

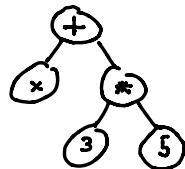
Ukazni programski jezik

Ponovimo:

aritmetični izrazi

- sintaksa (konkretna, abstraktna)

" $x + 3 * 5$ "



- operacijska/dinamična semantika

$$\frac{\text{predpostavke} \quad C_1 \quad C_2 \quad \dots \quad C_n}{C}$$

sklep

pravilo:

če imamo vse
predpostavke C_1, \dots, C_n ,
potem lahko sklepamo
 C

Podali smo pravila za

$$\eta \mid e \hookrightarrow n$$

"veliki koraki"

$\eta = [x \mapsto 5, y \mapsto 1, z \mapsto 42]$

okolje izraz vrednost izraza
(členilo)

$$\eta \mid e \mapsto e'$$

"mali koraki"

okolje izraz izraz
V okolju η se izraz e
v enem koraku
transformira v izraz e'

Primer: $\eta \mid (x + 3) \cdot 5 \hookrightarrow 40$

$$\eta \mid (x + 3) \cdot 5 \mapsto \eta \mid (5 + 3) \cdot 5 \mapsto \eta \mid 8 \cdot 5 \mapsto \eta \mid 40$$

Ukazni programske jezike:

- aritmetični izrazi
- boolovi izrazi
- ukazi: prirejanje, pogojni stavek, zanka while

Sintaksa:

Aritmetični izrazi:

$\langle \text{aritmetični-izraz} \rangle ::= \langle \text{aditivni-izraz} \rangle$

$\langle \text{aditivni-izraz} \rangle ::= \langle \text{multiplikativni-izraz} \rangle \mid \langle \text{aditivni-izraz} \rangle + \langle \text{multiplikativni-izraz} \rangle$

$\langle \text{multiplikativni-izraz} \rangle ::= \langle \text{osnovni-izraz} \rangle \mid \langle \text{multiplikativni-izraz} \rangle * \langle \text{osnovni-izraz} \rangle$

$\langle \text{osnovni-izraz} \rangle ::= \langle \text{spremenljivka} \rangle \mid \langle \text{številka} \rangle \mid (\langle \text{izraz} \rangle)$

$\langle \text{spremenljivka} \rangle ::= [a-zA-Z]^+$

$\langle \text{številka} \rangle ::= -? [0-9]^+$

konkretna sintaksa
kaj pa - ?!

$$\begin{aligned} a \neq b &\Leftrightarrow !(b = a) \\ &\Leftrightarrow a < b + 1 \\ &\Leftrightarrow a = b \text{ || } a < b \end{aligned}$$

Boolovi izrazi:

$\langle \text{boolov-izraz} \rangle ::= \text{true} \mid \text{false} \mid$
 $\quad \langle \text{aritmetični-izraz} \rangle = \langle \text{aritmetični-izraz} \rangle \mid$
 $\quad \langle \text{aritmetični-izraz} \rangle < \langle \text{aritmetični-izraz} \rangle \mid$
 $\quad \langle \text{boolov-izraz} \rangle \&& \langle \text{boolov-izraz} \rangle \mid$
 $\quad \langle \text{boolov-izraz} \rangle \mid\mid \langle \text{boolov-izraz} \rangle \mid$
 $\quad ! \langle \text{boolov-izraz} \rangle$

abstraktna
true && false || $7 < 5$
 $(\text{true} \&& \text{false}) \mid\mid (7 < 5)$
 $\text{true} \&& (\text{false} \mid\mid (7 < 5))$

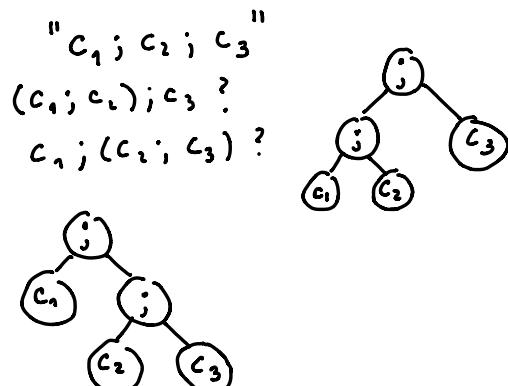
Ukazi:

$\langle \text{ukaz} \rangle ::= \text{skip} \mid$
 $\quad \langle \text{spremenljivka} \rangle := \langle \text{aritmetični-izraz} \rangle \mid$
 $\quad \langle \text{ukaz} \rangle ; \langle \text{ukaz} \rangle \mid$
 $\quad \text{while } \langle \text{boolov-izraz} \rangle \text{ do } \langle \text{ukaz} \rangle \text{ done} \mid$
 $\quad \text{if } \langle \text{boolov-izraz} \rangle \text{ then } \langle \text{ukaz} \rangle \text{ else } \langle \text{ukaz} \rangle$

abstraktna sintaksa

Primer programa:

```
s := 0;
i := 0;
while i < 101 do
  s := s + i;
  i := i + 1
done
```



Asociativnost operatorja:

$$a \otimes b \otimes c = (a \otimes b) \otimes c \text{ levo asociativen}$$

$$a \oplus b \oplus c = a \oplus (b \oplus c) \text{ desno asociativen}$$

$$7 - 5 - 3 = (7 - 5) - 3 \text{ levo}$$

$$70 : 10 : 7 = (70 : 10) : 7 \quad \text{levo}$$

$$2^3 \cdot 3^2 = 2^3 \cdot (3^2 \cdot 2) \quad \text{desno} \quad 2^{3^2} = 2^{(3^2)}$$

- $p \Leftrightarrow q \Leftrightarrow r$ v resnik: $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r)$
- $a \leq b \leq c$ v resnik: $a \leq b \wedge b \leq c$

Operacijska semantika

Okolje: preslikava iz spremenljivk v njihove vrednosti

$$\eta = [x \mapsto 1, y \mapsto 2, z \mapsto 42]$$

$\eta(x)$... vrednost spremenljivke x v okolju η

$$\text{Primer: } \eta(y) = 2$$

$\eta(t)$ nedeterminirano

$\eta[v \mapsto n]$... v okolju η nastavi spremenljivko v na n , vrni novo okolje

Primer:

$$\eta[x \mapsto 7] = [x \mapsto 7, y \mapsto 2, z \mapsto 42]$$

Operacijska semantika

- aritmetični izrazi: $\eta \models e \hookrightarrow n$ veliki koraki
- boolovi izrazi: $\eta \models b \hookrightarrow N$
 - boolov izraz
 - boolova vrednost: true, false

- ukaz: mali koraki

$$(\eta, C) \mapsto \eta' \quad \begin{array}{l} \text{V okolju } \eta \text{ ukaz } C \text{ konča v enem koraku} \\ \text{in novo okolje je } \eta' \end{array}$$

$$(\eta, C) \mapsto (\eta', C') \quad \begin{array}{l} \text{V okolju } \eta \text{ ukaz } C \text{ naredi en korak,} \\ \text{ičkanje se nato nadaljuje v okolju } \eta', \\ \text{izvesti moramo še ukaz } C' \end{array}$$

shift

Aritmetični izrazi:

$$\frac{}{\eta \mid n \hookrightarrow n}$$

$$\frac{\eta(x) = n}{\eta \mid x \hookrightarrow n}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 + e_2 \hookrightarrow n_1 + n_2}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 - e_2 \hookrightarrow n_1 - n_2}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2}{\eta \mid e_1 * e_2 \hookrightarrow n_1 * n_2}$$

Boolovi izrazi:

$$\frac{}{\eta \mid \text{true} \hookrightarrow \text{true}}$$

$$\frac{}{\eta \mid \text{false} \hookrightarrow \text{false}}$$

$$\frac{\eta \mid b \hookrightarrow \text{false}}{\eta \mid !b \hookrightarrow \text{true}}$$

$$\frac{\eta \mid b \hookrightarrow \text{true}}{\eta \mid !b \hookrightarrow \text{false}}$$

$$\frac{}{\eta \mid b_1 \hookrightarrow \text{false}}$$

$$\frac{}{\eta \mid b_1 \&& b_2 \hookrightarrow \text{false}}$$

$$\frac{\eta \mid b_1 \hookrightarrow \text{true} \quad \eta \mid b_2 \hookrightarrow \text{vz}}{\eta \mid b_1 \&& b_2 \hookrightarrow \text{vz}}$$

$$\frac{}{\eta \mid b_1 \hookrightarrow \text{true}}$$

$$\frac{}{\eta \mid b_1 \mid\mid b_2 \hookrightarrow \text{true}}$$

$$\frac{\eta \mid b_1 \hookrightarrow \text{false} \quad \eta \mid b_2 \hookrightarrow \text{vz}}{\eta \mid b_1 \mid\mid b_2 \hookrightarrow \text{vz}}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 = n_2}{\eta \mid e_1 = e_2 \hookrightarrow \text{true}}$$

$$\frac{}{\eta \mid e_1 = e_2 \hookrightarrow \text{true}}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 \neq n_2}{\eta \mid e_1 = e_2 \hookrightarrow \text{false}}$$

$$\frac{}{\eta \mid e_1 = e_2 \hookrightarrow \text{false}}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 < n_2}{\eta \mid e_1 < e_2 \hookrightarrow \text{true}}$$

$$\frac{}{\eta \mid e_1 < e_2 \hookrightarrow \text{true}}$$

$$\frac{\eta \mid e_1 \hookrightarrow n_1 \quad \eta \mid e_2 \hookrightarrow n_2 \quad n_1 \geq n_2}{\eta \mid e_1 < e_2 \hookrightarrow \text{false}}$$

$$\frac{}{\eta \mid e_1 < e_2 \hookrightarrow \text{false}}$$

Ukazi:

$$(\eta, \text{skip}) \rightarrow \eta$$

$$\eta \mid e \hookrightarrow \eta$$

$$(\eta, (x := e)) \rightarrow \eta[x \mapsto \eta]$$

$$(\eta, c_1) \rightarrow (\eta', c_1')$$

$$(\eta, (c_1 ; c_2)) \rightarrow (\eta', (c_1' ; c_2))$$

$$(\eta, c_1) \rightarrow \eta'$$

$$(\eta, (c_1 ; c_2)) \rightarrow (\eta', c_2)$$

$$\eta \mid b \hookrightarrow \text{false}$$

$$(\eta, (\text{while } b \text{ do } c \text{ done})) \rightarrow \eta$$

$$\eta \mid b \hookrightarrow \text{true}$$

$$(\eta, (\text{while } b \text{ do } c \text{ done})) \rightarrow (\eta, (c ; \text{while } b \text{ do } c \text{ done}))$$

$$\eta \mid b \hookrightarrow \text{true}$$

$$(\eta, (\text{if } b \text{ then } c_1 \text{ else } c_2)) \rightarrow (\eta, c_1)$$

$$\eta \mid b \hookrightarrow \text{false}$$

$$(\eta, (\text{if } b \text{ then } c_1 \text{ else } c_2)) \rightarrow (\eta, c_2)$$

Primer: $\eta := [x \mapsto 2, y \mapsto 3, z \mapsto 10]$

Evaluiramo:

$$(\eta, (\text{if } x < y \text{ then } z := x \text{ else } z := y)) \rightarrow \\ (\eta, z := x) \rightarrow$$

$$\eta \mid x < y \hookrightarrow \text{true}$$

$$\eta \mid x \hookrightarrow 2$$

$$[x \mapsto 2, y \mapsto 3, z \mapsto 2]$$

Primer: $\xrightarrow{\text{ratumeju hot}}$ $(\eta, (z := x + 7 ; y := z ; \underbrace{\text{skip} ; x := 10}_{C_1})) \rightarrow$ $\eta \mid x + 7 \hookrightarrow 9$

$$([\underbrace{x \mapsto 2, y \mapsto 3, z \mapsto 2}_{C_1}], (y := z ; \underbrace{\text{skip} ; x := 10}_{C_2})) \rightarrow \eta \mid z \hookrightarrow 9$$

$$\frac{\eta \mid b \hookrightarrow \text{true} \quad (\eta, c_1) \rightarrow (\eta', c_1')} {(\eta, (\text{if } b \text{ then } c_1 \text{ else } c_2)) \rightarrow (\eta', c_1')}$$

dva koraka!

to pravilo je slabše (je ok, a nema grdo)

$$\begin{aligned} ([x \mapsto 2, y \mapsto 9, z \mapsto 9], (\text{skip} ; x := 10)) &\mapsto \\ ([x \mapsto 2, y \mapsto 9, z \mapsto 9], (x := 10)) &\mapsto \\ [x \mapsto 10, y \mapsto 9, z \mapsto 9] \end{aligned}$$

Primer:

$$\begin{aligned} (\eta, \underbrace{((z := x + 7 ; y := z) ; \text{skip})}_{C_1} ; \underbrace{x := 10}_{C_2}) &\mapsto \\ C_1 \downarrow C'_1 & \\ (\eta[z \mapsto 9], (((y := z) ; \text{skip}) ; x := 10)) &\mapsto \dots \\ &\text{isti rezultat, a} \\ &\text{drug vrstni red uporabe} \\ &\text{pravil.} \end{aligned}$$

Primer: $[x \mapsto 1]$

$$\begin{aligned} ([x \mapsto 1], (\text{while } x > 0 \text{ do } x := x + 1 \text{ done})) &\mapsto \begin{array}{l} [x \mapsto 1] \mid x > 0 \hookrightarrow \text{true} \\ [x \mapsto 2] \mid x + 1 \hookrightarrow 2 \end{array} \\ ([x \mapsto 1], (x := x + 1 ; \text{while } x > 0 \text{ do } x := x + 1 \text{ done})) &\mapsto \\ ([x \mapsto 2], (\text{while } x > 0 \text{ do } x := x + 1 \text{ done})) &\mapsto \\ ([x \mapsto 2], (x := x + 1 ; \text{while } x > 0 \text{ do } x := x + 1 \text{ done})) &\mapsto \\ ([x \mapsto 3], (\text{while } x > 0 \text{ do } x := x + 1 \text{ done})) &\mapsto \dots \\ \text{in taku naprej za vedno} & \text{nestekano zaporedje krovakov} \end{aligned}$$

Dilema o sintaksi:

Ali je $\text{if } b \text{ then } c_1 \text{ else } c_2 ; c_3$

tole: $(\text{if } b \text{ then } c_1 \text{ else } c_2) ; c_3$

tole: $\text{if } b \text{ then } c_1 \text{ else } (c_2 ; c_3)$?

Temu se izognemo tako, da popravimo sintaksco:

programmers of this app are idiots

Ukazi:

```
<ukaz> ::= skip |
    <spremenljivka> := <aritmetični-izraz> |
    <ukaz> ; <ukaz> |
    while <boolov-izraz> do <ukaz> done |
    if <boolov-izraz> then <ukaz> else <ukaz> end
```

Dileme ni več:

if b then c_1 else c_2 ; c_3 maybe end

if b then c_1 else c_2 ; c_3 end

if b then c_1 else end; c_3

Kdaj sta dva programa "ekvivalentna"?

$X := X + 1 ; X := X + 1$



$X := X + 2$