CHAPTER - 20 Vector Algebra and Three Dimensional Geometry

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1 A. Fill in the Blanks

1) Let $OA = a$, $OB = 10a + 2b$ and $OC = b$ where **O**, **A** and **C** are non-collinear points. Let **p** denote the area of the quadrilateral $OABC$, and let **q** denote the area of the parallelogram with OA and OC as adjacent sides. If $p = kq$, then **k**= ...... (1997 - 2 Marks)

2 B. True/False

1) Let **A**, **B** and **C** be unit vectors suppose that $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$, and that the angle between **B** and **C** is $\frac{\pi}{6}$. Then $\mathbf{A} = \pm 2 (\mathbf{B} \times \mathbf{C})$. (1981 - 2 Marks)
2) If $\mathbf{X} \cdot \mathbf{A} = 0$, $\mathbf{X} \cdot \mathbf{B} = 0$, $\mathbf{X} \cdot \mathbf{C} = 0$ for some non-zero vector **X**, then $[\mathbf{A}\ \mathbf{B}\ \mathbf{C}] = 0$ (1983 - 1 Mark)
3) The points with position vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$ and $\mathbf{a} + \mathbf{kb}$ are collinear for all real values of **k**. (1984 - 1 Mark)
4) For any three vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$, $(\mathbf{a} - \mathbf{b}) \cdot ((\mathbf{b} - \mathbf{c}) \times (\mathbf{c} - \mathbf{a})) = 2\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. (1989 - 1 Mark)

3 C. MCQs with One Correct Answer

1) The scalar $\mathbf{A} \cdot ((\mathbf{B} + \mathbf{C}) \times (\mathbf{A} + \mathbf{B} + \mathbf{C}))$ equals:
   a) 0
   b) $[\mathbf{A}\ \mathbf{B}\ \mathbf{C}] + [\mathbf{B}\ \mathbf{C}\ \mathbf{A}]$
   c) $[\mathbf{A}\ \mathbf{B}\ \mathbf{C}]$
   d) None of these

   (1981 - 2 Marks)
2) For non-zero vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a}|\,|\mathbf{b}|\,|\mathbf{c}|$ holds if and only if
   a) $\mathbf{a} \cdot \mathbf{b} = 0$, $\mathbf{b} \cdot \mathbf{c} = 0$
   b) $\mathbf{b} \cdot \mathbf{c} = 0$, $\mathbf{c} \cdot \mathbf{a} = 0$
   c) $\mathbf{c} \cdot \mathbf{a} = 0$, $\mathbf{a} \cdot \mathbf{b} = 0$
   d) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

   (1982 - 2 Marks)
3) The volume of the parallelopiped whose sides are given by $OA = 2\mathbf{i} - 2\mathbf{j}$, $OB = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $OC = 3\mathbf{i} - \mathbf{k}$, is
   a) $\frac{4}{13}$

b) 4
c) $\frac{2}{7}$
d) None of these

(1983 - 1 Mark)

4) The points with position vectors $\mathbf{60i} + \mathbf{3j}$, $\mathbf{40i} - \mathbf{8j}$, $\mathbf{ai} - \mathbf{52j}$ are collinear if
a) $a = -40$
b) $a = 40$
c) $a = 20$
d) None of these

(1983 - 1 Mark)

5) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three non coplanar vectors and $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are vectors defined by the relations $\mathbf{p} = \frac{\mathbf{b}\times\mathbf{c}}{[\mathbf{a\ b\ c}]}, \mathbf{q} = \frac{\mathbf{c}\times\mathbf{a}}{[\mathbf{a\ b\ c}]}, \mathbf{r} = \frac{\mathbf{a}\times\mathbf{b}}{[\mathbf{a\ b\ c}]}$ then the value of the expression $(\mathbf{a}+\mathbf{b})\cdot\mathbf{p}+(\mathbf{b}+\mathbf{c})\cdot\mathbf{q}+(\mathbf{c}+\mathbf{a})\cdot\mathbf{r}$ is equal to
a) 0
b) 1
c) 2
d) 3

(1988 - 2 Marks)

6) Let **a**, **b**, **c** be distinct non-negative numbers. If the vectors $\mathbf{ai} + \mathbf{aj} + \mathbf{ck}$, $\mathbf{i} + \mathbf{k}$ and $\mathbf{ci} + \mathbf{cj} + \mathbf{bk}$ lie in a plane, then **c** is
a) the Arithmetic Mean of **a** and **b**
b) the Geometric Mean of **a** and **b**
c) the Harmonic Mean of **a** and **b**
d) equal to zero

(1993 - 1 Mark)

7) Let **p** and **q** be the position vectors of **P** and **Q** respectively, with respect to **O** and $|\mathbf{p}| = p$, $|\mathbf{q}| = q$. The points **R** and **S** divide PQ internally and externally in the ratio $2:3$ respectively. If OR and OS are perpendicular then
a) $9p^2 = 4q^2$
b) $4p^2 = 9q^2$
c) $9p = 4q$
d) $4p = 9q$

(1994)

8) Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\mathbf{i} + \beta\mathbf{j} + \gamma\mathbf{k}$, $\beta\mathbf{i} + \gamma\mathbf{j} + \alpha\mathbf{k}$, $\gamma\mathbf{i} + \alpha\mathbf{j} + \beta\mathbf{k}$
a) are collinear
b) form an equilateral triangle
c) form a scalene triangle
d) form a right angles triangle

(1994)

9) Let $\mathbf{a} = \mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{j} - \mathbf{k}$, $\mathbf{c} = \mathbf{k} - \mathbf{i}$. If **d** is a unit vector such that $\mathbf{a}\cdot\mathbf{d} = 0 = [\mathbf{b\ c\ d}]$, then **d** equals
a) $\pm\frac{\mathbf{i}+\mathbf{j}-2\mathbf{k}}{\sqrt{6}}$

b) $\pm \frac{\mathbf{i}+\mathbf{k}-\mathbf{k}}{\sqrt{3}}$
c) $\pm \frac{\mathbf{i}+\mathbf{j}+\mathbf{k}}{\sqrt{3}}$
d) $\pm\mathbf{k}$

(1995S)

10) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non coplanar unit vectors such that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{(\mathbf{b}+\mathbf{c})}{\sqrt{2}}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is
a) $\frac{3\pi}{4}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) π

(1995S)