CHAPTER - 3
Pair of Linear Equations in Two Variables

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Exercise : 3.3

1.1) Solve the following pair of linear equations using LU decomposition:
Solution:

$$x + y = 14 \tag{1}$$
$$x - y = 4 \tag{2}$$

First, we rewrite the question as a system of linear equations.

$$x_1 \implies x, \tag{3}$$
$$x_2 \implies y \tag{4}$$

Converting into matrix form, we get:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{5}$$
$$\mathbf{A}x = \mathbf{b} \tag{6}$$

To solve the above equation, we apply LU decomposition to matrix $\mathbf{A}$.
We decompose $\mathbf{A}$ as:

$$\mathbf{A} = \mathbf{L}\mathbf{U}, \tag{7}$$
$$\mathbf{L} = \text{Lower Triangular Matrix}, \tag{8}$$
$$\mathbf{U} = \text{Upper Triangular Matrix}. \tag{9}$$

Let $y = \mathbf{U}x$, then we can rewrite the above equation as:

$$\mathbf{A}x = \mathbf{b} \implies \mathbf{L}\mathbf{U}x = \mathbf{b} \implies \mathbf{L}y = \mathbf{b} \tag{10}$$

Now, the above equation can be solved using forward substitution since $\mathbf{L}$ is lower triangular, thus we get the solution vector y. Using this, we solve for x in $y = \mathbf{U}x$ using back substitution knowing that $\mathbf{U}$ is upper triangular.

LU Factorizing $\mathbf{A}$, we get:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}, \tag{11}$$

$$\mathbf{L} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \tag{12}$$

$$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \tag{13}$$

The solution can now be obtained as:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{14}$$

Solving for y, we get:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -10 \end{pmatrix} \tag{15}$$

Now, solving for x via back substitution:

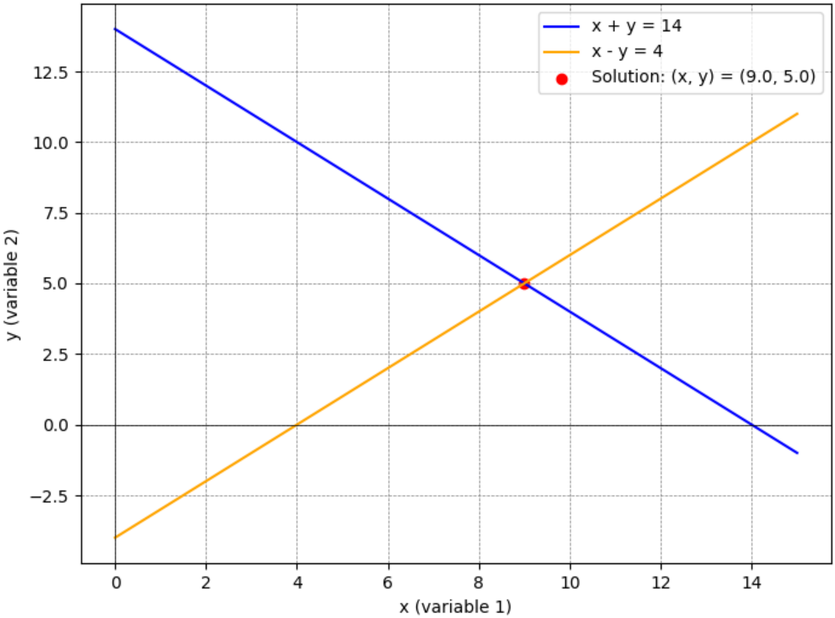
$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -10 \end{pmatrix} \tag{16}$$

$$x_2 = 5, \tag{17}$$

$$x_1 + x_2 = 14 \implies x_1 = 9 \tag{18}$$

Thus, the solution is:

$$x = 9, \; y = 5 \tag{19}$$



x + y = 14
x - y = 4
Solution: (x, y) = (9.0, 5.0)
y (variable 2)
x (variable 1)
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
0
2
4
6
8
10
12
14