Recurrence Relation Solving : Substitution Method

Substitution Method : Substitute the given function repeatedly until the given function is removed.

Problem 1 :

T(n) = 1   if n = 1

      T(n-1) + n   if n>1

T(n)   = T(n-1) + n   1st time

       = T(n-2) + n-1 + n   2nd time

       = T(n-3) + n-2 + n-1 + n   3rd time

k times = n-1

n-k = 1

n-1 = k

       = T(n-k) + (n-k+1) + (n-k+2) +.........+n-2+n-1+n   k times

       = T(n-(n-1)) + (n-(n-1)+1) + (n-(n-1)+2) +.........+n-2+n-1+n

       = T(n-n+1) + (n-n+1+1) + (n-n+1+2) + ... + n-2+n-1+n

       = T(1) + 2 + 3 + 4 + 5 + ... + n-2+n-1+n

       = 1 + 2 + 3 + 4 + 5 + ... + n-2 + n-1 + n => Sum of n natural numbers

       = n(n+1)/2

       = (n^2 + n)/2

       = O(n^2)

Problem 2 :

$T(n) = 1$ if n = 1

       $T(n-1).n$ if n>1

$T(n)$ $= T(n-1).n$ **1st time**

       $= T(n-2) (n-1) n$ **2nd time**

       $= T(n-3) (n-2) (n-1) n$ **3rd time**

       $= T(n-k) (n-k+1) (n-k+2) .... (n-2)(n-1)n$ **k times = n-1**

n-k = 1

n-1 = k

       $= T(n-(n-1)) (n-(n-1)+1) (n-(n-1)+2) .... (n-2)(n-1)n$

       $= T(n-n+1) (n-n+2) (n-n+3)..........(n-2)(n-1)n$

       $= T(1).2.3.4.........(n-1).n$

       $= 1.2.3..........(n-1).n = n! = O(n!)$

$5! = 5.4.3.2.1 = 120$

       $n^n > n!$

       $= O(n^n)$

Problem 3 :

$T(n) = 1$ if n = 0

       $T(n-2) + n^2$ if n>0