Rohit Mandavia
Section 0101

10-17-19

Problem #1:

1) look at graphs below ↓.

Givens: $m = 1$ kg → $F_N = 9.81$ N.

$F_{applied} = t \cdot \sin(t)\,\hat{i} + \cos(t)\,\hat{j}$

Find: path of cube for intervals t=10, 20, 30 equations of motion of system. Behavior of system with friction

2) coefficient of kinetic friction is μ_K.

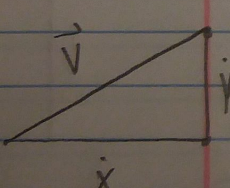
→ we know $\vec{F}_{friction}$ points in the direction opposite to the velocity.

→ $|F_{friction}| = \mu_K \cdot m \cdot g$

→ $\vec{F}_{friction} = -\vec{u}_v \cdot \mu_K \cdot m \cdot g$

> $\vec{u}_v$ = unit vector of velocity.

> $\vec{u}_v$ can also be written as:



$$\frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}}\,\hat{i} + \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}\,\hat{j}$$

where $\dot{x} = \frac{F_{applied\_x}}{m}$ and $\dot{y} = \frac{F_{applied\_y}}{m}$

$$\sum F = \vec{F}_{applied} + \vec{F}_{friction}$$

$$= t \cdot \sin(t) - \left(\frac{\dot{x} \cdot \mu_K \cdot m \cdot g}{\sqrt{\dot{x}^2 + \dot{y}^2}}\right)\hat{i} + \cos(t) - \left(\frac{\dot{y} \cdot \mu_K \cdot m \cdot g}{\sqrt{\dot{x}^2 + \dot{y}^2}}\right)\hat{j}$$

$$\therefore \boxed{\ddot{x} = \frac{1}{m}\left(t \cdot \sin(t) - \frac{\dot{x} \cdot \mu_K \cdot m \cdot g}{\sqrt{\dot{x}^2 + \dot{y}^2}}\right) \qquad \ddot{y} = \frac{1}{m}\left(\cos(t) - \frac{\dot{y} \cdot \mu_K \cdot m \cdot g}{\sqrt{\dot{x}^2 + \dot{y}^2}}\right)}$$

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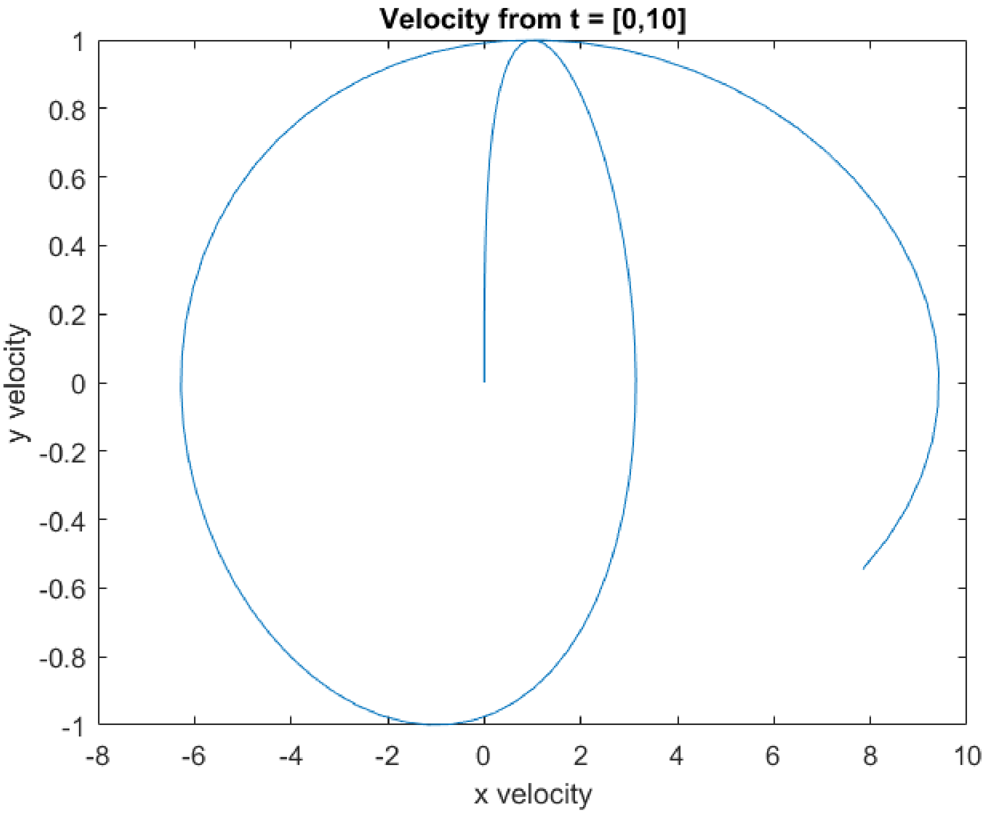
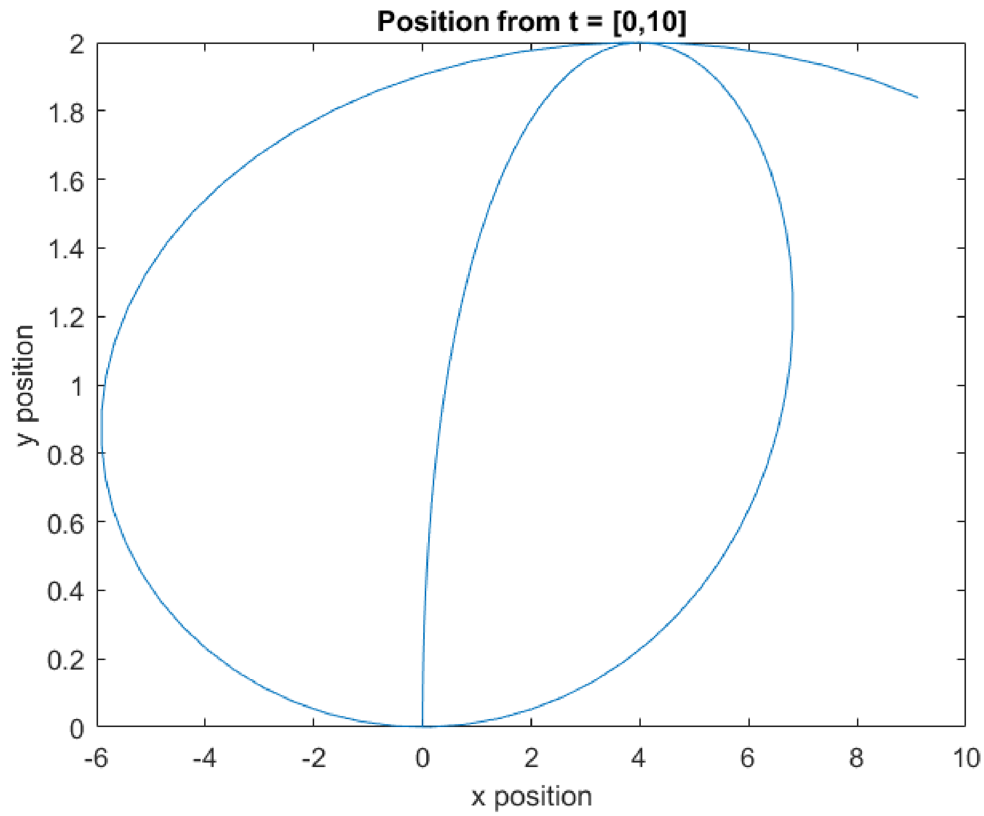
Problem # 1 (continued)

4.) It is important to note that the direction of the Frictional force points in the direction opposite to the velocity and NOT force. This means the frictional force is always acting to limit the speed, and therefor displacement, of the cube. Because of this, we notice that the cube moves in a similar, but smaller path when friction is added.

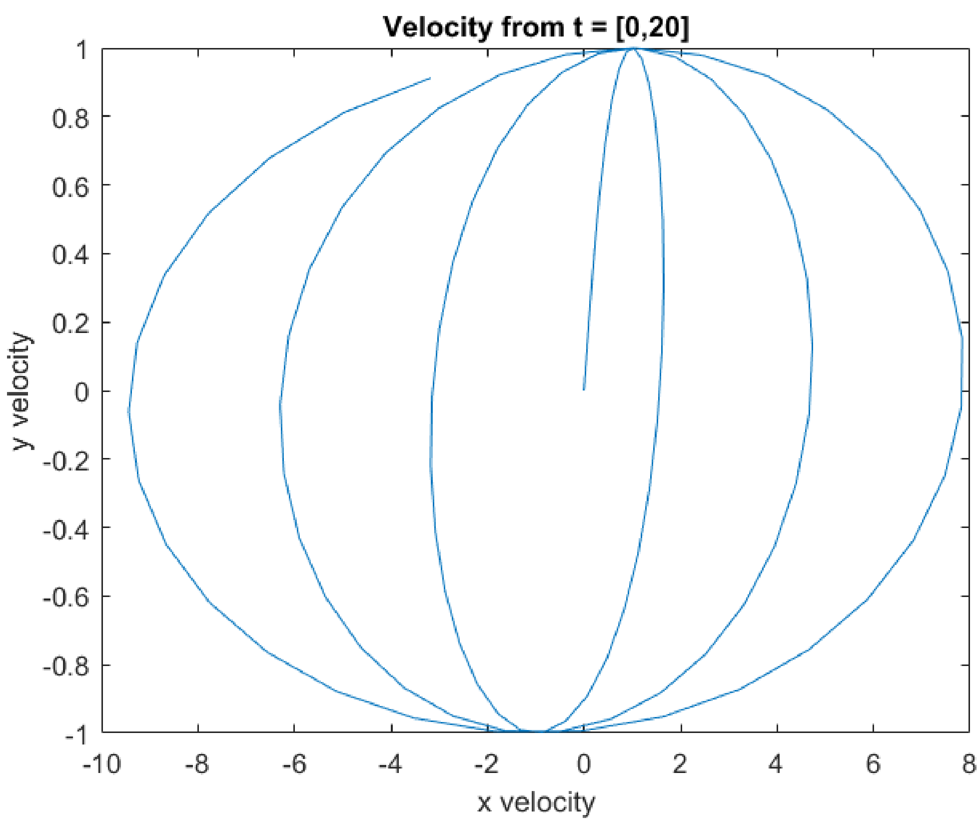
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Problem 1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

syms t

t = linspace(0,10);

F_x = t.*sin(t);
F_y = cos(t);
V_x = sin(t) - t.*cos(t);
V_y = sin(t);
X_x = - 2.*cos(t) - t.*sin(t) + 2;
X_y = -cos(t) + 1;
figure;
plot(V_x, V_y)
title 'Velocity from t = [0,10]'
xlabel("x velocity")
ylabel("y velocity")
figure;
plot(X_x, X_y)
title 'Position from t = [0,10]'
xlabel("x position")
ylabel("y position")


t2 = linspace(0,20);
F_x = t.*sin(t2);
F_y = cos(t2);
V_x = sin(t2) - t.*cos(t2);
V_y = sin(t2);
X_x = - 2.*cos(t2) - t.*sin(t2) + 2;
X_y = -cos(t2) + 1;
figure;
plot(V_x, V_y)
title 'Velocity from t = [0,20]'
xlabel("x velocity")
ylabel("y velocity")
figure;
plot(X_x, X_y)
title 'Position from t = [0,20]'
xlabel("x position")
ylabel("y position")


t3 = linspace(0,30);
F_x = t3.*sin(t3);
F_y = cos(t3);
V_x = sin(t3) - t.*cos(t3) ;
V_y = sin(t3);
X_x = - 2.*cos(t3) - t.*sin(t3) + 2;
X_y = -cos(t3) + 1;
```

```
figure;
plot(V_x, V_y)
title 'Velocity from t = [0,30]'
xlabel("x velocity")
ylabel("y velocity")
figure;
plot(X_x, X_y)
title 'Position from t = [0,30]'
xlabel("x position")
ylabel("y position")
```

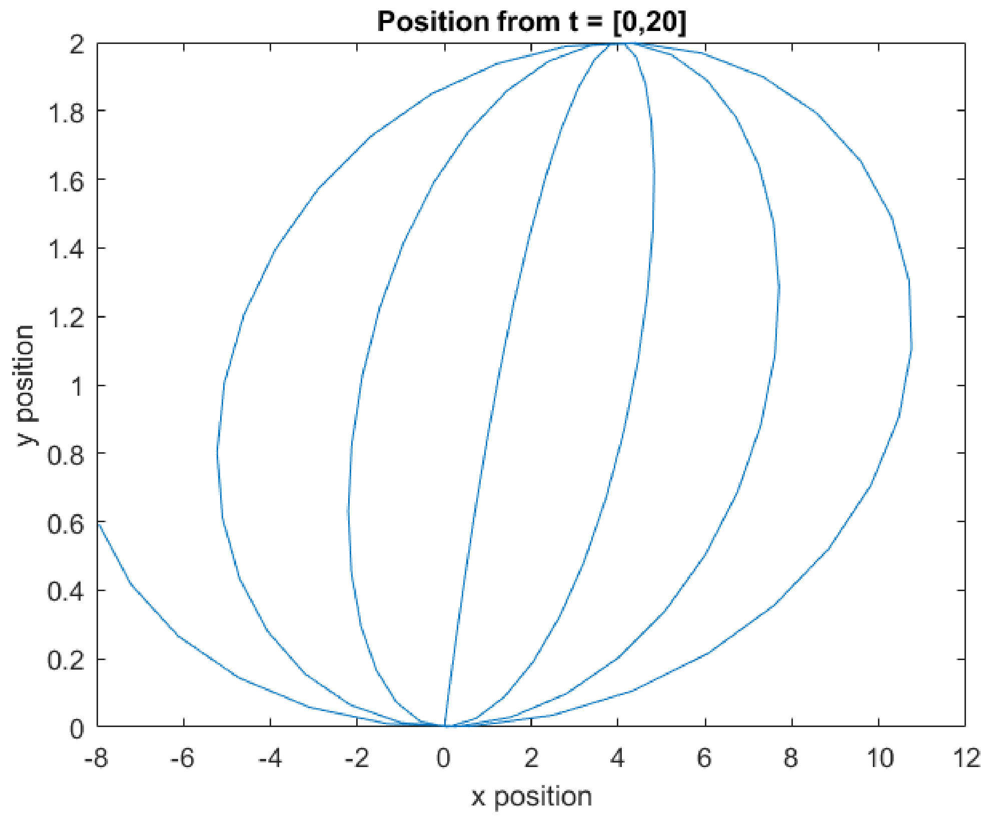


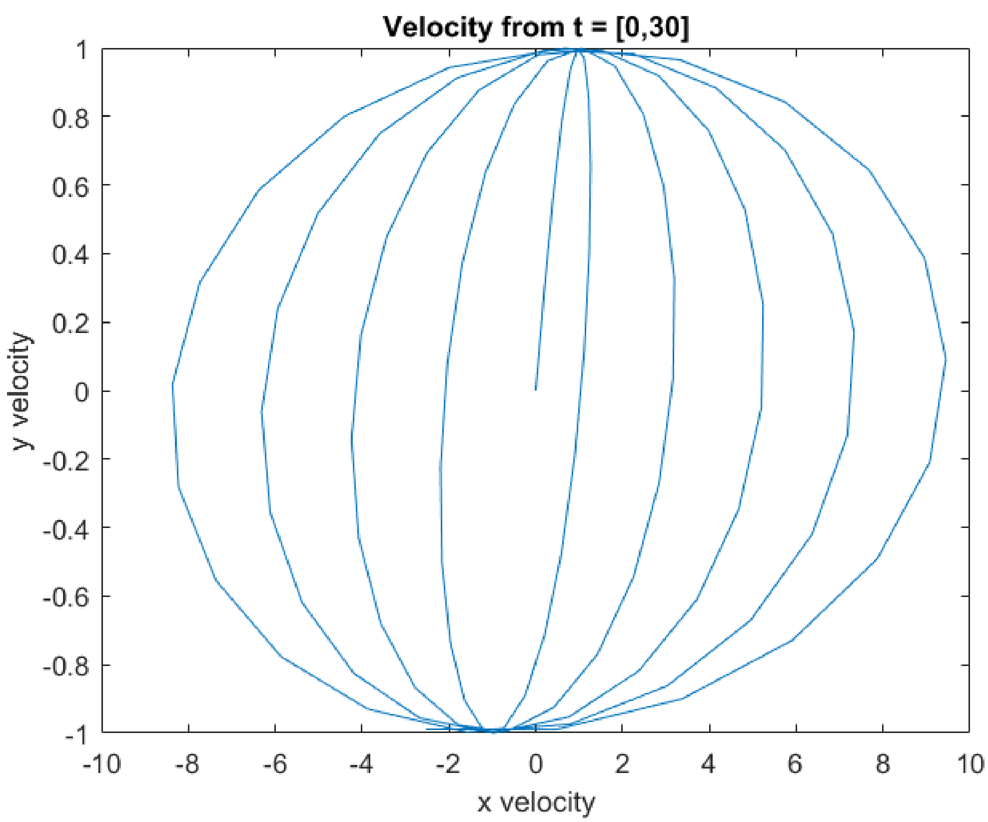


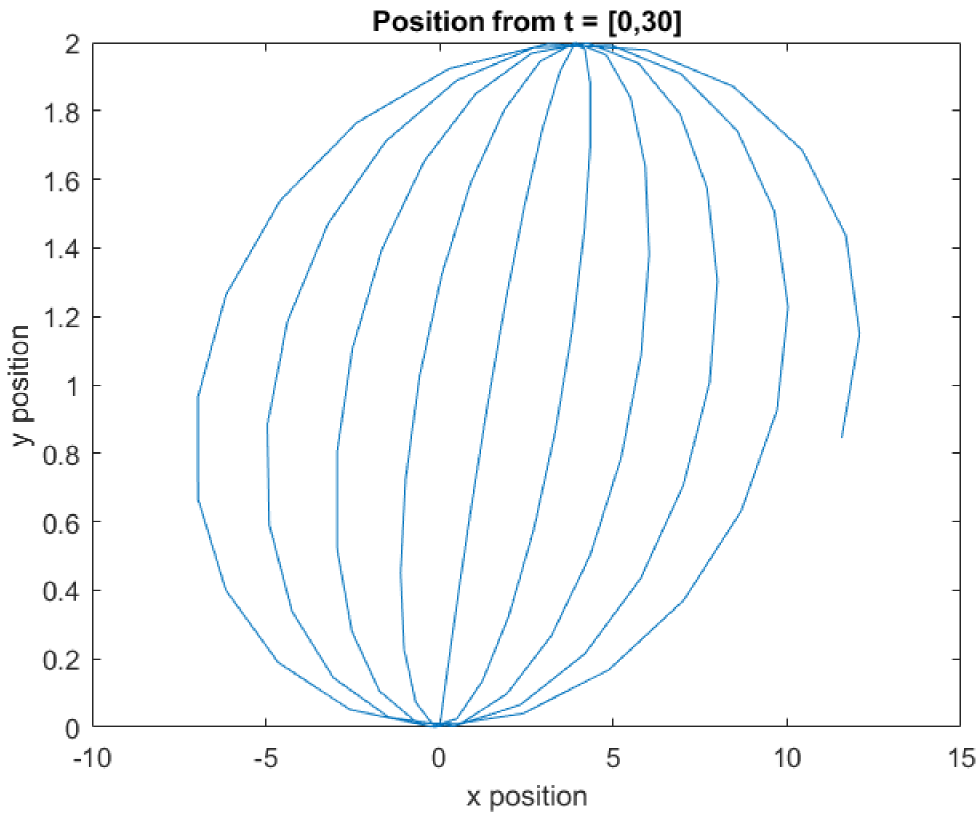
Position from t = [0,10]
y position
x position



Velocity from t = [0,20]
y velocity
x velocity


Position from t = [0,20]
y position
x position
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-8
-6
-4
-2
0
2
4
6
8
10
12


Velocity from t = [0,30]
y velocity
x velocity
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-10
-8
-6
-4
-2
0
2
4
6
8
10



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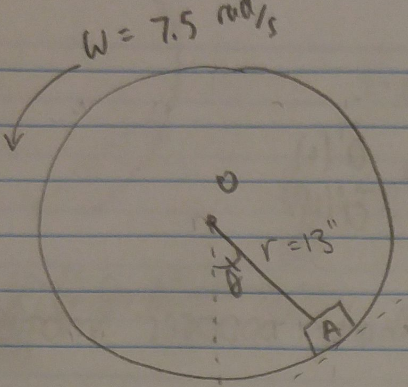
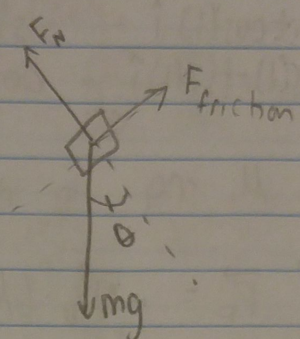
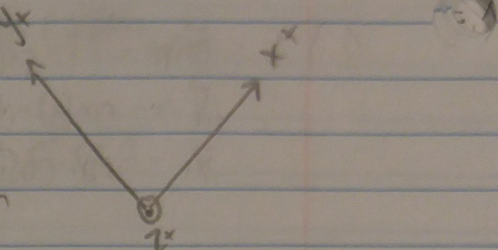
Problem #2







$F_N = \frac{mv^2}{r} + mg\cos\theta$, $\boxed{\begin{matrix} F_F = \mu \cdot F_N. \\ F_F \geq mg \cdot \sin\theta \end{matrix}}$ → equations that govern friction

$\therefore \mu\left(\frac{mv^2}{r} + mg\cos\theta\right) \geq mg\sin\theta$

$\rightarrow \mu = \dfrac{g \cdot \sin\theta}{\frac{v^2}{r} + g\cos\theta} = \dfrac{386.1 \cdot \sin\theta}{13 \cdot (7.5)^2 + 386.1\cos\theta}$

$\frac{v^2}{r} = r \cdot \omega^2$

Givens: $\omega = +7.5 \frac{rad}{s}$, $r = 13''$, $\theta = 0^\circ$ at bottom

1.) Plot of μ_s from $[0°, 180°]$
please look at screenshot provided below.

2.) μ_{min} = max of #1 because this means the block would be able to rotate from the bottom of the ring to the top without slipping. Going from the top to bottom ($\theta = [180, 360]$) would require the same μ_{min} because the velocity is the same magnitude and the angles are just flipped

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Problem #2 (cont.)

To find μ_{min}, I set the derivative of the μ_k function equal to zero. Becaus the graph only has one critical point, I knew it would be the max.

$$\text{solve}\left(\frac{d\mu_s}{dt} = 0\right) \rightarrow \theta = 2.12704 \text{ rad} = 121.87^\circ$$

$$\mu_s(\theta) = \mu_s(2.12704) = 0.6217$$

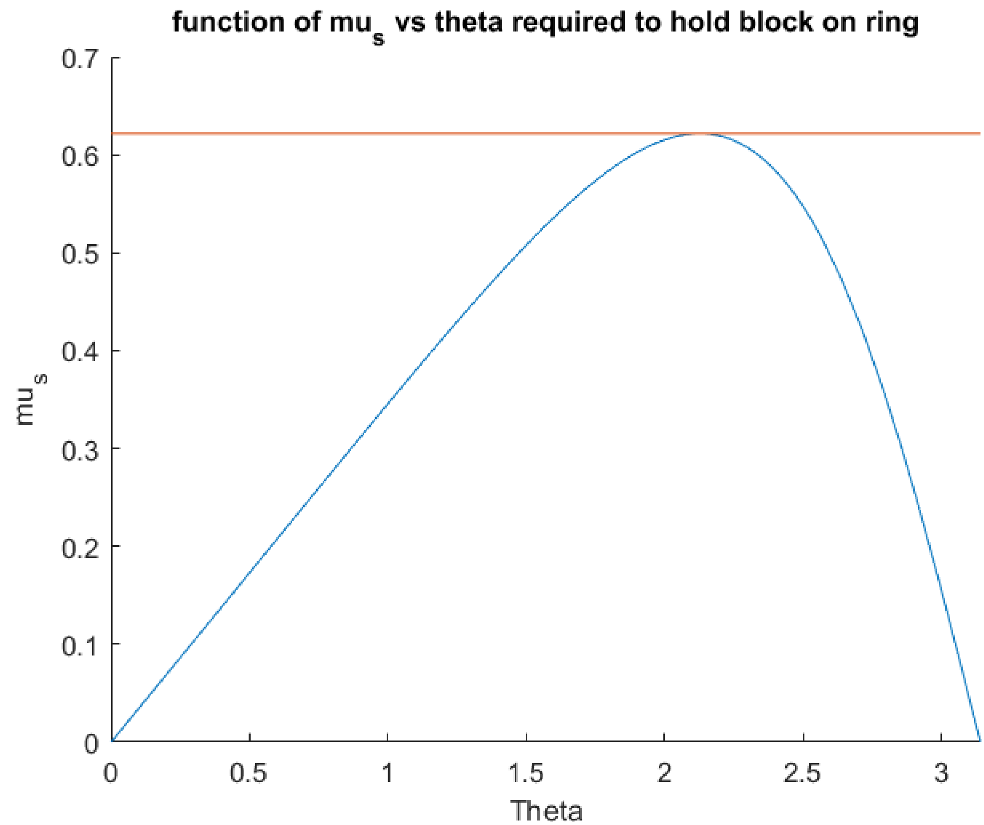
$$\therefore \boxed{\mu_{s\,min} = 0.6217}$$

3.) I actually already solved for the θ where the slippage would occur in #2. μ_s slightly less than μ_{min}. It would be at 2.12704 rad (121.87°) because this is where the greatest value of μ_s is needed.

$\therefore$ slippage would occur at 2.12704 rad

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Problem 2
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

syms t
figure;
hold on
M = (386.1.*sin(t))./(13*7.5^2 + 386.1*cos(t));
title 'function of mu_s vs theta required to hold block on ring'
xlabel ("Theta")
ylabel ("mu_s")
fplot(M)
axis([0 pi 0 .7]);
dt = diff(M, t)
vpa(solve(dt==0))
%use the one that is in the region [0 - pi] so theta = 2.12704
%plug theta into M to get Mmax.
%this max M is the minimum requried Ms to get the block aroun the
 entire
%circle.
Mmin = 0.6217
fplot(Mmin)
```

```
dt =

(3861*cos(t))/(10*((3861*cos(t))/10 + 2925/4)) + (14907321*sin(t)^2)/
(100*((3861*cos(t))/10 + 2925/4)^2)


ans =

 4.1561451786156195558748842543849
 2.1270401285639669210504025121741


Mmin =

    0.6217
```



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