JEE MAINS 2022
June 27 - Shift 2

EE24BTECH11061 - Rohith Sai

1) The number of points of intersection of $|z-(4+3\iota)|=2$ and $|z|+|z-4|=6$, $z\in\mathbb{C}$ is:

a) 0
b) 1
c) 2
d) 3

2) Let $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a\in\mathbb{R}$. Then the sum of which the squares of all the values of a for $2f'(10)-f'(5)+100=0$ is:

a) 117
b) 106
c) 125
d) 136

3) Let for some real numbers α and β, $a=\alpha-\iota\beta$. If the system of equations $4\iota x+(1+\iota)y=0$ and $8\left(\cos\frac{2\pi}{3}+\iota\sin\frac{2\pi}{3}\right)x+\overline{a}y=0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to:

a) $-2+\sqrt{3}$
b) $2-\sqrt{3}$
c) $2+\sqrt{3}$
d) $-2-\sqrt{3}$

4) Let A and B be two 3×3 matrices such that AB = I and $\det A=\frac{1}{8}$ then $\det\{adj(Badj(2A))\}$ is equal to

a) 16
b) 32
c) 64
d) 128

5) Let $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+\ldots$ then 4S is equal to

a) $\left(\frac{7}{3}\right)^2$
b) $\left(\frac{7}{3}\right)^3$
c) $\frac{7^3}{3^2}$
d) $\frac{7^2}{3^3}$

6) If $a_1,a_2,a_3,\ldots$ and $b_1,b_2,b_3,\ldots$ are in A.P. and $a_1=2$, $a_{10}=3$, $a_1b_1=1=a_{10}b_{10}$ then a_4b_4 is equal to

a) $\frac{35}{27}$
b) 1
c) $\frac{27}{28}$
d) $\frac{28}{27}$

7) If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t}, dt$, then the ordered pair (m,n) is equal to

a) $(3, 2)$
b) $(2, 3)$
c) $(2, 2)$
d) $(3, 4)$

8) Let f be a differentiable function in $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^1 t^2 f(t), dt = \sin^3 x + \cos x$ then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to:

a) $6 - 9\sqrt{2}$
b) $6 - \frac{9}{\sqrt{2}}$
c) $\frac{9}{2} - 6\sqrt{2}$
d) $\frac{9}{\sqrt{2}} - 6$

9) The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}, dx$, where [.] denotes the greatest integer function is equal to

a) $1 + 6\log_e \frac{6}{7}$
b) $1 - 6\log_e \frac{6}{7}$
c) $\log_e \frac{7}{6}$
d) $1 - 7\log_e \frac{6}{7}$

10) If the solution curve of the differential equation $\left(\left(\tan^{-1} y\right) - x\right) dy = \left(1 + y^2\right) dx$ passes through the point $(1, 0)$ then the abscissa of the point on the curve whose ordinate is $\tan 1$ is:

a) $2e$
b) $\frac{2}{e}$
c) 2
d) $\frac{1}{e}$

11) If the equation of the parabola, whose vertex is at $(5, 4)$ and the directrix is $3x+y-29 = 0$, id $x^2 + ay^2 + bxy + cx + dy + k = 0$ then $a + b + c + d + k$ is equal to

a) 575
b) -575
c) 576
d) -576

12) The set of values of k for which the circle C : $4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is:

a) An empty set
b) $(6, \frac{95}{9}]$
c) $[\frac{80}{9}, 10)$
d) $(9, \frac{92}{9}]$

13) Let the foot of the perpendicular from the point $(1, 2, 4)$ on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x + 4y + 12z + 23 = 0$

a) 5
b) $\frac{50}{13}$
c) 4
d) $\frac{63}{13}$

14) The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$ is:

a) $\frac{18}{\sqrt{5}}$
b) $\frac{22}{3\sqrt{5}}$
c) $\frac{46}{3\sqrt{5}}$
d) $6\sqrt{3}$

15) Let $\mathbf{a}$ and $\mathbf{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\mathbf{a}$ and $\mathbf{b}$ be acute. $|\mathbf{a}| = 1$ and $|\mathbf{a} \cdot \mathbf{b}| = |\mathbf{a} \times \mathbf{b}|$. If $\mathbf{c} = 2\sqrt{2}(\mathbf{a} \times \mathbf{b}) - 2\mathbf{b}$, then an angle between $\mathbf{b}$ and $\mathbf{c}$ is:

a) $\frac{\pi}{4}$
b) $-\frac{\pi}{4}$
c) $\frac{5\pi}{6}$
d) $\frac{3\pi}{4}$