CHAPTER - 3
Pair of Linear Equations in Two Variables

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Exercise : 3.3

1.1) Solve the following pair of linear equations using LU decomposition:
Solution:

$$x + y = 14 \tag{1}$$
$$x - y = 4 \tag{2}$$

First, we rewrite the question as a system of linear equations.

$$x_1 \implies x, \tag{3}$$
$$x_2 \implies y \tag{4}$$

Converting into matrix form, we get:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{5}$$
$$\mathbf{A}x = \mathbf{b} \tag{6}$$

To solve the above equation, we apply LU decomposition to matrix $\mathbf{A}$.

Step 2: LU Factorization Using Update Equations

Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:
Step-by-Step Procedure:
1. **Initialization:** - Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. **Iterative Update:** - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of $\mathbf{U}$ using the first update equation. - Compute the entries of $\mathbf{L}$ using the second update equation.
3. **Result:** - After completing the iterations, the matrix $\mathbf{A}$ is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where $\mathbf{L}$ is a lower triangular matrix with ones on the diagonal, and $\mathbf{U}$ is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of **U***)*

For each column $j \geq k$, the entries of **U** in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of **L***)*

For each row $i > k$, the entries of **L** in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

LU Factorizing **A**, we get:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}, \tag{7}$$

$$\mathbf{L} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \tag{8}$$

$$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \tag{9}$$

The solution can now be obtained as:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 14 \\ 4 \end{pmatrix} \tag{10}$$

Solving for y, we get:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -10 \end{pmatrix} \tag{11}$$

Now, solving for x via back substitution:

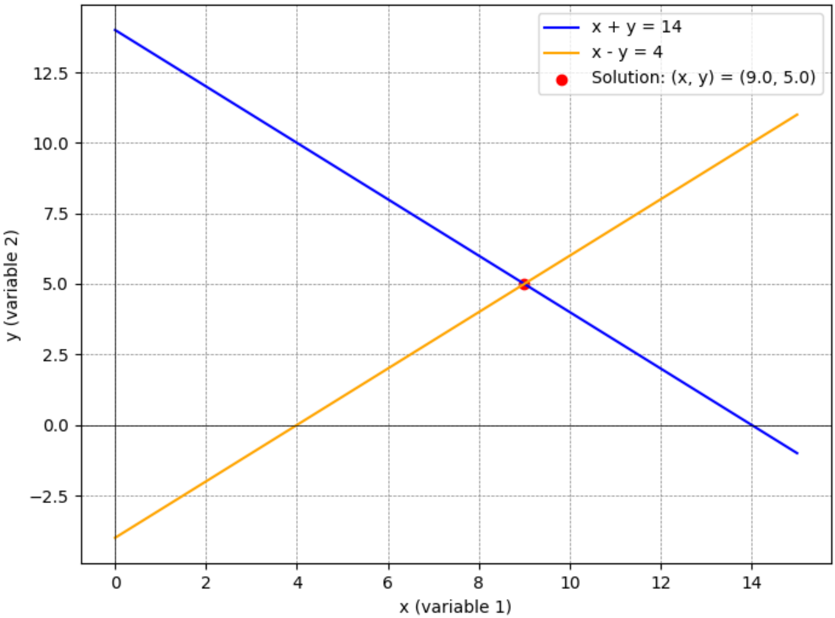
$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -10 \end{pmatrix} \tag{12}$$

$$x_2 = 5, \tag{13}$$

$$x_1 + x_2 = 14 \implies x_1 = 9 \tag{14}$$

Thus, the solution is:

$$x = 9, \; y = 5 \tag{15}$$


x + y = 14
x - y = 4
Solution: (x, y) = (9.0, 5.0)
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
y (variable 2)
0
2
4
6
8
10
12
14
x (variable 1)