CHAPTER - 9
Differential Equations

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EXERCISE : 9.5

9) Solve the differential equation $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$
Solution (using the Method of Finite Differences):
The given differential equation is

$$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0 \tag{1}$$

$$y\,dx = \left(2x - x\log\left(\frac{y}{x}\right)\right)dy \tag{2}$$

$$\implies \frac{dx}{dy} = \frac{2x - x\log\left(\frac{y}{x}\right)}{y} \tag{3}$$

$$\implies \frac{dy}{dx} = \frac{\frac{y}{x}}{2 - \log\left(\frac{y}{x}\right)} \tag{4}$$

Using the method of finite differences, the next value of y can be computed as:

$$y_{n+1} = y_n + h \cdot f(x_n, y_n) \tag{5}$$

$$f(x,y) = \frac{\frac{y}{x}}{2 - \log\left(\frac{y}{x}\right)} \tag{6}$$

Let x_0 be the initial x value and y_0 be the initial value and let the step size h be 0.001. The first few iterations are:

$$\begin{aligned}
x_1 &= x_0 + h, & y_1 &= y_0 + h \cdot \frac{\frac{y_0}{x_0}}{2 - \log\left(\frac{y_0}{x_0}\right)} \\
x_2 &= x_1 + h, & y_2 &= y_1 + h \cdot \frac{\frac{y_1}{x_1}}{2 - \log\left(\frac{y_1}{x_1}\right)} \\
&\vdots & &\vdots \\
x_n &= x_{n-1} + h, & y_n &= y_{n-1} + h \cdot \frac{\frac{y_{n-1}}{x_{n-1}}}{2 - \log\left(\frac{y_{n-1}}{x_{n-1}}\right)}
\end{aligned}$$

Solution (using the general method):
Let $\frac{y}{x} = v$, i.e., $y = vx$

$$\implies \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting in equation (4), we get:

$$\begin{aligned} v + x\frac{dv}{dx} &= \frac{v}{2 - \log v} \\ \implies x\frac{dv}{dx} &= \frac{v}{2 - \log v} - v \\ \implies x\frac{dv}{dx} &= \frac{v(\log v - 1)}{2 - \log v} \\ \implies \frac{dx}{x} &= \left[\frac{1}{v(\log v - 1)} - \frac{1}{v}\right]dv \end{aligned}$$

Integrating on both sides, we get:

$$\begin{aligned} \log|x| + \log|c| &= \int \left[\frac{1}{v(\log v - 1)} - \frac{1}{v}\right]dv \\ \implies \log|x| + \log|c| &= \log\left|\log v - 1\right| - \log|v| \\ \implies Cxv &= \log v - 1 \quad [\text{where, } C = \pm c] \end{aligned}$$

Replacing v by $\frac{y}{x}$ and rearranging, we have:

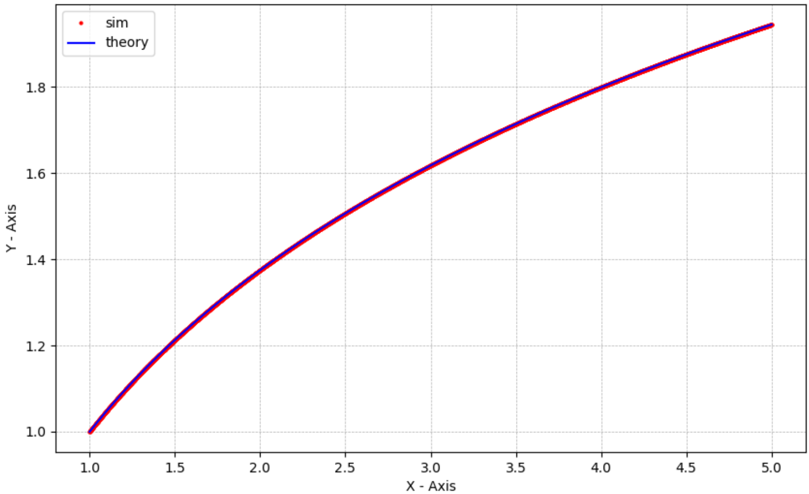
$$\log\left(\frac{y}{x}\right) - 1 = Cx\left(\frac{y}{x}\right)$$

$$\log\left(\frac{y}{x}\right) - 1 = Cy \tag{7}$$

Let's assume, $C = -1$ and $x = 1$. On substituting these values in equation (7), we get, $y = 1$.
Therefore, the equation is as follows:

$$y = xe^{(1-y)}$$

Therefore, the curve generated using both the above mentioned methods for the given differential equation (1) is shown below:


sim
theory
1.8
1.6
1.4
1.2
1.0
Y - Axis
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
X - Axis