CHAPTER - 16
Probability

EE24BTECH11061 - Rohith Sai

Exercise : 16.3

8.1) Three coins are tossed once, what is the probability of getting three heads?
Solution:
Define a discrete random variable X = number of heads
We will take our random variable as a sum of outcomes of three Bernoulli random variables

$$X = X_1 + X_2 + X_3 \tag{1}$$

Where

$$X_i = \begin{cases} 1, & \text{Outcome in Heads} \\ 0, & \text{Outcome in Tails} \end{cases} \tag{2}$$

$$p_{X_i}(n) = \begin{cases} 1-p, & n = 0 \\ p, & n = 1 \end{cases} \tag{3}$$

Where $p = \frac{1}{2}$
Using properties of Z-Transform of PMF

$$M_X(z) = M_{X_1}(z)M_{X_2}(z)M_{X_3}(z) \tag{4}$$

$$M_{X_1}(z) = \sum_{n=-\infty}^{\infty} p_{X_1}(n)z^{-n} = (1-p) + pz^{-1} \tag{5}$$

$$M_{X_2}(z) = \sum_{n=-\infty}^{\infty} p_{X_2}(n)z^{-n} = (1-p) + pz^{-1} \tag{6}$$

$$M_{X_3}(z) = \sum_{n=-\infty}^{\infty} p_{X_3}(n)z^{-n} = (1-p) + pz^{-1} \tag{7}$$

$$M_X(z) = ((1-p) + pz^{-1})^3 \tag{8}$$

$$= \sum_{n=-\infty}^{\infty} {}^3C_n(1-p)^{3-n}p^n z^{-n} \tag{9}$$

$$p_X(n) = {}^3C_n p^n (1-p)^{3-n} \tag{10}$$

$$p_X(n) = \frac{{}^3C_n}{8} \tag{11}$$

The Probability Mass Function (PMF) for the given random variable is

$$p_X(n) = \begin{cases} \frac{1}{8}, & n = 0 \\ \frac{3}{8}, & n = 1 \\ \frac{3}{8}, & n = 2 \\ \frac{1}{8}, & n = 3 \end{cases} \tag{12}$$

The probability of getting three heads is

$$p_X(3) = \frac{1}{8} \tag{13}$$

Simulation:

To run a simulation we need to generate random numbers with uniform probability, which is done as shown below:

a) Generate a random number by calling rand(). It generates a random number between 0 and RANDMAX
b) Divide the generated number by RANDMAX so that it becomes a real number in the range [0, 1)
c) If the number is less than p, take it as event happened, else the event did not happen

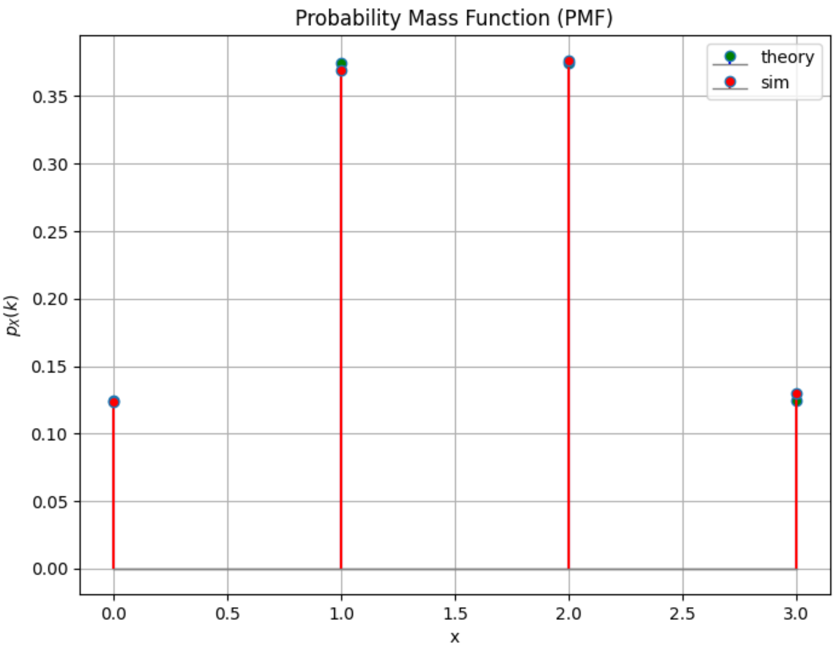

Fig. 0.1: Probability Mass Function of given Random variable

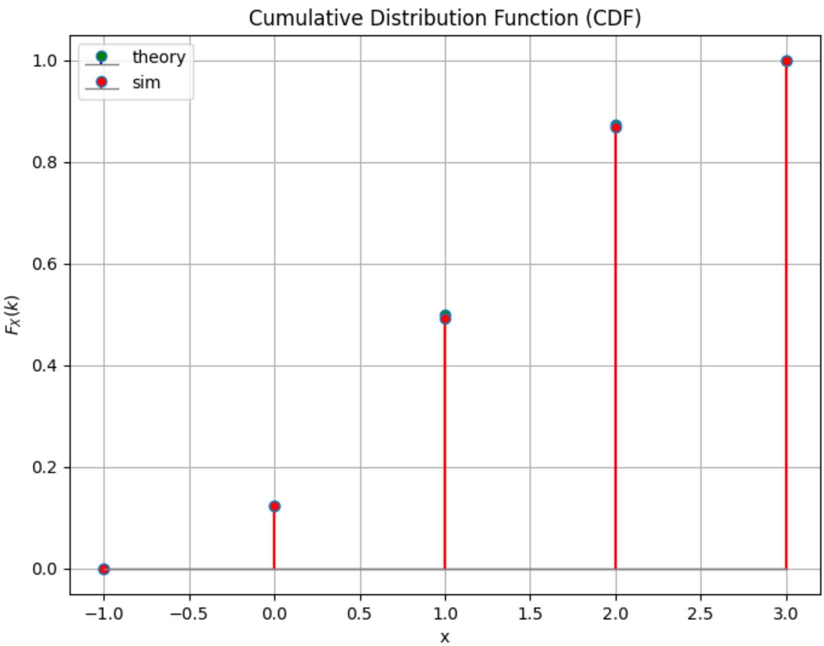

Fig. 0.2: Cumulative Distribution Function of given Random variable