Practice Assignments 1

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a) Test for Consistency and Solve.

(i) $2x-3y+7z=5$
$3x+y-3z=13$
$2x+19y-47z=32$.

Here $AX=B$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

let make this A. matrix in Augomented form.

$(A:B)$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 3 & 1 & -3 & : & 13 \\ 2 & 19 & -47 & : & 32 \end{bmatrix}$$

let's make it in Echlon form using Elementary Operation

$R_2 \to R_2 - \frac{3}{2}R_1$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 5.5 & -13.50 & : & 5.50 \\ 0 & 22 & -54 & : & 27 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2$

$$\begin{bmatrix} 2 & -3 & 7 & : & 5 \\ 0 & 5.5 & -13.5 & : & 5.5 \\ 0 & 0 & 0 & : & 5 \end{bmatrix}$$

Here

$\rho(A)=2$ $\quad\because \rho(A:B) \neq \rho(A)$

$\rho(A:B)=3$ $\quad$ It is Inconsistent
No Solution.

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(ii) $2x - y + 3z = 8$
$-x + 2y + z = 4$
$3x + y - 4z = 0$

$AX = B$

$$\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix}$$

Augmented form

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ -1 & 2 & 1 & 4 \\ 3 & 1 & -4 & 0 \end{array}\right]$$

$R_2 \to R_2 + \frac{1}{2} R_1, \quad R_3 \to R_3 - \frac{3}{2} R_1$

$-\frac{3}{2} \times 8$
-12

$-4 \cdot \frac{9}{2}$
$\frac{-8-9}{2}$
$\frac{-17}{2}$
$1+\frac{3}{2}$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 5/2 & -17/2 & 0 \end{array}\right]$$

$R_3 \to R_3 - \frac{5}{3} R_1$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 8 \\ 0 & 3/2 & 5/2 & 8 \\ 0 & 0 & -76/6 & -\frac{40}{3} \end{array}\right]$$

$\frac{5/2}{3/2}$
$\frac{5 \times 2}{2 \times 3}$
$\frac{5}{3}$
$\frac{-17}{2} - \frac{5}{3} \times \frac{5}{2}$
$\frac{-17}{2} - \frac{25}{6}$
$\frac{-51-25}{6}$
$\frac{-76}{6}$
$-\frac{5}{3} \cdot 8$
$-\frac{40}{3}$

Here
$\rho(A:B) = \rho(A) = n$
$3 = 3 = 3$
$\therefore$ it is consistent
unique solution

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$2x - y + 3z = 7$

$\frac{3}{2}y + \frac{5}{2}z = 9$

$\frac{76}{62}z = \frac{140}{8}$

$z = \frac{80}{76} = 1.053$

$\frac{3}{2}y = 8 - \frac{5}{2} \times \frac{\cancel{80}\,40}{76}$

$\frac{3}{2}y = 8 - \frac{200}{76}$

$\frac{3}{2}y = \frac{608 - 200}{\cancel{76}\,38}$

$y = \frac{409}{114} = 3.579$

$2x - \frac{409}{114} + \frac{3 \times 80}{76} = 8$

$2x - \frac{409}{114} + \frac{240}{76} = 7$

$2x - 0.421 = 7$

$2x = 8 + 0.421$

$2x = 8.421$

$x = \frac{8.421}{2}$

$x = 4.211$

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iii) $4x - y = 12,\ -x + 5y - 2z = 0\ ,\ -2x + 4z = -8$

$AX = B$

$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

Augomented form.

$$\left[\begin{array}{ccc:c} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

~~$R_2 = R_2 -$~~

$R_2 \leftrightarrow R_1$

$$\left[\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 4 & -1 & 0 & 12 \\ -2 & 0 & 4 & -8 \end{array}\right]$$

$R_2 \to R_2 + 4R_1$

$R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & -10 & 8 & -8 \end{array}\right]$$

$R_3 \to R_3 + \frac{10}{19} R_2$

$$\left[\begin{array}{ccc:c} -1 & 5 & -2 & 0 \\ 0 & 19 & -8 & 12 \\ 0 & 0 & -28/19 & -32/19 \end{array}\right]$$

Here

$\rho(A:B) = \rho(A) = n$

$3 = 3 = 3$

$\therefore$ Consistent

unique solution

$z = \frac{4}{9}, y = \frac{4}{9}, x = \frac{27}{9}$

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$-x + 5y - 2z = 0$

$19y - 8z = 12$

$\frac{+27}{\cancel{19}} z = \frac{+32}{\cancel{19}}$

$z = \frac{32}{28} = 1.143$

$19y - \cancel{8}^{2} \times \frac{32}{\cancel{28}_{7}} = 12$

$19y - \frac{64}{7} = 12$

$19y = 12 + \frac{64}{7}$

$19y = \frac{84 + 64}{7}$

$19y = \frac{148}{7}$

~~19y =~~

$y = \frac{148}{7 \times 19}$

$y = \frac{148}{133} = 1.133.$

$-x + 5 \times 1.133 - 2 \times 1.143 = 0.$

$-x + 5.665 - 2.286 = 0$

$-x + 3.379 = 0$

$x = 3.379.$

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b. for what values of λ and μ the given system of equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ has (i) No Solution; (ii) a unique solution and (iii) infinite number of solutions

Solution

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

$AX=B$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 = R_2 - R_1$
$R_3 = R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_3 = R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

(i) for No solution

$\rho(A:B) \neq \rho(A)$

$\therefore \lambda-3=0; \quad \mu-10 \neq 0$

$\lambda=3, \quad \mu \neq 10.$

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(iii) a unique solution

$\rho(A:B) = \rho(A) = n.$

$\delta - 3 \neq 0$, $\mu \in R$, $n = 3$

$\delta \neq 3$

(iv) infinite number of solutions

$\rho(A:B) = \rho(A) \neq n$

$\delta - 3 = 0$, $\mu - 10 = 0$; $n = 3$

$\delta = 3$, $\mu = 10$

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(E) Find for what values of λ the given equations $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a solution and Solve the completely in each case.

Soln

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

~~Aug~~ Augmented form

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{array}\right] \quad R_3 \to R_3 - 3R_2$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{array}\right]$$

~~Here~~

~~$\rho(A:B) \neq \rho(A)$~~

Can't be uniqe solution because $\rho(A)=2$

for a infinite solution

$$\lambda^2-3\lambda+2=0$$
$$\lambda^2-2\lambda-\lambda+2=0$$
$$(\lambda-1)(\lambda-2)=0$$
$$\lambda=1$$
$$\lambda=2$$

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now

let $\lambda = 1$

$\cancel{x + y + z = 1}$
$\cancel{x + 2y + 4z = 1}$
$\cancel{x + 4y + 10z = 1}$

$x + y + z = 1$
$y + 3z = 0$
let $z = k$
$y + 3k = 0$
$y = -3k$
$x - 3k + k = 1$
$x = 1 + 2k$

$\therefore x = 1 + 2k$
$y = -3k$
$z = k.$

let $\lambda = 2$,

$x + y + z = 1$
$y + 3z = 1$
let $z = k$
$y = 1 - 3k$

$x + 1 - 3k + k = 1$
$x = 2k$
$\therefore x = 2k$
$y = 1 - 3k$
$z = k.$

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(d) Find the solution of the System of equations $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$.

Sol.

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0.$$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -5 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array}\right]$$

$R_3 \to R_3 - \frac{14}{5} R_1$

$$\left[\begin{array}{ccc:c} 1 & 3 & -2 & 0 \\ 0 & -5 & 8 & 0 \\ 0 & 0 & \frac{-32}{5} & 0 \end{array}\right]$$

$\frac{-32}{5} z =$ $\quad \rho(A) = \rho(A:B) = n$

$\therefore$ Consistent

Unique Solution

$$\frac{-32}{5} z = 0$$
$$z = 0.$$

$-5y = 0$ $\quad x = 0.$

$y = 0$

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(e) Find for what value of δ the given equations $3x+y-\delta z=0$, $4x-2y-3z=0$, $2\delta x+4y+\delta z=0$, may posses non-trivial solution & solve them completely in each case.

Soln

$$3x+y-\delta z=0$$
$$4x-2y-3z=0$$
$$2\delta x+4y+\delta z=0.$$

Augomented form

$$\left[\begin{array}{ccc:c} 3 & 1 & -\delta & 0 \\ 4 & -2 & -3 & 0 \\ 2\delta & 4 & \delta & 0 \end{array}\right]$$

$R_2 \to R_2 - \frac{4}{3}R_1$

$R_3 \to R_3 - \frac{2\delta}{3}R_1$

$$\left[\begin{array}{ccc:c} 3 & \delta & -\delta & 0 \\ 0 & \frac{-10}{3} & \frac{4\delta}{3}-3 & 0 \\ 0 & \frac{12-2\delta}{3} & \frac{2\delta^2+\delta}{3} & 0 \end{array}\right]$$

$\frac{12-2\delta}{3}=0$

$12-2\delta=0$

$\delta=+12$

$\delta=6$

$\frac{2\delta^2+\delta}{3}=0$

$2\delta^2+\delta=0$

$\delta(2\delta+1)$

$\delta=0,\ \delta=\frac{-1}{2}$

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$$\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$$

$$3(-2\lambda+12)-1(4\lambda+6\lambda)-\lambda(16+4\lambda)=0$$

$$-6\lambda+36-10\lambda-16\lambda-4\lambda^2=0$$

$$-4\lambda^2-32\lambda+36=0$$

$$4\lambda^2+32\lambda-36=0$$

$$\lambda^2+8\lambda-9=0$$

$$\lambda^2+9\lambda-\lambda-9=0$$

$$\lambda(\lambda+9)-1(\lambda+9)=0$$

$$(\lambda-1)(\lambda+9)=0$$

$$\lambda=1$$

$$\lambda=-9$$