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Assignment

Q.1 → Find the rank of the matrix A by reducing in Row reduced echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \Rightarrow \frac{-R_2}{4}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

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$R_1 = R_1 - 2R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 = R_4 + 4R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 = -R_3/3$

$$A = \begin{bmatrix} 1 & 0 & -1 & 3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_1 = R_1 + R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

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$R_2 = R_2 - 2R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 = R_4 + 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & 5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of Matrix is '3'.

Q.2 Let w be the vector space of all symmetric 2×2 matrices & let $T: w \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$.

find rank & nullity of T.

Solution

Since the maximum degree of polynomial $T \Rightarrow 2$. So $\dim(P_2) \Rightarrow 3$.

Kernel

So a subset of kernel T is $T(A) = 0$.

$(a-b) + (b-c)x + (c-a)x^2 = 0$

$\boxed{a = b = c = t \text{ (let)}}$

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new matrix → $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

dimension of kernel is 1, because there's only one independent parameter as 't'.

Acc. to rank nullity Theorem →

$$\text{rank}(T) + \text{nullity}(T) \Rightarrow \dim(W).$$

$$\text{rank}(T) + 1 = 4$$

rank of T is 3, & nullity is 1.

Q.3 Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigen values & eigen vectors of A^{-1} and $A + 4I$.

$$A - \lambda I \Rightarrow 0$$

$$\det\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)^2 - 1 = 0$$

$$(2-\lambda) = \pm 1$$

$$\lambda = 1, 3.$$

for $\lambda = 1$

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$x - y = 0$$

$$x \Rightarrow y$$

let $x = t$,
$y \Rightarrow t$

Eigen vector v_1, $t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ Ans

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$\boxed{\text{for } \lambda = 3}$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x - y = 0$ let $x = t$

$x = -y$ $y = -t$

so, eigen value $V_2 \Rightarrow t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Now, find for A^{-1}.

→ eigen values of A^{-1} will be $\frac{1}{\lambda}$ & $\frac{1}{\lambda^2} \Rightarrow 1, \frac{1}{3}$.

→ and eigen vectors are same as of A.

$v_1 \Rightarrow t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

Now for $A + 4I$

→ eigen values for $A + 4I$ will be $\lambda_1 + 4$, $\lambda_2 + 4 \Rightarrow 5, 7$.

→ and eigen vectors are same as of A

$v_1 = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v_2 = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

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Q.4 Solve by Gauss-Siedel Method
(Take three Iteration)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

With initial values $x(0) = 0, y(0) = 0, z(0) = 0$

equation

$$\Rightarrow x^{k+1} \Rightarrow \frac{7.85 + 0.1y^{k} + 0.2z^{k}}{3}$$

$$y^{k+1} \Rightarrow \frac{-19.3 - 0.1x^{k+1} - 0.3z^{k}}{-7}$$

$$z^{k+1} = \frac{71.4 - 0.3x^{k+1} + 0.2y^{k+1}}{10}$$

we know

$$x(0) = 0, y(0) = 0, z(0) = 0$$

Iteration - 1

$$x(1) \Rightarrow \frac{7.85 + 0.1(0) + 0.2(0)}{3} \Rightarrow 2.6167$$

$$y(1) \Rightarrow \frac{-19.3 - 0.1(2.6167) - 0.3(0)}{-7} = 2.7956$$

$$z(1) \Rightarrow \frac{71.4 - 0.3(2.6167) - 0.2(2.9756)}{10} \Rightarrow 7.1373$$

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Iteration-2

$$x(2) \Rightarrow \frac{7.85 + 0.1(2.7956) + 0.2(7.1373)}{3} = 3$$

$$y(2) \Rightarrow \frac{-19.3 - 0.1(3) - 0.3(7.1373)}{-7} = 3$$

$$z(2) = \frac{71.4 - 0.3(3) - 0.2(3)}{10} = 3$$

Iteration-3

$$x(3) = \frac{7.85 + 0.1(3) + 0.2(3)}{3} = 3$$

$$y(3) = \frac{(-19.3 - 0.1(3) - 0.3(3))}{-7} = 3$$

$$z(3) = \frac{71.4 - 0.3(3) + 0.2(3)}{10} = 3$$

After, three ~~iteration~~ iteration $x, y, z \approx 3$

so value of $x = 3, y = 3$ & $z = 3$.

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Q.5 Define consistent and inconsistent of equations. Hence solve the following system of equations if consistent $x+3y+2z=0$, $2x-y+3z=0$, $3x-5y+4z=0$, $x+17y+4z=0$

Consistent
(atleast one solution)

Dependent
(infinite solution)

Independent
(unique solution)

Inconsistent
(No solution)

Now

$$A \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$$\Rightarrow \begin{array}{cccc} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{array}$$

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$R_3 \to R_3 - 2R_2$, $R_4 \to R_4 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(A) \Rightarrow 2$
$\rho(A:B) \Rightarrow 2$
$n \Rightarrow 3$
$\rho(A) = \rho(A:B) \neq n.$

Consistent, but infinite solⁿ.

Q.6 Determine whether the function $T: P_2 \to P_2$ is linear transformation or not.
where $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

1) Additive

$T(u+v) \Rightarrow T(u)+T(v)$
$u = a_1 + b_1 x + c_1 x^2$
$v = a_2 + b_2 x + c_2 x^2$

$T(u+v) \Rightarrow T\{(a_1+a_2)+(b_1+b_2)x+(c_1+c_2)x^2\}$
$\Rightarrow (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$\Rightarrow (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x$
$+(c_2+1)x^2$
$\Rightarrow T(u)+T(v)$

Hence, proved

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(2) Homogeneity →

$T(ku) \Rightarrow kT(u)$

$T(k(a+bx+cx^2))$

$T(ka+kbx+kcx^2)$

$\Rightarrow (ka+kb+kc+1)+(ka+kb+kc+1)x$
$+(ka+kb+kc+1)x^2$

$\Rightarrow k(a+1)+k(b+1)x+k(c+1)x^2$

$\Rightarrow kT(u)$

Hence proved.

~~Hence, It~~

Hence It's a Linear Transformation.

Q.7 Determine whether set $S \to \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In case S is not a basis, determine the dim^n & basis of subspace spanned by S.

$$a(1,2,3)+b(3,1,0)+c(-2,1,3)=(0,0,0)$$

$$a+3b-2c=0$$

$$2a+b+c=0$$

$$3a+3c=0$$

$$c=-a, \quad b \Rightarrow -a.$$

Only one sol^n is possible is $a=b=c=0$
So linearly independent.

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Since dim^n of $V_3(R)$ is 3 and S also Contains 3 vector and S is LI then it Spans $V_3(R)$ making it a basis for $V_3(R)$.

Q.8 Using Jacob's method (perform 3 iterations), Solve $3x-6y+2z=23$, $-4x+y-z=-15$, $x-3y+7z=16$, with initial values $x_0=1, y_0=1, z_0=1$

$\Rightarrow$ first eq^n $x = 1/3 \; (23+6y-2z)$

second eq^n $y \Rightarrow (-15+4x+z)$

third eq^n $z \Rightarrow \frac{1}{7}(16-x+3y)$.

$x(0)=1$, $y(0)=1$, $z(0)=1$.

Iteration-1 $\rightarrow$

$x(1) \Rightarrow (23+6-2)/3 \Rightarrow 9$

$y(1) \Rightarrow (-15+4+1) \Rightarrow -10$

$z(1) \Rightarrow (16-1+3)/7 \Rightarrow 18/7$.

Iteration 2

$x(2) \Rightarrow \frac{1}{3}(23+6(-10)-36/7) = 10.62$

$y(2) \Rightarrow (-15+4(10.62)+18/7) \Rightarrow 30.04$

$z(2) \Rightarrow \frac{1}{7}(16-10.62+3(30.4))$

$= 13.8$.

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Iteration 3

$$x(3) \Rightarrow \frac{[23 + 6(30.4) - 2(13.8)]}{3} = 59.27$$

$$y(3) = \frac{-15 + 4(59.27) + 13.8}{} = 235.88$$

$$z(3) \Rightarrow \frac{16 - 59.27 + 3(235.88)}{7} = 67.4$$

So after three iteration $x(3) \Rightarrow 59.27$, $y(3) \Rightarrow 235.88$, $z(3) \Rightarrow 67.4$.

Ans

Q.9 Explain one application of matrix operations in image processing with example.

Affline Transformation
Rotation

Suppose we have a 2-D image represented as grid or pixels we can use AT matrix to rotate around centre.

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

to rotate it around centre

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(1) Translation to origin
→ Translate the image, So that it Centre aligns with origin.

(2) Rotation
→ Apply rotation matrix.

(3) Translation Back.

→ Translate it back with its original position by adding coordinates of centre.

Q-10 Give a brief description of Linear transformation for Computer Vision for rotating 2D image.

=) Linear transformation for rotating 2D images involves applying a rotation matrix to each pixel coordinate, This matrix rotates points Counter clockwise by an angle θ around the origin. It preserves geometric properties like parallelism and distances. Rotation is essential in tasks like image allignment and object detection in Computer vision.