CHAPTER - 20
Vector Algebra and Three Dimensional Geometry

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I. Fill in the Blanks

1) Let $OA = a, OB = 10a + 2b$ and $OC = b$ where O, A and C are non-collinear points. Let p denote the area of the quadrilateral $OABC$, and let q denote the area of the parallelogram with OA and OC as adjacent sides. If $p = kq$, then k=....... **(1997 - 2 Marks)**

II. True/False

1) Let $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ be unit vectors suppose that $\mathbf{A}.\mathbf{B} = \mathbf{A}.\mathbf{C} = 0$, and that the angle between $\mathbf{B}$ and $\mathbf{C}$ is $\frac{\pi}{6}$. Then $\mathbf{A} = \pm 2(\mathbf{B} \times \mathbf{C})$. **(1981 - 2 Marks)**
2) If $\mathbf{X}.\mathbf{A} = 0, \mathbf{X}.\mathbf{B} = 0, \mathbf{XC} = 0$ for some non-zero vector $\mathbf{X}$, then $[ABC] = 0$ **(1983 - 1 Mark)**
3) The points with position vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + \mathbf{kb}$ are collinear for all real values of k. **(1984 - 1 Mark)**
4) For any three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, $(\mathbf{a} - \mathbf{b}).(\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a}) = 2\mathbf{a}.(\mathbf{b} \times \mathbf{c})$. **(1989 - 1 Mark)**

III. MCQs with One Correct Answer

1) The scalar $\mathbf{A}.(\mathbf{B} + \mathbf{C}) \times (\mathbf{A} + \mathbf{B} + \mathbf{C})$ equals:
   (a) 0
   (b) $[\mathbf{A}\ \mathbf{B}\ \mathbf{C}] + [\mathbf{B}\ \mathbf{C}\ \mathbf{A}]$
   (c) $[\mathbf{A}\ \mathbf{B}\ \mathbf{C}]$
   (d) None of these
   (1981 - 2 Marks)
2) For non-zero vectors $\mathbf{a}, \mathbf{b}, \mathbf{c} |(\mathbf{a} \times \mathbf{b}).\mathbf{c}| = |\mathbf{a}||\mathbf{b}||\mathbf{c}|$ holds if and only if
   (a) $\mathbf{a}.\mathbf{b} = 0,\ \mathbf{b}.\mathbf{c} = 0$
   (b) $\mathbf{b}.\mathbf{c} = 0,\ \mathbf{c}.\mathbf{a} = 0$
   (c) $\mathbf{c}.\mathbf{a} = 0,\ \mathbf{a}.\mathbf{b} = 0$
   (d) $\mathbf{a}.\mathbf{b} =\ \mathbf{b}.\mathbf{c} =\ \mathbf{c}.\mathbf{a} = 0$
   (1982 - 2 Marks)
3) The volume of the parallelopiped whose sides are given by $\mathbf{OA} = \mathbf{2i} - \mathbf{2j}$, $\mathbf{OB} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{OC} = \mathbf{3i} - \mathbf{k}$, is
   (a) $\frac{4}{13}$
   (b) 4
   (c) $\frac{2}{7}$
   (d) None of these
   (1983 - 1 Mark)
4) The points with position vectors $\mathbf{60i} + \mathbf{3j}$, $\mathbf{40i} - \mathbf{8j}$, $\mathbf{ai} - \mathbf{52j}$ are collinear if
   (a) $a = -40$
   (b) $a = 40$
   (c) $a = 20$
   (d) None of these
   (1983 - 1 Mark)
5) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three non coplanar vectors and $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are vectors defined by the relations $\mathbf{p} = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}, \mathbf{q} = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}, \mathbf{r} = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]}$ then the value of the expression $(\mathbf{a} + \mathbf{b}).\mathbf{p} + (\mathbf{b} + \mathbf{c}).\mathbf{q} + (\mathbf{c} + \mathbf{a}).\mathbf{r}$ is equal to
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (1988 - 2 Marks)
6) Let a, b, c be distinct non-negative numbers. If the vectors $\mathbf{ai} + \mathbf{aj} + \mathbf{ck}$, $\mathbf{i} + \mathbf{k}$ and $\mathbf{ci} + \mathbf{cj} + \mathbf{bk}$ lie in a plane, then c is
   (a) the Arithmetic Mean of a and b
   (b) the Geometric Mean of a and b
   (c) the Harmonic Mean of a and b
   (d) equal to zero
   (1993 - 1 Mark)
7) Let $\mathbf{p}$ and $\mathbf{q}$ be the position vectors of P and Q respectively, with respect to O and $|\mathbf{p}| = p$, $|\mathbf{q}| =$

q. The points R and S divide PQ internally and externally in the ratio 2 : 3 respectively. If OR and OS are perpendicular then

(a) $9p^2 = 4q^2$
(b) $4p^2 = 9q^2$
(c) $9p = 4q$
(d) $4p = 9q$

(1994)

8) Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$, $\beta\mathbf{i} + \gamma\mathbf{j} + \alpha\mathbf{k}$, $\gamma\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$

(a) are collinear
(b) form an equilateral triangle
(c) form a scalene triangle
(d) form a right angles triangle

(1994)

9) Let $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{j} - \mathbf{k}$, $\mathbf{c} = \mathbf{k} - \mathbf{i}$. If $\mathbf{d}$ is a unit vector such that $\mathbf{a}.\mathbf{d} = 0 = [\mathbf{b}\ \mathbf{c}\ \mathbf{d}]$, then $\mathbf{d}$ equals

(a) $\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$
(b) $\pm\frac{\mathbf{i}+\mathbf{k}-\mathbf{k}}{\sqrt{3}}$
(c) $\pm\frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$
(d) $\pm\mathbf{k}$

(1995S)

10) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non coplanar unit vectors such that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{(\mathbf{b}+\mathbf{c})}{\sqrt{2}}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

(a) $\frac{3\pi}{4}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$
(d) π

(1995S)