

natural language understanding and multiagent systems.

Horvitz *et al.* (1988) specifically suggest the use of rationality conceived as the maximization of expected utility as a basis for AI. The text by Pearl (1988) was the first in AI to cover probability and utility theory in depth; its exposition of practical methods for reasoning and decision making under uncertainty was probably the single biggest factor in the rapid shift towards utility-based agents in the 1990s (see Part IV).

The general design for learning agents portrayed in Figure 2.15 is classic in the machine learning literature (Buchanan *et al.*, 1978; Mitchell, 1997). Examples of the design, as embodied in programs, go back at least as far as Arthur Samuel's (1959, 1967) learning program for playing checkers. Learning agents are discussed in depth in Part V.

Interest in agents and in agent design has risen rapidly in recent years, partly because of the growth of the Internet and the perceived need for automated and mobile **softbot** (Etzioni and Weld, 1994). Relevant papers are collected in *Readings in Agents* (Huhns and Singh, 1998) and *Foundations of Rational Agency* (Wooldridge and Rao, 1999). Texts on multiagent systems usually provide a good introduction to many aspects of agent design (Weiss, 2000a; Wooldridge, 2002). Several conference series devoted to agents began in the 1990s, including the International Workshop on Agent Theories, Architectures, and Languages (ATAL), the International Conference on Autonomous Agents (AGENTS), and the International Conference on Multi-Agent Systems (ICMAS). In 2002, these three merged to form the International Joint Conference on Autonomous Agents and Multi-Agent Systems (AAMAS). The journal *Autonomous Agents and Multi-Agent Systems* was founded in 1998. Finally, *Dung Beetle Ecology* (Hanski and Cambefort, 1991) provides a wealth of interesting information on the behavior of dung beetles. YouTube features inspiring video recordings of their activities.

EXERCISES

- 2.1** Suppose that the performance measure is concerned with just the first T time steps of the environment and ignores everything thereafter. Show that a rational agent's action may depend not just on the state of the environment but also on the time step it has reached.
- 2.2** Let us examine the rationality of various vacuum-cleaner agent functions.
- Show that the simple vacuum-cleaner agent function described in Figure 2.3 is indeed rational under the assumptions listed on page 38.
 - Describe a rational agent function for the case in which each movement costs one point. Does the corresponding agent program require internal state?
 - Discuss possible agent designs for the cases in which clean squares can become dirty and the geography of the environment is unknown. Does it make sense for the agent to learn from its experience in these cases? If so, what should it learn? If not, why not?
- 2.3** For each of the following assertions, say whether it is true or false and support your answer with examples or counterexamples where appropriate.
- An agent that senses only partial information about the state cannot be perfectly rational.

- b. There exist task environments in which no pure reflex agent can behave rationally.
- c. There exists a task environment in which every agent is rational.
- d. The input to an agent program is the same as the input to the agent function.
- e. Every agent function is implementable by some program/machine combination.
- f. Suppose an agent selects its action uniformly at random from the set of possible actions. There exists a deterministic task environment in which this agent is rational.
- g. It is possible for a given agent to be perfectly rational in two distinct task environments.
- h. Every agent is rational in an unobservable environment.
- i. A perfectly rational poker-playing agent never loses.

2.4 For each of the following activities, give a PEAS description of the task environment and characterize it in terms of the properties listed in Section 2.3.2.

- Playing soccer.
- Exploring the subsurface oceans of Titan.
- Shopping for used AI books on the Internet.
- Playing a tennis match.
- Practicing tennis against a wall.
- Performing a high jump.
- Knitting a sweater.
- Bidding on an item at an auction.

2.5 Define in your own words the following terms: agent, agent function, agent program, rationality, autonomy, reflex agent, model-based agent, goal-based agent, utility-based agent, learning agent.

2.6 This exercise explores the differences between agent functions and agent programs.

- a. Can there be more than one agent program that implements a given agent function? Give an example, or show why one is not possible.
- b. Are there agent functions that cannot be implemented by any agent program?
- c. Given a fixed machine architecture, does each agent program implement exactly one agent function?
- d. Given an architecture with n bits of storage, how many different possible agent programs are there?
- e. Suppose we keep the agent program fixed but speed up the machine by a factor of two. Does that change the agent function?

2.7 Write pseudocode agent programs for the goal-based and utility-based agents.



The following exercises all concern the implementation of environments and agents for the vacuum-cleaner world.

2.8 Implement a performance-measuring environment simulator for the vacuum-cleaner world depicted in Figure 2.2 and specified on page 38. Your implementation should be modular so that the sensors, actuators, and environment characteristics (size, shape, dirt placement, etc.) can be changed easily. (*Note:* for some choices of programming language and operating system there are already implementations in the online code repository.)

2.9 Implement a simple reflex agent for the vacuum environment in Exercise 2.8. Run the environment with this agent for all possible initial dirt configurations and agent locations. Record the performance score for each configuration and the overall average score.

2.10 Consider a modified version of the vacuum environment in Exercise 2.8, in which the agent is penalized one point for each movement.

- a. Can a simple reflex agent be perfectly rational for this environment? Explain.
- b. What about a reflex agent with state? Design such an agent.
- c. How do your answers to **a** and **b** change if the agent's percepts give it the clean/dirty status of every square in the environment?

2.11 Consider a modified version of the vacuum environment in Exercise 2.8, in which the geography of the environment—its extent, boundaries, and obstacles—is unknown, as is the initial dirt configuration. (The agent can go *Up* and *Down* as well as *Left* and *Right*.)

- a. Can a simple reflex agent be perfectly rational for this environment? Explain.
- b. Can a simple reflex agent with a *randomized* agent function outperform a simple reflex agent? Design such an agent and measure its performance on several environments.
- c. Can you design an environment in which your randomized agent will perform poorly? Show your results.
- d. Can a reflex agent with state outperform a simple reflex agent? Design such an agent and measure its performance on several environments. Can you design a rational agent of this type?

2.12 Repeat Exercise 2.11 for the case in which the location sensor is replaced with a “bump” sensor that detects the agent's attempts to move into an obstacle or to cross the boundaries of the environment. Suppose the bump sensor stops working; how should the agent behave?

2.13 The vacuum environments in the preceding exercises have all been deterministic. Discuss possible agent programs for each of the following stochastic versions:

- a. Murphy's law: twenty-five percent of the time, the *Suck* action fails to clean the floor if it is dirty and deposits dirt onto the floor if the floor is clean. How is your agent program affected if the dirt sensor gives the wrong answer 10% of the time?
- b. Small children: At each time step, each clean square has a 10% chance of becoming dirty. Can you come up with a rational agent design for this case?

order even with an inadmissible heuristic. The idea of keeping track of the best alternative path appeared earlier in Bratko's (1986) elegant Prolog implementation of A^* and in the DTA* algorithm (Russell and Wefald, 1991). The latter work also discusses metalevel state spaces and metalevel learning.

The MA* algorithm appeared in Chakrabarti *et al.* (1989). SMA*, or Simplified MA*, emerged from an attempt to implement MA* as a comparison algorithm for IE (Russell, 1992). Kaindl and Khorsand (1994) have applied SMA* to produce a bidirectional search algorithm that is substantially faster than previous algorithms. Korf and Zhang (2000) describe a divide-and-conquer approach, and Zhou and Hansen (2002) introduce memory-bounded A^* graph search and a strategy for switching to breadth-first search to increase memory-efficiency (Zhou and Hansen, 2006). Korf (1995) surveys memory-bounded search techniques.

The idea that admissible heuristics can be derived by problem relaxation appears in the seminal paper by Held and Karp (1970), who used the minimum-spanning-tree heuristic to solve the TSP. (See Exercise 3.30.)

The automation of the relaxation process was implemented successfully by Prieditis (1993), building on earlier work with Mostow (Mostow and Prieditis, 1989). Holte and Hernadvolgyi (2001) describe more recent steps towards automating the process. The use of pattern databases to derive admissible heuristics is due to Gasser (1995) and Culberson and Schaeffer (1996, 1998); disjoint pattern databases are described by Korf and Felner (2002); a similar method using symbolic patterns is due to Edelkamp (2009). Felner *et al.* (2007) show how to compress pattern databases to save space. The probabilistic interpretation of heuristics was investigated in depth by Pearl (1984) and Hansson and Mayer (1989).

By far the most comprehensive source on heuristics and heuristic search algorithms is Pearl's (1984) *Heuristics* text. This book provides especially good coverage of the wide variety of offshoots and variations of A^* , including rigorous proofs of their formal properties. Kanal and Kumar (1988) present an anthology of important articles on heuristic search, and Rayward-Smith *et al.* (1996) cover approaches from Operations Research. Papers about new search algorithms—which, remarkably, continue to be discovered—appear in journals such as *Artificial Intelligence* and *Journal of the ACM*.

PARALLEL SEARCH

The topic of **parallel search** algorithms was not covered in the chapter, partly because it requires a lengthy discussion of parallel computer architectures. Parallel search became a popular topic in the 1990s in both AI and theoretical computer science (Mahanti and Daniels, 1993; Grama and Kumar, 1995; Crauser *et al.*, 1998) and is making a comeback in the era of new multicore and cluster architectures (Ralphs *et al.*, 2004; Korf and Schultze, 2005). Also of increasing importance are search algorithms for very large graphs that require disk storage (Korf, 2008).

EXERCISES

- 3.1 Explain why problem formulation must follow goal formulation.
- 3.2 Your goal is to navigate a robot out of a maze. The robot starts in the center of the maze

facing north. You can turn the robot to face north, east, south, or west. You can direct the robot to move forward a certain distance, although it will stop before hitting a wall.

- a. Formulate this problem. How large is the state space?
- b. In navigating a maze, the only place we need to turn is at the intersection of two or more corridors. Reformulate this problem using this observation. How large is the state space now?
- c. From each point in the maze, we can move in any of the four directions until we reach a turning point, and this is the only action we need to do. Reformulate the problem using these actions. Do we need to keep track of the robot's orientation now?
- d. In our initial description of the problem we already abstracted from the real world, restricting actions and removing details. List three such simplifications we made.

3.3 Suppose two friends live in different cities on a map, such as the Romania map shown in Figure 3.2. On every turn, we can simultaneously move each friend to a neighboring city on the map. The amount of time needed to move from city i to neighbor j is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible.

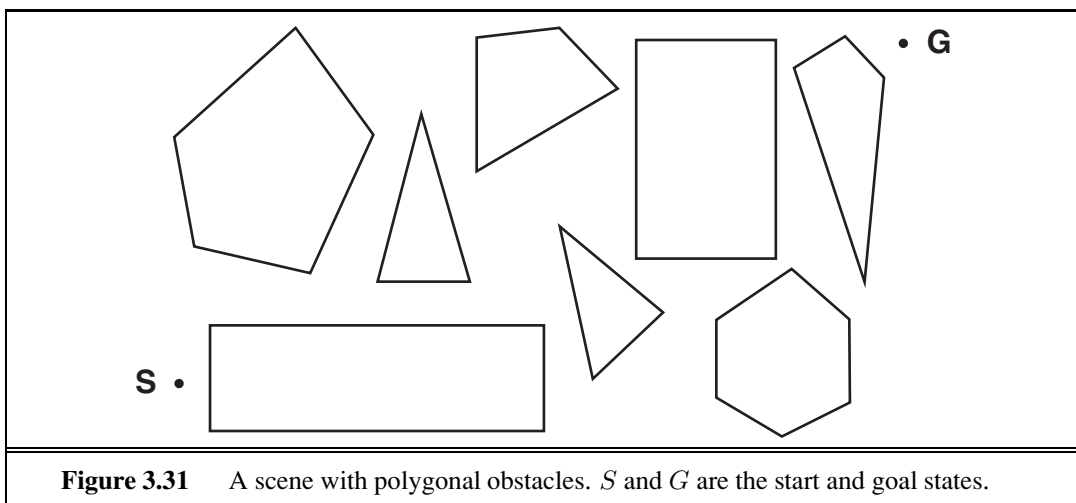
- a. Write a detailed formulation for this search problem. (You will find it helpful to define some formal notation here.)
- b. Let $D(i, j)$ be the straight-line distance between cities i and j . Which of the following heuristic functions are admissible? (i) $D(i, j)$; (ii) $2 \cdot D(i, j)$; (iii) $D(i, j)/2$.
- c. Are there completely connected maps for which no solution exists?
- d. Are there maps in which all solutions require one friend to visit the same city twice?

3.4 Show that the 8-puzzle states are divided into two disjoint sets, such that any state is reachable from any other state in the same set, while no state is reachable from any state in the other set. (*Hint*: See Berlekamp *et al.* (1982).) Devise a procedure to decide which set a given state is in, and explain why this is useful for generating random states.

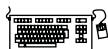
3.5 Consider the n -queens problem using the “efficient” incremental formulation given on page 72. Explain why the state space has at least $\sqrt[3]{n!}$ states and estimate the largest n for which exhaustive exploration is feasible. (*Hint*: Derive a lower bound on the branching factor by considering the maximum number of squares that a queen can attack in any column.)

3.6 Give a complete problem formulation for each of the following. Choose a formulation that is precise enough to be implemented.

- a. Using only four colors, you have to color a planar map in such a way that no two adjacent regions have the same color.
- b. A 3-foot-tall monkey is in a room where some bananas are suspended from the 8-foot ceiling. He would like to get the bananas. The room contains two stackable, movable, climbable 3-foot-high crates.



- c. You have a program that outputs the message “illegal input record” when fed a certain file of input records. You know that processing of each record is independent of the other records. You want to discover what record is illegal.
- d. You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet. You can fill the jugs up or empty them out from one to another or onto the ground. You need to measure out exactly one gallon.



3.7 Consider the problem of finding the shortest path between two points on a plane that has convex polygonal obstacles as shown in Figure 3.31. This is an idealization of the problem that a robot has to solve to navigate in a crowded environment.

- a. Suppose the state space consists of all positions (x, y) in the plane. How many states are there? How many paths are there to the goal?
- b. Explain briefly why the shortest path from one polygon vertex to any other in the scene must consist of straight-line segments joining some of the vertices of the polygons. Define a good state space now. How large is this state space?
- c. Define the necessary functions to implement the search problem, including an ACTIONS function that takes a vertex as input and returns a set of vectors, each of which maps the current vertex to one of the vertices that can be reached in a straight line. (Do not forget the neighbors on the same polygon.) Use the straight-line distance for the heuristic function.
- d. Apply one or more of the algorithms in this chapter to solve a range of problems in the domain, and comment on their performance.

3.8 On page 68, we said that we would not consider problems with negative path costs. In this exercise, we explore this decision in more depth.

- a. Suppose that actions can have arbitrarily large negative costs; explain why this possibility would force any optimal algorithm to explore the entire state space.

- b. Does it help if we insist that step costs must be greater than or equal to some negative constant c ? Consider both trees and graphs.
- c. Suppose that a set of actions forms a loop in the state space such that executing the set in some order results in no net change to the state. If all of these actions have negative cost, what does this imply about the optimal behavior for an agent in such an environment?
- d. One can easily imagine actions with high negative cost, even in domains such as route finding. For example, some stretches of road might have such beautiful scenery as to far outweigh the normal costs in terms of time and fuel. Explain, in precise terms, within the context of state-space search, why humans do not drive around scenic loops indefinitely, and explain how to define the state space and actions for route finding so that artificial agents can also avoid looping.
- e. Can you think of a real domain in which step costs are such as to cause looping?



3.9 The **missionaries and cannibals** problem is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place. This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint (Amarel, 1968).

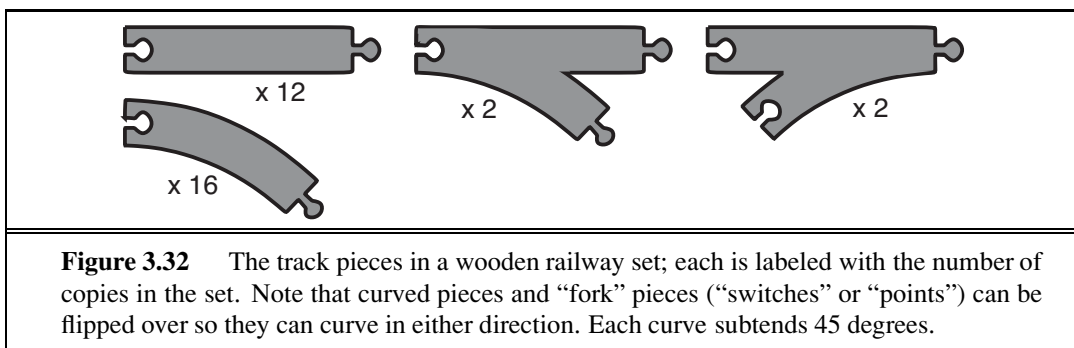
- a. Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.
- b. Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?
- c. Why do you think people have a hard time solving this puzzle, given that the state space is so simple?

3.10 Define in your own words the following terms: state, state space, search tree, search node, goal, action, transition model, and branching factor.

3.11 What's the difference between a world state, a state description, and a search node? Why is this distinction useful?

3.12 An action such as *Go(Sibiu)* really consists of a long sequence of finer-grained actions: turn on the car, release the brake, accelerate forward, etc. Having composite actions of this kind reduces the number of steps in a solution sequence, thereby reducing the search time. Suppose we take this to the logical extreme, by making super-composite actions out of every possible sequence of *Go* actions. Then every problem instance is solved by a single super-composite action, such as *Go(Sibiu)Go(Rimnicu Vilcea)Go(Pitesti)Go(Bucharest)*. Explain how search would work in this formulation. Is this a practical approach for speeding up problem solving?

3.13 Prove that GRAPH-SEARCH satisfies the graph separation property illustrated in Figure 3.9. (*Hint*: Begin by showing that the property holds at the start, then show that if it holds before an iteration of the algorithm, it holds afterwards.) Describe a search algorithm that violates the property.



3.14 Which of the following are true and which are false? Explain your answers.

- Depth-first search always expands at least as many nodes as A^* search with an admissible heuristic.
- $h(n) = 0$ is an admissible heuristic for the 8-puzzle.
- A^* is of no use in robotics because percepts, states, and actions are continuous.
- Breadth-first search is complete even if zero step costs are allowed.
- Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

3.15 Consider a state space where the start state is number 1 and each state k has two successors: numbers $2k$ and $2k + 1$.

- Draw the portion of the state space for states 1 to 15.
- Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search with limit 3, and iterative deepening search.
- How well would bidirectional search work on this problem? What is the branching factor in each direction of the bidirectional search?
- Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?
- Call the action going from k to $2k$ Left, and the action going to $2k + 1$ Right. Can you find an algorithm that outputs the solution to this problem without any search at all?

3.16 A basic wooden railway set contains the pieces shown in Figure 3.32. The task is to connect these pieces into a railway that has no overlapping tracks and no loose ends where a train could run off onto the floor.

- Suppose that the pieces fit together *exactly* with no slack. Give a precise formulation of the task as a search problem.
- Identify a suitable uninformed search algorithm for this task and explain your choice.
- Explain why removing any one of the “fork” pieces makes the problem unsolvable.

- d. Give an upper bound on the total size of the state space defined by your formulation. (*Hint*: think about the maximum branching factor for the construction process and the maximum depth, ignoring the problem of overlapping pieces and loose ends. Begin by pretending that every piece is unique.)



3.17 On page 90, we mentioned **iterative lengthening search**, an iterative analog of uniform cost search. The idea is to use increasing limits on path cost. If a node is generated whose path cost exceeds the current limit, it is immediately discarded. For each new iteration, the limit is set to the lowest path cost of any node discarded in the previous iteration.

- Show that this algorithm is optimal for general path costs.
- Consider a uniform tree with branching factor b , solution depth d , and unit step costs. How many iterations will iterative lengthening require?
- Now consider step costs drawn from the continuous range $[\epsilon, 1]$, where $0 < \epsilon < 1$. How many iterations are required in the worst case?
- Implement the algorithm and apply it to instances of the 8-puzzle and traveling salesperson problems. Compare the algorithm's performance to that of uniform-cost search, and comment on your results.

3.18 Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, $O(n^2)$ vs. $O(n)$).



3.19 Write a program that will take as input two Web page URLs and find a path of links from one to the other. What is an appropriate search strategy? Is bidirectional search a good idea? Could a search engine be used to implement a predecessor function?

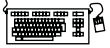


3.20 Consider the vacuum-world problem defined in Figure 2.2.

- Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?
- Apply your chosen algorithm to compute an optimal sequence of actions for a 3×3 world whose initial state has dirt in the three top squares and the agent in the center.
- Construct a search agent for the vacuum world, and evaluate its performance in a set of 3×3 worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.
- Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.
- Consider what would happen if the world were enlarged to $n \times n$. How does the performance of the search agent and of the reflex agent vary with n ?

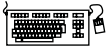
3.21 Prove each of the following statements, or give a counterexample:

- Breadth-first search is a special case of uniform-cost search.
- Depth-first search is a special case of best-first tree search.
- Uniform-cost search is a special case of A^* search.



3.22 Compare the performance of A^* and RBFS on a set of randomly generated problems in the 8-puzzle (with Manhattan distance) and TSP (with MST—see Exercise 3.30) domains. Discuss your results. What happens to the performance of RBFS when a small random number is added to the heuristic values in the 8-puzzle domain?

3.23 Trace the operation of A^* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic. That is, show the sequence of nodes that the algorithm will consider and the f , g , and h score for each node.



3.24 Devise a state space in which A^* using GRAPH-SEARCH returns a suboptimal solution with an $h(n)$ function that is admissible but inconsistent.

HEURISTIC PATH
ALGORITHM

3.25 The **heuristic path algorithm** (Pohl, 1977) is a best-first search in which the evaluation function is $f(n) = (2 - w)g(n) + wh(n)$. For what values of w is this complete? For what values is it optimal, assuming that h is admissible? What kind of search does this perform for $w = 0$, $w = 1$, and $w = 2$?

3.26 Consider the unbounded version of the regular 2D grid shown in Figure 3.9. The start state is at the origin, $(0,0)$, and the goal state is at (x, y) .

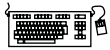
- What is the branching factor b in this state space?
- How many distinct states are there at depth k (for $k > 0$)?
- What is the maximum number of nodes expanded by breadth-first tree search?
- What is the maximum number of nodes expanded by breadth-first graph search?
- Is $h = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v) ? Explain.
- How many nodes are expanded by A^* graph search using h ?
- Does h remain admissible if some links are removed?
- Does h remain admissible if some links are added between nonadjacent states?

3.27 n vehicles occupy squares $(1, 1)$ through $(n, 1)$ (i.e., the bottom row) of an $n \times n$ grid. The vehicles must be moved to the top row but in reverse order; so the vehicle i that starts in $(i, 1)$ must end up in $(n - i + 1, n)$. On each time step, every one of the n vehicles can move one square up, down, left, or right, or stay put; but if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

- Calculate the size of the state space as a function of n .
- Calculate the branching factor as a function of n .
- Suppose that vehicle i is at (x_i, y_i) ; write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n - i + 1, n)$, assuming no other vehicles are on the grid.
- Which of the following heuristics are admissible for the problem of moving all n vehicles to their destinations? Explain.
 - $\sum_{i=1}^n h_i$.
 - $\max\{h_1, \dots, h_n\}$.
 - $\min\{h_1, \dots, h_n\}$.

3.28 Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.) Prove that if h never overestimates by more than c , A^* using h returns a solution whose cost exceeds that of the optimal solution by no more than c .

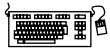
3.29 Prove that if a heuristic is consistent, it must be admissible. Construct an admissible heuristic that is not consistent.



3.30 The traveling salesperson problem (TSP) can be solved with the minimum-spanning-tree (MST) heuristic, which estimates the cost of completing a tour, given that a partial tour has already been constructed. The MST cost of a set of cities is the smallest sum of the link costs of any tree that connects all the cities.

- a. Show how this heuristic can be derived from a relaxed version of the TSP.
- b. Show that the MST heuristic dominates straight-line distance.
- c. Write a problem generator for instances of the TSP where cities are represented by random points in the unit square.
- d. Find an efficient algorithm in the literature for constructing the MST, and use it with A^* graph search to solve instances of the TSP.

3.31 On page 105, we defined the relaxation of the 8-puzzle in which a tile can move from square A to square B if B is blank. The exact solution of this problem defines **Gaschnig's heuristic** (Gaschnig, 1979). Explain why Gaschnig's heuristic is at least as accurate as h_1 (misplaced tiles), and show cases where it is more accurate than both h_1 and h_2 (Manhattan distance). Explain how to calculate Gaschnig's heuristic efficiently.



3.32 We gave two simple heuristics for the 8-puzzle: Manhattan distance and misplaced tiles. Several heuristics in the literature purport to improve on this—see, for example, Nilsson (1971), Mostow and Prieditis (1989), and Hansson *et al.* (1992). Test these claims by implementing the heuristics and comparing the performance of the resulting algorithms.

for belief-state search; these were refined by Bryce *et al.* (2006). The incremental approach to belief-state search, in which solutions are constructed incrementally for subsets of states within each belief state, was studied in the planning literature by Kurien *et al.* (2002); several new incremental algorithms were introduced for nondeterministic, partially observable problems by Russell and Wolfe (2005). Additional references for planning in stochastic, partially observable environments appear in Chapter 17.

EULERIAN GRAPH

Algorithms for exploring unknown state spaces have been of interest for many centuries. Depth-first search in a maze can be implemented by keeping one's left hand on the wall; loops can be avoided by marking each junction. Depth-first search fails with irreversible actions; the more general problem of exploring **Eulerian graphs** (i.e., graphs in which each node has equal numbers of incoming and outgoing edges) was solved by an algorithm due to Hierholzer (1873). The first thorough algorithmic study of the exploration problem for arbitrary graphs was carried out by Deng and Papadimitriou (1990), who developed a completely general algorithm but showed that no bounded competitive ratio is possible for exploring a general graph. Papadimitriou and Yannakakis (1991) examined the question of finding paths to a goal in geometric path-planning environments (where all actions are reversible). They showed that a small competitive ratio is achievable with square obstacles, but with general rectangular obstacles no bounded ratio can be achieved. (See Figure 4.20.)

REAL-TIME SEARCH

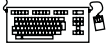
The LRTA* algorithm was developed by Korf (1990) as part of an investigation into **real-time search** for environments in which the agent must act after searching for only a fixed amount of time (a common situation in two-player games). LRTA* is in fact a special case of reinforcement learning algorithms for stochastic environments (Barto *et al.*, 1995). Its policy of optimism under uncertainty—always head for the closest unvisited state—can result in an exploration pattern that is less efficient in the uninformed case than simple depth-first search (Koenig, 2000). Dasgupta *et al.* (1994) show that online iterative deepening search is optimally efficient for finding a goal in a uniform tree with no heuristic information. Several informed variants on the LRTA* theme have been developed with different methods for searching and updating within the known portion of the graph (Pemberton and Korf, 1992). As yet, there is no good understanding of how to find goals with optimal efficiency when using heuristic information.

EXERCISES

4.1 Give the name of the algorithm that results from each of the following special cases:

- a. Local beam search with $k = 1$.
- b. Local beam search with one initial state and no limit on the number of states retained.
- c. Simulated annealing with $T = 0$ at all times (and omitting the termination test).
- d. Simulated annealing with $T = \infty$ at all times.
- e. Genetic algorithm with population size $N = 1$.

4.2 Exercise 3.16 considers the problem of building railway tracks under the assumption that pieces fit exactly with no slack. Now consider the real problem, in which pieces don't fit exactly but allow for up to 10 degrees of rotation to either side of the "proper" alignment. Explain how to formulate the problem so it could be solved by simulated annealing.



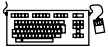
4.3 In this exercise, we explore the use of local search methods to solve TSPs of the type defined in Exercise 3.30.

- a. Implement and test a hill-climbing method to solve TSPs. Compare the results with optimal solutions obtained from the A^* algorithm with the MST heuristic (Exercise 3.30).
- b. Repeat part (a) using a genetic algorithm instead of hill climbing. You may want to consult Larrañaga *et al.* (1999) for some suggestions for representations.



4.4 Generate a large number of 8-puzzle and 8-queens instances and solve them (where possible) by hill climbing (steepest-ascent and first-choice variants), hill climbing with random restart, and simulated annealing. Measure the search cost and percentage of solved problems and graph these against the optimal solution cost. Comment on your results.

4.5 The AND-OR-GRAPH-SEARCH algorithm in Figure 4.11 checks for repeated states only on the path from the root to the current state. Suppose that, in addition, the algorithm were to store *every* visited state and check against that list. (See BREADTH-FIRST-SEARCH in Figure 3.11 for an example.) Determine the information that should be stored and how the algorithm should use that information when a repeated state is found. (*Hint:* You will need to distinguish at least between states for which a successful subplan was constructed previously and states for which no subplan could be found.) Explain how to use labels, as defined in Section 4.3.3, to avoid having multiple copies of subplans.



4.6 Explain precisely how to modify the AND-OR-GRAPH-SEARCH algorithm to generate a cyclic plan if no acyclic plan exists. You will need to deal with three issues: labeling the plan steps so that a cyclic plan can point back to an earlier part of the plan, modifying OR-SEARCH so that it continues to look for acyclic plans after finding a cyclic plan, and augmenting the plan representation to indicate whether a plan is cyclic. Show how your algorithm works on (a) the slippery vacuum world, and (b) the slippery, erratic vacuum world. You might wish to use a computer implementation to check your results.

4.7 In Section 4.4.1 we introduced belief states to solve sensorless search problems. A sequence of actions solves a sensorless problem if it maps every physical state in the initial belief state b to a goal state. Suppose the agent knows $h^*(s)$, the true optimal cost of solving the physical state s in the fully observable problem, for every state s in b . Find an admissible heuristic $h(b)$ for the sensorless problem in terms of these costs, and prove its admissibility. Comment on the accuracy of this heuristic on the sensorless vacuum problem of Figure 4.14. How well does A^* perform?

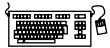
4.8 This exercise explores subset–superset relations between belief states in sensorless or partially observable environments.

- a. Prove that if an action sequence is a solution for a belief state b , it is also a solution for any subset of b . Can anything be said about supersets of b ?

- b. Explain in detail how to modify graph search for sensorless problems to take advantage of your answers in (a).
- c. Explain in detail how to modify AND–OR search for partially observable problems, beyond the modifications you describe in (b).

4.9 On page 139 it was assumed that a given action would have the same cost when executed in any physical state within a given belief state. (This leads to a belief-state search problem with well-defined step costs.) Now consider what happens when the assumption does not hold. Does the notion of optimality still make sense in this context, or does it require modification? Consider also various possible definitions of the “cost” of executing an action in a belief state; for example, we could use the *minimum* of the physical costs; or the *maximum*; or a cost *interval* with the lower bound being the minimum cost and the upper bound being the maximum; or just keep the set of all possible costs for that action. For each of these, explore whether A* (with modifications if necessary) can return optimal solutions.

4.10 Consider the sensorless version of the erratic vacuum world. Draw the belief-state space reachable from the initial belief state $\{1, 2, 3, 4, 5, 6, 7, 8\}$, and explain why the problem is unsolvable.



4.11 We can turn the navigation problem in Exercise 3.7 into an environment as follows:

- The percept will be a list of the positions, *relative to the agent*, of the visible vertices. The percept does *not* include the position of the robot! The robot must learn its own position from the map; for now, you can assume that each location has a different “view.”
 - Each action will be a vector describing a straight-line path to follow. If the path is unobstructed, the action succeeds; otherwise, the robot stops at the point where its path first intersects an obstacle. If the agent returns a zero motion vector and is at the goal (which is fixed and known), then the environment teleports the agent to a *random location* (not inside an obstacle).
 - The performance measure charges the agent 1 point for each unit of distance traversed and awards 1000 points each time the goal is reached.
- a. Implement this environment and a problem-solving agent for it. After each teleportation, the agent will need to formulate a new problem, which will involve discovering its current location.
 - b. Document your agent’s performance (by having the agent generate suitable commentary as it moves around) and report its performance over 100 episodes.
 - c. Modify the environment so that 30% of the time the agent ends up at an unintended destination (chosen randomly from the other visible vertices if any; otherwise, no move at all). This is a crude model of the motion errors of a real robot. Modify the agent so that when such an error is detected, it finds out where it is and then constructs a plan to get back to where it was and resume the old plan. Remember that sometimes getting back to where it was might also fail! Show an example of the agent successfully overcoming two successive motion errors and still reaching the goal.

- d. Now try two different recovery schemes after an error: (1) head for the closest vertex on the original route; and (2) replan a route to the goal from the new location. Compare the performance of the three recovery schemes. Would the inclusion of search costs affect the comparison?
- e. Now suppose that there are locations from which the view is identical. (For example, suppose the world is a grid with square obstacles.) What kind of problem does the agent now face? What do solutions look like?

4.12 Suppose that an agent is in a 3×3 maze environment like the one shown in Figure 4.19. The agent knows that its initial location is (1,1), that the goal is at (3,3), and that the actions *Up*, *Down*, *Left*, *Right* have their usual effects unless blocked by a wall. The agent does *not* know where the internal walls are. In any given state, the agent perceives the set of legal actions; it can also tell whether the state is one it has visited before.

- a. Explain how this online search problem can be viewed as an offline search in belief-state space, where the initial belief state includes all possible environment configurations. How large is the initial belief state? How large is the space of belief states?
- b. How many distinct percepts are possible in the initial state?
- c. Describe the first few branches of a contingency plan for this problem. How large (roughly) is the complete plan?

Notice that this contingency plan is a solution for *every possible environment* fitting the given description. Therefore, interleaving of search and execution is not strictly necessary even in unknown environments.



4.13 In this exercise, we examine hill climbing in the context of robot navigation, using the environment in Figure 3.31 as an example.

- a. Repeat Exercise 4.11 using hill climbing. Does your agent ever get stuck in a local minimum? Is it *possible* for it to get stuck with convex obstacles?
- b. Construct a nonconvex polygonal environment in which the agent gets stuck.
- c. Modify the hill-climbing algorithm so that, instead of doing a depth-1 search to decide where to go next, it does a depth- k search. It should find the best k -step path and do one step along it, and then repeat the process.
- d. Is there some k for which the new algorithm is guaranteed to escape from local minima?
- e. Explain how LRTA* enables the agent to escape from local minima in this case.

4.14 Like DFS, online DFS is incomplete for reversible state spaces with infinite paths. For example, suppose that states are points on the infinite two-dimensional grid and actions are unit vectors $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$, tried in that order. Show that online DFS starting at $(0, 0)$ will not reach $(1, -1)$. Suppose the agent can observe, in addition to its current state, all successor states and the actions that would lead to them. Write an algorithm that is complete even for bidirected state spaces with infinite paths. What states does it visit in reaching $(1, -1)$?