CHAPTER - 9
Intersection of Conics

EE24BTECH11061 - Rohith Sai

1 9.3 CBSE

9.3.22 If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

Solution:

The given conic parameters are

Variable	Description
$\mathbf{V}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}$	$\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}$
$\mathbf{f}$	0
$\mathbf{O}(vertex)$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\mathbf{F}(focus)$	$\begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}$

TABLE 0

The parameters of the line $x = a$ are

$$\mathbf{q_2} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{m_2} = e_2 \tag{1}$$

We get

$$\mu_i = -\sqrt{a}, \sqrt{a} \tag{2}$$

The points of intersection of the line $x = a$ with the parabola $x = y^2$ is

$$\mathbf{a_0} = \begin{pmatrix} a \\ -\sqrt{a} \end{pmatrix}, \mathbf{a_1} = \begin{pmatrix} a \\ \sqrt{a} \end{pmatrix} \tag{3}$$

Similarly for the line $x - 4 = 0$

$$\mathbf{q_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{m_1} = e_2 \tag{4}$$

We get

$$\mu_i = -2, 2 \tag{5}$$

The points of intersection of the lines $x = 4$ with the parabola $x = y^2$ is

$$\mathbf{a_2} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}, \mathbf{a_3} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{6}$$

Area between the parabola and the line $x = 4$ is divided equally by the line $x = a$. Thus from Fig. 9.3.22.1,

$$A_1 = \int_0^a \sqrt{x}, dx \tag{7}$$

$$A_2 = \int_a^4 \sqrt{x}, dx \tag{8}$$

$$\text{and } A_1 = A_2 \tag{9}$$

$$\implies a = 4^{\frac{2}{3}} \tag{10}$$

Therefore, we get the following graph

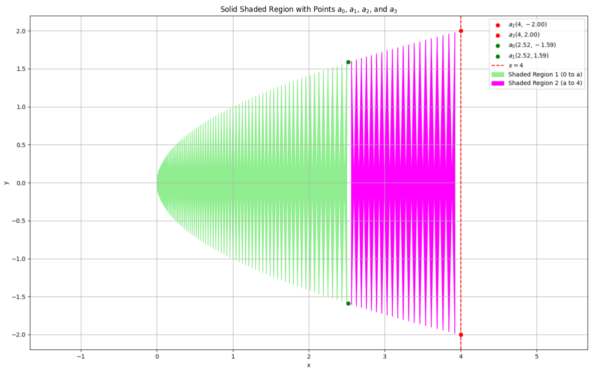

Fig. 9.3.22.1