

GATE: CH-62 2023

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Question : The transfer function of a measuring instrument is

$$G_m(s) = \frac{1.05}{2s + 1} \exp(-s)$$

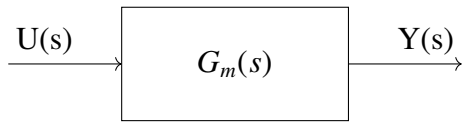
At time $t = 0$, a step change of +1 unit is introduced in the input of this instrument. The time taken by the instrument to show an increase of 1 unit in its output is (rounded off to two decimal places).

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Solution:

Parameter	Description	Value
$G_m(s)$	Transfer function	$\frac{Y(s)}{U(s)}$
$Y(s)$	Laplace transform of the output	?
$U(s)$	Laplace transform of the input	$\frac{1}{s}$

TABLE 0: Given parameters



$$G_m(s) = \frac{1.05}{2s + 1} e^{-s} \quad (1)$$

$$\therefore Y(s) = G_m(s) \cdot U(s) \quad (2)$$

$$\Rightarrow Y(s) = \frac{1}{s} \cdot \frac{1.05}{2s + 1} e^{-s} \quad (3)$$

By splitting into partial fractions, we get

$$Y(s) = \left[\frac{1.05}{s} - \frac{1.05}{s + 0.5} \right] e^{-s} \quad (4)$$

As we know,

$$\mathcal{L}[e^{-at}] \longleftrightarrow \frac{1}{s + a} \quad (5)$$

$$\mathcal{L}[f(t - 1)] \longleftrightarrow e^{-s} F(s) \quad (6)$$

By taking inverse laplace we get,

$$y(t) = 1.05 \left[1 - e^{-\frac{t-1}{2}} \right] u(t - 1) \quad (7)$$

$$\frac{1}{1.05} = 1 - e^{-\frac{t-1}{2}} \quad (8)$$

$$\frac{-(t-1)}{2} = \ln\left(\frac{0.05}{1.05}\right) \quad (9)$$

$$\Rightarrow t = 7.073 \quad (10)$$

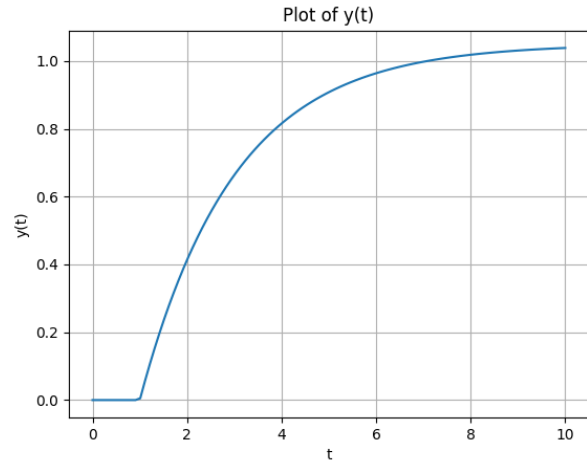


Fig. 0: $y(t) = 1.05[1 - e^{-\frac{t-1}{2}}]$