

lecture 10:-

$A = \{a, b, c\}$

$P(SA) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$

$$R \quad |P(SA) \times P(SA)| = |P(SA)| \times |P(SA)| = 8 \times 8 = 2^{64}$$

$|A| = 3$

$$3 \times 3 = 9$$

$$2^9 = 512$$

$$P(SA) \times P(SA) = \{(\emptyset, \emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), \dots, (\{a\}, \emptyset), (\{a\}, \{a\}), \dots, \dots\}$$

$$R = \{(a, b) \mid a \subseteq b\} = ? \quad P(SA)$$

HW.

$$= \{(\emptyset, \emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), (\emptyset, \{c\}), (\emptyset, \{a, b\}), (\emptyset, \{b, c\}), (\emptyset, \{a, c\}), (\emptyset, \{a, b, c\}), (\{a\}, \{a\}), (\{a\}, \{a, b\}), (\{a\}, \{a, c\}), (\{a\}, \{a, b, c\}), (\{b\}, \{b\}), (\{b\}, \{b, c\}), (\{b\}, \{a, b, c\}), (\{c\}, \{c\}), (\{c\}, \{a, b, c\})\}$$

$$R = \{(a, b) \mid |a \cap b| \geq 1\} \quad \text{HW.}$$

SET BUILDER NOTATION.

$$\{ \text{Syntax} \mid \text{Semantics Condition} \}$$

Ex 5 = let $A = \{1, 2, 3, 4\}$
 P461

$$R_1 = \{(a, b) \mid a \leq b\}.$$

$$= \{(1, 1), (1, 2), (1, 3), (1, 4),$$

$$(2, 2), (2, 3), (2, 4),$$

$$(3, 3), (3, 4), (4, 4)\}.$$

$$A \times A.$$

$$= \{(1, 1), (1, 2), (1, 3), (1, 4),$$

$$(2, 1), (2, 2), (2, 3), (2, 4),$$

$$(3, 1), (3, 2), (3, 3), (3, 4),$$

$$(4, 1), (4, 2), (4, 3), (4, 4)\}.$$

$$R_2 = \{(a, b) \mid a > b\} = ? \text{ HW. } R_2 = \{(a, b) \mid a \text{ divides } b\}.$$

$$R_3 = \{(a, b) \mid a = b\}. \text{ HW}$$

$$R_4 = \{(a, b) \mid a = b + 1\}.$$

$$R_5 = \{(a, b) \mid a + b \leq 3\}. \text{ HW.}$$

PROPERTIES OF RELATIONS.

1- Reflexive. $\forall a \in A (a, a) \in R.$

Ex 7 $A = \{1, 2, 3, 4\}.$
 P462.

$$\begin{array}{l} \checkmark \\ (1, 1) \in R \wedge (2, 2) \in R \wedge \\ (3, 3) \in R \wedge (4, 4) \in R. \\ \checkmark \end{array}$$

$$R_1 = \{(1, 1), (2, 2)\}. \quad X$$

$$R_2 = \{(\underline{1}, \underline{1}), (\underline{2}, \underline{2}), (\underline{3}, \underline{3}), (\underline{4}, \underline{4})\}. \quad \checkmark$$

$$R_3 = \{(\underline{1}, \underline{1}), (\underline{2}, \underline{2}), (\underline{3}, \underline{3}), (\underline{4}, \underline{4}), (3, 1), (3, 2)\}. \quad \checkmark$$

$$R_4 = \{\}. \quad X$$

$$A = \{1\}. \quad A \times A = \{(1, 1)\}.$$

$$|A \times A| = |A| \times |A| = 1 \times 1 = 1.$$

$$P(A \times A) = 2^{|A \times A|} = 2^{1 \times 1} = 2^1 = 2.$$

$$= \{ \emptyset, \{ (1,1) \} \}$$

$$\forall a \in A \quad (a,a) \in R.$$

$$A = \{ 1 \}.$$

$$(1,1) \in R$$

$$\begin{array}{c} \emptyset \\ \{ (1,1) \} \end{array} \quad \begin{array}{c} X \\ \checkmark \end{array}$$

$$A = \{ 1, 2 \}.$$

Reflexive = ?

$$\forall a \in A \quad (a,a) \in R.$$

$$(1,1) \in R \wedge (2,2) \in R.$$

$$A \times A = \{ (1,1), (1,2), (2,1), (2,2) \}.$$

$$P(A \times A) = \{ \emptyset, \{ (1,1) \}, \{ (1,2) \}, \{ (2,1) \}, \{ (2,2) \},$$

$$\{ (1,1), (1,2) \}, \{ (1,1), (2,1) \}, \{ (1,1), (2,2) \},$$

$$\{ (1,2), (2,1) \}, \{ (1,2), (2,2) \}, \{ (2,1), (2,2) \}$$

$$\{ (1,1), (1,2), (2,1) \}, \{ (1,1), (1,2), (2,2) \},$$

$$\{ (1,2), (2,1), (2,2) \}, \{ (1,1), (2,1), (2,2) \},$$

$$\{ (1,1), (1,2), (2,1), (2,2) \}.$$

$$A = \{ 1, 2, 3, 4 \}.$$

$$R = \{ (a,b) \mid a = b \} = \{ (1,1), (2,2), (3,3), (4,4) \}.$$

$$\bar{R} = \{ (a,b) \mid (a,b) \notin R \} = \text{HW.}$$

$$= A \times A - R.$$

$$R^{-1} = \{ (a,b) \mid (b,a) \in R \} = \{ (b,a) \mid (a,b) \in R \}.$$

$$R = \{ (2,2), (2,1), (3,1) \}. \quad \text{Find } R^{-1}.$$

$$R^{-1} = \{ (2,1), (1,2), (1,3) \}.$$

Symmetric. $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

Ex 7: $A = \{ 1, 2, 3, 4 \}$.
P462

$$R_1 = \{ (\underline{1}, \underline{1}), (1,2), (2,1), (2,2), (\underline{3}, \underline{4}), (4,1), (4,4) \}. X.$$

$$R_2 = \{ \}$$

$$R_3 = \{ (1,1) \}.$$

$$R_4 = \{ (1,2), (2,1), (1,1) \}. \checkmark$$

$$A = \{ 1 \}.$$

$$A \times A = \{ (1,1) \}.$$

$$PSCA = \{ \varnothing, \{ (\underline{1}, \underline{1}) \} \}.$$

$$A = \{ 1, 2 \}.$$

Symmetric.