1 WHOLE NUMBERS

Figure 1.1 Purchasing pounds of fruit at a fruit market requires a basic understanding of numbers. (credit: Dr. Karl-Heinz Hochhaus, Wikimedia Commons)

Chapter Outline

Introduction

Even though counting is first taught at a young age, mastering mathematics, which is the study of numbers, requires constant attention. If it has been a while since you have studied math, it can be helpful to review basic topics. In this chapter, we will focus on numbers used for counting as well as four arithmetic operations—addition, subtraction, multiplication, and division. We will also discuss some vocabulary that we will use throughout this book.

1.1 Introduction to Whole Numbers

Learning Objectives

By the end of this section, you will be able to:

- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Round whole numbers

Identify Counting Numbers and Whole Numbers

Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

Algebra uses numbers and symbols to represent words and ideas. Let’s look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5, \ldots$ and so on. These are called the **counting numbers**. The notation “…” is called an ellipsis, which is another way to show “and so on”, or that the pattern continues endlessly. Counting numbers are also called natural numbers.

Counting Numbers

The counting numbers start with 1 and continue.

$$1, 2, 3, 4, 5\ldots$$

Counting numbers and whole numbers can be visualized on a **number line** as shown in Figure 1.2.

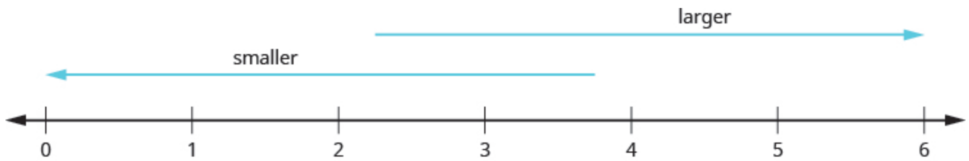


Figure 1.2 The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the **origin**. The points are equally spaced to the right of 0 and labeled with the counting numbers. When a number is paired with a point, it is called the **coordinate** of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the **whole numbers**.

Whole Numbers

The whole numbers are the counting numbers and zero.

$$0, 1, 2, 3, 4, 5\ldots$$

We stopped at 5 when listing the first few counting numbers and whole numbers. We could have written more numbers if they were needed to make the patterns clear.

EXAMPLE 1.1

Which of the following are ⓐ counting numbers? ⓑ whole numbers?

$0, \frac{1}{4}, 3, 5.2, 15, 105$

Solution

ⓐ The counting numbers start at 1, so 0 is not a counting number. The numbers $3, 15,$ and 105 are all counting numbers.

ⓑ Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ and 105 are whole numbers.

The numbers $\frac{1}{4}$ and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

TRY IT 1.1 Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{2}{3}, 2, 9, 11.8, 241, 376$

TRY IT 1.2 Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{5}{3}, 7, 8.8, 13, 201$

Model Whole Numbers

Our number system is called a **place value system** because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the 7 and the 5.

Money gives us a familiar model of place value. Suppose a wallet contains three $\$100$ bills, seven $\$10$ bills, and four $\$1$

bills. The amounts are summarized in Figure 1.3. How much money is in the wallet?

Figure 1.3

Find the total value of each kind of bill, and then add to find the total. The wallet contains $\$374$.

$300 + $70 + $4

$374

Base-10 blocks provide another way to model place value, as shown in Figure 1.4. The blocks can be used to represent hundreds, tens, and ones. Notice that the tens rod is made up of 10 ones, and the hundreds square is made of 10 tens, or 100 ones.

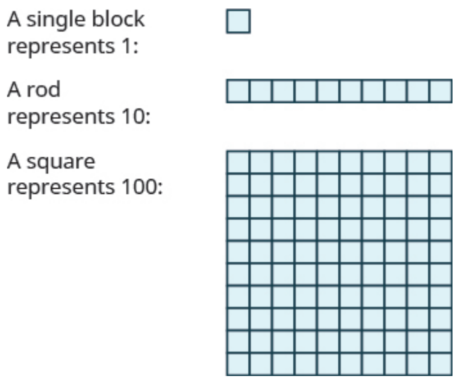


Figure 1.4

Figure 1.5 shows the number 138 modeled with $\text{base-}10$ blocks.

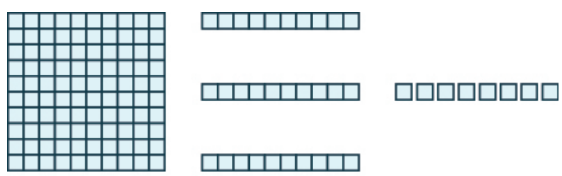


Figure 1.5 We use place value notation to show the value of the number 138.

100 + 30 + 8

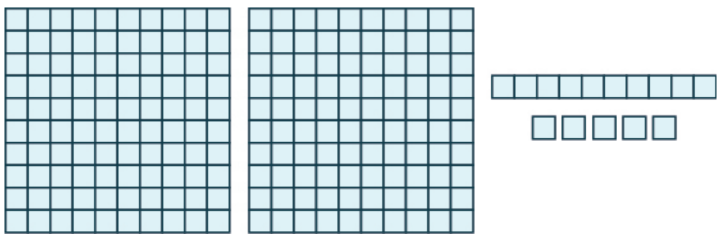
138

Digit	Place value	Number	Value	Total value
1	hundreds	1	100	100
3	tens	3	10	30

Digit	Place value	Number	Value	Total value
8	ones	8	1	+ 8
				Sum =138

EXAMPLE 1.2

Use place value notation to find the value of the number modeled by the base-10 blocks shown.

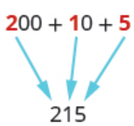


Solution

There are 2 hundreds squares, which is 200.

There is 1 tens rod, which is 10.

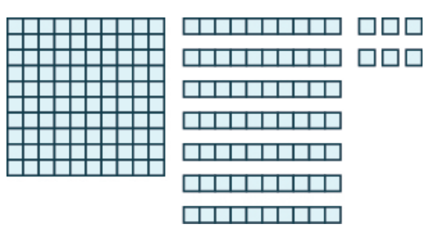
There are 5 ones blocks, which is 5.



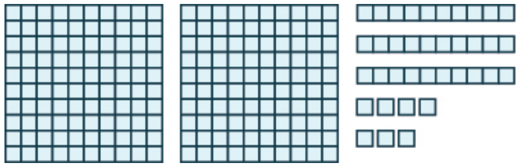
Digit	Place value	Number	Value	Total value
2	hundreds	2	100	200
1	tens	1	10	10
5	ones	5	1	+ 5
				215

The base-10 blocks model the number 215.

TRY IT 1.3 Use place value notation to find the value of the number modeled by the base-10 blocks shown.



TRY IT 1.4 Use place value notation to find the value of the number modeled by the base-10 blocks shown.



MANIPULATIVE MATHEMATICS

Doing the Manipulative Mathematics activity Number Line-Part 1 will help you develop a better understanding of the counting numbers and the whole numbers.

Identify the Place Value of a Digit

By looking at money and $\text{base-}10$ blocks, we saw that each place in a number has a different value. A place value chart is a useful way to summarize this information. The place values are separated into groups of three, called periods. The periods are *ones, thousands, millions, billions, trillions*, and so on. In a written number, commas separate the periods.

Just as with the $\text{base-}10$ blocks, where the value of the tens rod is ten times the value of the ones block and the value of the hundreds square is ten times the tens rod, the value of each place in the place-value chart is ten times the value of the place to the right of it.

Figure 1.6 shows how the number $5{,}278{,}194$ is written in a place value chart.

Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

Figure 1.6

- The digit 5 is in the millions place. Its value is $5{,}000{,}000$.
- The digit 2 is in the hundred thousands place. Its value is $200{,}000$.
- The digit 7 is in the ten thousands place. Its value is $70{,}000$.
- The digit 8 is in the thousands place. Its value is $8{,}000$.
- The digit 1 is in the hundreds place. Its value is 100.
- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

EXAMPLE 1.3

In the number $63{,}407{,}218$; find the place value of each of the following digits:

ⓐ 7 ⓑ 0 ⓒ 1 ⓓ 6 ⓔ 3

Solution

Write the number in a place value chart, starting at the right.

Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

ⓐ The 7 is in the thousands place.
ⓑ The 0 is in the ten thousands place.
ⓒ The 1 is in the tens place.
ⓓ The 6 is in the ten millions place.
ⓔ The 3 is in the millions place.

TRY IT 1.5 For each number, find the place value of digits listed: $27,493,615$

ⓐ 2 ⓑ 1 ⓒ 4 ⓓ 7 ⓔ 5

TRY IT 1.6 For each number, find the place value of digits listed: $519,711,641,328$

ⓐ 9 ⓑ 4 ⓒ 2 ⓓ 6 ⓔ 7

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the ‘s’ at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.

$$\underbrace{37}_{\text{millions}}\ ,\ \underbrace{519}_{\text{thousands}}\ ,\ \underbrace{248}_{\text{ones}} \longleftarrow \textit{periods}$$

37 → Thirty-seven million
519 → Five hundred nineteen thousand
248 → Two hundred forty-eight

So the number $37,519,248$ is written thirty-seven million, five hundred nineteen thousand, two hundred forty-eight.

Notice that the word *and* is not used when naming a whole number.

HOW TO

Name a whole number in words.

Step 1. Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
Step 2. Use commas in the number to separate the periods.

EXAMPLE 1.4

Name the number $8,165,432,098,710$ in words.

Solution

Begin with the leftmost digit, which is 8. It is in the trillions place.	eight trillion
The next period to the right is billions.	one hundred sixty-five billion
The next period to the right is millions.	four hundred thirty-two million
The next period to the right is thousands.	ninety-eight thousand
The rightmost period shows the ones.	seven hundred ten

$$\underbrace{8}_{\textit{trillions}}\;,\;\underbrace{165}_{\textit{billions}}\;,\;\underbrace{432}_{\textit{millions}}\;,\;\underbrace{098}_{\textit{thousands}}\;,\;\underbrace{710}_{\textit{ones}}$$

$8 \longrightarrow$ Eight trillion,
$165 \longrightarrow$ One hundred sixty-five billion,
$432 \longrightarrow$ Four hundred thirty-two million,
$098 \longrightarrow$ Ninety-eight thousand,
$710 \longrightarrow$ Seven hundred ten

Putting all of the words together, we write $8{,}165{,}432{,}098{,}710$ as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

TRY IT 1.7 Name each number in words: $9{,}258{,}137{,}904{,}061$

TRY IT 1.8 Name each number in words: $17{,}864{,}325{,}619{,}004$

EXAMPLE 1.5

A student conducted research and found that the number of mobile phone users in the United States during one month in 2014 was $\mathbf{327{,}577{,}529}$. Name that number in words.

Solution

Identify the periods associated with the number.

$$\underbrace{327}_{\textit{millions}}\;,\;\underbrace{577}_{\textit{thousands}}\;,\;\underbrace{529}_{\textit{ones}}$$

Name the number in each period, followed by the period name. Put the commas in to separate the periods.

Millions period: three hundred twenty-seven million

Thousands period: five hundred seventy-seven thousand

Ones period: five hundred twenty-nine

So the number of mobile phone users in the Unites States during the month of April was three hundred twenty-seven million, five hundred seventy-seven thousand, five hundred twenty-nine.

TRY IT 1.9 The population in a country is $316{,}128{,}839$. Name that number.

TRY IT 1.10 One year is $31{,}536{,}000$ seconds. Name that number.

Use Place Value to Write Whole Numbers

We will now reverse the process and write a number given in words as digits.

HOW TO

Use place value to write a whole number.

Step 1. Identify the words that indicate periods. (Remember the ones period is never named.)
Step 2. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Step 3. Name the number in each period and place the digits in the correct place value position.

EXAMPLE 1.6

Write the following numbers using digits.

ⓐ fifty-three million, four hundred one thousand, seven hundred forty-two
ⓑ nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine

Solution

ⓐ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.

millions		thousands		ones
fifty-three million	,	four hundred one thousand	,	seven hundred forty-two
_ 5 3		4 0 1		7 4 2

Put the numbers together, including the commas. The number is $53,401,742$.

ⓑ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.

billions		millions		thousands		ones
nine billion	,	two hundred forty-six million	,	seventy-three thousand	,	one hundred eighty-nine
_ _ 9		2 4 6		0 7 3		1 8 9

The number is $9,246,073,189$.

Notice that in part ⓑ, a zero was needed as a place-holder in the hundred thousands place. Be sure to write zeros as needed to make sure that each period, except possibly the first, has three places.

TRY IT 1.11 Write each number in standard form:

fifty-three million, eight hundred nine thousand, fifty-one.

TRY IT 1.12 Write each number in standard form:

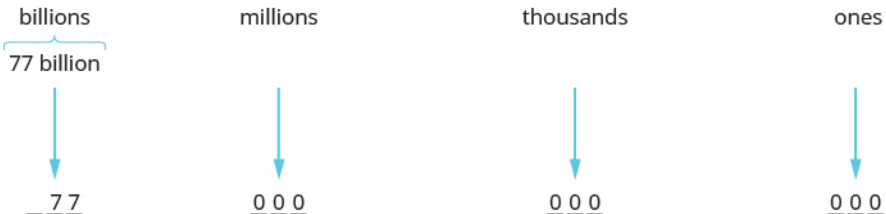
two billion, twenty-two million, seven hundred fourteen thousand, four hundred sixty-six.

EXAMPLE 1.7

A state budget was about $77 billion. Write the budget in standard form.

Solution

Identify the periods. In this case, only two digits are given and they are in the billions period. To write the entire number, write zeros for all of the other periods.



So the budget was about $77,000,000,000.

TRY IT 1.13 Write each number in standard form:

The closest distance from Earth to Mars is about 34 million miles.

TRY IT 1.14 Write each number in standard form:

The total weight of an aircraft carrier is 204 million pounds.

Round Whole Numbers

In 2013, the U.S. Census Bureau reported the population of the state of New York as 19,651,127 people. It might be enough to say that the population is approximately 20 million. The word *approximately* means that 20 million is not the exact population, but is close to the exact value.

The process of approximating a number is called **rounding**. Numbers are rounded to a specific place value depending on how much accuracy is needed. 20 million was achieved by rounding to the millions place. Had we rounded to the one hundred thousands place, we would have 19,700,000 as a result. Had we rounded to the ten thousands place, we would have 19,650,000 as a result, and so on. The place value to which we round to depends on how we need to use the number.

Using the number line can help you visualize and understand the rounding process. Look at the number line in Figure 1.7. Suppose we want to round the number 76 to the nearest ten. Is 76 closer to 70 or 80 on the number line?

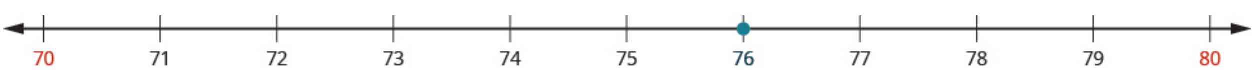


Figure 1.7 We can see that 76 is closer to 80 than to 70. So 76 rounded to the nearest ten is 80.

Now consider the number 72. Find 72 in Figure 1.8.

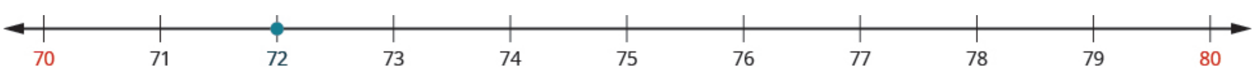


Figure 1.8 We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

How do we round 75 to the nearest ten. Find 75 in Figure 1.9.

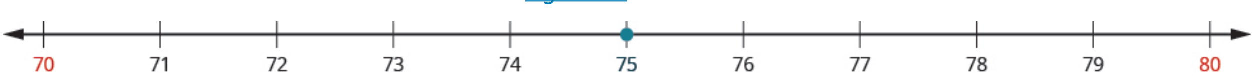


Figure 1.9 The number 75 is exactly midway between 70 and 80.

So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

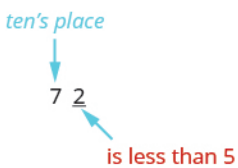
Now that we have looked at this process on the number line, we can introduce a more general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.

So, for example, to round 76 to the nearest ten, we look at the digit in the ones place.

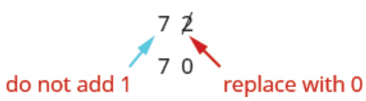
The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

76 rounded to the nearest ten is 80.

Let’s look again at rounding 72 to the nearest 10. Again, we look to the ones place.



The digit in the ones place is 2. Because 2 is less than 5, we keep the digit in the tens place the same and replace the digits to the right of it with zero. So 72 rounded to the nearest ten is 70.



HOW TO

Round a whole number to a specific place value.

Step 1. Locate the given place value. All digits to the left of that place value do not change unless the digit immediately to the left is 9, in which case it may. (See Step 3.)

Step 2. Underline the digit to the right of the given place value.

Step 3. Determine if this digit is greater than or equal to 5.

- Yes—add 1 to the digit in the given place value. If that digit is 9, replace it with 0 and add 1 to the digit immediately to its left. If that digit is also a 9, repeat.
- No—do not change the digit in the given place value.

Step 4. Replace all digits to the right of the given place value with zeros.

EXAMPLE 1.8

Round 843 to the nearest ten.

Solution

Locate the tens place.	*tens place* 843
Underline the digit to the right of the tens place.	84$\underline{3}$
Since 3 is less than 5, do not change the digit in the tens place.	84$\underline{3}$
Replace all digits to the right of the tens place with zeros.	84$\underline{0}$
	Rounding 843 to the nearest ten gives 840.

TRY IT 1.15 Round to the nearest ten: 157.

TRY IT 1.16 Round to the nearest ten: 884.

EXAMPLE 1.9

Round each number to the nearest hundred:

ⓐ 23,658 ⓑ 3,978

Solution

ⓐ

Locate the hundreds place.	hundreds place 23,658
The digit to the right of the hundreds place is 5. Underline the digit to the right of the hundreds place.	23,6$\underline{5}$8
Since 5 is greater than or equal to 5, round up by adding 1 to the digit in the hundreds place. Then replace all digits to the right of the hundreds place with zeros.	23,658 add 1 replace with 0s 23,700 So 23,658 rounded to the nearest hundred is 23,700.

ⓑ

Locate the hundreds place.	hundreds place 3,978

Underline the digit to the right of the hundreds place.	$3{,}9\underline{7}8$
The digit to the right of the hundreds place is 7. Since 7 is greater than or equal to 5, round up by adding 1 to the 9. Then place all digits to the right of the hundreds place with zeros.	$3{,}9\underline{7}8$ add 1 $(9 + 1 = 10)$ Write 0 in the hundreds place. Add 1 to the thousands place. replace with 0s $4{,}000$ So 3,978 rounded to the nearest hundred is 4,000.

TRY IT 1.17 Round to the nearest hundred: $17{,}852$.

TRY IT 1.18 Round to the nearest hundred: $4{,}951$.

EXAMPLE 1.10

Round each number to the nearest thousand:

ⓐ $147{,}032$ ⓑ $29{,}504$

Solution

ⓐ

Locate the thousands place. Underline the digit to the right of the thousands place.	thousands place $147{,}032$
The digit to the right of the thousands place is 0. Since 0 is less than 5, we do not change the digit in the thousands place.	$147{,}\underline{0}32$
We then replace all digits to the right of the thousands pace with zeros.	$147{,}000$ So 147,032 rounded to the nearest thousand is 147,000.

ⓑ

Locate the thousands place.	thousands place $29{,}504$