

lecture 8:-

Question 39 p57.

$$\forall x \exists y (x^2 \neq y^2)$$

$$x, y \in \mathbb{Z}.$$

x

✓1 2

$$2^2 \neq 2^2$$

T.

✓2

✓3

✓4

✓5

⋮

⋮

⋮

⋮

✓∞.

$$30 \neq 900$$

$$40 \neq 1600.$$

$$\forall x \exists y (y^2 - x < 100).$$

$$(30^2 - 1000 < 100).$$

$$900 - 1000 < 100$$

$$-100 < 100.$$

Rules of Inference.

"If you have Current password P .
then you can log into System Q ."

- افلاطون

- سقراط

- ارسطو 2500

"You have Current password" P .

therefore,

"You can log into System" Q

دو سوالات

1- وجود کیا ہے؟

2- علم کیا ہے؟

$$P1: P \rightarrow Q$$

$$P2: P$$

$$C: \therefore Q$$

premises = facts.

Conclusion:-

Valid Argument:- $((P1 \wedge P2 \wedge \dots \wedge PN) \rightarrow Q)$ tautology

then Argument is Valid.

P	Q	$P \rightarrow Q$	$(P \rightarrow Q) \wedge P$	$((P \rightarrow Q) \wedge P) \rightarrow Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

P61 Table 1.

$$\begin{array}{l} 1) \quad P \rightarrow Q \\ \quad P \\ \hline \therefore Q \end{array} \quad \text{Modus Ponens.}$$

$$\begin{array}{l} 5) \quad P \\ \hline \therefore P \vee Q \end{array} \quad \text{Addition.}$$

$$\begin{array}{l} 2) \quad P \rightarrow Q \\ \quad \neg Q \\ \hline \therefore \neg P \end{array} \quad \text{Modus Tollens.}$$

$$\begin{array}{l} 6) \quad P \wedge Q \\ \hline \therefore P \end{array} \quad \text{Simplification.}$$

$$\begin{array}{l} 7) \quad P \\ \hline P \end{array} \quad \text{Conjunction.}$$

$$\frac{\neg q}{\therefore \neg p} \text{ Tollens.}$$

$$\frac{P}{\therefore p \wedge q} \text{ Conjunction.}$$

$$\frac{P \rightarrow q \quad q \rightarrow r}{\therefore P \rightarrow r} \text{ Hypothetical Syllogism.}$$

$$\frac{P \vee q \quad \neg P \vee r}{\therefore q \vee r} \text{ Resolution.}$$

$$\frac{P \vee q \quad \neg P}{\therefore q} \text{ Disjunctive Syllogism.}$$

Ex. 6:- Show that the hypothesis

"It is not ^PSunny this afternoon and it is ^qcolder than yesterday"

"we will go ^rSwimming only if it is ^PSunny".

"If we do not go swimming then ^Swe will take a canoe trip".

"If we take a canoe trip ^Sthen we will be home by ^tsunset".

leads to Conclusion.

"we will be home by ^tSunset".

$$\begin{array}{l} P1:- \neg p \wedge q \\ P2:- r \rightarrow p \\ P3:- \neg r \rightarrow s \\ P4:- s \rightarrow t \\ C: \therefore t \end{array}$$

$$\frac{P \rightarrow q \quad P}{\therefore q} \text{ Modus Ponens.}$$

$$\frac{P}{\therefore P \vee q} \text{ Addition.}$$

$$\frac{P \wedge q}{\therefore P} \text{ Simplification.}$$

$$\frac{P \rightarrow q \quad \neg q}{\therefore \neg P} \text{ Modus Tollens.}$$

$$\frac{P}{\therefore p \wedge q} \text{ Conjunction.}$$

$$\frac{P \rightarrow q \quad q \rightarrow r}{\therefore P \rightarrow r} \text{ Hypothetical Syllogism.}$$

$$\frac{P \vee q \quad \neg P \vee r}{\therefore q \vee r} \text{ Resolution.}$$

$$\frac{P \vee q \quad \neg P}{\therefore q} \text{ Disjunctive Syllogism.}$$

$$\begin{array}{l} P1:- \neg p \wedge q \checkmark \\ P2:- r \rightarrow p \checkmark \\ P3:- \neg r \rightarrow s \checkmark \\ P4:- s \rightarrow t \checkmark \\ C: \therefore t \end{array}$$

$$\begin{array}{l} P5:- \neg p \checkmark \\ P6:- \neg r \\ P7:- s \checkmark \\ P8:- t \end{array}$$

from p1 by Simplification.
from p5, p2 by Modus Tollens.
from p3, p6 by Modus Ponens.
from p7, p4 by " "

which is Conclusion.

$\frac{p_1}{p_2} \div$
 $p_2 \div p \rightarrow q \checkmark$
 $p_2 \div \neg p \rightarrow \neg q \checkmark$

$p3: \gamma \rightarrow S \checkmark$

$$\therefore \neg q \rightarrow s.$$

P4: $\neg q \rightarrow \neg p$ ✓ by Contrapositive from P2.

P5:- $\neg q \rightarrow r$ ✓ by Hypothetical for p4, p2.
 Syllogism.

$p_0 \vdash \neg q_i \rightarrow S$

which is Conclusion.

2)
$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$
 Modus Ponens.

2)
$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$
 Modus
Tollens.

2)
$$\begin{array}{l} P \rightarrow q \\ q \rightarrow r \\ \hline \therefore P \rightarrow r \end{array}$$
 Hypothetical
Syllogism

4) $p \vee q$ Disjunctive
p.z. $\frac{\neg p}{\therefore q}$ Syllogism.

5) $\frac{P}{\therefore PVq}$ Addition.

6) $\frac{p \wedge q}{\therefore p}$ Simplification.

7) $\frac{P}{q}$ Conjunction.
 $\therefore p \wedge q$

8) $p \vee q$, Resolution.
 $\frac{\neg p \vee r}{\therefore q \vee r}$

Ex 8 $P1 \vdash T \rightarrow MVB \checkmark$
 $P63 \vdash S \rightarrow \neg FE \checkmark$

$p2 \div S \rightarrow \neg E \checkmark$

Q3:- TMS ✓
C ∴ M.

2)
$$\frac{p \rightarrow q \quad p}{\therefore q}$$
 Modus Ponens.

2)
$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$
 Modus
Tollens.

2)
$$\begin{array}{l} P \rightarrow Q \\ Q \rightarrow R \\ \hline \therefore P \rightarrow R \end{array}$$
 Hypothetical
Syllogism.

4) $\frac{p \vee q}{\neg p} \therefore q$ Disjunctive Syllogism.

5) $\frac{P}{\therefore PVq_1}$ Addition.

6) $\frac{PAQ}{\therefore P}$ Simplification.

7) $\frac{p}{q}$ Conjunction.
 $\therefore p \wedge q$

8) $p \vee q$ resolution.
 $\neg p \vee r$

 $\therefore q \vee r$

P4:- T from P3 by Simplification ✓

PS:- S u u u u . ✓

P6:- MVE from P1, P4 by M9. ✓

P7: TE " \$2.95 N " ✓

P8:- M " P6, P7

which is Conclusion.