Quantitative Reasoning & Aptitude

CALENDAR

1 ordinary year → 365 days → 52 weeks + 1 day

1 Leap year → 366 days → 52 weeks + 2 days

* Every 4th century is leap year (Not every century)
  100th year is not leap year

100
- 24 (leap year) → 48 (odd) → 6 odd
- 76 (Ordinary year) → 6 odd

6 odd + 6 odd → 12 → 5 odd days

100 years → 5 odd days

200 years → 2(5) = 10 → 3 odd

300 years → 3(5) = 15 → 1 odd

400 years → 4(5) = 20 → 6
+1 (since leap year)
= 7 → 0 odd days

0	1	2	3	4	5	6
Sun	Mon	Tue	Wed	Thu	Fri	Sat

Example :-

* Nov 3, 2003

2002 + (1st Jan – 3rd Nov 2003) → 3 odd days

2002 → 2000 + 2 years
- 2000 → 0 odd days (CLY)
- 2 years → 2 odd days

J – 3	Jul – 3
F – 0	Aug – 3
M – 3	Sept – 2
A – 2	Oct – 3
May – 3	Nov
Jun – 2	

24 + 3 = 27

Total : 27 + 2 = 29 → 1 odd day → Monday.

Example :-

10 Nov, 1981 → 3 odd

1980 + 1st Jan – 10 Nov 1981

1900 + 80

1600 + 300 + 80

0 → 1 odd → 20 odd → 3 odd

80 → 20 LY, 60 OY

20 LY → 40 → 5 odd

60 OY → 4 odd

5 odd + 4 odd = 9 → 2 odd

J – 3
F – 0
M – 3
Apr – 2
May – 3
Jun – 2
Jul – 3
Aug – 3
Sept – 2
Oct – 3

24 + 3 = 27

27 + 3 = 30 → 2 odd days → Tuesday

Example :-

16 Sep, 2345 → 2 odd

2344 + (1st Jan – 16 Sep 2345)

→ 19 + 2 = 21

2300 + 44

2000 + 300 + 44

0 → 1 → 6

44 → 11 (LY) → 22 → 1; 33 (OY) → 5

1 + 6 = 7

Total → 7 + 21 = 28 → 0 odd days → Sunday

To check if a number is prime or not

1. Check if given number is perfect square or not
2. Check if given number is cube or not.
3. Take nearest highest square number.
   Write prime numbers less than square root.
   Check if these prime numbers are factors of given number.
   If not divisible, then number is prime number.

Eg:- 161

1. $11^2 = 121$ $\quad 13^2 = 169$
   $12^2 = 144$
   Not a perfect square.
2. $5^3 = 125$
   $6^3 = 216$
   Not a cube.
3. Nearest square number $\rightarrow$ $13^2 = 169$
   Prime nos less than 13 $\rightarrow$ 2, 3, 5, 7, 11
   $\frac{161}{7} = 23$ (divisible by 7)
   So not prime.

Day 2

Squares of 2-Digit Numbers:-

→ Ending with zero.

→ Ending with 5

$(45)^2 = [4(5)][5^2] = 2025$

$(75)^2 = [7(8)][5^2] = 5625$

$(nm)^2 = [n(n+1)][m^2]$

→ Ending with other digit

$(72)^2 \rightarrow (7)^2(2)^2$

$\rightarrow 4904$

$+ 280 \rightarrow 7 \times 2 = 14$ ⟩ double

5184 28

add zero at end

280

$(69)^2 \rightarrow (6)^2(9)^2$

$\rightarrow 3681$ $54 \times 2 = 108$

$+1080$

4761

Cubes of 2 Digit Numbers:-

→ Ending with zero

→ Starting with 1

$(12)^3 = 1 \quad 2 \quad 4 \quad 8$ ⟩ double

$(+) \quad 4 \quad 8$

$1 \quad 7 \quad 2 \quad 8$

$(13)^3 = 1 \quad 3 \quad 9 \quad 27$

$6 \quad 18$

$2 \quad 1 \quad 9 \quad 7$

→ Ending with 1 :—

$(41)^3$

(4) (1)
64 16 4 1

32 8

68 9 2 1

$(51)^3$

(7) (1)
125 25 5 1

50 10

132 6 5 1

→ Doublets :—

$(66)^3$ = 6^3 6^3 6^3 6^3

(71) (66) (21)
216 216 216 216

432 432

287 4 9 6

→ Ending with any other digit :—

$(43)^3$ = a^3 a^2b ab^2 b^3

(15) (11) (2)
64 48 36 27

a, b

96 72

795 0 7

$(19)^3$ =

(5) (31) (72)
1 9 81 729

18 162

6 8 5 9

$(81)^3$ =

(19) (2)
512 64 8 1

128 16

531 4 4 1

$(72)^3$ → a^3 a^2b ab^2 b^3

30 8

343 98 28 8

196 56

3 7 3 2 4 8

21 19 6

$(44)^3$ → 64 64 64 64

128 128

4

8 5 1 8

19 54 21

$(36)^3$ → 27 54 108 216

108 216

4 6 6 5 6

Squares of 3 Digit Number:-

→ Less than 316 / Greater than 317

$316^2 = 99856$

$317^2 = 100489$

→ Type A →. Round off to less than nearest 50.

→ Type B → Round off beyond 50.

Eg:- $(203)^2$

4 1 2 0 9

↓

200 + 3 = 203

Square of 3 (add)

Add 3 again

203 + 3 = 206 (Since there are 2 hundreds)

multiply with 2.

× 2

412

(*) 208

200 → 208 (+8)

208 + 8 = 216

216 × 2 = 432

4 3 2 6 4

8

(*) (124)^

100 → 124 (+24)

(24)^ = 576

124 + 24 = 148 (one hundred)

148 × 1 = 148

⑧

1 4 5 7 6

+

1 5 3 7 6

(*) (132)^

100 → 132 (+32)

(32)^ = (3)^(2)^ = 904 + 120 = 1024

(3)(2)^(2) + 0.

132 + 32 = 164

164 × 1 = 164

⑥ ④

1 1 0 2 4

1 7 4 2 4

(*) (404)^

400 → 404 (+4)

404 + 4 = 408

408 × 4 = 1632

1 6 3 2 1 6

(*) (508)$^{\sim}$

500 → 508

+8

508 + 8 = 516

× 5

2580

2 5 8 0 6 4

(*) (729)$^{\sim}$

700 + 29

(29)$^{\sim}$ → 481

+360

841

729 + 29 = 758

× 7

5306

(6)

5 3 0 8 4 1

5 3 1 4 4 1

TYPE B :—

(*) 298 → 300

−2

298 − 2 = 296

296 × 3 → (since rounding off around 50 i.e to 300)

888

8 8 8 0 4

(*) 296 → 300

−4

296 − 4 = 292

× 3

876

8 7 6 1 6

Find Number of factors :-

$N = a^p b^q c^r$

No. of factors $= (p+1)(q+1)(r+1)$

Sum of factors $= [a^{p+1} - 1/a - 1][b^{q+1} - 1/b - 1]$
$[c^{r+1} - 1/c - 1]$

Product of factors
$= N^{(p+1)(q+1)(r+1)/2}$ (including 1 & N).

6 $\longrightarrow$ $2^1 \times 3^1$ $(a^p b^q)$

$(1+1)(1+1) = 4$

7 $\rightarrow$ $7^1 \rightarrow (1+1) = 2$

8 $\rightarrow$ $2^3 \rightarrow (3+1) = 4$

9 $\rightarrow$ $3^2 \rightarrow (2+1) = 3$

10 $\rightarrow$ $2^1 \times 5^1 \rightarrow (2)(2) = 4$

12 $\rightarrow$ $2^2 \times 3^1 \rightarrow (3)(2) = 6$

52900 $\longrightarrow$ $2^2 \times 5^2 \times 23^2 = (3)(3)(3)$

No. of zeros that many 2's & 5's $= 27$

4800 $\rightarrow$ $2^5 \times 5^2 \times 2^4 \times 3^1$

$= 2^6 \times 5^2 \times 3^1$

$\rightarrow (7)(3)(2) = 42$

2	48
2	24
2	12
2	6
	3

(*) 54400 $\rightarrow 2^5 \times 5^2 \times 2^5 \times 17$

$\rightarrow 2^7 \times 5^2 \times 17$

$(8)(3)(2) = 48$

2	544
2	272
2	136
2	68
2	34
	17

(*) 54400×30600

$\downarrow$ $\qquad$ $\downarrow$

$2^7 \times 5^2 \times 17$ $\qquad$ $2^2 \times 5^2 \times 2 \times 3^2 \times 17$

$= 2^{10} \times 5^4 \times 3^2 \times 17^2$

$(11)(5)(3)(3) = 495$

2 | 306
3 | 153
3 | 51
17

$+ 2$
48
$\times 11$
$\overline{495}$

(*) $444 \times 888 \rightarrow 4 \times 111 \times 8 \times 111$

$2^2 \times 3 \times 37 \times 2^3 \times 3 \times 37$

$= 2^5 \times 3^2 \times 37^2$

$\rightarrow (6)(3)(3) \rightarrow 54$

(*) $120 \times 240 \times 360$

$\downarrow$ $\qquad$ $\downarrow$ $\qquad$ $\rightarrow$

$2 \times 5 \times 2^2 \times 3$ $\qquad$ $2 \times 5 \times 2^3 \times 3$ $\qquad$ $2 \times 5 \times 2^2 \times 3^2$

$= 2^{10} \times 3^4 \times 5^3 \rightarrow (11)(5)(4)$

$= 220$

(*) 557×699

Check if 557 is prime.

→ Nearest square → $(24)^2$

→ Primes less than 24 → 2, 3, 5, 7, 11, 13, 17, 19, 23

Check if 557 is divisible by any nos.

No → prime.

$= 557 \times 3 \times 233$

$\rightarrow (16)^2 = 256$

2, 3, 5, 7, 11, 13.

→ $(2)(2)(2) = 8$

(*) 3220×5290

$5 \times 2 \times 7 \times 23$ $\qquad$ $5 \times 2 \times 23^2$

$= 2^2 \times 5^2 \times 7 \times 23^3$

$= (3)(3)(2)(4) = 72$

LAST DIGIT :-

$N = (xy|z)^n$

* Divide the number with 4 (since for every 4 multiple last digit reapeats)

Remainder $R = 1$ then last digit is z

$R = 2 \longrightarrow z^2$

$R = 3 \longrightarrow z^3$

$R = 0$; z is even $\longrightarrow 6$

z is odd (except 5) $\longrightarrow 1$

Eg:- $(17)^{18} \times (18)^{19} \times (19)^{20}$

$\frac{18}{4} \rightarrow R = 2$

$\therefore z^2 = 7^2 = 49$

$\frac{19}{4} \rightarrow R = 3$

$L = 8^3 = \ldots 2$

$\frac{20}{4} \rightarrow R = 0$

$z = 9$ (odd)

$L = 1$

$L \rightarrow (9)(2)(1) = 18$

(*) $(5234)^{4325}$

$\frac{4325}{4} \rightarrow R = 1$ $\qquad$ last digit : 4 (2)

(*) $(645)^{640}$

$\frac{640}{4} \rightarrow R = 0$

$Z = 5 \Rightarrow$ Last digit = 5

(*) Magical numbers $\rightarrow$ 1, 5, 6, 0 $\longrightarrow$ $(n^{\text{any no}} = n)$

(*) $(999)^{888}$

$\frac{888}{4} \rightarrow R = 0 \quad Z = 9 \quad , L = 1$

(*) $(544)^{688} \times (306)^{405}$

$R = 0$ $\qquad$ $R = 1 \rightarrow L = 6$

$Z = 4$ $\qquad$ $(1)(6) = 6$

$L = 6$

Last two Digits :-

Case 1 :- End with 1

Case 2 :- End with 3, 7, 9

Case 3 :- End with 5

Case 4 :- End with even numbers.

$(41)^{88}$ $\qquad$ $\frac{2}{\quad}$ $\frac{1}{\quad}$

$4(8) = 32$

$(81)^{43}$ $\qquad$ $\frac{4}{\quad}$ $\frac{1}{\quad}$

$8(3) = 24$

$(61)^{66} = \frac{6}{\quad}$ $\frac{1}{\quad}$

$6(6) = 36$

Case 3 :- Possibilities $\qquad$ Examples

End with (75) $\in (OD\ 5)^{OD} = (35)^{77}$

any even cases end with (25)

$(OD\ 5)^{EV} = (35)^{76}$

$(EV\ 5)^{OD} = (45)^{79}$

$(EV\ 5)^{EV} = (45)^{78}$

Case 2 :-

$(43)^{162} = ((43)^4)^{40} \times (43)^2$

$= (01)^{40} \times (49)$

$= 01 \times 49 \rightarrow 49.$

162
$\downarrow$
$(40 \times 4) + 2$

$3^4 = 1$
$7^4 = 1$
$9^2 = 1$

$(43)^2 \rightarrow (16)(09)$
1609
240
49

(*) $(53)^{84} = [(53)^4]^{21} \times (53)$

$= (81)^{21} \rightarrow 01$

$(49)^2 \rightarrow 1681$
$+720$
01

(*) $(49)^{162} = [(49)^2]^{81}$

$= (01)^{81} \rightarrow 01$

$(53)^2 = (9+)(09)$
300
09
$(83)^2 \rightarrow$

(*) $(119)^{456} \rightarrow [(119)^2]^{228} \rightarrow 0$

$(61)^{228} \rightarrow 0$

$6(8) = 48$

$8 \quad 1$

$(119)^2$
$100 \quad 119 \quad +19 = 138$
19

$1 \quad 3 \quad 3 \quad 6 \quad 1$

Case 4 :-

$(24)^{odd} = 24$

$(24)^{even} = 76$

$(76)^{any\ number} = 76$

$2^{10} \rightarrow 24$

$(46)^{250}$

Part-3

Ratios and properties

Mid-1
Numbers,
Ratios, proportion
Alligations,
Averages, Ages
Partnership
Percentages

(3) Bridge three components

$A:B = 2:3$

$B:C = 4:5$

$A:B:C = 2(4) : 4(3) : 3(5)$

$= 8:12:15$

Rs. 7000/-

B's share $\Rightarrow \frac{12}{35} \times 7000 = 2400$

200
1000

(4)

Given		Find Out
$A:B = 4:5$	$B:C = 6:7$	$A:C = 24:35$
$A:B = 6:7$	$B:C = 8:9$	$A:B:C = 48:56:63$
$BC:AC:AB = 1:2:3$		$A:B:C = 6:3:2$
$\frac{1}{A}:\frac{1}{B}:\frac{1}{C} = 2:3:5$		$A:B:C = 15:10:6$

$BC : AC : AB = 1:2:3$

$\frac{BC}{AC} - \frac{B}{A}$

$A:B = 2:1$

$B:C = 3:2$

$A:B:C = 6:3:2$

$\frac{1}{A}:\frac{1}{B}:\frac{1}{C} = 2:3:5$

$A:B = 3:2$

$B:C = 5:3$

$A:B:C = 15:10:6$

(*) Bridge 4 components

A : B = 2 : 5

B : C = 3 : 1

C : D = 3 : 5

A : B : C : D = ?

A : B = 2 : 5

B : C = 3 : 1

C : D = 3 : 5

A : B : C : D = 2×3×3 : 5×3×3 : 5×1×3 : 5×1×5

= 18 : 45 : 15 : 25

(*) Rs. 6300

A : B = 2 : 3

B : C = 4 : 5

C : D = 3 : 7

A : B : C : D = 2×4×3 : 3×4×3 : 3×5×3 : 3×5×7

= 24 : 36 : 45 : 105

B's share = $\frac{36}{210} \times 6300$

= 1080

(*) Find A : D if

A : B = 2 : 5

B : C = 4 : 3

C : D = 1 : 7

A : C = 8 : 15

A : D = 8 : 105

(*) Two nos Ratio → 3 : 5 New → 12 : 23 after subtracting 9 from numbers

Method-1:-

$\frac{3x-9}{5x-9} = \frac{12}{23}$

$69x - 207 = 60x - 108$

$9x = 99 \Rightarrow x = 11$

$\therefore 3x = 33$

$5x = 55$

Method II :-

(Choose options)

3 : 5 12 : 23

(a) 27 $27-9=18$

$45-9=36$

(b) 30 $30-9=21$

$50-9=41$

(c) 33 $33-9=24$) $\frac{12}{23}$

$55-9=46$

(*) M, P, B → 5 : 7 : 8

40% 50% 75%

Ratio of increased seats ?

$5 \times 140 : 7 \times 150 : 8 \times 175$

(100+40) (100+50) (100+75)

2 : 3 : 4

(*) Rs. 782

$\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ Find 1st number.

$$\frac{6x+8x+9x}{12} = 782$$

$$\Rightarrow \frac{23x}{12} = 782$$

$$\Rightarrow x = 34 \times 12$$

$$\frac{1}{2}(x) : 34 \times 6 \times \frac{1}{x}$$

$= 204$

(or)

$\frac{1}{2}^{\times 12} : \frac{2}{3}^{\times 12} : \frac{3}{4}^{\times 12} = 6 : 8 : 9$

$$\frac{6}{6+8+9} \times 782 = \frac{6}{23} \times 34$$

$= 204$

Income	Expenditure	Savings
$I_A : I_B = 3:4$	$E_A : E_B = 4:5$	$S_A : S_B = ?$

Method I :–

$I_A = 3x \qquad I_B = 4x$

Income = Savings + Expenditure

$E_A = 4y \qquad E_B = 5y$

$S_A = 3x - 4y \qquad S_B = 4x - 5y$

$$\frac{S_A}{S_B} = \frac{3x-4y}{4x-5y}$$

Also given → $3x - 4y = \frac{1}{4}(3x)$ $\quad S_A = \frac{1}{4} I_A$

$$9x = 16y$$

$$\Rightarrow x = \left(\frac{16}{9}\right)y$$

$$\frac{3\left(\frac{16}{9}\right)y - 4y}{4\left(\frac{16}{9}\right)y - 5y} = \frac{\frac{y}{9}[48-36]}{\frac{y}{9}[64-45]}$$

$$= \frac{12}{19} \rightarrow 12:19$$

Method 2 :–

Assumptions.

$I_A = 300 \quad , I_B = 400$

$S_A = \frac{1}{4}(300) = 75$

$\therefore E_A = 225$

$4 : 5 = 225 : E_B$

$E_B = \frac{1125}{4}$

$S_B = 400 - \frac{1125}{4} = \frac{475}{4}$

$$\frac{S_A}{S_B} = \frac{\cancel{75}^{3} \times 4}{\cancel{475}_{19}}$$

$= 12:19$

(*) A:B = 2:3
B:C = 2:3

Same expenditure for all A, B, C.

One third of income of C exceeds 80 than half of income of A.

$\frac{1}{3} I_C = 80 + \frac{I_A}{2}$

$I_A : I_B : I_C = 4 : 6 : 9$

$S_A : S_B : S_C = 1 : 9 : 21$

$\frac{1}{3}(21x) = 80 + \frac{4x}{2}$

$x = 80$

$I_A = 4(80) = 320$

$I_B = 480$

$I_C = 720$

$320 - x = y$
$480 - x = 9y$
$720 - x = 21y$

$320 - x = y$.
$480 - x = 9y$
$8y = 160 \Rightarrow y = 20$

$320 - x = 20$
$\Rightarrow x = 300$

Combined expenditure $= 300(3) = 900$

Averages :—

(*) Average of new items added

$A +/- (1 + N/n)x$

+ → increase
− → decrease.

Average of items removed

$A +/- (1 - N/n)x$

A → Original average
N → Original items
n → Added or removed
x → by which avg increased or ~~dec~~ decreased

(d) $A \pm \left(1 + \frac{N}{n}\right) x$

$15 + \left(1 + \frac{40}{10}\right) \times 0.2$

$= 15 + 5 \times 0.2$

$= 16$

(e) $N = 15$, $A = 4500$

$4800 - \left(1 - \frac{15}{3}\right) 1750$

$= 4500 + 700$

Avg salary = 5200

Total salary of 3 teachers = $3(5200)$

$= 15600$

(f) $N = 50$, $A = 45$

$45 - \left(1 - \frac{50}{1}\right) \times 100$

$= 45 + 4900$

$= 4945\ g \rightarrow 49.45\ kg$

Replacement of items :-

(*) For N items in a group,

Sum of new items added − Sum of new items removed

$= +/- Nx$

$A - R = \pm Nx$

(c) $N = 5$, $x = 3\ kg$

$A - R = \pm Nx$

$= -5(3)$

$A - 80 = -15$

$A = 65\ kg$

(*) $N = 15$

$A - 40 = (15)(1.5)$

$A - 40 = 22.5$

$\Rightarrow A = 62.5\ kg$.

(*) $N = 8 \qquad x = 2.5$

$A - 65 = +20$

$A = 85\ kg$

(*) Avg age of hus, wife, child 3 years ago → 27 years

Avg age of wife, child 5 years ago → 20 years.

Present husband age?

$$\frac{H-3+W-3+C-3}{3} = 27$$

$$\Rightarrow H + W + C = 90$$

$$\frac{W-5+C-5}{2} = 20$$

$$\Rightarrow W + C = 50$$

$$H + 50 = 90 \Rightarrow H = 40.$$

(*)

$$P + Q - 10 = 30$$

$$P + Q = 40$$

$$\frac{P+Q+20}{2} = \frac{40+20}{2} = 30$$

A B

(*) Present age → 9 : 5

6 years ago.

A is twice B.

$$9x - 6 = 2(5x - 6)$$

$$\Rightarrow 9x - 6 = 10x - 12$$

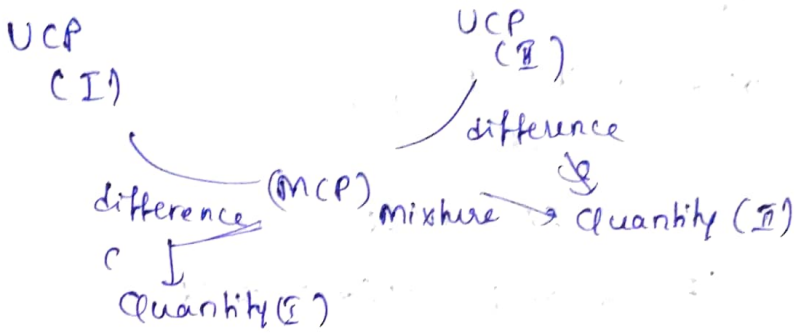
$$\Rightarrow x = 6$$

$9(6) = 54 \qquad 5(6) = 30$

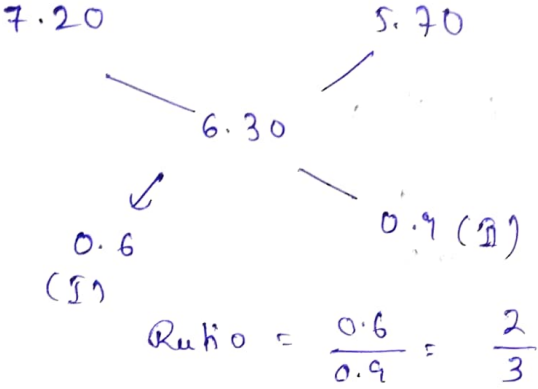
Difference = 24.

⊛ Aligation Rule :-

Unit cost price of each variety.



→

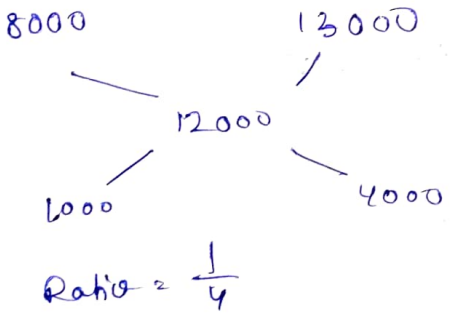


$$\text{Ratio} = \frac{0.6}{0.9} = \frac{2}{3}$$

⊛ $A = 12000$

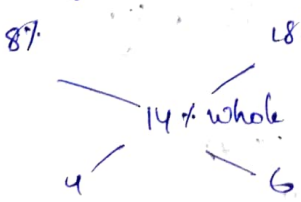
$A_v = 8000$

$A_m = 13000$



$$\text{Ratio} = \frac{1}{4}$$

⊛ 1000 kg sugar



$$\frac{4}{6} = \frac{2}{3}$$

$$18\% \rightarrow \frac{3}{5} \times 1000 = 600$$

(*) SP − CP = Gain

CP − SP = loss

$\frac{G}{CP} \times 100 = G\%$ $\qquad \frac{L}{CP} \times 100 \rightarrow L\%$

If $10\%.G \rightarrow \frac{SP - CP}{CP} = 0.1$

$\therefore \Rightarrow SP = 1.1\,CP$

if $20\%.G \rightarrow SP = 1.2\,CP$

If $20\%.L \rightarrow SP = 0.8\,CP$

(*) Gain = 10% $\qquad SP = 1.1\,CP$

CP $\qquad$ CP

60 $\qquad$ 65

62

3 $\qquad$ 2

3 : 2

$68.20 = 1.1\,CP$

$\Rightarrow CP = \frac{682}{11} = 62$

(*) SP = 40 P/kg

$SP = 1.25\,(CP)$

$\frac{40}{1.25} = CP$

$\frac{40}{125} \times 100 = 32$ P/kg

~~20kg~~ ? $\qquad$ (25 kg)

42 $\qquad$ 24

32

8 $\qquad$ 10

8 : 10 $\rightarrow$ 4 : 5

$4:5 :: x:25$

$\Rightarrow 5x = 100 \Rightarrow x = 20$ kg

Partnership:-

Investment Ratio = Profit Ratio

$$IR = \frac{(\text{Amount invested})_A \times (\text{No. of months})_A}{(\text{Amount})_B \times (\text{No. of months})_B}$$

$$\frac{25000 \times 12}{30000 \times 8} = \frac{5}{4}$$

(*) A → 25000

After 4 months

B → 30 k

After 6 months → A withdraw 5k

$$\therefore IR = \frac{25000 \times 6 + 20000 \times 6}{30000 \times 8}$$

$$= \frac{6 \times 45000}{30000 \times 8} = \frac{9}{8}$$

Profit sharing ratio = 9:8