

### Assignment-3

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$$\textcircled{1} \quad A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$-(2+\lambda)[\lambda^2 - \lambda - 12] - (2)[-2\lambda - 6] - 3[-4 + (1-\lambda)]$$

$$-(2\lambda^2 - 2\lambda + 24 + \lambda^3 - \lambda^2 - 12\lambda) + 4\lambda + 12 + 12 - 3 + 3\lambda = 0$$

$$-\lambda^3 - \lambda^2 + 21\lambda + 3 = 0 \Rightarrow \lambda^3 + \lambda^2 - 21\lambda + 3 = 0$$

# Eigenvalues  $\lambda = -5.4, -0.14, 4$



$$d=4$$

$$\left\{ \begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\left\{ \begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right\} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -6 & 2 & 3 \\ 2 & -3 & -6 \\ -1 & -2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-6x + 2y + 3z = 0$$

$$2x - 3y - 6z = 0$$

$$-x - 2y - 4z = 0$$

$$-6x + 2y + 3z = 0$$

$$6x - 9y - 18z = 0$$

$$-7y - 15z = 0$$

$$-7y = -15z$$

$$y = \frac{-15}{7} z$$



$$(2) \quad A \Rightarrow \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I \Rightarrow \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix}$$

$$\Rightarrow (4-\lambda)((1-\lambda)^2) + 1(2(1-\lambda)) = 0$$

$$(4-\lambda)(1-\lambda^2) + 2(1-\lambda) = 0$$

$$(1-\lambda)[(4-\lambda)(1-\lambda) + 2] = 0$$

$$(1-\lambda)[6 - 5\lambda + \lambda^2] = 0$$

$$(\lambda-1)(\lambda-2)(\lambda-3) = 0$$

Eigen values  $= \lambda = 1, 2, 3$

for  $\lambda = 1$

$$\begin{bmatrix} 3 & 0 & 1 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

$$-2x = 0$$

$$x = 0, z = 0, y = k$$

$$3x + 2 = 0$$

$$\text{Eigen vector} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

for  $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}$$

$$-2x - y = 0$$

$$2x = -y$$

$$-2x - z = 0$$

$$z = -2x$$

$$x = k, y = -2k, z = -2k$$

$$\text{Eigen vector} \Rightarrow k \begin{bmatrix} 1 \\ -2 \\ -2 \end{bmatrix}$$



for  $\lambda = -3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}$$

$$-2x - 2z = 0$$

$$x = -z$$

$$-2x - 2y = 0$$

$$x = -y$$

eigen vectors =  $k \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

(3)  $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 5, 0, 3$$

for  $\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}$$

$$-5y = 0$$

$$-x - 2z = 0$$

$$y = 0$$

$$x = -2z$$

$$x = -2k, y = 0, z = k$$

Eigen vectors =  $k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

for  $\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$x = 0$$

$$-x + 3z = 0$$

$$z = 0$$

$$x = 0, y = k, z = 0$$

Eigen vectors =  $k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$



for  $\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$y = 0$$

$$x = 0$$

$$z = k$$

Eigen vectors  $\Rightarrow k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$