

# GATE MA - 2009

EE24BTECH11061 - Rohith Sai

SINGLE CORRECT 2 MARKS EACH

1) A simple pendulum, consisting of a bob of mass  $m$  connected with a string of length  $a$ , is oscillating in a vertical plane. If the string is making an angle  $\theta$  with the vertical, then the expression for the Lagrangian is given as

- |                                                                                |                                                                                          |
|--------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| a) $ma^2 \left( \dot{\theta}^2 - \frac{2g}{a} \sin^2 \frac{\theta}{2} \right)$ | c) $ma^2 \left( \frac{\dot{\theta}^2}{2} - \frac{2g}{a} \sin^2 \frac{\theta}{2} \right)$ |
| b) $2mga \sin^2 \frac{\theta}{2}$                                              | d) $\frac{ma}{2} \left( \dot{\theta}^2 - \frac{2g}{a} \cos \theta \right)$               |

2) The extremal of the functional  $\int_0^1 \left( y + x^2 + \frac{y'^2}{4} \right) dx$ ,  $y(0) = 0$ ,  $y(1) = 0$  is

- |                 |                 |
|-----------------|-----------------|
| a) $4(x^2 - x)$ | c) $2(x^2 - x)$ |
| b) $3(x^2 - x)$ | d) $x^2 - x$    |

COMMON DATA QUESTIONS

*Common Data for Questions 51 & 52:*

Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation defined by

$$T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3, 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$$

3) The dimension of the range space of  $T^2$  is

- |      |      |
|------|------|
| a) 0 | c) 2 |
| b) 1 | d) 3 |

4) The dimension of the null space of  $T^3$  is

- |      |      |
|------|------|
| a) 0 | c) 2 |
| b) 1 | d) 3 |

*Common Data for Questions 53 & 54:*

Let  $y_1(x) = 1+x$  and  $y_2(x) = e^x$  be two solutions of  $y''(x) + P(x)y'(x) + Q(x)y(x) = 0$

5)  $P(x) =$

a)  $1 + x$

b)  $-1 - x$

c)  $\frac{1+x}{x}$

d)  $\frac{1-x}{x}$

6) The set of initial conditions for which the above differential equation has NO solution is

a)  $y(0) = 2, y'(0) = 1$

c)  $y(1) = 1, y'(1) = 0$

b)  $y(1) = 0, y'(1) = 1$

d)  $y(2) = 1, y'(2) = 2$

*Common Data for Questions 55 & 56:*

Let  $X$  and  $Y$  be random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\sqrt{2\pi y}} e^{\frac{-1}{2y}(x-y)^2}, & \text{if } -\infty < x < \infty, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

7) The variance of the random variable  $X$  is

a)  $\frac{1}{12}$

c)  $\frac{7}{12}$

b)  $\frac{1}{4}$

d)  $\frac{5}{12}$

8) The covariance between the random variables  $X$  and  $Y$  is

a)  $\frac{1}{3}$

c)  $\frac{1}{6}$

b)  $\frac{1}{4}$

d)  $\frac{1}{12}$

### LINKED ANSWER QUESTIONS

*Statement for Linked Answer Questions 57 & 58:*

Consider the function  $f(z) = \frac{e^{iz}}{z(z^2+1)}$

9) The residue of  $f$  at the isolated singular point in the upper half plane  $\{z = x + iy \in \mathbb{C} : y > 0\}$  is

a)  $\frac{-1}{2e}$

c)  $\frac{e}{2}$

b)  $\frac{-1}{e}$

d)  $1$

10) The Cauchy Principal Value of the integral  $\int_{-\infty}^{\infty} \frac{\sin x dx}{x(x^2+1)}$  is

a)  $-2\pi(1 + 2e^{-1})$

c)  $2\pi(1 + e)$

b)  $\pi(1 - e^{-1})$

d)  $-\pi(1 + e^{-1})$

*Statement for Linked Answer Questions 59 & 60:*

Let  $f(x, y) = kxy - x^3y - xy^3$  for  $(x, y) \in \mathbb{R}$ , where  $k$  is a real constant. The directional derivative of  $f$  at the point  $(1, 2)$  in the direction of the unit vector  $u = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$  is

$$\frac{15}{\sqrt{2}}.$$

11) The value of  $k$  is

a) 2

c) 1

b) 4

d) -1

12) The value of  $f$  at a local minimum in the rectangular region  $R = \{(x, y) \in \mathbb{R}^2: |x| < \frac{3}{2}, |y| < \frac{3}{2}\}$  is

a) -2

c)  $-\frac{7}{8}$ 

b) -3

d) 0