Lecture 14:-   Representing   Relations   Using   Graphs.

○   V = Set of Vertices.

(⟶ , ⟶)   E = Set of Edges.   G = (V, E).

syntax.   Semantics.

Let   R   be   a   relation   on   Set   A.
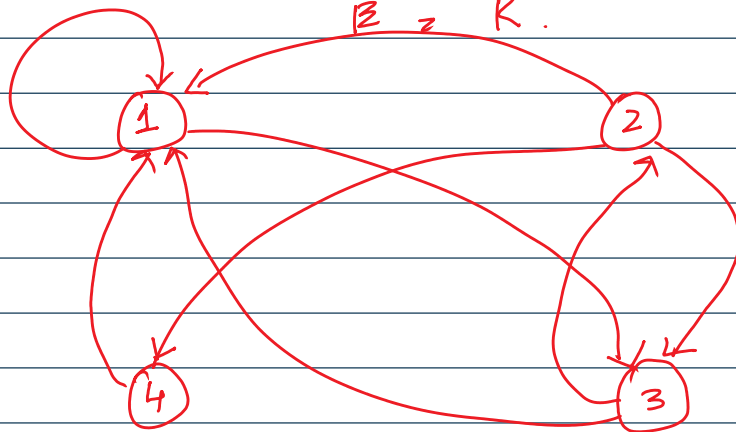
V =   A
E =   R.

Ex 8 | 489:-   R = { (1,1), (1,3), (2,1), (2,3), (2,4), (3,1)
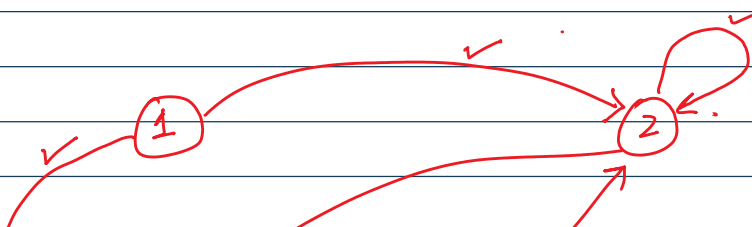   (3,2), (4,1) }.

V = A = { 1,2,3,4 }.
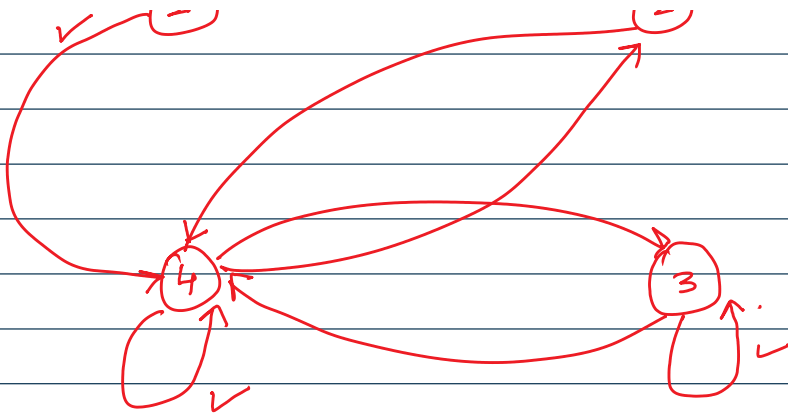
E = R.

HW.
Do the Reverse.

HW.
Represent the Same
Using matrix.

Relation in terms of Set ⟷ Relation in terms of matrices
                          ⟷   "   "   "   Graphs.
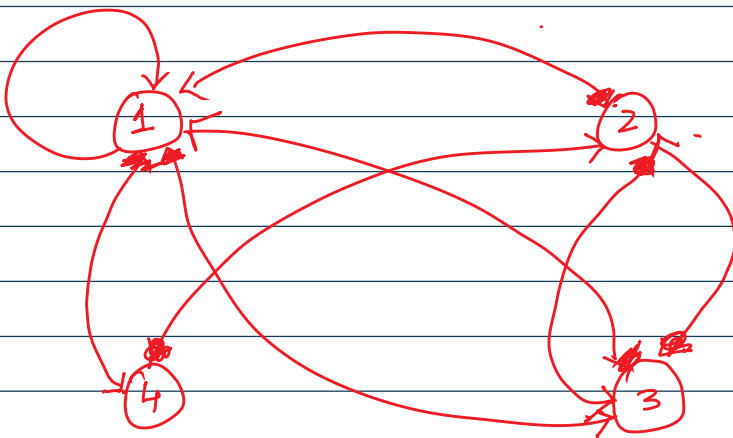
Find $\overline{R}$ = { (a,b) | (a,b) ∉ R }.

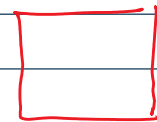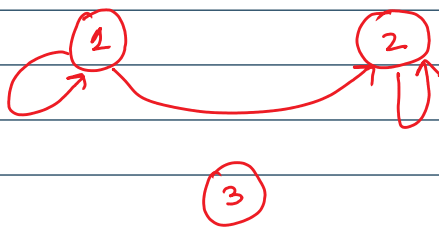$|R| \cup |\overline{R}| = |A \times A|$

$$R^{-1} = \{ (b, a) \mid (a, b) \in R \}.$$

R^{-1}.

Reflexive :- $\forall a \in A$ $(a, a) \in R$.

1 -   ②   ②   X

$V = A = \{1, 2, 3\}.$

$|A \times A| = 9.$

1 –   ①   ②   ✗ $|A \times A| = 4$
                        $2^4 = 16$

2 –   ①   ②   ✗

3 –   ① → ②   ✗

4 –   ①   ②   ✗

5 –   ①   ②   ✗

6 –   ①   ②   ✗

7 –   ①   ②   ✗

8 –   ①   ②   ✓

9 –   ①   ②   ✗

10   ① → ②   ✗

11 – ①   ②   ✗

12 – ①   ②   ✗

13 – ①   ②   ✓

14 – ①   ②   ✓

15 – ①   ②   ✗

16 – ①   ②   ✓

16 —

Symmetric. $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.

$R = \{ \ \}.$

1 — ② ② ✓

2 — ① ② ✓

3 — ① → ② ✗

4 — ① ② ✗

5 — ② ② ✓

6 — ① ② ✗

7 — ① ② ✗

8 — ① ② ✓

9 — ② ② ✓

10   ✗

11 —   ✗

12 —   ✓

13 —   ✗

14 —   ✗

15 —   ✓

16 —   ✓
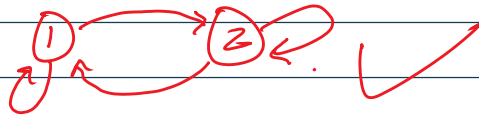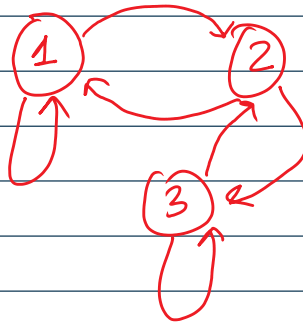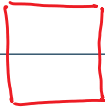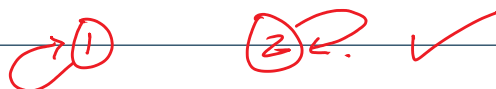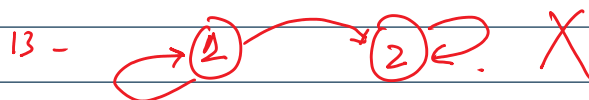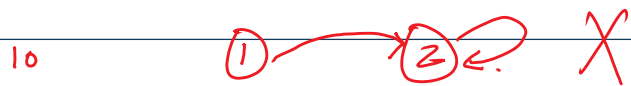
Anti Symmetric:- $\forall a, b \in A$ if $(a,b) \in R \wedge (b,a) \in R \Rightarrow a = b.$

 ✗.

1 —   ✓

2 —   ✓

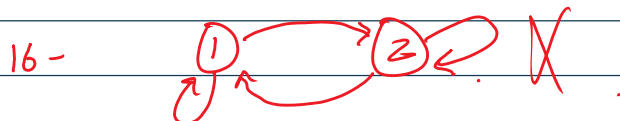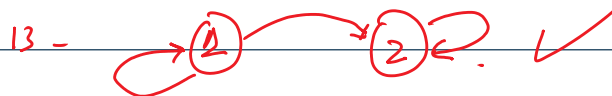3 —   ✓

4 - (1) ⇄ (2) ✓

5 - (1) (2) ↺ ✓

6 - (1) ↺ → (2) ✓

7 - (1) ↺ ← (2) ✓

8 - (1) ↺ (2) ↺ ✓

9 - (1) ⇄ (2) ✗

10 - (1) → (2) ↺ ✓

11 - (1) ← (2) ↺ ✓

12 - (1) ↺ ⇄ (2) ✗

13 - (1) ↺ → (2) ↺ ✓

14 - (1) ↺ (2) ↺ ✓

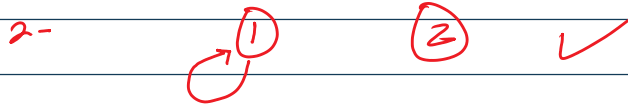15 - (1) ⇄ (2) ↺ ✗

16 - (1) ↺ ⇄ (2) ↺ ✗
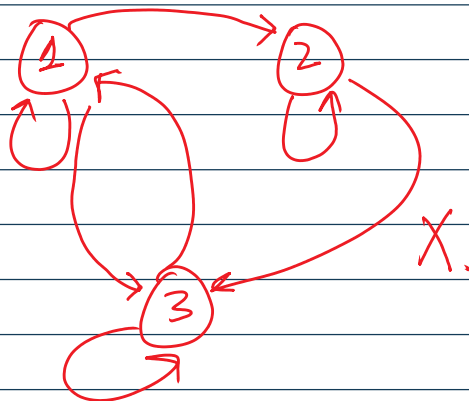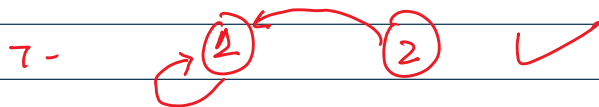
Transitive: $\forall a, b, c \in A$ if $(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R$.
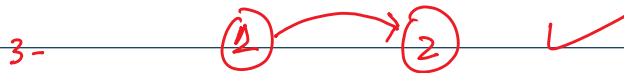
☐   (1) → (2)

1 – ✓ $R_2 \{ \}$ $\Delta_2 \{ (1,1), (2,2) \}$

$R \cup \Delta_2 \{ (1,1), (2,2) \}$
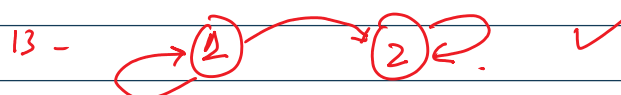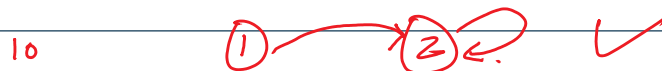
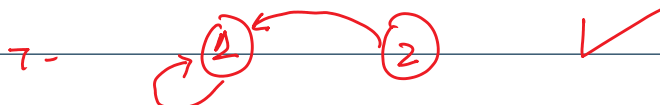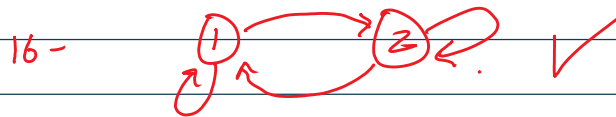2 – ✓ $R_2 \{ (1,1) \}$

$R \cup \Delta_2 \quad \{ (1,1), (2,2) \}$

3 – ✓ $R_2 \{ (1,2) \}$

$R \cup \Delta_2 \quad \{ (1,1), (1,2), (2,2) \}$

4 – ✓

5 – ✓

6 – ✓

7 – ✓

8 – ✓

9 – ✗

10 ✓

11 – ✓

12 – ✗

13 – ✓

13 –

14 – ✓

15 – ✗

16 – ✓
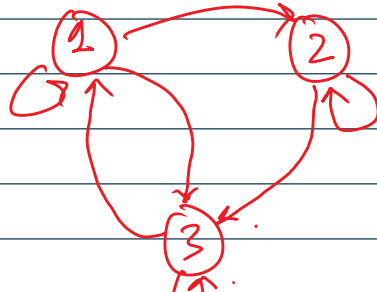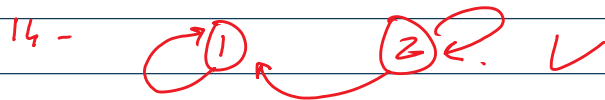
$(a,a) \in A$.

Closure :-

Closure w.r.t Reflexive.

P483: $\quad R = \{(1,1), (1,2), (2,1), (3,2)\}$. $\quad A = \{1,2,3\}$.

$\Delta = \{(1,1), (2,2), (3,3)\}$. $\quad = \{(a,a) \mid \forall a \in A\}$.

__Ex 1__
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$R = \{(a,b) \mid a < b\}$. $\qquad A = \mathbb{Z}$.

$R = \{(a,b) \mid a < b\}$. $\qquad\qquad \Delta = \{(a,a) \mid \forall a \in \mathbb{Z}\}$.

$\Delta = \{(a,a) \mid \forall a \in \mathbb{Z}\}$.

$= \{(a,b) \mid a = b\}$. $\qquad \{(-\infty,-\infty), (-\infty+1, -\infty+1), \cdots (0,0)$

$\qquad\qquad\qquad\qquad (1,1), \cdots\cdots (+\infty, +\infty)\}$.

$R \cup \Delta = \{(a,b) \mid a < b \lor a = b\}$.

$= \{(a,b) \mid a \leq b\}$.

Symmetric Closure :-

$R \cup R^{-1}$ $\qquad\qquad\qquad R = \{(a,b) \mid \underline{\qquad}\}$.

$\qquad\qquad\qquad\qquad\qquad R^{-1} = \{(a,b) \mid (a,b) \in R\}$.

__Ex 2 :-__ $\qquad\qquad$ b a $\qquad$ b a.

Ex2:-

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$$R^{-1} = \{(a,b) \mid (a,b) \in R\}$$

$$R = \{(a,b) \mid a > b\}.$$

$b > a \qquad a < b.$

$$R^{-1} = \{(a,b) \mid (b,a) \in R\}.$$

$$R^{-1} = \{(a,b) \mid a < b\}.$$

$$R \cup R^{-1} = \{(a,b) \mid a > b \lor a < b\}$$
$$= \{(a,b) \mid a \neq b\}$$