NCERT 11.9.5 Q4

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Question : Find the sum of all numbers between 200 and 400 which are divisible by 7.
Solution:

Parameter	Description	Value
$x(n)$	n^{th} term of the AP	$(x(0)+nd)u(n)$
$x(0)$	1st term of the AP	?
d	Common Difference of the AP	7

TABLE 0

INPUT PARAMETERS

The first and last term of the AP are 203 and 399 respectively.

$$\implies x(n) = (203 + 7n)u(n) \tag{1}$$

To calculate the number of terms in the AP,

$$399 = 203 + 7n \tag{2}$$

$$\implies n = 28 \tag{3}$$

From (**??**)

$$X(z) = \frac{203}{1 - z^{-1}} + \frac{7.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{4}$$

$$\because y(n) = x(n) * u(n) \tag{5}$$

$$Y(z) = X(z)U(z) \tag{6}$$

$$\implies Y(z) = \frac{203}{(1 - z^{-1})^2} + \frac{7.z^{-1}}{(1 - z^{-1})^3}; |z| > 1 \tag{7}$$

Using Contour integration for inverse Z transform,

$$y(28) = \frac{1}{2\pi j} \oint_c Y(z)z^{27} dz \tag{8}$$

$$= \frac{1}{2\pi j} \int \frac{203.z^{29}}{(z - 1)^2} dz + \frac{1}{2\pi j} \int \frac{7.z^{29}}{(z - 1)^3} dz \tag{9}$$

$$\therefore R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \tag{10}$$

$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 . \frac{203.z^{29}}{(z - 1)^2} \right) \tag{11}$$

$$= 203 \times 29 = 5887 \tag{12}$$

$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 . \frac{7.z^{29}}{(z - 1)^3} \right) \tag{13}$$

$$= \frac{7 \times 29 \times 28}{2!} = 2842 \tag{14}$$
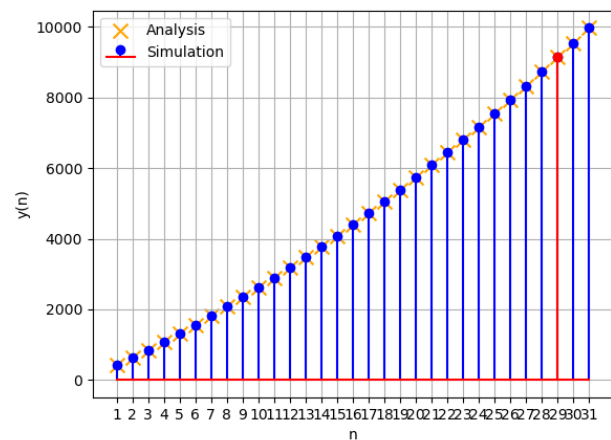
$$y(28) = R_1 + R_2 \tag{15}$$

$$\implies y(28) = 8729 \tag{16}$$

Fig. 0. $y(n) = 199.5n + 3.5n^2$