In $[ICl_4]^-$ Cl—I—Cl bond angle is 90°
(*d*) In OBr_2, the bond angle is less than that in OCl_2.

MULTIPLE CORRECT TYPE QUESTIONS

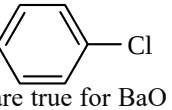
10. The pair of species/molecule having same shape but different hybridisation is/are:
(*a*) XeF_2, CO_2 (*b*) I_3^-, $HgCl_2$
(*c*) BCl_3, SO_3 (*d*) SO_2, OCl_2

11. Which of the following have planar structure?
(*a*) I_3^- (*b*) ICl_3 (*c*) Cl_2O_6 (*d*) Be_2Cl_4

12. Correct order about bond angle is/are :
(*a*) $H_2O > H_2S > H_2Se > H_2Te$
(*b*) $C_2H_2 > C_2H_4 > CH_4 > NH_3$
(*c*) $SF_6 < NH_3 < H_2O < OF_2$
(*d*) $ClO_2 > H_2O > H_2S > SF_6$

13. N_2O has a linear, unsymmetrical structure that may be thought of as a hybrid of two resonance forms. If a resonance form must have a satisfactory Lewis structure, which two of the five structures shown below are the two resonance foms of N_2O?
(*a*) :N≡N=Ö: (*b*) :N̈=N=Ö:
(*c*) :N̈—N≡O: (*d*) :N̈≡N—Ö:

14. Identify the **correct** option(s).
(*a*) $NH_4^+ > NH_3 > NH_2^-$ (order of bond angle.)
(*b*) $(CH_3)_3B$ is a trigonal planar molecule (not considering the H-atoms on 'C').
(*c*) In NH_4Cl 'N' atom is in sp^3d hybridisation.
(*d*) In S_8 molecule a total of 16 electrons are left on all the 'S' atoms after bonding.

15. Which has (have) zero value of dipole moment?
(*a*) I_2Cl_6 (*b*) $CHCl_3$
(*c*) CO_2 (*d*) 

16. Which of the following statements is/are true for BaO and MgO?
(*a*) BaO is more ionic than MgO
(*b*) MgO is more ionic than BaO
(*c*) BaO has a higher melting point than MgO
(*d*) MgO has a higher melting point than BaO

17. A, B, C are three substances. A does not conduct electricity in the solid, molten state and aqueous solution. B conducts electricity both in the fused and aqueous states, while C conducts electricity only in the aqueous state. In solid state neither B nor C conducts electricity. Which of the following statements is/are true regarding A, B and C?
(*a*) A has polar covalent linkage
(*b*) A has nonpolar covalent linkage
(*c*) B is ionic in nature
(*d*) Cation formed by C is highly polarizing

18. Which of the following have identical bond order?
(*a*) O_2^{2+} (*b*) NO^+ (*c*) CN^- (*d*) CN^+

COMPREHENSION BASED QUESTIONS

Comprehension (Q. 19 to 21): Chemical bonding between two atoms is necessarily associated with an electrical moment arising out of the difference in electronegativity of two atoms. This means that every bond carries with it an electrical moment called the "bond moment". The dipole moment of a molecule is really the vectorial sum of the individual bond moment present in it. To compute the dipole moment it is necessary to find out the values of various bond moment. in the following table dipole moment of different bonds are as given.

Bond :	$\overrightarrow{H—C}$	$\overrightarrow{C—Cl}$	$\overrightarrow{C=O}$
Bond moments :	0.4D	1.5D	2.5D

The group moments of few groups are given:

Group	NO_2	OH	CN	CH_3
Direction of dipole	towards N	towards O	towards N	away from CH_3
Dipole moment	4D	1.6 D	3.8 D	0.4 D

19. The bond angle in H_2S is 97º and its dipole moment is 1.5D. The S – H bond distance is 0.15nm. Therefore approximate percentage ionic character of S – H bond is (neglect the effect of dipole moment of lone pair on sulphur atom in H_2S). (Given [cos 97º = – 0.12] and $\sqrt{0.88}$ = 0.94)

(*a*) 32 % (*b*) 16 % (*c*) 84 % (*d*) 10 %

20. In CH_3CCl_3 (I), $CHCl_3$ (II) and CH_3Cl (III) the normal tetrahedral bond angle is maintained. $\left(\cos 70.5° = \frac{1}{3}\right)$

Therefore dipole moments of the given compounds are. (due to – I effect of Cl, the Bond moment of H—C bond are directed toward the H in $CHCl_3$)

(*a*) I = 1.9 D, II = 1.9 D, III = 1.7 D

(*b*) I = 1.9 D, II = 1.7 D, III = 1.9 D

(*c*) I = 1.9 D, II = 1.7 D, III = 1.7 D

(*d*) I = 1.9 D, II = 1.1 D, III = 1.9 D

21. In the acetone molecule considering the normal planar structure, the observed dipole moment of acetone molecule is

(*a*) 2.9 D (*b*) 2.75 D (*c*) 3 D (*d*) None of these

MATCH THE COLUMN TYPE QUESTIONS

22. Match the species given in column-I with the type of hybridisation given in column-II.

Column-I		Column-II	
A.	$IO_2F_2^-$	p.	sp^3d
B.	F_2SeO	q.	sp^3
C.	$ClOF_3$	r.	sp^2
D.	XeF_5^+	s.	sp^3d^2

(*a*) A → s; B → q; C → p; D → r

(*b*) A → p; B → q; C → p; D → s

(*c*) A → p; B → p; C → q; D → s

(*d*) A → s; B → p; C → q; D → r

INTEGER TYPE QUESTIONS

23. Count the total number of X–O bonds having equal bond length in HSO_4^- and $S_3O_6^{2-}$ respectively. (If the answer is 5 and 3 then represent as 53)

24. How many compounds among the following contains N–N bond?

N_2O_3, N_2O_4, N_2O_5. N_2O

25. Find the number of species in which bond angle(s) at central atom is less than idealised bond angle as per hybridisation.

NOF, XeF_4, XeF_3^+, $SbCl_5^{2-}$, IF_2^+, XeF_5^-, O_3, $O(CH_3)_2$

26. Calculate (x + y) for XeO_6^{4-}, where:

x = Total number of 'd' orbitals of xenon atom which participate in bond formation.

y = Number of triangular faces.

27. Consider the following species

NO_3^-, SO_4^{2-}, ClO_3^-, SO_3, PO_4^{3-}, XeO_3, CO_3^{2-}, SO_3^{2-}

Then calculate value of |x – y|, where

x : Total number of species which have bond order 1.5 or greater than 1.5

y : Total number of species which have bond order less than 1.5

28. For the following molecules:

PCl_5, BrF_3, ICl_2^-, XeF_5^-, NO_3^-, XeO_2F_2, PCl_4^+, CH_3^+

Calculate the value of $\frac{a+b}{c}$

a = Number of species having sp^3 d-hybridisation

b = Number of species which are planar

c = Number of species which are non-planar

29. The sum of bond order of O_2, O_2^- and O_2^{2-} is ____.

30. Write sum of the bond order of the following species: C_2^{2-}, B_2, NO, O_2^+, O_2^-, Cl_2, S_2

PYQ'S (PAST YEAR QUESTIONS)

KOSSEL LEWIS THEORY & LEWIS DOT STRUCTURE

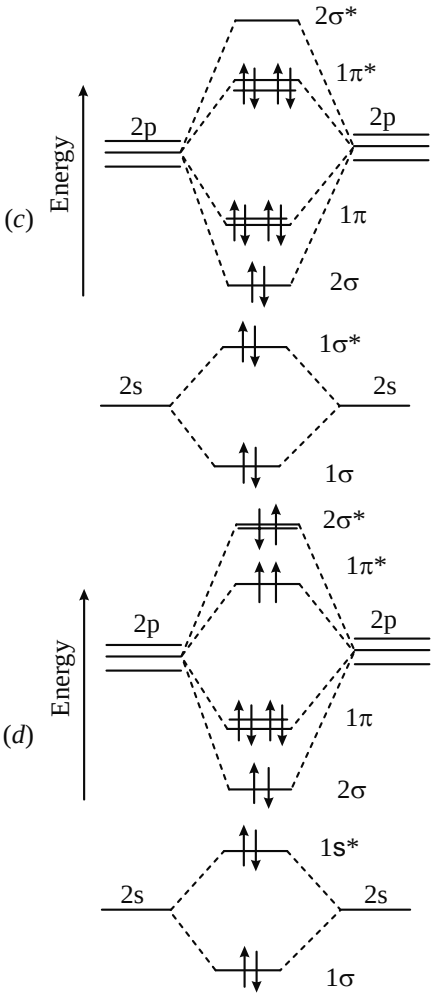
1. Which of the following pair of molecules contain odd electron molecule and an expanded octet molecule?

[29 July, 2022 (Shift-I)]

(*a*) BCl_3 and SF_6 (*b*) NO and H_2SO_4

(*c*) SF_6 and H_2SO_4 (*d*) BCl_3 and NO

2. Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$ and $H_2S_2O_8$, and total number of molecules containing covalent bond between two atoms of the same kind is_____. **[JEE Adv 2019]**



28. For diatomic molecules, the correct statement(s) about the molecular orbitals formed by the overlap of two $2p_z$ orbitals is(are)

(*a*) σ orbital has a total of two nodal planes.

(*b*) σ* orbital has one node in the xz-plane containing the molecular axis.

(*c*) π orbital has one node in the plane which is perpendicular to the molecular axis and goes through the center of the molecule.

(*d*) π* orbital has one node in the xy-plane containing the molecular axis. **[JEE Adv 2022]**

29. At 143 K. the reaction of XeF_4 with O_2F_2, produces a xenon compound **Y**. The total number of lone pair(s) of electrons present on the whole molecule of **Y** is _____.
[JEE Adv 2019]

30. A tin chloride **Q** undergoes the following reactions (not balanced) **[JEE Adv 2019]**

$Q + Cl^- \rightarrow X$

$Q + Me_3N \rightarrow Y$

$Q + CuCl_2 \rightarrow Z + CuCl$

X is a monoanion having pyramidal geometry. Both Y and Z are neutral compounds.

Choose the correct option(s).

(*a*) The central atoms in X is sp^3 hybridized

(*b*) The oxidation state of the central atom in Z is +2

(*c*) The central atom in Z has one lone pair of electrons

(*d*) There is a coordinate bond in Y

PW CHALLENGERS

1. Mark by the "+" in a corresponding window those molecules for which the assertions written on the left side of the Table, are true.

Molecule	Assertion			
	C_2H_4	N_2H_4	H_2O_2	H_2F_2
There is a covalent bond between two equal atoms				
The molecule contains a double bond				
The molecule is planar				
The molecule is polar				
There is also a hydrogen bond in the molecule				
It has basic properties in relation to water				

2. The following assertion and Table are incomplete. Fill in the dotted places by a missing word and the missing formulas, respectively.

Assertion: The electronic structures of the molecules (ions) being placed in the table one under the other are .....

CH_4	C_2H_6	CO_3^{2-}	.....	$C_2O_4^{2-}$	.....
NH_4^+	$N_2H_6^{2+}$	.....	NO_2^+	.....	N_2

PASSAGE-I

Compare three salts of a composition $M_2S_2O_x$ where x are three different small integers and M is an alkali metal. To each of the three salts apply some of the following assertions:

(*a*) The O−O bond is characteristic for the anion.

(*b*) The S−S bond is characteristic for the anion.

(*c*) The S−O−S bond is characteristic for the anion.

(*d*) It is formed by thermal decomposition of hydrogen sulphate.

(*e*) It is formed by anodic oxidation of hydrogen sulphate.

(*f*) It is formed by the reaction of an aqueous solution of sulphite with sulphur.

(*g*) Its aqueous solution dissolves silver bromide.

(*h*) Neutralisation of its aqueous solution with hydroxide MOH yields sulphate M_2SO_4.

(*i*) In aqueous solution, it is able to oxidise Mn(II) salt to permanganate.

Answer Key

CONCEPT APPLICATION

1. (*d*)	**2.** (*d*)	**3.** (*d*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*b*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)
11. (*a*)	**12.** (*c*)	**13.** (*a*)	**14.** (*c*)	**15.** (*d*)	**16.** (*a*)	**17.** (*b*)	**18.** (*c*)	**19.** (*b*)	**20.** (*b*)
21. (*c*)	**22.** (*b*)	**23.** (*b*)	**24.** (*c*)	**25.** (*c*)	**26.** (*a*)	**27.** (*b*)	**28.** (*b*)	**29.** (*a*)	**30.** (*b*)
31. (*c*)	**32.** (*a*)	**33.** (*b*)	**34.** (*a*)	**35.** (*d*)	**36.** (*b*)	**37.** (*c*)			

SCHOOL LEVEL PROBLEMS

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*d*)	**8.** (*b*)	**9.** (*a*)	**10.** (*a*)
11. (*b*)	**12.** (*a*)	**24.** (i)-(*c*), (ii)-(*b*), (iii)-(*c*), (iv)-(*b*)							

PRARAMBH (TOPICWISE)

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*a*)	**5.** (*c*)	**6.** (*b*)	**7.** (*a*)	**8.** (*b*)	**9.** (*d*)	**10.** (*c*)
11. (*d*)	**12.** (*b*)	**13.** (*d*)	**14.** (*b*)	**15.** (*b*)	**16.** (*a*)	**17.** (*d*)	**18.** (*a*)	**19.** (*c*)	**20.** (*b*)
21. (*d*)	**22.** (*d*)	**23.** (*d*)	**24.** (*d*)	**25.** (*d*)	**26.** (*a*)	**27.** (*b*)	**28.** (*b*)	**29.** (*c*)	**30.** (*c*)
31. (*c*)	**32.** (*d*)	**33.** (*b*)	**34.** (*c*)	**35.** (*d*)	**36.** (*a*)	**37.** (*a*)	**38.** (*a*)	**39.** (*a*)	**40.** (*a*)
41. (*c*)	**42.** (*d*)	**43.** (*b*)	**44.** (*c*)	**45.** (*a*)	**46.** (*c*)	**47.** (*c*)	**48.** (*c*)	**49.** (*a*)	**50.** (*c*)
51. (*b*)	**52.** (*b*)	**53.** (*a*)							

PRABAL (JEE MAIN LEVEL)

1. (*d*)	**2.** (*d*)	**3.** (*c*)	**4.** (*d*)	**5.** (*c*)	**6.** (*d*)	**7.** (*b*)	**8.** (*b*)	**9.** (*c*)	**10.** (*d*)
11. (*d*)	**12.** (*d*)	**13.** (*b*)	**14.** (*c*)	**15.** (*c*)	**16.** (*b*)	**17.** (*a*)	**18.** (*a*)	**19.** (*a*)	**20.** (*c*)
21. (*b*)	**22.** (*c*)	**23.** (*c*)	**24.** (*a*)	**25.** (*b*)	**26.** (*c*)	**27.** (*c*)	**28.** (*b*)	**29.** (*d*)	**30.** (*a*)
31. (*b*)	**32.** (*d*)	**33.** (*a*)	**34.** (*c*)	**35.** (*c*)	**36.** (*b*)	**37.** (*a*)	**38.** (*d*)	**39.** (*d*)	**40.** (*d*)
41. [2]	**42.** [3]	**43.** [6]	**44.** [5]	**45.** [3]	**46.** [3]	**47.** [3]			

PARIKSHIT (JEE ADVANCED LEVEL)

1. (*b*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)	**6.** (*a*)	**7.** (*d*)	**8.** (*c*)	**9.** (*d*)	**10.** (*a*,*b*,*c*,*d*)
11. (*a*,*b*,*d*)	**12.** (*a*,*b*,*d*)	**13.** (*b*,*d*)	**14.** (*a*,*b*)	**15.** (*a*,*c*,*d*)	**16.** (*a*,*d*)	**17.** (*b*,*c*,*d*)	**18.** (*a*,*b*,*c*)	**19.** (*b*)	**20.** (*d*)
21. (*a*)	**22.** (*b*)	**23.** [36]	**24.** [3]	**25.** [5]	**26.** [10]	**27.** [0]	**28.** [3]	**29.** [4.5]	**30.** [13.5]

PYQ'S (PAST YEAR QUESTIONS)

1. (*b*)	**2.** [4.00]	**3.** (*c*)	**4.** (*a*)	**5.** (*d*)	**6.** (*d*)	**7.** (*)	**8.** (*a*)	**9.** (*d*)	**10.** [5]
11. (*b*)	**12.** (*b*)	**13.** (*c*)	**14.** [5]	**15.** (*b*,*d*)	**16.** [6]	**17.** (*a*)	**18.** (*b*)	**19.** (*c*)	**20.** (*c*)
21. (*c*)	**22.** (*a*)	**23.** [2]	**24.** [1]	**25.** [15]	**26.** (*a*)	**27.** (*c*)	**28.** (*a*,*d*)	**29.** [19]	**30.** (*a*,*d*)

PW CHALLENGERS

9. [5]	**10.** [20]	**11.** [173]	**12.** [83]	**13.** [6]