Lecture 16:- Equivalence Relation.

- Reflexive
- Symmetric
- Transitive.

Ex2:- 494. $R = \{(a,b) \mid a-b \in Z\}$. $A = \mathbb{R}$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$.
$\forall a \in \mathbb{R} \quad a-a \in Z$. ✓

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.
$\forall a,b \in \mathbb{R}$ if $(a-b) \in Z \rightarrow (b-a) \in Z$. ✓

Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{R}$ if $(a-b) \in Z \wedge (b-c) \in Z \rightarrow (a-c) \in Z$. ✓
$1-0.5$

EQUIVALENCE ✓.

Ex3. 494.   Congruence Modulo.

$R = \{(a,b) \mid a \equiv b \mod m\}$. $A = Z$.

Reflexive:- $\forall a \in A \quad (a,a) \in R$
$\forall a \in Z \quad a \equiv a \mod m$. ✓

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.
$\forall a,b \in Z$ if $\exists_{k \in Z} (a-b) = km \rightarrow \exists_{k \in Z} (b-a) = km$. .

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in Z$ if $a-b = km \wedge b-c = lm \rightarrow a-c = (k+l)m$ ✓

EQUIVALENCE.

Ex 6
4a5

$R = \{(a,b) \mid a \div b\}$. $A = Z^+$.

Reflexive: $\forall a \in A$ $(a,a) \in R$

$\forall a \in Z^+$ $a \div a$ ✓.

Symmetric : $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.

$\forall a,b \in Z^+$ if $a \div b \rightarrow b \div a$. X

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

X.

EQUIVALENCE X.

Ex 7
4a5

$R = \{(x,y) \mid |x-y| < 1\}$. $A = \mathbb{R}$.

Reflexive: $\forall a \in A$ $(a,a) \in R$

$\forall a \in R$ $|a-a| < 1$.

Symmetric : $\forall a,b \in A$ if $(a,b) \in R \rightarrow (a,b) \in R$.

$\forall a,b \in \mathbb{R}$ if $|a-b| < 1 \rightarrow |b-a| < 1$.

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$\forall a,b,c \in \mathbb{R}$ if $|a-b| < 1 \wedge |b-c| < 1 \rightarrow |a-c| < 1$.

$|0.2-0.8| < 1 \wedge |0.8-1.3| < 1 \rightarrow |0.2-1.3| \not< 1$.

$\forall a,b,c \in \mathbb{R}$ if $|a-b|<1 \wedge |b-c|<1 \rightarrow |a-c|<1$.

$|0.2-0.8|<1 \wedge |0.8-1.3|<1 \rightarrow |0.2-1.3| \not< 1$.

$a = 0.2$.
$b = 0.8$.
$c = 1.3$.

✗ EQUIVALENCE.

EQUIVALENCE CLASS.

$$[a] = \{ s \mid (a,s) \in R \}$$

Ex8 : 496 $\quad R = \{(a,b) \mid a=b \vee a=-b\}$. $\quad A = \mathbb{Z}$.

$7=7$

$[7] = \{7, -7\}$.

$(7,7)$.
$(7,-7)$.

Ex9 : 496 $\quad R = \{(a,b) \mid a \equiv b \bmod m\}$. $\quad A = \mathbb{Z}$.

$[0] = \{0, \pm 4, \pm 8, \pm 12, \pm 16 \ldots\}$. $\quad 0 \equiv b \bmod 4$.

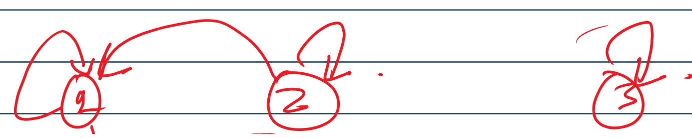
$(0,0)$
$(0,4)$
$(0,8)$

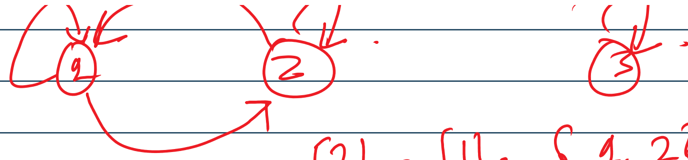
$[1] = \{1, 5, 9, 13, \ldots, -3, -7, -11, -15 \ldots\}$. $\quad 1 \equiv b \bmod 4$.

$(1,1), (1,5), (1,9) \ldots$

$\left.\begin{array}{l}[2] = ? \\ {[3]} = ?\end{array}\right\}$ HW.

$$\begin{array}{r} -1 \\ 4 \overline{)\,-3} \\ -4 \\ + \\ \hline 1 \end{array}$$





$[2] = [1] = \{1, 2\}$.

$[3] = \{3\}$.

$$\begin{array}{c c} & \begin{array}{ccc} a & b & c \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

$[c] = [a] = \{a, c\}$.

$[b] = \{b\}$.

PARTITION. Let S be set. A partition $P = \{P_1, P_2, \dots P_n\}$ where $P_i \subseteq S$.

if.

i) $\forall i \; P_i \neq \phi$. $= P_1 \neq \phi \wedge P_2 \neq \phi \wedge \dots \wedge P_n \neq \phi$.

ii) $\forall i,j \; P_i \cap P_j = \phi \quad i \neq j$.

iii). $\bigcup_{i=1}^{n} P_i = S$.

Ex12 498 $\quad S = \{1, 2, 3, 4, 5, 6\}$.

$P_1 = \{1, 2, 3\} \qquad P_2 = \{4, 5\}, \qquad P_3 = \{6\}$.

Check (i) $\quad P_1 \neq \phi \wedge P_2 \neq \phi \wedge P_3 \neq \phi$

$T \wedge T \wedge T = T$.

Check (ii) $\quad P_1 \cap P_2 = \phi \wedge P_1 \cap P_3 = \phi \wedge P_2 \cap P_3 = \phi$.

$T \wedge T \wedge T = T$.

Check (iii) $\quad P_1 \cup P_2 \cup P_3 = S$.

Hence a Partition.

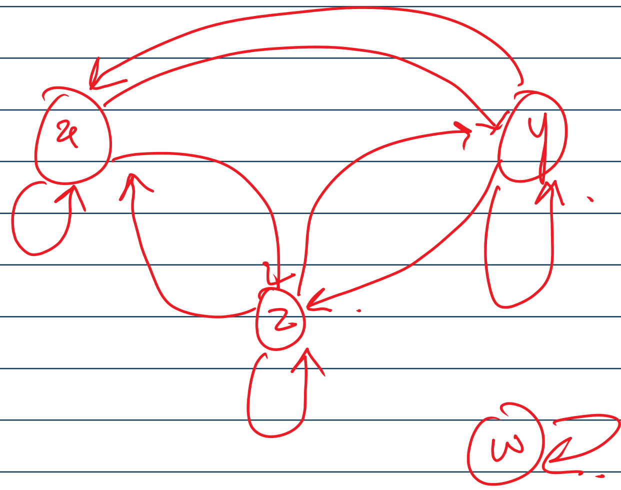
Theorem: Equivalence classes creates a partition of the set based on which the relation is defined.

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 1 \\ b & 0 & 1 & 0 \\ c & 1 & 0 & 1 \end{array}$$

$[c] = [a] = \{a, c\}$.

$[b] = \{b\}$.

$P = \{\{a, c\}, \{b\}\}$



$[x] = [y] = [z] = \{x, y, z\}$.

$[w] = \{w\}$.

$P = \{\{x, y, z\}, \{w\}\}$.

$A = \{x, y, z, w\}$.

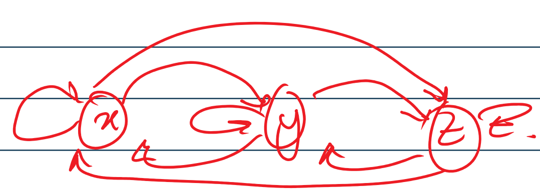
$R = \{(x,x), (x,y), (x,z),$
$(y,x), (y,y), (y,z),$
$(z,x), (z,y), (z,z),$
$(w,w)\}$.

G.
M

ER $\rightarrow$ EC $\rightarrow$ P

ER $\leftarrow$ EC $\leftarrow$ P

$$\begin{array}{c|cccc} & x & y & z & w \\ \hline x & 1 & 1 & 1 & 0 \\ y & 1 & 1 & 1 & 0 \\ z & 1 & 1 & 1 & 0 \\ w & 0 & 0 & 0 & 1 \end{array}$$



w