(364-1-1441) Foundations of Artificial Intelligence

Problem Set 4: *what game spy ye?*

Due: 26/1/2023

Title from the poem Rosalind *by Alfred, Lord Tennyson*

You need to submit only written answers. You will submit one `answers.pdf` file containing your typewritten answers (in English or Hebrew).

1 Problem 1: Formalizing Games

Model the game Rock-Paper-Scissors-Lizard-Spock (see explanation here: `https://en.wikipedia.org/wiki/Rock-paper-scissors#Additional_weapons` or here `http://www.samkass.com/theories/RPSSL.html`) as a normal-form game (i.e., table form).

- Is there a dominant strategy? If there is, prove it. If there isn't, make changes to the utilities that will make one of the players have a dominant strategy.
- Now make changes such that your game will have an equilibrium in dominant strategies.
- Now change utilities such that in addition to that equilibrium, the game will have a Nash equilibrium that is not an equilibrium in dominant strategies.
- Is there a Pareto state? If there is, prove it. If there isn't, make changes to the utilities that will make the game have a Pareto state without being a Nash equilibrium state as well. Now suggest changes to the utilities that will make the game have a Pareto state that is the Nash equilibrium.

2 Problem 2: Voting

Prove no scoring rule can be Condorcet consistent.

(*hint*: A proof can be found in Peter C. Fishburn, "Paradoxes of Voting", The American Political Science Review 68(2):537-546 (June 1974), which you can access through the university internet network)

3 Problem 3: Cake Cutting

The *moving knife* cake-cutting algorithm for n participants works in the following way:

1. A knife begins to move from the edge of the cake (from point 0 to point 1).

2. Once the knife has reached a point v for which if the knife cut there, the cake $[0, v]$ would be worth $\frac{1}{n}$ for some player, the player calls out and they get their cake and leave the game.

3. return to step 1, with the cake being $[v, 1]$ (and 1 less participant).

- Is this mechanism proportional? (prove or disprove)
- Is this mechanism envy-free? (prove or disprove)
- What is the complexity of this algorithm (using the query counting we defined in class)?