

## The first week of class:

In week 1 we leaned about Lean as a programming language, and it's correlation to discrete math. We also learnt about other proof assistants. We then shifted our focus to the nng practice questions as you can see below.

### LEVEL 5 EXPLANATION:

Discrete math's lemmas tell us that anything added to 0 will give the result of that number itself. So;  $A+0=A$ .

Using this we can bring the left hand side down to  $a+b+c$ .

From here we can use the property of reflexivity to show that both sides are equal, hence solving the puzzle.

## NNG Solutions

### Level 5 / 8 : Adding zero

Active Goal

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + (b + 0) + (c + 0) = a + b + c$

rw [add\_zero]

Active Goal

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + b + (c + 0) = a + b + c$

rw [add\_zero]

Active Goal

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + b + c = a + b + c$

refl

level completed! 🎉

### Level 6 / 8 : Precision rewriting

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + (b + 0) + (c + 0) = a + b + c$

rw [add\_zero c]

Active Goal

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + (b + 0) + c = a + b + c$

rw [add\_zero b]

Active Goal

Objects:

$a\ b\ c : \mathbb{N}$

Goal:

$a + b + c = a + b + c$

refl

level completed! 🎉

## Level 7 / 8 : add\_succ

**Theorem** `succ_eq_add_one`: For all natural numbers  $a$ , we have  $\text{succ}(a) = a + 1$ .

Active Goal

Objects:

$n : \mathbb{N}$

Goal:

$\text{succ } n = n + 1$

`rw [one_eq_succ_zero]`

Active Goal

Objects:

$n : \mathbb{N}$

Goal:

$\text{succ } n = n + \text{succ } 0$

`rw [add_succ]`

Active Goal

Objects:

$n : \mathbb{N}$

Goal:

$\text{succ } n = \text{succ } (n + 0)$

`rw [add_zero]`

## Level 8 / 8 : 2+2=4

$2 + 2 = 4$ .

Active Goal

Goal:

$2 + 2 = 4$

`rw[four_eq_succ_three]`

Active Goal

Goal:

$2 + 2 = \text{succ } 3$

`rw[three_eq_succ_two]`

Active Goal

Goal:

$2 + 2 = \text{succ } (\text{succ } 2)$

`rw[two_eq_succ_one]`

Active Goal

Goal:

$\text{succ } 1 + \text{succ } 1 = \text{succ } (\text{succ } (\text{succ } 1))$

`rw[one_eq_succ_zero]`

## Level 8 / 8 : 2+2=4

Active Goal

Goal:

$\text{succ } (\text{succ } 0) + \text{succ } (\text{succ } 0) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0)))$

`rw[succ_eq_add_one]`

Active Goal

Goal:

$\text{succ } 0 + 1 + (\text{succ } 0 + 1) = \text{succ } (\text{succ } (\text{succ } 0 + 1))$

`rw[one_eq_succ_zero]`

Active Goal

Goal:

$\text{succ } 0 + \text{succ } 0 + (\text{succ } 0 + \text{succ } 0) = \text{succ } (\text{succ } (\text{succ } 0 + \text{succ } 0))$

`rw[add_succ]`

Active Goal

Goal:

$\text{succ } (\text{succ } 0 + 0) + \text{succ } (\text{succ } 0 + 0) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0 + 0)))$

`rw[add_zero]`

## Level 8 / 8 : 2+2=4

Active Goal

Goal:

$\text{succ } (\text{succ } 0) + \text{succ } (\text{succ } 0) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0)))$

`rw[add_succ]`

Active Goal

Goal:

$\text{succ } (\text{succ } (\text{succ } 0) + \text{succ } 0) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0)))$

`rw[add_succ]`

Active Goal

Goal:

$\text{succ } (\text{succ } (\text{succ } (\text{succ } 0) + 0)) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0)))$

`rw[add_zero]`

Active Goal

Goal:

$\text{succ } (\text{succ } (\text{succ } (\text{succ } 0))) = \text{succ } (\text{succ } (\text{succ } (\text{succ } 0)))$

`refl`

level completed! 🎉

