## EXERCISES

**8.1** A logical knowledge base represents the world using a set of sentences with no explicit structure. An **analogical** representation, on the other hand, has physical structure that corresponds directly to the structure of the thing represented. Consider a road map of your country as an analogical representation of facts about the country—it represents facts with a map language. The two-dimensional structure of the map corresponds to the two-dimensional surface of the area.

**a**. Give five examples of *symbols* in the map language.

**b**. An *explicit* sentence is a sentence that the creator of the representation actually writes down. An *implicit* sentence is a sentence that results from explicit sentences because of properties of the analogical representation. Give three examples each of *implicit* and *explicit* sentences in the map language.

**c**. Give three examples of facts about the physical structure of your country that cannot be represented in the map language.

**d**. Give two examples of facts that are much easier to express in the map language than in first-order logic.

**e**. Give two other examples of useful analogical representations. What are the advantages and disadvantages of each of these languages?

**8.2** Consider a knowledge base containing just two sentences: $P(a)$ and $P(b)$. Does this knowledge base entail $\forall\, x\ P(x)$? Explain your answer in terms of models.

**8.3** Is the sentence $\exists\, x, y \;\; x = y$ valid? Explain.

**8.4** Write down a logical sentence such that every world in which it is true contains exactly one object.

**8.5** Consider a symbol vocabulary that contains $c$ constant symbols, $p_k$ predicate symbols of each arity $k$, and $f_k$ function symbols of each arity $k$, where $1 \leq k \leq A$. Let the domain size be fixed at $D$. For any given model, each predicate or function symbol is mapped onto a relation or function, respectively, of the same arity. You may assume that the functions in the model allow some input tuples to have no value for the function (i.e., the value is the invisible object). Derive a formula for the number of possible models for a domain with $D$ elements. Don't worry about eliminating redundant combinations.

**8.6** Which of the following are valid (necessarily true) sentences?

**a**. $(\exists x\; x = x) \;\Rightarrow\; (\forall\, y \;\; \exists z\; y = z)$.

**b**. $\forall\, x \;\; P(x) \lor \neg P(x)$.

**c**. $\forall\, x \;\; \mathit{Smart}(x) \lor (x = x)$.

**8.7** Consider a version of the semantics for first-order logic in which models with empty domains are allowed. Give at least two examples of sentences that are valid according to the standard semantics but not according to the new semantics. Discuss which outcome makes more intuitive sense for your examples.

**8.8** Does the fact $\neg Spouse(George, Laura)$ follow from the facts $Jim \neq George$ and $Spouse(Jim, Laura)$? If so, give a proof; if not, supply additional axioms as needed. What happens if we use $Spouse$ as a unary function symbol instead of a binary predicate?

**8.9** This exercise uses the function $MapColor$ and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

**a**. Paris and Marseilles are both in France.

(i) $In(Paris \land Marseilles, France)$.
(ii) $In(Paris, France) \land In(Marseilles, France)$.
(iii) $In(Paris, France) \lor In(Marseilles, France)$.

**b**. There is a country that borders both Iraq and Pakistan.

(i) $\exists\, c \quad Country(c) \land Border(c, Iraq) \land Border(c, Pakistan)$.
(ii) $\exists\, c \quad Country(c) \;\Rightarrow\; [Border(c, Iraq) \land Border(c, Pakistan)]$.
(iii) $[\exists\, c \quad Country(c)] \;\Rightarrow\; [Border(c, Iraq) \land Border(c, Pakistan)]$.
(iv) $\exists\, c \quad Border(Country(c), Iraq \land Pakistan)$.

**c**. All countries that border Ecuador are in South America.

(i) $\forall\, c \;\; Country(c) \land Border(c, Ecuador) \;\Rightarrow\; In(c, SouthAmerica)$.
(ii) $\forall\, c \;\; Country(c) \;\Rightarrow\; [Border(c, Ecuador) \;\Rightarrow\; In(c, SouthAmerica)]$.
(iii) $\forall\, c \;\; [Country(c) \;\Rightarrow\; Border(c, Ecuador)] \;\Rightarrow\; In(c, SouthAmerica)$.
(iv) $\forall\, c \;\; Country(c) \land Border(c, Ecuador) \land In(c, SouthAmerica)$.

**d**. No region in South America borders any region in Europe.

(i) $\neg[\exists\, c, d \;\; In(c, SouthAmerica) \land In(d, Europe) \land Borders(c, d)]$.
(ii) $\forall\, c, d \;\; [In(c, SouthAmerica) \land In(d, Europe)] \;\Rightarrow\; \neg Borders(c, d)]$.
(iii) $\neg\forall\, c \;\; In(c, SouthAmerica) \;\Rightarrow\; \exists\, d \;\; In(d, Europe) \land \neg Borders(c, d)$.
(iv) $\forall\, c \;\; In(c, SouthAmerica) \;\Rightarrow\; \forall\, d \;\; In(d, Europe) \;\Rightarrow\; \neg Borders(c, d)$.

**e**. No two adjacent countries have the same map color.

(i) $\forall\, x, y \;\; \neg Country(x) \lor \neg Country(y) \lor \neg Borders(x, y) \lor$
$\neg(MapColor(x) = MapColor(y))$.
(ii) $\forall\, x, y \;\; (Country(x) \land Country(y) \land Borders(x, y) \land \neg(x = y)) \;\Rightarrow$
$\neg(MapColor(x) = MapColor(y))$.
(iii) $\forall\, x, y \;\; Country(x) \land Country(y) \land Borders(x, y) \land$
$\neg(MapColor(x) = MapColor(y))$.
(iv) $\forall\, x, y \;\; (Country(x) \land Country(y) \land Borders(x, y)) \;\Rightarrow\; MapColor(x \neq y)$.

**8.10** Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person $p$ has occupation $o$.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor$, $Surgeon$, $Lawyer$, $Actor$: Constants denoting occupations.
$Emily$, $Joe$: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

**a**. Emily is either a surgeon or a lawyer.
**b**. Joe is an actor, but he also holds another job.
**c**. All surgeons are doctors.
**d**. Joe does not have a lawyer (i.e., is not a customer of any lawyer).
**e**. Emily has a boss who is a lawyer.
**f**. There exists a lawyer all of whose customers are doctors.
**g**. Every surgeon has a lawyer.

**8.11** Complete the following exercises about logical senntences:

**a**. Translate into *good, natural* English (no $x$s or $y$s!):

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \land SpeaksLanguage(y, l) \Rightarrow \; Understands(x, y) \land Understands(y, x).$$

**b**. Explain why this sentence is entailed by the sentence

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \land SpeaksLanguage(y, l) \Rightarrow \; Understands(x, y).$$

**c**. Translate into first-order logic the following sentences:

(i) Understanding leads to friendship.
(ii) Friendship is transitive.

Remember to define all predicates, functions, and constants you use.

**8.12** Rewrite the first two Peano axioms in Section 8.3.3 as a single axiom that defines $NatNum(x)$ so as to exclude the possibility of natural numbers except for those generated by the successor function.

**8.13** Equation (8.4) on page 306 defines the conditions under which a square is breezy. Here we consider two other ways to describe this aspect of the wumpus world.

DIAGNOSTIC RULE

**a**. We can write **diagnostic rules** leading from observed effects to hidden causes. For finding pits, the obvious diagnostic rules say that if a square is breezy, some adjacent square must contain a pit; and if a square is not breezy, then no adjacent square contains a pit. Write these two rules in first-order logic and show that their conjunction is logically equivalent to Equation (8.4).

CAUSAL RULE

**b**. We can write **causal rules** leading from cause to effect. One obvious causal rule is that a pit causes all adjacent squares to be breezy. Write this rule in first-order logic, explain why it is incomplete compared to Equation (8.4), and supply the missing axiom.

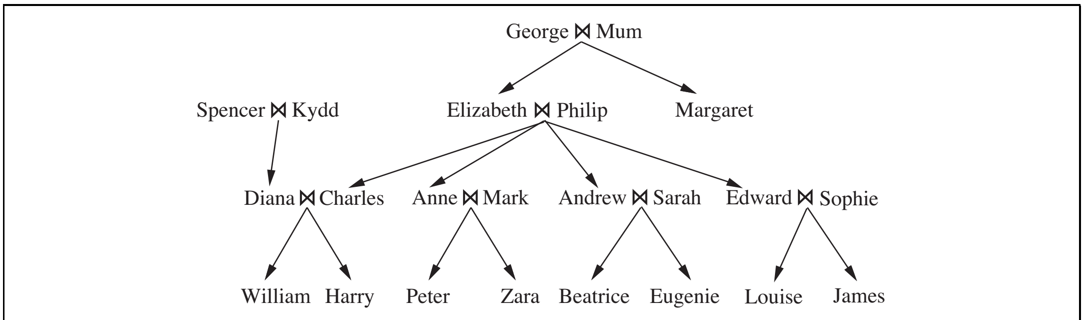


**Figure 8.7** A typical family tree. The symbol "⋈" connects spouses and arrows point to children.

**8.14** Write axioms describing the predicates $Grandchild$, $Greatgrandparent$, $Ancestor$, $Brother$, $Sister$, $Daughter$, $Son$, $FirstCousin$, $BrotherInLaw$, $SisterInLaw$, $Aunt$, and $Uncle$. Find out the proper definition of $m$th cousin $n$ times removed, and write the definition in first-order logic. Now write down the basic facts depicted in the family tree in Figure 8.7. Using a suitable logical reasoning system, TELL it all the sentences you have written down, and ASK it who are Elizabeth's grandchildren, Diana's brothers-in-law, Zara's great-grandparents, and Eugenie's ancestors.

**8.15** Explain what is wrong with the following proposed definition of the set membership predicate $\in$:

$$\forall x, s \;\; x \in \{x|s\}$$
$$\forall x, s \;\; x \in s \;\Rightarrow\; \forall y \;\; x \in \{y|s\} \,.$$

**8.16** Using the set axioms as examples, write axioms for the list domain, including all the constants, functions, and predicates mentioned in the chapter.

**8.17** Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:

$$\forall x, y \;\; Adjacent([x, y], [x + 1, y]) \wedge Adjacent([x, y], [x, y + 1]) \,.$$

**8.18** Write out the axioms required for reasoning about the wumpus's location, using a constant symbol $Wumpus$ and a binary predicate $At(Wumpus, Location)$. Remember that there is only one wumpus.

**8.19** Assuming predicates $Parent(p, q)$ and $Female(p)$ and constants $Joan$ and $Kevin$, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists^1$ to mean "there exists exactly one.")

**a**. Joan has a daughter (possibly more than one, and possibly sons as well).
**b**. Joan has exactly one daughter (but may have sons as well).
**c**. Joan has exactly one child, a daughter.
**d**. Joan and Kevin have exactly one child together.
**e**. Joan has at least one child with Kevin, and no children with anyone else.

**8.20** Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $\times$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

**a.** Represent the property "$x$ is an even number."

**b.** Represent the property "$x$ is prime."

**c.** Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

**8.21** In Chapter 6, we used equality to indicate the relation between a variable and its value. For instance, we wrote $WA = red$ to mean that Western Australia is colored red. Representing this in first-order logic, we must write more verbosely $ColorOf(WA) = red$. What incorrect inference could be drawn if we wrote sentences such as $WA = red$ directly as logical assertions?

**8.22** Write in first-order logic the assertion that every key and at least one of every pair of socks will eventually be lost forever, using only the following vocabulary: $Key(x)$, $x$ is a key; $Sock(x)$, $x$ is a sock; $Pair(x, y)$, $x$ and $y$ are a pair; $Now$, the current time; $Before(t_1, t_2)$, time $t_1$ comes before time $t_2$; $Lost(x, t)$, object $x$ is lost at time $t$.

**8.23** For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)

**a.** No two people have the same social security number.

$$\neg \exists\, x, y, n \;\; Person(x) \land Person(y) \;\Rightarrow\; [HasSS\#(x, n) \land HasSS\#(y, n)].$$

**b.** John's social security number is the same as Mary's.

$$\exists\, n \;\; HasSS\#(John, n) \land HasSS\#(Mary, n).$$

**c.** Everyone's social security number has nine digits.

$$\forall\, x, n \;\; Person(x) \;\Rightarrow\; [HasSS\#(x, n) \land Digits(n, 9)].$$

**d.** Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$.

**8.24** Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

**a.** Some students took French in spring 2001.

**b.** Every student who takes French passes it.

**c.** Only one student took Greek in spring 2001.

**d.** The best score in Greek is always higher than the best score in French.

**e.** Every person who buys a policy is smart.

**f.** No person buys an expensive policy.

**g.** There is an agent who sells policies only to people who are not insured.

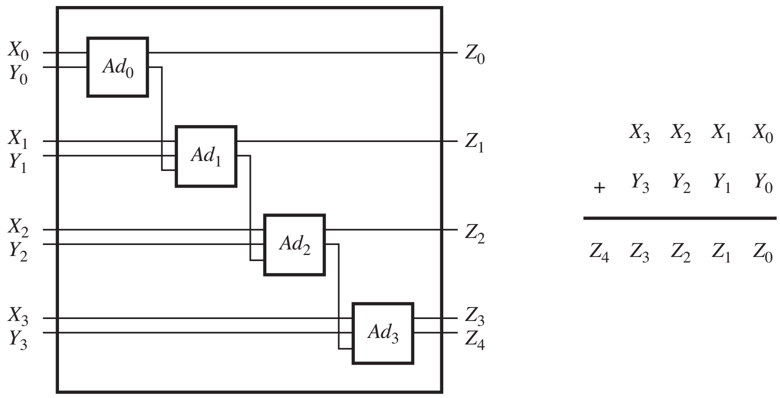


**Figure 8.8** A four-bit adder. Each $Ad_i$ is a one-bit adder, as in Figure 8.6 on page 309.

**h**. There is a barber who shaves all men in town who do not shave themselves.

**i**. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

**j**. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

**k**. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

**l**. All Greeks speak the same language. (Use $Speaks(x, l)$ to mean that person $x$ speaks language $l$.)

**8.25** Write a general set of facts and axioms to represent the assertion "Wellington heard about Napoleon's death" and to correctly answer the question "Did Napoleon hear about Wellington's death?"

**8.26** Extend the vocabulary from Section 8.4 to define addition for $n$-bit binary numbers. Then encode the description of the four-bit adder in Figure 8.8, and pose the queries needed to verify that it is in fact correct.

**8.27** Obtain a passport application for your country, identify the rules determining eligibility for a passport, and translate them into first-order logic, following the steps outlined in Section 8.4.

**8.28** Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

$CopyOf(d, a)$: Predicate. Disk $d$ is a copy of album $a$.
$Owns(p, d)$: Predicate. Person $p$ owns disk $d$.
$Sings(p, s, a)$: Album $a$ includes a recording of song $s$ sung by person $p$.
$Wrote(p, s)$: Person $p$ wrote song $s$.
$McCartney$, $Gershwin$, $BHoliday$, $Joe$, $EleanorRigby$, $TheManILove$, $Revolver$: Constants with the obvious meanings.

Express the following statements in first-order logic:

**a**. Gershwin wrote “The Man I Love.”

**b**. Gershwin did not write “Eleanor Rigby.”

**c**. Either Gershwin or McCartney wrote “The Man I Love.”

**d**. Joe has written at least one song.

**e**. Joe owns a copy of *Revolver*.

**f**. Every song that McCartney sings on *Revolver* was written by McCartney.

**g**. Gershwin did not write any of the songs on *Revolver*.

**h**. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)

**i**. There is a single album that contains every song that Joe has written.

**j**. Joe owns a copy of an album that has Billie Holiday singing “The Man I Love.”

**k**. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

**l**. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.