# GATE: AG-26 2021

EE23BTECH11038 - Rohith Madhani*

**Question :** Solution of differential equation $y'' + y' + 0.25y = 0$ with initial values $y(0) = 3$ and $y'(0) = -3.5$ is

(A) $y = (3 - 2x)e^{0.5x}$
(B) $y = (3 - 2x)e^{-0.25x}$
(C) $y = (3 - 2x)e^{-0.5x}$
(D) $y = (2 - 3x)e^{-0.5x}$

(GATE AG 2021)

**Solution:**

| Parameter | Description | Value |
|-----------|-------------|-------|
| $Y(s)$ | y in laplace domain | ? |
| $y(x)$ | y in x domain | ? |
| $y(0)$ | y at $x = 0$ | 3 |
| $y'(0)$ | $y'$ at $x = 0$ | -3.5 |

TABLE 0: Given parameters

By applying laplace transform to the differential equation,

$$y'' + y' + 0.25y \overset{\mathcal{L}}{\longleftrightarrow} s^2Y(s) - sy(0) - y'(0) + sY(s) - y( \tag{1}$$



Plot of y(x)

$$Y(s)(s^2 + s + 0.25) = 3s - 0.5 \tag{2}$$

$$\implies Y(s) = \frac{3s - 0.5}{s^2 + s + 0.25} \tag{3}$$
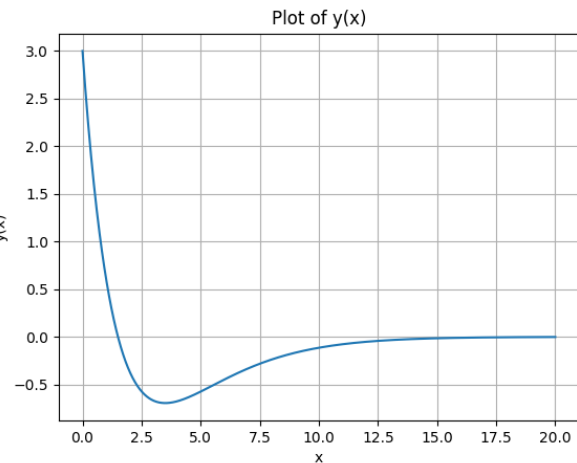
$$= \frac{3}{s + 0.5} - \frac{2}{(s + 0.5)^2} \tag{4}$$

Fig. 0: $y(x) = \left[(3 - 2x)e^{-0.5x}\right]u(x)$

As we know,

$$\frac{b}{(s + a)^n} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} \frac{b}{(n - 1)!} \cdot x^{n-1}e^{-ax} \cdot u(x) \tag{5}$$

By taking inverse laplace of (**??**), we get

$$y(x) = \frac{3}{0!}.e^{-0.5x}.u(x) - \frac{2}{1!}x.e^{-0.5x}.u(x) \tag{6}$$

$$\implies y(x) = \left[(3 - 2x)e^{-0.5x}\right]u(x) \tag{7}$$

Hence the correct answer is option (C)