

ToC 2021S -- Mid Exam -- 20 Marks

1. (i) Find the DFA for binary integers divisible by 2 having exactly two states q_0 for remainder 0 and q_1 for remainder 1. (ii) Similarly find the DFA for binary integers divisible by 3 having exactly 3 states (notation used to label the states should be p_0, p_1, p_2). (iii) Now create the product machine which can recognize binary integers divisible by 2 and 3. Give the 5 tuple representation of the DFA in which the states are represented like (q_i, p_j) . (1+1+3 = 5 Marks)

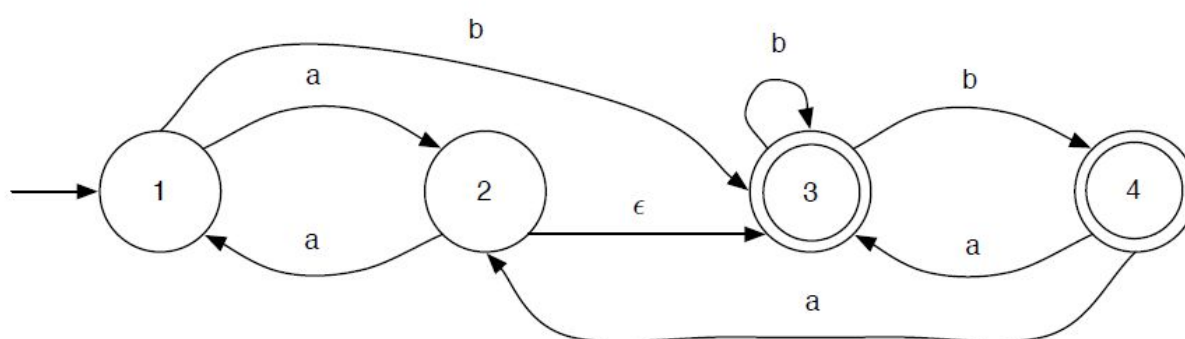
2 a. Let $L = \{a^k w \mid w \in \{a, b\}^*, |w| = k\}$

Use Pumping lemma for regular languages to prove that L is not regular. (4 marks)

b. Give regular expressions for the following language over $\Sigma = \{0,1\}$

$L = \{w \in \{0, 1\}^* \mid w \text{ does not end with } 00\}$ (1 mark)

3. Using state elimination convert the following Finite Automata to Regular Expression. (5 marks)



4. (a) Prove that 9 divides $n^3 + (n+1)^3 + (n+2)^3$, for $n \geq 0$. Use proof by induction.

(b) Consider the following grammar and give leftmost and rightmost derivation for the strings (i) abbbbaa, (ii) abbaabbaba. (2+3 = 5 Marks)

$S \rightarrow aB \mid bA$

$B \rightarrow b \mid bS \mid aBB$

$A \rightarrow a \mid aS \mid bAA$