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GATE: CH-62 2023

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Question: The transfer function of a measuring instrument is

$$G_m(s) = \frac{1.05}{2s+1} exp(-s)$$

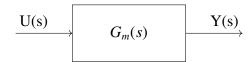
At time t = 0, a step change of +1 unit is introduced in the input of this instrument. The time taken by the instrument to show an increase of 1 unit in its output is (rounded off to two decimal places).

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Solution:

Parameter	Description	Value
$G_m(s)$	Transfer function	$\frac{Y(s)}{U(s)}$
Y(s)	Laplace transform of the output	?
U(s)	Laplace transform of the input	1 5

TABLE 0: Given parameters



$$G_m(s) = \frac{1.05}{2s+1}e^{-s} \tag{1}$$

$$Y(s) = G_m(s).U(s)$$
 (2)

$$\implies Y(s) = \frac{1}{s} \cdot \frac{1.05}{2s+1} e^{-s}$$
 (3)

By splitting into partial fractions, we get

$$Y(s) = \left[\frac{1.05}{s} - \frac{1.05}{s + 0.5} \right] e^{-s} \tag{4}$$

As we know,

$$\mathcal{L}[e^{-at}] \longleftrightarrow \frac{1}{s+a} \tag{5}$$

$$\mathcal{L}[f(t-1)] \longleftrightarrow e^{-s}F(s)$$
 (6)

By taking inverse laplce we get,

$$y(t) = 1.05 \left[1 - e^{\frac{-(t-1)}{2}} \right] u(t-1)$$
 (7)

$$\frac{1}{1.05} = 1 - e^{\frac{-(t-1)}{2}} \tag{8}$$

$$\frac{-(t-1)}{2} = \ln(\frac{0.05}{1.05})\tag{9}$$

$$\implies t = 7.073 \tag{10}$$

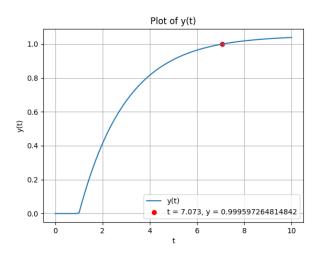


Fig. 0: $y(t) = 1.05 \left[1 - e^{\frac{t-1}{2}} \right]$