EE2703 : Applied Programming Lab

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Assignment 4

Abstract

We will t two functions, exp(x) and cos(cos(x)) over the interval [0,2) using the fourier series

$$a_o + \sum_{n=1}^{infinity} \{a_n cos(nx) + b_n sin(nx)\} \tag{1}$$

As you know from earlier courses, the coefcients an and bn are given by

$$a_o = \frac{1}{2\pi} \int_0^{2\pi} f(x)dx \tag{2}$$

$$a_n = \frac{1}{1\pi} \int_0^{2\pi} f(x)cosnxdx \tag{3}$$

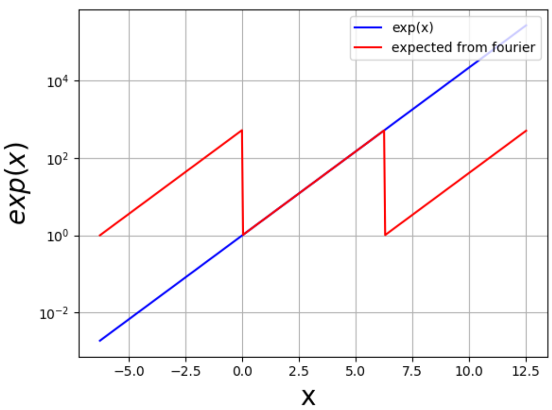
$$b_n = \frac{1}{1\pi} \int_0^{2\pi} f(x)sinnxdx \tag{4}$$

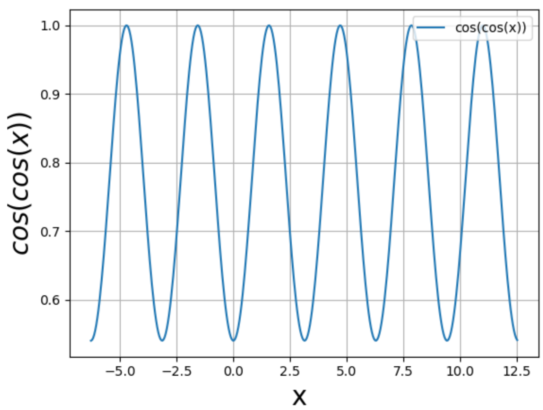
Q1:

CODE:

```
def epower(p):
        y=[]
        for i in range(len(p)):
                y.append(exp(p[i]))                # exp(x) function
        return y
def coscos(p):
        z=[]
        for i in range(len(p)):                # cos(cos(x))
                z.append(cos(cos(p[i])))
        return z
t=linspace(-2*(pi),4*(pi),401)
t=t[:-1]
ft=linspace(0,2*(pi),401)
ft=ft[:-1]
figure(1)
```

```
semilogy(t,epower(t),color='blue')
semilogy(ft,epower(ft),ft-2*math.pi,epower(ft),ft+2*pi,epower(ft),color='red')
xlabel('x ',size=20)
ylabel(r'$exp(x)$',size=20)
grid(True)
figure(2)
plot(t,coscos(t))
xlabel('x ',size=20)
ylabel(r'$cos(cos(x))$',size=20)
grid(True)
show()
```





Q2

CODE:

```
u1=lambda x,k: (exp(x))*cos(k*x)
v1=lambda x,k: (exp(x))*sin(k*x)
u2=lambda x,k: (cos(cos(x)))*cos(k*x)
v2=lambda x,k: (cos(cos(x)))*sin(k*x)
a1=[(c/2)*integrate.quad(u1,0,2/c,args=(0,))[0]]
b1=[(c/2)*integrate.quad(v1,0,2/c,args=(0,))[0]]
a2=[(c/2)*integrate.quad(u2,0,2/c,args=(0,))[0]]
b2=[(c/2)*integrate.quad(v2,0,2/c,args=(0,))[0]]
for i in range(1,26):
    a1.append(c*integrate.quad(u1,0,2/c,args=(i,))[0])
    a2.append(c*integrate.quad(u2,0,2/c,args=(i,))[0])
```

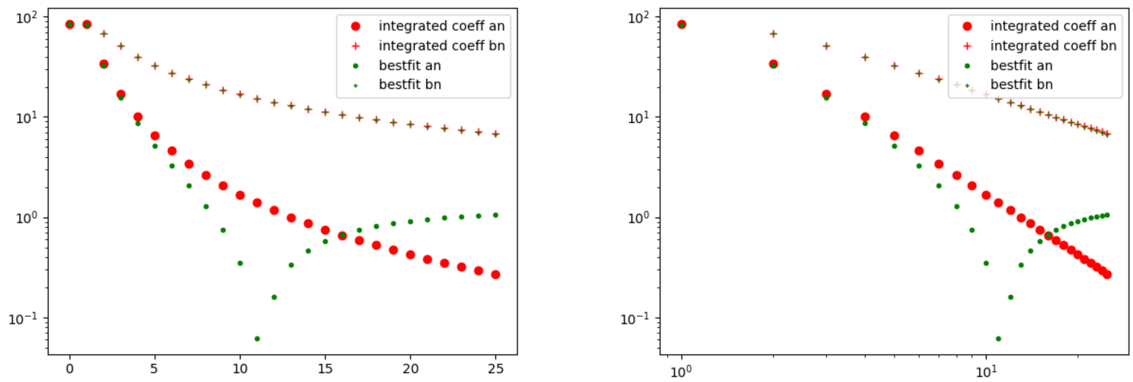
```
    b1.append(c*integrate.quad(v1,0,2/c,args=(i,))[0])
    b2.append(c*integrate.quad(v2,0,2/c,args=(i,))[0])
```

Q3,Q4,Q5,Q6:

CODE for getting coefficients from bestfirt method

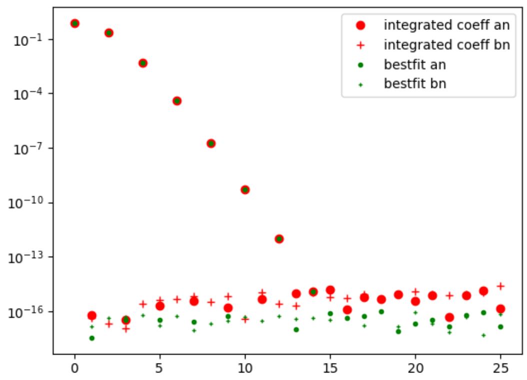
```
ans1=[a1[0]]
ans2=[a2[0]]
for i in range(1,len(a1)):
    ans1.append(a1[i])
    ans1.append(b1[i])
    ans2.append(a2[i])
    ans2.append(b2[i])
p=linspace(0,2*pi,401)
p=p[:-1]
B1=epower(p)
B2=coscos(p)
A=zeros((400,2*no_of_coefficients+1))
A[:,0]=1
for k in range(1,no_of_coefficients+1):
    A[:,2*k-1]=cos(k*p)
    A[:,2*k]=sin(k*p)
c1=lstsq(A,c_[B1],rcond=1)[0]
c2=lstsq(A,c_[B2],rcond=1)[0]
besta1=[c1[0]]
bestb1=[c1[0]]
besta2=[c2[0]]
bestb2=[c2[0]]
for k in range(1,no_of_coefficients+1):
    besta1.append(c1[2*k-1])
    besta2.append(c2[2*k-1])
    bestb1.append(c1[2*k])
    bestb2.append(c2[2*k])
gsize=3
\textbf{CODE for plotting fourier coefficients of exp(x)}
figure(3)
semilogy(abs(array(a1)),'ro',label='integrated coeff an')
semilogy(abs(array(b1)),'r+',label='integrated coeff bn')
semilogy(abs(array(besta1)),'go',markersize=gsize,label='bestfit an')
semilogy(abs(array(bestb1)),'g+',markersize=gsize,label='bestfit bn')
```

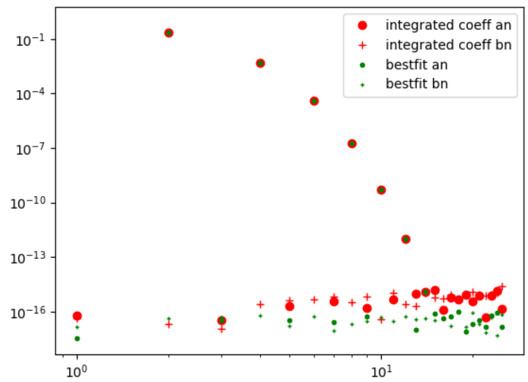
```
legend(loc='upper right')
figure(4)
loglog(abs(array(a1)),'ro',label='integrated coeff an')
loglog(abs(array(b1)),'r+',label='integrated coeff bn')
loglog(abs(array(besta1)),'go',markersize=gsize,label='bestfit an')
loglog(abs(array(bestb1)),'g+',markersize=gsize,label='bestfit bn')
legend(loc='upper right')
```



CODE for plotting fourier coefficients of cos(cos(x)

```
figure(5)
semilogy(abs(array(a2)),'ro',label='integrated coeff an')
semilogy(abs(array(b2)),'r+',label='integrated coeff bn')
semilogy(abs(array(besta2)),'go',markersize=gsize,label='bestfit an')
semilogy(abs(array(bestb2)),'g+',markersize=gsize,label='bestfit bn')
legend(loc='upper right')
figure(6)
loglog(abs(array(a2)),'ro',label='integrated coeff an')
loglog(abs(array(b2)),'r+',label='integrated coeff bn')
loglog(abs(array(besta2)),'go',markersize=gsize,label='bestfit an')
loglog(abs(array(bestb2)),'g+',markersize=gsize,label='bestfit bn')
legend(loc='upper right')
```





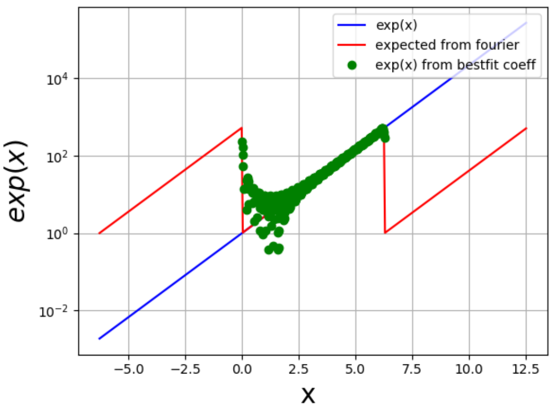
Finding deviation in coefficients

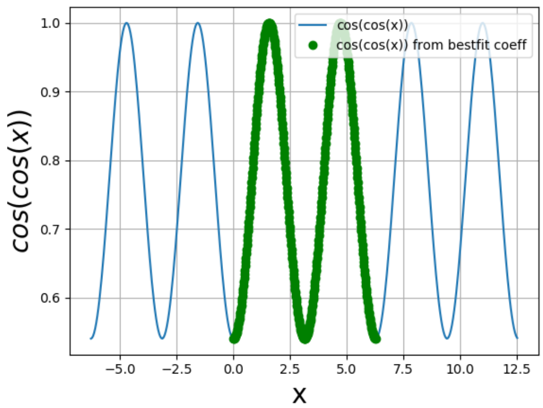
```
absdiff1=abs(c_[ans1]-c1)
absdiff2=abs(c_[ans2]-c2)
print('for exp(x) largest deviation '+str(max(absdiff1)))
print('for cos(cos(x)) largest deviation '+str(max(absdiff2)))
```

Q7:comparing function values with 51 bestfit fourier coefficients and general function

CODE:

```
figure(1)
semilogy(t,e**t,color='blue',label='exp(x)')
semilogy(t,epower(abs(t%(2*pi))),color='red',label='expected from fourier')
semilogy(p,dot(A,c1),'go',label='exp(x) from bestfit coeff')
xlabel('x ',size=20)
ylabel(r'$exp(x)$',size=20)
legend(loc='upper right')
grid(True)
figure(2)
plot(t,coscos(t),label='cos(cos(x))')
plot(p,dot(A,c2),'go',label='cos(cos(x)) from bestfit coeff')
xlabel('x ',size=20)
ylabel(r'$cos(cos(x))$',size=20)
legend(loc='upper right')
grid(True)
show()
```





Results

- For Q1,cos(cos(t)) is naturally periodic with period 2 π .Where as e t is not naturally periodic but we are making it to be periodic with period 2 π.

- For Q3,As cos(cos(t)) is an even function the values of bn are zero ideally.And e t's coefficients are not decaying faster than cos(cos(t)) because e t needs many more frequencies than cos(cos(t)) which can be represented with much fewer frequencies.

- For Q6,The Coefficients obtained from integration and from the Fitting need not be same as One considers infinite terms and other considers Finite terms only.

- For Q7,As the function e t is discontinuous at the boundaries there will always a ringing in the output summation.But if we consider more and more coefficients then this ringing is also reduced.

Conclusion

- In this assignment we learnt how to calculate some definite integrals using quad() function.

- We learnt about Fourier series and how to calculate its coefficients analytically.

- We also calculated some error plots and saw upto which degree are the results matching with analytical solution(of course approximated).