GATE: CH-62 2023

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Question : The transfer function of a measuring instrument is

$$G_m(s) = \frac{1.05}{2s+1} exp(-s)$$

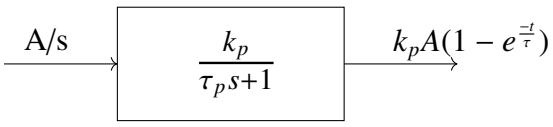
At time $t = 0$, a step change of +1 unit is introduced in the input of this instrument.The time taken by the instrument to show an increase of 1 unit in its output is(rounded off to two decimal places).
(GATE CH 2023)

Solution:

Parameter	Description	Value
k_p	Gain of the system	1.05
A	The initial value of the response at $t = 0$	1
τ_p	Time constant of the system	2
y(t)	Output of the system	1

TABLE 0: Given parameters

$$\mathcal{L}^{-\infty}(\frac{1}{s+a}) \longleftrightarrow e^{-at} \tag{1}$$



$$G_m(s) = \frac{1.05}{2s+1} exp(-s) \tag{2}$$

$$\because Y(s) = G_m(s).U(s) \tag{3}$$

$$\implies Y(s) = \frac{1}{s}.\frac{1.05}{2s+1} exp(-s) \tag{4}$$

By splitting into partial fractions,we get

$$Y(s) = [\frac{1.05}{s} - \frac{2.10}{2s+1}] exp(-s) \tag{5}$$

By taking inverse laplace transform,

$$y(t) = 1.05[1 - e^{\frac{-(t-1)}{2}}] \tag{6}$$

$$\frac{1}{1.05} = 1 - e^{\frac{-(t-1)}{2}} \tag{7}$$

$$\frac{-(t-1)}{2} = ln(\frac{0.05}{1.05}) \tag{8}$$

$$\implies t = 7.073 \tag{9}$$

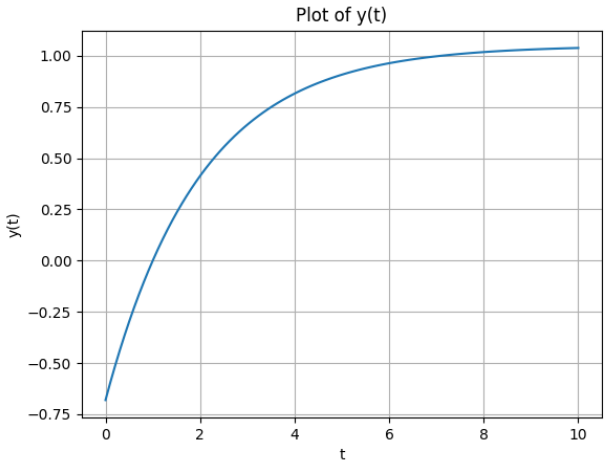


Fig. 0: y(t) = $1.05[1 - e^{\frac{t-1}{2}}]$