**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B (0.2676)

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

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| Ans: |
|  | P(X > 44) = 1 - P(X ≤ 44). |
|  | Z = (X - )/ = (X - 38)/6 |
|  | Thus the question can be answered by using the normal table to find |
|  | P(X ≤ 44) = P(Z ≤ (44 - 38)/6) = P(Z ≤ 1)=84.1345% |
|  | probability that the employee will be greater than age of 44 = 100-84.1345=15.86% |
|  | So the probability of number of employees between 38-44 years of age = P(X<44)-0.5=84.1345-0.5= 34.1345% |
|  | Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is FALSE. |
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1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: As we know X1~N(μ1, σ12) and X2~N(μ2, σ22) are two independent random variables then X1+X2~N(μ1+ μ2, σ12+σ22) and X1-X2~N(μ1- μ2, σ12- σ22).

Similarly if Z = aX + bY , where X and Y are as defined above, i.e. Z is linear combination X1 and X2

Then Z~N(aμ1+bμ2,a2σ12+b2σ22)

Therefore in the question,

2X1~N(2μ,4σ2) and

X1+X2~N(μ+μ, σ2+ σ2)~N(2μ, 2σ2)

2X1-(X1+X2) = N(4μ, 6σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D (48.5, 151.5)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Rupee ranges in between [9.1 to 98.1] crore rupees, 95% of the time for the annual profit of the company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: The 5th percentile of profit for the company is 17 crore rupees.

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: The Division #2 (Profit2~N(7,42)) has a larger probability of making a loss in the given year.