

CHAPTER - 1

Vector Arithmetic

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1 UNIT VECTOR

1.10.5 If $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, find the unit vector in the direction of

a) $6\mathbf{a}$

b) $2\mathbf{a} - \mathbf{b}$

Solution:

Variable	Description	Formula
\mathbf{a}	First point	$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$
\mathbf{b}	Second point	$\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

TABLE 0

a) According to the question:

$$6\mathbf{a} = 6 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \quad (1)$$

$$\|6\mathbf{a}\| = 6 \|\mathbf{a}\| = 6 \sqrt{(\mathbf{a})^T (\mathbf{a})}, \quad (2)$$

$$\Rightarrow \|6\mathbf{a}\| = 6 \sqrt{6} \quad (3)$$

We know that, to find the unit vector:

$$\Rightarrow \frac{6\mathbf{a}}{\|6\mathbf{a}\|} = \frac{1}{6 \sqrt{6}} \begin{pmatrix} 6 \\ 6 \\ 12 \end{pmatrix} \quad (4)$$

Therefore, the required unit vector is:

$$\Rightarrow \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad (5)$$

b) According to the question:

$$2\mathbf{a} - \mathbf{b} = 2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}, \quad (6)$$

$$\|2\mathbf{a} - \mathbf{b}\| = \sqrt{(2\mathbf{a} - \mathbf{b})^T (2\mathbf{a} - \mathbf{b})}, \quad (7)$$

$$\Rightarrow \|2\mathbf{a} - \mathbf{b}\| = \sqrt{37} \quad (8)$$

We know that, to find the unit vector:

$$\Rightarrow \frac{2\mathbf{a} - \mathbf{b}}{\|2\mathbf{a} - \mathbf{b}\|} = \frac{1}{\sqrt{37}} \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} \quad (9)$$

Therefore, the required unit vector is:

$$\Rightarrow \frac{1}{\sqrt{37}} \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} \quad (10)$$

