Lecture 17:- PARTIAL ORDER.

1- Reflexive.
2- Anti Symmetric
3- Transitive.

if R is defined on S. The generalized notation for partial order $(S, \preceq_R)$.

$(a,b) \in R$

$a \preceq_R b$

Ex1
504

$R = \{(a,b) \mid a \geq b\}$. $A = \mathbb{Z}$.

Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z} \quad a \geq a$

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in \mathbb{Z}$ if $a \geq b \wedge b \geq a \rightarrow a = b$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $a \geq b \wedge b \geq c \rightarrow a \geq c$.

$-1 \geq -5 \wedge -5 \geq -10 \rightarrow -1 \geq -10$.

$\div$ ✓
$>$ ✗
$=$ ✓
$\leq$ ✓
$<$ ✗

HW.

Ex 3:-
504

$R = \{(a,b) \mid a \subseteq b\}$ $PS(S) \times PS(S)$.

Reflexive $\forall a \in A \quad (a,a) \in R$.

$\forall a \in PS(S) \quad a \subseteq a$

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in PS(S)$ if $a \subseteq b \wedge b \subseteq a \rightarrow a = b$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$

$\forall a,b \in ps(S)$ if $a \subseteq b \wedge b \subseteq a \rightarrow a = b$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in ps(A)$. if $a \subseteq b \wedge b \subseteq c \rightarrow a \subseteq c$

$(S, \preceq)$. POSET.

$(\mathbb{Z}, \geq)$ $R = \{(a,b) \mid a \geq b\}$. $A = \mathbb{Z}$.

$(\mathbb{Z}, =)$. = ".

$(\mathbb{Z}, \div)$.

$(\mathbb{Z}, \leq)$.

$(ps(S), \subseteq)$.

COMPARABLE.

Two elements $a,b \in S$ in a $(S, \preceq_R)$ are Comparable if $a \preceq_R b$ or $b \preceq_R a$.

$(a,b) \in R$ $(b,a) \in R$.

Ex 5 $(\mathbb{Z}^+, /)$ tell us if $5, 7$ are Comparable?

Sol

$= 5 \preceq_R 7$ or $7 \preceq_R 5$

$= 5 / 7 \vee 7 / 5$

$= F \vee F$

$= F$

Not Comparable.

$3 \preceq_R 9$ or $9 \preceq_R 3$.

$3 / 9$ or $9 / 3$.

$F \vee T$. $= T$.

Total Order:- If all elements in S based on the Poset $(S, \preceq_R)$ are Comparable then the relation $\preceq_R$ is Total Order.

Ex 6:- $(\mathbb{Z}, \leq)$. Total order ?

$16 \preceq_R -7$ or $-7 \preceq_R 16$.

$16 \leq -7$ or $-7 \leq 16$.

$F \lor T = T$.

$-\infty \leq -\infty + 1 \leq \cdots \quad -1 \leq 0 \leq 1 \leq 2 \cdots$

Ex7 $(\mathbb{Z}^+, |)$.

$2 \preceq_R 3$ or $3 \preceq_R 2$.

$2 \mid 3 \lor 3 \mid 2$.

$F \lor F = F$.

GRAPHICAL REPRESENTATION OF PARTIAL ORDER.
"HASSE DIAGRAM".

Ex12 $(\{1, 2, 3, 4, 6, 8, 12\}, \div)$.
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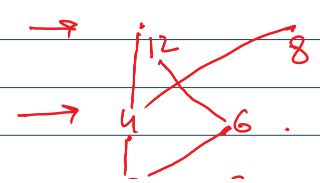
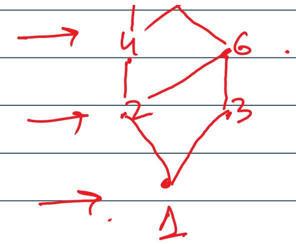
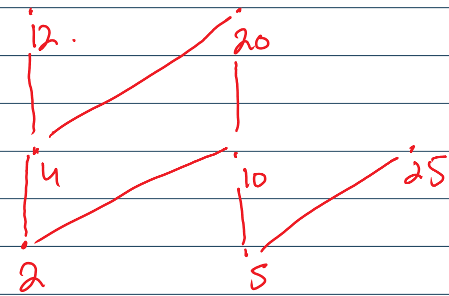
$R = \{\cancel{(1,1)}, (1,2), (1,3), (1,4), (1,6), (1,8), (1,12),$
$\cancel{(2,2)}, (2,4), (2,6), (2,8), (2,12),$
$\cancel{(3,3)}, (3,6), (3,12),$
$\cancel{(4,4)}, (4,8), (4,12),$
$\cancel{(6,6)}, (6,12),$
$\cancel{(8,8)},$
$\cancel{(12,12)}\}$.





$R = \{$ reflexive elements
Move upwards without lifting pen $\}$.

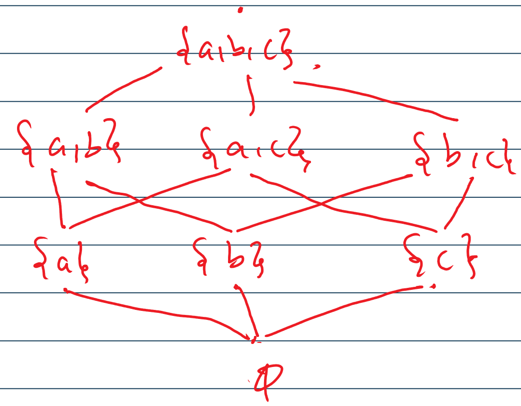
Ex14:- $(\{2, 4, 5, 10, 12, 20, 25\}, \ |\ )$.



Ex13: 509 $(PS(\{a, b, c\}), \subseteq)$.

$PS(\{a, b, c\})$.

$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$.

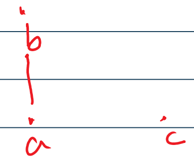


$$\begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

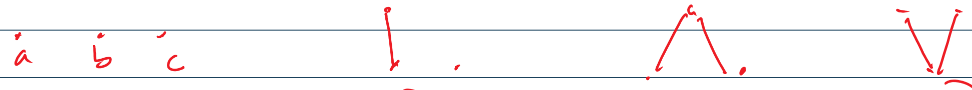
$R = \{(a, a), (a, b), (b, b), (c, c)\}$.

$$\begin{array}{c|ccc|} b & 0 & 1 & 0 \\ c & 0 & 0 & 1 \end{array}$$

$(c,c)\}.$



Consider three elements. Construct possible Hasse diagram.



1   2   3   4

5

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$