

# CHAPTER - 20

## Vector Algebra and Three Dimensional Geometry

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### 1 A. FILL IN THE BLANKS

- 1) Let  $OA = a$ ,  $OB = 10a + 2b$  and  $OC = b$  where  $\mathbf{O}$ ,  $\mathbf{A}$  and  $\mathbf{C}$  are non-collinear points. Let  $\mathbf{p}$  denote the area of the quadrilateral  $OABC$ , and let  $\mathbf{q}$  denote the area of the parallelogram with  $OA$  and  $OC$  as adjacent sides. If  $p = kq$ , then  $\mathbf{k} = \dots\dots$  (1997 - 2 Marks)

### 2 B. TRUE/FALSE

- 1) Let  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  be unit vectors suppose that  $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$ , and that the angle between  $\mathbf{B}$  and  $\mathbf{C}$  is  $\frac{\pi}{6}$ . Then  $\mathbf{A} = \pm 2(\mathbf{B} \times \mathbf{C})$ . (1981 - 2 Marks)
- 2) If  $\mathbf{X} \cdot \mathbf{A} = 0$ ,  $\mathbf{X} \cdot \mathbf{B} = 0$ ,  $\mathbf{X} \cdot \mathbf{C} = 0$  for some non-zero vector  $\mathbf{X}$ , then  $[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = 0$  (1983 - 1 Mark)
- 3) The points with position vectors  $\mathbf{a} + \mathbf{b}$ ,  $\mathbf{a} - \mathbf{b}$  and  $\mathbf{a} + \mathbf{k}\mathbf{b}$  are collinear for all real values of  $\mathbf{k}$ . (1984 - 1 Mark)
- 4) For any three vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ ,  $(\mathbf{a} - \mathbf{b}) \cdot ((\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a})) = 2\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ . (1989 - 1 Mark)

### 3 C. MCQs WITH ONE CORRECT ANSWER

- 1) The scalar  $\mathbf{A} \cdot ((\mathbf{B} + \mathbf{C}) \times (\mathbf{A} + \mathbf{B} + \mathbf{C}))$  equals:  
 a) 0  
 b)  $[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] + [\mathbf{B} \ \mathbf{C} \ \mathbf{A}]$   
 c)  $[\mathbf{A} \ \mathbf{B} \ \mathbf{C}]$   
 d) None of these  
 (1981 - 2 Marks)
- 2) For non-zero vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$ ,  $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a}||\mathbf{b}||\mathbf{c}|$  holds if and only if  
 a)  $\mathbf{a} \cdot \mathbf{b} = 0$ ,  $\mathbf{b} \cdot \mathbf{c} = 0$   
 b)  $\mathbf{b} \cdot \mathbf{c} = 0$ ,  $\mathbf{c} \cdot \mathbf{a} = 0$   
 c)  $\mathbf{c} \cdot \mathbf{a} = 0$ ,  $\mathbf{a} \cdot \mathbf{b} = 0$   
 d)  $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$   
 (1982 - 2 Marks)
- 3) The volume of the parallelopiped whose sides are given by  $OA = 2\mathbf{i} - 2\mathbf{j}$ ,  $OB = \mathbf{i} + \mathbf{j} - \mathbf{k}$ ,  $OC = 3\mathbf{i} - \mathbf{k}$ , is  
 a)  $\frac{4}{13}$

- b) 4
- c)  $\frac{2}{7}$
- d) None of these

(1983 - 1 Mark)

4) The points with position vectors  $60\mathbf{i} + 3\mathbf{j}$ ,  $40\mathbf{i} - 8\mathbf{j}$ ,  $\mathbf{ai} - 52\mathbf{j}$  are collinear if

- a)  $a = -40$
- b)  $a = 40$
- c)  $a = 20$
- d) None of these

(1983 - 1 Mark)

5) Let  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  be three non coplanar vectors and  $\mathbf{p}, \mathbf{q}, \mathbf{r}$  are vectors defined by the relations

$$\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]}, \mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]}, \mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]} \text{ then the value of the expression } (\mathbf{a} + \mathbf{b}) \cdot \mathbf{p} + (\mathbf{b} + \mathbf{c}) \cdot \mathbf{q} + (\mathbf{c} + \mathbf{a}) \cdot \mathbf{r} \text{ is equal to}$$

- a) 0
- b) 1
- c) 2
- d) 3

(1988 - 2 Marks)

6) Let  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  be distinct non-negative numbers. If the vectors  $\mathbf{ai} + \mathbf{aj} + \mathbf{ck}$ ,  $\mathbf{i} + \mathbf{k}$  and  $\mathbf{ci} + \mathbf{cj} + \mathbf{bk}$  lie in a plane, then  $\mathbf{c}$  is

- a) the Arithmetic Mean of  $\mathbf{a}$  and  $\mathbf{b}$
- b) the Geometric Mean of  $\mathbf{a}$  and  $\mathbf{b}$
- c) the Harmonic Mean of  $\mathbf{a}$  and  $\mathbf{b}$
- d) equal to zero

(1993 - 1 Mark)

7) Let  $\mathbf{p}$  and  $\mathbf{q}$  be the position vectors of  $\mathbf{P}$  and  $\mathbf{Q}$  respectively, with respect to  $\mathbf{O}$  and  $|\mathbf{p}| = p$ ,  $|\mathbf{q}| = q$ . The points  $\mathbf{R}$  and  $\mathbf{S}$  divide  $PQ$  internally and externally in the ratio 2: 3 respectively. If  $OR$  and  $OS$  are perpendicular then

- a)  $9p^2 = 4q^2$
- b)  $4p^2 = 9q^2$
- c)  $9p = 4q$
- d)  $4p = 9q$

(1994)

8) Let  $\alpha, \beta, \gamma$  be distinct real numbers. The points with position vectors  $\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$ ,  $\beta\mathbf{i} + \gamma\mathbf{j} + \alpha\mathbf{k}$ ,  $\gamma\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$

- a) are collinear
- b) form an equilateral triangle
- c) form a scalene triangle
- d) form a right angles triangle

(1994)

9) Let  $\mathbf{a} = \mathbf{i} - \mathbf{j}$ ,  $\mathbf{b} = \mathbf{j} - \mathbf{k}$ ,  $\mathbf{c} = \mathbf{k} - \mathbf{i}$ . If  $\mathbf{d}$  is a unit vector such that  $\mathbf{a} \cdot \mathbf{d} = 0 = [\mathbf{b} \ \mathbf{c} \ \mathbf{d}]$ , then  $\mathbf{d}$  equals

- a)  $\pm \frac{\mathbf{i} + \mathbf{j} - 2\mathbf{k}}{\sqrt{6}}$

- b)  $\pm \frac{\mathbf{i}+\mathbf{k}-\mathbf{k}}{\sqrt{3}}$
- c)  $\pm \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$
- d)  $\pm \mathbf{k}$

(1995S)

10) If  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  are non coplanar unit vectors such that  $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{(\mathbf{b}+\mathbf{c})}{\sqrt{2}}$ , then the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is

- a)  $\frac{3\pi}{4}$
- b)  $\frac{\pi}{4}$
- c)  $\frac{\pi}{2}$
- d)  $\pi$

(1995S)