lecture 15 :- Transitive Closure.

P484 :- $R = \{(1,3),(1,4),(2,1),(3,2)\}$. $A = \{1,2,3,4\}$.

$R_m = \{(1,2),(2,3),(2,4),(3,1)\}$.

$R \cup R_m = \{(1,3),(1,4),(2,1),(3,2)\} \cup \{(1,2),(2,3),(2,4),(3,1)\}$.

$= \{(1,3),(\overset{b}{1},\overset{c}{4}),(2,1),(3,2),(1,2),(2,3),(2,4),(\overset{a}{3},\overset{b}{1})\}$

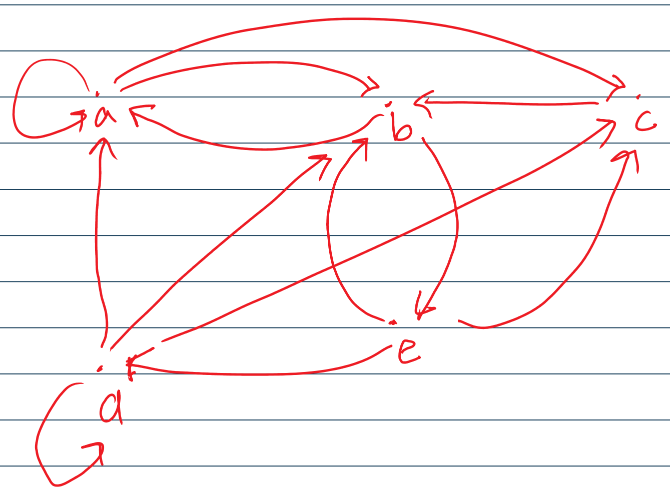
$(3,1),(1,4) \rightarrow (3,4) \notin R \cup R_m$.

R Insert missing $\rightarrow$ More Missing.

Paths in a Directed Graph.

A path from a to b in a directed Graph G when there is a sequence of edges such that $(a,x_1),(x_1,x_2),(x_2,x_3)\dots(x_n,b)$.

Ex3 :-
P484



a to e.	Length
(a,b) (b,e).	2.
a,b,e	3-1=2
a to a	
(a,a)	1.
(a,a)(a,a)(a,a)	3.

P485 / Theorem :- Let R be a relation on A.

there is a path of length n $n \in Z^+$ from a to b

iff $(a, b) \in R^n$.

$R^2 = R \circ R$.

Find R in the above Example.

then u R^2.

Confirm length 2 paths in the Graph.

Connectivity Relation: Let R be a relation on A.

the Connectivity Relation R^* Consist of pairs (a, b)

Such that $\exists$ a path from a to b in R.

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cup R^4 \cup \dots \cup R^{\infty}.$$

Ex4 p485 $R = \{(a, b) \mid a \text{ has met } b\}$. A = Set of all people of world.

R^*.

$\overset{R}{S} \circ R$. R $(a, b) \in R$ $A \times B \; A$

$R \; S$ $(b, c) \in R$. $B \times C$.

x_1 A A.

$(a, c) \in S \circ R$. iff $(a, b) \in R \wedge$.

$(b, c) \in S$.

$R^2 = R \circ R$. a has met x_1

x_1 has met b.

$(a, b) \in R^2 \Rightarrow \exists$ x_1 Such That

a has met x_1 & x_1 has met b.

$(a, b) \in R^3 \Rightarrow \exists$ x_1, x_2, Such that

a has met x_1, x_1 has met x_2

and x_2 has met b.

$(a,b) \in R^n \Rightarrow \exists\ x_1, x_2, x_3, \ldots x_{n-1}$ such that
a has met x_1
x_1 " " x_2
- - - - - - -
x_{n-2} has met x_{n-1}.
x_{n-1} " " b.

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cup \cdots \cup R^\infty.$$

$= \exists$ atleast any persons between a & b.

Ex 6
486

$R = \{(a,b) \mid a$ has a common border with $b\}$.
$A =$ Set of States in US.

R^*.

The transitive closure equals to R^*.

Ex 491 – 493 Hw.

EQUIVALENCE RELATIONS.
1- Reflexive.
2- Symmetric
3- Transitive.

Ex 1:
P494

$R = \{(a,b) \mid a = b \lor a = -b\}$ $A = \mathbb{Z}$

Reflexive. $\forall a \in A$ $(a,a) \in R$.

$\forall a \in \mathbb{Z}$ $a = a \lor a = -a$.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a, b \Rightarrow$ if $a = b \lor a = -b \rightarrow b = a \lor b = -a$.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in \mathbb{Z}$ if $a=b \vee a=-b \rightarrow b=a \vee b=-a$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $(a=b \vee a=-b) \wedge (b=c \vee b=-c) \rightarrow$
$a=c \vee a=-c$.

Ex2, Ex3 Hw.