lecture 18 :-

PARTIAL ORDER.

1- Reflexive
2- Anti Symmetric
3- Transitive.

POSET.

$(S, \preceq_R)$

$S \times S$

Set — Generalized Notation for representing any Partial Order.

Same: $(a,b) \in R$ / $a \preceq_R b$.

Ex1
504

$R = \{(a,b) \mid a \geq b\}$ $\quad A = \mathbb{Z}$.

Reflexive $\quad \forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad a \geq a$

Anti Symmetric $\quad \forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in \mathbb{Z}$ if $a \geq b \wedge b \geq a \rightarrow a = b$.

Transitive $\quad \forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $a \geq b \wedge b \geq c \rightarrow a \geq c$.

$(\mathbb{Z}, \geq)$ $\quad$ $>$ X

$<$ X.

$(\mathbb{Z}, =)$

$(\mathbb{Z}, \leq)$

$(\mathbb{Z}, \div)$

Ex3
504

$R = \{(a,b) \mid a \subseteq b\}$. $\quad PS(S)$.

Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in PS(S) \quad a \subseteq a$

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in PS(S)$ if $a \subseteq b \wedge b \subseteq a \rightarrow a = b$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in PS(S)$ if $a \subseteq b \wedge b \subseteq c \rightarrow a \subseteq c$.

$R = \{(a,b) \mid a \subseteq b\}$. $\quad PS(S)$. $\quad S = \{a,b\}$.

$PS(S) = \{\phi, \{a\}, \{b\}, \{a,b\}\}$.

$R = \{(\phi,\phi), (\phi, \{a\}), (\phi, \{b\}),$
$(\phi, \{a,b\}),$

$(\{a\}, \{a\}), (\{a\}, \{a,b\}),$
$(\{b\}, \{b\}), (\{b\}, \{a,b\}),$
$(\{a,b\}, \{a,b\})\}$.

$(PS(S), \subseteq)$. $\quad (S, \preceq)$.

$(\mathbb{Z}, \geq)$

$(\{1,2,3,4,6,8,10\}, \div)$.

$(\mathbb{Z}, =)$

$(\mathbb{Z}, \leq)$ $\quad R = \{(a,b) \mid a \div b\}$.

$(\mathbb{Z}, \div)$

Comparable :- Two elements 'a' & 'b' in S based on $(S, \preceq)$ are Comparable if .

$a \preceq b$ or $b \preceq a$.

EX5 Show if 5, 7 (3, 9.) are Comparable in The

Ex5 Show if 5, 7 (3, 9.) are Comparable in the
Sol Poset $(\mathbb{Z}, |)$.

$5 \preceq 7$ or $7 \preceq 5$
$5 | 7$ $\vee$ $7 | 5$
F $\vee$ F $= F$.

$3 \preceq 9$ or $9 \preceq 3$.
$3 | 9$ $\vee$ $9 | 3$.
F $\vee$ T $= T$.

Total Order:- if all elements in S based on $(S, \preceq)$ are Comparable then it is total Order.

Ex6 $(\mathbb{Z}, \leq)$ Total order.
Sol

$-2 \leq 5$ or $5 \leq -2$.
T $\vee$ F ✓.

$2 \leq 2$ or $2 \leq 2$.
T T ✓

$-\infty \leq -\infty + 1 \leq \dots -2 \leq 1 \leq 0 \leq 1 \leq 2 \leq \dots \leq \infty$.

Ex. $(\mathbb{Z}^+, |)$.
$4 | 3$ or $3 | 4$.
F $\vee$ F Not a total order.

$R \rightarrow PO \rightarrow TO$.

GRAPHICAL/VISUAL REPRESENTATION OF A PO.

HASSE DIAGRAM :-

Ex12 $\quad (\{1,2,3,4,6,8,12\}, \mid)$.

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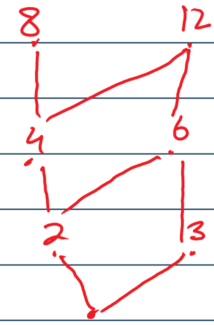
$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12),$
$(2,2), (2,4), (2,6), (2,8), (2,12),$
$(3,3), (3,6), (3,12),$
$(4,4), (4,8), (4,12),$
$(6,6), (6,12),$
$(8,8),$
$(12,12)\}$.



$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12),$
$(2,2), (2,4), (2,6), (2,8), (2,12),$
$(3,3), (3,6), (3,12),$
$(4,4), (4,8), (4,12),$
$(6,6), (6,12)$
$(8,8)$
$(12,12)\}$.

Ex14 :- $\quad (\{2,4,5,10,12,20,25\}, \mid)$.

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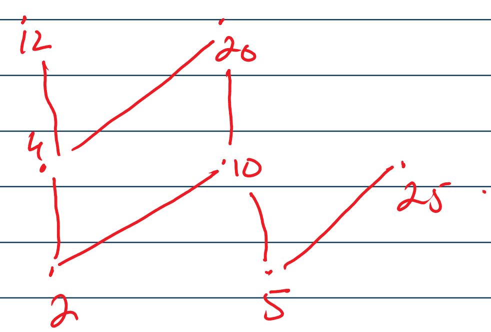

Ex 13

S09.

$(PS(\{a,b,c\}), \subseteq)$.

$PS = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$.

$R = \{(\emptyset,\emptyset), (\emptyset,\{a\})$ - - —— - - - .

$(\{a\},\{a\}), (\{a\},\{a,b\}), (\{a\},\{a,c\}), (\{a\},\{a,b,c\}),$

$(\{b\},$ — $)$

$(\{c\},$ — $)$

$(\{a,b\},\{a,b\}), (\{a,b\},\{a,b,c\})$

$(\{b,c\}$ — $)$

$(\{a,c\}$ — $).$

$(\{a,b,c\}, (\{a,b,c\})$