Multiple Linear Regression

What is Multiple Linear Regression

19 April 2023 19:24

1 input

exp | salary

city | age | gender | edu

more than 1 predictors

→ SLR ✗

↳ MLR

1000 students → low

(β_1) x_1 cgpa	x_2 (β_2) iq	placement
8	80	8
9	90	9
5	120	15
⋮		

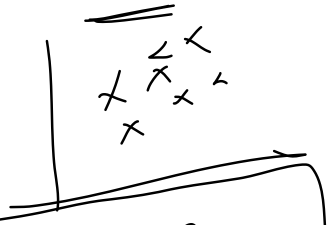
→ model

[simple (linear reg)] {multiple linear reg}

cgpa → [ ] → lpa

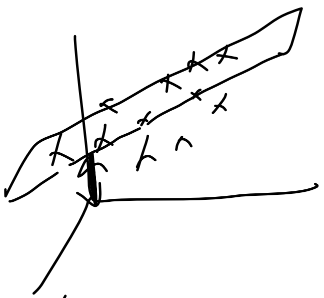
iq ↗

line



$y = mx + b$

$y = \beta_0 + \beta_1 X_1$



$$\boxed{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2}$$

$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$

$$\boxed{y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n}$$

$x_1 \rightarrow$ cgpa

$x_2 \rightarrow$ iq

$\beta_0 \rightarrow b$

$\beta_1 \rightarrow m$

$\beta_2 \rightarrow$

hyperplane in n-dim coordinate

cgpa | iq | 12th marks

m, b

$$\boxed{\beta_0, \beta_1 \ldots \beta_n}$$

coefficients

3d data → 2 input, 1 output

Python Code

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Mathematical Formulation

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x_1 cgpa	x_2 iq	y placement
8 x_{11}	80 x_{12}	8
7 x_{21}	70 x_{22}	7
5 x_{31}	120 x_{32}	15

β_0 (β_1) (β_2)

3) predict

m cols → n students

$$\hat{y}_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + \beta_3 x_{13} + \beta_4 x_{14} + \dots + \beta_m x_{1m}$$
$$\hat{y}_2 = \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + \dots + \beta_m x_{2m}$$
$$\hat{y}_3 = \beta_0 + \beta_1 x_{31} + \beta_2 x_{32} + \dots + \beta_m x_{3m}$$
$$\hat{y}_4$$
$$\vdots$$
$$\hat{y}_n = \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + \dots + \beta_m x_{nm}$$

$y_1 = 8 \quad y_2 = 7 \quad y_3 = 15$

$\hat{y}_1 = ? \quad \hat{y}_2 = ? \quad \hat{y}_3 = ?$

$$\begin{bmatrix} \hat{y}_1 = \beta_0 + \beta_1 8 + \beta_2 80 \\ \hat{y}_2 = \beta_0 + \beta_1 7 + \beta_2 70 \\ \hat{y}_3 = \beta_0 + \beta_1 5 + \beta_2 120 \end{bmatrix}$$

$$\hat{Y} = \underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}}_{n \times 1} = \begin{bmatrix} \beta_0 + \beta_1 x_{11} + \beta_2 x_{12} + \beta_3 x_{13} + \beta_4 x_{14} + \dots + \beta_m x_{1m} \\ \beta_0 + \beta_1 x_{21} + \beta_2 x_{22} + \dots + \beta_m x_{2m} \\ \beta_0 + \beta_1 x_{31} + \beta_2 x_{32} + \dots + \beta_m x_{3m} \\ \vdots \\ \beta_0 + \beta_1 x_{n1} + \beta_2 x_{n2} + \dots + \beta_m x_{nm} \end{bmatrix}$$

$(n \times 1)$

$$= \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1m} \\ 1 & x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & & & & \\ 1 & x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_m \end{bmatrix}$$

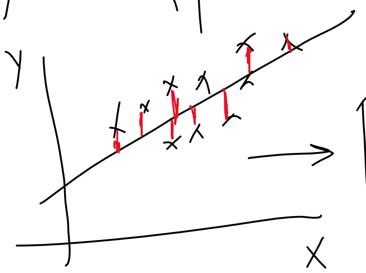
$$\boxed{\hat{Y} = X\beta}$$

$n \times (m+1) \longleftrightarrow (m+1) \times 1$

$\boxed{n \times 1}$

$$\hat{Y} = X\beta$$

$$\boxed{d_1^2 + d_2^2 + d_3^2 + \dots d_n^2}$$



$$\boxed{\min E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2}$$

→ minimize

matrix form

→ matrix form

$$Y = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}}_{n \times 1} \qquad \hat{Y} = \underbrace{\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}}_{n \times 1}$$

$$e = Y - \hat{Y} = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \end{bmatrix}$$

$$e = y - \hat{y} = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}_{n \times 1}$$

$$e^T e = \begin{bmatrix} y_1 - \hat{y}_1 & y_2 - \hat{y}_2 & \cdots & y_n - \hat{y}_n \end{bmatrix}_{1 \times n} \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}_{n \times 1} =$$

1×1

$$e^T e = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + \dots + (y_n - \hat{y}_n)^2$$

$$= \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$\to$ II nd eq^n

$$\boxed{E = e^T e}$$

$\uparrow$ minimize

$$E = (y - \hat{y})^T (y - \hat{y}) = (y^T - \hat{y}^T)(y - \hat{y})$$

$$E = y^T y - y^T \hat{y} - \hat{y}^T y + \hat{y}^T \hat{y}$$

$$\boxed{E = y^T y - 2 y^T \hat{y} + \hat{y}^T \hat{y}} \to \text{eq}^n \ 3$$

$\hat{y} = X\beta$

$y = f(x) \to x$

$$E = y^T y - 2 y^T X\beta + (X\beta)^T (X\beta)$$

$$\boxed{E = y^T y - 2 y^T X \beta + \beta^T X^T X \beta} \leftarrow \text{eq}^n \ 4$$

$y \to$ data output

$X \to$ data input

$f(x) = x^2$

$\boxed{\beta^T X^T X \beta}$

$\frac{d}{d\beta}$

$E(\beta) =$ find such value of β matrix for which E is min

$$\boxed{\frac{dE}{d\beta} = 0}$$

$$\frac{dE}{d\beta} = 0 - 2 y^T X + 2 \beta^T X^T X = 0$$

$$\cancel{2} \beta^T X^T X = \cancel{2} y^T X$$

matrix diff

$A^T = A$

$$\boxed{\beta^T X^T X = y^T X}$$

$$\frac{d}{d\beta} \beta^T (X^T X) \beta$$

A is symmetric

$$\beta^T X^T X (X^T X)^{-1} = y^T X (X^T X)^{-1}$$

$$\frac{d}{d\beta}\, \beta^T (X^T X) \beta$$

$x^T A x$ $\quad 2x^T A$

$$= \boxed{2\beta^T X^T X}$$

A is symmetric

$$(X^T X)^T = X^T X$$

$$A^T = A$$

$$\beta = [(X^T X)^{-1}]^T (y^T X)^T$$

$$\beta = [(X^T X)^{-1}]^T X^T y$$

$$\boxed{\beta = (X^T X)^{-1} X^T y}$$

eqn 5

$\boxed{\text{OLS}}$

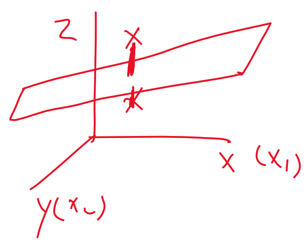
β = values $\quad (m+1 \times 1) \rightarrow (m+1) \times 1$

$$(X^T X)^{-1}$$

$m+1 \times n \quad n \times (m+1)$

$(m+1) \times (m+1) \quad (m+1) \times n$

$(m+1) \times n \quad n \times 1$

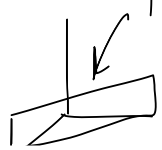


exp | 12th marks | salary

$\beta_1 \quad \beta_2$

$$\text{salary} = \text{exp} \times \beta_1 + 12^{th} \times \beta_2 + \boxed{\beta_0}$$

0 $\quad$ 0



$\beta_0 = 0$

$$(X^T X)^{-1} X^T y$$

$$\beta^T X^T X (X^T X)^{-1} = y^T X (X^T X)^{-1}$$

$$\beta^T I = y^T X (X^T X)^{-1}$$

$$\beta^T = y^T X (X^T X)^{-1}$$

$$(\beta^T)^T = [\underset{A}{y^T X} \underset{B}{(X^T X)^{-1}}]^T$$

$(X^T X)^{-1}$ symmetric

$$[(X^T X)^{-1}]^T = (X^T X)^{-1}$$

$$(X^T X)^T = X^T X$$

$$A^T = A$$

$$X^T X = A$$

$$A A^{-1} = I$$

$$(A A^{-1})^T = I^T$$

global minima

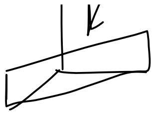
$$(A^{-1})^T A^T = I$$

$$(A^{-1})^T A = I$$

$$(A^{-1})^T A A^{-1} = I A^{-1}$$

$$(A^{-1})^T I = A^{-1}$$

$$(A^{-1})^T = A^{-1}$$



$\beta_0 = 0$

$$(X^T X)^{-1} X^T Y$$

$1 \quad x_1 \quad x_2 \quad x_3$

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$y = A \quad \hat{y} = B$

$(AB)^T = B^T A^T$

$(A^T)^T = A$

$$\boxed{y^T \hat{y} = \hat{y}^T y}$$

$A^T B = (A^T B)^T$

$\boxed{A^T B} = C$ symmetric matrix

$A^T B = B^T A$

$\boxed{C = C^T}$

$(A^T B)^T = B^T A$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\hat{y} = X\beta$

$n \times (m+1)$

$A^T B$ is symmetric

$1 \times 1 \rightarrow$ Scaler

$(1) \quad [2]$

$y^T \hat{y} = y^T X \beta$

$A^T B$ is sym

$(1 \times n) \quad n \times (m+1) \quad (m+1) \times 1$

$1 \times n \quad n \times 1$

Code From Scratch

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Problem with OLS solution

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linear reg

$\boxed{OLS}$ (closed

$\boxed{\text{gradient descent}}$

10 input

$$OLS \rightarrow \boxed{(X^T X)^{-1}}$$

inverse

$O(n^3)$

$X \rightarrow n \times (m+1)$

$(m+1) \times n \quad n \times (m+1)$

$(m+1) \times (m+1)$

11×11

$\boxed{X} \rightarrow n$ row $(m+1)$

$n \times (m+1)$

$(m+1) \times n \quad n \times (m+1)$

$(m+1) \quad (m+1)$

$(100)^3$

$[1000000]$

$(100 \times 100) \quad (1000)^3$