**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Given

*μ* = 45; *σ* = 8;

x= 60 – 10 = 50

Z =

From z- scores table,

P (50>x) = 0.7324

But we need P(x>50) = 1- P (50>x)

= 1 - 0.7324

= 0.2676

The probability that the service manager cannot meet his commitment is 26.76%

From python

from scipy.stats import norm

x= 50

m=45

s=8

P= 1 - norm.cdf(x,m,s)

P

Output: 0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

Given

N = 400; *μ* = 38; *σ* =6

1. More employees at the processing center are older than 44 than between 38 and 44.

We know that probability of form mean to high extreme is 0.5 and in range of μ ± σ we have 0.68 probability.

Probability of employees is in between age of 38 and 44 **P(38<x<44)** = 0.68/2 = 0.34

No of employees is in between age of 38 and 44 = 400\*0.34 = 136

Probability of employees above age of 44**P(x>44)** = 0.5 - P(38<x<44) = 0.5-0.34 = 0.16

No of employees above age of 44 = 400\*0.16 = 64.

So, the above statement is ***False***

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Z =

P(x<30) = 0.0918

No of employees under the age of 30 = 0.0918\*400 = 36.712 i.e., 36 employees

So, the above statement is True.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

We know that if X ∼ N (µ1, σ1^2), and Y ∼ N (µ2, σ2^2) are two independent random variables then X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2), and X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y, then Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2 µ,2 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 µ, 2σ^2)

***2X1-(X1+X2) ~ N (0*** ***µ,4 σ^2)***

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Given µ = 100, σ = 20

We have to use inverse of cumulative probability function to get Z-score.

from scipy.stats import norm

norm.ppf(0.99)

Output:

2.3263478740408408

From z-score to range

Z =

Z= 2.326347874

Then,

X1= z\* + = 2.326347874\*20 +100

= 146.5269574808168

X2= -z\* + = -2.326347874\*20 +100

= 53.47304251918318

Probability of 0.99 is in (53.4730425, 146.526957).

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Given:

Profit1 ~ N (5, 32); Profit2 ~ N (7, 42)

Total profits = Profit1 + Profit2

~ N (5+7,32+ 42)

~ N (13, 52)

µ = 13

σ = 5

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

We use inverse of cumulative probability function or z- score tables to get z-score.

(100-95)/2 = 2.5

From z-score table z-score for 97.5% is 1.96 and for 2.5 is -1.96

1.96 = 🡪 x1 = 22.8

-1.96 = 🡪 x2 = 3.2

The range that contains 95% probability for the annual profit of the company is 3.2 to 22.8 $ million or **14.4 ₹ crores to 102.6 ₹ crores.**

1. Specify the 5th percentile of profit (in Rupees) for the company

From z-score table z-score for 5% is -1.64

Or

From python

from scipy.stats import norm

norm.ppf(0.09)

Output:

-1.6448536269514729

z-score = -1.6448536269514729

Z =

-1.6448536269514729= 🡪 x = 4.776 $ million

i.e., The 5th percentile of profit (in Rupees) for the company is 21.49079 ₹ crores

1. Which of the two divisions has a larger probability of making a loss in a given year?

Making loss mean profits are negative. i.e., x<0

From python P(x<0)

from scipy.stats import norm

# Division -1

norm.cdf (0,5,3)

Out: 0.0477903522728147

# Division -2

norm.cdf (0,7,4)

Out: 0.040059156863817086

We can see that probability of division-1 in making loss is greater than probability of division-2.

i.e., 0.04779 >0.040059