Q: There exists an integer x, such that

$$3x \equiv 347 \pmod{453}.$$

Sol:

$$\boxed{ax \equiv b \bmod m \text{ has a solution iff } (a, m) \mid b}$$

$a = 3$, $b = 347$, $m = 453$

$(a, m) = 3$

$3 \nmid 347$

$\therefore$ No such x exists.

Q: Find the remainder of 7^{2012} upon division by 2011

Sol:

2011 is prime.

Fermat's Theorem, $7^{2010} \bmod 2011 = 1$

$$7^{2012} = \underline{\underline{7^{2010}}} * \underline{\underline{7^{2}}} \bmod 2011 = 49$$

Q: Find last two decimal digits of 413^{402}.

Hint: The last two decimal digits of a positive integer n are given by the least non-negative residue of $n \bmod 100$.

Sol:

$413 \equiv 13 \mod 100$

$13^{402} \mod \underline{100}$

$13^{\phi(100)} \equiv 1 \mod 100$

$13^{402} = (13^{40})^{10} \times 13^{2} \mod 100$

$= 69$

$100 = 2^2 \times 5^2$

$\phi(100) = 2^1(2-1) \times 5^1(5-1)$

$= 40$

$402 = 40 * 10 + 2$

Q: Find all integer solutions (x, y) of the equation $13x + 11y = 7$.

Hint: first find solution for $13x + 11y = 1$
then generalize.

Sol: By Euclidean algorithm, we will get $x = -5$, $y = 6$.

that $(x_0, y_0) = (-5, 6)$ for $13x + 11y = 1$.

$(x_1, y_1) = 7(x_0, y_0) = (-35, 42)$ is solution for $13x + 11y = 7$.

$$\boxed{(x, y) = (-35 + 11k,\ 42 - 13k),\ k \in \mathbb{Z}}$$

Q: prove that, for all integers $n \geq 2$, the number $n^{40}+1$ is Composite.

and find a non-trivial divisors of this number.

Hint: try modulo n^8+1, $40 = 8 \times 5$

$$n^8 \equiv -1 \bmod (n^8+1)$$

$$(n^8)^5 \equiv (-1)^5 \bmod (n^8+1)$$

$$n^{40} \equiv -1 \bmod (n^8+1)$$

$$n^{40}+1 \equiv 0 \bmod \underline{(n^8+1)}$$

Q: prove that, for any integer n, the number $n^{13}-n$ is divisible by 35.

fill ??

$$\boxed{\begin{array}{l} a^{p-1} \equiv 1 \bmod p, \\ p \nmid a \end{array}}$$

$35 = 7 \times 5$

$p=5$, $\left.\begin{array}{l} 5 \nmid n \\ 5 \mid n \end{array}\right\}$

$p=7$, $\left.\begin{array}{l} 7 \nmid n \\ 7 \mid n \end{array}\right\}$