

Agenda for today's live session

Analysis

Algorithms : It is a combination of **sequence of finite steps** to solve a particular problem.

for example : Multiplication of two numbers

Mul() {

1. Take two numbers (a,b)

2. Multiply two numbers a and b and store the value of result in c

3. return c

}

Properties of Algorithms :

- It should terminate after finite time.
- It should produce atleast one output.
- It is independent of any sort of programming language.
- It should be unambiguous (Deterministic)

Deterministic - For the same input same output will come always.

Not Deterministic - For the same input different output will come. And this is not at all preferable whenever we write any sort of algorithms.

Question : Based on the above mentioned properties, can you determine whether the below program is an algorithm or not ?

```
while(true){
```

```
System.out.println("Hello World");
```

```
}
```

Hello World

Hello World

Hello Worldinfinite times.....not a algorithm

Steps Required to construct an algorithm :

- Problem Definition -> what is the problem they are asking
- **Design algorithm** -> Out of existing algorithms which algorithm is more suitable to this problem statement.

Existing algorithms - Divide and Conquer, Greedy Technique, Dynamic Programming, Backtracking and so on.

- Draw flow chart
- Testing -> For every input correct output is coming or not
- Implementation -> Coding Part
- **Analysis**

2 1 6 3 8 => 1 2 3 6 8

Note : Design algorithm and analysis are the two major steps.

Analysis : If any problem contains more than one solution, then best one will be decided by the **analysis** based on mainly two factors :

1. **Time Complexity** - CPU time
2. **Space Complexity** - Main Memory Space

Note : **Time Complexity** is more powerful than **Space Complexity** because processor cost is more costly.

Time Complexity : $T(P) = C(P) + R(P)$

C(P) -> Compile-time is the time at which the **source code is converted into an executable code.**

R(P) -> Run time is the time at which the **executable code is started running.**

Types of analysis :

1. Apostiary Analysis (Relative Analysis) :

- Dependent on language of compiler and the type of hardware
- Exact answer
- Different answer
- Program run fast because of the type of hardware used

2. Apriori Analysis (Absolute Analysis) :

- Independent on language of compiler and the type of hardware
- Approximate answers
- same answer
- Program run fast because of nice logic

Apriori Analysis : It is a determination of order of magnitude of a statement.

O(magnitude)

Examples for better understanding of the concepts :

Problem 1 :

```
main() {
```

```
x = y + z;          O(1) - constant time - best case time complexity
```

```
}
```

Problem 2 :

```
main() {
```

```
  x = y + z;          O(1) - constant time
```

```
  for(i = 1; i <= n ; i++){
```

```
    x = y + z;          O(n) time
```

```
  }
```

```
}
```

Overall time complexity = $O(1) + O(n) = O(n)$

$O(n+1) = O(n)$

$n = 1000000000000 + 1 = 1000000000000$

$n = 5 = 5$ times

$n = 10 = 10$ times

$n = 1000 = 1000$ times and so on

$i = 1 ; 1 \leq 5 = \text{true}$

$x = y + z$ 1st time

$i++ = i + 1 = 1 + 1 = 2 ; 2 \leq 5 = \text{true}$

$x = y + z$ 2nd time

$i++ = i + 1 = 2 + 1 = 3 ; 3 \leq 5 = \text{true}$

$x = y + z$ 3rd time

$i++ = i + 1 = 3 + 1 = 4 ; 4 \leq 5 = \text{true}$

$x = y + z$ 4th time

$i++ = i + 1 = 4 + 1 = 5 ; 5 \leq 5 = \text{true}$

$x = y + z$ 5th time

$i++ = i + 1 = 5 + 1 = 6$; $6 \leq 5 = \text{false}$

no statement is executed and for loop is terminated here

Problem 3 :

main(){

$x = y + z;$ $O(1)$ - constant time

for($i = 1$; $i \leq n$; $i++$){

$x = y + z;$ $O(n)$ - time

}

for($i = 1$; $i \leq n$; $i++$){

for($j = 1$; $j \leq n$; $j++$){

$x = y + z;$ $O(n^2)$ time

}

}

}

So, overall time complexity of the above code : $1 + n + n^2 = O(n^2)$ time complexity.

Execution of code :

for($i = 1$; $i \leq n$; $i++$){

for($j = 1$; $j \leq n$; $j++$){

$x = y + z;$ $O(n^2)$ time

}

}

$n = 3 \Rightarrow 3 * 3 = 9$ times ($x = y + z$)

$n = 10 \Rightarrow 10 * 10 = 100$ times ($x = y + z$)

$n = 100000 \Rightarrow 100000 * 100000$ ($x = y + z$)

n^2

$i = 1; 1 \leq 3 = \text{true}$

$j = 1; 1 \leq 3 = \text{true}$

$x = y + z$ 1st time

$j++ = j + 1 = 1 + 1 = 2 \leq 3 = \text{true}$

$x = y + z$ 2nd time

$j++ = j + 1 = 2 + 1 = 3 \leq 3 = \text{true}$

$x = y + z$ 3rd time

$j++ = j + 1 = 3 + 1 = 4 \leq 3 = \text{false}$

// no statement is executed now

$i = 2; 2 \leq 3 = \text{true}$

$j = 1; 1 \leq 3 = \text{true}$

$x = y + z$ 4th time

$j++ = j + 1 = 1 + 1 = 2 \leq 3 = \text{true}$

$x = y + z$ 5th time

$j++ = j + 1 = 2 + 1 = 3 \leq 3 = \text{true}$

$x = y + z$ 6th time

$j++ = j + 1 = 3 + 1 = 4 \leq 3 = \text{false}$

// no statement is executed now

$i = 3; 3 \leq 3 = \text{true}$

$j = 1; 1 \leq 3 = \text{true}$

$x = y + z$ 7th time

$j++ = j + 1 = 1 + 1 = 2 \leq 3 = \text{true}$

$x = y + z$ 8th time

$j++ = j + 1 = 2 + 1 = 3 \leq 3 = \text{true}$

$x = y + z$ 9th time

 $j++ = j + 1 = 3 + 1 = 4 \leq 3 = \text{false}$

// no statement is executed now

$i++ = i + 1 = 3 + 1 = 4 ; 4 \leq 3 = \text{false}$

//no statement is executed now

Problem 4 :

main(){

$i = n;$

while($i > 1$){

$i = i - 1;$ $O(n-1) = O(n)$

}

}

$n = 10000000000000000 - 1$ is it effect?

n = 5 (Assume) 4 times (n-1) times

n = 10 9 times

n = 100 99 times

i = 5

5 > 1 = true

i = i - 1 = 5 - 1 = 4 1st time

i = 4

4 > 1 = true

i = i - 1 = 4 - 1 = 3 2nd time

i = 3

3 > 1 = true

i = i - 1 = 3 - 1 = 2 3rd time

i = 2

2 > 1 = true

i = i - 1 = 2 - 1 = 1 4th time

i = 1

1 > 1 = false no execution inside while loop

//now it will not enter into the while loop statements

Problem 5 :

```
main(){  
  i = n;  
  while(i >= 1){  
    i = i - 2;    // statement  
  }  
}
```

n = 10 assumption	5 times
n = 100	50 times
n = 1000	500 times
n	n/2 times

Overall time complexity of above program : $O(n/2) = O(n)$

i = 10

10 >= 1 true

i = 10 - 2 = 8 1st time

8 >= 1 true

i = 8 - 2 = 6 2nd time

6 >= 1 true

i = 6 - 2 = 4 3rd time

4 >= 1 true

i = 4 - 2 = 2 4th time

$2 \geq 1$ true

$i = 2 - 2 = 0$ 5th time

$0 \geq 1$ false

// no statement will be executed inside the while loop

Problem 6 :

```
main(){
```

```
  i = n;
```

```
  while(i >= 1){
```

```
    i = i - 30;                      // i = i - 35
```

```
    i = i - 5;
```

```
  }
```

```
}
```

n n/35 times $O(n/35) = O(n)$

Problem 7 :

```
main() {
```

```
  i = 1;
```

```
    while(i < n){
```

```
      i = 2 * i;      // statement
```

```
    }
```

```
}
```

n = 64 assumption 6 time

n = 32 5 time

n $\log_2 n$

Overall time complexity of above program : $O(\log_2 n)$

$i = 1$ given

$1 < 64$ true

$i = 2 * i = 2 * 1 = 2$ 1st time

$2 < 64$ true

$i = 2 * i = 2 * 2 = 4$ 2nd time

$4 < 64$ true

$i = 2 * i = 2 * 4 = 8$ 3rd time

$8 < 64$ true

$i = 2 * i = 2 * 8 = 16$ 4th time

$16 < 64$ true

$i = 2 * i = 2 * 16 = 32$ 5th time

$32 < 64$ true

$i = 2 * i = 2 * 32 = 64$ 6th time

$64 < 64$ false

// no statement will be executed after this

Conclusion :

- Time complexity is loop only
- Not only loop but larger loop

$$n + n^2 + n^3 = O(n^3)$$

- And if in a program there is no loop at all - $O(1)$ - best case scenario
