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Q-1 :- $y[n] = x[n] * h[n]$

(a) $x[n] = u[n]$, $h[n] = 2^n u[-n]$

$$y[n] = \sum_{K=-\infty}^{\infty} h[K] * x[n-K]$$

$h[K]$

$K \leq 0 \Rightarrow -K \geq 0$

$2^K u[-K]$ is non zero

$u[n-K]$ to be non zero

$n-K \geq 0$

$\boxed{K \leq n}$

case ① $n < 0$

$$y[n] = \sum_{K=-\infty}^{n} 2^K$$

$$= \sum_{K=-\infty}^{-1} 2^K + \sum_{K=0}^{n} 2^K$$

$\downarrow$ $K$

$\hookrightarrow |\alpha| > 1$ GP $= \frac{2^{n+1}-1}{2-1} = (2^{n+1}-1)$

replace $K \to -K$

$$= \sum_{K=1}^{\infty} \left(\frac{1}{2}\right)^K = \sum_{K=0}^{\infty} \left(\frac{1}{2}\right)^K - 1$$

$= 1 + 2^{n+1} - 1$

$= 2^{n+1}$

$= |\alpha| < 1$

$$\left(\frac{1}{1-\frac{1}{2}}\right) - 1 = 2 - 1 = 1$$

case ② $n \geq 0$

$$y[n] = \sum_{K=-\infty}^{0} 2^K = \sum_{K=0}^{\infty} \left(\frac{1}{2}\right)^K$$

replace $K \to -K$ $= \frac{1}{1-\frac{1}{2}}$ $= 2$

$$y[n] = \begin{cases} 2^{n+1} & n < 0 \\ 2 & n \geq 0 \end{cases}$$

b) $x[n] = u[n] - u[n-N]$

$h[n] = a^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k].$$

$x[k]$ equals 1 for $k = 0, 1 \dots N-1$ else 0

$h[n-k] = a^{n-k} u[n-k]$ $\quad u[n-k] = 1 \quad n-k \geq 0$

$k \leq n$

~~k=0,1, ---~~

Let $m = \min(n, N-1)$

$$y[n] = \sum_{k=0}^{m} a^{n-k}$$

$$y[n] = a^{n-m} \sum_{k=0}^{m} a^{m-k}$$

$$\sum_{k=0}^{m} a^{m-k} = \sum_{j=0}^{m} a^j$$

$$y[n] = a^{n-m} \sum_{j=0}^{m} a^j$$

$$\sum_{j=0}^{m} a^j = 1 + a + a^2 + \dots + a^m = \frac{1 - a^{m+1}}{1-a} \quad (a \neq 1)$$

$$y[n] = a^{n-m} \frac{1 - a^{m+1}}{1-a}$$

$m = \min(n, N-1)$

$y[n] = a^{n-N+1} \dfrac{1-a^N}{1-a}, \; n \geq N$

$(0 < a < 1)$

**Case A:** $n < 0$ $\quad$ for $N > 0$

$x[k]$ & $h[n-k]$

$y[n] = 0, \; n < 0$

**Case B:** $0 \leq n \leq N-1$

Then $m = n$

$$y[n] = a^{n-n} \frac{1 - a^{n+1}}{1-a} = \frac{1 - a^{n+1}}{1-a}$$

**Case C:** $n \geq N$

then $m = N-1$. Substitute:

$$y[n] = a^{n-(N-1)} \frac{1 - a^N}{1-a} = a^{n-N+1} \frac{1-a^N}{1-a}$$

$$y[n] = \begin{cases} 0 & n < 0 \\ \dfrac{1 - a^{n+1}}{1-a} & 0 \leq n \leq N-1, \end{cases}$$

b) for $N=0$

$x[n]=0$

the convolution is 0

$\boxed{y[n]=0}$

for $N<0$ with

$(M=-N)$

$x[k] \rightarrow$ non-zero

$x[k]=-1 \leftarrow (-M \leq k \leq -1)$

$$y[n]=\sum_{k=-M}^{-1}\underbrace{(-1)}_{x[k]}\cdot\underbrace{\alpha^{n-k}u[n-k]}_{h[n-k]}$$

$u[n-k] \rightarrow$ non-zero

$n-k \geq 0$

$k \leq n$

Interval (1)

$n \leq -m$

$-m \leq k \leq -1$

$k < n$

$k < -m$

$y[n]=0 \leftarrow$ the summation range is empty

Interval (2) $-m \leq n < 0$

$\min(n,-1)=n$

$$y[n]=-\sum_{k=-m}^{n}\alpha^{n-k}$$

$j=n-k$

$k=-m$

$k=n$

$$=-\sum_{j=0}^{n+m}\alpha^j=-\left(\frac{\alpha^0-\alpha^{n+m+1}}{1-\alpha}\right)$$

$$=\left(\frac{\alpha^{n+m+1}-1}{1-\alpha}\right)$$

Interval (3) $n \geq 0$

$\min(n,-1)=-1$

$$y[n]=-\sum_{k=-m}^{-1}\alpha^{n-k} \rightarrow -\sum_{j=n+1}^{n+m}\alpha^j=\left(\frac{\alpha^{n+1}-\alpha^{n+m+1}}{1-\alpha}\right)$$

$j=n-k$

$k=-m \quad j=n+M$

$k=-1 \quad j=n+1$

$$=\alpha^{n+1}\left(\frac{\alpha^m-1}{1-\alpha}\right)$$

this is the case for $N<0$

& $M=-N$

$M>0$

c) $x(n) = \left(\frac{1}{2}\right)^n u(n), \quad h(n) = \delta(n) - \frac{1}{2}\delta(n-1)$

$y(n) = x(n) * h(n)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

step-2 substitute $h(n)$

$h(n) = \delta(n) - \frac{1}{2}\delta(n-1)$

$y(n) = x(n) - \frac{1}{2}x(n-1)$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$

$y(n) = \left(\frac{1}{2}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1)$

for $n \geq 1$

$y(n) = \left(\frac{1}{2}\right)^n - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n = 0$

for $n = 0$

$y(0) = \left(\frac{1}{2}\right)^0 = 1$

$y(n) = \delta(n)$

Q-2 :- $y_1(t) = x(t) * h_1(t)$

$y_2(t) = x(t) * h_2(t)$

$\therefore$ Parallel output $y(t) = y_1(t) + y_2(t)$

$y(t) = x(t) * h_1(t) + x(t) * h_2(t)$

by the dist prop

$y(t) = x(t) * (h_1(t) + h_2(t))$

$h(t) = h_1(t) + h_2(t)$

$h(t) = \left(e^{-2t} + 2e^{-t}\right) u(t)$

b) stability

$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty$$

$$\int_0^{\infty} (e^{-2t} + 2e^{-t}) \, dt$$

$$= \left[ -\frac{1}{2}e^{-2t} - 2e^{-t} \right]_0^{\infty}$$

$$= [0 - 0] - \left[ -\frac{1}{2} - 2 \right] = \frac{5}{2}$$

$= 2.5 < \infty$ BIBO stable.

Q3) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$x[n] = 2\delta[n] + \delta(n-3]$

$y[1]$

$y[n] = x[n] * h[n]$

$= \sum_{k=-\infty}^{\infty} x[k] h[n-k]$. ←

$n=0$ $x[0] = 2$

$n=3$ $x[3] = 1$

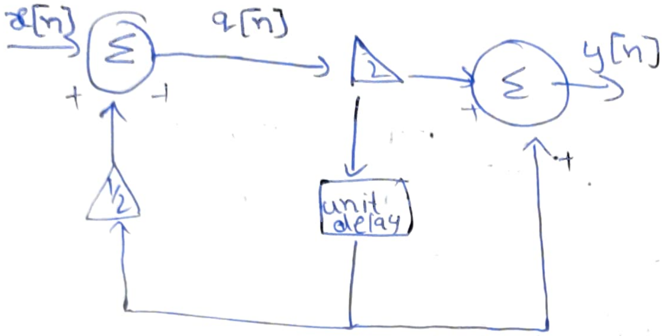
$y[n] = x[0] h[n] + x[3] h[n-3]$

$y[n] = 2h[n] + h[n-3] = 2\left(\left(\frac{1}{2}\right)^n u[n]\right) + \left(\frac{1}{2}\right)^{n-3} u[n-3]$

$y[1] = 2\left(\frac{1}{2} u[1]\right) + \left(\frac{1}{2}\right)^{-2} u[-2] = 1$

$y[4] = 2\left(\left(\frac{1}{2}\right)^4 u[4]\right) + \frac{1}{2} u[1] = \frac{1}{8} + \frac{1}{2} = \frac{5}{8} = 0.625$

Q-4 :




$$q[n] = x[n] + \frac{1}{2} q[n-1] \rightarrow (1)$$

$$y[n] = 2q[n] + q[n-1] \rightarrow (2)$$

$1 \times 2$

$$2q[n] = 2x[n] + q[n-1]$$

$$(-)\ y[n] = 2q[n] + q[n-1]$$

$$2q[n] - y[n] = 2x[n] - 2q[n]$$

$$4q[n] = y[n] + 2x[n]$$

$$\boxed{q[n] = \frac{y[n]}{4} + \frac{x[n]}{2}}$$

: sub $n \rightarrow n-1$

$$q[n-1] = \frac{y[n-1]}{4} + \frac{x[n-1]}{2}$$

sub in (1)

$$\frac{y[n]}{4} + \frac{x[n]}{2} = x[n] + \frac{y[n-1]}{8} + \frac{x[n-1]}{4}$$

$$\boxed{y[n] - \frac{y[n-1]}{2} = 2x[n] + x[n-1]}$$