Ke Wang
903058889
kwang337

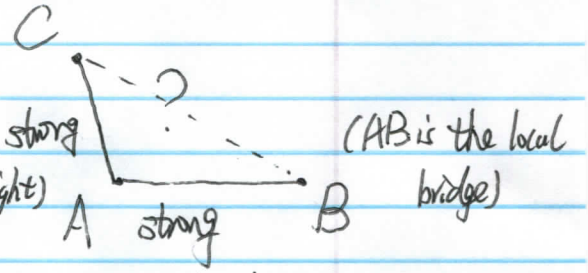
CSE 6240
HW 7

1. Strong and weak ties in the network

a) A node "A" violates the Strong Triadic Closure Property if it has strong ties to two other nodes "B" and "C" and there is no edge at all between B and C. A node "A" satisfied with the Strong Triadic Closure Property if it does not violate it.

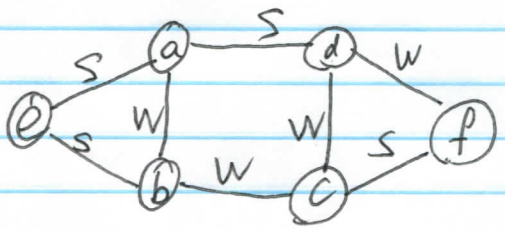
The theorem is that: If a node A satisfies the STCP and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie.

We can prove it by contradiction.
Assume that there exists a local bridge which is a strong tie (like the one on the right)
Then what about the edge BC?
If BC doesn't exist, then AC. AB cannot be strong ties because of the property.
If BC does exist, then AB is not a local bridge because A and B have common neighbor "C"
In both cases contradicting the assumption.



(AB is the local bridge)

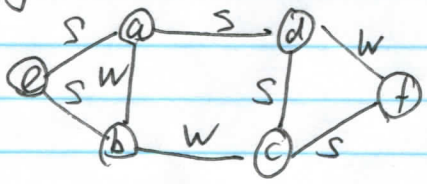
b) Figure 1 (a)



"a" violates because there is no edge between e and d

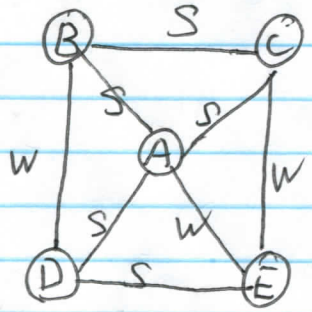
Ke Wang
9030588889
kwang337

Figure 1 (b)



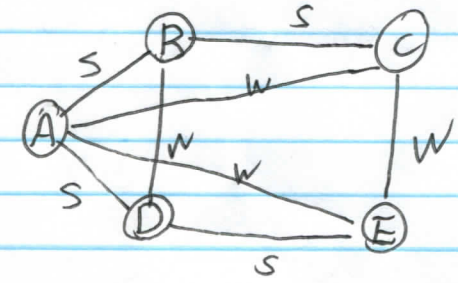
"a" violates because no edge between e and d
"d" violates because no edge between a and c

Figure 2



"A" violates because no edge between C and D

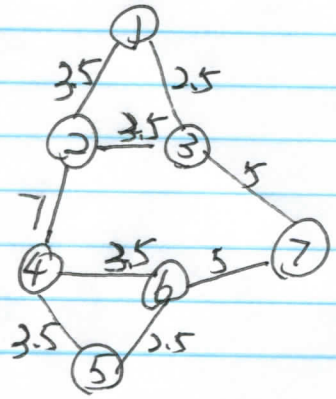
Figure 3



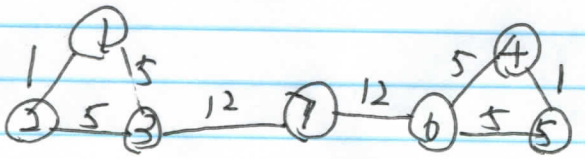
No nodes violate the property.

Ke Wang
9030588889
KWang337

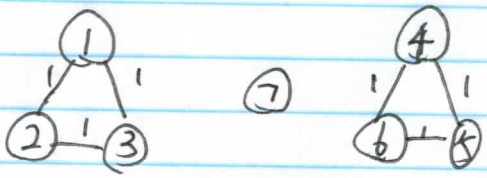
c) I. calculate betweeness



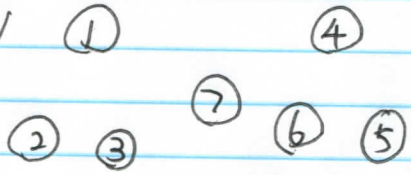
II remove and re-calculate



III



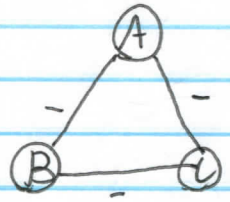
IV



Ke Wang
9030588879
kwang337

2. Structural Balance in Social Networks

a) Figure 5



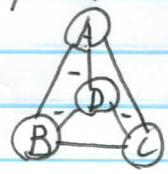
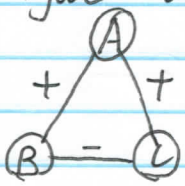
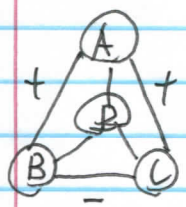
There is not a way for D to do this.
Because all signs are negative (they are the same), so the sign between D and any of ABC should be the same. But neither  nor  meet the requirement.

Figure 6



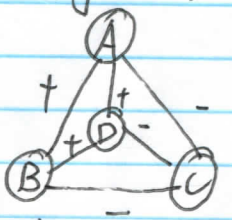
There is not a way for D to do this



AD cannot be −, otherwise BD, DC must be +, △BDC is unbalanced
If AD is + BD, DC must have exactly one −, either way there exists an unbalanced △.

Ke Wang
9030588 89
kwang337

Figure 7



There is only one way
Because AD, DC must have one and only one -.
BD, DC must have one and only one -.
This is the only graph.

b) It's not possible for X to join the network and at the same time not become involved in an unbalanced Δ.
Because the question said that "it contains at least one unbalanced triangle", so let one of the triangle be ΔABC.
From a), there is no way for X to be not involved in unbalanced with respect to A.B.C.
So, there is no way for X to do so.