lecture 13:- $R$ $A \times B$.

$M_R$. $A = \{a_1, a_2, \dots a_m\}$.

$m_{ij}$ $B = \{b_1, b_2, \dots b_n\}$.

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

$m_{ij}$ $(a_i, b_j)$

1) Reflexive:- $\forall a_i \in A$ $(a_i, a_i) \in R$.

$\forall_i$ $m_{ii} = 1$.

$A = \{1\}$.

$A \times A = \{(1,1)\}$.

$[\ ]$ $\checkmark$ $[0]$ $[1]$.

$pow(A \times A) = \{\phi, \{(1,1)\}\}$.

$A = \{1, 2\}$. $A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$.

$Pow(A \times A) = \{\phi \quad - - - - \quad \}$.

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$[\ ]$ ?

Symmetric:- $\forall a_i, b_j \in A$ if $(a_i, b_j) \in R \rightarrow (b_j, a_i) \in R$.

$\forall_{ij}$ if $m_{ij} = 1 \rightarrow m_{ji} = 1$

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$$M_R^T = M_R$$

Anti Symmetric $\forall a,b \in A$ if $(a_i, b_j) \in R \wedge (b_j, a_i) \in R \rightarrow a = b$, $i = j$.

$\forall i,j$ if $m_{ij} = 1 \wedge m_{ji} = 1 \longrightarrow i = j$.

$m_{12} = 1 \wedge m_{21} = 1 \longrightarrow 1 \neq 2$.

$m_{11} = 1 \wedge m_{11} = 1 \longrightarrow 1 = 1$.

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$[\;]$ ✓ $[0]$ ✓ $[1]$ ✓.

Ex4 :-
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$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_{R_1 \cup R_2} = \begin{bmatrix} & \text{HW.} & \\ & & \\ & & \end{bmatrix}$$

Quiz #6.

$A = \{a\}.$

$PS(A) \times PS(A).$

$PS(A) = \{\phi, \{a\}\}.$

$PS(A) \times PS(A) = \{(\phi, \phi), (\phi, \{a\}), ( - - - - - - \}.$

$PS(\ PS(A) \times PS(A)) = \{\phi \ - - - - \}.$

1) Symmetric. ?
2.) Anti Symmetric ?