CS335 Introduction to AI

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One Sherlock Holmes

Another Sherlock

BBC One's Sherlock (2010-) with Benedict Cumberbach and Martin Freeman.

- Whorf (1956) suggest that communities determine lang. categories
- Wanner (1975) subject remember the **content** of what they read better than the actual words.
- Mitchell et al. (2008) could predict –with above chance accuracy– the areas of the brain that would activate with certain words(fMRI)

- Objects (cat, dog, house, John, etc.)
- Relations (has color, bigger than, comes between, etc.)
- Facts: (One value for a given input: has father, has head, can swim)

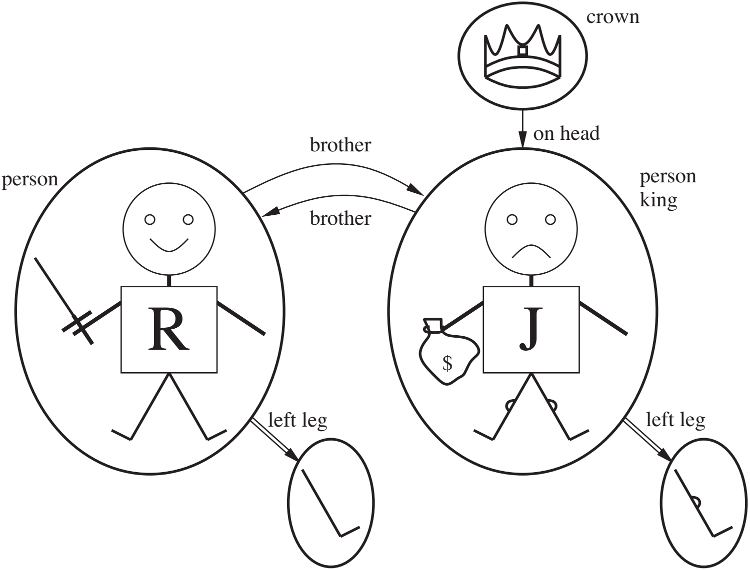
Facts have a truth value. *true or* false

Language	Ontological Commitment[1]	Epistemological Commitment[2]
Propositional Logic	facts	true/false/unknown
First-Order Logic	facts, objects, relations	true/false/unknown
Temporal Logic	facts,objects, relations, time	true/false/unknown
Probability Theory	facts	degree of belief $\in [0, 1]$
Fuzzy Logic	facts with degree of truth	known interval value

[1] What exists in the world

[2] Agent's beliefs about facts

Relationships
Models for first order logic



First-Order-Logic
Syntax

$$
\begin{array}{rcl}
\textit{Sentence} & \rightarrow & \textit{AtomicSentence} | \textit{ComplexSentence} \\
\textit{AtomicSentence} & \rightarrow & \textit{Predicate} | \textit{Predicate}(\textit{Term}, \ldots) | \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} & \rightarrow & (\textit{Sentence}) | [\textit{Sentence}] \\
 & | & \neg\ \textit{Sentence} \\
 & | & \textit{Sentence} \wedge \textit{Sentence} \\
 & | & \textit{Sentence} \vee \textit{Sentence} \\
 & | & \textit{Sentence} \Rightarrow \textit{Sentence} \\
 & | & \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
 & | & \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\textit{Term} & \rightarrow & \textit{Function}(\textit{Term}, \ldots) \\
 & | & \textit{Constant} \\
 & | & \textit{Variable} \\
\textit{Quantifier} & \rightarrow & \forall | \exists \\
\textit{Constant} & \rightarrow & A | X_1 | \textit{John} | \ldots \\
\textit{Variable} & \rightarrow & a | x | s | \ldots \\
\textit{Predicate} & \rightarrow & \textit{True} | \textit{False} | \textit{After} | \textit{Loves} | \textit{Raining} | \ldots \\
\textit{Function} & \rightarrow & \textit{Mother} | \textit{Left\ leg} | \ldots \\
\text{Operator Precedence}^3 & : & \neg, \wedge, \vee, \Rightarrow, \leftrightarrow
\end{array}
$$

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[3] Otherwise the grammar is ambiguous

- Three kinds of symbols
  - Constant: objects
  - Predicate: relations
  - Function: funcions (i.e. can return values other than truth vals.)
- Predicate and Function have **arity.**
- Symbols have an interpretation
- Terms: $LeftLeg(John)$
- Atomic Sentences state facts: $Brother(Richard, John)$
- Complex Sentence: $Brother(R, J) \wedge Brother(J, R)$ or $\neg King(Richard) \Rightarrow King(John)$
- Universal Quantifiers: $\forall King(x) \Rightarrow Person(x)$
- Existential Quantifiers: $\exists Crown(x) \wedge OnHead(x, John)$

What is the interpretation for:

- $King(Richard) \lor King(John)$
- $\neg Brother(LeftLeg(Richard), John)$
- $\forall x \forall y Brother(x, y) \Rightarrow Sibling(x, y)$
- $In(Paris, France) \land In(Marseilles, France)$
- $\forall c\ Country(c) \land Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$
- $\exists Country(c) \land Border(c, Spain) \land Border(c, Italy)$

First Order Logic
More Facts

- Richard has only two brothers, John and Geoffrey:

$$Brother(John, Richard) \wedge Brother(Geoffrey, Richard) \wedge John \neq Geoffrey \wedge$$
$$\forall x Brother(x, Richard) \Rightarrow (x = John \vee x = Geoffrey)$$

- No Region in South America borders any region in Europe

$$\forall c, d \; In(c, SouthAmerica) \wedge In(d, Europe) \Rightarrow \neg Border(c, d)$$

- No two adjacent countries have the same map color

$$\forall x, y \; Country(x) \wedge Country(y) \wedge Border(x, y) \Rightarrow$$
$$\neg(Color(x) = Color(y)) \wedge \neg(x = y)$$

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- $TELL(KB, King(John))$
- $TELL(KB, \forall x\ King(x) \Rightarrow Person(x))$

- $ASK(KB, King(John))$ return True
- $ASK(KB, \exists x\ Person(x))$ return True

- $ASKVARS(KB, Person(x))$ yields $\{x/John, x/Richard\}$, a binding list

"The son of my father is my brother"; "One's grandmother is the mother of one's parent"; etc.

- Domain: People.
- Unary predicates: *Male*, *Female*
- Relations:
  Parent, *Sibling*, *Brother*, *Sister*, *Child*, *Daughter*, *Son*, *Spouse*, *Wife*, *Husband*, *Grandparent*, *Grandchild*, *Cousin*, *Aunt*, *Uncle*
- Functions: *Mother*, *Father*

"One's mother is one's female parent"
$\forall m, c\ Mother(c) = m \leftrightarrow Female(m) \wedge Parent(m, c)$

"A sibling is another child of one's parents"
$\forall x, y\ Sibling(x, y) \leftrightarrow x \neq y \wedge \exists p\ Parent(p, x) \wedge Parent(p, y)$

"Wendy is female"

$Female(wendy)$

These are **axioms**

"One's mother is one's female parent"
$\forall m, c \; Mother(c) = m \leftrightarrow Female(m) \land Parent(m, c)$

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These are **axioms**

Can be represented more concisely

- at time step 3:$Percept([Stench, Breeze, Glitter, None, None], 3)$
- at time step 6:$Percept([None, Breeze, None, None, Scream], 6)$
- Actions can be: $Turn(Right), Turn(Left), Forward, Shoot, Grab, Climb$

And we can *ASK* the best action at time step 5
$ASKVARS(\exists a\ BestAction(a, 5))$

Can encode:

- Raw percepts:
  $\forall t, s, g, m, c \; Percept([s, b, Glitter, m, c], t) \Rightarrow Glitter(t)$
- Reflex actions: $\forall t \; Glitter(t) \Rightarrow BestAction(Grab, t)$

Instead of encoding stuff like:

$Adjacent(Square_{1,2}, Square_{1,1})$
$Adjacent(Square_{3,4}, Square_{4,4})$

Encode:

$$\forall x, y, a, b \; Adjacent([x, y], [a, b]) \Leftrightarrow$$

$$(x = a \wedge (y = b-1 \vee y = b+1)) \vee (y = b \wedge (x = a-1 \vee x = a+1))$$

- Define pieces: $Long(p), Short(p), etc.$
- and restrictions: $\forall p\ Long(p) \leftrightarrow \neg(Long(p) \wedge Short(p))$
- Define rules: $\forall x, y, z\ CanConnectWithOverlap(x, y, z) \leftrightarrow x \neq y \wedge Piece(x) \wedge Piece(y) \wedge Number(z) \wedge Value(z) \leq Overlap(x, y)$...
- $\forall x, y\ Short(x) \wedge Short(y) \wedge Overlap(x, y) < 1 \Rightarrow WeakLink(x, y)$

Prolog
Example

Alain Colmerauer (1972)

Download SWI Prolog

And a quick tutorial

Creating a Knowledge Base

- Identify the Task
- Assemble the relevant knowledge
- Decide on a vocabulary of predicates, functions and constants
- Encode general knowledge about the domain (rules)
- Encode a description of the problem
- Pose queries to the inference procedure and get answers
- Debug the KB

Given $\forall x \; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$

One can infer

- $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
- $King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$
- $King(Father(John)) \wedge Greedy(Father(John)) \Rightarrow Evil(Father(John))$

- Universal Instantiation (in a $\forall$ rule, substitute all symbols)
- Existential Instantiation (in a $\exists$ rule, substitute one symbol, use the rule and discard)
- Skolem Constants is a new variable that represents a new inference.

Suppose KB:

- $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
- $King(John)$
- $Greedy(John)$
- $Brother(Richard, John)$

Apply UI using $\{x/John\} and \{x/Richard\}$

- $King(John) \wedge Greedy(John) \Rightarrow Evil(John)$
- $King(Richard) \wedge Greedy(Richard) \Rightarrow Evil(Richard)$

And discard the Universally quantified sentence. We can get the KB to be propositions.

Suppose KB:

- $\forall x\ King(x) \wedge Greedy(x) \Rightarrow Evil(x)$
- $King(John)$
- $\forall y\ Greedy(y)$

Apply UI using $\{x/John\} and \{y/John\}$

for atomic sentences $p_i, p_i^{'}$ and q, where there is a substitution θ such that $SUBST(\theta, p_i^{'}) = SUBST(\theta, p_i)$, for all i

$$\frac{p_1^{'}, p_2^{'}, \ldots, p_n^{'}, (p_1 \wedge p_2 \wedge \ldots \wedge p_n \Rightarrow q)}{SUBST(\theta, q)}$$

$$\begin{array}{ll} p_1^{'} = King(John) & p_1 = King(x) \\ p_2^{'} = Greedy(y) & p_2 = Greedy(x) \\ \theta = \{x/John, y/John\} & q = Evil(x) \\ SUBST(\theta, q) & . \end{array}$$

$UNIFY(p, q) = \theta$ Where $SUBST(\theta, p) = SUBST(\theta, q)$

For example:

- We ask $ASKVARS(Knows(John, x))$ (Whom does John know?)
- $UNIFY(Knows(John, x), Knows(John, Jane)) = \{x/Jane\}$
- $UNIFY(Knows(John, x), Knows(y, Bill)) = \{y/John, x/Bill\}$
- $UNIFY(Knows(John, x), Knows(y, Mother(y))) = \{y/John, x/Mother(John)\}$

Algorithm in the book (goes variable by variable recursively unifying)

Inference
Putting it all together

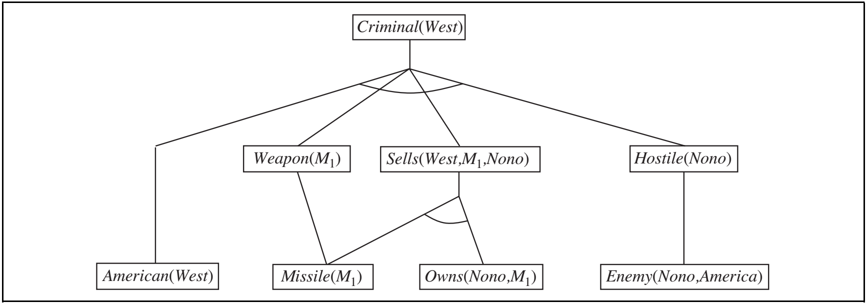
"The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles , and all of its missiles were sold to it by Colonel West, who is American"

Prove that Colonel Wes is a Criminal

"The Law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, and enemy of America, has some missiles , and all of its missiles were sold to it by Colonel West, who is American"

- R1: $American(x) \wedge Weapon(y) \wedge Sells(x, y.z) \wedge Hostile(z) \Rightarrow Criminal(x)$
- R2: $Owns(Nono, M_1)$ Nono has some missiles
- R3: $Missile(M_1)$
- R4: $Missile(x) \Rightarrow Weapon(x)$ A missile is a weapon
- R5: $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$ All missiles sold by west
- R6: $Enemy(x, America) \Rightarrow Hostile(x)$ Enemies of America are hostile
- R7: $American(West)$ West is american
- R8: $Enemy(Nono, America)$

Inference
Graph



Iteration 1:

- R5 satisfied with $\{x/M_1\}$ and R9: $Sells(West, M_1, Nono)$ is added
- R4 satisfied with $\{x/M_1\}$ and R10: $Weapon(M_1)$ is added
- R6 satisfied with $\{x/Nono\}$ and R11: $Hostile(Nono)$ is added

Iteration 2:

- R1 is satisfied with $\{x/West, y/M_1, z/Nono\}$ and $Criminal(West)$ is added.

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- Once we have facts that evaluate to T or F
- We can apply Forward Chaining, Backwards Chaining and Resolution
- The key is to understand Unification
- Very similar to Logical agents.