

qr_pivoting_python

November 19, 2023

```
[ ]: import pandas as pd
import numpy as np

df_ = pd.read_csv("dataset.csv")
print(df_)
for i in df_.columns:
    df_[i] = df_[i].astype("float64")
df = np.array(df_)
```

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	...	Q91	Q92	Q93	Q94	\
0	325	72	184	405	431	354	302	190	415	492	...	329	30	441	458	
1	229	235	141	482	170	242	219	313	173	148	...	224	139	221	25	
2	37	172	78	319	220	332	422	334	314	488	...	248	153	131	441	
3	224	465	421	101	236	252	142	216	432	218	...	330	65	50	86	
4	474	395	110	178	270	165	405	491	125	32	...	246	441	418	322	
5	282	432	241	146	67	327	373	162	354	166	...	175	211	212	6	
6	141	206	40	45	287	488	181	471	113	441	...	363	341	12	339	
7	110	92	462	373	298	7	315	88	387	282	...	30	375	189	440	
8	245	325	299	79	480	395	137	60	141	431	...	382	385	469	106	
9	291	290	114	344	399	369	87	52	196	163	...	367	127	391	116	

	Q95	Q96	Q97	Q98	Q99	Q100
0	406	307	125	201	282	260
1	27	187	316	238	384	134
2	407	223	21	51	228	466
3	120	441	9	43	45	107
4	461	45	199	176	457	20
5	113	485	25	133	52	105
6	471	481	137	271	345	286
7	20	160	129	294	440	149
8	59	305	105	54	122	283
9	83	52	184	444	304	32

[10 rows x 100 columns]

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[ ]: def qr_factorization_gram_schmidt_pivoting(A):
    m, n = A.shape
    Q = np.zeros((m, n))
    R = np.zeros((n, n))
    P = np.eye(n)
    for k in range(n):
        max_col_index = np.argmax(np.linalg.norm(A[:, k:], axis=0))
        max_col_index += k
        A[:, [k, max_col_index]] = A[:, [max_col_index, k]]
        P[:, [k, max_col_index]] = P[:, [max_col_index, k]]
        v = A[:, k]
        for j in range(k):
            R[j, k] = Q[:, j] @ A[:, k]
            v -= R[j, k] * Q[:, j]
        R[k, k] = np.linalg.norm(v)
        Q[:, k] = v / R[k, k]
    return Q, P, R
```

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[ ]: qr_factorization_gram_schmidt_pivoting(df)
```

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