

## lecture 20

## GRAPHS



Set of Vertices.



Set of Edges.

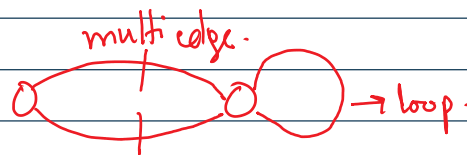


Syntax

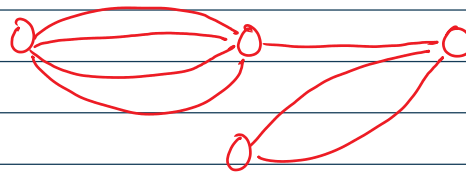


Semantics.

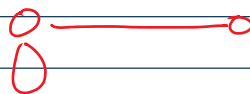
Types: Simple Graph. 1)  $\neg$  multi edge. and.  
2)  $\neg$  loops.



Multi Graph: which contain multi edges.

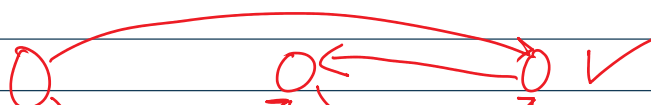


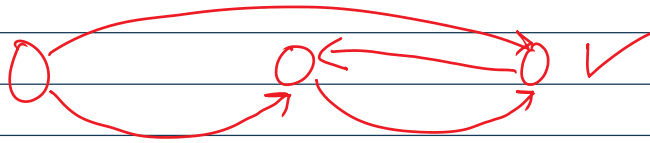
Pseudograph: 1) loop. OR.  
2) multiedge.



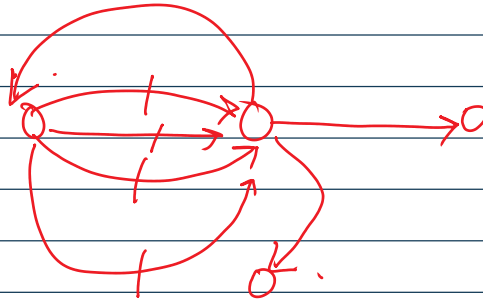
Undirected Graph: edges do not have arrows.

directed Simple Graph:



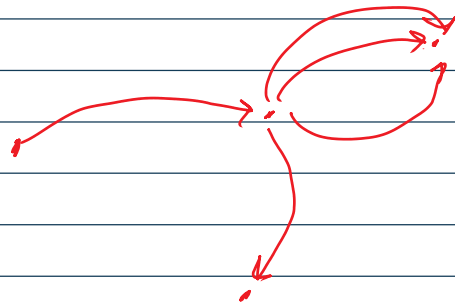


multiplicity -



§ 4.

Facebook

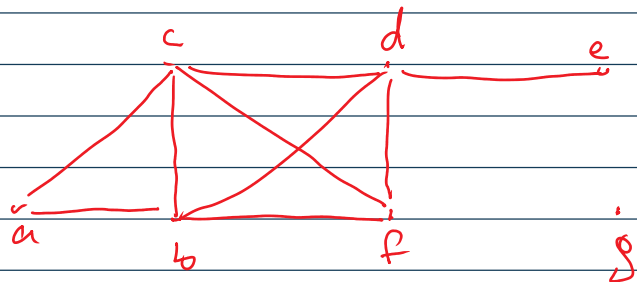


Call Graphs-



Degree of a Graphs-

Ex 2  
536



$$\deg(a) = 2$$

$$\deg(b) = 4$$

$$\deg(c) = 4$$

$$\deg(d) = 4$$

$$\begin{array}{rcl}
 " (e) & = & 2 \\
 " (f) & = & 3 \\
 " (g) & = & 0 \\
 \hline
 & & 18.
 \end{array}$$

HandShaking Theorem:

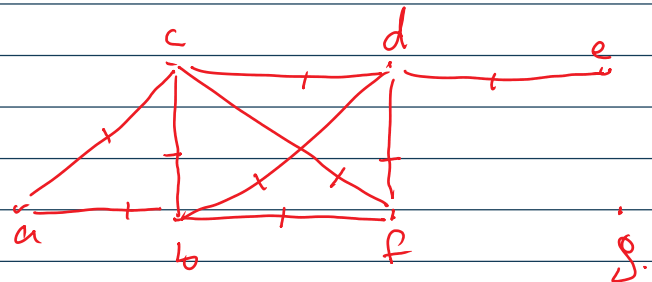
$$2e = \sum_{v \in V} \deg(v).$$

$$2e = \deg(a) + \deg(b) + \dots + \deg(g).$$

$$= 2 + 4 + 4 + 4 + 1 + 3 + 0.$$

$$2e = 18$$

$$\Rightarrow e = 9.$$



$$V = \{a, b, c, d, e, f, g\}.$$

$$\deg(a) = 2$$

$$" (b) = 4$$

$$" (c) = 4$$

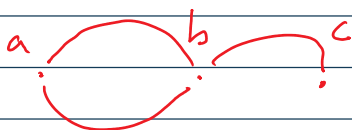
$$" (d) = 4$$

$$" (e) = 2$$

$$" (f) = 3$$

$$" (g) = 0$$

Theorem An Undirected graph has even number of vertices having odd degree.



$$\deg(a) = 2.$$

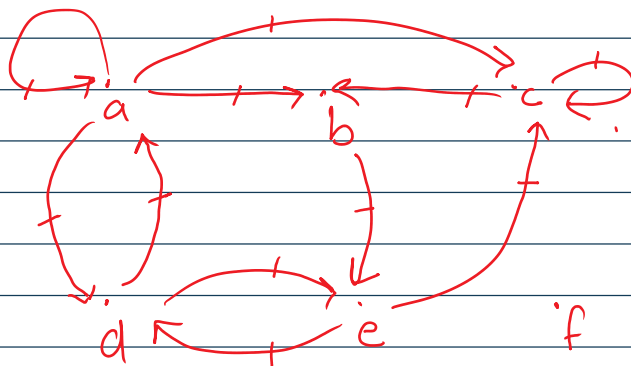
$$(b) = 3 \checkmark$$

$$(c) = 1. \checkmark$$

In degree & out degree.

# Directed Graph.

Ex 4 :-  
PS38



$$\begin{aligned} \deg^-(a) &= 2 \\ \deg^-(b) &= 2 \\ \deg^-(c) &= 3 \\ \deg^-(d) &= 2 \\ \deg^-(e) &= 2 \\ \deg^-(f) &= 0/11. \end{aligned}$$

$$e = \sum_{u \in V} \deg^-(u) = \sum_{u \in V} \deg^+(u).$$

$$\begin{aligned} \deg^+(a) &= 4 \\ \deg^+(b) &= 1 \\ \deg^+(c) &= 2 \\ \deg^+(d) &= 2 \\ \deg^+(e) &= 2 \\ \deg^+(f) &= 0/11. \end{aligned}$$

## Special types of Simple Graphs.

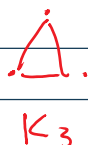
1. Complete Graph.  $K_n$ .  $n = 1, 2, \dots, \infty$



$K_1$



$K_2$



$K_3$

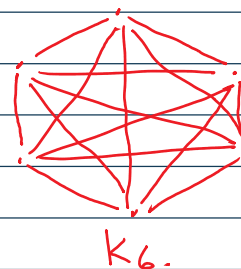


$K_4$



$K_5$

	Vertices	Edges -
$K_1$	1	0
$K_2$	2	1
$K_3$	3	3
$K_4$	4	6
$K_5$	5	10
	$n$	$\frac{n(n-1)}{2}$



$K_6$

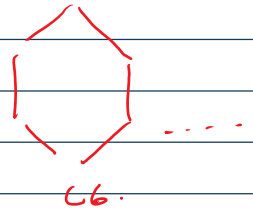
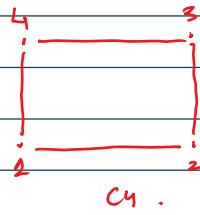
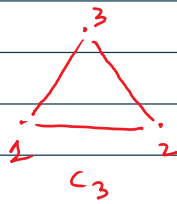
2. Cycles  $C_n$   $n \geq 3, 4, 5, \dots$

$C_n$

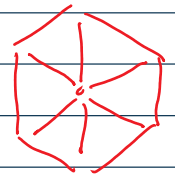
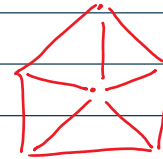
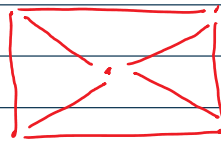
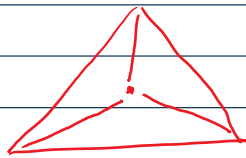
Vertices  $n$   
Edges  $n$

$1, 2, 3, \dots, n$   
 $(1,2), (2,3), (3,4), \dots, (n-1,n), (n,1)$

Edges  $n$ .



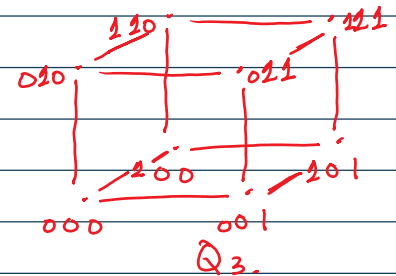
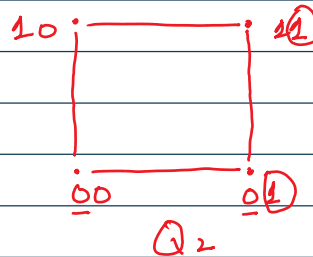
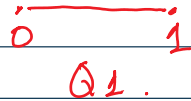
3- Wheels.  $W_n$   $n \geq 3, 4, \dots$



$W_n$ :

Vertices  $n+1$   
Edges  $2n$ .

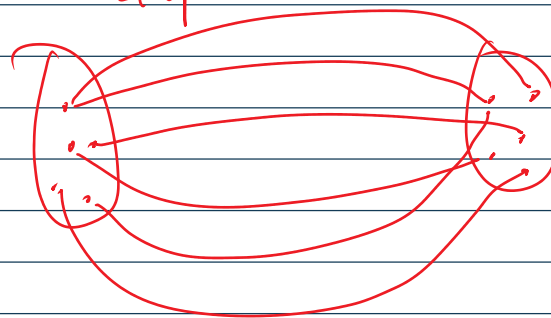
4- Quboid.  $Q_n$   $n = 1, 2, 3, \dots$



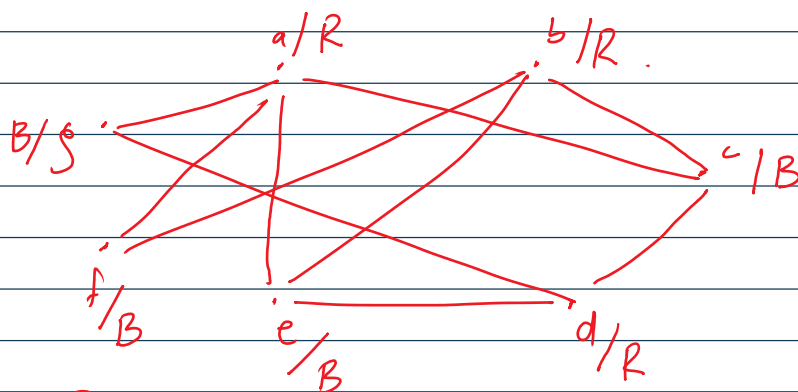
$Q_n$

Vertices  $= 2^n$   
Edges  $= 2^{n-1} \cdot n$ .

Bipartite Graph.



Ex 11  
p542.



$$G_1 = \{a, b, d\}$$

$$G_2 = \{g, f, e, c\}$$

