CHAPTER - 1
Vector Arithmetic

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1 1.10 Unit Vector

1.10.5 If $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, find the unit vector in the direction of
  a) $6\mathbf{a}$
  b) $2\mathbf{a} - \mathbf{b}$
  Solution:

Variable	Description	Formula
a	First point	$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$
b	Second point	$\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

TABLE 0

a) According to the question:

$$6\mathbf{a} = 6 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \tag{1}$$

$$\|6\mathbf{a}\| = 6 \|\mathbf{a}\| = 6 \sqrt{(\mathbf{a})^\top (\mathbf{a})}, \tag{2}$$

$$\implies \|6\mathbf{a}\| = 6 \sqrt{6} \tag{3}$$

We know that, to find the unit vector:

$$\implies \frac{6\mathbf{a}}{\|6\mathbf{a}\|} = \frac{1}{6\sqrt{6}} \begin{pmatrix} 6 \\ 6 \\ 12 \end{pmatrix} \tag{4}$$

Therefore, the required unit vector is:

$$\implies \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{5}$$

b) According to the question:

$$2\mathbf{a} - \mathbf{b} = 2\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix}, \tag{6}$$

$$\|2\mathbf{a} - \mathbf{b}\| = \sqrt{(2\mathbf{a} - \mathbf{b})^\top (2\mathbf{a} - \mathbf{b})}, \tag{7}$$

$$\implies \|2\mathbf{a} - \mathbf{b}\| = \sqrt{37} \tag{8}$$

We know that, to find the unit vector:

$$\implies \frac{2\mathbf{a} - \mathbf{b}}{\|2\mathbf{a} - \mathbf{b}\|} = \frac{1}{\sqrt{37}} \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} \tag{9}$$

Therefore, the required unit vector is:

$$\implies \frac{1}{\sqrt{37}} \begin{pmatrix} 0 \\ 1 \\ 6 \end{pmatrix} \tag{10}$$