NCERT 11.9.5 Q4

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Question : Find the sum of all numbers between 200 and 400 which are divisible by 7.
Solution:

Parameter	Description	Value
$x(n)$	n^{th} term of the AP	$(x(0)+nd)u(n)$
$x(0)$	1st term of the AP	?
$x(l)$	last term of the AP	?
$y(n)$	Sum of n terms of the AP	$x(n)*u(n)$
d	Common Difference of the AP	7

TABLE 0

INPUT PARAMETERS

To calculate the first number between 200 and 400 divisible by 7 we can use modulo and calculate the remainder when 200 is divided by 7.

$$\implies 200 mod 7 = 4 \tag{1}$$
$$(200+3) mod 7 = 4+3 \tag{2}$$
$$203 mod 7 = 0 \tag{3}$$
$$\therefore x(0) = 203 \tag{4}$$

Similarly for last number between 200 and 400 divisible by 7 we can use modulo and calculate the remainder when 400 is divided by 7.

$$\implies 400 mod 7 = 1 \tag{5}$$
$$(400-1) mod 7 = 1-1 \tag{6}$$
$$399 mod 7 = 0 \tag{7}$$
$$\therefore x(l) = 399 \tag{8}$$

To calculate the number of terms in the AP,

$$x(l) = x(0) + 7l \tag{9}$$
$$399 = 203 + 7l \tag{10}$$
$$\implies l = 28 \tag{11}$$
$$\therefore x(n) = (203 + 7n)u(n) \tag{12}$$

From (**??**)

$$X(z) = \frac{203}{1-z^{-1}} + \frac{7.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{13}$$
$$\because y(n) = x(n) * u(n) \tag{14}$$
$$Y(z) = X(z)U(z) \tag{15}$$
$$Y(z) = \frac{203}{(1-z^{-1})^2} + \frac{7.z^{-1}}{(1-z^{-1})^3}; |z| > 1 \tag{16}$$

Using Contour integration for inverse Z transform,

$$y(28) = \frac{1}{2\pi j} \oint_c Y(z)z^{27} dz \tag{17}$$
$$= \frac{1}{2\pi j} \int \frac{203.z^{29}}{(z-1)^2} dz + \frac{1}{2\pi j} \int \frac{7.z^{29}}{(z-1)^3} dz \tag{18}$$
$$\because R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \tag{19}$$
$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 . \frac{203.z^{29}}{(z-1)^2} \right) \tag{20}$$
$$= 203 \times 29 = 5887 \tag{21}$$
$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 . \frac{7.z^{29}}{(z-1)^3} \right) \tag{22}$$
$$= \frac{7 \times 29 \times 28}{2!} = 2842 \tag{23}$$
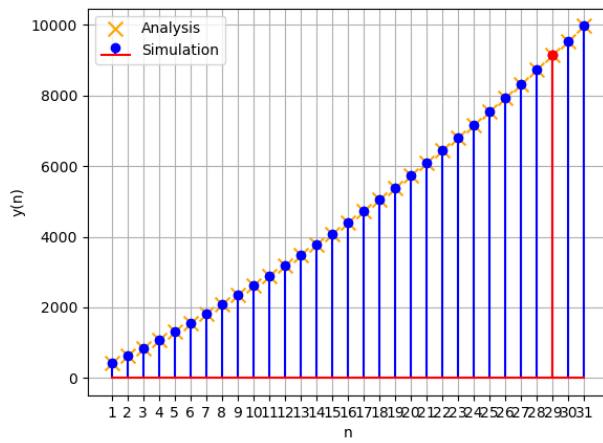$$\implies y(28) = R_1 + R_2 \tag{24}$$
$$\therefore y(28) = 8729 \tag{25}$$

Fig. 0. y(n) = $199.5n + 3.5n^2$