

NCERT 11.9.5 Q4

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Question : Find the sum of all numbers between 200 and 400 which are divisible by 7.

Solution:

Parameter	Description	Value
$x(n)$	n^{th} term of the AP	$(x(0)+nd)u(n)$
$x(0)$	1 st term of the AP	?
d	Common Difference of the AP	7

TABLE 0
INPUT PARAMETERS

The first and last term of the AP are 203 and 399 respectively.

$$\Rightarrow x(n) = (203 + 7n)u(n) \quad (1)$$

To calculate the number of terms in the AP,

$$\begin{aligned} 399 &= 203 + 7n & (2) \\ \Rightarrow n &= 28 & (3) \end{aligned}$$

From (??)

$$X(z) = \frac{203}{1 - z^{-1}} + \frac{7 \cdot z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \quad (5)$$

$$\therefore y(n) = x(n) * u(n) \quad (6)$$

$$Y(z) = X(z)U(z) \quad (7)$$

$$\Rightarrow Y(z) = \frac{203}{(1 - z^{-1})^2} + \frac{7 \cdot z^{-1}}{(1 - z^{-1})^3}; |z| > 1 \quad (8)$$

Using Contour integration for inverse Z transform,

$$y(28) = \frac{1}{2\pi j} \oint_c Y(z)z^{27} dz \quad (9)$$

$$= \frac{1}{2\pi j} \int \frac{203 \cdot z^{29}}{(z-1)^2} dz + \frac{1}{2\pi j} \int \frac{7 \cdot z^{29}}{(z-1)^3} dz \quad (10)$$

$$\therefore R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \cdot \frac{203 \cdot z^{29}}{(z-1)^2} \right) \quad (12)$$

$$= 203 \times 29 = 5887 \quad (13)$$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{7 \cdot z^{29}}{(z-1)^3} \right) \quad (14)$$

$$= \frac{7 \times 29 \times 28}{2!} = 2842 \quad (15)$$

$$\Rightarrow y(28) = R_1 + R_2 \quad (16)$$

$$\Rightarrow y(28) = 8729 \quad (17)$$

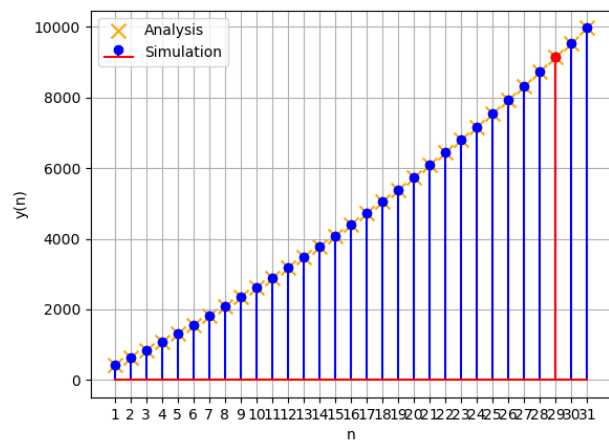


Fig. 0. $y(n) = 199.5n + 3.5n^2$