Lecture 6:- Negating Quantifiers.

DeMorgans law.

$$\neg(P \wedge Q) = \neg P \vee \neg Q.$$

$$\neg(P1 \wedge P2 \wedge P3 \wedge \cdots \wedge PN) = \neg P1 \vee \neg P2 \vee \neg P3 \vee \cdots \vee \neg PN.$$

$$\neg(P \vee Q) = \neg P \wedge \neg Q.$$

$$\neg(P1 \vee P2 \vee P3 \vee \cdots \vee P_N) = \neg P1 \wedge \neg P2 \wedge \neg P3 \wedge \cdots \wedge \neg P_N.$$

$$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(N). \qquad x \in \{1,2,3 \cdots N\}.$$

Taking Negation on both Sides.

$$\neg(\forall x P(x)) = \neg(P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(N)).$$
$$= \underbrace{\neg P(1)} \vee \underbrace{\neg P(2)} \vee \underbrace{\neg P(3)} \vee \cdots \vee \underbrace{\neg P(N)}.$$
$$= \exists x \neg P(x).$$

① $\neg \forall x P(x) = \exists x \neg P(x).$

$$\exists x P(x) = \overline{P(1)} \vee \overline{P(2)} \vee \overline{P(3)} \vee \cdots \vee \overline{P(N)}$$

$$\exists x P(x) = P(1) \vee P(2) \vee P(3) \vee \cdots \vee P(N).$$

Taking Negation on both Sides.

$$\neg(\exists x P(x)) = \neg(P(1) \vee P(2) \vee P(3) \vee \cdots \vee P(N))$$
$$= \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \cdots \wedge \neg P(N).$$
$$= \forall x \neg P(x).$$

$$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(N).$$

① $\neg \forall x \boxed{P(x)} = \exists x \neg P(x).$

② $\neg \exists x P(x) = \forall x \neg P(x).$

Question:- $\underline{\neg \forall x} \boxed{\forall y \neg P(x,y)}$ — $P(x)$.

$$= \exists x \neg \forall y \boxed{\neg P(x,y)}.$$

$$= \exists x \exists y \neg \neg P(x,y).$$

$$= \exists x \exists y P(x,y).$$

$$= \exists x \exists y \neg \neg P(x,y).$$
$$= \exists x \exists y\, P(x,y).$$

Question:- $\forall x \neg \exists y \underbrace{\neg \forall z\, P(x,y,z)}_{P(x)}$

$$= ?$$

$$= \forall x \forall y \neg \neg \forall z\, P(x,y,z).$$
$$= \forall x \forall y \forall z\, P(x,y,z).$$

① $\neg \forall x\, P(x). = \exists x \neg P(x).$

② $\neg \exists x\, P(x) = \forall x \neg P(x).$

Ex 1–40
P 43–46 HW.

Nested Quantifiers.

$\forall x \forall y\, P(x,y).$ $\qquad x,y \in \{1,2,\cdots N\}$

$\forall x P(x) = P(1) \wedge P(2) \wedge \cdots \wedge P(N).$

$$= \forall x \left(P(x,1) \wedge P(x,2) \wedge P(x,3) \wedge \cdots \wedge P(x,N) \right).$$
$$= \underbrace{\forall x\, P(x,1)}_{\star} \wedge \underbrace{\forall x P(x,2)}_{\star\star} \wedge \forall x P(x,3) \wedge \cdots \wedge \forall x P(x,N).$$
$$= \left(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \right) \wedge$$
$$\left(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \right) \wedge.$$
$$\vdots$$
$$\left(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N) \right).$$

$\forall x \exists y\, P(x,y).$

$$= \forall x \left(P(x,1) \vee P(x,2) \vee P(x,3) \vee \cdots \vee P(x,N) \right).$$
$$= \forall x\, P(x,1) \vee \forall x P(x,2) \vee \forall x\, P(x,3) \vee \cdots \vee \forall x P(x,N)$$
$$= \left(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \right) \vee$$
$$\left(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \right) \vee$$
$$\vdots$$

$(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N))$.

HW. $\exists x \forall y \; P(x,y) = ?$

HW. $\exists x \exists y \; P(x,y) = ?$

Ex:- Question 30 | p45

Find $\exists x \, P(x, 1)$. $x \in \{0, 1, 2\}$.

in terms of $\vee, \wedge$ & $\neg$ only.

Solution:- $P(0,1) \vee P(1,1) \vee P(2,1)$.

$\neg \forall x \exists y \, P(x,y)$.

$= \exists x \neg \exists y \, P(x,y)$

$= \exists x \forall y \neg P(x,y)$. $x, y \in \{a, b\}$.

$= \exists x \, [\neg P(x,a) \wedge \neg P(x,b)]$.

$= \exists x \neg P(x,a) \wedge \exists x \neg P(x,b)$.

$= (\neg P(a,a) \vee \neg P(b,a)) \wedge (\neg P(a,b) \vee \neg P(b,b))$

Meaning of Quantifiers (Nested).

Ex 4 | p48. $Q(x,y) = x + y = 0$.

find $\exists y \forall x \; Q(x,y) = ?$ F $x, y \in \mathbb{R}$.

$\forall x \exists y \; Q(x,y) = ?$ T

$\exists z$ أي

$\forall z$ لأي

Ex 5 $Q(x,y,z) = x + y = z$

$\forall x \forall y \exists z \; Q(x,y,z) = T$ $x, y, z \in \mathbb{R}$.

$\exists z \forall x \forall y \; Q(x,y,z) = F$

$Q(x,y) \;\; x = y$. $x, y \in \mathbb{R}$.

$\forall x \exists y \; Q(x,y) = ?$ T

$\exists y \forall x\ Q(x,y) = ?$ F

Expressing English Statements Using Quantifiers.

Ex23 :- Every Student in this class has Studied DM.
P40

Quantifier + Subject + Predicate.

for all x, x is a student in this class, x has studied DM.

$x \in$ { Students in this class }.

$\forall x P(x)$.

$P(x)$ = x has studied DM.

All lions are fierce.

for all x, x is a lion, x is fierce.

$\forall x f(x)$.

$x \in$ { lions }.

$f(x)$ = x is fierce.

Q25/P46/Ex:- All your friends are perfect.

for all x, x is your friend, x is perfect.

$x \in$ { friends }

$P(x)$ = x is perfect.

$\forall x\ P(x)$.

Not everyone is perfect.

It is not true that for all x, x is a person, x is perfect.

$\neg \forall x\ P(x)$.

$= \exists x \neg P(x)$.

$x \in$ { persons }.

$P(x)$ = x is perfect.

everyone is not perfect.

for all x, x is a person, x is not perfect.

everyone is not perfect.

for all x, x is a person, x is not perfect.

$\forall x\ \neg P(x)$.

$x \in \{persons\}$.

$P(x) =$ x is perfect.

Ex 11 :- If a person is a female, and is a parent then this person is someone's mother".
P51

If for all x, x is a person. x is a female and. x is a parent then there exist y, y is a person, x is the mother of y".

$\forall x\ (F(x) \land P(x)) \rightarrow \exists y\ M(x,y)$.

$x, y \in \{persons\}$.

$F(x) =$ x is a female.

$P(x) =$ " " " parent.

$M(x,y) =$ x is the mother of y.

Everyone has atleast one friend.

for all x, x is a person, there exist y, y is also a person. such that x is the friend of y.

$\forall x \exists y\ F(x,y)$.

$x, y \in \{person\}$.

$F(x,y) =$ x is the friend of y.

Question 20 / P54.

$F(x,y) =$ " x can fool y "

$x, y \in \{$people in world$\}$.

a) Everyone can fool fred.

Sol:- for all x, x is a person, x can fool fred.

$\forall x\ F(x, Fred)$.

No one can fool fred and jerry.

It is not the case for all x, x is a person, x can fool fred and. x can fool jerry.

$$\neg \forall x \left(F(x, Fred) \land F(x, Jerry) \right).$$

$$= \exists x \neg \left(\underbrace{F(x, Fred)} \land F\underbrace{(x, Jerry)} \right).$$

$$= \exists x \; \neg F(x, Fred)^{P} \lor \neg F(x, \overset{Q}{Jerry}) .$$