GATE: AG-26 2021

EE23BTECH11038 - Rohith Madhani*

Question : Solution of differential equation $y'' + y' + 0.25y = 0$ with initial values $y(0) = 3$ and $y'(0) = -3.5$ is

(A) $y = (3 - 2x)e^{0.5x}$
(B) $y = (3 - 2x)e^{-0.25x}$
(C) $y = (3 - 2x)e^{-0.5x}$
(D) $y = (2 - 3x)e^{-0.5x}$

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Solution:

Parameter	Description	Value
$y(t)$	y in t domain	?
$y(0)$	y at $t = 0$	3
$y'(0)$	y' at $t = 0$	-3.5

TABLE 0: Given parameters

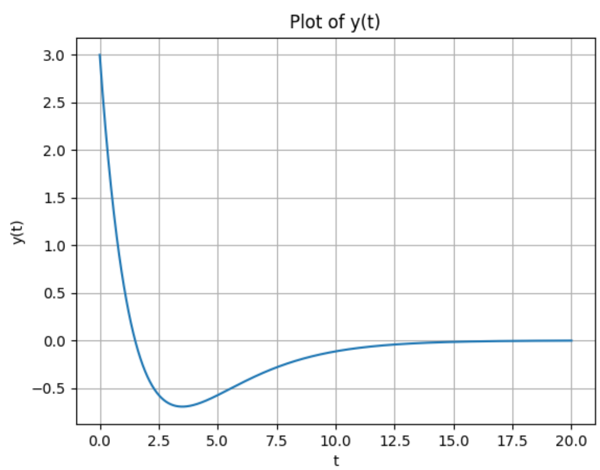

Fig. 0: $y(t) = \left[(3 - 2t)e^{-0.5t}\right]u(t)$

By applying laplace transform to the differential equation,

$$y'' + y' + 0.25y \xleftrightarrow{\mathcal{L}} s^2Y(s) - sy(0) - y'(0) + sY(s) - y(0) + 0.25Y(s) \tag{1}$$

$$Y(s)(s^2 + s + 0.25) = 3s - 0.5 \tag{2}$$

$$\implies Y(s) = \frac{3s - 0.5}{s^2 + s + 0.25} \tag{3}$$

$$= \frac{3}{s + 0.5} - \frac{2}{(s + 0.5)^2} \tag{4}$$

As we know,

$$\frac{b}{(s + a)^n} \xleftrightarrow{\mathcal{L}^{-1}} \frac{b}{(n - 1)!} \cdot t^{n-1}e^{-at} \cdot u(t) \tag{5}$$

By taking inverse laplace of (4), we get

$$y(t) = \frac{3}{0!}.e^{-0.5t}.u(t) - \frac{2}{1!}t.e^{-0.5t}.u(t) \tag{6}$$

$$\implies y(t) = \left[(3 - 2x)e^{-0.5t}\right]u(t) \tag{7}$$

Hence the correct answer is option (C)