lecture 12:- Composite. $\circ$

$a \in A$

R $(a,b) \in A \times B$ $b \in B$

S $(b,c) \in B \times C$ $c \in C$

if $(a,b) \in R \wedge (b,c) \in S$.

Then $(a,c) \in S \circ R$

Ex:-20:-
P465

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ $A \times B$. $A = \{1,2,3\}$.

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$. $B \times C$. $B = \{1,2,3,4\}$.

$C = \{0,1,2\}$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$.

$R \circ S = ?$

$S \circ R \neq R \circ S$. HW.

$R \circ R = ? = R^2$ HW

$R^2 \circ R = R^3$ "

⋮

Theorem 1:- A relation R on A. is transitive.
P466. iff $R^n \subseteq R$ $n = 1,2,3 \dots$.

Exercise 1-30 HW.
P466-P468.

N-ary Relations with Application.

Ex1: $R = \{(a,b,c) \mid a < b < c\}$ $N \times N \times N$.
P469

$(1,2,3) \in R$ $= T$

$(2,4,3) \in R = F.$

Ex2 ÷ $R = \{(a,b,c) \mid b = a+K \wedge c = a+2K\}$.
469 if $\exists K\ K \in \mathbb{Z}$ $\quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

$(2,5,9) \notin R.$ $\quad -2+5 = \cancel{2}+K-\cancel{2}$ $\quad 9 = 2+2.3$
$\downarrow\ \downarrow\ \downarrow$ $\quad K = 3.$ $\quad 9 = 2+6$
a b c. $\quad 9 = 8$

$(1,3,5) \in R = ?$ Hw.

$|A| = n \quad |B| = m \quad |C| = r$
How many Relations $\quad 2^{n \times m \times r}$

Ex3
$R = \{(a,b,m) \mid a \equiv b \mod m\}$.

$$4 \overline{)\,-17\,}\ ?$$
-5, ∓ 20, 3.

$(-1,9,5) \in R.$ ✓ $\quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+.$

$(8,2,3) \in R$ ✓
$(4,0,7) \in R$ ✗

Ex4:- 5-ary Relation.

$A \times N \times S \times D \times T.$

(PIA, PK515, PEW, ISB, 2:45)

A = Set of Airlines
N = " " Flight Numbers.
S = " " Starting Points.
D = " " Destination ".
T = " " Departure Time.

Relational Database.

Airline Flight Numbers Start Point Destination DT.

PIA PK515 PEW ISB 2:45pm

Representing Relations

m_{ij} M_R

$A = \{a_1, a_2, \dots a_m\}$.

$B = \{b_1, b_2, \dots b_n\}$.

$$m_{ij}^{12} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

M_R Size

Rows $= |A|$

Col $= |B|$.

Ex1. :-

P476

$A = \{\underset{a_1}{1}, \underset{a_2}{2}, \underset{a_3}{3}\}$ $B = \{\underset{b_1}{1}, \underset{b_2}{2}\}$.

$R = \{(2,1), (3,1), (3,2)\}$. $A \times B$.

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$