CHAPTER - 16
Probability

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Exercise : 16.3

8.1) Three coins are tossed once, what is the probability of getting three heads?

Solution:

Textual solution:

When three coins are tossed, the sample space is:

$$S = \{\text{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}\}$$

where: - H represents heads. - T represents tails.
The total number of outcomes in the sample space is:

$$|S| = 8$$

The event A of getting 3 heads (HHH) contains only one favorable outcome:

$$A = \{\text{HHH}\}$$

Thus, the probability of A is:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{8}$$

Computational solution:

Introduction

This document explains the computational process of determining the probability distribution of the number of heads when a coin is tossed three times. The implementation involves two components:

- A **C program** to perform the simulation, calculate the probabilities (PMF), and the cumulative distribution function (CDF).
- A **Python script** to use the results from the C program and generate a plot of the probability.

Definitions

Let the random variable X represent the number of heads when three fair coins are tossed. The possible values of X are:

$$X \in \{0, 1, 2, 3\}.$$

- **Probability Mass Function (PMF):** The PMF of X is defined as:

$$P(X = x) = \frac{\text{Number of occurrences of } x}{\text{Total number of trials}},$$

where $x \in \{0, 1, 2, 3\}$.

- **Cumulative Distribution Function (CDF):** The CDF of X is defined as:

$$F(X = x) = \sum_{k=0}^{x} P(X = k),$$

which gives the cumulative probability of obtaining k or fewer heads.

C Program Implementation

The C program performs the following steps:

a) Simulate n trials of tossing three coins using the `rand()` function to generate random outcomes (head or tail).
b) Count the number of occurrences of 0, 1, 2, and 3 heads across all trials.
c) Compute the PMF:

$$P(X = x) = \frac{\text{Number of occurrences of } x}{n}, \quad \text{for } x \in \{0, 1, 2, 3\}.$$

d) Compute the CDF using the PMF:

$$F(X = x) = \sum_{k=0}^{x} P(X = k).$$

e) Expose the results (PMF and CDF) as arrays for use in Python.

Python Script Implementation

The Python script:

a) Loads the shared object file (`.so`) created by the C program.
b) Calls the `calculate_probabilities` function in the C program, passing the number of trials (n) and retrieving the PMF and CDF as arrays.
c) Plots the PMF as a stem plot, showing the probabilities for each outcome (0, 1, 2, and 3 heads).

Use of CDF in Computation

The CDF is computed in the C program using the PMF:

$$F(X = x) = F(X = x - 1) + P(X = x), \quad \text{for } x \geq 1,$$

with the initial condition:

$$F(X = 0) = P(X = 0).$$

The Python script does not directly use the CDF but focuses on visualizing the PMF.

Visualization

The Python script generates a stem plot of the PMF, illustrating the probabilities for each outcome:

$$P(X = 0), P(X = 1), P(X = 2), P(X = 3).$$

Conclusion

This computational process combines the efficiency of C for numerical simulation and the versatility of Python for visualization. The CDF plays a key role in ensuring the correctness of cumulative probabilities, which validates the PMF computation.