

# lecture 21:-

Bipartite  
 $K_{m,n}$

Vertices  
 $m+n$

Edges -  
 $mn$ .

PS45 | 546

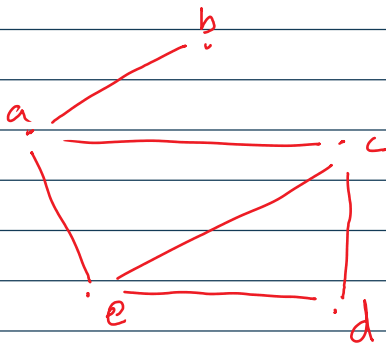
"Subgraph"

"Graph Intersection"

Ex 545-549. (1-40).

## 1- Adjacency List.

Ex1  
SSO



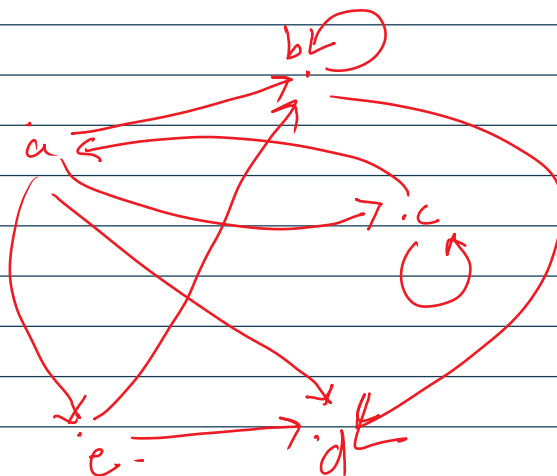
Vertices

Adjacent Vertices.

a  
b  
c  
d  
e

b, c, e  
a  
a, e, d  
c, e  
a, c, d.

Ex2  
SSO.



Initial  
Vertex

Terminal  
Vertex.

a  
b  
c  
d  
e

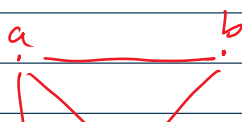
b, c, d, e  
b, d  
c, a  
-  
d, b.

## 2- Adjacency Matrix.

$|V| \times |V|$ .

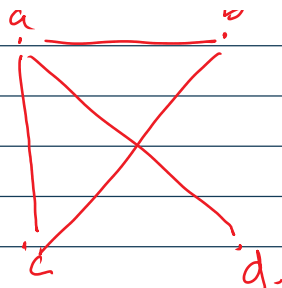
$V =$  Set of Vertices.

Ex3 :-  
PSSD



	a	b	c	d	
a	0	1	1	1	3

Ex 3 :-  
PSSD



	a	b	c	d	
a	0	1	1	1	3
b	1	0	1	0	2
c	1	1	0	0	2
d	1	0	0	0	1

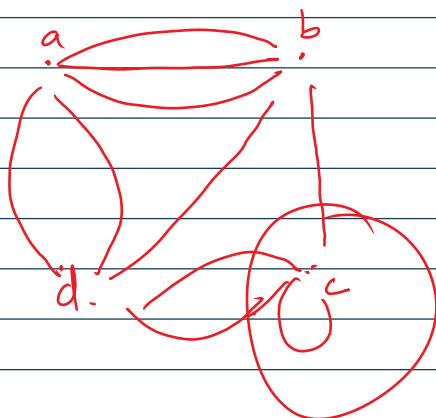
Properties: 1. No loop.

2- Zero-one matrix  
then No multiedges.

4- Symmetric matrix.

3- Row-wise / Col-wise  
sum = degrees of  
vertices.

Ex 5

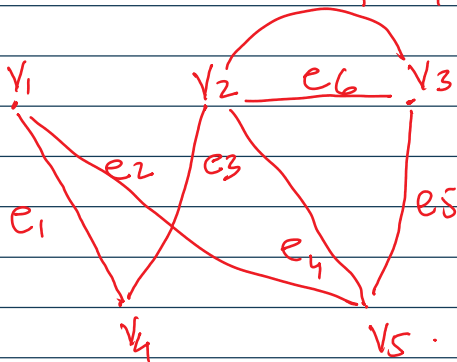


	a	b	c	d	
a	0	3	0	2	
b	3	0	1	1	
c	0	1	1	2	4
d	2	1	2	0	

1- while counting degree -  
if main diagonal contain value -  
x by 2. to count degrees.

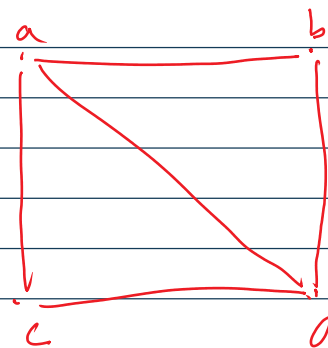
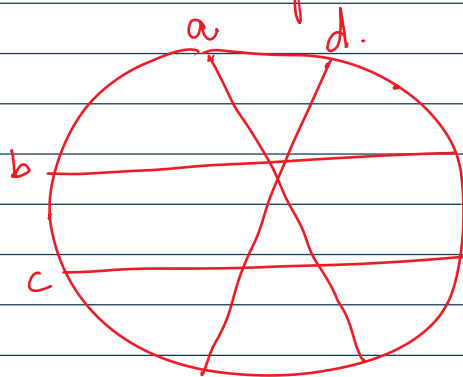
Incident Matrix:  $e_7$ .  $|V| \times |E|$ .

Ex 6  
SSD.

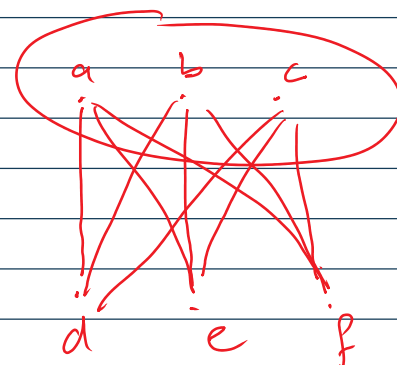
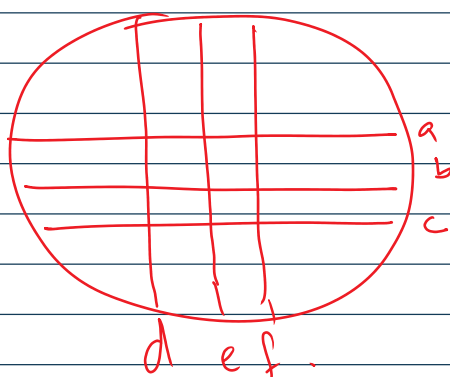


	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	
$v_1$	1	1	0	0	0	0	2
$v_2$	0	0	1	1	0	1	3
$v_3$	0	0	0	0	1	1	2
$v_4$	1	0	1	0	0	0	2
$v_5$	0	1	0	1	1	0	3

Circular Graph.



$K_{3,3}$ .



Ex 556-559.

1-50.

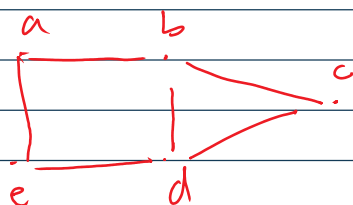
Connectivity:-

path

'a' to 'b'

$$(a, x_1) (x_1, x_2) (x_2, x_3) \dots (x_{n-1}, x_n), (x_n, b).$$

Ex 1  
560

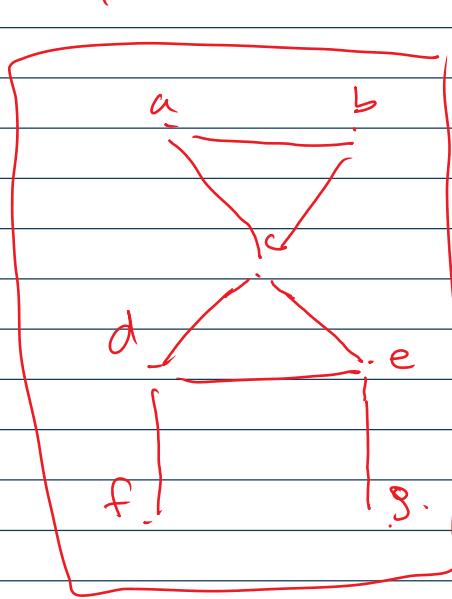


Connected Graph:- If  $\exists$  a path b/w all pairs of vertices.

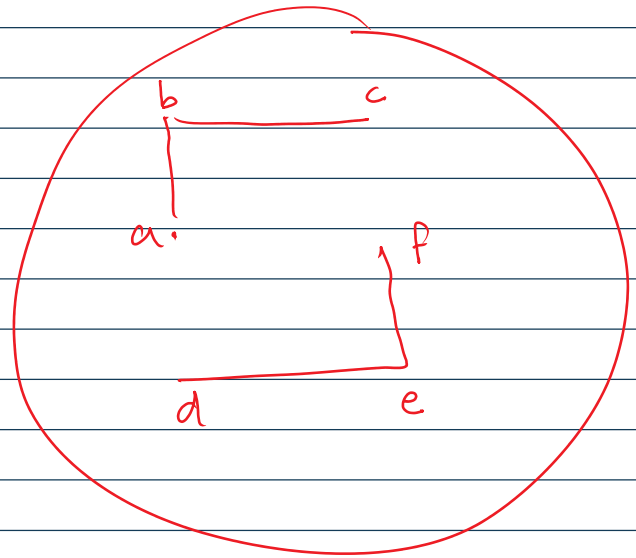


... part of ...

Ex 6  
563.



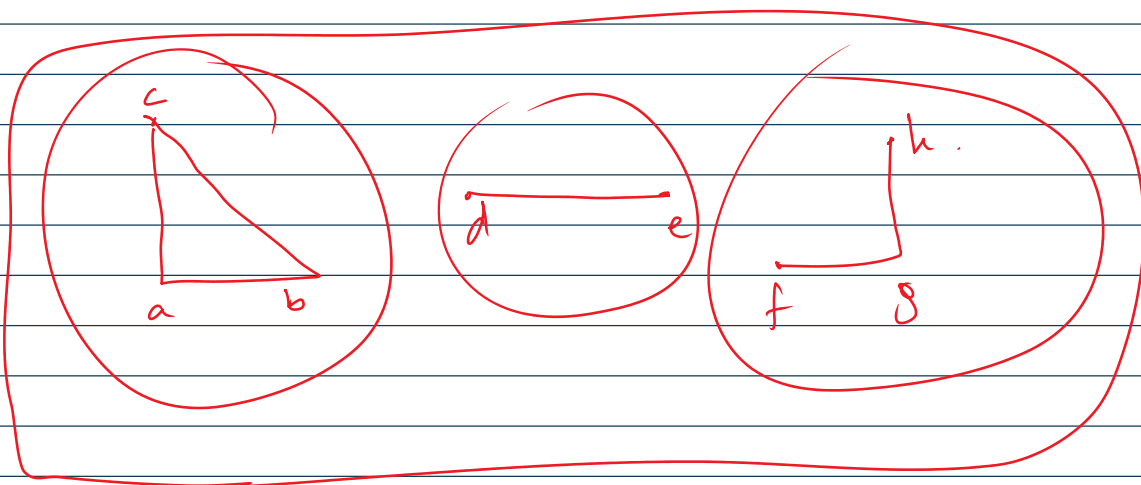
$G$  ✓



$H$  ✗

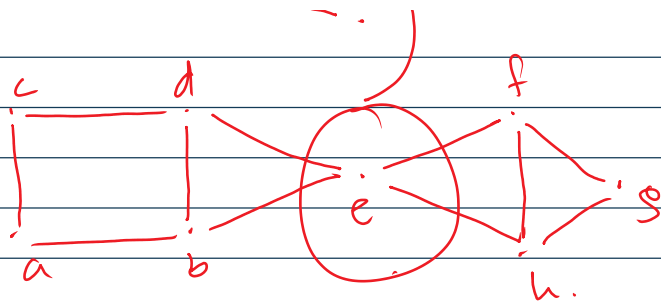
Connected Component:-

- 1- Subgraph.
- 2- Connected.
- 3- Maximally Connected.



Cut Vertex ←

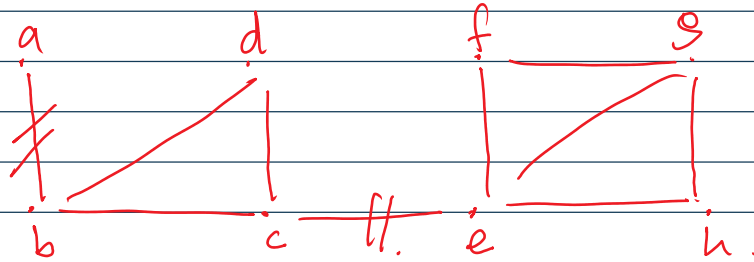
c d f



Cut Vertices = e

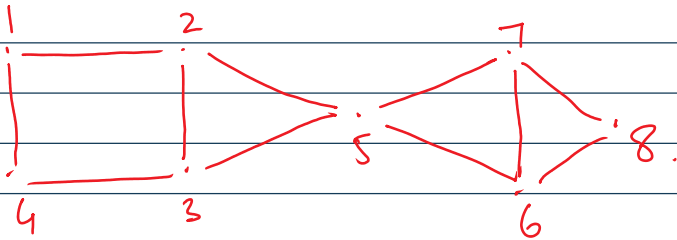
Cut Edges:-

Ex 8  
564.



Cut Edges =  $(a,b)$   
 $(c,e)$ .

Cut Set  
=  $\{(2,5), (3,5)\}$ .

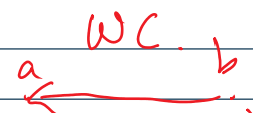
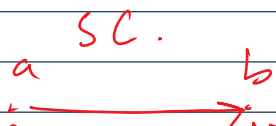


Strongly Connected / weakly Connected.  
- directed.

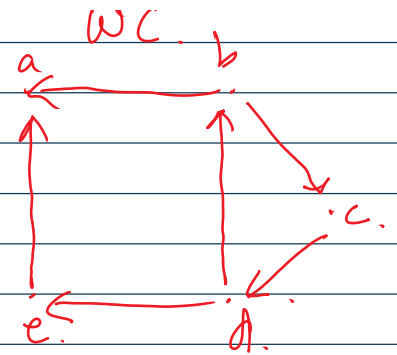
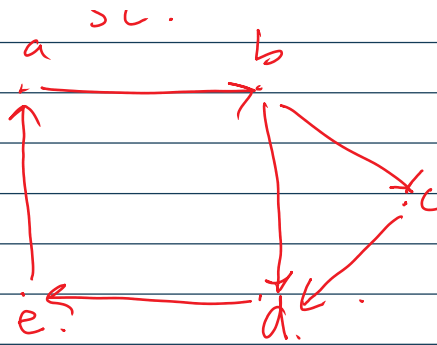
-  $\forall a, b \in V$  there must  
be path from a to b  
^  
b to a.

-  $\forall a, b \in V$  there  
exist a path from  
a to b  $\vee$   
b to a.

Ex 12.



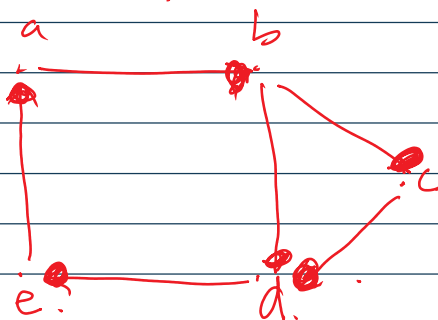
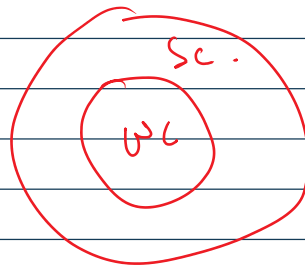
Ex 12.  
565



a to b ✓      b to a ✓  
 a to c ✓      c to a ✓  
 a to d ✓      d to a ✓  
 a to e ✓      e to a ✓

a to b X.

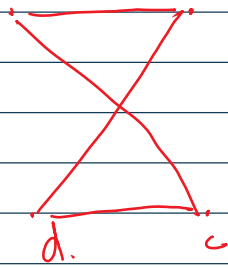
b to c ✓      c to b ✓  
 b to d ✓      d to b ✓  
 b to e ✓      e to b ✓



How many paths.



	a	b	c	d
a	0	1	1	0
b	1	0	0	1



$$A_2^2 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

a to a.

① a b d c a.

② a b a b a.

③ a b d b a.

④ a b a c a.

⑤ a c a c a. ⑦ a c a b a.

⑥ a c d b a. ⑧ a c d c a.

$A_2^4 = 2$

$$A_2^4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 8 & 0 & 0 & 8 \\ b & 0 & 8 & 8 & 0 \\ c & 0 & 8 & 8 & 0 \\ d & 8 & 0 & 0 & 8 \end{array}$$

Ex PS 67-569.

(1-45).