

Mum \equiv Ber's

→ Digital Root :- Nothing but add all the digits.

* Digital root of perfect square 1, 4, 7, 9

Eg:- 1, 4, 9, 16, 25, 36

* Not any prime number except 3 will have 3, 6 or 9 as its digital root

Eg:- $\frac{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}{2 \ 4 \ 8 \ 1 \ 5 \ 2 \ 4 \ 1}$

Simplify:-

$$1) \overline{1777} - 2349 - 1345 + 6523 = 1777 + 6523 - 2349 - 1345 = 17 + 7 + 7 + 7 - 2 - 3 - 4 - 5 = 4$$

a) $4706 = 8$

b) $4606 = 7$

c) $4\cancel{0}76 = 8$ $4-0-4+7=7$

d) $4176 = 9$

$$2) \overline{5016} \times 1001 - 333 \times 77 + 22 = ? \times 11$$

a) $435570 = 6$

$3 \times 2 - 0 \times 5 + 22 = 22$

b) $\cancel{4}54127 = 5$

$6 + 22 = 28$

c) $527240 = 2$

$28 = 2x$

d) $3\cancel{6}6531 = 6$

$x = 14 = 5$

$$3) \frac{2387}{2} - 123 + 980 = ? - 145 + 945$$

$$= 6 + 8 = x - 1 + 0$$

$$4 = x - 1 \Rightarrow \boxed{x = 5}$$

a) 2244 , b) 2355 , c) 2434 , ~~d) 2444~~ ① ⑤

4) 135% of 342 - 342.1% of 15.3

a) 411.13 b) 412.23 c) 414.43 ~~d) 415.53~~

\Rightarrow Squares of two digit number

- (i) Ending with 0 or 5
- (ii) Ending with 5
- (iii) Ending with any other digits.

Q:- (ii) Ending with 5

$$(15)^2 = 225$$

$$\begin{array}{r} 1 \\ \times 1 \\ \hline 225 \end{array}$$

$$(25)^2 = 625$$

$$\begin{array}{r} 2 \\ \times 2 \\ \hline 25 \\ 25 \\ \hline 625 \end{array}$$

$$(35)^2 = 1225$$

$$\begin{array}{r} 3 \\ \times 3 \\ \hline 25 \\ 12 \\ \hline 25 \\ 12 \\ \hline 25 \end{array}$$

$$(45)^2 = 2025$$

$$\begin{array}{r} 4 \\ \times 4 \\ \hline 25 \\ 20 \\ \hline 25 \\ 20 \\ \hline 25 \end{array}$$

$$(55)^2 = 3025$$

$$(65)^2 = 4225$$

$$(75)^2 = 5625$$

$$c) (i) (76)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 49 \quad 36 \\ \hline 840 \\ \hline \end{array}$$

square the each number

multiple and double
Add both.

$$(ii) (28)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 464 \\ 320 \\ \hline 784 \\ \hline \end{array}$$

$$(iii) (73)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 4909 \\ 1420 \\ \hline 5329 \\ \hline \end{array}$$

$$(iv) (66)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 3636 \\ 720 \\ \hline 356 \\ \hline \end{array}$$

$$(v) (51)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 2501 \\ 100 \\ \hline 2601 \\ \hline \end{array}$$

$$(vi) (44)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 1616 \\ 32 \\ \hline 1936 \\ \hline \end{array}$$

$$(vii) (97)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 8149 \\ 126 \\ \hline 9409 \\ \hline \end{array}$$

$$(viii) (82)^2$$

$$\begin{array}{r} \text{^ } \text{^} \\ 6404 \\ 32 \\ \hline 6724 \\ \hline \end{array}$$

$$\begin{array}{r} 45 \\ +4 \\ 63 \\ \hline 126 \end{array}$$

\Rightarrow Squares of 3 Digit numbers

(i) less than 316 / greater than 317

(ii) Type A \rightarrow Rounding off to less than nearest 50

Type B \rightarrow Rounding off beyond 50.

Type A eg :- 203, 307, 412, 521

Type B eg :- 196, 291, 389, 492

* Less than $= 316$ greater than $= 317$

$\rightarrow (316)^2$ will be a 5 digit number.

$\rightarrow (317)^2$ will be a 6 digit number.

Type A \rightarrow Rounding off to less than nearest 50

$$\text{eg:- } (203)^2$$

$200 + 03 \quad (\text{Nearest to } 200)$

$203 + 03 = \begin{array}{r} 206 \\ \times 2 \\ \hline 412 \end{array}$

$(203)^2 = 4 \underline{1} - 2 \boxed{09} \rightarrow \text{square of } 03$

(ii) $(207)^2 = 200 + 07 \Rightarrow$

$207 + 07 = \begin{array}{r} 214 \\ \times 2 \\ \hline 428 \end{array}$

$$(207)^2 = 42849.$$

(iii) $(303)^2 = 300 + 03 \Rightarrow 303 + 3 = \begin{array}{r} 306 \\ \times 31 \\ \hline 918 \end{array}$

$$(303)^2 = 91809$$

(iv) $(311)^2 = 300 + 11 \Rightarrow 311 + 11 \Rightarrow$

$\begin{array}{r} 322 \\ \times 3 \\ \hline 966 \end{array}$

$$(311)^2 = \cancel{604} \frac{966}{11} \Rightarrow \cancel{604} \frac{966721}{966721}$$

$$(V) \quad (223)^2 = 200 + 23 \Rightarrow 223 + 23 = \frac{246}{\begin{array}{r} \times 2 \\ \hline 492 \end{array}}$$

$$(223)^2 = \frac{492 - -}{\begin{array}{r} 49729 \\ \hline \end{array}}$$

$$(VI) \quad (249)^2 = 200 + 49 \Rightarrow \frac{298}{\begin{array}{r} \times 2 \times 1 \\ \hline 596 \end{array}} \quad (249)^2 = \frac{244}{\begin{array}{r} \wedge \wedge \\ 1681 \\ + 72 \\ \hline 2401 \end{array}}$$

$$(249)^2 = \frac{596 - -}{\begin{array}{r} 2401 \\ \hline 62001 \end{array}}$$

$$(VII) \quad (302)^2 = \frac{304}{\begin{array}{r} \times 3 \\ \hline 912 \end{array}} \quad (302)^2 = 91204$$

Type - Rounding off beyond 50

$$(I) \quad (196)^2 = 200 - 04 \Rightarrow 196 - 4 = \frac{192}{\begin{array}{r} \times 2 \\ \hline 384 \end{array}}$$

$$(196)^2 = 384 \textcircled{16}$$

$$(II) \quad (254)^2 = \cancel{250} \quad 300 - 46 \Rightarrow 254 - 46 = \frac{208}{\begin{array}{r} \times 3 \times \\ \hline 624 \end{array}} \quad (46)^2 = \frac{1636}{\begin{array}{r} \times 48 \\ \hline 2116 \end{array}}$$

$$(254)^2 = 624 \underline{- 6} \Rightarrow 6456.$$

$$(III) \quad (275)^2 = 300 - 25 \Rightarrow 275 - 25 = \frac{250}{\begin{array}{r} \times 3 \times \\ \hline 750 \end{array}}$$

$$(275)^2 = \frac{750 - -}{\begin{array}{r} 75625 \\ \hline \end{array}}$$

\Rightarrow numbers greater than 317 (six digit number)

$$(i) (913)^2 = 900 + 13 \quad \Rightarrow 913 + 13 = \begin{array}{r} 926 \\ \times 98 \\ \hline 8334 \end{array}$$

$$(913)^2 = \begin{array}{r} 8334 \\ - 169 \\ \hline 833569 \end{array}$$

$$(ii) (956)^2 = 1000 - 46 \Rightarrow \begin{array}{r} 956 \\ - 46 \\ \hline 910 \end{array}$$

$$(956)^2 = \begin{array}{r} 9000 \\ - 2126 \\ \hline 902126 \end{array}$$

$$(956)^2 = \begin{array}{r} 9000 \\ - 2126 \\ \hline 902126 \end{array}$$

$$(ii) (956)^2 = 1000 - 44 \Rightarrow \begin{array}{r} 956 \\ - 44 \\ \hline 912 \end{array}$$

$$(956)^2 = \begin{array}{r} 9120 \\ - 1936 \\ \hline 9120 \end{array}$$

$$(956)^2 = \begin{array}{r} 913936 \\ \hline \end{array}$$

$$(46)^2 \begin{array}{r} 900 \\ \times 10 \\ \hline 9000 \end{array}$$

$$(44)^2 \begin{array}{r} 1616 \\ \times 10 \\ \hline 1616 \end{array}$$

⇒ Divisibility Rules

- 2 → last digit is 0, 2, 4, 6 or 8.
- 3 → sum of the digits is divisible by 3.
- 4 → last two digits is divisible by 4.
- 5 → last digit is 0 or 5.
- 6 → Number is divisible by 2 and 3
- 7 → Double the value of the last digit and subtract the result from the rest of the number. The answer is divisible by 7.
- 8 → last three digits of a number is divisible by 8.
- 9 → sum of the digits is divisible by 9.
- 10 → last digit is 0.
- 11 → Modulus of odd - even digits leaves 0 or 11
- 12 → leaves 4 and 3.
- Example:- For 7
- (i) $\begin{array}{r} 443 \\ \underline{-} 268 \\ \hline 175 \end{array}$
- $|25 - 268 + 443 - 5| = 195$
- $$\begin{array}{r} 443 \\ \underline{-} 25 \\ \hline 198 \\ \underline{-} 18 \\ \hline 15 \end{array}$$
- Steps:- (i) Divide the number in blocks of 3 from the right end
(ii) Addition & subtraction operators
(iii) Check divisible by 7

$$195 = 19 \cancel{(5 \times 2)} \rightarrow \text{double the last digit.}$$

$$= 19 (10) \rightarrow \text{subtract from the first two digits.}$$

$$\Rightarrow 19 - 10 \cancel{= 9} \rightarrow \text{if this is divisible by 7 then the number is divisible by 7.}$$

$$\begin{aligned}
 \text{(ii) } 343 &\Rightarrow 34 - (3 \cdot 2) = 34 - 6 = \underline{28} \text{ (Divisible by 7)} & \frac{29}{2} \frac{14}{36} \\
 \text{(iii) } 25637 &\Rightarrow 2563 - (14) \Rightarrow 2549 \Rightarrow 254 - 18 = 236 \\
 236 &= 23 - (12) = 11 \text{ (not divisible by 7). } \quad \boxed{236}
 \end{aligned}$$

Divisibility rule for 11 (Diff b/w odd & even digits)

Ex:- 544667732

$$\left| (5+4+6+7+2) - (4+6+7+3) \right| = |24 - 20| = \underline{4} \text{ (Remainder)}$$

Step 1:- Odd digits - Even digits in modulus.

Step 2:- If rem is 0 or 11, then it is divisible by 11.

* Find out divisibility rule for 24.

In order to do that you should consider co-primes

Factors of 24	\rightarrow	1	24	X
2		12	X	
3		8	\rightarrow Co-primes (✓)	
4		6	X	

\therefore Divisibility rule for 24 leaves rem of 3 & 8.

(ii) Find out divisibility rule for 12

Factors of 12	\rightarrow	1	12	X
2		6	X	
3		4	✓	

\Rightarrow Arithmetic progression.

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

a = 1st term, d = common diff

t_n = n^{th} term, S_n = sum of terms

\Rightarrow Geometric progression.

$$t_n = ar^{(n-1)}$$

$$S_n = \frac{a(r^{n-1})}{r-1}, \text{ when } r > 1$$

$$S_n = \frac{a(r^{n-1})}{1-r}, \text{ when } r < 1$$

$$S_n = n \times a, \text{ when } r = 1$$

a is 1st term

r is common ratio

* Sum of first n natural numbers = $n(n+1)/2$

* Sum of the squares of first n natural numbers

$$= n(n+1)(2n+1)/6$$

* Sum of the cubes of first n natural

$$\text{numbers} = [n^2(n+1)^2]/4$$

* Sum of first n natural odd numbers = n^2

$\Rightarrow \text{H.C.F} \& \text{L.C.M}$

(i) Find the greatest number that will exactly divide x, y & z .

\rightarrow Approach :- Required number = H.C.F of x, y & z .

(ii) find the greatest number that will divide x, y and z .
leaving remainders a, b & c respectively.

\rightarrow Required number = H.C.F of $(x-a), (y-b)$ & $(z-c)$.

(iii) Find the least number that is exactly divisible by x, y & z .

\rightarrow Required number = L.C.M of x, y & z

(iv) Find the least number which when divide by x, y & z
leaves remainder a, b & c respectively.

$$\rightarrow x-a = y-b = z-c = k$$

Required number = (L.C.M of x, y, z) - k .

(v) Find the least number which is divided by x, y & z
leaves the remainder 'r'

\rightarrow Required number = (L.C.M of x, y, z) + r .

(vi) Find the greatest number that will divide x, y and z
leaving same remainder in each case

\rightarrow Required number = H.C.F of $(x-y), (y-z)$ and $(z-x)$.

H.C.F and L.C.M of Fractions

$$\rightarrow \text{H.C.F of fractions} = \frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$$

$$\rightarrow \text{L.C.M of fractions} = \frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$$

Examples.

Number no. of factors

$$6 \quad 1, 2, 3, 6 = 4$$

$$7 \quad 1, 7 = 2$$

$$8 \quad 1, 2, 4, 8 = 4$$

$$10 \quad 1, 2, 5, 10 = 4$$

$$12 \quad 1, 2, 3, 4, 6, 12 = 6$$

$$52900 \rightarrow ?$$

If N is a number then $N = a^p \cdot b^q \cdot c^r \dots$

$$\text{No of factors} = (p+1)(q+1)(r+1) \dots$$

$$\text{Ex: (i)} \quad 6 = 2^1 \times 3^1 \Rightarrow (1+1)(1+1) = 2 \cdot 2 = 4$$

$$\text{(ii)} \quad 24 = 2^3 \cdot 3^1 \Rightarrow (3+1)(1+1) = 4 \cdot 2 = 8$$

$$\text{(iii)} \quad 480 \Rightarrow 2^4 \times 3^1 \times 2^1 \times 5^1 \Rightarrow 2^5 \times 3^1 \times 5^1 \Rightarrow 6 \cdot 2 \cdot 2$$

* 0 can be written as $2^0 \times 5^1 = 24$.

If 00 it can be written as $2^2 \times 5^2$

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \end{array}$$

$$\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \\ \end{array}$$

$$\begin{array}{r} 24 \\ \times 1 \\ \hline 24 \\ \end{array}$$

$$\begin{array}{r} 24 \\ \times 2 \\ \hline 48 \\ \end{array}$$

$$\text{(iv)} \quad 52900 = \\ 00 = \cancel{2^2} \times 5^2 \times \cancel{23}^2$$

$529 =$ check the digital root to find if 529 is P.F

529 lies between 400 & 625

$21^{\text{st}} \rightarrow$ last digit = 1

22^{nd} → last digit = 4

23^{rd} → last digit = 9

* Perfect square - If the total number of factors is a odd number, then given number is a perfect square.

Example:-

$$\text{(i)} \quad 54400 = 2^2 \times 5^2 \times 2^5 \times 17^1 \Rightarrow 2^7 \times 5^2 \times 17^1$$

$$54400 = (7+1)(2+1)(1+1) = 8 \cdot 3 \cdot 2 = 48.$$

$$\begin{array}{r} 2 | 544 \\ 2 | 272 \\ 2 | 136 \\ 2 | 68 \\ 2 | 34 \\ \hline 17 \end{array}$$

$$\text{(ii)} \quad 557 \times 699 = (\text{ref}) \quad 557^1 \times 699^1 \Rightarrow 2^0 \cdot 2 = 4$$

$$\text{(iii)} \quad 54400 \times 30600 = 2^4 \times 5^4 \times 2^5 \times 17^1 \times 2^3 \times 13^1 \times 5^1 \\ \Rightarrow 2^{12} \times 5^5 \times 13^1 \times 17^1$$

$$\Rightarrow (13) \cdot (6) \cdot (2) \cdot (2) \Rightarrow 332.$$

$$\begin{array}{r} 2 | 544 \\ 2 | 272 \\ 2 | 136 \\ 2 | 68 \\ 2 | 34 \\ 17 | 17 \\ 17 | 13 \\ 13 | 13 \\ 13 | 4 \\ 13 | 2 \\ 2 | 2 \\ \hline 1 \end{array}$$

$$\text{(iv)} \quad 444 \times 888 = 2^2 \times 3^1 \times 37^1 \times 2^3 \times 3^1 \times 37^1 \\ \Rightarrow 2^5 \times 3^2 \times 37^2$$

$$\Rightarrow (5+1)(2+1)(2+1) = 6 \cdot 3 \cdot 3 = 54$$

$$(v) 120 \times 240 \times 360 = 2^3 \times 5^3 \times 2^2 \times 3^1 \times 2^2 \times 3^1 \times 2^2 \times 3^2$$

$$\Rightarrow 2^{10} \times 3^4 \times 5^3 \Rightarrow 11 \times 5 \times 4$$

$$\Rightarrow 220.$$

$$(vi) (3230)^2 = 2^2 \times 5^2 \times 17^2 \times 19^2$$

$$\Rightarrow 3 \cdot 3 \cdot 3 \cdot 3 = 81.$$

\Leftrightarrow To find the number of factors:

(i) Express the number as $N = a^p \times b^q \times c^r$

(ii) No of factors = $(p+1)(q+1)(r+1)$

(iii) sum of the factors = $[a^{p+1}-1/a-1][b^{q+1}-1/b-1][c^{r+1}-1/c-1]$

(iv) Product of the factors = $N^{(p+1)(q+1)(r+1)/2}$

To find last digit:-

Calculate the last digit for $23 \times 34 \times 77 \times 32$.

$$\Rightarrow 3 \times 4 \times 7 \times 2 = 168$$

$\Rightarrow 8$ is the last digit.

$$\Rightarrow N = (xyz)^n \quad z \text{ is even} = 6$$

$$n \div 4 \rightarrow R=0 \rightarrow z \text{ is odd (except 5)} = 1$$

$$R=1 \rightarrow z^0$$

$$R=2 \rightarrow z^2$$

$$R=3 \rightarrow z^3$$

$$\begin{array}{r} 12 \\ \times 14 \\ \hline 84 \\ 84 \\ \hline 168 \end{array}$$

Ex :- (i) $(23)^{267}$

4) $\frac{24}{27}(66)$ $R = 3,$

$\frac{24}{27}$
 $\frac{24}{(3)} \rightarrow R$

$\Rightarrow (23)^3 = \underbrace{23 \cdot 23 \cdot 23}_{3 \times 3 \times 3} = 27$

$\frac{27}{\times 3} \underline{2}$

\therefore last digit of $(23)^{267}$ is 7

(ii) $(777)^{444}$

4) $444(111)$

$\frac{44}{4}$
 $\frac{04}{04}$
 $\frac{04}{(0)} \rightarrow R$

Here $R = 0$ & 777 is odd

~~so remainder is 1.~~

So, last digit = 1

(iii) $2^8 = 256$ (last digit = 6)

2 is even so, last digit = 6

4) $8(2)$

$\frac{8}{(0)} \rightarrow R$

$R = 0, z \rightarrow \text{even}, 6$

$R = 1, z \rightarrow 2$

$R = 2, z^2 \rightarrow 4$

$R = 3, z^3 \rightarrow 8$

* $z^n \rightarrow z^8 = 256$

$2^9 = 512$, last digit = 2

$2^{10} = \text{Rem } 2$, last digit =

Last digit examples:-

$$(i) (999)^{888} \Rightarrow 4) 888(222 \text{ Rem } 0)$$

(0)

999 \rightarrow 9 is odd, so last digit = 1

(ii) $(544)^{688} \times (306)^{405}$

$$4) 688(172$$

$$\begin{array}{r} 4 \\ 28 \\ \hline 28 \\ \hline 8 \\ \hline 0 \end{array}$$

$R=0$

$$4) 405(101$$

$$\begin{array}{r} 4 \\ 5 \\ \hline 4 \\ \hline 0 \end{array}$$

(1)

$R=1$

$$l=0, z' (\text{even}) 6; R=1, \Rightarrow (306)^1 = 6$$

$$6 \times 6 = 3\underline{6}$$

\therefore last digit = 6

(iii) $(445)^{975} \times (888)^{932}$

$$4) 975(243 \quad 4) 888($$

$$\begin{array}{r} 8 \\ 19 \\ \hline 16 \\ \hline 15 \\ \hline 12 \\ \hline 3 \end{array}$$

$R=0 \Rightarrow 8$ is even, last digit = 6

$R=3$

$$(445)^3 \Rightarrow 5^3 = 125$$

last digit = 5

$$\therefore \text{overall last digit} = 5 \times 6 = 3\underline{0}$$

\therefore zero is last digit.

$$(iv) (122)^{244} \times (244)^{366} \times (366)^{122}$$

$$4) \frac{244}{244} (6)$$

$$4) \frac{366}{366} (9)$$

$$4) \frac{122}{122} (6)$$

$$\text{Rem} = 0$$

$$\text{Rem} = 2$$

$$\text{Rem} = 2$$

2 is even \Rightarrow last digit = 6

$$\frac{36}{2} \times$$

$$(244)^2 \Rightarrow 4 \times 4 = 1\cancel{6} \Rightarrow \text{last digit} = 6$$

$$(366)^2 \Rightarrow 6 \times 6 = 3\cancel{6} \Rightarrow \text{last digit} = 6$$

$$\Rightarrow \text{last digit} = 6 \times 6 \times 6 = 6$$

$$\frac{36}{2} \times$$

$$(v) (32)^{22} \times (88)^{44} \times (66)^{66}$$

$$(32)^2 \times (88)^0 \times (66)^2 \Rightarrow 4 \times 6 \times 6 = 144 = 4$$

\Rightarrow Cubes of 2 Digit Number

(i) Ending with zero - 30, 40, $(50)^3 = 25000$

(ii) Starting with 1 - $(12)^3$

(iii) Ending with 1

(iv) Doublets

(v) Ending with any other digit.

(i) ending with zero

$$\text{eg: } (20)^3 = 8000, (30)^3 = 27000, (50)^3 = 625000$$

(ii) starting with 1

$$\text{eg: } (12)^3 = \begin{array}{r} 1 \\ - 2 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 1 \rightarrow 1 \\ 2 \rightarrow 2 \text{ same} \\ 4 \rightarrow (2^2) \text{ square} \\ 8 \rightarrow (2^3) \text{ cube} \end{array}$$

1 2 8
Double the center digit

$$(12)^3 = 1448$$

② ②

$$(ii) (13)^3 = 1 \ 3 \ 9 \ 27$$

$$\begin{array}{r} 1 \ 6 \ 12 \\ \hline 2 \ 4 \ 1 \ 9 \ 7 \\ \hline 3 \ 6 \end{array}$$

$$(13)^3 = 2497$$

$$(iii) (14)^3 = 1 \ 4 \ 16 \ 64$$

$$\begin{array}{r} 1 \ 8 \ 32 \\ \hline 2 \ 7 \ 4 \ 4 \\ \hline 1 \end{array}$$

$$(14)^3 = 2744$$

$$(iv) (19)^3 = 1 \ 9 \ 81 \ 729$$

$$\begin{array}{r} 18 \ 162 \\ \hline 6 \ 8 \ 5 \ 9 \\ \hline 1 \end{array}$$

$$(19)^3 = 6859$$

21

$$\begin{array}{r} 16 \\ \times 42 \\ \hline 64 \\ 64 \\ \hline 68 \end{array}$$

$$\begin{array}{r} 81 \\ 18 \\ \times 9 \\ \hline 72 \\ 72 \\ \hline 729 \end{array}$$

$$\begin{array}{r} 9 \ 9 \ 9 \\ \times 9 \\ \hline 81 \\ 81 \\ \hline 81 \end{array}$$

$$\begin{array}{r} 81 \\ 61 \\ \times 2 \\ \hline 162 \\ 12 \\ \hline 162 \end{array}$$

$$\begin{array}{r} 162 \\ 72 \\ \times 9 \\ \hline 281 \\ 315 \end{array}$$

(iii) Ending with 1

$$\text{eg: } (51)^3 = 525 \ 25 \ 5 \ 1$$

$$\begin{array}{r} ② \\ ① \\ \hline 132 \end{array} \quad \begin{array}{r} 8 \\ 1 \\ \hline 65 \end{array} \quad 1$$

$$(51)^3 = 132651$$

1 →

→ same

5 →

25 → square of 2nd num

525 → square of 2nd num
cube

4

$$(ii) (61)^3 = 216 \ 36 \ 6 \ 1$$

$$(61)^3 = \begin{array}{r} 226 \\ 2107981 \end{array}$$

$$\begin{array}{r} 10 \\ ② \\ \hline 246 \end{array} \quad \begin{array}{r} 72 \\ 12 \\ \hline 98 \end{array} \quad 1$$

$$\begin{array}{r} 36 \\ \times 1 \\ \hline 36 \\ 72 \\ \hline 108 \end{array}$$

$$(iii) (31)^3 = 27 \ 9 \ 3 \ 1$$

$$(31)^3 = 29791.$$

$$\begin{array}{r} ② \cdot 12 \ 6 \\ \hline 29791 \end{array}$$

$$\begin{array}{r} 72 \\ \cdots 36 \\ \hline 108 \end{array}$$

(iv) Doublets :-

..... Cubes of number

$$\text{eg: } (22)^3 = 8 \ 8 \ 8 \ 8 \quad (22)^3 = 10648$$

$$\begin{array}{r} 2 \ 16 \ 16 \\ \hline 10 \ 6 \ 4 \ 8 \end{array}$$

$$\begin{array}{r} 84 \\ 51 \\ 83 \\ 58 \\ 52 \\ \hline 27 \\ 4 \end{array}$$

$$(ii) (33)^3 = 27 \ 27 \ 27 \ 27$$

$$\begin{array}{r} ⑨ \ ⑧ \ ④ \\ \hline 36 \ 2 \ 3 \ 1 \end{array}$$

$$\begin{array}{r} 27 \\ 56 \\ 33 \end{array}$$

$$(33)^3 = 36231.$$

(V) finding with Any other Digit:

eg: $(32)^3 = a^3 \ a^2 b \ ab^2 \ b^3$

$\begin{array}{r} 32 \\ \times 32 \\ \hline 27 \end{array}$

$\begin{array}{r} 9 \cdot 2 \\ \times 32 \\ \hline 18 \end{array}$

$\begin{array}{r} 3 \cdot 4 \\ \times 32 \\ \hline 12 \end{array}$

$\begin{array}{r} 8 \\ \times 32 \\ \hline 8 \end{array}$

$\begin{array}{r} 36 \\ \times 32 \\ \hline 24 \end{array}$

$\begin{array}{r} 5 \\ \times 32 \\ \hline 10 \end{array}$

$\begin{array}{r} 36 \\ 38 \\ \hline 74 \end{array}$

$\begin{array}{r} 74 \\ + 3 \\ \hline 77 \end{array}$

$\begin{array}{r} 27 \\ 12 \\ \hline 39 \end{array}$

$(32)^3 = 32768$

⇒ Averages And Ages

↳ Basic notations, Add / remove items, Replacement, Average with Ages, Ages with Ratio, Framing equations with years.

→ Basic notations and Average of Different Groups

eg:- A class A of 50 students scoring average marks of 45 and Class B has 45 students scoring average marks of 50. What is the average of both classes together.

General Way:- $45 = \frac{TA}{50}$ $TB = 45 \times 50$

$\Rightarrow \frac{45 \times 50 + 45 \times 50}{45 + 45}$

Eg:- The avg weight of 17 girls is 20kg and that of 23 boys is 22 kg. find the weight of the class.

$$\Rightarrow \frac{17 \times 20 + 23 \times 22}{20+22} = \frac{846}{42} \Rightarrow 20.15$$

→ Addition or Removal of items

(i) Avg of new items added (ii) Avg of items removed

$$A + l - (l+N/n)x \quad A + l - (l-N/n)x$$

Where, A = original average

N = original number of items

n = number of items added or removed.

x = By which the avg is increased or decreased.

Eg:- The average age of 40 students in a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.

$$\rightarrow A + l - (l+N/n)x$$

$$15 + \left(1 + \frac{40}{10}\right) 0.2 \Rightarrow 15 + (5) 0.2$$

$$= 15 + 1$$

$$= \underline{16}$$

Q:- Avg salary of 15 teachers is 4500, 3 teachers left the school and the avg salary of remaining dropped by 175. Find the total salary of the teacher left out.

$$A \pm \left(1 - \frac{N}{n}\right)x \Rightarrow 4500 - \left(1 - \frac{15}{3}\right) \times 175$$

$$\Rightarrow 4500 - (1-5) \times 175 = 4500 + 700 = 5200.$$

$$\text{Total salary} = 5200 \times 3 = 15600.$$

(iii) 50 boys in class. The Avg weight 45kg. 1 boy leaves reduced by 100g. Find the weight of the boy who left the class.

$$A \pm \left(1 - \frac{N}{n}\right)x = 45 - \left(1 - \frac{50}{1}\right) 100g$$

$$\Rightarrow 45 - (-49) \frac{100g}{0.1} = 45 + \frac{4.9}{0.001} = 49.9$$

→ Replacement of items:-

For N items in a group, sum of new added - sum of new item removed = $\pm 1 - Nx$.

Q:- A man weigh 80 kg is replaced by another man in a grp of 5, the avg weight is ↓ by 3kg. What is the weight of new man?

$$\text{Added} - \text{Removed} = \pm Nx$$

$$A - 80 = -5 \times 3$$

$$A = -15 + 80$$

$$A = 65 \text{ kg.}$$

(ii) The avg weight of 15 students in a class is 70 kg by it, when one of the stud weight 40 kg is rep by a new stud . Find the weight of new stud .

$$\text{Added} - \text{Removed} = \pm N \times$$

$$A - 40 = + 15 \times 1.5 \Rightarrow A - 40 = 22.5 \Rightarrow A = 62.5.$$

(iii) Avg weight of 8 ↑ by 2.5 kg , 1 person 65 kg,
weight of new people

$$A - R = \pm N \times \Rightarrow$$

Ages and Averages

eg: The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of husband is ?

$$\frac{H-3 + W-3 + C-3}{3} = 27 \Rightarrow H-3 + W-3 + C-3 = 27 \times 3$$

$$H + W + C = 81 + 9 = 90$$

$$\frac{W+C}{2} = \frac{50}{2} \Rightarrow \boxed{W+C=50}$$

$$H = 90 - 50 = 40,$$

a) 35

b) 38

c) 39

d) 40

eg 2 years ago, Avg age of P & Q was 15.
Avg age of P, Q & R is 20 now. How old will R be
after 10 years.

$$P+Q = 15 \times 2 + 10 \quad \left(\frac{P+Q}{2} = 15 \right)$$

$$P+Q = 40$$

$$P+Q+R = 20 \times 3 \Rightarrow P+Q+R = 60$$

$$R = 60 - 40 = 20$$

$$\text{But after 10 years } = 20 + 10 = 30.$$

→ Comparison of Ages

eg: A's age 6 years back was half of the total of B & C's
if C is 2 years older than B, what is A's present age.

$$A - 6 = \frac{1}{2} (B+C) \quad \Rightarrow A - 12 = B + 2C$$

- a) 20 b) 24 c) 27 d) CBD

(ii) In 10 years, A will be twice as old as B was 10 years ago. If A is now 9 years older than B, then present age of B is

- a) 35 b) 38 c) 39 d) 40

$$A = 9 + B$$

$$A + 10 = 2(B - 10) \Rightarrow A + 10 = 2B - 20 \quad A - 2B = -10$$

→ Alligations

when even there are average's and ratios blended
then you will use alligations.

⇒ Ratios:-

Bridge three Components

eg:- $A:B = 4:5$, $B:C = 6:7$, find $A:C = ?$

$$\frac{A}{B} = \frac{4}{5} \times \frac{B}{C} = \frac{6}{7} \Rightarrow \frac{A}{C} = \frac{4}{5} \times \frac{6}{7} \Rightarrow \frac{A}{C} = \frac{24}{35}$$

$$\therefore A:C = 24:35$$

$$(ii) \quad \frac{A}{B} = \frac{6}{7} : \frac{B}{C} = \frac{8}{9} : \quad A:B:C = ?$$

$$A:B:C = 48:56:63 \quad \begin{array}{r} 6 \\ \downarrow \\ A:B = 6:7 \end{array} \quad \begin{array}{r} 8 \\ \downarrow \\ B:C = 8:9 \end{array}$$

$$(iii) \quad BC:AC:AB = 1:2:3, \text{ find } A:B:C = ?$$

$$\frac{AC}{BC} = \frac{2}{1} : \quad B:C = 3:2$$

$$\begin{array}{r} 2:1 \\ \downarrow \\ A:B = 2:1 \end{array} \quad \begin{array}{r} 3:2 \\ \downarrow \\ B:C = 3:2 \end{array}$$

$$A:B:C = 6:3:2$$

$$(iv) \quad \frac{1}{A} : \frac{1}{B} : \frac{1}{C} = 2:3:5, \text{ find } A:B:C = ?$$

$$A:B:C = \frac{1}{2} : \frac{1}{3} : \frac{1}{5}$$

$$A:B:C = 15:10:6$$

→ Bridge four components:

e.g.: given $A:B = 2:5$, $B:C = 3:1$, $C:D = 3:5$, find $A:B:C:D$

$$\begin{aligned} A:B &= 2:5 \\ B:C &= 3:1 \\ C:D &= 3:5 \end{aligned}$$

$$\frac{15}{45}$$

$$A:B:C:D = 18: 5 \times 3 \times 3 : 5 \times 1 \times 3 : 5 \times 1 \times 5$$

$$A:B:C:D = 18: 45: 15: 25$$

(ii) Find the B's share in Rs 6300. if $A:B = 2:3$,

$$B:C = 4:5 \text{ & } C:D = 3:7$$

$$\begin{aligned} A:B &= 2:3 \\ B:C &= 4:5 \\ C:D &= 3:7 \end{aligned}$$

$$\frac{10}{15}$$

$$\frac{8}{10}$$

$$A:B:C:D = 2 \times 4 \times 3 : 3 \times 4 \times 3 : 3 \times 5 \times 3 : 3 \times 5 \times 7$$

$$A:B:C:D = 24: 36: 45: 105 = 8: 12: 15: 35$$

$$\frac{85}{105}$$

$$\frac{9}{10}$$

$$\underline{20}$$

$$B's = \frac{12}{8+12+15+35} \times 6300$$

$$B's = \frac{12}{70} \times 6300 \Rightarrow 1080$$

(iii) $A:B = 2:5$, $B:C = 4:3$; $C:D = 1:7$

$$\frac{2}{5} \times \frac{4}{3} \times \frac{1}{7} = \frac{8}{105}$$

→ Bridge four components:

e.g.: given $A:B = 2:5$, $B:C = 3:1$, $C:D = 3:5$, find $A:B:C:D$

$$A:B = 2:5$$

$$B:C = 3:1$$

$$C:D = 3:5$$

$$C:D = 3:5$$

$$\frac{15}{45}$$

$$A:B:C:D = 18:5 \times 3 \times 3 : 5 \times 1 \times 3 : 5 \times 1 \times 5$$

$$A:B:C:D = 18:45:15:25$$

(ii) Find the B's share in Rs 6300. If $A:B = 2:3$,

$$B:C = 4:5 \text{ & } C:D = 3:7$$

$$A:B = 2:3$$

$$B:C = 4:5$$

$$C:D = 3:7$$

$$\frac{10}{35}$$

$$\frac{35}{35}$$

$$A:B:C:D = 2 \times 4 \times 3 : 3 \times 4 \times 3 : 3 \times 5 \times 3 : 3 \times 5 \times 7$$

$$A:B:C:D = 24:36:45:105 = 8:12:15:35$$

$$B's = \frac{12}{8+12+15+35} \times 6300$$

$$B's = \frac{12}{70} \times 6300 \Rightarrow 1080$$

(iii) $A:B = 2:5$, $B:C = 4:3$; $C:D = 1:7$

$$\frac{2}{5} \times \frac{4}{3} \times \frac{1}{7} = \frac{8}{105}$$

(iv) Two numbers are in the ratio 3:5, if 9 is subtracted from each, the new number are in the ratio 12:23, then the smaller number is

- a) 27 b) 30 c) 33 d) 36.

$$\frac{3x - 9}{5x - 9} = \frac{12}{23} \Rightarrow \frac{23(3x - 9)}{12(5x - 9)} = \frac{69x - 207}{60x - 108}$$

$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$

$\begin{array}{r} 23 \\ \times 9 \\ \hline 207 \\ + 12 \\ \hline 327 \end{array}$

$\begin{array}{r} 140 \\ \times 5 \\ \hline 700 \end{array}$

$\begin{array}{r} 150 \\ \times 8 \\ \hline 120 \\ + 40 \\ \hline 160 \end{array}$

$\begin{array}{r} 175 \\ \times 8 \\ \hline 140 \\ + 15 \\ \hline 175 \end{array}$

$\begin{array}{r} 175 \\ \times 25 \\ \hline 125 \\ + 350 \\ \hline 475 \end{array}$

$\begin{array}{r} 207 \\ \times 25 \\ \hline 1035 \\ + 4140 \\ \hline 6175 \end{array}$

(v) Ratio = 5:7:8, increased seats by 401,501 & 751
Find out the ratio of seats.
 $5x 140 : 7x 150 : 8x 175$
 $20 : 150 : 200$
 $\therefore 2:3:4$.

(vi) Ratio A:B = 3:4, Ratio of monthly exp = 4:5.
find the ratio of their monthly savings, if the savings of A is $\frac{1}{4}$ th of income.

$$I_A = 800$$

$$S_A = \frac{800}{4} = 200$$

$$E_A = 800 - 200 = 600$$

$$I_B = 400$$

$$S_B = \frac{400}{4}$$

$$E_B = 400 - \frac{400}{4} \Rightarrow \frac{1200}{4}$$

$$4:5 = 600 : E_B$$

$$\Rightarrow \frac{75 \times 4}{475} = \frac{12}{19}$$

$$E_B = \frac{1125}{4}$$

- \Rightarrow Partnership
 \Rightarrow If there are two people

\Rightarrow (Blood) Relations

+ve / -ve
 $\underline{\text{Male}}$ $\underline{\text{Female}}$

1) Gender - Male & Female

2) Generation Gr-2 (Grand parents)

Gr-1 (Anty, Uncle, mother, father ...)

Gr-0 (Brothers, Cousins, Spouses,

Gr-1 \rightarrow (Sons, kids, daughter, nephew, nieg)

Question - types :-

- (i) Pointing a picture / person
- (ii) Paragraph - small data's
- (iii) Paragraph - large data's
- (iv) Defined operators
- (v) Different sets of Verticals of data.

notations:- A is the brother of B

$$B \rightarrow A^+$$

A is the father of B

$$\begin{array}{c} A^+ \\ \uparrow \\ B \end{array}$$

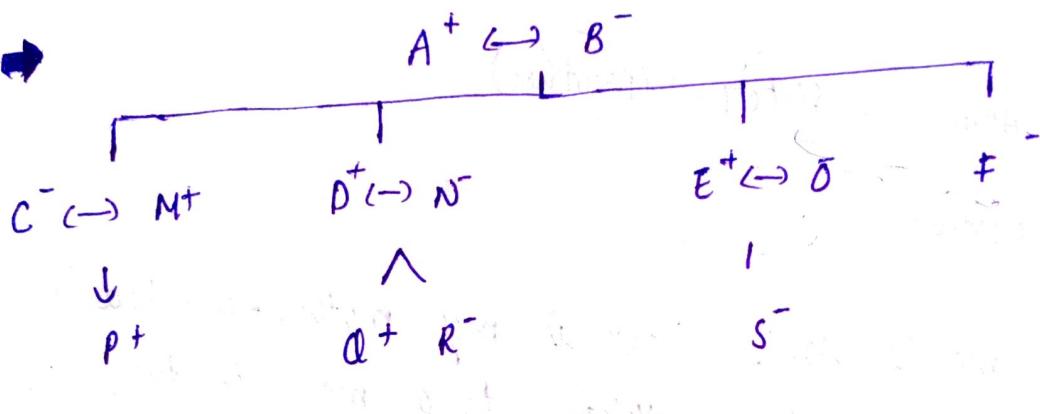
(ii) A is the daughter of B

B



A-

(iv) Couple (Imagine) $A^+ \leftrightarrow B^-$



No of males = 6

No of Coupled = 4

Question: How is P related to B.

Ans: P is the Grand son.

(ii) How is O related to A.

Ans: Daughter in law

(iii) How is M related to F

| O related F |
Nephew

Ans: Brother in law

(iv) How is P related to F

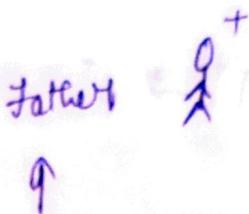
| O related D |
Sister in law

Ans: Maternal Aunt

| P related S |
Cousin

→ Pointing a picture / Person

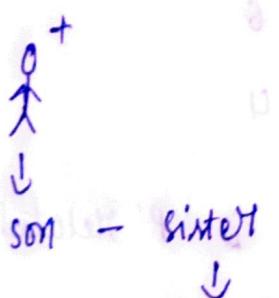
e.g. Arun said "He is the only son of the father of my sister's brother". How is that person related to Arun.



Arun - sister - **Brother**

Ans :- brother

(ii) "This man's son's sister is my mother-in-law". How's women husband related to a man in the photograph



son (\rightarrow) Mother in law
= (me)

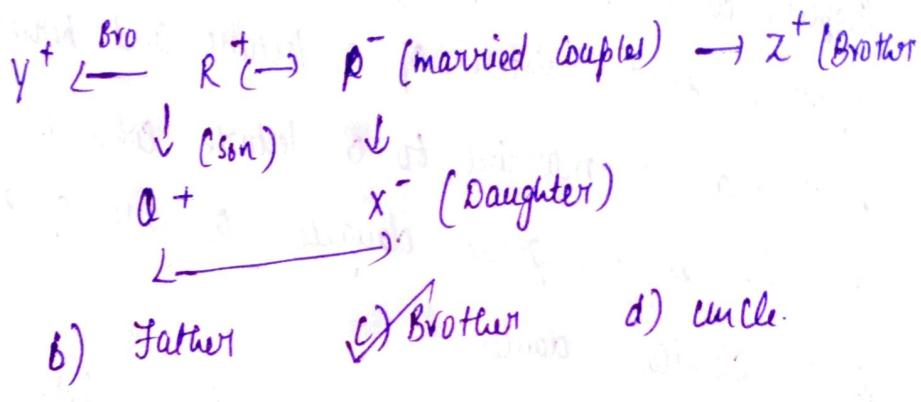
(iii) He is maternal grandfather of children of my husband's sister. How is the man related to the woman

Ans → Father
Me (\rightarrow) My husband → Sister
↓
children.

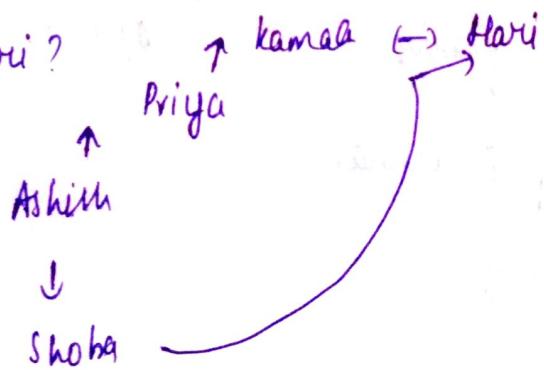
(iv) His mother is the wife of my father's son. Brother and sister I have none

→ Paragraphs - small Data's

e.g:- A family consists of six members P, Q, R, X, Y and Z.
Q is son of R but R is not mother of Q. P and Q are
married couple. Y is the brother of R. X is the daughter of P.
Z is the brother of P. Now is Q related to X?



(ii) Shoba is the niece of Ashish. Ashish' mother is Priya. Kamala is Priya's mother. Kamala's husband is Hari. Krish is the mother-in-law of Hari. How is Shoba related to Hari?



- a) Daughter
b) Great grand daughter
c) Grand niece.

→ Paragraph's large data:

e.g.: In a family of seven, three generations are living together.
The family consists of two married couples having two children.
Kannan is Gyothika's elder brother. Gopal is lucky to have two
grand children. There are two housewives in the family.
Gopal, who is Manoj's father, is a lawyer and Manoj's mother
Gyothika is the sister of a lecture and herself is a nurse.
Anuradha is married to a lecture who is Neelhi's son.
Gyothika is the grand daughter of one of the housewives
is a classical dancer.

(i) What is Manoj's profession

A) Lecture

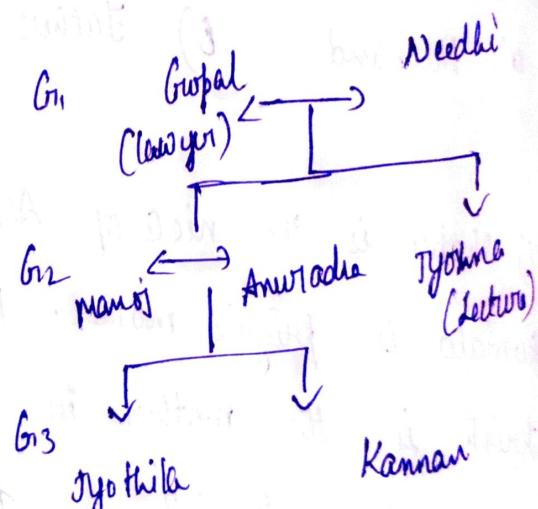
(ii) How many male members
are there in family?

A) Gopal, Manoj & Kannan

Total = 3

(iii) Who are Neelhi's children

A) Gopal & Neelhi
(Lawyer)



(iv) Which of the following statements is not true

- a) The nurse is sister in law of the house wife
- b) Gopal has 2 grand children
- c) Nidhi has a son and a daughter
- d) Gopal has two children
- e) Anwadha has a son and a daughter

-> Defined Operations
 $\text{a} \text{ } \text{b}$ mean a is the son of b

$\text{A} - \text{B}$ mean A is the wife of B

$\text{A} \times \text{B}$ mean A is the brother of B

A/B mean A is the mother of B

$\text{A} = \text{B}$ mean A is the sister of B .

} \rightarrow They will give

eg:- (i) What does $P + Q - R$ mean?
 $\downarrow \quad \downarrow \quad \downarrow$
 $R \rightarrow Q$
 $P +$

Q is the son father of P .

(BODMAS)

(ii) what does $P + R/Q$ mean.
 $\downarrow \quad \downarrow$
brother mother.

$P + \rightarrow R^-$
 $\swarrow \quad \downarrow$
 Q .

P is uncle of Q .