CHAPTER - 1
Vector Arithmetic

EE24BTECH11061 - Rohith Sai

1 1.8 LENGTH

1.8.9 Find the values of y for which the distance between the points $\mathbf{P}(2, -3)$ and $\mathbf{Q}(10, y)$ is 10 units.

Solution:

Variable	Description	Formula
P	It is one end of the line segment	$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$
Q	It is other end of line segment	$\mathbf{Q} = \begin{pmatrix} 10 \\ y \end{pmatrix}$

TABLE 0

Let the distance between the two points be d. Given that the distance between the two points $\mathbf{P}$ and $\mathbf{Q}$ is 10 units:

$$d = 10 \tag{1}$$

We know that d is defined as:

$$d = \|\mathbf{P} - \mathbf{Q}\| \tag{2}$$

$$\implies d = \sqrt{(\mathbf{P} - \mathbf{Q})^{\top} (\mathbf{P} - \mathbf{Q})} \tag{3}$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -3 - y \end{pmatrix} \tag{4}$$

$$\tag{5}$$

From (3):

$$d = \|\mathbf{P} - \mathbf{Q}\| = 10 \tag{6}$$

$$\implies (\mathbf{P} - \mathbf{Q})^{\top} (\mathbf{P} - \mathbf{Q}) = 100 \tag{7}$$

Therefore:

$$y = 3 \text{ and } y = -9 \tag{8}$$

Distance between P(2, -3) and Q(10, y)