JEE MAINS 2020
September 2 - Shift 1

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1) A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1). Then $x_1^2 + 5y_1^2$ is equal to:

a) 6
c) 8
b) 10
d) 5

2) The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$. Then a is equal to:

a) $\frac{\sqrt{17}-1}{2}$
c) $\frac{1+\sqrt{17}}{2}$
b) $\frac{\sqrt{17}}{2}$
d) $\frac{\sqrt{17}}{2} + 1$

3) If a function $f(x)$ defined by

$$f(x) = \begin{cases} ae^x + be^{-x}, & \text{if } -1 \le x < 1 \\ cx^2, & \text{if } 1 \le x \le 3 \\ ax^2 + 2cx, & \text{if } 3 < x \le 4 \end{cases}$$

be continuous for some $a, b, c \in \mathbb{R}$ and $f'(0) + f'(2) = e$, then the value of a is:

a) $\frac{1}{e^2-3e+13}$
c) $\frac{e}{e^2+3e+13}$
b) $\frac{e}{e^2-3e-13}$
d) $\frac{e}{e^2-3e+13}$

4) The sum of the first three terms of G.P is S and their product is 27. Then all such S lie in

a) $(-\infty, -9] \cup [3, \infty]$
c) $(-\infty, -9]$
b) $[-3, \infty)$
d) $(-\infty, -3] \cup [9, \infty)$

5) If $R = \{(x, y) : x, y \in \mathbb{Z},\ x^2 + 3y^2 \le 8\}$ is relation on the set of integers $\mathbb{Z}$, then the domain of R^{-1} is:

a) {-1, 0, 1}
c) {0, 1}
b) {-2, -1, 1, 2}
d) {-2, -1, 0, 1, 2}

6) The value of $\left(\frac{1+\sin\frac{2\pi}{9}+\iota\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-\iota\cos\frac{2\pi}{9}}\right)^3$

a) $\frac{-1}{2}\left(1-\iota\sqrt{3}\right)$
b) $\frac{1}{2}\left(1-\iota\sqrt{3}\right)$
c) $\frac{-1}{2}\left(\sqrt{3}-\iota\right)$
d) $\frac{1}{2}\left(\sqrt{3}-\iota\right)$

7) Let $\mathbf{P}(h,k)$ be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at $\mathbf{P}$ is:

a) $x + 3y - 62 = 0$
b) $x - 3y - 11 = 0$
c) $x - 3y + 22 = 0$
d) $x + 3y + 26 = 0$

8) Let A be a 2×2 real matrix with entries from {0,1} and A≠0. Consider the following two statements:
(P) If A≠ I_2, then A = -1
(Q) If $\det A = 1$, then tr(A) = 2,
where I_2 denotes 2×2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then:

a) Both (P) and (Q) are false
b) (P) is true and (Q) is false
c) Both (P) and (Q) are false
d) (P) is false and (Q) is true

9) Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

a) $\frac{4}{17}$
b) $\frac{8}{17}$
c) $\frac{2}{5}$
d) $\frac{2}{3}$

10) If $p(x)$ be a polynomial of degree three that has a local maximum value 8 at $x = 1$ and a local minimum value 4 at $x = 2$; then $p(0)$ is equal to:

a) 12
b) -12
c) -24
d) 6

11) The contra-positive of the statement ”If I reach teh station in time, then i will catch the train” is:

a) If I will catch the train, then I reach the station in time.
b) If I do not reach the station in time, then I will catch the train.
c) If I do not reach the station in time, then I will not catch the train.
d) If I will not catch the train, then I do not reach the station in time.

12) Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, $n = 1, 2, 3, \ldots$ then:

a) $5S_6 + 6S_5 + 2S_4 = 0$
b) $6S_6 + 5S_5 = 2S_4$
c) $6S_6 + 5S_5 + 2S_4 = 0$
d) $5S_6 + 6S_5 = 2S_4$

13) If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then:

a) $b = \left(\frac{\pi}{2} + a\right)$
b) $|a + b| = 1$
c) $|b - a| = 1$
d) $b = a$

14) Area (in sq. units) of the region outside $\frac{x}{2} + \frac{y}{3} = 1$ and inside the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is:

a) $3(\pi - 2)$
b) $6(\pi - 2)$
c) $6(4 - \pi)$
d) $3(4 - \pi)$

15) If $|x| < 1$, $|y| < 1$, and xy, then the sum to infinity of the following series

$$(x + y) + \left(x^2 + xy + y^2\right) + \left(x^3 + x^2y + xy^2 + y^3\right) + \ldots$$

is:

a) $\frac{x+y+xy}{(1-x)(1-y)}$
b) $\frac{x+y-xy}{(1-x)(1-y)}$
c) $\frac{x+y+xy}{(1+x)(1+y)}$
d) $\frac{x+y-xy}{(1+x)(1+y)}$