

NCERT 10.5.3 Q16

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Question : A sum of Rs.700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs.20 less than its preceding prize, find the value of each of the prizes.

Solution:

Parameter	Description	Value
$x(n)$	n^{th} term of discrete signal	$(x(0)+20n)u(n)$
$x(0)$	1 st term of the AP	

TABLE 0: Given parameters

Taking Z transform of $x(n)$,

$$X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{20 \cdot z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \quad (1)$$

$$\because y(n) = x(n) * u(n) \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

$$Y(z) = \frac{x(0)}{(1 - z^{-1})^2} + \frac{20 \cdot z^{-1}}{(1 - z^{-1})^3} \quad (4)$$

Using contour integration for inverse Z transform,

$$y(6) = \frac{1}{2\pi j} \oint_c Y(z)z^5 dz \quad (5)$$

$$= \frac{1}{2\pi j} \int \frac{x(0) \cdot z^7}{(z-1)^2} dz + \frac{1}{2\pi j} \int \frac{20 \cdot z^7}{(z-1)^3} dz \quad (6)$$

$$\because R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (7)$$

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \cdot \frac{x(0) \cdot z^7}{(z-1)^2} \right) \quad (8)$$

$$= 7 \times x(0) \quad (9)$$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{20 \cdot z^7}{(z-1)^3} \right) \quad (10)$$

$$= 420 \quad (11)$$

$$\Rightarrow y(6) = R_1 + R_2 \quad (12)$$

$$700 = 420 + 7 \times x(0) \quad (13)$$

$$\therefore x(0) = 40 \quad (14)$$

The value of each of the prizes is

$$x(0) = 40 \quad (15)$$

$$x(1) = 40 + 20(1) = 60 \quad (16)$$

$$x(2) = 40 + 20(2) = 80 \quad (17)$$

$$x(3) = 40 + 20(3) = 100 \quad (18)$$

$$x(4) = 40 + 20(4) = 120 \quad (19)$$

$$x(5) = 40 + 20(5) = 140 \quad (20)$$

$$x(6) = 40 + 20(6) = 160 \quad (21)$$

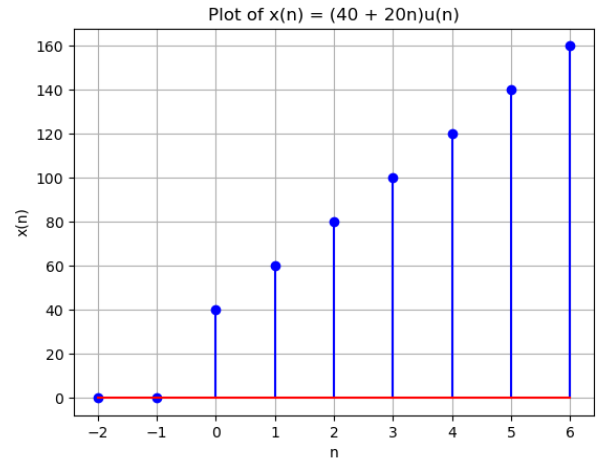


Fig. 0: $x(n) = (40 + 20n)u(n)$

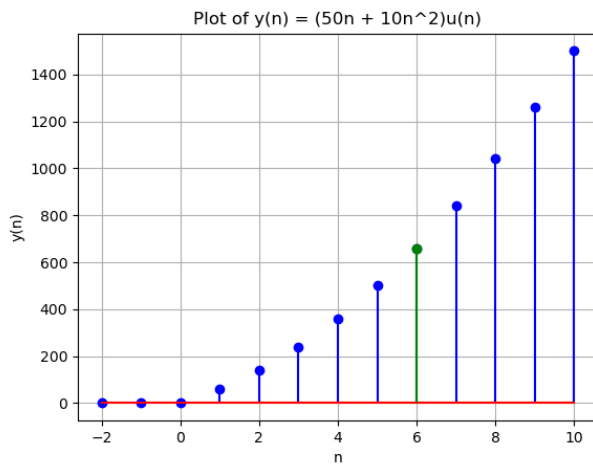


Fig. 0: $y(n) = (50n + 10n^2)u(n)$