CHAPTER - 1
Vector Arithmetic

EE24BTECH11061 - Rohith Sai

1 1.9 CBSE

1.9.25 If the point $\mathbf{P}(x, y)$ is equidistant from the points $\mathbf{A}(a+b, b-a)$ and $\mathbf{B}(a-b, a+b)$, prove that $bx = ay$.

Solution:

Variable	Description	Formula
$\mathbf{A}$	It is one end of the line segment	$\mathbf{A} = \begin{pmatrix} a+b \\ b-a \end{pmatrix}$
$\mathbf{B}$	It is other end of line segment	$\mathbf{B} = \begin{pmatrix} a-b \\ a+b \end{pmatrix}$
$\mathbf{P}$	It is the point equidistant from $\mathbf{A}$ and $\mathbf{B}$	$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 0

Given that the point $\mathbf{P}$ is equidistant from both $\mathbf{A}$ and $\mathbf{B}$:

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \tag{1}$$

$$\implies \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{2}$$

On expanding, we get:

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top\mathbf{B} + \|\mathbf{B}\|^2 \tag{3}$$

On further simplifying, we get:

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \tag{4}$$

$$\tag{5}$$

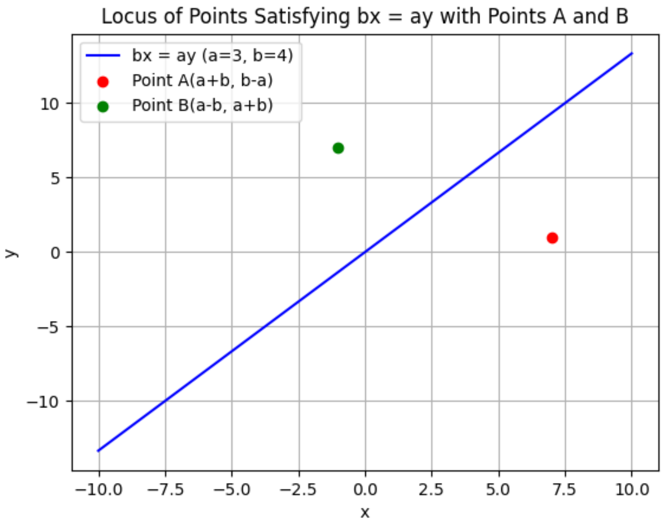
From equation (3):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top} \tag{6}$$

Substituting from equations (1) and (2). Thus,

$$bx = ay \tag{7}$$

Hence, proved.


Locus of Points Satisfying bx = ay with Points A and B
bx = ay (a=3, b=4)
Point A(a+b, b-a)
Point B(a-b, a+b)
10
5
0
−5
−10
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x