CHAPTER - 9
Differential Equations

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Exercise : 9.5

9) Solve the differential equation $ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$
Solution: The given differential equation is

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

On rearranging, we get:

$$\frac{dy}{dx} = \frac{\frac{y}{x}}{2 - \log\left(\frac{y}{x}\right)} \tag{1}$$

Let $\frac{y}{x} = v$, i.e., $y = vx$

$$\implies \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting in equation (1), we get:

$$v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$$
$$\implies x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$$
$$\implies x\frac{dv}{dx} = \frac{v(\log v - 1)}{2 - \log v}$$
$$\implies \frac{dx}{x} = \left[\frac{1}{v(\log v - 1)} - \frac{1}{v}\right]dv$$

Integrating on both sides, we get:

$$\log|x| + \log|c| = \int \left[\frac{1}{v(\log v - 1)} - \frac{1}{v}\right]dv$$
$$\implies \log|x| + \log|c| = \log\left|\log v - 1\right| - \log|v|$$
$$\implies Cxv = \log v - 1 \quad [\text{where, } C = \pm c]$$

Replacing v by $\frac{y}{x}$ and rearranging, we have:

$$\log\left(\frac{y}{x}\right) - 1 = Cx\left(\frac{y}{x}\right)$$

$$\log\left(\frac{y}{x}\right) - 1 = Cy \tag{2}$$

To obtain a plot for this, let's assume, $C = -1$ and $x = 1$. On substituting these values in equation (2), we get, $y = 1$.
Let the x value be limited to 5.

$$\implies x_0 = 1.0, y_0 = 1.0, x_{end} = 5.0$$

The plot of the differential equation

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0$$

is shown below.