CHAPTER - 9
Differential Equations

EE24BTECH11061 - Rohith Sai

EXERCISE : 9.6

12) Solve the differential equation $\left(x+3y^2\right)\frac{dy}{dx}=y$
Solution (using the Method of Finite Differences):
The given differential equation is:

$$\left(x+3y^2\right)\frac{dy}{dx}=y \tag{1}$$

Rearranging the equation to express $\frac{dx}{dy}$ for Euler's method:

$$\frac{dx}{dy}=\frac{x+3y^2}{y} \tag{2}$$

Using the method of finite differences, the next value of x can be computed as:

$$x_{n+1}=x_n+h\cdot f\left(y_n,x_n\right) \tag{3}$$

$$f\left(y,x\right)=\frac{x+3y^2}{y} \tag{4}$$

Let y_0 be the initial y value and x_0 be the initial x value. Let the step size h be 0.001. The first few iterations are:

$$\begin{aligned} y_1 &= y_0+h, & x_1 &= x_0+h\cdot\frac{x_0+3y_0^2}{y_0} \\ y_2 &= y_1+h, & x_2 &= x_1+h\cdot\frac{x_1+3y_1^2}{y_1} \\ &\vdots & &\vdots \\ y_n &= y_{n-1}+h, & x_n &= x_{n-1}+h\cdot\frac{x_{n-1}+3y_{n-1}^2}{y_{n-1}} \end{aligned}$$

Solution (using the general method):
From equation (2),

$$\frac{dx}{dy}=\frac{x+3y^2}{y} \tag{5}$$

$$\implies \frac{dx}{dy}-\frac{x}{y}=3y \tag{6}$$

On comparing equation (6) with the standard form of differential equation

$$\frac{dx}{dy} + Px = Q \tag{7}$$

we get
$P = -\frac{1}{y}$ and $Q = 3y$.
Now we find the Integrating Factor (I.F.) in the following manner:

$$\text{I.F.} = e^{\int P\,dy} \tag{8}$$

$$\implies \text{I.F.} = e^{\int -\frac{1}{y}\,dy} \tag{9}$$

$$\implies \text{I.F.} = \frac{1}{y} \tag{10}$$

Therefore, solution is

$$x \cdot (\text{I.F.}) = \int Q \cdot (\text{I.F.})\ dy + C \tag{11}$$

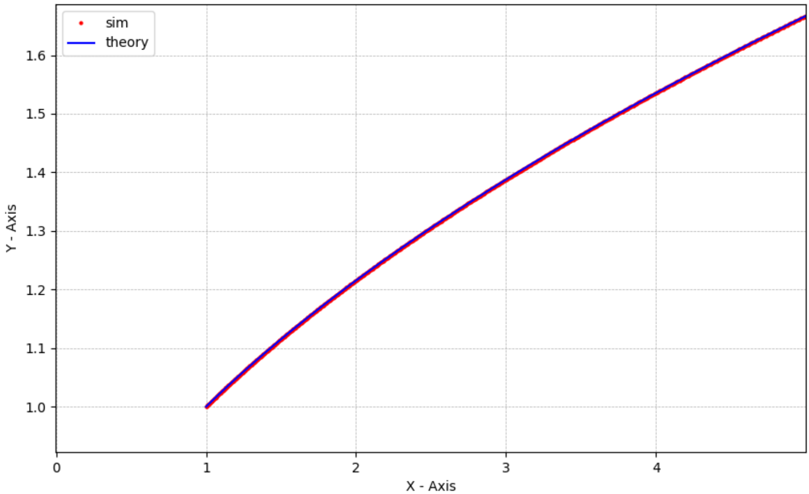
On substituting the value of I.F. in equation (11) and solving, we get:

$$x = 3y^2 + Cy \tag{12}$$

Let's assume $C = -2$ and $y = 1$. On substituting these values in equation (12), we get $x = 1$.
Therefore, the equation is

$$x = 3y^2 - 2y$$

Therefore, the curve generated using both the above mentioned methods for the given differential equation (1) is shown below:


sim
theory
1.6
1.5
1.4
1.3
1.2
1.1
1.0
Y - Axis
0
1
2
3
4
X - Axis