CHAPTER - 6
Application of Derivatives

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Exercise : 6.5

5.2) Find the absolute maximum value and the absolute minimum value of the function $f(x) = \sin x + \cos x, x \in [0, \pi]$

Theoretical Solution:

Given the function:

$$f(x) = \sin x + \cos x, \text{ where } x \in [0, \pi] \tag{1}$$

To find the critical points, we differentiate $f(x)$ with respect to x:

$$\frac{df}{dx} = \cos x - \sin x \tag{2}$$

Setting the derivative equal to zero:

$$\cos x - \sin x = 0 \tag{3}$$

$$\cos x = \sin x \tag{4}$$

This occurs when:

$$x = \frac{\pi}{4} \tag{5}$$

Now, let's evaluate $f(x)$ at the critical point and the boundaries of the interval:

- At $x = 0$:

$$f(0) = \sin(0) + \cos(0) = 0 + 1 = 1 \tag{6}$$

- At $x = \frac{\pi}{4}$:

$$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2} \tag{7}$$

- At $x = \pi$:

$$f(\pi) = \sin(\pi) + \cos(\pi) = 0 - 1 = -1 \tag{8}$$

Therefore, the maximum value of $f(x)$ in the interval $[0, \pi]$ is $\sqrt{2}$, and the minimum value is -1.

Computational Solution (Solved):

To find the maximum and minimum values of the function $f(x) = \sin x + \cos x$ on the interval $x \in [0, \pi]$, we use iterative methods like gradient ascent and gradient descent.

The gradient ascent method is used to find the maximum value of a function by following the direction of the positive gradient. The iterative formula for gradient ascent is given by:

$$x_{n+1} = x_n + \eta \cdot \frac{df}{dx} \tag{9}$$

where x_n is the current value of x, x_{n+1} is the next value of x, η is the learning rate (a small positive number), and $\frac{df}{dx}$ is the derivative of the function at x_n.
First, we compute the derivative of $f(x)$:

$$\frac{df}{dx} = \cos x - \sin x \tag{10}$$

To apply gradient ascent, choose an initial point x_0 within the interval $[0, \pi]$, and apply the gradient ascent formula iteratively until convergence. The iteration stops when the gradient becomes zero. We find the maximum at $x = \frac{\pi}{4}$ because the gradient changes sign around this point.
The function value at $x = \frac{\pi}{4}$ is:

$$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) = \sqrt{2} \tag{11}$$

Therefore, the absolute maximum value of $f(x)$ is $\sqrt{2}$ at $x = \frac{\pi}{4}$.

The gradient descent method is used to find the minimum value of a function by following the direction of the negative gradient. The iterative formula for gradient descent is given by:

$$x_{n+1} = x_n - \eta \cdot \frac{df}{dx} \tag{12}$$

Starting with an initial point x_0 within the interval $[0, \pi]$, we apply the gradient descent formula iteratively until convergence. The iteration stops when the gradient becomes zero. We find the minimum at $x = \pi$ because the gradient changes sign around this point.
The function value at $x = \pi$ is:

$$f(\pi) = \sin(\pi) + \cos(\pi) = -1 \tag{13}$$

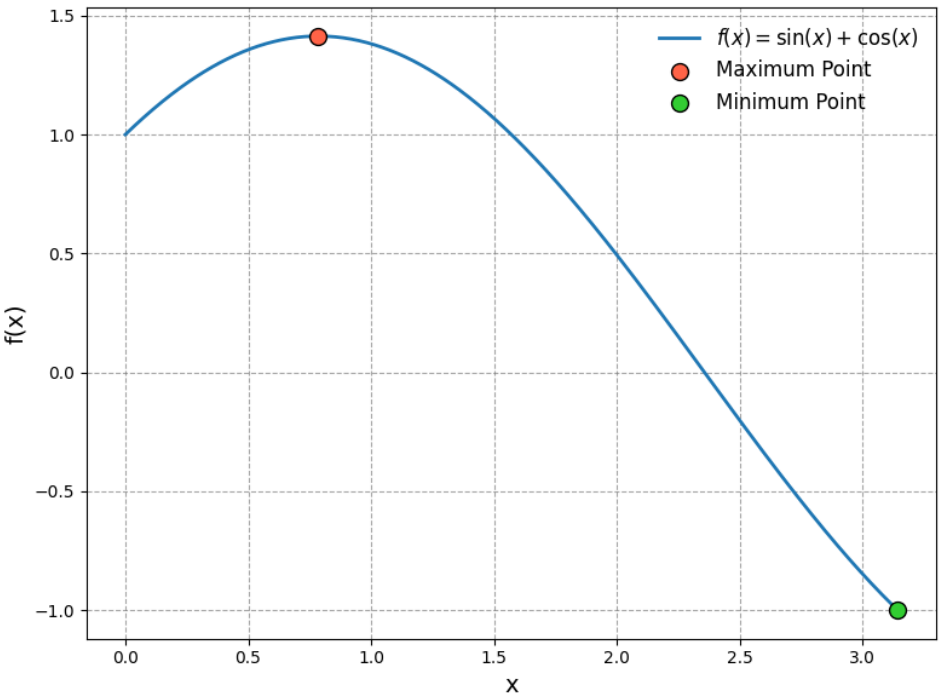
Therefore, the absolute minimum value of $f(x)$ is -1 at $x = \pi$.

The computational results are summarized as follows:

$$\text{Absolute Maximum:} \quad f(x) = \sqrt{2} \text{ at } x = \frac{\pi}{4} \tag{14}$$

$$\text{Absolute Minimum:} \quad f(x) = -1 \text{ at } x = \pi \tag{15}$$

The graph of the function $f(x) = \sin x + \cos x$ representing the maximum and minimum values is shown below:



$f(x) = \sin(x) + \cos(x)$
Maximum Point
Minimum Point
f(x)
x
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0