GATE
MA - 2007

EE24BTECH11061 - Rohith Sai

Single Correct

1) For which of the following pair of functions $y_1(x)$ and $y_2(x)$, continuous functions $p(x)$ and $q(x)$ can be determined on $[-1,1]$ such that $y_1(x)$ and $y_2(x)$ give two linearly independent solutions of

$$y'' + p(x)y' + q(x)y = 0, x \in [-1,1].$$

a) $y_1(x) = x\sin(x)$, $y_2(x) = \cos(x)$
b) $y_1(x) = xe^x$, $y_2(x) = \sin(x)$
c) $y_1(x) = e^{x-1}$, $y_2(x) = e^x - 1$
d) $y_1(x) = x^2$, $y_2(x) = \cos(x)$

2) Let $J_0(\cdot)$ and $J_1(\cdot)$ be the Bessel functions of the first kind of orders zero and one, respectively. If

$$\mathfrak{L}(J_0(t)) = \frac{1}{\sqrt{s^2+1}}$$

then $\mathfrak{L}(J_1(t)) =$

a) $\frac{s}{\sqrt{s^2+1}}$
b) $\frac{1}{\sqrt{s^2+1}} - 1$
c) $1 - \frac{s}{\sqrt{s^2+1}}$
d) $\frac{s}{\sqrt{s^2+1}} - 1$

Common Data Questions

Common Data for Questions 71, 72, 73:

Let $P[0,1] = \{p : p \text{ is a polynomial function on } [0,1]\}$. For $p \in P[0,1]$, define

$$\|p\| = \sup\{|p(x)| : 0 \le x \le 1\}.$$

Consider the map $T : P[0,1] \to P[0,1]$ defined by

$$(Tp)(x) = \frac{d}{dx}(p(x)).$$

Then $P[0,1]$ is a normed linear space and T is a linear map. The map T is said to be closed if the set $G = \{(p, Tp) : p \in P[0,1]\}$ is a closed subset of $P[0,1] \times P[0,1]$.

3) The linear map T is

a) one to one and onto
b) one to one but NOT onto
c) onto but NOT one to one
d) neither one to one nor onto

4) The normed linear space $P[0,1]$ is

a) a finite, dimensional normed linear space which is NOT a Banach space
b) a finite dimensional Banach space
c) an infinite dimensional normed linear space which is NOT a Banach space
d) an infinite dimensional Banach space

5) The map T is

a) closed and continuous
b) neither continuous nor closed
c) continuous but NOT closed
d) closed but NOT continuous

Common Data for Questions 74, 75:

Let X and Y be jointly distributed random variables such that the conditional distribution of Y, given $X = x$, is uniform on the interval $(x-1, x+1)$. Suppose $E(X) = 1$ and $Var(X) = \frac{5}{3}$.

6) The mean of the random variable Y is

a) $\frac{1}{2}$
b) 1
c) $\frac{3}{2}$
d) 2

7) The variance of the random variable Y is

a) $\frac{1}{2}$
b) $\frac{2}{3}$
c) 1
d) 2

Statement for Linked Answer Questions 76 & 77:

Suppose the equation

$$x^2 y'' - xy' + \left(1 + x^2\right) y = 0$$

has a solution of the form

$$y = x^r \sum_{n=0}^{\infty} c_n x^n, c_0 \neq 0.$$

8) The indicial equation for r is

a) $r^2 - 1 = 0$
b) $(r-1)^2 = 0$
c) $(r+1)^2 = 0$
d) $r^2 + 1 = 0$

9) For $n \geq 2$, the coefficients c_n will satisfy the relation

a) $n^2 c_n - c_{n-2} = 0$
b) $n^2 c_n + c_{n-2} = 0$
c) $c_n - n^2 c_{n-2} = 0$
d) $c_n + n^2 c_{n-2} = 0$

Statement for Linked Answer Questions 78 & 79:

A particle of mass m slides down without friction along a curve $x = 1 + \frac{x^2}{2}$ in the $xz-plane$ under the action of constant gravity. Suppose the $z-axis$ points vertically upwards. Let $\dot{x}$ and $\ddot{x}$ denote $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$, respectively.

10) The Lagrangian of the motion is

a) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

b) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) + mg\left(1+\frac{x^2}{2}\right)$

c) $\frac{1}{2}m\dot{x}^2x^2 - mg\left(1+\frac{x^2}{2}\right)$

d) $\frac{1}{2}m\dot{x}^2\left(1-x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

11) The Lagrangian equation of motion is

a) $\ddot{x}\left(1+x^2\right) = -x\left(g+\dot{x}^2\right)$

b) $\ddot{x}\left(1+x^2\right) = x\left(g-\dot{x}^2\right)$

c) $\ddot{x} = -gx$

d) $\ddot{x}\left(1-x^2\right) = -x\left(g-\dot{x}^2\right)$

Statement for Linked Answer Questions 80 & 81:

Let $u(x,t)$ be the solution of the one dimensional wave equation

$$u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0, u(x,0) = \begin{cases} 16 - x^2, & |x| \leq 4, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$u_t(x,0) = \begin{cases} 1, & |x| \leq 2, \\ 0, & otherwise. \end{cases}$$

12) For $1 < t < 3$, $u(2,t) =$

a) $\frac{1}{2}\left[16-(2-2t)^2\right] + \frac{1}{2}\left[1 - min\{1, t-1\}\right]$

b) $\frac{1}{2}\left[32-(2-2t)^2-(2+2t)^2\right] + t$

c) $\frac{1}{2}\left[32-(2-2t)^2-(2+2t)^2\right] + 1$

d) $\frac{1}{2}\left[16-(2-2t)^2\right] + \frac{1}{2}\left[1 - max\{1-t, -1\}\right]$

13) The value of $u_t(2,2)$

a) equals -15

b) equals -16

c) equals 0

d) does NOT exist

Statement for Linked Answer Questions 82 & 83:

Suppose $E = \{(x,y) : xy \neq 0\}$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} 0, & \text{if } xy = 0, \\ y\sin\left(\frac{1}{x}\right) + x\sin\left(\frac{1}{y}\right), & \text{otherwise.} \end{cases}$$

Let S_1 be the set of points in $\mathbb{R}^2$ where f_x exists and S_2 be the set of points in $\mathbb{R}^2$ where f_y exists. Also, let E_1 be the set of points where f_x is continuous and E_2 be the set of points where f_y is continuous.

14) S_1 and S_2 are given by

a) $S_1 = E \cup \{(x, y) : y = 0\}, S_2 = E \cup \{(x, y) : x = 0\}$
b) $S_1 = E \cup \{(x, y) : x = 0\}, S_2 = E \cup \{(x, y) : y = 0\}$
c) $S_1 = S_2 = \mathbb{R}^2$
d) $S_1 = S_2 = E \cup \{(0, 0)\}$

15) E_1 and E_2 are given by

a) $E_1 = E_2 = S_1 \cap S_2$
b) $E_1 = E_2 = S_1 \cap S_2 \setminus \{(0, 0)\}$
c) $E_1 = S_1, E_2 = S_2$
d) $E_1 = S_2, E_2 = S_1$

Statement for Linked Answer Questions 84 & 85:

Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 6 & 2 \\ 0 & 2 & 6 \end{pmatrix}$$

and let $\lambda_1 \geq \lambda_2 \geq \lambda_3$ be the eigenvalues of A.

16) The triple $(\lambda_1, \lambda_2, \lambda_3)$ equals

a) $(9, 4, 2)$
b) $(8, 4, 3)$
c) $(9, 3, 3)$
d) $(7, 5, 3)$

17) The matrix P such that

$$P^\top AP = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

is

a) $\begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}$

b) $\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$

c) $\begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix}$