Assignment-02

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Q. Are the following sets of vectors linearly independent or dependent?

Solution

(1) $[1,0,0]$, $[1,1,0]$, $[1,1,1]$

Solution: let $v_1 = [1,0,0]$, $v_2 = [1,1,0]$ & $v_3 = [1,1,1]$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

$c_1 + c_2 + c_3 = 0$

$c_2 + c_3 = 0$

$c_3 = 0$

Here

$c_3 = 0$

then

$c_2 + c_3 = 0$

$c_2 = 0$

Also

$c_1 = 0$

$\Rightarrow c_1 = c_2 = c_3 = 0$ (This system has only the trivial solution)

$\Rightarrow$ S is linearly independent.

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② [7, -3, 11, -6] , [-56, 24, -88, 48]

solution

Lets

$v_1 = [7, -3, 11, -6]$

$v_2 = [-56, 24, -88, 48]$

now

$7c_1 - 56c_2 = 0$

$-3c_1 + 24c_2 = 0$

$11c_1 - 88c_2 = 0$

$-6c_1 - 48 = 0$.

$$\begin{bmatrix} 7 & -56 \\ -3 & 24 \\ 11 & -88 \\ -6 & -48 \end{bmatrix}$$

we

$R_4 \leftrightarrow \frac{1}{7} R_1$

$$\begin{bmatrix} 1 & -8 \\ -3 & 24 \\ 11 & -88 \\ -6 & -48 \end{bmatrix}$$

$R_2 \to R_2 + 3R_1$

$R_3 \to R_3 - 11R_1$

$R_4 \to R_4 + 6R_1$

$$\begin{bmatrix} 1 & -8 \\ 0 & 0 \\ 0 & -172 \\ 0 & 0 \end{bmatrix}$$

Here

$\rho(A) \neq n$

$1 \neq 4$

∴ infinite solution

∴ dependent.

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(3) $[-1 \ 5 \ 0], [16 \ 8 \ -3], [-64 \ 56 \ 9]$

Soln

$v_1 = [-1 \ 5 \ 0]$

$v_2 = [16 \ 8 \ -3]$

$v_3 = [-64 \ 56 \ 9]$

$-c_1 + 16c_2 - 64c_3 = 0$

$5c_1 + 8c_2 + 56c_3 = 0$

$-3c_2 + 9c_3 = 0$

$$\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$$

$R_2 \to R_2 + 5R_1$

~~$R_3 \to R_3 - 3R_1$~~

$$\begin{bmatrix} -1 & 16 & -64 \\ 5 & 8 & 56 \\ 0 & -3 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 16 & -64 \\ 0 & 88 & -264 \\ 0 & -3 & 9 \end{bmatrix}$$

$-1(72+168) - 16(45) - 64(-15)$

$\rho(A) < 3$

$\rho(A) = n$

$= -240 - 720 + 960$

$= -960 + 960$

$= 0$

$\rho(A) \neq n$

$\therefore$ Infinite solution

linearly dependent.

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(4) $[1\ -1\ 1], [1\ \ 1\ -1], [-1\ 1\ 1], [0\ 1\ 0]$

Solution

$$c_1 + c_2 - c_3 + 0 = 0$$
$$-c_1 + c_2 + c_3 + c_4 = 0$$
$$c_1 - c_2 + c_3 = 0$$

Adding 1 & 3

$$c_1 + c_2 - c_3 = 0$$
$$c_1 - c_2 + c_3 = 0$$
$$2c_1 = 0$$
$$c_1 = 0.$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$
$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & -2 & 2 & 0 \end{bmatrix}$$

$R_2 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

$\rho(A) \neq n$

$\therefore$ dependent

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(5) $[2, -4]$, $[1, a]$, $[3, 5]$

~~$c_1 v_1 + c_2 v_2$~~

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$$

$$c_1(2, -4) + c_2(1, a) + c_3(3, 5) = 0$$

$$\left.\begin{aligned} 2c_1 + 2c_2 + 3c_3 &= 0 \\ -4c_1 + ac_2 + 5c_3 &= 0 \end{aligned}\right\} \text{no solution.}$$