

CHAPTER - 1

Vector Arithmetic

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1 1.9 CBSE

- 1) If the point $\mathbf{P}(x, y)$ is equidistant from the points $\mathbf{A}(a + b, b - a)$ and $\mathbf{B}(a - b, a + b)$, prove that $bx = ay$.

Solution:

Variable	Description	Formula
A	It is one end of the line segment	$\mathbf{A} = \begin{pmatrix} a + b \\ b - a \end{pmatrix}$
B	It is other end of line segment	$\mathbf{B} = \begin{pmatrix} a - b \\ a + b \end{pmatrix}$
P	It is the point equidistant from A and B	$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 1

Given that the point \mathbf{P} is equidistant from both \mathbf{A} and \mathbf{B} :

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \quad (1)$$

$$\Rightarrow \|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (2)$$

On expanding, we get:

$$\Rightarrow \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{B} + \|\mathbf{B}\|^2 \quad (3)$$

On further simplifying, we get:

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (4)$$

(5)

From equation (3):

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top} \quad (6)$$

Substituting from equations (1) and (2). Thus,

$$bx = ay \quad (7)$$

Hence, proved.

