Lecture 19:-

Maximal :- $a \in S$ is maximal in $(S, \leq)$ if $\neg \exists_{b \in S}\ a < b$.

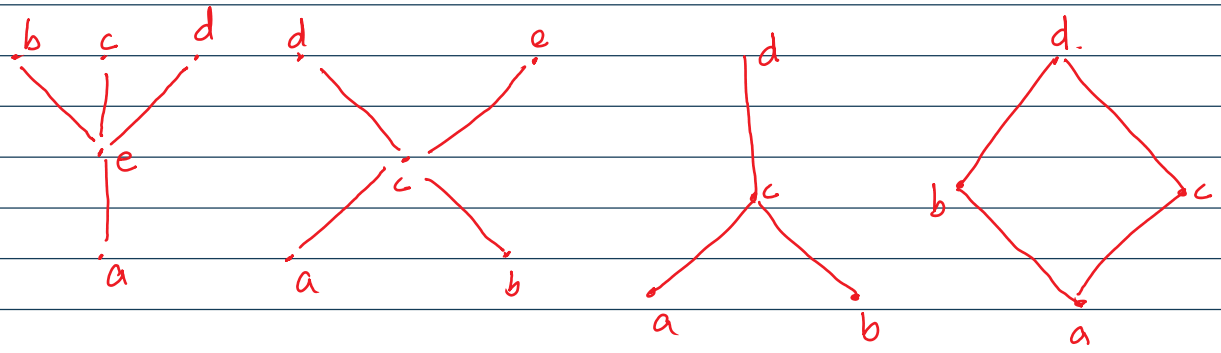
Minimal :- $a \in S$ is minimal in $(S, \leq)$ if $\neg \exists_{b \in S}\ b < a$.

Greatest :- $a \in S$ is Greatest in $(S, \leq)$ if $\forall_{b \in S}\ b \leq a$.

Least :- $a \in S$ is least in $(S, \leq)$ if $\forall_{b \in S}\ a \leq b$.

Ex 17

PSlo.



(a) (b) (c) (d)

	(a)	(b)	(c)	(d)
Maximal	b, c, d	d, e	d	d
Minimal	a	a, b	a, b	a
Greatest	X.	X	d	d
least	a.	X.	X.	a.

PSlo

Upper Bound. $U \in S$ in $(S, \leq)$. $A \subseteq S$.
if $\forall_{a \in A}\ a \leq U$

Upper Bound. $U \in S$ in $(S, \leq)$. $A \subseteq S$.

if $\forall a \in A \quad a \leq U$

then U is in the Upper Bound of A.

lower Bound:- $l \in S$ in $(S, \leq)$.

if $\forall a \in A \quad l \leq a$

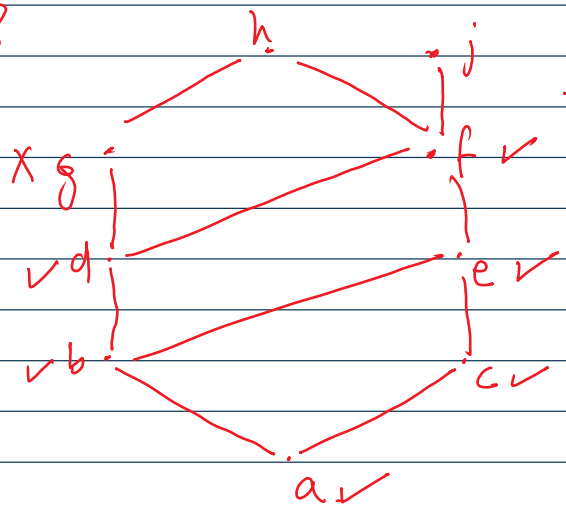
then l is in the lower Bound of A.

least Upper Bound:-

Greatest lower Bound:-

PSII.

Ex 18



$\{a, b, c\}$.

→ least Upper Bound
Greatest lower Bound.

Upper Bound $(\{j, h\}) = \phi$.

Upper Bound $(\{a, b, c\}) = \{e, f, j, h\}$.

least Upper Bound $(\{a, b, c\}) = e$

lower Bound $(\{a, b, c\}) = \{a\}$.

Greatest " " " $= a$.

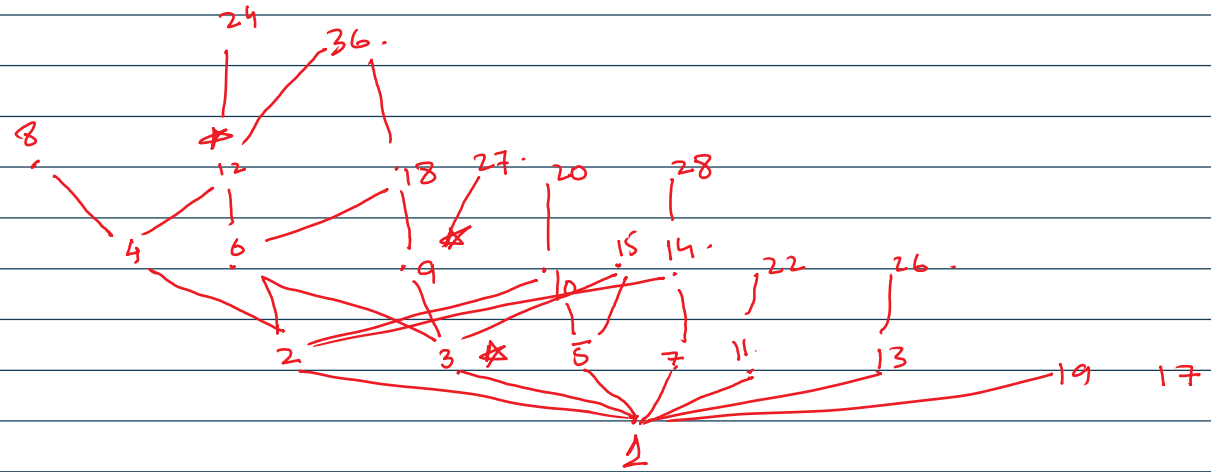
Ex 19. (Hw)

Ex 20 :- (Z+, |). {3, 9, 12}.

P 511

Greatest lower Bound.

Least Upper " .



Lower Bound ({3, 9, 12}) = {1, 3}.

Greatest " " " = 3. = GCD.

Upper Bound {{3, 9, 12}} = {36, 72, 108, 144 - - -}.

Least " " " = 36. = LCM.

LATTICE :- A poset in which every pair of elements has least Upper and greatest lower Bounds.

Ex 21 :-

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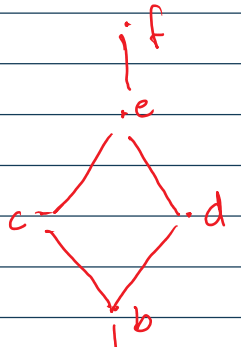
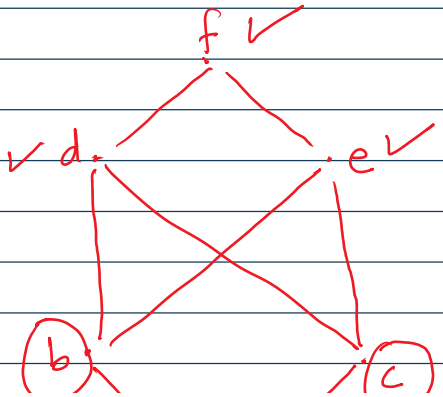
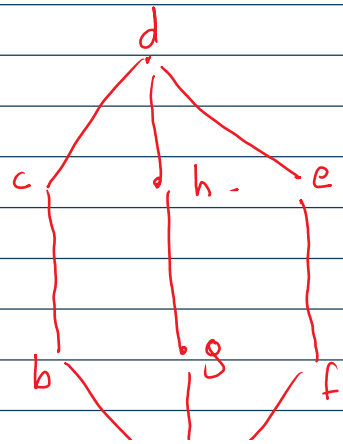
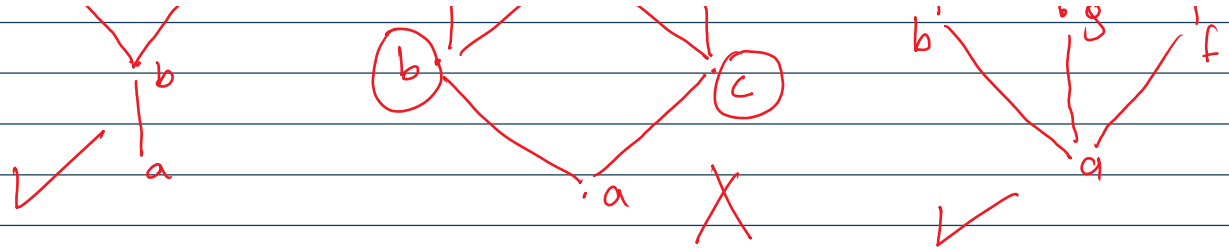
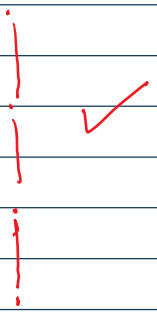




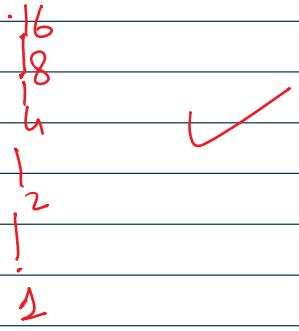




{d, e, f}



Ex 23: PS12 $(\{1, 2, 4, 8, 16\}, |)$ Lattice?



$(\{1, 2, 3, 4, 5\}, |)$ Lattice?

{3, 5}