CHAPTER - 1
Vector Arithmetic

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1 1.9 CBSE

1) If the point $\mathbf{P}(x, y)$ is equidistant from the points $\mathbf{A}(a+b, b-a)$ and $\mathbf{B}(a-b, a+b)$, prove that $bx = ay$.
Solution: Given points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{P}$ are represented as:

$$\mathbf{A} = \begin{pmatrix} a+b \\ b-a \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} a-b \\ a+b \end{pmatrix} \tag{2}$$

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{3}$$

Given that the point $\mathbf{P}$ is equidistant from both $\mathbf{A}$ and $\mathbf{B}$:

$$||\mathbf{P} - \mathbf{A}|| = ||\mathbf{P} - \mathbf{B}|| \tag{4}$$

$$\implies ||\mathbf{P} - \mathbf{A}||^2 = ||\mathbf{P} - \mathbf{B}||^2 \tag{5}$$

One expanding, we get:

$$\implies ||\mathbf{P}||^2 - 2\mathbf{P}^\top \mathbf{A} + ||\mathbf{A}||^2 = ||\mathbf{P}||^2 - 2\mathbf{P}^\top \mathbf{B} + ||\mathbf{B}||^2 \tag{6}$$

On further simplifying, we get:

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{||\mathbf{A}||^2 - ||\mathbf{B}||^2}{2} \tag{7}$$

$$\tag{8}$$

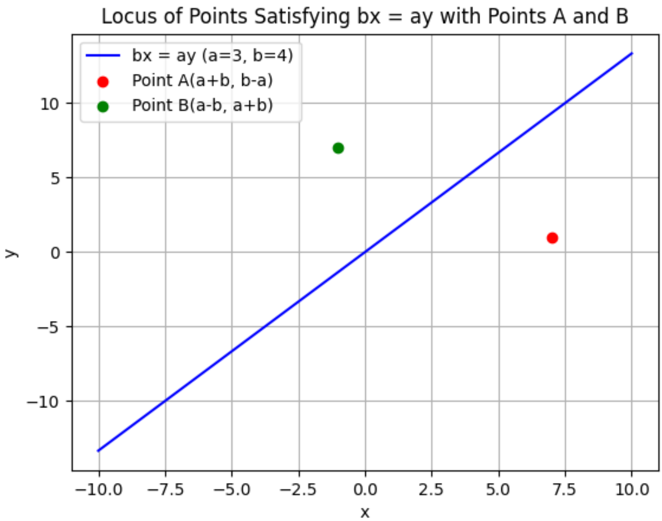
From equation (3):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{||\mathbf{A}||^2 - ||\mathbf{B}||^2}{2(\mathbf{A} - \mathbf{B})^\top} \tag{9}$$

Substituting from equations (1) and (2). Thus,

$$bx = ay \tag{10}$$

Hence, proved.


Locus of Points Satisfying bx = ay with Points A and B
bx = ay (a=3, b=4)
Point A(a+b, b-a)
Point B(a-b, a+b)
10
5
0
−5
−10
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x