CHAPTER - 7
Circle

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1 7.3 MISCELLANEOUS

7.3.6 The equation of the circle circumscribing the triangle whose sides are the lines
$y = x + 2$, $3y = 4x$, $2y = 3x$ is
Solution: The given lines are

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -2 \tag{1}$$

$$\begin{pmatrix} 4 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

$$\begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{x} = 0 \tag{3}$$

The points of intersection of the lines are given as A, B, C

$$\mathbf{a} = \begin{pmatrix} 1 & -1 & | & -2 \\ 4 & -3 & | & 0 \end{pmatrix} \tag{4}$$

$$\implies \mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{5}$$

$$\mathbf{b} = \begin{pmatrix} 4 & -3 & | & 0 \\ 3 & -2 & | & 0 \end{pmatrix} \tag{6}$$

$$\implies \mathbf{b} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{7}$$

$$\mathbf{c} = \begin{pmatrix} 3 & -2 & | & 0 \\ 1 & -1 & | & -2 \end{pmatrix} \tag{8}$$

$$\implies \mathbf{c} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{9}$$

Variable	Description	Value
a	1st vertex of triangle	$\begin{pmatrix} 6 \\ 8 \end{pmatrix}$
b	2nd vertex of triangle	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
c	3rd vertex of triangle	$\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

TABLE 0

Now we need to find the equation of the circle passing these three vertices A, B, C

$$\begin{pmatrix} 2\mathbf{a} & 2\mathbf{b} & 2\mathbf{c} \\ 1 & 1 & 1 \end{pmatrix}^\top \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{a}\|^2 \\ \|\mathbf{b}\|^2 \\ \|\mathbf{c}\|^2 \end{pmatrix} \tag{10}$$

Substituting the numerical values, we get

$$\begin{pmatrix} 12 & 16 & 1 \\ 0 & 0 & 1 \\ 8 & 12 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -100 \\ 0 \\ -52 \end{pmatrix} \tag{11}$$

$$\implies \mathbf{u} = \begin{pmatrix} -23 \\ 11 \end{pmatrix} \tag{12}$$

$$f = 0 \tag{13}$$
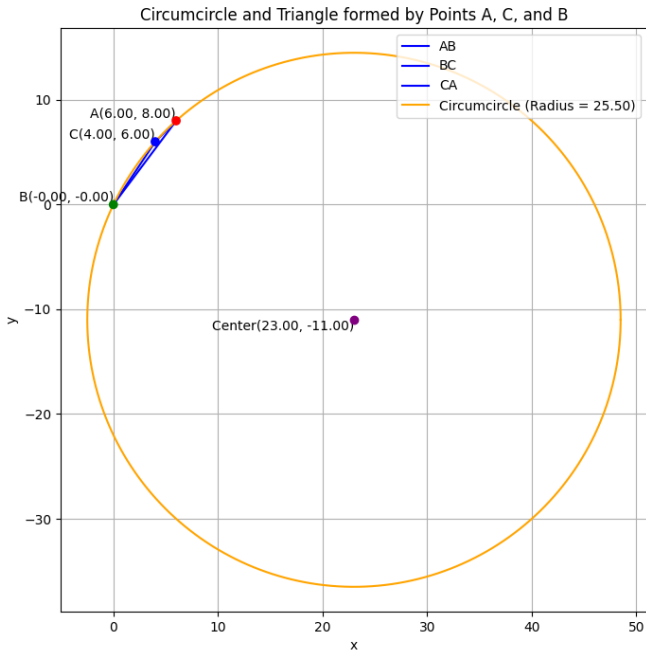
For the given values, the circle is represented as

Fig. 0.1