GATE: CH-62 2023

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Question : The transfer function of a measuring instrument is

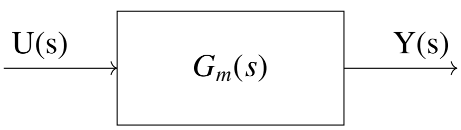
$$G_m(s) = \frac{1.05}{2s+1}exp(-s)$$

At time $t = 0$, a step change of $+1$ unit is introduced in the input of this instrument.The time taken by the instrument to show an increase of 1 unit in its output is(rounded off to two decimal places).
(GATE CH 2023)

Solution:

Parameter	Description	Value
$G_m(s)$	Transfer function	$\frac{Y(s)}{U(s)}$
Y(s)	Laplace transform of the output	?
U(s)	Laplace transform of the input	$\frac{1}{s}$

TABLE 0: Given parameters



$$G_m(s) = \frac{1.05}{2s+1}e^{-s} \tag{1}$$

$$\because Y(s) = G_m(s).U(s) \tag{2}$$

$$\implies Y(s) = \frac{1}{s}.\frac{1.05}{2s+1}e^{-s} \tag{3}$$

By splitting into partial fractions,we get

$$Y(s) = [\frac{1.05}{s} - \frac{1.05}{s+0.5}]e^{-s} \tag{4}$$

As we know,

$$\mathcal{L}[e^{-at}] \longleftrightarrow \frac{1}{s+a} \tag{5}$$

$$\mathcal{L}[f(t-1)] \longleftrightarrow e^{-s}F(s) \tag{6}$$

By taking inverse laplce we get,

$$y(t) = 1.05[1 - e^{\frac{-(t-1)}{2}}]u(t-1) \tag{7}$$

$$\implies y(t) = \begin{cases} 0 & t < 1, \\ 1.05[1 - e^{\frac{-(t-1)}{2}}] & t \geq 1 \end{cases} \tag{8}$$

$$\frac{1}{1.05} = 1 - e^{\frac{-(t-1)}{2}} \tag{9}$$

$$\frac{-(t-1)}{2} = ln(\frac{0.05}{1.05}) \tag{10}$$

$$\implies t = 7.073 \tag{11}$$

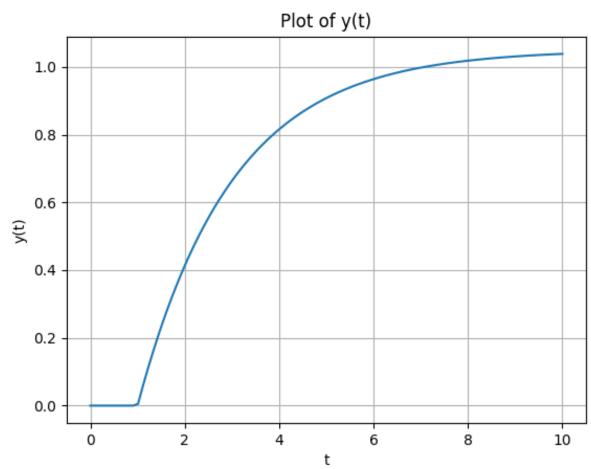


Fig. 0: y(t) = $1.05[1 - e^{\frac{t-1}{2}}]$