Lecture # 14
AVL Deletion

Deletion in AVL Tree

- Delete is the inverse of insert: given a value X and an AVL tree T, delete the node containing X and rebalance the tree, if necessary.
- Turns out that deletion of a node is considerably more complex than insert

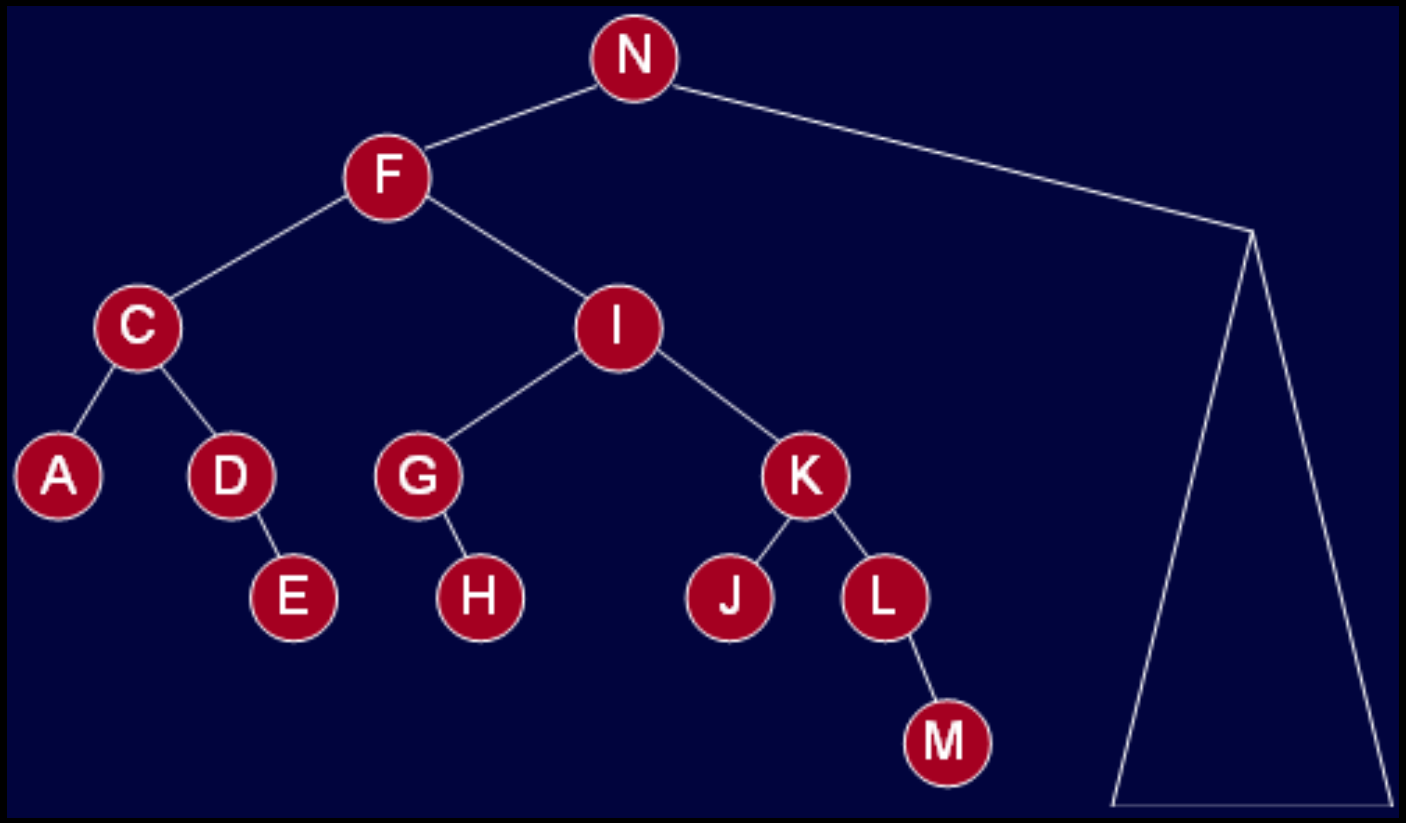
Deletion in AVL Tree

- Insertion in a height-balanced tree requires at most one single rotation or one double rotation.
- We can use rotations to restore the balance when we do a deletion.

- We may have to do a rotation at every level of the tree.

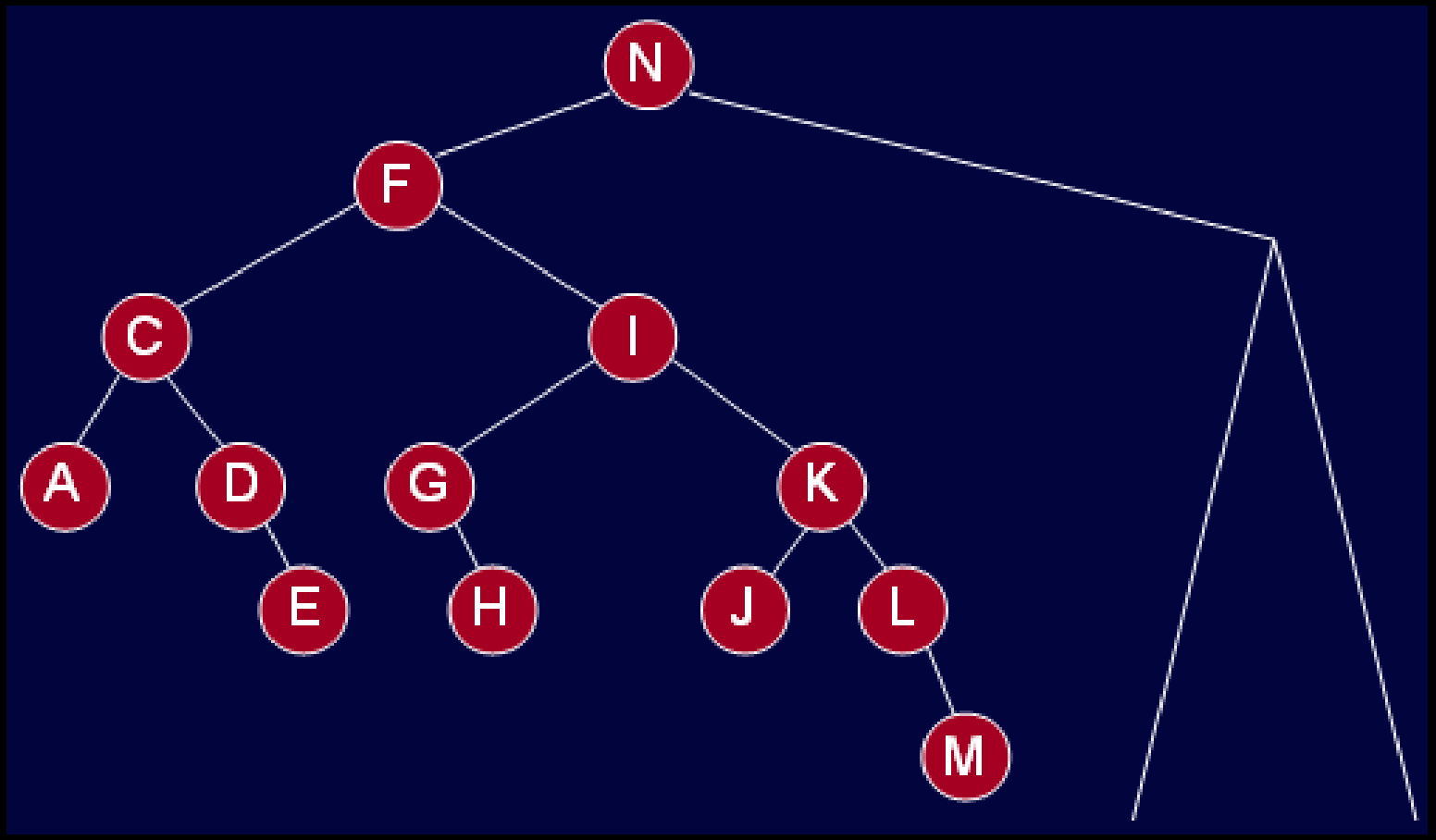
Deletion in AVL Tree

- Here is a tree that causes this worse case number of rotations when we delete A. At every node in N's left subtree, the left subtree is one shorter than the right subtree.



Deletion in AVL Tree

- Deleting A upsets balance at C. When rotate (D up, C down) to fix this



Deletion in AVL Tree

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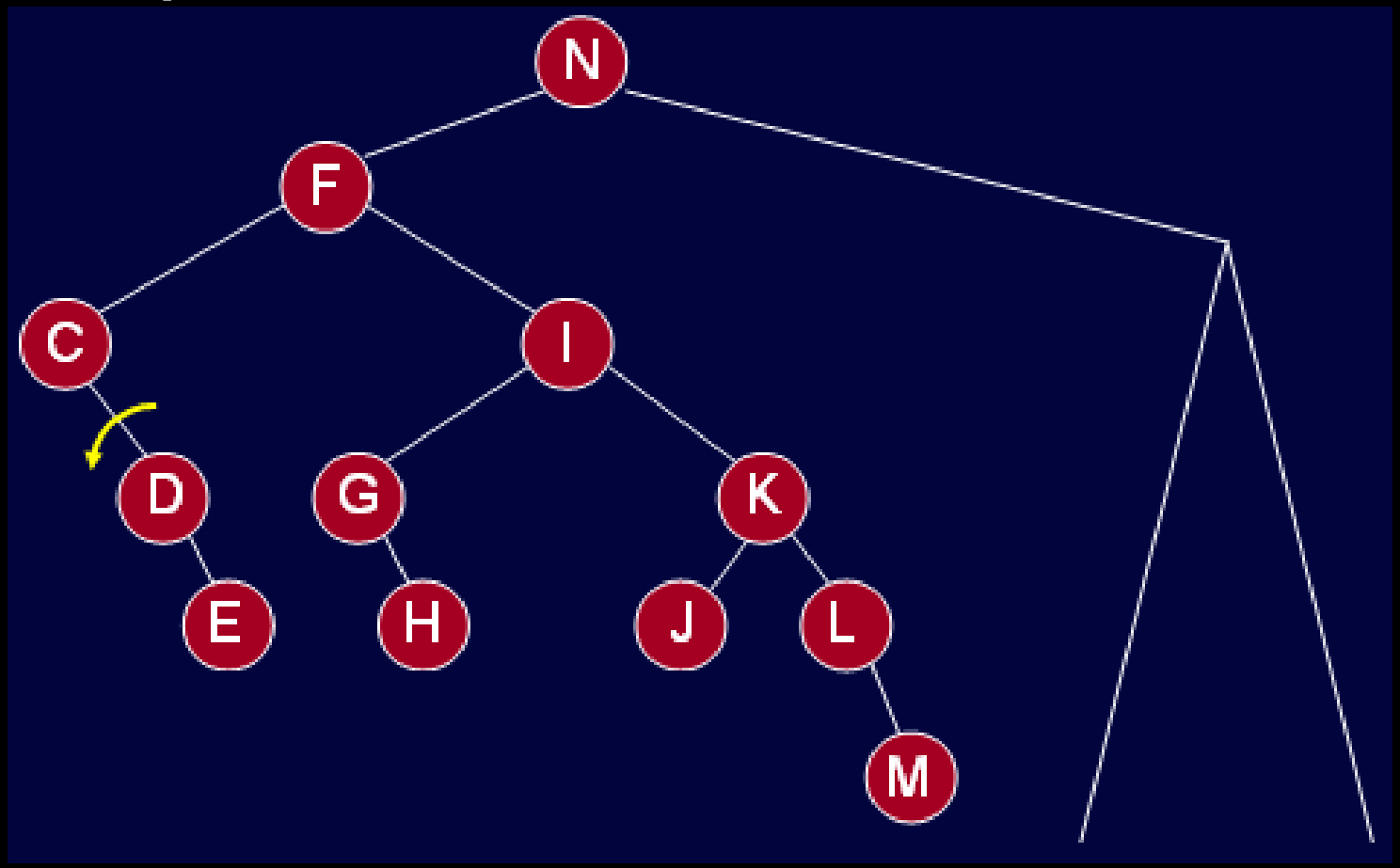
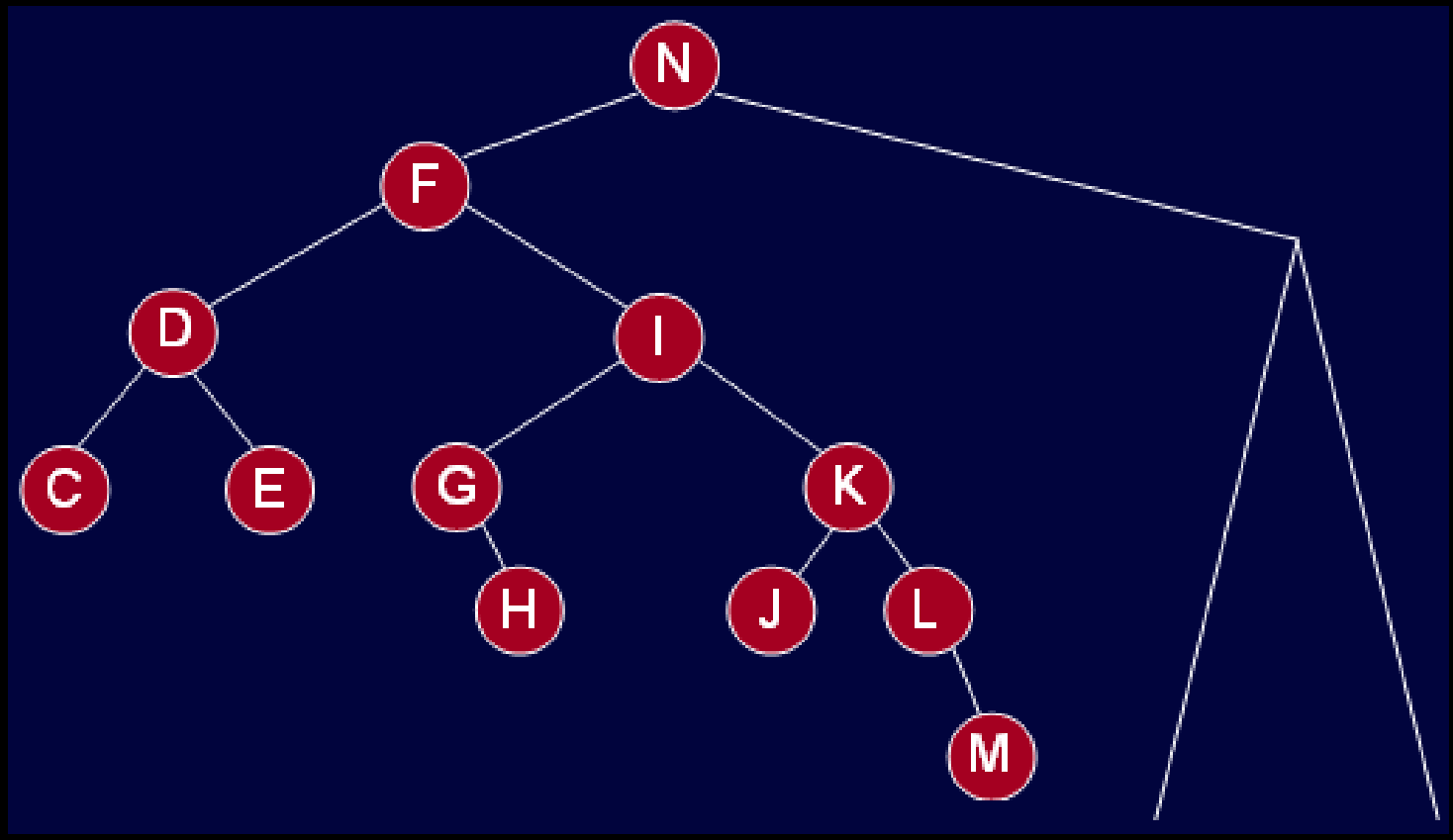


Deletion in AVL Tree

- The whole of F's left subtree gets shorter. We fix this by rotation about F-I: F down, I up.



Deletion in AVL Tree

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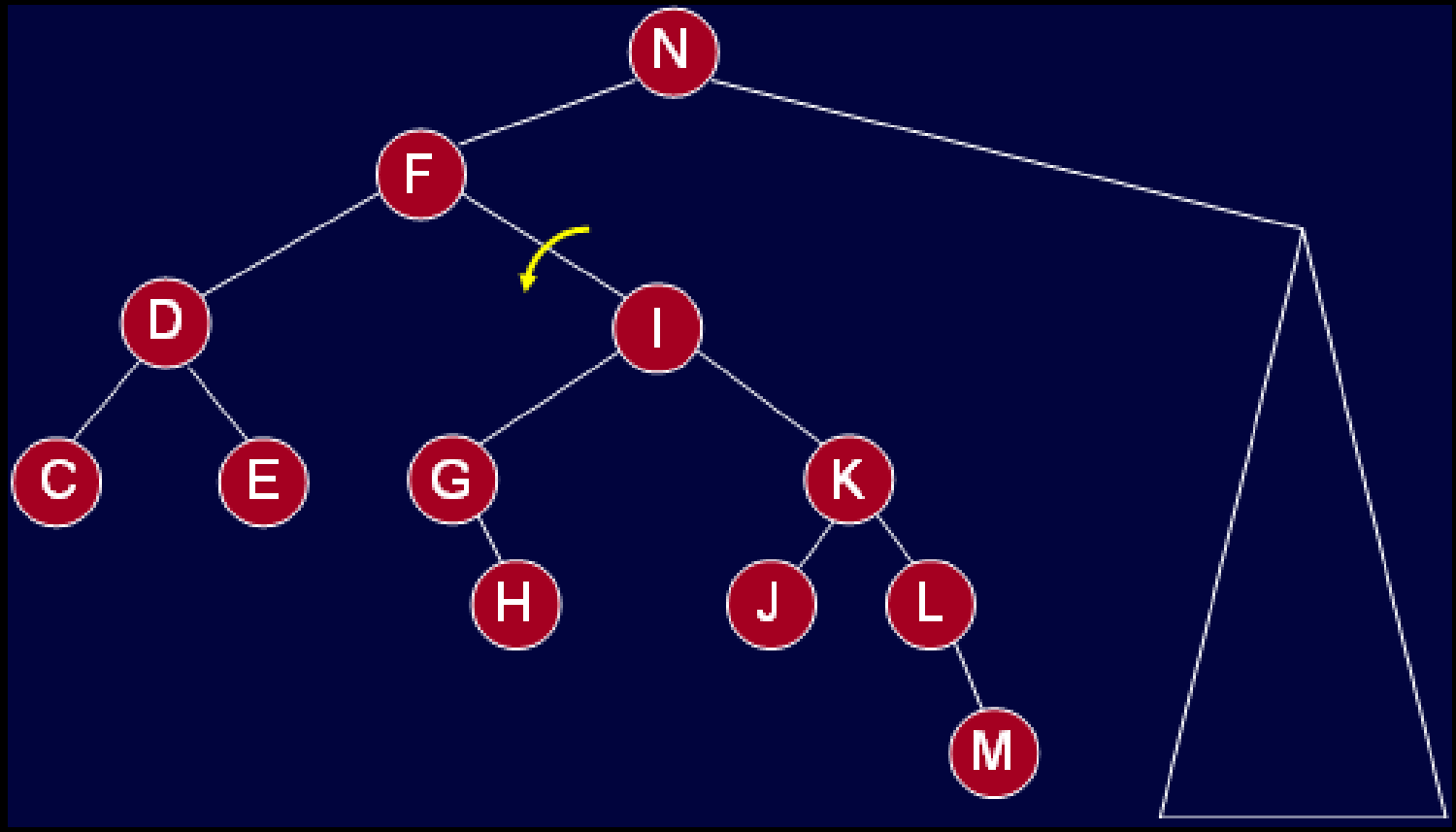
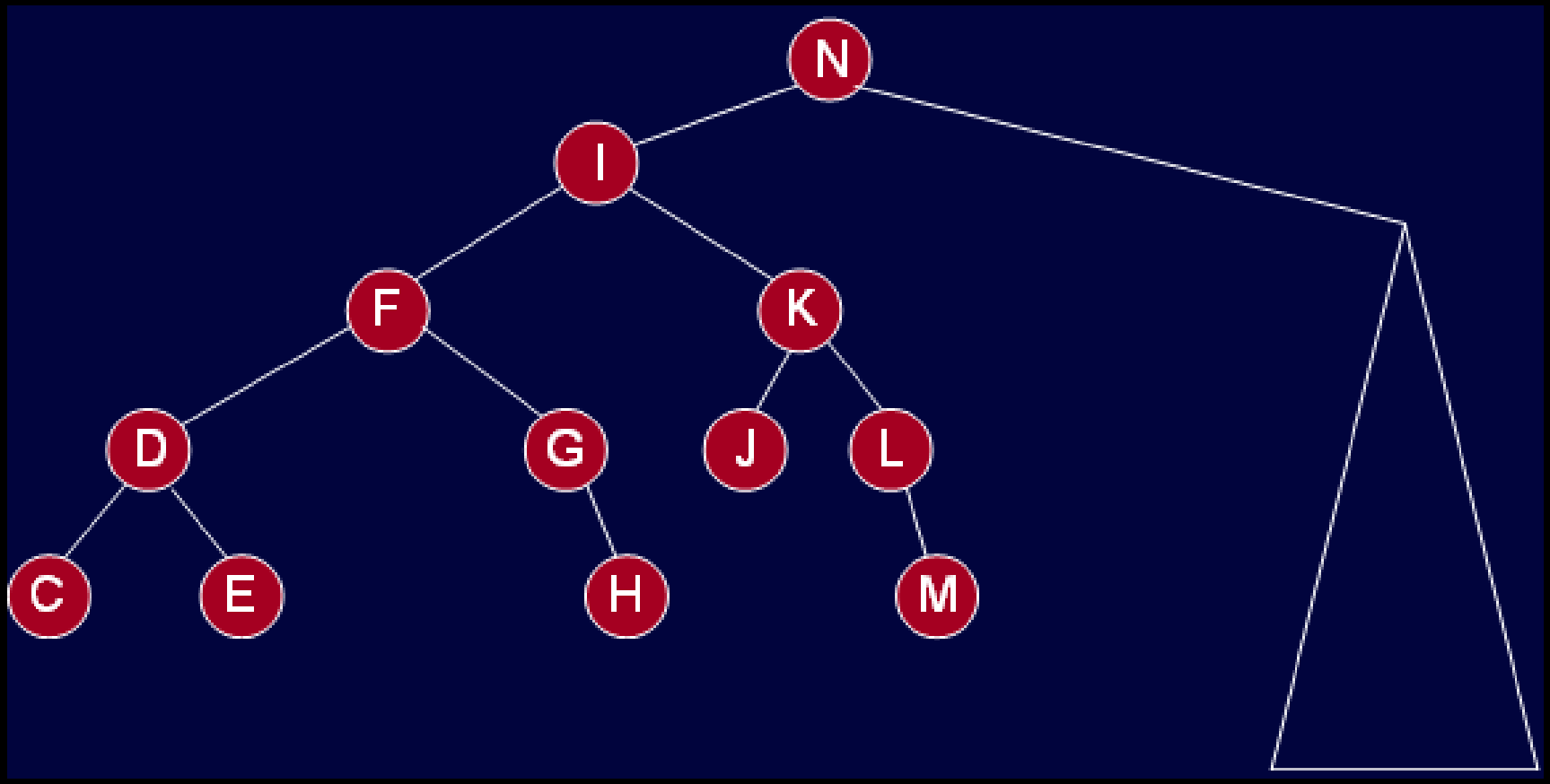


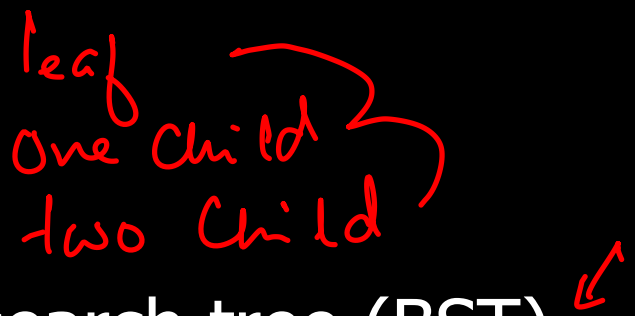
Deletion in AVL Tree

- This could cause imbalance at N.
- The rotations propagated to the root.



Deletion in AVL Tree

Procedure

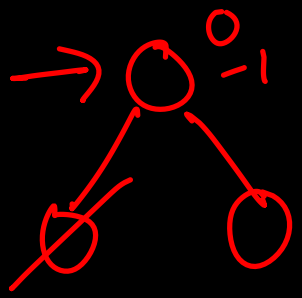


- Delete the node as in binary search tree (BST).
- The node deleted will be either a leaf or have just one subtree.
- *Since this is an AVL tree, if the deleted node has one subtree, that subtree contains only one node (***why?***)*
- Traverse up the tree from the deleted node checking the balance of each node.

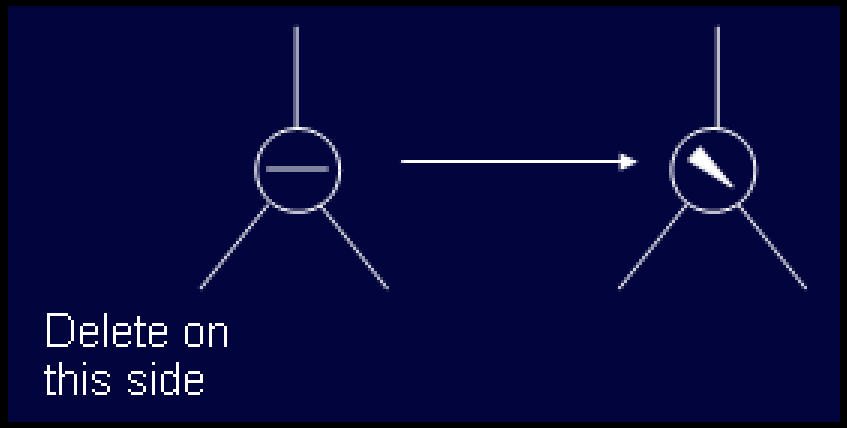
Deletion in AVL Tree

- There are 5 cases to consider.
- Let us go through the cases graphically and determine what action to take.

Deletion in AVL Tree



- *Case 1a*: the parent of the deleted node had a balance of 0 and the node was deleted in the parent's *left* subtree.

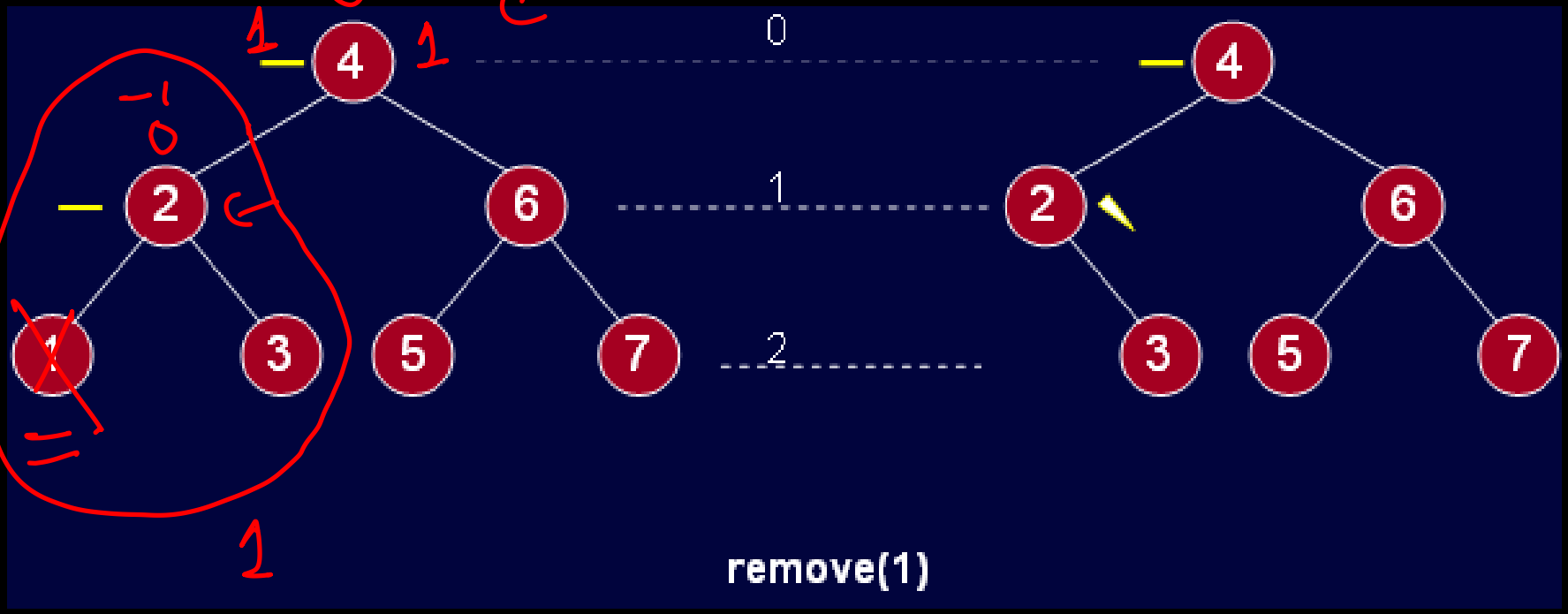


- *Action*: change the balance of the parent node and stop. No further effect on balance of any higher node.

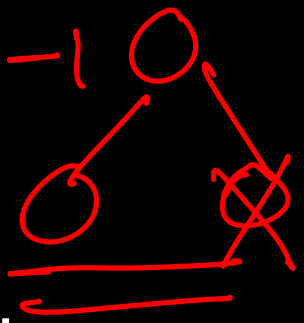
No Rotation

Deletion in AVL Tree

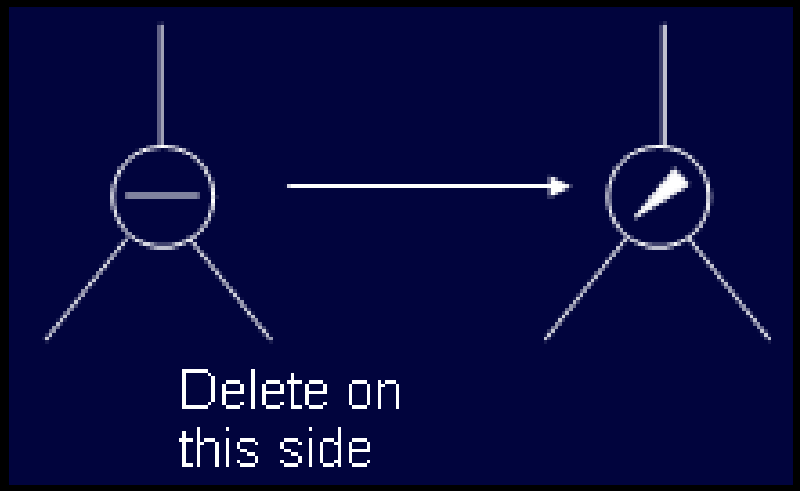
- Here is why; the height of left tree does not change.

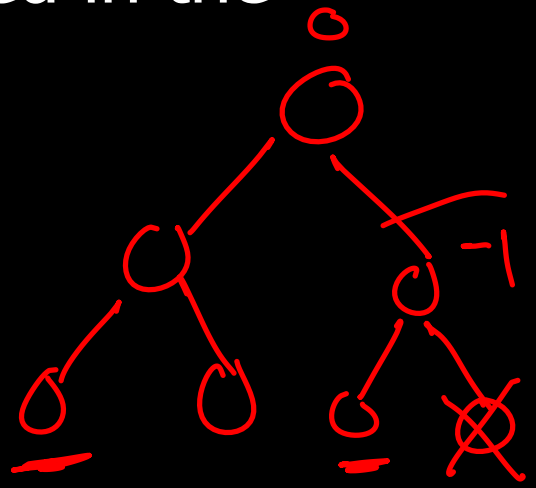


Deletion in AVL Tree



- *Case 1b*: the parent of the deleted node had a balance of 0 and the node was deleted in the parent's *right* subtree.

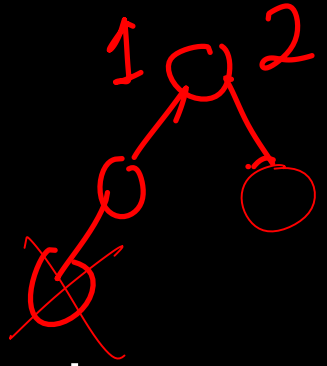




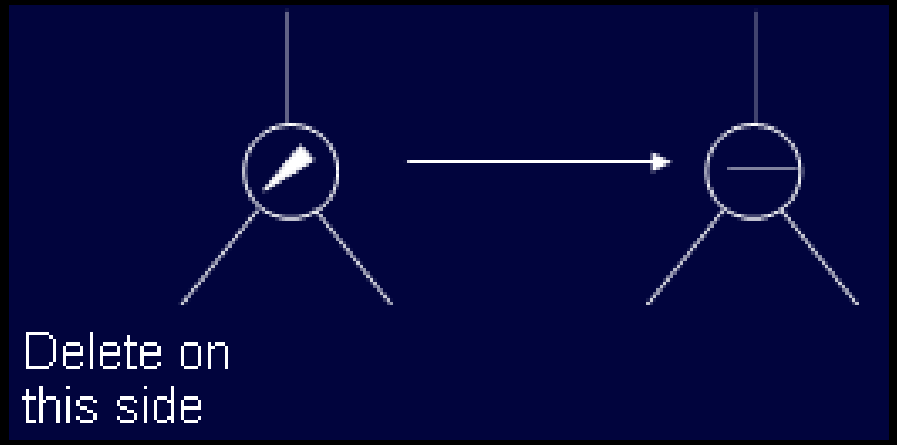
- *Action*: (same as *1a*) change the balance of the parent node and stop. No further effect on balance of any higher node.

No Rotation req.

Deletion in AVL Tree

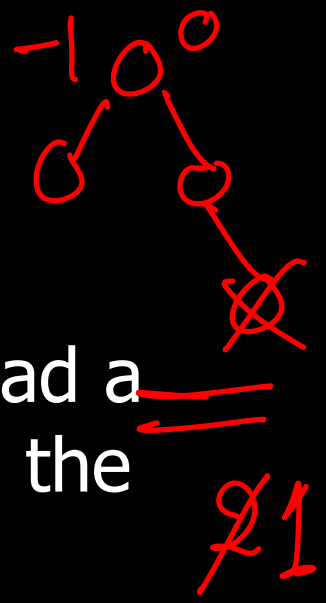


- *Case 2a*: the parent of the deleted node had a balance of 1 and the node was deleted in the parent's *left* subtree.

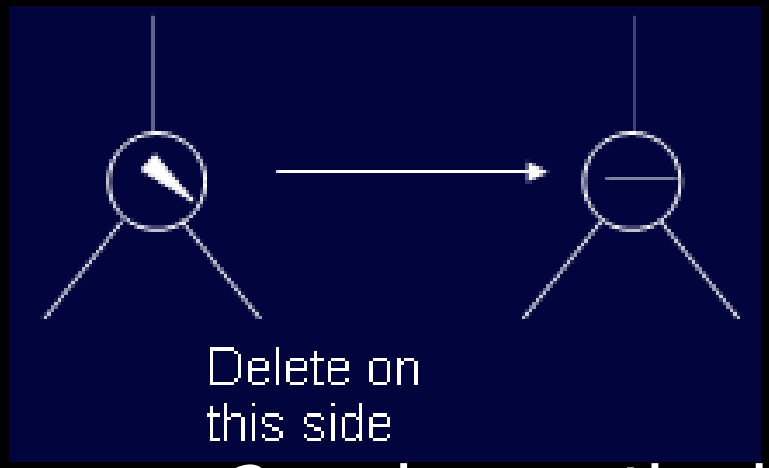


- *Action*: change the balance of the parent node. May have caused imbalance in higher nodes so continue up the tree.

Deletion in AVL Tree



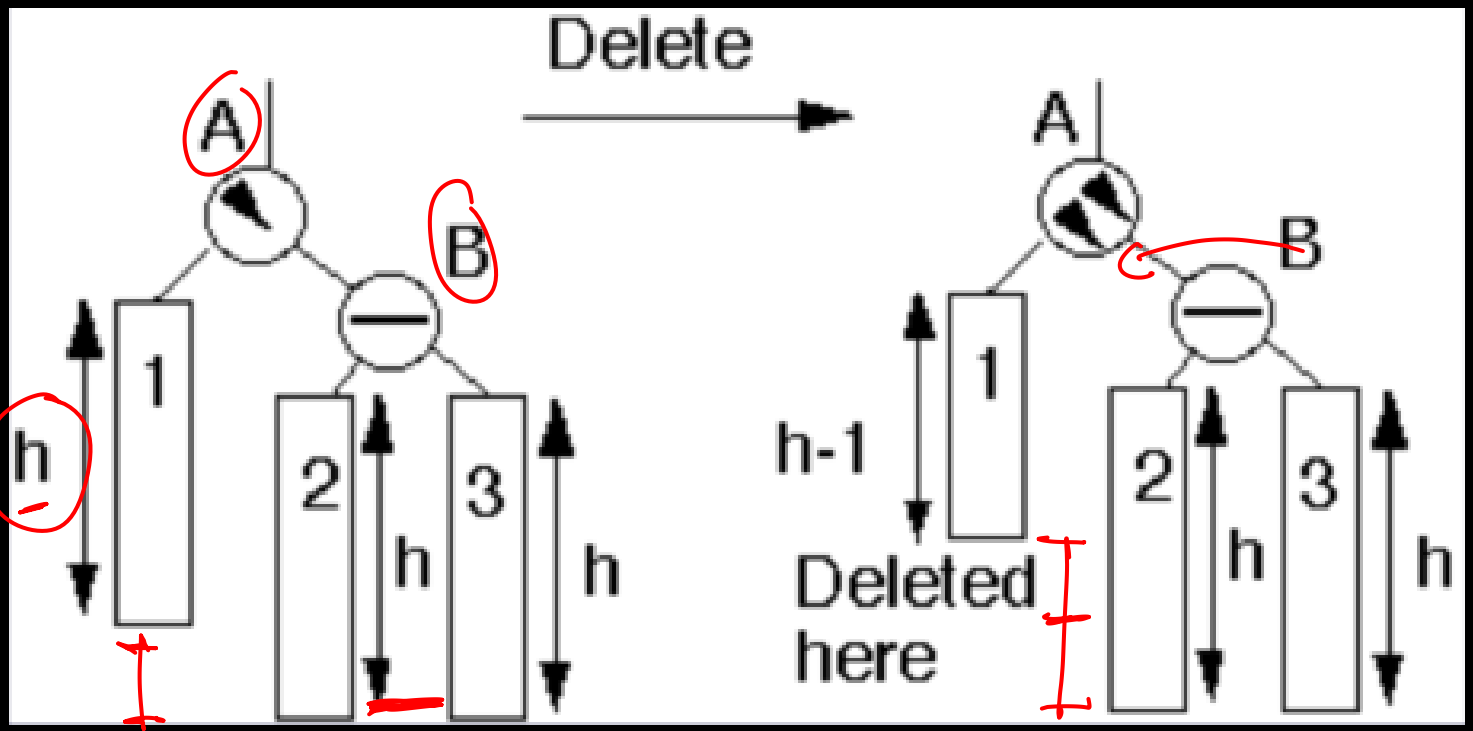
- *Case 2b*: the parent of the deleted node had a balance of -1 and the node was deleted in the parent's *right* subtree.



- *Action*: same as 2a: change the balance of the parent node. May have caused imbalance in higher nodes so continue up the tree.

Deletion in AVL Tree

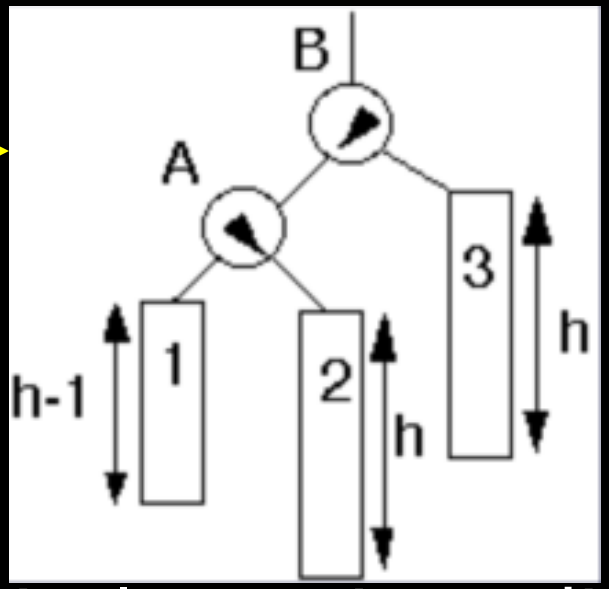
- *Case 3a*: the parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was balanced



Deletion in AVL Tree

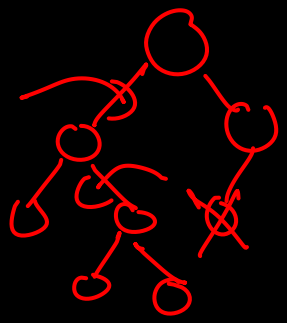
- *Case 3a*: the parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was balanced.

Single Rotate

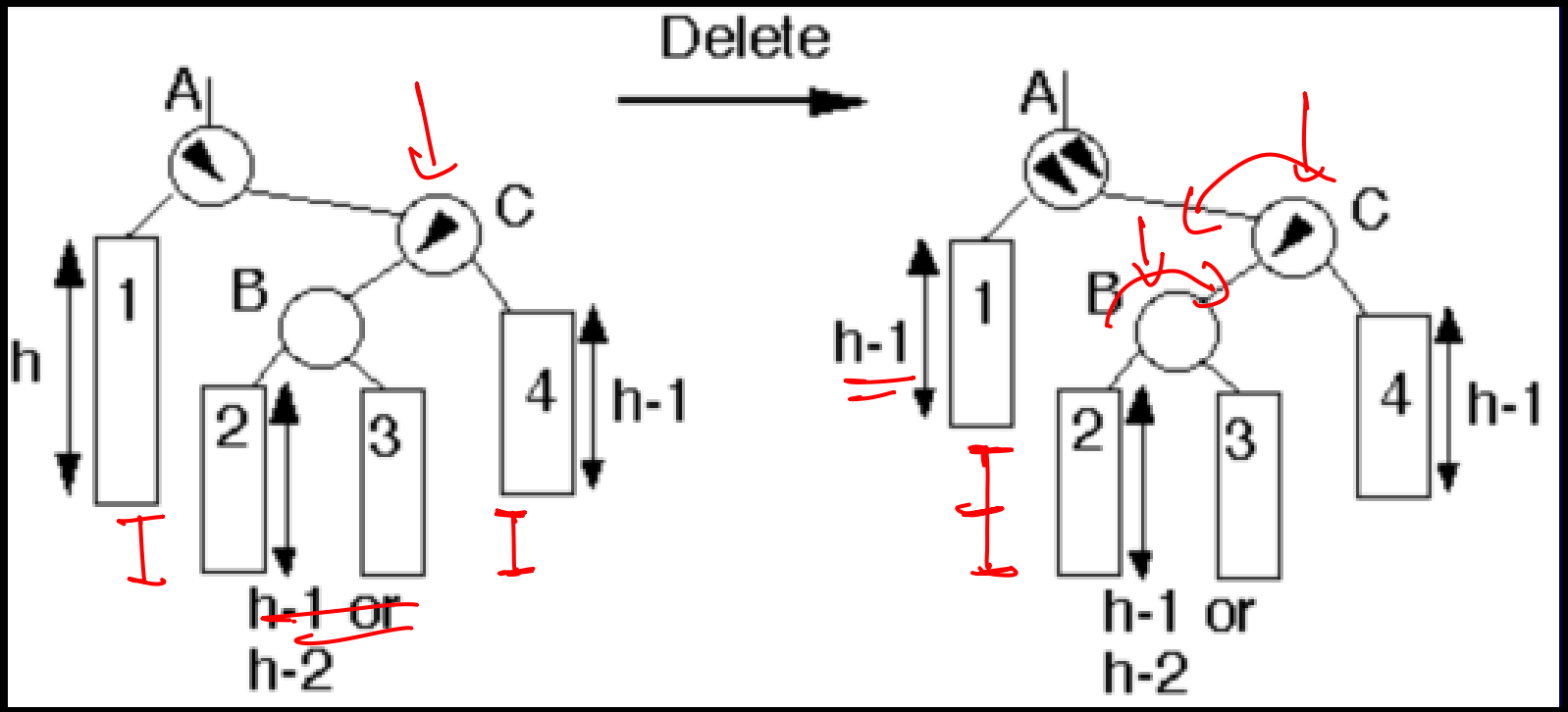


- *Action*: perform single rotation, adjust balance. No effect on balance of higher nodes so stop here.

Deletion in AVL Tree



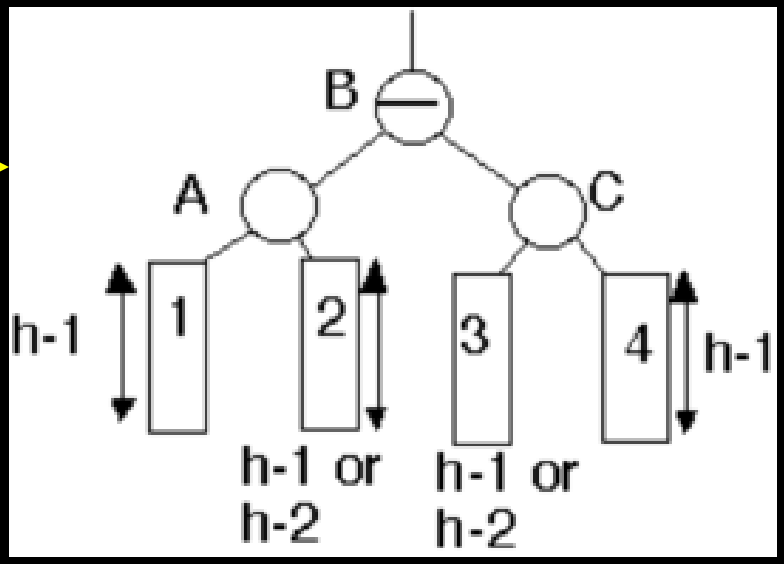
- *Case 4a*: parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was unbalanced. (not exact same levels)



Deletion in AVL Tree

- *Case 4a*: parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was unbalanced.

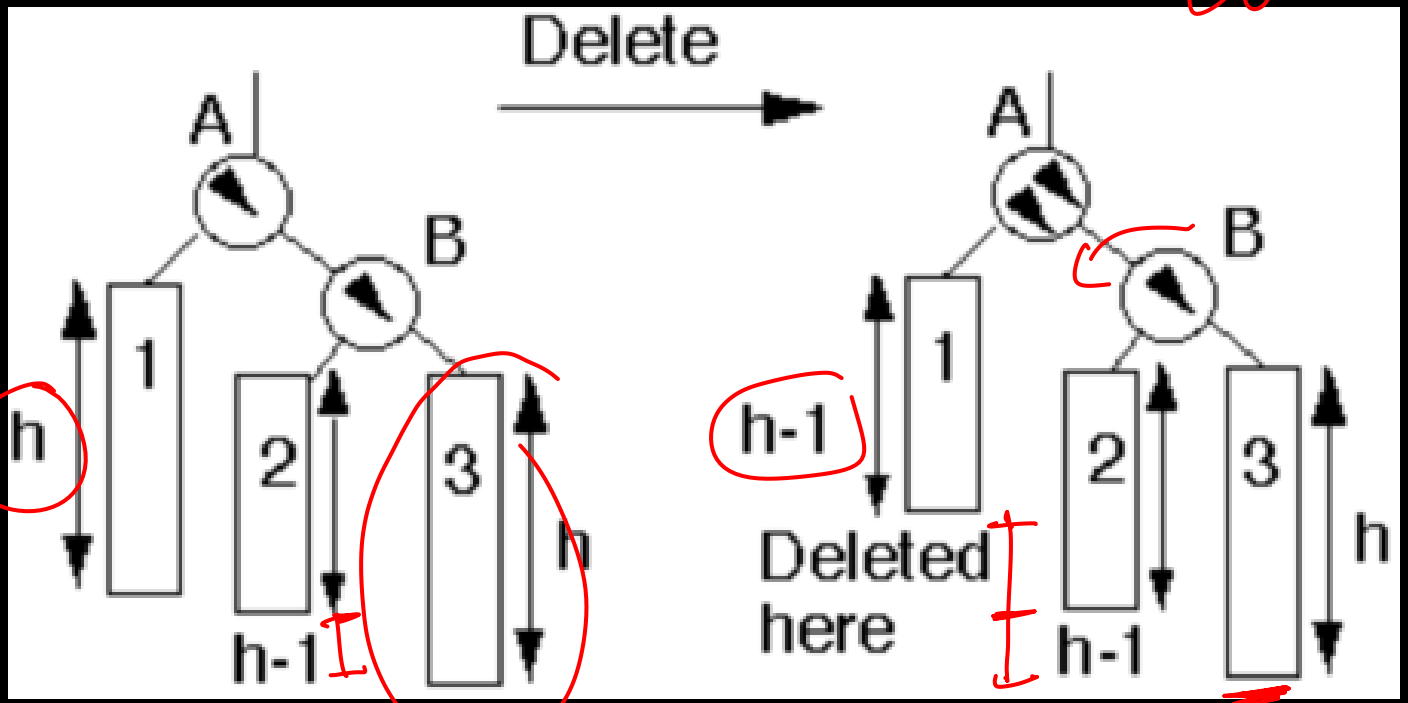
Double Rotate



- *Action*: Double rotation at B. May have effected the balance of higher nodes, so continue up the tree.

Deletion in AVL Tree

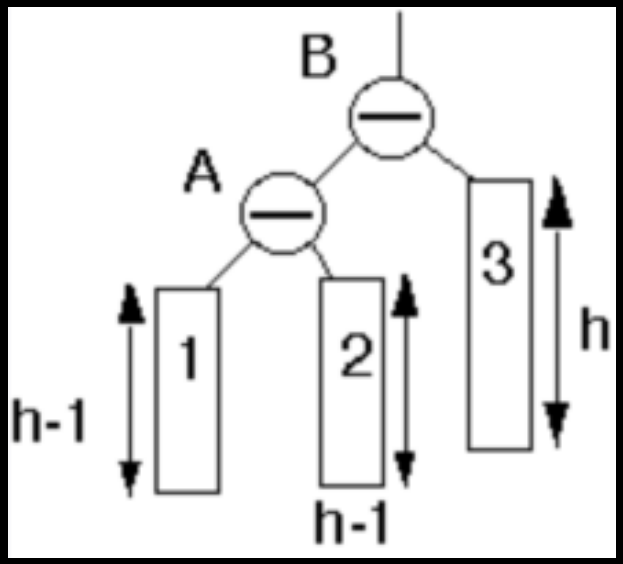
- *Case 5a*: parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was unbalanced. (one level difference)



Deletion in AVL Tree

- *Case 5a*: parent had balance of -1 and the node was deleted in the parent's *left* subtree, right subtree was unbalanced.

Single Rotate



- *Action*: Single rotation at B. May have effected the balance of higher nodes, so continue up the tree.

Thanks …