EE2703 : Applied Programming Lab

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Assignment 8

Abstract

In this Assignment we implement DFT in python and plot the Magnitude and Phase of the spectrum of the given signal.

Requirments:

Modules to be imported,

```
from pylab import *
```

DFT

The Discrete time fourier transform converts a finite sequence of equally spaced samples of a signal into a same length sequence of equally spaced samples of DTFT of the same signal.It is often used on machines because it is capable of all frequencies in limited memory unlike other transforms.the equations corresponding to the DFT are as follows.

$$DFT : X[k] = \sum_{n=0}^{N-1} x[n]W^{k}$$

$$inverseDFT : x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W^{-k}$$

where,

$$W = e^{-\frac{2\pi jn}{N}}$$

Implementing DFT in python

First the given continuous signal is sampled with sufficiently high frequency to capture the frequency components of the signal and stored it in y(t).The transform of y(t) is calculated by using fft command imported from pylab.the transform is given by Y=(fft(y)).but we use Y=fftshift(fft(y)) which rearranges the output of fft(y) such that zero and positive frequency components are shifted to rightside of the set.

- fft - Calculates DFT
- ifft - Calculates inverse DFT
- fftshift - Make the transform symmetrical w.r.t axis.

We use the above commands and plot the Magnitude and Phase plots of DFT obtained for given signal.

Q2:Spectrum of $sin^3(t)$ and $cos^3(t)$

Spectrum of $sin^3(t)$:

$$y = sin^3(t) = \frac{3sin(t) - sin(3t)}{4}$$

$$y = sin^3(t) = \frac{-j}{8}e^{-3jt} + \frac{3j}{8}e^{-jt} - \frac{3j}{8}e^{jt} + \frac{j}{8}e^{+3jt}$$

$$Y(\omega) = \frac{-j}{8}\delta(\omega+3) + \frac{3j}{8}\delta(\omega+1) - \frac{3j}{8}\delta(\omega-1) + \frac{j}{8}\delta(\omega-3)$$

Spectrum of $cos^3(t)$:

$$y = cos^3(t) = \frac{3cos(t) + cos(3t)}{4}$$

$$y = cos^3(t) = \frac{1}{8}e^{-3jt} + \frac{3}{8}e^{-jt} + \frac{3}{8}e^{jt} + \frac{1}{8}e^{+3jt}$$

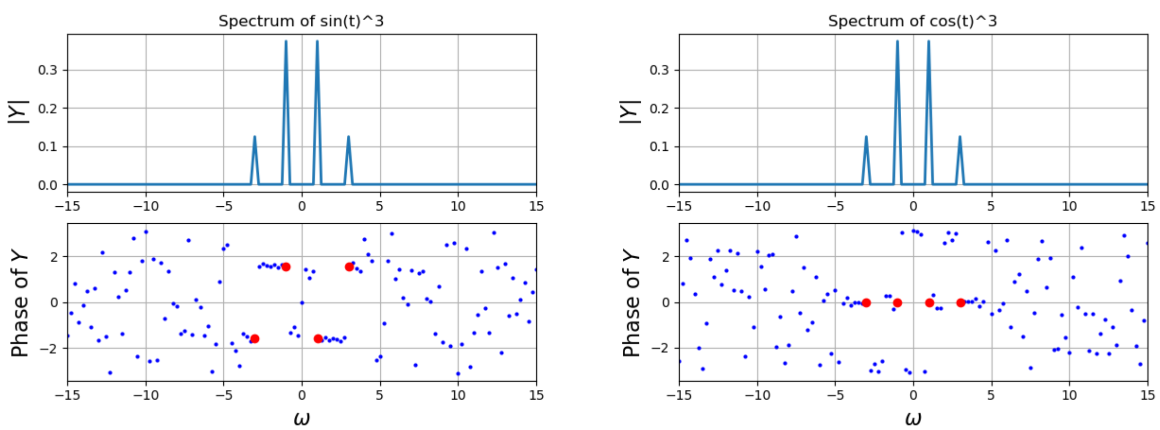
$$Y(\omega) = \frac{1}{8}\delta(\omega+3) + \frac{3}{8}\delta(\omega+1) + \frac{3}{8}\delta(\omega-1) + \frac{1}{8}\delta(\omega-3)$$

Code:

```
from pylab import *


tm=linspace(-4*pi,4*pi,513)
tm=tm[:-1]
w=linspace(-64,64,513)
w=w[:-1]
```

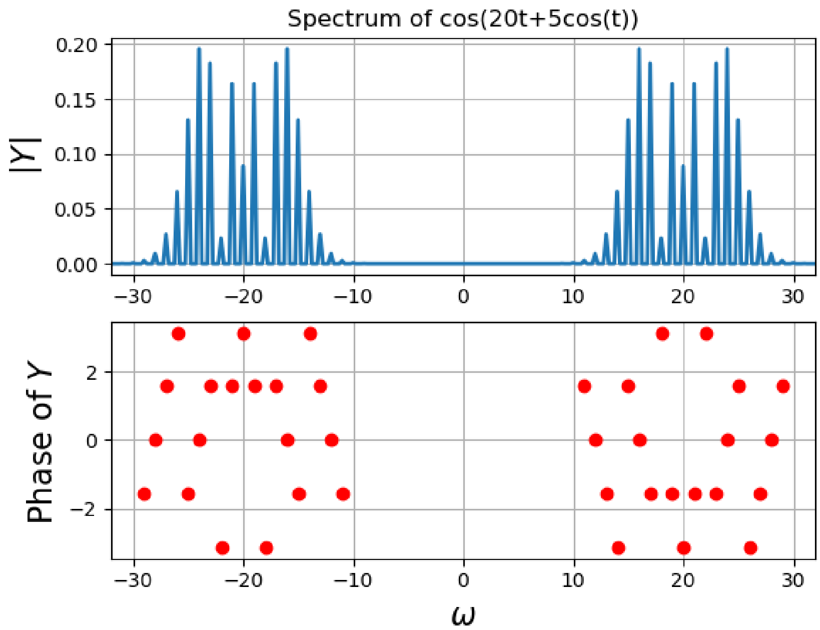
```
y=[sin(5*tm),(1+0.1*cos(tm))*cos(10*tm),sin(tm)**3,cos(tm)**3]
yn=['sin(5t)','(1+0.1cos(t))cos(10t)','sin(t)^3','cos(t)^3']
for i in range(len(y)):
        Y=fftshift(fft(y[i]))/512.0
        figure(i)
        subplot(2,1,1)
        plot(w,abs(Y),lw=2)
        xlim([-15,15])
        ylabel(r"$|Y|$",size=16)
        title(r"Spectrum of "+ yn[i])
        grid(True)
        subplot(2,1,2)
        plot(w,angle(Y),'bo',markersize=2,lw=2)
        ii=where(abs(Y)>1e-3)
        plot(w[ii],angle(Y[ii]),'ro',lw=2)
        xlim([-15,15])
        ylabel(r"Phase of $Y$",size=16)
        xlabel(r"$\omega$",size=16)
        grid(True)
show()
```



Q3:

Spectrum of cos(20t+5cos(t)):
Code:

```
y3=cos(20*tm+5*cos(tm))
Y=fftshift(fft(y3))/512.0
figure(len(y))
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-32,32])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of "+ 'cos(20t+5cos(t))')
grid(True)
subplot(2,1,2)
#plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
xlim([-32,32])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
```



Q4:Spectrum of the Guassian $e^{-t^2/2}$

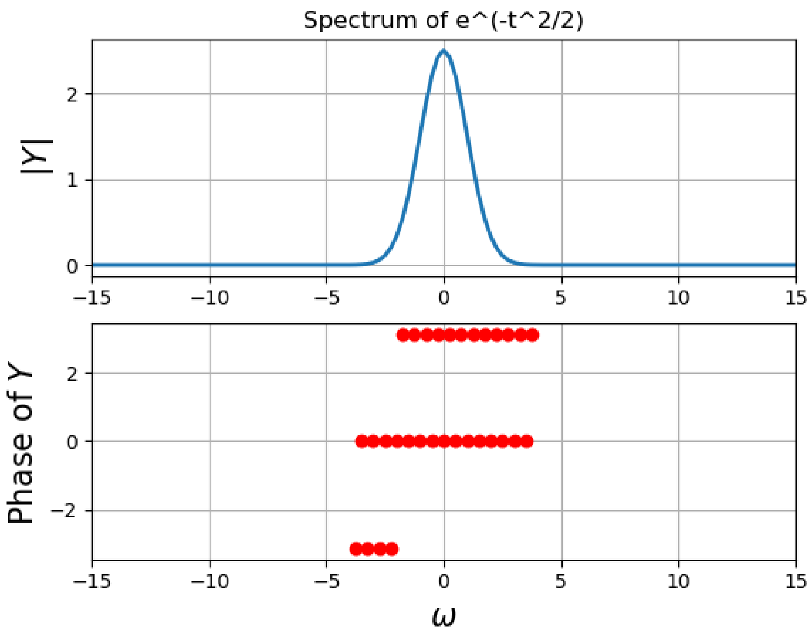
The true fourier transform of this is

$$Y(\omega) = \sqrt{2\pi}e^{-\omega^2/2}$$

Now we have to find the fourier transform for this using python and approximate this to the true transform.
Code:

```
y4=e**(-tm*tm/2)
Y=fftshift(fft(y4))*(8*pi)/(512)     # Multiply time period/no. of samples
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of "+'e^(-t^2/2)')
grid(True)
subplot(2,1,2)
#plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)

show()
```



Results

- For Q1, It is just an example and results are already mentioined in the PDF.
- For Q2(a),We can clearly see in the magnitude spectrum there are 4 peaks at frequencies 3, 1, 1, 3 which are expected from the below expansion and also phases are 180 degree apart of same frequency(Magnitude) as the signal is a sin function.(Peak Values are also as expected).
- ForQ2(b),Similar to the above question We can clearly see in the magnitude spectrum there are 4 peaks at frequencies 3, 1, 1, 3 which are expected as clearly shown in the below expansion and also phases 0 degree apart of same frequency(Magnitude) as the signal is a cos function.(Peak Values are also as expected)
- For Q3,As the signal is a phase modulated signal the center frequency among the peaks is 20rad/s which is kind of a carrier wave and other peaks are because of the oscillating phase.

- For Q4,We can clearly see the frequency spectrum is also a Gaussian which verifies the transform.As previously mentioned for T=8π(@Nsample=512) The error is in order of 10^{-15} Which says that the transform is pretty much accurate.