

MOSEK ApS

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## Chapter 1

## Introduction

The **MOSEK** Optimization Suite 9.1.13 is a powerful software package capable of solving large-scale optimization problems of the following kind:

- linear,
- conic:
  - conic quadratic (also known as second-order cone),
  - involving the exponential cone,
  - involving the power cone,
  - semidefinite.
- convex quadratic and quadratically constrained,
- integer.

In order to obtain an overview of features in the MOSEK Optimization Suite consult the product introduction guide.

The most widespread class of optimization problems is *linear optimization problems*, where all relations are linear. The tremendous success of both applications and theory of linear optimization can be ascribed to the following factors:

- The required data are simple, i.e. just matrices and vectors.
- Convexity is guaranteed since the problem is convex by construction.
- Linear functions are trivially differentiable.
- There exist very efficient algorithms and software for solving linear problems.
- Duality properties for linear optimization are nice and simple.

Even if the linear optimization model is only an approximation to the true problem at hand, the advantages of linear optimization may outweigh the disadvantages. In some cases, however, the problem formulation is inherently nonlinear and a linear approximation is either intractable or inadequate. *Conic optimization* has proved to be a very expressive and powerful way to introduce nonlinearities, while preserving all the nice properties of linear optimization listed above.

The fundamental expression in linear optimization is a linear expression of the form

$$Ax - b \ge 0$$
.

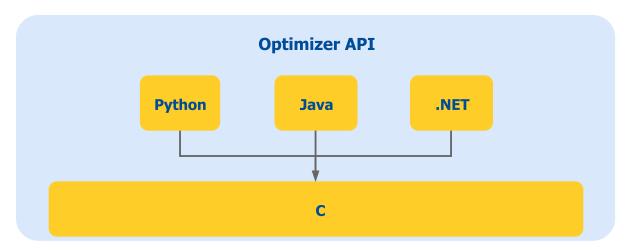
In conic optimization this is replaced with a wider class of constraints

$$Ax - b \in \mathcal{K}$$

where  $\mathcal{K}$  is a *convex cone*. For example in 3 dimensions  $\mathcal{K}$  may correspond to an ice cream cone. The conic optimizer in **MOSEK** supports a number of different types of cones  $\mathcal{K}$ , which allows a surprisingly large number of nonlinear relations to be modeled, as described in the **MOSEK** Modeling Cookbook, while preserving the nice algorithmic and theoretical properties of linear optimization.

## 1.1 Why the Optimizer API for .NET?

The Optimizer API for .NET provides an object-oriented interface to the **MOSEK** optimizers. This object oriented design is common to Java, Python and .NET and is based on a thin class-based interface to the native C optimizer API. The overhead introduced by this mapping is minimal.



The Optimizer API for .NET can be used with any application running on the Microsoft .NET platform (and other .NET implementations like Mono and .NET Core). It consists of a single library, mosekdotnet9\_1.dll, containing classes and more in the mosek namespace.

The Optimizer API for .NET provides access to:

- Linear Optimization (LO)
- Conic Quadratic (Second-Order Cone) Optimization (CQO, SOCO)
- Power Cone Optimization
- Conic Exponential Optimization (CEO)
- Convex Quadratic and Quadratically Constrained Optimization (QO, QCQO)
- Semidefinite Optimization (SDO)
- Mixed-Integer Optimization (MIO)

as well as to additional functions for

- problem analysis,
- sensitivity analysis,
- infeasibility diagnostics,
- BLAS/LAPACK linear algebra routines.

# Chapter 2

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You can get in touch with  $\mathbf{MOSEK}$  using popular social media as well:

Blogger	https://blog.mosek.com/
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$\mathbf{Google} +$	https://plus.google.com/+Mosek/posts
Linkedin	https://www.linkedin.com/company/mosek-aps

In particular  $\mathbf{Twitter}$  is used for news, updates and release announcements.

## Chapter 3

# License Agreement

Before using the **MOSEK** software, please read the license agreement available in the distribution at <MSKHOME>/mosek/9.1/mosek-eula.pdf or on the **MOSEK** website https://mosek.com/products/license-agreement.

MOSEK uses some third-party open-source libraries. Their license details follows.

#### zlib

**MOSEK** includes the *zlib* library obtained from the zlib website. The license agreement for *zlib* is shown in Listing 3.1.

### Listing 3.1: zlib license.

zlib.h -- interface of the 'zlib' general purpose compression library version 1.2.7, May 2nd, 2012

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### fplib

**MOSEK** includes the floating point formatting library developed by David M. Gay obtained from the netlib website. The license agreement for *fplib* is shown in Listing 3.2.

Listing 3.2: fplib license.

#### **Zstandard**

**MOSEK** includes the *Zstandard* library developed by Facebook obtained from github/zstd. The license agreement for *Zstandard* is shown in Listing 3.3.

Listing 3.3: Zstandard license.

```
BSD License
```

For Zstandard software

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## Chapter 4

# Installation

In this section we discuss how to install and setup the MOSEK Optimizer API for .NET.

Important: Before running this MOSEK interface please make sure that you:

- Installed **MOSEK** correctly. Some operating systems require extra steps. See the Installation guide for instructions and common troubleshooting tips.
- Set up a license. See the Licensing guide for instructions.

## Compatibility

The Optimizer API for .NET is compatible with the Microsoft .NET framework version 4.5 and later, Mono v.1.2 and later, and .NET Core 2.0 and later.

## Locating files in the MOSEK Optimization Suite

The relevant files of the Optimizer API for .NET are organized as reported in Table 4.1.

Table 4.1: Relevant files for the Optimizer API for .NET.

Relative Path	Description	Label
<pre><mskhome>/mosek/9.1/tools/platform/<platform>/bin</platform></mskhome></pre>	Libraries	<libdir></libdir>
<mskhome>/mosek/9.1/tools/examples/dotnet</mskhome>	Examples	<exdir></exdir>
<mskhome>/mosek/9.1/tools/examples/data</mskhome>	Additional data	<miscdir></miscdir>

## where

- <MSKHOME> is the folder in which the MOSEK Optimization Suite has been installed,
- <PLATFORM> is the actual platform among those supported by the MOSEK, i.e. win32x86, win64x86.

## Setting up paths

To compile a .NET program using **MOSEK** the correct path to mosekdotnet.dll must be provided. For example, using the Microsoft .NET compiler this is done with the command line option

```
csc /r:"<LIBDIR>\mosekdotnet.dll" lo1.cs
```

To run applications the system must be able to locate mosekdotnet.dll, either in the current directory or in the Global Assembly Cache.

## 4.1 Testing the Installation and Compiling Examples

This section describes how to verify that **MOSEK** has been installed correctly, and how to build and execute the .NET examples distributed with **MOSEK**.

## Compiling and running from the command line

To compile an example, say 101, with the Microsoft .NET compiler, open a DOS box with paths for Visual Studio set up (usually in the Start menu, the sub-menu for Visual Studio contains an entry that starts a DOS box with everything set up).

To compile the example lol.cs distributed with MOSEK:

- Go to the examples directory <EXDIR>.
- To compile the code and produce an executable, type:

```
csc /r:"<LIBDIR>\mosekdotnet.dll" lo1.cs
```

or for Visual Basic:

```
vbc /r:"<LIBDIR>\mosekdotnet.dll" lo1.vb
```

• Copy mosekdotnet.dll into the directory where lo1.exe was created, and run the program with:

```
lo1
```

## Compiling the examples using nmake

A makefile for use with nmake, named Makefile is available in <EXDIR>. To compile all examples using this makefile use the command

make /f Makefile all

## 4.2 Other platforms: .NET Core, Mono

The library mosekdotnet.dll may be used from any .NET compatible language such as Visual Basic, Microsoft C# or Microsoft Managed C++ and with .NET Core and Mono. Both the examples and the library should also work with Mono on most 32-bit platforms. If the file mosekdotnet.dll is not included in the MOSEK distribution for your platform, use mosekdotnet.dll included in the Windows distribution.

Note that the library accesses methods in the native **MOSEK** library, which is considered *unsafe* from a .NET point of view. This means that use of the library in certain restricted contexts is not possible — building an ordinary application and running it from a local drive should not be a problem.

## 4.2.1 Mono

Mono is a free implementation if the .NET platform available at http://mono-project.com/. To use it install MOSEK as described in the Installation Manual. Set the environment variable

```
MONO_PATH
```

to point to mosekdotnet9\_1.dll for the 64-bit Mono. You should now be able to compile and run the distributed .NET examples using Mono.

### 4.2.2 IronPython

It is possible to use the  $\mathbf{MOSEK}$  .NET API interactively from .NET languages which implement a command-line interpreter, for example IronPython, available at <a href="http://ironpython.net/">http://ironpython.net/</a>. This can be used to create and examine the problems and solutions from  $\mathbf{MOSEK}$  more easily.

## 4.2.3 MOSEK and .NET Core

The MOSEK NuGet package Mosek.9.1.13.nupkg is a complete cross-platform .NET Core compatible distribution that works on Windows, Linux and OS X.

The NuGet package Mosek.9.1.13.nupkg is available for download from https://mosek.com/downloads. Assuming that the Mosek.9.1.13.nupkg file has been downloaded in a directory local-nupkgs, modify the configuration file \*.csproj to add the following entry

```
<PropertyGroup>
    <RestoreSources>$(RestoreSources);local-nupkgs</RestoreSources>
</PropertyGroup>
```

Now, add the dependency on MOSEK to the project:

```
dotnet add package Mosek
```

and the project using  $\mathbf{MOSEK}$  API can be built:

```
dotnet build dotnet run
```

Installation instructions for different .NET Core compatible environments may vary.

## Chapter 5

# Design Overview

## 5.1 Modeling

Optimizer API for .NET is an interface for specifying optimization problems directly in matrix form. It means that an optimization problem such as:

minimize 
$$c^T x$$
  
subject to  $Ax \leq b$ ,  
 $x \in \mathcal{K}$ 

is specified by describing the matrix A, vectors b, c and a list of cones K directly.

The main characteristics of this interface are:

- Simplicity: once the problem data is assembled in matrix form, it is straightforward to input it into the optimizer.
- Exploiting sparsity: data is entered in sparse format, enabling huge, sparse problems to be defined and solved efficiently.
- Efficiency: the Optimizer API incurs almost no overhead between the user's representation of the problem and MOSEK's internal one.

Optimizer API for .NET does not aid with modeling. It is the user's responsibility to express the problem in **MOSEK**'s standard form, introducing, if necessary, auxiliary variables and constraints. See Sec. 12 for the precise formulations of problems **MOSEK** solves.

## 5.2 "Hello World!" in MOSEK

Here we present the most basic workflow pattern when using Optimizer API for .NET.

## Creating an environment and task

Every interaction with **MOSEK** using Optimizer API for .NET begins by creating a **MOSEK** environment. It coordinates the access to **MOSEK** from the current process.

In most cases the user does not interact directly with the environment, except for creating optimization **tasks**, which contain actual problem specifications and where optimization takes place. An environment can host multiple tasks.

## **Defining tasks**

After a task is created, the input data can be specified. An optimization problem consists of several components; objective, objective sense, constraints, variable bounds etc. See Sec. 6 for basic tutorials on how to specify and solve various types of optimization problems.

## Retrieving the solutions

When the model is set up, the optimizer is invoked with the call to *Task.optimize*. When the optimization is over, the user can check the results and retrieve numerical values. See further details in Sec. 7.

We refer also to Sec. 7 for information about more advanced mechanisms of interacting with the solver.

## Source code example

Below is the most basic code sample that defines and solves a trivial optimization problem

```
minimize x
subject to 2.0 \le x \le 3.0.
```

For simplicity the example does not contain any error or status checks.

Listing 5.1: "Hello World!" in MOSEK

```
using mosek;
using System;
public class helloworld {
 public static void Main() {
   double[] x = new double[1];
                                                // Create Environment
   using (Env env = new Env()) {
      using (Task task = new Task(env, 0, 1)) { // Create Task
       task.appendvars(1);
                                                     // 1 variable x
        task.putcj(0, 1.0);
                                                    // c_0 = 1.0
        task.putvarbound(0, boundkey.ra, 2.0, 3.0); // 2.0 <= x <= 3.0
        task.putobjsense(objsense.minimize);
                                              // Optimize
        task.optimize();
        task.getxx(soltype.itr, x);
                                                    // Get solution
        Console.WriteLine("Solution x = " + x[0]); // Print solution
   }
 }
}
```

## Chapter 6

# Optimization Tutorials

In this section we demonstrate how to set up basic types of optimization problems. Each short tutorial contains a working example of formulating problems, defining variables and constraints and retrieving solutions.

## 6.1 Linear Optimization

The simplest optimization problem is a purely linear problem. A *linear optimization problem* is a problem of the following form:

Minimize or maximize the objective function

$$\sum_{j=0}^{n-1} c_j x_j + c^f$$

subject to the linear constraints

$$l_k^c \le \sum_{j=0}^{n-1} a_{kj} x_j \le u_k^c, \quad k = 0, \dots, m-1,$$

and the bounds

$$l_j^x \le x_j \le u_j^x, \quad j = 0, \dots, n - 1.$$

The problem description consists of the following elements:

- $\bullet$  m and n the number of constraints and variables, respectively,
- x the variable vector of length n,
- ullet c the coefficient vector of length n

$$c = \left[ \begin{array}{c} c_0 \\ \vdots \\ c_{n-1} \end{array} \right],$$

- $c^f$  fixed term in the objective,
- A an  $m \times n$  matrix of coefficients

$$A = \begin{bmatrix} a_{0,0} & \cdots & a_{0,(n-1)} \\ \vdots & \cdots & \vdots \\ a_{(m-1),0} & \cdots & a_{(m-1),(n-1)} \end{bmatrix},$$

- $l^c$  and  $u^c$  the lower and upper bounds on constraints,
- $l^x$  and  $u^x$  the lower and upper bounds on variables.

Please note that we are using 0 as the first index:  $x_0$  is the first element in variable vector x.

## 6.1.1 Example LO1

The following is an example of a small linear optimization problem:

maximize 
$$3x_0 + 1x_1 + 5x_2 + 1x_3$$
  
subject to  $3x_0 + 1x_1 + 2x_2 = 30$ ,  
 $2x_0 + 1x_1 + 3x_2 + 1x_3 \ge 15$ ,  
 $2x_1 + 3x_3 \le 25$ , (6.1)

under the bounds

### Solving the problem

To solve the problem above we go through the following steps:

- 1. Create an environment.
- 2. Create an optimization task.
- 3. Load a problem into the task object.
- 4. Optimization.
- 5. Extracting the solution.

Below we explain each of these steps.

### Create an environment.

Before setting up the optimization problem, a **MOSEK** environment must be created. All tasks in the program should share the same environment.

```
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
{
```

## Create an optimization task.

Next, an empty task object is created:

```
// Create a task object.
using (mosek.Task task = new mosek.Task(env, 0, 0))
{
    // Directs the log task stream to the user specified
    // method msgclass.streamCB
    task.set_Stream (mosek.streamtype.log, new msgclass (""));
```

We also connect a call-back function to the task log stream. Messages related to the task are passed to the call-back function. In this case the stream call-back function writes its messages to the standard output stream. See Sec. 7.3.

## Load a problem into the task object.

Before any problem data can be set, variables and constraints must be added to the problem via calls to the functions Task. appendoons and Task. appendours.

```
// Append 'numcon' empty constraints.
// The constraints will initially have no bounds.
task.appendcons(numcon);
// Append 'numvar' variables.
// The variables will initially be fixed at zero (x=0).
task.appendvars(numvar);
```

New variables can now be referenced from other functions with indexes in  $0, \ldots, numvar - 1$  and new constraints can be referenced with indexes in  $0, \dots, numcon - 1$ . More variables and/or constraints can be appended later as needed, these will be assigned indexes from numvar/numcon and up.

Next step is to set the problem data. We loop over each variable index  $j = 0, \dots, numvar - 1$  calling functions to set problem data. We first set the objective coefficient  $c_j = c[j]$  by calling the function Task.putcj.

```
task.putcj(j, c[j]);
```

## Setting bounds on variables

boundkey.up

The bounds on variables are stored in the arrays

```
mosek.boundkey[] bkx = {mosek.boundkey.lo,
                          mosek.boundkey.ra,
                          mosek.boundkey.lo,
                          mosek.boundkey.lo
double[] blx = \{0.0,
                  0.0,
                  0.0
                 };
double[] bux = { +infinity,
                   10.0.
                   +infinity,
                   +infinity
                 };
```

and are set with calls to Task.putvarbound.

 $\overline{\cdots} \leq u_i$ 

```
// Set the bounds on variable j.
// blx[j] <= x_j <= bux[j]
task.putvarbound(j, bkx[j], blx[j], bux[j]);
```

The Bound key stored in bkx specifies the type of the bound according to Table 6.1.

Bound key	Type of bound	Lower bound	Upper bound
boundkey.fx	$u_j = l_j$	Finite	Identical to the lower bound
boundkey.fr	Free	$-\infty$	$+\infty$
boundkey.lo	$l_j \leq \cdots$	Finite	$+\infty$
boundkeu.ra	$l_i < \cdots < u_i$	Finite	Finite

Finite

Table 6.1: Bound keys as defined in the enum boundkey.

For instance bkx[0] = boundkey. lo means that  $x_0 \ge l_0^x$ . Finally, the numerical values of the bounds on variables are given by

$$l_j^x = \mathtt{blx}[\mathtt{j}]$$

and

$$u_i^x = \text{bux}[j].$$

## Defining the linear constraint matrix.

Recall that in our example the A matrix is given by

$$A = \left[ \begin{array}{cccc} 3 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 3 \end{array} \right].$$

This matrix is stored in sparse format in the arrays:

```
int[][] asub = {
    new int[] {0, 1},
    new int[] {0, 1, 2},
    new int[] {0, 1},
    new int[] {1, 2}
};
double[][] aval = {
    new double[] {3.0, 2.0},
    new double[] {1.0, 1.0, 2.0},
    new double[] {2.0, 3.0},
    new double[] {1.0, 3.0}
};
```

The array aval[j] contains the non-zero values of column j and asub[j] contains the row indices of these non-zeros.

Using the function Task.putacol we set column j of A

There are many alternative formats for entering the A matrix. See functions such as Task.putarow, Task.putarowlist, Task.putaijlist and similar.

Finally, the bounds on each constraint are set by looping over each constraint index  $i=0,\ldots,\mathtt{numcon}-1$ 

```
// Set the bounds on constraints.
// blc[i] <= constraint_i <= buc[i]
for (int i = 0; i < numcon; ++i)
  task.putconbound(i, bkc[i], blc[i], buc[i]);</pre>
```

## Optimization

After the problem is set-up the task can be optimized by calling the function Task.optimize.

```
task.optimize();
```

## Extracting the solution.

After optimizing the status of the solution is examined with a call to Task.getsolsta. If the solution status is reported as solsta.optimal the solution is extracted in the lines below:

The Task.getxx function obtains the solution. **MOSEK** may compute several solutions depending on the optimizer employed. In this example the  $basic\ solution$  is requested by setting the first argument to soltype.bas.

## **Catching exceptions**

We catch any exceptions thrown by **MOSEK** in the lines:

```
catch (mosek.Exception e) {
   mosek.rescode res = e.Code;
   Console.WriteLine("Response code {0}\nMessage {1}", res, e.Message);
}
```

The types of exceptions that MOSEK can throw can be seen in Sec. 15.5. See also Sec. 7.2.

#### Source code

The complete source code loi.cs of this example appears below. See also loi.cs for a version where the A matrix is entered row-wise.

Listing 6.1: Linear optimization example.

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
   string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
     Console.Write ("{0}{1}", prefix, msg);
   }
 }
 public class 101
   public static void Main ()
     const int numcon = 3;
     const int numvar = 4;
      // Since the value of infinity is ignored, we define it solely
      // for symbolic purposes
     double infinity = 0;
     double[] c = {3.0, 1.0, 5.0, 1.0};
      int[][] asub = {
           new int[] {0, 1},
           new int[] {0, 1, 2},
           new int[] {0, 1},
           new int[] {1, 2}
     };
     double[][] aval = {
               new double[] {3.0, 2.0},
               new double[] {1.0, 1.0, 2.0},
              new double[] {2.0, 3.0},
               new double[] {1.0, 3.0}
     };
     mosek.boundkey[] bkc = {mosek.boundkey.fx,
                               mosek.boundkey.lo,
```

```
mosek.boundkey.up
double[] blc = \{30.0,
                 15.0,
                 -infinity
                };
double[] buc = \{30.0,
                 +infinity,
                 25.0
                };
mosek.boundkey[] bkx = {mosek.boundkey.lo,
                          mosek.boundkey.ra,
                          mosek.boundkey.lo,
                          mosek.boundkey.lo
double[] blx = \{0.0,
                  0.0,
                  0.0,
                  0.0
                 };
double[] bux = { +infinity,
                   10.0,
                   +infinity,
                   +infinity
                 };
try
  // Make mosek environment.
  using (mosek.Env env = new mosek.Env())
    // Create a task object.
    using (mosek.Task task = new mosek.Task(env, 0, 0))
      // Directs the log task stream to the user specified
      // method msgclass.streamCB
      task.set_Stream (mosek.streamtype.log, new msgclass (""));
      // Append 'numcon' empty constraints.
      // The constraints will initially have no bounds.
      task.appendcons(numcon);
      // Append 'numvar' variables.
      // The variables will initially be fixed at zero (x=0).
      task.appendvars(numvar);
      for (int j = 0; j < numvar; ++j)
        // Set the linear term c_j in the objective.
        task.putcj(j, c[j]);
        // Set the bounds on variable j.
        // blx[j] <= x_j <= bux[j]
        task.putvarbound(j, bkx[j], blx[j], bux[j]);
        // Input column j of A
                                            /* Variable (column) index.*/
        task.putacol(j,
                                            /* Row index of non-zeros in column j.*/
                     asub[j],
                                            /* Non-zero Values of column j. */
                     aval[j]);
      }
```

```
// Set the bounds on constraints.
          // blc[i] <= constraint_i <= buc[i]</pre>
          for (int i = 0; i < numcon; ++i)</pre>
            task.putconbound(i, bkc[i], blc[i], buc[i]);
          // Input the objective sense (minimize/maximize)
          task.putobjsense(mosek.objsense.maximize);
          // Solve the problem
          task.optimize();
          // Print a summary containing information
          // about the solution for debugging purposes
          task.solutionsummary(mosek.streamtype.msg);
          // Get status information about the solution
          mosek.solsta solsta;
          task.getsolsta(mosek.soltype.bas, out solsta);
          switch (solsta)
            case mosek.solsta.optimal:
              double[] xx = new double[numvar];
              task.getxx(mosek.soltype.bas, // Request the basic solution.
                         xx);
              Console.WriteLine ("Optimal primal solution\n");
              for (int j = 0; j < numvar; ++j)
                Console.WriteLine (x[0]: \{1\}, j, xx[j]);
              break;
            case mosek.solsta.dual_infeas_cer:
            case mosek.solsta.prim_infeas_cer:
              Console.WriteLine("Primal or dual infeasibility certificate found.\n");
              break;
            case mosek.solsta.unknown:
              Console.WriteLine("Unknown solution status.\n");
            default:
              Console.WriteLine("Other solution status");
              break;
       }
      }
    catch (mosek.Exception e) {
      mosek.rescode res = e.Code;
      Console.WriteLine("Response code {0}\nMessage
                                                      {1}", res, e.Message);
  }
}
```

## 6.2 Quadratic Optimization

**MOSEK** can solve quadratic and quadratically constrained problems, as long as they are convex. This class of problems can be formulated as follows:

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
subject to 
$$l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$$

$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$$

$$(6.2)$$

Without loss of generality it is assumed that  $Q^o$  and  $Q^k$  are all symmetric because

$$x^T Q x = \frac{1}{2} x^T (Q + Q^T) x.$$

This implies that a non-symmetric Q can be replaced by the symmetric matrix  $\frac{1}{2}(Q+Q^T)$ .

The problem is required to be convex. More precisely, the matrix  $Q^o$  must be positive semi-definite and the kth constraint must be of the form

$$l_k^c \le \frac{1}{2} x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \tag{6.3}$$

with a negative semi-definite  $Q^k$  or of the form

$$\frac{1}{2}x^T Q^k x + \sum_{j=0}^{n-1} a_{k,j} x_j \le u_k^c.$$

with a positive semi-definite  $Q^k$ . This implies that quadratic equalities are *not* allowed. Specifying a non-convex problem will result in an error when the optimizer is called.

A matrix is positive semidefinite if all the eigenvalues of Q are nonnegative. An alternative statement of the positive semidefinite requirement is

$$x^T Qx > 0, \quad \forall x.$$

If the convexity (i.e. semidefiniteness) conditions are not met **MOSEK** will not produce reliable results or work at all.

## 6.2.1 Example: Quadratic Objective

We look at a small problem with linear constraints and quadratic objective:

minimize 
$$x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2$$
 subject to 
$$1 \le x_1 + x_2 + x_3$$
 
$$0 < x.$$
 (6.4)

The matrix formulation of (6.4) has:

$$Q^o = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

with the bounds:

$$l^c = 1, u^c = \infty, l^x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
 and  $u^x = \begin{bmatrix} \infty \\ \infty \\ \infty \end{bmatrix}$ 

Please note the explicit  $\frac{1}{2}$  in the objective function of (6.2) which implies that diagonal elements must be doubled in Q, i.e.  $Q_{11}=2$  even though 1 is the coefficient in front of  $x_1^2$  in (6.4).

### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

## Setting up the quadratic objective

The quadratic objective is specified using the function Task.putqobj. Since  $Q^o$  is symmetric only the lower triangular part of  $Q^o$  is inputted. In fact entries from above the diagonal may not appear in

The lower triangular part of the matrix  $Q^{o}$  is specified using an unordered sparse triplet format (for details, see Sec. 15.1.4):

```
2,
int[]
             qsubi = \{0,
                                                     };
                                 1,
int[] qsubj = {0, 1, 0, 2 };
double[] qval = {2.0, 0.2, -1.0, 2.0};
                                                2 };
```

Please note that

- only non-zero elements are specified (any element not specified is 0 by definition),
- the order of the non-zero elements is insignificant, and
- only the lower triangular part should be specified.

Finally, this definition of  $Q^o$  is loaded into the task:

```
task.putqobj(qsubi, qsubj, qval);
```

### Source code

Listing 6.2: Source code implementing problem (6.4).

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
   string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
 public class qo1
   public static void Main ()
      // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
     const double infinity = 0;
     const int numcon = 1; /* Number of constraints.
     const int numvar = 3; /* Number of variables.
                                                                     */
```

```
double[] c = \{0.0, -1.0, 0.0\};
mosek.boundkey[] bkc = {mosek.boundkey.lo};
double[] blc = {1.0};
double[] buc = {infinity};
mosek.boundkey[] bkx = {mosek.boundkey.lo,
                          mosek.boundkey.lo,
                          mosek.boundkey.lo
                         };
double[] blx = \{0.0,
                0.0,
                0.0
               };
double[] bux = { +infinity,
                 +infinity,
                 +infinity
               };
          int[][]
double[][] aval = { new double[] {1.0}, new double[] {1.0}, new double[] {1.0}};
mosek.Task task = null;
mosek.Env env = null;
double[] xx = new double[numvar];
try
{
  // Make mosek environment.
  env = new mosek.Env ();
  // Create a task object linked with the environment env.
  task = new mosek.Task (env, 0, 0);
  // Directs the log task stream to the user specified
  // method task_msg_obj.streamCB
  task.set_Stream (mosek.streamtype.log, new msgclass (""));
  /* Give MOSEK an estimate of the size of the input data.
       This is done to increase the speed of inputting data.
       However, it is optional. */
  /* Append 'numcon' empty constraints.
       The constraints will initially have no bounds. */
  task.appendcons(numcon);
  /* Append 'numvar' variables.
       The variables will initially be fixed at zero (x=0). */
  task.appendvars(numvar);
  for (int j = 0; j < numvar; ++j)
    /* Set the linear term c_j in the objective.*/
   task.putcj(j, c[j]);
   /* Set the bounds on variable j.
            blx[j] \leftarrow x_j \leftarrow bux[j] */
   task.putvarbound(j, bkx[j], blx[j], bux[j]);
    /* Input column j of A */
   task.putacol(j,
                                       /* Variable (column) index.*/
                asub[j],
                                       /* Row index of non-zeros in column j.*/
                aval[j]);
                                       /* Non-zero Values of column j. */
  /* Set the bounds on constraints.
         for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
```

```
for (int i = 0; i < numcon; ++i)</pre>
      task.putconbound(i, bkc[i], blc[i], buc[i]);
     * The lower triangular part of the Q
     * matrix in the objective is specified.
    int[]
             qsubi = {0, 1, 2, 2};
    int[] qsubj = {0, 1, 0, 2 };
double[] qval = {2.0, 0.2, -1.0, 2.0};
    /* Input the Q for the objective. */
    task.putobjsense(mosek.objsense.minimize);
    task.putqobj(qsubi, qsubj, qval);
    task.optimize();
    // Print a summary containing information
    // about the solution for debugging purposes
    task.solutionsummary(mosek.streamtype.msg);
    mosek.solsta solsta;
    /* Get status information about the solution */
    task.getsolsta(mosek.soltype.itr, out solsta);
    switch (solsta)
      case mosek.solsta.optimal:
        task.getxx(mosek.soltype.itr, // Interior point solution.
        Console.WriteLine ("Optimal primal solution\n");
        for (int j = 0; j < numvar; ++j)
          Console.WriteLine ("x[{0}]:", xx[j]);
      case mosek.solsta.dual_infeas_cer:
      case mosek.solsta.prim_infeas_cer:
        Console.WriteLine("Primal or dual infeasibility.\n");
      case mosek.solsta.unknown:
        Console.WriteLine("Unknown solution status.\n");
        Console.WriteLine("Other solution status");
        break;
    }
  }
  catch (mosek.Exception e)
   Console.WriteLine (e);
    throw;
  finally
    if (task != null) task.Dispose ();
    if (env != null) env.Dispose ();
} /* Main */
```

}

## 6.2.2 Example: Quadratic constraints

In this section we show how to solve a problem with quadratic constraints. Please note that quadratic constraints are subject to the convexity requirement (6.3).

Consider the problem:

$$\begin{array}{ll} \text{minimize} & x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2 \\ \text{subject to} & 1 & \leq & x_1 + x_2 + x_3 - x_1^2 - x_2^2 - 0.1x_3^2 + 0.2x_1x_3, \\ & x \geq 0. \end{array}$$

This is equivalent to

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}x^TQ^ox + c^Tx \\ \text{subject to} & \frac{1}{2}x^TQ^0x + Ax & \geq & b, \\ & & x \geq 0, \end{array} \tag{6.5}$$

where

$$Q^o = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 0.2 & 0 \\ -1 & 0 & 2 \end{bmatrix}, c = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T, A = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, b = 1.$$
 
$$Q^0 = \begin{bmatrix} -2 & 0 & 0.2 \\ 0 & -2 & 0 \\ 0.2 & 0 & -0.2 \end{bmatrix}.$$

The linear parts and quadratic objective are set up the way described in the previous tutorial.

## Setting up quadratic constraints

To add quadratic terms to the constraints we use the function Task.putqconk.

While Task.putqconk adds quadratic terms to a specific constraint, it is also possible to input all quadratic terms in one chunk using the Task.putqcon function.

### Source code

Listing 6.3: Implementation of the quadratically constrained problem (6.5).

```
using System;
namespace mosek.example
{
```

```
class msgclass : mosek.Stream
 string prefix;
 public msgclass (string prfx)
   prefix = prfx;
 public override void streamCB (string msg)
   Console.Write ("{0}{1}", prefix, msg);
 }
public class qcqo1
 public static void Main ()
    const double inf = 0.0; /* We don't actually need any value for infinity */
    const int numcon = 1; /* Number of constraints.
    const int numvar = 3; /* Number of variables.
    mosek.boundkey[]
    bkc = { mosek.boundkey.lo },
    bkx = { mosek.boundkey.lo, mosek.boundkey.lo, mosek.boundkey.lo };
    int[][] asub = { new int[] {0}, new int[] {0}, new int[] {0} };
    double[][] aval = { new double[]{1.0}, new double[]{1.0}, new double[]{1.0}};
    double[]
    blc = \{1.0\},
    buc = { inf },
   c = { 0.0, -1.0, 0.0 },
blx = { 0.0, 0.0, 0.0 },
    bux = { inf, inf, inf },
    xx = new double[numvar];
    try
      using (mosek.Env env = new mosek.Env())
        using (mosek.Task task = new mosek.Task(env))
          task.set_Stream (mosek.streamtype.log, new msgclass (""));
          /* Give MOSEK an estimate of the size of the input data.
               This is done to increase the speed of inputting data.
               However, it is optional. */
          /* Append 'numcon' empty constraints.
               The constraints will initially have no bounds. */
          task.appendcons(numcon);
          /* Append 'numvar' variables.
               The variables will initially be fixed at zero (x=0). */
          task.appendvars(numvar);
          for (int j = 0; j < numvar; ++j)
            /* Set the linear term c_j in the objective.*/
            task.putcj(j, c[j]);
            /* Set the bounds on variable j.
```

```
blx[j] <= x_j <= bux[j] */
 task.putvarbound(j, bkx[j], blx[j], bux[j]);
  /* Input column j of A */
 task.putacol(j,
                                       /* Variable (column) index.*/
               asub[j],
                                       /* Row index of non-zeros in column j.*/
                                       /* Non-zero Values of column j. */
               aval[j]);
/* Set the bounds on constraints.
       for i=1, ..., numcon: blc[i] \leftarrow constraint i \leftarrow buc[i] */
for (int i = 0; i < numcon; ++i)</pre>
 task.putconbound(i, bkc[i], blc[i], buc[i]);
 * The lower triangular part of the Q
 * matrix in the objective is specified.
 int[]
 qsubi = \{ 0, 1, 2, 2 \},
 qsubj = { 0, 1, 0, 2 };
 double[]
 qval = \{ 2.0, 0.2, -1.0, 2.0 \};
 /* Input the Q for the objective. */
 task.putqobj(qsubi, qsubj, qval);
}
* The lower triangular part of the Q^0
* matrix in the first constraint is specified.
 * This corresponds to adding the term
 * - x0^2 - x1^2 - 0.1 x2^2 + 0.2 x0 x2
 */
 int[]
 qsubi = \{ 0, 1, 2, 2 \},
 qsubj = \{ 0, 1, 2, 0 \};
 double[]
 qval = \{ -2.0, -2.0, -0.2, 0.2 \};
 /* put Q^0 in constraint with index 0. */
 task.putqconk (0,
                 qsubi,
                 qsubj,
                 qval);
}
task.putobjsense(mosek.objsense.minimize);
task.optimize();
// Print a summary containing information
// about the solution for debugging purposes
task.solutionsummary(mosek.streamtype.msg);
mosek.solsta solsta;
/* Get status information about the solution */
task.getsolsta(mosek.soltype.itr, out solsta);
task.getxx(mosek.soltype.itr, // Basic solution.
```

```
xx);
            switch (solsta)
              case mosek.solsta.optimal:
                Console.WriteLine ("Optimal primal solution\n");
                for (int j = 0; j < numvar; ++j)
                  Console.WriteLine ("x[{0}]:", xx[j]);
                break;
              case mosek.solsta.dual_infeas_cer:
              case mosek.solsta.prim_infeas_cer:
                Console.WriteLine("Primal or dual infeasibility.\n");
                break;
              case mosek.solsta.unknown:
                Console.WriteLine("Unknown solution status.\n");
              default:
                Console.WriteLine("Other solution status");
                break;
          }
        }
      }
      catch (mosek.Exception e)
      {
        Console.WriteLine (e);
    } /* Main */
 }
}
```

## 6.3 Conic Quadratic Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic quadratic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the two types of quadratic cones defined as:

• Quadratic cone:

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{0} \ge \sqrt{\sum_{j=1}^{n-1} x_{j}^{2}} \right\}.$$

• Rotated quadratic cone:

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_0 x_1 \ge \sum_{j=2}^{n-1} x_j^2, \quad x_0 \ge 0, \quad x_1 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.4, Sec. 6.5, Sec. 6.6. See *Task.appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

For example, the following constraint:

$$(x_4, x_0, x_2) \in \mathcal{Q}^3$$

describes a convex cone in  $\mathbb{R}^3$  given by the inequality:

$$x_4 \ge \sqrt{x_0^2 + x_2^2}.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

## 6.3.1 Example CQO1

Consider the following conic quadratic problem which involves some linear constraints, a quadratic cone and a rotated quadratic cone.

minimize 
$$x_4 + x_5 + x_6$$
  
subject to  $x_1 + x_2 + 2x_3 = 1$ ,  
 $x_1, x_2, x_3 \ge 0$ ,  
 $x_4 \ge \sqrt{x_1^2 + x_2^2}$ ,  
 $2x_5x_6 \ge x_3^2$  (6.6)

## Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

### Setting up the conic constraints

A cone is defined using the function Task.appendcone:

The first argument selects the type of quadratic cone, in this case either *conetype.quad* for a *quadratic cone* or *conetype.rquad* for a *rotated quadratic cone*. The second parameter is currently ignored and passing 0.0 will work.

The last argument is a list of indexes of the variables appearing in the cone.

Variants of this method are available to append multiple cones at a time.

## Source code

Listing 6.4: Source code solving problem (6.6).

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
    string prefix;
    public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
 }
 public class cqo1
   public static void Main ()
      const int numcon = 1;
      const int numvar = 6;
      // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
      double infinity = 0;
      mosek.boundkey[] bkc = { mosek.boundkey.fx };
      double[] blc = { 1.0 };
      double[] buc = { 1.0 };
      mosek.boundkey[] bkx = {mosek.boundkey.lo,
                              mosek.boundkey.lo,
                              mosek.boundkey.lo,
                              mosek.boundkey.fr,
                              mosek.boundkey.fr,
                              mosek.boundkey.fr
      double[] blx = { 0.0,}
                       0.0,
                       0.0,
                       -infinity,
                       -infinity,
                       -infinity
                     };
      double[] bux = { +infinity,
                       +infinity,
                       +infinity,
                       +infinity,
                       +infinity,
                       +infinity
                     };
      double[]c = {0.0,}
                       0.0,
                       0.0,
                       1.0,
                       1.0,
```

```
1.0
               };
double[][] aval = {
        new double[] {1.0},
        new double[] {1.0},
        new double[] {2.0}
};
int[][] asub = {
      new int[] {0},
      new int[] {0},
      new int[] {0}
};
int[] csub = new int[3];
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
  // Create a task object.
 using (mosek.Task task = new mosek.Task(env, 0, 0))
    // Directs the log task stream to the user specified
    // method msgclass.streamCB
    task.set_Stream (mosek.streamtype.log, new msgclass (""));
    /* Append 'numcon' empty constraints.
       The constraints will initially have no bounds. */
    task.appendcons(numcon);
    /* Append 'numvar' variables.
       The variables will initially be fixed at zero (x=0). */
    task.appendvars(numvar);
    for (int j = 0; j < numvar; ++j)
      /* Set the linear term c_j in the objective.*/
      task.putcj(j, c[j]);
      /* Set the bounds on variable j.
             blx[j] \leftarrow x_j \leftarrow bux[j] */
      task.putvarbound(j, bkx[j], blx[j], bux[j]);
    for (int j = 0; j < aval.Length; ++j)
      /* Input column j of A */
                               /* Variable (column) index.*/
      task.putacol(j,
                               /* Row index of non-zeros in column j.*/
                   asub[j],
                               /* Non-zero Values of column j. */
                   aval[j]);
    /* Set the bounds on constraints.
         for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
    for (int i = 0; i < numcon; ++i)</pre>
      task.putconbound(i, bkc[i], blc[i], buc[i]);
    csub[0] = 3;
    csub[1] = 0;
    csub[2] = 1;
    task.appendcone(mosek.conetype.quad,
                    0.0, /* For future use only, can be set to 0.0 */
                    csub):
```

```
csub[0] = 4;
          csub[1] = 5;
          csub[2] = 2;
          task.appendcone(mosek.conetype.rquad, 0.0, csub);
          task.putobjsense(mosek.objsense.minimize);
          task.optimize();
          // Print a summary containing information
          // about the solution for debugging purposes
          task.solutionsummary(mosek.streamtype.msg);
          mosek.solsta solsta;
          /* Get status information about the solution */
          task.getsolsta(mosek.soltype.itr, out solsta);
          double[] xx = new double[numvar];
          task.getxx(mosek.soltype.itr, // Basic solution.
          switch (solsta)
            case mosek.solsta.optimal:
              Console.WriteLine ("Optimal primal solution\n");
              for (int j = 0; j < numvar; ++j)
                Console.WriteLine ("x[{0}]: {1}", j, xx[j]);
              break;
            case mosek.solsta.dual_infeas_cer:
            case mosek.solsta.prim_infeas_cer:
              Console.WriteLine("Primal or dual infeasibility.\n");
              break;
            case mosek.solsta.unknown:
              Console.WriteLine("Unknown solution status.\n");
            default:
              Console.WriteLine("Other solution status");
       }
     }
   }
 }
}
```

## 6.4 Power Cone Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$
,

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the power cone. The primal power cone of dimension n with parameter  $0 < \alpha < 1$  is defined as:

$$\mathcal{P}_n^{\alpha, 1 - \alpha} = \left\{ x \in \mathbb{R}^n : x_0^{\alpha} x_1^{1 - \alpha} \ge \sqrt{\sum_{i=2}^{n-1} x_i^2}, \ x_0, x_1 \ge 0 \right\}.$$

In particular, the most important special case is the three-dimensional power cone family:

$$\mathcal{P}_3^{\alpha,1-\alpha} = \left\{ x \in \mathbb{R}^3 : x_0^{\alpha} x_1^{1-\alpha} \ge |x_2|, \ x_0, x_1 \ge 0 \right\}.$$

For example, the conic constraint  $(x, y, z) \in \mathcal{P}_3^{0.25, 0.75}$  is equivalent to  $x^{0.25}y^{0.75} \ge |z|$ , or simply  $xy^3 \ge z^4$  with x, y > 0.

**MOSEK** also supports the dual power cone:

$$\left(\mathcal{P}_n^{\alpha,1-\alpha}\right)^* = \left\{ x \in \mathbb{R}^n : \left(\frac{x_0}{\alpha}\right)^\alpha \left(\frac{x_1}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{i=2}^{n-1} x_i^2}, \ x_0, x_1 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.3, Sec. 6.5, Sec. 6.6. See *Task.appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

## 6.4.1 Example POW1

Consider the following optimization problem which involves powers of variables:

$$\begin{array}{lll} \text{maximize} & x^{0.2}y^{0.8} + z^{0.4} - x \\ \text{subject to} & x + y + \frac{1}{2}z & = & 2, \\ & x, y, z & \geq & 0. \end{array} \tag{6.7}$$

With  $(x, y, z) = (x_0, x_1, x_2)$  we convert it into conic form using auxiliary variables as bounds for the power expressions:

maximize 
$$x_3 + x_4 - x_0$$
  
subject to  $x_0 + x_1 + \frac{1}{2}x_2 = 2$ ,  
 $(x_0, x_1, x_3) \in \mathcal{P}_3^{0.2, 0.8}$ ,  
 $(x_2, x_5, x_4) \in \mathcal{P}_3^{0.4, 0.6}$ ,  
 $x_5 = 1$ . (6.8)

### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

### Setting up the conic constraints

A cone is defined using the function Task.appendcone:

```
task.appendcone(mosek.conetype.ppow, 0.2, new int[3] {0, 1, 3});
task.appendcone(mosek.conetype.ppow, 0.4, new int[3] {2, 5, 4});
```

The first argument selects the type of power cone, that is conetype.ppow. The second argument is the cone parameter  $\alpha$ . The remaining arguments list the variables which form the cone. Variants of this method are available to append multiple cones at a time.

The code below produces the answer of (6.7) which is

#### Source code

Listing 6.5: Source code solving problem (6.7).

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
   string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
     Console.Write ("{0}{1}", prefix, msg);
 public class ceo1
   public static void Main ()
     const int numcon = 1;
     const int numvar = 6;
     // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
     double infinity = 0;
     mosek.boundkey[] bkx = new mosek.boundkey[numvar];
     double[] blx
                   = new double[numvar];
     double[] bux
                          = new double[numvar];
     double[] val
                   = { 1.0, 1.0, -1.0 };
      int[]
              sub = { 3, 4, 0 };
     double[] aval = { 1.0, 1.0, 0.5 };
      int[]
              asub = \{ 0, 1, 2 \};
      double[] xx = new double[numvar];
      // Make mosek environment.
     using (mosek.Env env = new mosek.Env())
       // Create a task object.
       using (mosek.Task task = new mosek.Task(env, 0, 0))
          // Directs the log task stream to the user specified
          // method msgclass.streamCB
```

```
task.set_Stream (mosek.streamtype.log, new msgclass (""));
    /* Append 'numcon' empty constraints.
       The constraints will initially have no bounds. */
    task.appendcons(numcon);
    /* Append 'numvar' variables.
       The variables will initially be fixed at zero (x=0). */
    task.appendvars(numvar);
    /* Set up the linear part of the problem */
    task.putclist(sub, val);
    task.putarow(0, asub, aval);
    task.putconbound(0, mosek.boundkey.fx, 2.0, 2.0);
    for(i=0;i<5;i++) {
      bkx[i] = mosek.boundkey.fr;
      blx[i] = -infinity;
      bux[i] = infinity;
    bkx[5] = mosek.boundkey.fx;
    blx[5] = bux[5] = 1.0;
    task.putvarboundslice(0, numvar, bkx, blx, bux);
    /* Add a conic constraint */
    task.appendcone(mosek.conetype.ppow, 0.2, new int[3] {0, 1, 3});
    task.appendcone(mosek.conetype.ppow, 0.4, new int[3] {2, 5, 4});
    task.putobjsense(mosek.objsense.maximize);
    task.optimize();
    // Print a summary containing information
    // about the solution for debugging purposes
    task.solutionsummary(mosek.streamtype.msg);
    mosek.solsta solsta;
    /* Get status information about the solution */
    task.getsolsta(mosek.soltype.itr, out solsta);
    task.getxx(mosek.soltype.itr, // Basic solution.
               xx);
    switch (solsta)
      case mosek.solsta.optimal:
        Console.WriteLine ("Optimal primal solution\n");
        for (int j = 0; j < 3; ++j)
          Console.WriteLine ("x[{0}]: {1}", j, xx[j]);
        break:
      case mosek.solsta.dual_infeas_cer:
      case mosek.solsta.prim_infeas_cer:
        Console.WriteLine("Primal or dual infeasibility.\n");
        break:
      case mosek.solsta.unknown:
        Console.WriteLine("Unknown solution status.\n");
        Console.WriteLine("Other solution status");
        break;
    }
 }
}
```

```
}
}
```

# 6.5 Conic Exponential Optimization

Conic optimization is a generalization of linear optimization, allowing constraints of the type

$$x^t \in \mathcal{K}_t$$

where  $x^t$  is a subset of the problem variables and  $\mathcal{K}_t$  is a convex cone. Since the set  $\mathbb{R}^n$  of real numbers is also a convex cone, we can simply write a compound conic constraint  $x \in \mathcal{K}$  where  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_l$  is a product of smaller cones and x is the full problem variable.

MOSEK can solve conic optimization problems of the form

where the domain restriction,  $x \in \mathcal{K}$ , implies that all variables are partitioned into convex cones

$$x = (x^0, x^1, \dots, x^{p-1}), \text{ with } x^t \in \mathcal{K}_t \subseteq \mathbb{R}^{n_t}.$$

In this tutorial we describe how to use the primal exponential cone defined as:

$$K_{\text{exp}} = \left\{ x \in \mathbb{R}^3 : x_0 \ge x_1 \exp(x_2/x_1), \ x_0, x_1 \ge 0 \right\}.$$

MOSEK also supports the dual exponential cone:

$$K_{\text{exp}}^* = \left\{ s \in \mathbb{R}^3 : s_0 \ge -s_2 e^{-1} \exp(s_1/s_2), \ s_2 \le 0, s_0 \ge 0 \right\}.$$

For other types of cones supported by **MOSEK** see Sec. 6.3, Sec. 6.4, Sec. 6.6. See *Task.appendcone* for a list and definitions of available cone types. Different cone types can appear together in one optimization problem.

For example, the following constraint:

$$(x_4, x_0, x_2) \in K_{\text{exp}}$$

describes a convex cone in  $\mathbb{R}^3$  given by the inequalities:

$$x_4 \ge x_0 \exp(x_2/x_0), \ x_0, x_4 \ge 0.$$

Furthermore, each variable may belong to one cone at most. The constraint  $x_i - x_j = 0$  would however allow  $x_i$  and  $x_j$  to belong to different cones with same effect.

### 6.5.1 Example CEO1

Consider the following basic conic exponential problem which involves some linear constraints and an exponential inequality:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + x_1 + x_2 = 1$ ,  $x_0 \ge x_1 \exp(x_2/x_1)$ ,  $x_0, x_1 \ge 0$ . (6.9)

The conic form of (6.9) is:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + x_1 + x_2 = 1$ ,  $(x_0, x_1, x_2) \in K_{\text{exp}}$ ,  $x \in \mathbb{R}^3$ . (6.10)

#### Setting up the linear part

The linear parts (constraints, variables, objective) are set up using exactly the same methods as for linear problems, and we refer to Sec. 6.1 for all the details. The same applies to technical aspects such as defining an optimization task, retrieving the solution and so on.

#### Setting up the conic constraints

A cone is defined using the function Task.appendcone:

The first argument selects the type of exponential cone, that is *conetype.pexp*. The second parameter is currently ignored and passing 0.0 will work.

The last argument is a list of indexes of the variables appearing in the cone.

Variants of this method are available to append multiple cones at a time.

#### Source code

Listing 6.6: Source code solving problem (6.9).

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
 {
   string prefix;
   public msgclass (string prfx)
      prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
 }
 public class ceo1
   public static void Main ()
      const int numcon = 1;
      const int numvar = 3;
      // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
      double infinity = 0;
      mosek.boundkey bkc = mosek.boundkey.fx;
      double blc = 1.0 ;
      double buc = 1.0 ;
      mosek.boundkey[] bkx = {mosek.boundkey.fr,
                              mosek.boundkey.fr,
```

```
mosek.boundkey.fr
                       };
double[] blx = { -infinity,
                 -infinity,
                 -infinity
               };
double[] bux = { +infinity,
                 +infinity,
                 +infinity
               };
double[] c = { 1.0,
                1.0,
                0.0
               };
double[] a = { 1.0, 1.0, 1.0 };
int[] asub = { 0, 1, 2 };
int[] csub = new int[3];
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
  // Create a task object.
  using (mosek.Task task = new mosek.Task(env, 0, 0))
    // Directs the log task stream to the user specified
    // method msgclass.streamCB
    task.set_Stream (mosek.streamtype.log, new msgclass (""));
    /* Append 'numcon' empty constraints.
       The constraints will initially have no bounds. */
    task.appendcons(numcon);
    /* Append 'numvar' variables.
       The variables will initially be fixed at zero (x=0). */
    task.appendvars(numvar);
    /* Set up the linear part of the problem */
    task.putcslice(0, numvar, c);
    task.putarow(0, asub, a);
    task.putconbound(0, bkc, blc, buc);
    task.putvarboundslice(0, numvar, bkx, blx, bux);
    /* Define the exponential cone */
    csub[0] = 0;
    csub[1] = 1;
    csub[2] = 2;
    task.appendcone(mosek.conetype.pexp,
                    0.0, /* For future use only, can be set to 0.0 */
                    csub);
    task.putobjsense(mosek.objsense.minimize);
    task.optimize();
    // Print a summary containing information
    // about the solution for debugging purposes
    task.solutionsummary(mosek.streamtype.msg);
    mosek.solsta solsta;
    /* Get status information about the solution */
```

```
task.getsolsta(mosek.soltype.itr, out solsta);
          double[] xx = new double[numvar];
          task.getxx(mosek.soltype.itr, // Basic solution.
          switch (solsta)
            case mosek.solsta.optimal:
              Console.WriteLine ("Optimal primal solution\n");
              for (int j = 0; j < numvar; ++j)
                Console.WriteLine (x[\{0\}]: \{1\}, j, xx[j]);
            case mosek.solsta.dual_infeas_cer:
            case mosek.solsta.prim_infeas_cer:
              Console.WriteLine("Primal or dual infeasibility.\n");
              break;
            case mosek.solsta.unknown:
              Console.WriteLine("Unknown solution status.\n");
              break:
            default:
              Console.WriteLine("Other solution status");
        }
      }
   }
 }
}
```

# 6.6 Semidefinite Optimization

Semidefinite optimization is a generalization of conic optimization, allowing the use of matrix variables belonging to the convex cone of positive semidefinite matrices

$$\mathcal{S}^r_{\perp} = \left\{ X \in \mathcal{S}^r : z^T X z \ge 0, \quad \forall z \in \mathbb{R}^r \right\},$$

where  $S^r$  is the set of  $r \times r$  real-valued symmetric matrices.

MOSEK can solve semidefinite optimization problems of the form

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
 subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1,$  
$$l_j^x \leq x_j \leq \mathcal{K}, \overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, n-1,$$
 
$$j = 0, \dots, p-1$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $A, B \in \mathbb{R}^{m \times n}$  we have

$$\langle A, B \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} A_{ij} B_{ij}.$$

We demonstrate the setup of semidefinite variables and the matrices  $\overline{C}$ ,  $\overline{A}$  on the following examples:

- Sec. 6.6.1: A problem with one semidefinite variable and linear and conic constraints.
- Sec. 6.6.2: A problem with two semidefinite variables with a linear constraint and bound.

# 6.6.1 Example SDO1

We consider the simple optimization problem with semidefinite and conic quadratic constraints:

The problem description contains a 3-dimensional symmetric semidefinite variable which can be written explicitly as:

$$\overline{X} = \begin{bmatrix} \overline{X}_{00} & \overline{X}_{10} & \overline{X}_{20} \\ \overline{X}_{10} & \overline{X}_{11} & \overline{X}_{21} \\ \overline{X}_{20} & \overline{X}_{21} & \overline{X}_{22} \end{bmatrix} \in \mathcal{S}_{+}^{3},$$

and a conic quadratic variable  $(x_0, x_1, x_2) \in \mathcal{Q}^3$ . The objective is to minimize

$$2(\overline{X}_{00} + \overline{X}_{10} + \overline{X}_{11} + \overline{X}_{21} + \overline{X}_{22}) + x_0$$

subject to the two linear constraints

#### Setting up the linear and conic part

The linear and conic parts (constraints, variables, objective, cones) are set up using the methods described in the relevant tutorials; Sec. 6.1, Sec. 6.3, Sec. 6.5, Sec. 6.4. Here we only discuss the aspects directly involving semidefinite variables.

#### Appending semidefinite variables

First, we need to declare the number of semidefinite variables in the problem, similarly to the number of linear variables and constraints. This is done with the function Task.appendbarvars.

```
task.appendbarvars(dimbarvar);
```

#### Appending coefficient matrices

Coefficient matrices  $\overline{C}_j$  and  $\overline{A}_{ij}$  are constructed as weighted combinations of sparse symmetric matrices previously appended with the function Task.appendsparsesymmat.

The arguments specify the dimension of the symmetric matrix, followed by its description in the sparse triplet format. Only lower-triangular entries should be included. The function produces a unique index of the matrix just entered in the collection of all coefficient matrices defined by the user.

After one or more symmetric matrices have been created using Task.appendsparsesymmat, we can combine them to set up the objective matrix coefficient  $\overline{C}_j$  using Task.putbarcj, which forms a linear combination of one or more symmetric matrices. In this example we form the objective matrix directly, i.e. as a weighted combination of a single symmetric matrix.

```
task.putbarcj(0, idx, falpha);
```

Similarly, a constraint matrix coefficient  $\overline{A}_{ij}$  is set up by the function Task.putbaraij.

```
task.putbaraij(0, 0, idx, falpha);
```

#### Retrieving the solution

After the problem is solved, we read the solution using Task. getbarxj:

The function returns the half-vectorization of  $\overline{X}_j$  (the lower triangular part stacked as a column vector), where the semidefinite variable index j is passed as an argument.

#### Source code

Listing 6.7: Source code solving problem (6.11).

```
using System;
namespace mosek.example
 public class sdo1
   public static void Main(string[] args)
                       = 2; /* Number of constraints.
      int
             numcon
                     = 3; /* Number of conic quadratic variables */
      int[] dimbarvar = { 3 };  /* Dimensions of semidefinite cones */
      int[] lenbarvar = { 3 * (3 + 1) / 2 }; /* Number of scalar SD variables */
      mosek.boundkey[] bkc = { mosek.boundkey.fx, mosek.boundkey.fx };
                   blc
      double[]
                        = { 1.0, 0.5 };
      double[]
                   buc
                           = \{ 1.0, 0.5 \};
      int∏
                   barc_i = \{ 0, 1, 1, 2, 2 \},
                   barc_j = { 0, 0, 1, 1, 2 };
barc_v = { 2.0, 1.0, 2.0, 1.0, 2.0 };
      double[]
                           = { new int[] {0}, new int[] {1, 2}}; /* column subscripts of A */
      int[][]
                           = { new double[] {1.0}, new double[] {1.0, 1.0}};
      double[][]
                   aval
                   bara_i = { new int[] {0, 1, 2}, new int[] {0, 1, 2, 1, 2, __
      int[][]
\stackrel{\smile}{\rightarrow} 2 } },
                   bara_j = { new int[] {0, 1, 2}, new int[] {0, 0, 0, 1, 1, __
→2 } }:
                   bara_v = { new double[] {1.0, 1.0, 1.0}, new double[] {1.0, 1.0, 1.0, 1.0, 1.0
      double[][]
\hookrightarrow1.0, 1.0}};
                   conesub = \{ 0, 1, 2 \};
      int[]
      using (mosek.Env env = new mosek.Env())
        // Create a task object.
        using (mosek.Task task = new mosek.Task(env, 0, 0))
          // Directs the log task stream to the user specified
```

```
// method msgclass.streamCB
task.set_Stream (mosek.streamtype.log, new msgclass (""));
/* Append 'NUMCON' empty constraints.
   The constraints will initially have no bounds. */
task.appendcons(numcon);
/* Append 'NUMVAR' variables.
   The variables will initially be fixed at zero (x=0). */
task.appendvars(numvar);
/* Append 'NUMBARVAR' semidefinite variables. */
task.appendbarvars(dimbarvar);
/* Optionally add a constant term to the objective. */
task.putcfix(0.0);
/* Set the linear term c_j in the objective.*/
task.putcj(0, 1.0);
for (int j = 0; j < numvar; ++j)
 task.putvarbound(j, mosek.boundkey.fr, -0.0, 0.0);
/* Set the linear term barc_j in the objective.*/
  long[] idx = new long[1];
  double[] falpha = { 1.0 };
  idx[0] = task.appendsparsesymmat(dimbarvar[0],
                                    barc_i,
                                    barc_j,
                                    barc_v);
  task.putbarcj(0, idx, falpha);
/* Set the bounds on constraints.
  for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
for (int i = 0; i < numcon; ++i)
                                 /* Index of constraint.*/
 task.putconbound(i,
                   bkc[i],
                                /* Bound key.*/
                   blc[i],
                                /* Numerical value of lower bound.*/
                                /* Numerical value of upper bound.*/
                   buc[i]);
/* Input A row by row */
for (int i = 0; i < numcon; ++i)
  task.putarow(i,
               asub[i],
               aval[i]);
/* Append the conic quadratic cone */
task.appendcone(mosek.conetype.quad,
                0.0,
                conesub);
/* Add the first row of barA */
  long[] idx = new long[1];
  double[] falpha = {1.0};
  task.appendsparsesymmat(dimbarvar[0],
                          bara_i[0],
                          bara_j[0],
                          bara_v[0],
```

```
out idx[0]);
          task.putbaraij(0, 0, idx, falpha);
        {
          long[] idx = new long[1];
          double[] falpha = {1.0};
          /* Add the second row of barA */
          task.appendsparsesymmat(dimbarvar[0],
                                  bara_i[1],
                                  bara_j[1],
                                  bara_v[1],
                                   out idx[0]);
          task.putbaraij(1, 0, idx, falpha);
        /* Run optimizer */
        task.optimize();
        /* Print a summary containing information
           about the solution for debugging purposes*/
        task.solutionsummary (mosek.streamtype.msg);
        mosek.solsta solsta;
        task.getsolsta (mosek.soltype.itr, out solsta);
        switch (solsta)
        {
          case mosek.solsta.optimal:
            double[] xx = new double[numvar];
            double[] barx = new double[lenbarvar[0]];
            task.getxx(mosek.soltype.itr, xx);
            task.getbarxj(mosek.soltype.itr,
                                               /* Request the interior solution. */
                          barx);
            Console.WriteLine("Optimal primal solution");
            for (int i = 0; i < numvar; ++i)</pre>
              Console.WriteLine("x[{0}] : {1}", i, xx[i]);
            for (int i = 0; i < lenbarvar[0]; ++i)</pre>
              Console.WriteLine("barx[{0}]: {1}", i, barx[i]);
          case mosek.solsta.dual_infeas_cer:
          case mosek.solsta.prim_infeas_cer:
            Console.WriteLine("Primal or dual infeasibility certificate found.");
          case mosek.solsta.unknown:
            Console.WriteLine("The status of the solution could not be determined.");
            break;
          default:
            Console.WriteLine("Other solution status.");
            break;
      }
   }
 }
}
```

```
class msgclass : mosek.Stream
{
    string prefix;
    public msgclass (string prfx)
    {
        prefix = prfx;
    }
    public override void streamCB (string msg)
    {
            Console.Write ("{0}{1}", prefix, msg);
      }
}
```

### 6.6.2 Example SDO2

We now demonstrate how to define more than one semidefinite variable using the following problem with two matrix variables and two types of constraints:

minimize 
$$\langle C_1, \overline{X}_1 \rangle + \langle C_2, \overline{X}_2 \rangle$$
  
subject to  $\langle A_1, \overline{X}_1 \rangle + \langle A_2, \overline{X}_2 \rangle = b,$   

$$\frac{(\overline{X}_2)_{01}}{\overline{X}_1, \overline{X}_2} \leq k,$$

$$\overline{X}_1, \overline{X}_2 \geq 0.$$
(6.12)

In our example  $\dim(\overline{X}_1) = 3$ ,  $\dim(\overline{X}_2) = 4$ , b = 23, k = -3 and

$$C_{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}, A_{1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \end{bmatrix},$$

$$C_{2} = \begin{bmatrix} 1 & -3 & 0 & 0 \\ -3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_{2} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix},$$

are constant symmetric matrices.

Note that this problem does not contain any scalar variables, but they could be added in the same fashion as in Sec. 6.6.1.

Other than in Sec. 6.6.1 we don't append coefficient matrices separately but we directly input all nonzeros in each constraint and all nonzeros in the objective at once. Every term of the form  $(\overline{A}_{i,j})_{k,l}(\overline{X}_j)_{k,l}$  is determined by four indices (i,j,k,l) and a coefficient value  $v=(\overline{A}_{i,j})_{k,l}$ . Here i is the number of the constraint in which the term appears, j is the index of the semidefinite variable it involves and (k,l) is the position in that variable. This data is passed in the call to Task. putbarablocktriplet. Note that only the lower triangular part should be specified explicitly, that is one always has  $k \geq l$ . Semidefinite terms  $(\overline{C}_j)_{k,l}(\overline{X}_j)_{k,l}$  of the objective are specified in the same way in Task.putbarcblocktriplet but only include (j,k,l) and v.

For explanations of other data structures used in the example see Sec. 6.6.1.

The code representing the above problem is shown below.

Listing 6.8: Implementation of model (6.12).

```
using (mosek.Env env = new mosek.Env())
{
    // Create a task object.
    using (mosek.Task task = new mosek.Task(env, 0, 0))
    {
```

```
// Directs the log task stream to the user specified
    // method task_msg_obj.stream
    task.set_Stream (mosek.streamtype.log, new msgclass (""));
    /* Append numcon empty constraints.
       The constraints will initially have no bounds. */
    task.appendcons(numcon);
    /* Append numbarvar semidefinite variables. */
    task.appendbarvars(dimbarvar);
    /* Set objective (6 nonzeros).*/
    task.putbarcblocktriplet(6, Cj, Ck, Cl, Cv);
    /* Set the equality constraint (6 nonzeros).*/
    task.putbarablocktriplet(6, Ai, Aj, Ak, Al, Av);
    /* Set the inequality constraint (1 nonzero).*/
    task.putbarablocktriplet(1, A2i, A2j, A2k, A2l, A2v);
    /* Set constraint bounds */
    task.putconboundslice(0, 2, bkc, blc, buc);
    /* Run optimizer */
    task.optimize();
    task.solutionsummary(mosek.streamtype.msg);
    mosek.solsta solsta = task.getsolsta(mosek.soltype.itr);
    switch (solsta) {
      case mosek.solsta.optimal:
        /* Retrieve the soution for all symmetric variables */
        Console.WriteLine("Solution (lower triangular part vectorized):");
        for(int i = 0; i < numbarvar; i++) {</pre>
          int dim = dimbarvar[i] * (dimbarvar[i] + 1) / 2;
          double[] barx = new double[dim];
          task.getbarxj(mosek.soltype.itr, i, barx);
          Console.Write("X" + (i+1) + ": ");
          for (int j = 0; j < dim; ++j)
            Console.Write(barx[j] + " ");
          Console.WriteLine();
        }
        break;
      case mosek.solsta.dual_infeas_cer:
      case mosek.solsta.prim_infeas_cer:
        Console.WriteLine("Primal or dual infeasibility certificate found.");
        break:
      case mosek.solsta.unknown:
        Console.WriteLine("The status of the solution could not be determined.");
        Console.WriteLine("Other solution status.");
        break;
    }
 }
}
```

# 6.7 Integer Optimization

An optimization problem where one or more of the variables are constrained to integer values is called a (mixed) integer optimization problem. **MOSEK** supports integer variables in combination with linear, quadratic and quadratically constrained and conic problems (except semidefinite). See the previous tutorials for an introduction to how to model these types of problems.

# 6.7.1 Example MILO1

We use the example

```
maximize x_0 + 0.64x_1

subject to 50x_0 + 31x_1 \le 250,

3x_0 - 2x_1 \ge -4,

x_0, x_1 \ge 0 and integer (6.13)
```

to demonstrate how to set up and solve a problem with integer variables. It has the structure of a linear optimization problem (see Sec. 6.1) except for integrality constraints on the variables. Therefore, only the specification of the integer constraints requires something new compared to the linear optimization problem discussed previously.

First, the integrality constraints are imposed using the function Task.putvartype:

```
for (int j = 0; j < numvar; ++j)
  task.putvartype(j, mosek.variabletype.type_int);</pre>
```

Next, the example demonstrates how to set various useful parameters of the mixed-integer optimizer. See Sec. 13.4 for details.

```
/* Set max solution time */
task.putdouparam(mosek.dparam.mio_max_time, 60.0);
```

The complete source for the example is listed Listing 6.9. Please note that when Task. getsolutionslice is called, the integer solution is requested by using soltype.itg. No dual solution is defined for integer optimization problems.

Listing 6.9: Source code implementing problem (6.13).

```
using System;
namespace mosek.example
{
   public class MsgClass : mosek.Stream
   {
      public MsgClass ()
      {
            /* Construct the object */
      }

      public override void streamCB (string msg)
      {
            Console.Write ("{0}", msg);
      }
   }

   public class milo1
   {
      public static void Main ()
      {
            const int numcon = 2;
            const int numvar = 2;
            // Since the value infinity is never used, we define
```

```
// 'infinity' symbolic purposes only
double infinity = 0;
mosek.boundkey[] bkc = { mosek.boundkey.up,
                        mosek.boundkey.lo
                       };
double[] blc = { -infinity,
                -4.0
              };
double[] buc = { 250.0,
                infinity
              };
mosek.boundkey[] bkx = { mosek.boundkey.lo,
                        mosek.boundkey.lo
double[] blx = { 0.0,}
               0.0
              };
double[] bux = { infinity,
                infinity
              };
double[] c = \{1.0, 0.64\};
int[][] asub = { new int[] {0, 1}, new int[] {0, 1} };
double[][] aval = { new double[] {50.0, 3.0}, new double[] {31.0, -2.0} };
double[] xx = new double[numvar];
mosek.Env env = null;
mosek.Task task = null;
try
  // Make mosek environment.
  env = new mosek.Env ();
  // Create a task object linked with the environment env.
  task = new mosek.Task (env, numcon, numvar);
  // Directs the log task stream to the user specified
  // method task_msg_obj.streamCB
 MsgClass task_msg_obj = new MsgClass ();
 task.set_Stream (mosek.streamtype.log, task_msg_obj);
  /* Give MOSEK an estimate of the size of the input data.
       This is done to increase the speed of inputting data.
       However, it is optional. */
  /* Append 'numcon' empty constraints.
       The constraints will initially have no bounds. */
  task.appendcons(numcon);
  /* Append 'numvar' variables.
       The variables will initially be fixed at zero (x=0). */
  task.appendvars(numvar);
  /* Optionally add a constant term to the objective. */
  task.putcfix(0.0);
  for (int j = 0; j < numvar; ++j)
   /* Set the linear term c_j in the objective.*/
```

```
task.putcj(j, c[j]);
  /* Set the bounds on variable j.
           blx[j] \le x_j \le bux[j] */
  task.putvarbound(j, bkx[j], blx[j], bux[j]);
  /* Input column j of A */
                                      /* Variable (column) index.*/
 task.putacol(j,
                                      /* Row index of non-zeros in column j.*/
               asub[j],
                                       /* Non-zero Values of column j. */
               aval[j]);
/* Set the bounds on constraints.
       for i=1, ..., numcon : blc[i] \leftarrow constraint i \leftarrow buc[i] */
for (int i = 0; i < numcon; ++i)
 task.putconbound(i, bkc[i], blc[i], buc[i]);
/* Specify integer variables. */
for (int j = 0; j < numvar; ++j)
 task.putvartype(j, mosek.variabletype.type_int);
task.putobjsense(mosek.objsense.maximize);
/* Set max solution time */
task.putdouparam(mosek.dparam.mio_max_time, 60.0);
task.optimize();
// Print a summary containing information
// about the solution for debugging purposes
task.solutionsummary(mosek.streamtype.msg);
mosek.solsta solsta;
/* Get status information about the solution */
task.getsolsta(mosek.soltype.itg, out solsta);
task.getxx(mosek.soltype.itg, // Integer solution.
switch (solsta)
  case mosek.solsta.optimal:
    Console.WriteLine ("Optimal primal solution\n");
    for (int j = 0; j < numvar; ++j)
      Console.WriteLine ("x[{0}]:", xx[j]);
   break:
  case mosek.solsta.prim_feas:
    Console.WriteLine ("Feasible primal solution\n");
    for (int j = 0; j < numvar; ++j)
      Console.WriteLine ("x[{0}]:", xx[j]);
  case mosek.solsta.unknown:
   mosek.prosta prosta;
   task.getprosta(mosek.soltype.itg, out prosta);
    switch (prosta)
      case mosek.prosta.prim_infeas_or_unbounded:
        Console.WriteLine("Problem status Infeasible or unbounded");
        break;
      case mosek.prosta.prim_infeas:
        Console.WriteLine("Problem status Infeasible.");
        break;
      case mosek.prosta.unknown:
        Console.WriteLine("Problem status unknown.");
        break:
```

```
Console.WriteLine("Other problem status.");
                break;
            break;
          default:
            Console.WriteLine("Other solution status");
            break;
        }
      }
      catch (mosek.Exception e)
      {
        Console.WriteLine (e.Code);
        Console.WriteLine (e);
        throw;
      finally
        if (task != null) task.Dispose ();
        if (env != null) env.Dispose ();
   }
 }
}
```

# 6.7.2 Specifying an initial solution

It is a common strategy to provide a starting feasible point (if one is known in advance) to the mixed-integer solver. This can in many cases reduce solution time.

It is not necessary to specify the whole solution. MOSEK will attempt to use it to speed up the computation. MOSEK will first try to construct a feasible solution by fixing integer variables to the values provided by the user (rounding if necessary) and optimizing over the continuous variables. The outcome of this process can be inspected via information items <code>iinfitem.mio\_construct\_solution</code> and <code>dinfitem.mio\_construct\_solution\_obj</code>, and via the Construct solution objective entry in the log. We concentrate on a simple example below.

maximize 
$$7x_0 + 10x_1 + x_2 + 5x_3$$
  
subject to  $x_0 + x_1 + x_2 + x_3 \le 2.5$   
 $x_0, x_1, x_2 \in \mathbb{Z}$   
 $x_0, x_1, x_2, x_3 \ge 0$  (6.14)

Solution values can be set using Task.putsolution.

Listing 6.10: Implementation of problem (6.14) specifying an initial solution.

```
// Assign values to integer variables.
// We only set a slice of xx
double[] values = {1.0, 1.0, 0.0};
task.putxxslice(mosek.soltype.itg, 0, 3, values);
```

The log output from the optimizer will in this case indicate that the inputted values were used to construct an initial feasible solution:

```
Construct solution objective : 1.950000000000e+01
```

The same information can be obtained from the API:

Listing 6.11: Retrieving information about usage of initial solution

```
int constr = task.getintinf(mosek.iinfitem.mio_construct_solution);
double constrVal = task.getdouinf(mosek.dinfitem.mio_construct_solution_obj);
Console.WriteLine("Initial solution utilization: " + constr);
Console.WriteLine("Initial solution objective: " + constrVal);
```

# 6.7.3 Example MICO1

Integer variables can also be used arbitrarily in conic problems (except semidefinite). We refer to the previous tutorials for how to set up a conic optimization problem. Here we present sample code that sets up a simple optimization problem:

minimize 
$$x^2 + y^2$$
  
subject to  $x \ge e^y + 3.8$ ,  
 $x, y$  integer. (6.15)

The canonical conic formulation of (6.15) suitable for Optimizer API for .NET is

minimize 
$$x_0$$
  
subject to  $(x_0, x_1, x_2) \in \mathcal{Q}^3$   $(x_0 \ge \sqrt{x_1^2 + x_2^2})$   
 $(x_3, x_4, x_5) \in K_{\text{exp}}$   $(x_3 \ge x_4 \exp(x_5/x_4))$   
 $-x_1 + x_3 = -3.8$   $(6.16)$   
 $x_4 = 1$   
 $x_2 - x_5 = 0$   
 $x_1, x_2$  integer.

Listing 6.12: Implementation of problem (6.16).

```
public class mico1
   public static void Main ()
     mosek.Env env = new mosek.Env ();
     mosek.Task task = new mosek.Task(env, 0, 0);
     // Directs the log task stream to the user specified
     // method task_msg_obj.streamCB
     MsgClass task_msg_obj = new MsgClass ();
     task.set_Stream (mosek.streamtype.log, task_msg_obj);
     task.appendvars(6);
     task.appendcons(3);
     task.putvarboundsliceconst(0, 6, mosek.boundkey.fr, -0.0, 0.0);
     // Integrality constraints
     task.putvartypelist(new int[]{1,2},
                          new mosek.variabletype[]{mosek.variabletype.type_int, mosek.
→variabletype.type_int});
     // Set up the three auxiliary linear constraints
     task.putaijlist(new int[]{0,0,1,2,2},
                      new int[]{1,3,4,2,5},
                      new double[]{-1,1,1,1,-1});
     task.putconboundslice(0, 3,
                            new mosek.boundkey[]{mosek.boundkey.fx, mosek.boundkey.fx, mosek.
\hookrightarrow boundkey.fx},
                            new double[]{-3.8, 1, 0}, new double[]{-3.8, 1, 0});
     // Objective
```

```
task.putobjsense(mosek.objsense.minimize);
task.putcj(0, 1);

// Conic part of the problem
task.appendconeseq(mosek.conetype.quad, 0, 3, 0);
task.appendconeseq(mosek.conetype.pexp, 0, 3, 3);

// Optimize the task
task.optimize();
task.solutionsummary(mosek.streamtype.msg);

double[] xx = {0, 0};
task.getxxslice(mosek.soltype.itg, 1, 3, xx);
Console.WriteLine ("x = {0}, y = {1}", xx[0], xx[1]);
}
}
```

Error and solution status handling were omitted for readability.

# 6.8 Geometric Programming

Geometric programs (GP) are a particular class of optimization problems which can be expressed in special polynomial form as positive sums of generalized monomials. More precisely, a geometric problem in canonical form is

minimize 
$$f_0(x)$$
  
subject to  $f_i(x) \le 1$ ,  $i = 1, \dots, m$ ,  $x_j > 0$ ,  $j = 1, \dots, n$ , 
$$(6.17)$$

where each  $f_0, \ldots, f_m$  is a posynomial, that is a function of the form

$$f(x) = \sum_{k} c_{k} x_{1}^{\alpha_{k1}} x_{2}^{\alpha_{k2}} \cdots x_{n}^{\alpha_{kn}}$$

with arbitrary real  $\alpha_{ki}$  and  $c_k > 0$ . The standard way to formulate GPs in convex form is to introduce a variable substitution

$$x_i = \exp(y_i).$$

Under this substitution all constraints in a GP can be reduced to the form

$$\log(\sum_{k} \exp(a_k^T y + b_k)) \le 0 \tag{6.18}$$

involving a log-sum-exp bound. Moreover, constraints involving only a single monomial in x can be even more simply written as a linear inequality:

$$a_k^T y + b_k < 0$$

We refer to the **MOSEK** Modeling Cookbook and to [BKVH07] for more details on this reformulation. A geometric problem formulated in convex form can be entered into **MOSEK** with the help of exponential cones.

#### 6.8.1 Example GP1

The following problem comes from [BKVH07]. Consider maximizing the volume of a  $h \times w \times d$  box subject to upper bounds on the area of the floor and of the walls and bounds on the ratios h/w and d/w:

maximize 
$$hwd$$
  
subject to  $2(hw + hd) \leq A_{\text{wall}},$   
 $wd \leq A_{\text{floor}},$   
 $\alpha \leq h/w \leq \beta,$   
 $\gamma \leq d/w \leq \delta.$  (6.19)

The decision variables in the problem are h, w, d. We make a substitution

$$h = \exp(x), w = \exp(y), d = \exp(z)$$

after which (6.19) becomes

```
 \begin{array}{ll} \text{maximize} & x+y+z \\ \text{subject to} & \log(\exp(x+y+\log(2/A_{\text{wall}}))+\exp(x+z+\log(2/A_{\text{wall}}))) \leq 0, \\ & y+z \leq \log(A_{\text{floor}}), \\ & \log(\alpha) \leq x-y \leq \log(\beta), \\ & \log(\gamma) \leq z-y \leq \log(\delta). \end{array}
```

Next, we demonstrate how to implement a log-sum-exp constraint (6.18). It can be written as:

$$u_k \ge \exp(a_k^T y + b_k), \quad (\text{equiv. } (u_k, 1, a_k^T y + b_k) \in K_{\text{exp}}),$$
  
 $\sum_k u_k = 1.$  (6.21)

This presentation requires one extra variable  $u_k$  for each monomial appearing in the original posynomial constraint. Another fixed variable  $t_k = 1$  stands for the second entry in the exponential cone.

Listing 6.13: Implementation of log-sum-exp as in (6.21).

```
// Add a single log-sum-exp constraint sum(log(exp(z_i))) <= 0
// Assume numExp variable triples are ordered as (u0, t0, z0, u1, t1, z1...)
// starting from variable with index expStart
double[] val = new double[numExp];
int[]
          sub = new int[numExp];
// sum(u_i) = 1 as constraint number c, u_i unbounded
for(int i = 0; i < numExp; i++)</pre>
  { sub[i] = expStart + 3*i; val[i] = 1.0; }
task.putarow(c, sub, val);
task.putconbound(c, boundkey.fx, 1.0, 1.0);
task.putvarboundlistconst(sub, boundkey.fr, -inf, inf);
// z_i unbounded
for(int i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 2;</pre>
task.putvarboundlistconst(sub, boundkey.fr, -inf, inf);
// t_i = 1
for(int i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 1;</pre>
task.putvarboundlistconst(sub, boundkey.fx, 1, 1);
// Every triple is in an exponential cone
conetype[] ct = new conetype[numExp];
         len = new int[numExp];
for(int i = 0; i < numExp; i++)</pre>
  { ct[i] = conetype.pexp; val[i] = 0.0; len[i] = 3; }
task.appendconesseq(ct, val, len, expStart);
```

We can now use this function to assemble all constraints in the model. The linear part of the problem is entered as in Sec. 6.1.

Listing 6.14: Source code solving problem (6.20).

```
// Linear part of the problem
double[] cval = {1, 1, 1};
         asubi = \{0, 0, 1, 1, 2, 2\};
int[]
         asubj = \{1, 2, 0, 1, 2, 1\};
double[] aval = {1.0, 1.0, 1.0, -1.0, 1.0, -1.0};
boundkey[] bkc = {boundkey.up, boundkey.ra, boundkey.ra};
double[] blc = {-inf, Math.Log(alpha), Math.Log(gamma)};
double[] buc = {Math.Log(Af), Math.Log(beta), Math.Log(delta)};
// Linear part setting up slack variables
// for the linear expressions appearing inside exps
// x_5 - x - y = log(2/Awall)
// x_8 - x - z = log(2/Awall)
// The slack indexes are convenient for defining exponential cones, see later
         a2subi = {3, 3, 3, 4, 4, 4};
         a2subj = \{5, 0, 1, 8, 0, 2\};
double[] a2val = {1.0, -1.0, -1.0, 1.0, -1.0, -1.0};
boundkey[] b2kc = {boundkey.fx, boundkey.fx};
double[] b2luc = {Math.Log(2/Aw), Math.Log(2/Aw)};
using (Env env = new Env())
  using (Task task = new Task(env, 0, 0))
    // Directs the log task stream to the user specified
    // method task_msg_obj.stream
    task.set_Stream (mosek.streamtype.log, new msgclass (""));
    // Add variables and constraints
    task.appendvars(numvar + 3*numExp);
    task.appendcons(numLinCon + numExp + 1);
    // Objective is the sum of three first variables
    task.putobjsense(objsense.maximize);
    task.putcslice(0, numvar, cval);
    task.putvarboundsliceconst(0, numvar, boundkey.fr, -inf, inf);
    // Add the three linear constraints
    task.putaijlist(asubi, asubj, aval);
    task.putconboundslice(0, numvar, bkc, blc, buc);
    // Add linear constraints for the expressions appearing in exp(...)
    task.putaijlist(a2subi, a2subj, a2val);
    task.putconboundslice(numLinCon, numLinCon+numExp, b2kc, b2luc, b2luc);
    int c = numLinCon + numExp;
    int expStart = numvar;
    // Add a single log-sum-exp constraint sum(log(exp(z_i))) <= 0
    // Assume numExp variable triples are ordered as (u0,t0,z0,u1,t1,z1...)
    // starting from variable with index expStart
    double[] val = new double[numExp];
    int[]
             sub = new int[numExp];
    // sum(u_i) = 1 as constraint number c, u_i unbounded
    for(int i = 0; i < numExp; i++)</pre>
      { sub[i] = expStart + 3*i; val[i] = 1.0; }
    task.putarow(c, sub, val);
    task.putconbound(c, boundkey.fx, 1.0, 1.0);
    task.putvarboundlistconst(sub, boundkey.fr, -inf, inf);
```

```
// z_i unbounded
      for(int i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 2;</pre>
      task.putvarboundlistconst(sub, boundkey.fr, -inf, inf);
      // t_i = 1
      for(int i = 0; i < numExp; i++) sub[i] = expStart + 3*i + 1;</pre>
      task.putvarboundlistconst(sub, boundkey.fx, 1, 1);
      // Every triple is in an exponential cone
      conetype[] ct = new conetype[numExp];
      int[]
                len = new int[numExp];
      for(int i = 0; i < numExp; i++)</pre>
        { ct[i] = conetype.pexp; val[i] = 0.0; len[i] = 3; }
      task.appendconesseq(ct, val, len, expStart);
      // Solve and map to original h, w, d
      task.optimize();
      double[] xyz = new double[numvar];
      double[] hwd = new double[numvar];
      task.getxxslice(soltype.itr, 0, numvar, xyz);
      for(int i = 0; i < numvar; i++) hwd[i] = Math.Exp(xyz[i]);</pre>
      return hwd;
  }
}
```

Given sample data we obtain the solution h, w, d as follows:

Listing 6.15: Sample data for problem (6.19).

```
public static void Main(String[] args)
{
    double Aw = 200.0;
    double Af = 50.0;
    double alpha = 2.0;
    double beta = 10.0;
    double gamma = 2.0;
    double delta = 10.0;

    double delta = 10.0;

    console.WriteLine("h={0:f4} w={1:f4} d={2:f4}", hwd[0], hwd[1], hwd[2]);
}
```

# 6.9 Library of basic functions

This section contains a library of small models of basic functions frequently appearing in optimization models. It is essentially an implementation of the mathematical models from the **MOSEK** Modeling Cookbook using Optimizer API for .NET. These short code snippets can be seen as illustrative examples, can be copy-pasted to other code, and can even be directly called when assembling optimization models as we show in Sec. 6.9.6 (although this may be more suitable for prototyping; also note that additional variables and constraints will be introduced and there is no error checking).

# 6.9.1 Variable and constraint management

#### **Append variables**

Adds a number of new variables. Returns the index of the first variable in the sequence.

Listing 6.16: New variables.

```
public static int msk_newvar(Task task, int num) { // free
  int v = task.getnumvar();
  task.appendvars(num);
  for(int i=0; i<num; i++)</pre>
    task.putvarbound(v+i, boundkey.fr, -inf, inf);
 return v;
public static int msk_newvar_fx(Task task, int num, double val) { // fixed
  int v = task.getnumvar();
  task.appendvars(num);
  for(int i=0; i<num; i++)</pre>
    task.putvarbound(v+i, boundkey.fx, val, val);
  return v;
public static int msk_newvar_bin(Task task, int num) { // binary
  int v = task.getnumvar();
  task.appendvars(num);
  for(int i=0; i<num; i++) {</pre>
    task.putvarbound(v+i, boundkey.ra, 0.0, 1.0);
    task.putvartype(v+i, variabletype.type_int);
  }
  return v;
}
```

#### Variable duplication

Declares equality of two variables, or returns an index of a new duplicate of an existing variable.

Listing 6.17: Duplicate variables.

```
// x = y
public static void msk_equal(Task task, int x, int y) {
   int c = msk_newcon(task, 1);
   task.putaij(c, x, 1.0);
   task.putaij(c, y, -1.0);
   task.putconbound(c, boundkey.fx, 0.0, 0.0);
}
public static int msk_dup(Task task, int x) {
   int y = msk_newvar(task, 1);
   msk_equal(task, x, y);
   return y;
}
```

#### **Append constraints**

Adds a number of new constraints. Returns the index of the first constraint in the sequence.

Listing 6.18: New constraints.

```
public static int msk_newcon(Task task, int num) {
   int c = task.getnumcon();
   task.appendcons(num);
   return c;
}
```

# 6.9.2 Linear operations

#### Absolute value

 $t \ge |x|$ 

Listing 6.19: Absolute value.

```
// t >= /x/
public static void msk_abs(Task task, int t, int x) {
   int c = msk_newcon(task, 2);
   task.putaij(c, t, 1.0);
   task.putaij(c, x, 1.0);
   task.putconbound(c, boundkey.lo, 0.0, inf);
   task.putaij(c+1, t, 1.0);
   task.putaij(c+1, x, -1.0);
   task.putaij(c+1, x, -1.0);
   task.putconbound(c+1, boundkey.lo, 0.0, inf);
}
```

#### 1-norm

 $t \ge \sum_{i} |x_i|$ 

Listing 6.20: 1-norm.

```
// t >= sum( |x_i| ), x is a list of variables
public static void msk_norm1(Task task, int t, int[] x) {
   int n = x.Length;
   int u = msk_newvar(task, n);
   for(int i=0; i<n; i++) msk_abs(task, u+i, x[i]);
   int c = msk_newcon(task, 1);
   for(int i=0; i<n; i++) task.putaij(c, u+i, -1.0);
   task.putaij(c, t, 1.0);
   task.putconbound(c, boundkey.lo, 0.0, inf);
}</pre>
```

# 6.9.3 Quadratic and power operations

# **Square**

 $t > x^2$ 

Listing 6.21: Square.

```
// t >= x^2
public static void msk_sq(Task task, int t, int x) {
   task.appendcone(conetype.rquad, 0.0, new int[]{msk_newvar_fx(task, 1, 0.5), t, x});
}
```

# 2-norm

$$t \ge \sqrt{\sum_i x_i^2}$$

#### Listing 6.22: 2-norm.

```
// t >= sqrt(x_1^2 + ... + x_n^2) where x is a list of variables
public static void msk_norm2(Task task, int t, int[] x) {
   int[] submem = new int[x.Length+1];
   submem[0] = t;
   for(int i=0; i<x.Length; i++) submem[i+1] = x[i];
   task.appendcone(conetype.quad, 0.0, submem);
}</pre>
```

#### **Powers**

 $t \ge |x|^p, \, p > 1$ 

## Listing 6.23: Power.

```
// t \ge |x|^p (where p>1)
public static void msk_pow(Task task, int t, int x, double p) {
   task.appendcone(conetype.ppow, 1.0/p, new int[]{t, msk_newvar_fx(task, 1, 1.0), x});
}
t \ge 1/x^p, \ x > 0, \ p > 0
```

Listing 6.24: Power reciprocal.

```
// t >= 1/x^p, x>0 (where p>0)
public static void msk_pow_inv(Task task, int t, int x, double p) {
   task.appendcone(conetype.ppow, 1.0/(1.0+p), new int[]{t, x, msk_newvar_fx(task, 1, 1.0)}

→);
}
```

#### p-norm

 $t \ge (\sum_i |x_i|^p)^{1/p}, p > 1$ 

#### Listing 6.25: p-norm.

```
// t >= \/x\/_p (where p>1), x is a list of variables
public static void msk_pnorm(Task task, int t, int[] x, double p) {
   int n = x.Length;
   int r = msk_newvar(task, n);
   for(int i=0; i<n; i++)
      task.appendcone(conetype.ppow, 1.0-1.0/p, new int[]{t, r+i, x[i]});
   int c = msk_newcon(task, 1);
   for(int i=0; i<n; i++)
      task.putaij(c, r+i, -1.0);
   task.putaij(c, t, 1.0);
   task.putconbound(c, boundkey.fx, 0.0, 0.0);
}</pre>
```

# Geometric mean

```
t \le (x_1 \cdot \dots \cdot x_n)^{1/n}, \, x_i > 0
```

#### Listing 6.26: Geometric mean.

```
// |t| <= (x_1...x_n)^(1/n), x_i>=0, x is a list of variables of Length >= 1
public static void msk_geo_mean(Task task, int t, int n, int[] x) {
  if (n==1) msk_abs(task, x[0], t);
  else {
    int t2 = msk_newvar(task, 1);
}
```

```
task.appendcone(conetype.ppow, 1.0-1.0/n, new int[]{t2, x[n-1], t});
    msk_geo_mean(task, msk_dup(task, t2), n-1, x);
}
```

# 6.9.4 Exponentials and logarithms

#### log

 $t \le \log x, \ x > 0$ 

# Listing 6.27: Logarithm.

```
// t <= log(x), x>=0
public static void msk_log(Task task, int t, int x) {
  task.appendcone(conetype.pexp, 0.0, new int[]{x, msk_newvar_fx(task, 1, 1.0), t});
}
```

#### exp

 $t \ge e^x$ 

#### Listing 6.28: Exponential.

```
// t >= exp(x)
public static void msk_exp(Task task, int t, int x) {
   task.appendcone(conetype.pexp, 0.0, new int[]{t, msk_newvar_fx(task, 1, 1.0), x});
}
```

#### **Entropy**

 $t \ge x \log x, \ x > 0$ 

#### Listing 6.29: Entropy.

```
// t >= x * log(x), x>=0
public static void msk_ent(Task task, int t, int x) {
   int v = msk_newvar(task, 1);
   int c = msk_newcon(task, 1);
   task.putaij(c, v, 1.0);
   task.putaij(c, t, 1.0);
   task.putconbound(c, boundkey.fx, 0.0, 0.0);
   task.appendcone(conetype.pexp, 0.0, new int[]{msk_newvar_fx(task, 1, 1.0), x, v});
}
```

# Relative entropy

 $t \ge x \log x/y, \ x, y > 0$ 

# Listing 6.30: Relative entropy.

```
// t >= x * log(x/y), x,y>=0
public static void msk_relent(Task task, int t, int x, int y) {
   int v = msk_newvar(task, 1);
   int c = msk_newcon(task, 1);
   task.putaij(c, v, 1.0);
   task.putaij(c, t, 1.0);
   task.putconbound(c, boundkey.fx, 0.0, 0.0);
   task.appendcone(conetype.pexp, 0.0, new int[]{y, x, v});
}
```

#### Log-sum-exp

```
\log \sum_{i} e^{x_i} \le t
```

Listing 6.31: Log-sum-exp.

```
// log( sum_i(exp(x_i)) ) <= t, where x is a list of variables
public static void msk_logsumexp(Task task, int t, int[] x) {
   int n = x.Length;
   int u = msk_newvar(task, n);
   int z = msk_newvar(task, n);
   for(int i=0; i<n; i++) msk_exp(task, u+i, z+i);
   int c = msk_newcon(task, n);
   for(int i=0; i<n; i++) {
      task.putarow(c+i, new int[]{x[i], t, z+i}, new double[]{1.0, -1.0, -1.0});
      task.putconbound(c+i, boundkey.fx, 0.0, 0.0);
   }
   int s = msk_newcon(task, 1);
   for(int i=0; i<n; i++) task.putaij(s, u+i, 1.0);
   task.putconbound(s, boundkey.up, -inf, 1.0);
}</pre>
```

# 6.9.5 Integer Modeling

#### Semicontinuous variable

```
x \in \{0\} \cup [a, b], b > a > 0
```

Listing 6.32: Semicontinuous variable.

```
// x = 0 or a <= x <= b
public static void msk_semicontinuous(Task task, int x, double a, double b) {
   int u = msk_newvar_bin(task, 1);
   int c = msk_newcon(task, 2);
   task.putarow(c, new int[]{x, u}, new double[]{1.0, -a});
   task.putconbound(c, boundkey.lo, 0.0, inf);
   task.putarow(c+1, new int[]{x, u}, new double[]{1.0, -b});
   task.putconbound(c+1, boundkey.up, -inf, 0.0);
}</pre>
```

#### Indicator variable

 $x \neq 0 \implies t = 1$ . We assume x is a priori normalized so  $|x_i| \leq 1$ .

# Listing 6.33: Indicator variable.

```
// x!=0 implies t=1. Assumes that |x|<=1 in advance.
public static int msk_indicator(Task task, int x) {
   int t = msk_newvar_bin(task, 1);
   msk_abs(task, t, x);
   return t;
}</pre>
```

#### Logical OR

At least one of the conditions is true.

Listing 6.34: Logical OR.

```
// x OR y, where x, y are binary
public static void msk_logic_or(Task task, int x, int y) {
   int c = msk_newcon(task, 1);
```

```
task.putarow(c, new int[]{x, y}, new double[]{1.0, 1.0});
task.putconbound(c, boundkey.lo, 1.0, inf);
}
// x_1 OR ... OR x_n, where x is sequence of variables
public static void msk_logic_or_vect(Task task, int[] x) {
  int c = msk_newcon(task, 1);
  for(int i=0; i<x.Length; i++) task.putaij(c, x[i], 1.0);
  task.putconbound(c, boundkey.lo, 1.0, inf);
}</pre>
```

#### **Logical NAND**

At most one of the conditions is true (also known as SOS1).

#### Listing 6.35: Logical NAND.

```
// at most one of x_1,...,x_n, where x is a binary vector (SOS1 constraint)
public static void msk_logic_sos1(Task task, int[] x) {
   int c = msk_newcon(task, 1);
   for(int i=0; i<x.Length; i++) task.putaij(c, x[i], 1.0);
   task.putconbound(c, boundkey.up, -inf, 1.0);
}

// NOT(x AND y), where x, y are binary
public static void msk_logic_nand(Task task, int x, int y) {
   int c = msk_newcon(task, 1);
   task.putarow(c, new int[]{x, y}, new double[]{1.0, 1.0});
   task.putconbound(c, mosek.boundkey.up, -inf, 1.0);
}</pre>
```

#### Cardinality bound

At most k of the continuous variables are nonzero. We assume x is a priori normalized so  $|x_i| \leq 1$ .

Listing 6.36: Cardinality bound.

```
// At most k of entries in x are nonzero, assuming in advance that |x_i|<=1.
public static void msk_card(Task task, int[] x, int k) {
   int n = x.Length;
   int t = msk_newvar_bin(task, n);
   for(int i=0; i<n; i++) msk_abs(task, t+i, x[i]);
   int c = msk_newcon(task, 1);
   for(int i=0; i<n; i++) task.putaij(c, t+i, 1.0);
   task.putconbound(c,boundkey.up, -inf, k);
}</pre>
```

# 6.9.6 Model assembly example

We now demonstrate how to quickly build a simple optimization model for the problem

maximize 
$$-\sqrt{x^2 + y^2} + \log y - x^{1.5}$$
, subject to  $x \ge y + 3$ , (6.22)

or equivalently

$$\begin{array}{ll} \text{maximize} & -t_0+t_1-t_2,\\ \text{subject to} & x\geq y+3,\\ & t_0\geq \sqrt{x^2+y^2},\\ & t_1\leq \log y,\\ & t_2\geq x^{1.5}. \end{array}$$

Listing 6.37: Modeling (6.22).

```
public static void testExample() {
    Env env = new Env();
    Task task = new Task(env);
    int x = msk_newvar(task, 1);
    int y = msk_newvar(task, 1);
    int t = msk_newvar(task, 3);

int c = msk_newcon(task, 1);
    task.putarow(c, new int[]{x, y}, new double[]{1.0, -1.0});
    task.putconbound(c, boundkey.lo, 3.0, inf);

msk_norm2(task, t+0, new int[]{x,y});
    msk_log (task, t+1, msk_dup(task, y));
    msk_pow (task, t+2, msk_dup(task, x), 1.5);

task.putclist(new int[]{t, t+1, t+2}, new double[]{-1.0, 1.0, -1.0});
    task.putobjsense(objsense.maximize);
```

# 6.10 Problem Modification and Reoptimization

Often one might want to solve not just a single optimization problem, but a sequence of problems, each differing only slightly from the previous one. This section demonstrates how to modify and re-optimize an existing problem. The example we study is a simple production planning model.

Problem modifications regarding variables, cones, objective function and constraints can be grouped in categories:

- add/remove,
- coefficient modifications,
- bounds modifications.

Especially removing variables and constraints can be costly. Special care must be taken with respect to constraints and variable indexes that may be invalidated.

Depending on the type of modification, **MOSEK** may be able to optimize the modified problem more efficiently exploiting the information and internal state from the previous execution. After optimization, the solution is always stored internally, and is available before next optimization. The former optimal solution may be still feasible, but no longer optimal; or it may remain optimal if the modification of the objective function was small. This special case is discussed in Sec. 14.3.

In general, **MOSEK** exploits dual information and availability of an optimal basis from the previous execution. The simplex optimizer is well suited for exploiting an existing primal or dual feasible solution. Restarting capabilities for interior-point methods are still not as reliable and effective as those for the simplex algorithm. More information can be found in Chapter 10 of the book |Chv83|.

Parameter settings (see Sec. 7.4) can also be changed between optimizations.

#### 6.10.1 Example: Production Planning

A company manufactures three types of products. Suppose the stages of manufacturing can be split into three parts: Assembly, Polishing and Packing. In the table below we show the time required for each stage as well as the profit associated with each product.

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
0	2	3	2	1.50
1	4	2	3	2.50
2	3	3	2	3.00

With the current resources available, the company has 100,000 minutes of assembly time, 50,000 minutes of polishing time and 60,000 minutes of packing time available per year. We want to know how many items of each product the company should produce each year in order to maximize profit?

Denoting the number of items of each type by  $x_0, x_1$  and  $x_2$ , this problem can be formulated as a linear optimization problem:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $2x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.23)

and

$$x_0, x_1, x_2 \ge 0.$$

Code in Listing 6.38 loads and solves this problem.

Listing 6.38: Setting up and solving problem (6.23)

```
// Since the value infinity is never used, we define
// 'infinity' symbolic purposes only
double
infinity = 0;
int numcon = 3;
int numvar = 3;
                      = \{1.5,
double[] c
                         2.5,
                         3.0
                        };
mosek.boundkey[] bkc = {mosek.boundkey.up,
                         mosek.boundkey.up,
                         mosek.boundkey.up
                        };
double[] blc
                      = { -infinity,
                          -infinity,
                          -infinity
                        };
double[] buc
                         {100000,
                          50000,
                          60000
                         };
mosek.boundkey[] bkx = {mosek.boundkey.lo,
                         mosek.boundkey.lo,
                         mosek.boundkey.lo
                        };
double[] blx
                       = \{0.0,
                          0.0,
                          0.0
                         };
double[] bux
                       = { +infinity,
                           +infinity,
                           +infinity
                         };
int[][] asub = new int[numvar][];
asub[0] = new int[] {0, 1, 2};
asub[1] = new int[] {0, 1, 2};
asub[2] = new int[] {0, 1, 2};
double[][] aval = new double[numvar][];
aval[0] = new double[] { 2.0, 3.0, 2.0 };
aval[1] = new double[] { 4.0, 2.0, 3.0 };
aval[2] = new double[] { 3.0, 3.0, 2.0 };
```

```
double[] xx = new double[numvar];
mosek.Task task = null;
mosek.Env env = null;
try
  // Create mosek environment.
  env = new mosek.Env ();
  // Create a task object linked with the environment env.
  task = new mosek.Task (env, numcon, numvar);
  /* Append the constraints. */
  task.appendcons(numcon);
  /* Append the variables. */
  task.appendvars(numvar);
  /* Put C. */
  task.putcfix(0.0);
  for (int j = 0; j < numvar; ++j)
    task.putcj(j, c[j]);
  /* Put constraint bounds. */
  for (int i = 0; i < numcon; ++i)</pre>
    task.putconbound(i, bkc[i], blc[i], buc[i]);
  /* Put variable bounds. */
  for (int j = 0; j < numvar; ++j)
    task.putvarbound(j, bkx[j], blx[j], bux[j]);
  /* Put A. */
  if ( numcon > 0 )
  {
    for (int j = 0; j < numvar; ++j)
      task.putacol(j,
                   asub[j],
                   aval[j]);
  task.putobjsense(mosek.objsense.maximize);
  try
    task.optimize();
  catch (mosek.Warning w)
    Console.WriteLine("Mosek warning:");
    Console.WriteLine (w.Code);
    Console.WriteLine (w);
  task.getxx(mosek.soltype.bas, // Request the basic solution.
```

# 6.10.2 Changing the Linear Constraint Matrix

Suppose we want to change the time required for assembly of product 0 to 3 minutes. This corresponds to setting  $a_{0,0} = 3$ , which is done by calling the function Task.putaij as shown below.

```
task.putaij(0, 0, 3.0);
```

The problem now has the form:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2$$
  
subject to  $3x_0 + 4x_1 + 3x_2 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 \le 60000$ , (6.24)

and

$$x_0, x_1, x_2 \ge 0.$$

After this operation we can reoptimize the problem.

# 6.10.3 Appending Variables

We now want to add a new product with the following data:

Product no.	Assembly (minutes)	Polishing (minutes)	Packing (minutes)	Profit (\$)
3	4	0	1	1.00

This corresponds to creating a new variable  $x_3$ , appending a new column to the A matrix and setting a new term in the objective. We do this in Listing 6.39

Listing 6.39: How to add a new variable (column)

```
/*************** Add a new variable ************/
/* Get index of new variable. */
int varidx;
task.getnumvar(out varidx);
/* Append a new varaible x_3 to the problem */
task.appendvars(1);
numvar++;
/* Set bounds on new varaible */
task.putvarbound(varidx,
                mosek.boundkey.lo,
                +infinity);
/* Change objective */
task.putcj(varidx, 1.0);
/* Put new values in the A matrix */
int[] acolsub = new int[] {0, 2};
double[] acolval = new double[] {4.0, 1.0};
task.putacol(varidx, /* column index */
            acolsub,
            acolval);
```

After this operation the new problem is:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 100000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 50000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 60000$ , (6.25)

and

$$x_0, x_1, x_2, x_3 \ge 0.$$

# 6.10.4 Appending Constraints

Now suppose we want to add a new stage to the production process called *Quality control* for which 30000 minutes are available. The time requirement for this stage is shown below:

Product no.	Quality control (minutes)
0	1
1	2
2	1
3	1

This corresponds to adding the constraint

$$x_0 + 2x_1 + x_2 + x_3 \le 30000$$

to the problem. This is done as follows.

Listing 6.40: Adding a new constraint.

```
/************** Add a new constraint *************/
/* Get index of new constraint */
int conidx;
task.getnumcon(out conidx);
/* Append a new constraint */
task.appendcons(1);
numcon++;
/* Set bounds on new constraint */
task.putconbound(conidx,
             mosek.boundkey.up,
             -infinity,
             30000);
/* Put new values in the A matrix */
int[] arowsub = new int[] {0, 1, 2, 3};
double[] arowval = new double[] {1.0, 2.0, 1.0, 1.0};
task.putarow(conidx, /* row index */
            arowsub,
            arowval);
```

Again, we can continue with re-optimizing the modified problem.

# 6.10.5 Changing bounds

One typical reoptimization scenario is to change bounds. Suppose for instance that we must operate with limited time resources, and we must change the upper bounds in the problem as follows:

Operation	Time available (before)	Time available (new)
Assembly	100000	80000
Polishing	50000	40000
Packing	60000	50000
Quality control	30000	22000

That means we would like to solve the problem:

maximize 
$$1.5x_0 + 2.5x_1 + 3.0x_2 + 1.0x_3$$
  
subject to  $3x_0 + 4x_1 + 3x_2 + 4x_3 \le 80000$ ,  
 $3x_0 + 2x_1 + 3x_2 \le 40000$ ,  
 $2x_0 + 3x_1 + 2x_2 + 1x_3 \le 50000$ ,  
 $x_0 + 2x_1 + x_2 + x_3 \le 22000$ . (6.26)

In this case all we need to do is redefine the upper bound vector for the constraints, as shown in the next listing.

Listing 6.41: Change constraint bounds.

Again, we can continue with re-optimizing the modified problem.

#### 6.10.6 Advanced hot-start

If the optimizer used the data from the previous run to hot-start the optimizer for reoptimization, this will be indicated in the log:

```
Optimizer - hotstart : yes
```

When performing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for **MOSEK** to restart the simplex optimizer.

# 6.11 Parallel optimization

In this section we demonstrate the simplest possible multi-threading setup to run multiple **MOSEK** optimizations in parallel. All tasks must be created using the same **MOSEK** environment. One license token checked out by the environment will be shared by the tasks.

We first define a simple method that runs a number of optimization tasks in parallel, using the standard multi-threading setup available in the language.

Listing 6.42: Parallel optimization of a list of tasks.

```
for (var i = 0; i < n; ++i)
  int num = i;
  jobs[i] = System.Threading.Tasks.Task.Factory.StartNew( () => {
    try
    {
      trm[num] = tasks[num].optimize();
      res[num] = mosek.rescode.ok;
    }
    catch (mosek.Exception e)
      trm[num] = mosek.rescode.err_unknown;
      res[num] = e.Code;
 });
}
// Join all threads
foreach (var j in jobs)
  j.Wait();
```

It remains to call the method with a few different tasks. When optimizing many task in parallel it usually makes sense to solve each task using one thread to avoid additional multitasking overhead. When all tasks complete we access the solutions in the standard way.

Listing 6.43: Calling the parallel optimizer.

```
/** Example of how to use ``paropt``.
    Optimizes tasks whose names were read from command line.
public static void Main(string[] argv)
  int n = argv.Length;
  mosek.Task[] t
                      = new mosek.Task[n];
  mosek.rescode[] res = new mosek.rescode[n];
  mosek.rescode[] trm = new mosek.rescode[n];
  using (var env = new mosek.Env())
    for(int i = 0; i < n; i++)
      t[i] = new mosek.Task(env);
      t[i].readdata(argv[i]);
      // Each task will be single-threaded
      t[i].putintparam(mosek.iparam.intpnt_multi_thread, mosek.onoffkey.off);
    paropt(t, res, trm);
    for(int i = 0; i < n; i++)</pre>
      Console.WriteLine("Task {0} res {1} trm {2} obj_val {3} time {4}",
        res[i],
        trm[i],
        t[i].getdouinf(mosek.dinfitem.intpnt_primal_obj),
        t[i].getdouinf(mosek.dinfitem.optimizer_time));
  }
}
```

Another, slightly more advanced application of the parallel optimizer is presented in Sec. 11.3. For a more in-depth treatment see the following sections:

- $\bullet$   $\it Case\ studies$  for more advanced and complicated optimization examples.
- Problem Formulation and Solutions for formal mathematical formulations of problems MOSEK can solve, dual problems and infeasibility certificates.

# Chapter 7

# Solver Interaction Tutorials

In this section we cover the interaction with the solver.

# 7.1 Accessing the solution

This section contains important information about the status of the solver and the status of the solution, which must be checked in order to properly interpret the results of the optimization.

#### 7.1.1 Solver termination

The optimizer provides two status codes relevant for error handling:

- Response code of type rescode. It indicates if any unexpected error (such as an out of memory error, licensing error etc.) has occurred. The expected value for a successful optimization is rescode.ok.
- **Termination code**: It provides information about why the optimizer terminated, for instance if a predefined time limit has been reached. These are not errors, but ordinary events that can be expected (depending on parameter settings and the type of optimizer used).

If the optimization was successful then the method *Task.optimize* returns normally and its output is the termination code. If an error occurs then the method throws an exception, which contains the response code. See Sec. 7.2 for how to access it.

If a runtime error causes the program to crash during optimization, the first debugging step is to enable logging and check the log output. See Sec. 7.3.

If the optimization completes successfully, the next step is to check the solution status, as explained below.

# 7.1.2 Available solutions

 $\mathbf{MOSEK}$  uses three kinds of optimizers and provides three types of solutions:

- basic solution from the simplex optimizer,
- interior-point solution from the interior-point optimizer,
- integer solution from the mixed-integer optimizer.

Under standard parameters settings the following solutions will be available for various problem types:

Simplex Interior-point Mixed-integer optioptioptimizer mizer mizer Linear problem soltype.bas soltype.itr soltype.itr Nonlinear continuous prob-Problem with integer varisoltype.itg ables

Table 7.1: Types of solutions available from MOSEK

For linear problems the user can force a specific optimizer choice making only one of the two solutions available. For example, if the user disables basis identification, then only the interior point solution will be available for a linear problem. Numerical issues may cause one of the solutions to be unknown even if another one is feasible.

Not all components of a solution are always available. For example, there is no dual solution for integer problems and no dual conic variables from the simplex optimizer.

The user will always need to specify which solution should be accessed.

#### 7.1.3 Problem and solution status

Assuming that the optimization terminated without errors, the next important step is to check the problem and solution status. There is one for every type of solution, as explained above.

#### **Problem status**

Problem status (prosta) determines whether the problem is certified as feasible. Its values can roughly be divided into the following broad categories:

- **feasible** the problem is feasible. For continuous problems and when the solver is run with default parameters, the feasibility status should ideally be prosta.prim\_and\_dual\_feas.
- **primal/dual infeasible** the problem is infeasible or unbounded or a combination of those. The exact problem status will indicate the type of infeasibility.
- unknown the solver was unable to reach a conclusion, most likely due to numerical issues.

#### **Solution status**

Solution status (solsta) provides the information about what the solution values actually contain. The most important broad categories of values are:

- optimal (solsta.optimal) the solution values are feasible and optimal.
- **certificate** the solution is in fact a certificate of infeasibility (primal or dual, depending on the solution).
- unknown/undefined the solver could not solve the problem or this type of solution is not available for a given problem.

Problem and solution status for each solution can be retrieved with Task. getprosta and Task. getsolsta, respectively.

The solution status determines the action to be taken. For example, in some cases a suboptimal solution may still be valuable and deserve attention. It is the user's responsibility to check the status and quality of the solution.

#### Typical status reports

Here are the most typical optimization outcomes described in terms of the problem and solution statuses. Note that these do not cover all possible situations that can occur.

Table 7.2: Continuous problems (solution status for interior-point and basic solution)

Outcome	Problem status	Solution status
Optimal	prosta.	solsta.optimal
	$prim\_and\_dual\_feas$	
Primal infeasible	prosta.prim_infeas	solsta.
		prim_infeas_cer
Dual infeasible (unbounded)	prosta.dual_infeas	solsta.
		$dual\_infeas\_cer$
Uncertain (stall, numerical issues, etc.)	prosta.unknown	solsta.unknown

Table 7.3: Integer problems (solution status for integer solution, others undefined)

Outcome	Problem status	Solution status
Integer optimal	prosta.prim_feas	$solsta.integer\_optimal$
Infeasible	prosta.prim_infeas	solsta.unknown
Integer feasible point	prosta.prim_feas	solsta.prim_feas
No conclusion	prosta.unknown	solsta.unknown

# 7.1.4 Retrieving solution values

After the meaning and quality of the solution (or certificate) have been established, we can query for the actual numerical values. They can be accessed using:

- Task. getprimalobj, Task. getdualobj the primal and dual objective value.
- Task. getxx solution values for the variables.
- Task. getsolution a full solution with primal and dual values

and many more specialized methods, see the API reference.

## 7.1.5 Source code example

Below is a source code example with a simple framework for assessing and retrieving the solution to a conic optimization problem.

Listing 7.1: Sample framework for checking optimization result.

```
using System;
using mosek;
using System. Text;
namespace mosek.example
  // A log handler class
 class msgclass : mosek.Stream
   public msgclass () {}
   public override void streamCB (string msg) { Console.Write ("{0}", msg); }
 public class response
    public static void Main(string[] argv)
      StringBuilder symname = new StringBuilder();
      StringBuilder desc = new StringBuilder();
      string filename;
      if (argv.Length >= 1) filename = argv[0];
                            filename = "../data/cqo1.mps";
      // Define environment and task
      using (Env env = new Env())
        using (Task task = new Task(env, 0, 0))
        {
          try
            // (Optionally) set a log handler
            // task.set_Stream (streamtype.log, new msgclass ());
```

```
// (Optionally) uncomment this to get solution status unknown
            // task.putintparam(iparam.intpnt_max_iterations, 1);
            // In this example we read data from a file
            task.readdata(filename);
            // Perform optimization
            rescode trm = task.optimize();
            task.solutionsummary(streamtype.log);
            // Handle solution status. We expect Optimal
            solsta solsta = task.getsolsta(soltype.itr);
            switch ( solsta )
              case solsta.optimal:
                // Optimal solution. Print variable values
                Console.WriteLine("An optimal interior-point solution is located.");
                int numvar = task.getnumvar();
                double[] xx = new double[numvar];
                task.getxx(soltype.itr, xx);
                for(int i = 0; i < numvar; i++)</pre>
                  Console.WriteLine("x[" + i + "] = " + xx[i]);
                break;
              case solsta.dual_infeas_cer:
                Console.WriteLine("Dual infeasibility certificate found.");
                break;
              case solsta.prim_infeas_cer:
                Console.WriteLine("Primal infeasibility certificate found.");
                break;
              case solsta.unknown:
                /* The solutions status is unknown. The termination code
                   indicates why the optimizer terminated prematurely. */
                Console.WriteLine("The solution status is unknown.");
                Env.getcodedesc(trm, symname, desc);
                Console.WriteLine(" Termination code: {0} {1}", symname, desc);
                break;
              default:
                Console.WriteLine("An unexpected solution status " + solsta);
                break;
          }
          catch (mosek.Error e)
            Console.WriteLine("Unexpected optimization error ({0}) {1}", e.Code, e.Message);
          }
       }
     }
   }
 }
}
```

# 7.2 Errors and exceptions

#### **Exceptions**

Almost every function in Optimizer API for .NET can throw an exception informing that the requested operation was not performed correctly, and indicating the type of error that occurred. This is the case in situations such as for instance:

- referencing a nonexisting variable (for example with too large index),
- defining an invalid value for a parameter,
- accessing an undefined solution,
- repeating a variable name, etc.

It is therefore a good idea to catch exceptions of type *Error*. The one case where it is *extremely important* to do so is when *Task.optimize* is invoked. We will say more about this in Sec. 7.1.

The exception contains a *response code* (element of the enum *rescode*) and short diagnostic messages. They can be accessed as in the following example.

```
try {
   task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, -1.0e-7);
}
catch (mosek.Exception e) {
   mosek.rescode res = e.Code;
   Console.WriteLine("Response code {0}\nMessage {1}", res, e.Message);
}
```

It will produce as output:

```
Response code rescode.err_param_is_too_small

Message The parameter value -1e-07 is too small for parameter 'MSK_DPAR_INTPNT_CO_TOL_

REL_GAP'.
```

Another way to obtain a human-readable string corresponding to a response code is the method *Env.* getcodedesc. A full list of exceptions, as well as response codes, can be found in the *API reference*.

#### Optimizer errors and warnings

The optimizer may also produce warning messages. They indicate non-critical but important events, that will not prevent solver execution, but may be an indication that something in the optimization problem might be improved. Warning messages are normally printed to a log stream (see Sec. 7.3). A typical warning is, for example:

```
MOSEK warning 53: A numerically large upper bound value 6.6e+09 is specified for constraint -- 'C69200' (46020).
```

Warnings can also be suppressed by setting the  $iparam.max\_num\_warnings$  parameter to zero, if they are well-understood.

## Error and solution status handling example

Below is a source code example with a simple framework for handling major errors when assessing and retrieving the solution to a conic optimization problem.

Listing 7.2: Sample framework for checking optimization result.

```
using System;
using mosek;
using System.Text;
namespace mosek.example
```

```
// A log handler class
class msgclass : mosek.Stream
 public msgclass () {}
 public override void streamCB (string msg) { Console.Write ("{0}", msg); }
public class response
 public static void Main(string[] argv)
    StringBuilder symname = new StringBuilder();
    StringBuilder desc = new StringBuilder();
    string filename;
    if (argv.Length >= 1) filename = argv[0];
                          filename = "../data/cqo1.mps";
    // Define environment and task
    using (Env env = new Env())
      using (Task task = new Task(env, 0, 0))
        try
        {
          // (Optionally) set a log handler
          // task.set_Stream (streamtype.log, new msgclass ());
          // (Optionally) uncomment this to get solution status unknown
          // task.putintparam(iparam.intpnt_max_iterations, 1);
          // In this example we read data from a file
          task.readdata(filename);
          // Perform optimization
          rescode trm = task.optimize();
          task.solutionsummary(streamtype.log);
          // Handle solution status. We expect Optimal
          solsta solsta = task.getsolsta(soltype.itr);
          switch (solsta)
            case solsta.optimal:
              // Optimal solution. Print variable values
              Console.WriteLine("An optimal interior-point solution is located.");
              int numvar = task.getnumvar();
              double[] xx = new double[numvar];
              task.getxx(soltype.itr, xx);
              for(int i = 0; i < numvar; i++)</pre>
                Console.WriteLine(x[" + i + "] = " + xx[i]);
              break;
            case solsta.dual_infeas_cer:
              Console.WriteLine("Dual infeasibility certificate found.");
            case solsta.prim_infeas_cer:
              Console.WriteLine("Primal infeasibility certificate found.");
              break:
```

```
case solsta.unknown:
                /* The solutions status is unknown. The termination code
                   indicates why the optimizer terminated prematurely. */
                Console.WriteLine("The solution status is unknown.");
                Env.getcodedesc(trm, symname, desc);
                Console.WriteLine(" Termination code: {0} {1}", symname, desc);
                break;
              default:
                Console.WriteLine("An unexpected solution status " + solsta);
                break:
          }
          catch (mosek.Error e)
            Console.WriteLine("Unexpected optimization error ({0}) {1}", e.Code, e.Message);
      }
   }
 }
}
```

# 7.3 Input/Output

The logging and I/O features are provided mainly by the **MOSEK** task and to some extent by the **MOSEK** environment objects.

# 7.3.1 Stream logging

By default the solver runs silently and does not produce any output to the console or otherwise. However, the log output can be redirected to a user-defined output stream or stream callback function. The log output is analogous to the one produced by the command-line version of **MOSEK**.

The log messages are partitioned in three streams:

- messages, streamtype.msg
- warnings, streamtype.wrn
- errors, streamtype.err

These streams are aggregated in the streamtype.log stream. A stream handler can be defined for each stream separately.

The Stream class is used to receive text strings emitted to MOSEK's output streams. Extending Stream is the way to customize the solver output. For example:

```
class myStream : mosek.Stream
{
   public override void streamCB(string msg)
      {
        Console.WriteLine("{0}", msg);
    }
}
```

When a *Stream* object is attached to a *Task* stream, any text that is printed to that stream will be passed to the *Stream.streamCB* method.

```
task.set_Stream(mosek.streamtype.log, new myStream());
```

The stream can be detached by calling

```
task.set_Stream(mosek.streamtype.log, null);
```

After optimization is completed an additional short summary of the solution and optimization process can be printed to any stream using the method *Task.solutionsummary*.

### 7.3.2 Log verbosity

The logging verbosity can be controlled by setting the relevant parameters, as for instance

```
iparam.log,
iparam.log_intpnt,
iparam.log_mio,
iparam.log_cut_second_opt,
iparam.log_sim, and
iparam.log_sim_minor.
```

Each parameter controls the output level of a specific functionality or algorithm. The main switch is *iparam.log* which affect the whole output. The actual log level for a specific functionality is determined as the minimum between *iparam.log* and the relevant parameter. For instance, the log level for the output produce by the interior-point algorithm is tuned by the *iparam.log\_intpnt*; the actual log level is defined by the minimum between *iparam.log* and *iparam.log\_intpnt*.

Tuning the solver verbosity may require adjusting several parameters. It must be noticed that verbose logging is supposed to be of interest during debugging and tuning. When output is no more of interest, the user can easily disable it globally with <code>iparam.log</code>. Larger values of <code>iparam.log</code> do not necessarily result in increased output.

By default **MOSEK** will reduce the amount of log information after the first optimization on a given problem. To get full log output on subsequent re-optimizations set  $iparam.log\_cut\_second\_opt$  to zero.

# 7.3.3 Saving a problem to a file

An optimization problem can be dumped to a file using the method *Task.writedata*. The file format will be determined from the extension of the filename. Supported formats are listed in Sec. 16 together with a table of problem types supported by each.

For instance the problem can be written to an OPF file with

```
task.writedata("data.opf");
```

All formats can be compressed with gzip by appending the .gz extension, for example

```
task.writedata("data.task.gz");
```

Some remarks:

- Unnamed variables are given generic names. It is therefore recommended to use meaningful variable names if the problem file is meant to be human-readable.
- The task format is MOSEK's native file format which contains all the problem data as well as solver settings.

### 7.3.4 Reading a problem from a file

A problem saved in any of the supported file formats can be read directly into a task using <code>Task.readdata</code>. The task must be created in advance. Afterwards the problem can be optimized, modified, etc. If the file contained solutions, then are also imported, but the status of any solution will be set to <code>solsta.unknown</code> (solutions can also be read separately using <code>Task.readsolution</code>). If the file contains parameters, they will be set accordingly.

```
task = new mosek.Task(env, 0, 0);
try {
   task.readdata("file.task.gz");
   task.optimize();
} catch (mosek.Exception) {
   Console.WriteLine("Problem reading the file");
}
```

# 7.4 Setting solver parameters

**MOSEK** comes with a large number of parameters that allows the user to tune the behavior of the optimizer. The typical settings which can be changed with solver parameters include:

- choice of the optimizer for linear problems,
- choice of primal/dual solver,
- turning presolve on/off,
- turning heuristics in the mixed-integer optimizer on/off,
- level of multi-threading,
- feasibility tolerances,
- solver termination criteria,
- behaviour of the license manager,

and more. All parameters have default settings which will be suitable for most typical users. The API reference contains:

- Full list of parameters
- List of parameters grouped by topic

#### **Setting parameters**

Each parameter is identified by a unique name. There are three types of parameters depending on the values they take:

- Integer parameters. They take either either simple integer values or values from an enumeration provided for readability and compatibility of the code. Set with Task.putintparam.
- Double (floating point) parameters. Set with Task.putdouparam.
- String parameters. Set with Task.putstrparam.

There are also parameter setting functions which operate fully on symbolic strings containing generic command-line style names of parameters and their values. See the example below. The optimizer will try to convert the given argument to the exact expected type, and will error if that fails.

If an incorrect value is provided then the parameter is left unchanged.

For example, the following piece of code sets up parameters which choose and tune the interior point optimizer before solving a problem.

Listing 7.3: Parameter setting example.

```
// Set log level (integer parameter)
task.putintparam(mosek.iparam.log, 1);
// Select interior-point optimizer... (integer parameter)
task.putintparam(mosek.iparam.optimizer, mosek.optimizertype.intpnt);
// ... without basis identification (integer parameter)
```

```
task.putintparam(mosek.iparam.intpnt_basis, mosek.basindtype.never);
// Set relative gap tolerance (double parameter)
task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, 1.0e-7);

// The same using explicit string names
task.putparam ("MSK_DPAR_INTPNT_CO_TOL_REL_GAP", "1.0e-7");
task.putnadouparam("MSK_DPAR_INTPNT_CO_TOL_REL_GAP", 1.0e-7 );

// Incorrect value
try
{
    task.putdouparam(mosek.dparam.intpnt_co_tol_rel_gap, -1.0);
}
catch (mosek.Error)
{
    Console.WriteLine("Wrong parameter value");
}
```

### Reading parameter values

The functions Task.getintparam, Task.getdouparam, Task.getstrparam can be used to inspect the current value of a parameter, for example:

```
double param = task.getdouparam(mosek.dparam.intpnt_co_tol_rel_gap);
Console.WriteLine("Current value for parameter intpnt_co_tol_rel_gap = " + param);
```

# 7.5 Retrieving information items

After the optimization the user has access to the solution as well as to a report containing a large amount of additional *information items*. For example, one can obtain information about:

- timing: total optimization time, time spent in various optimizer subroutines, number of iterations, etc.
- solution quality: feasibility measures, solution norms, constraint and bound violations, etc.
- problem structure: counts of variables of different types, constraints, nonzeros, etc.
- integer optimizer: integrality gap, objective bound, number of cuts, etc.

and more. Information items are numerical values of integer, long integer or double type. The full list can be found in the API reference:

- Double
- Integer
- Long

Certain information items make sense, and are made available, also during the optimization process. They can be accessed from a callback function, see Sec. 7.6 for details.

#### Remark

For efficiency reasons, not all information items are automatically computed after optimization. To force all information items to be updated use the parameter  $iparam.auto\_update\_sol\_info$ .

#### Retrieving the values

Values of information items are fetched using one of the methods

- Task. getdouinf for a double information item,
- Task. getintinf for an integer information item,
- Task. getlintinf for a long integer information item.

Each information item is identified by a unique name. The example below reads two pieces of data from the solver: total optimization time and the number of interior-point iterations.

Listing 7.4: Information items example.

```
double tm = task.getdouinf(mosek.dinfitem.optimizer_time);
int iter = task.getintinf(mosek.iinfitem.intpnt_iter);

Console.WriteLine("Time: " + tm);
Console.WriteLine("Iterations: " + iter);
```

# 7.6 Progress and data callback

Callbacks are a very useful mechanism that allow the caller to track the progress of the **MOSEK** optimizer. A callback function provided by the user is regularly called during the optimization and can be used to

- obtain a customized log of the solver execution,
- collect information for debugging purposes or
- ask the solver to terminate.

Optimizer API for .NET has the following callback mechanisms:

- progress callback, which provides only the basic status of the solver.
- data callback, which provides the solver status and a complete set of information items that describe the progress of the optimizer in detail.
- integer solution callback, for reporting progress on a mixed-integer problem.

### Warning

The callbacks functions *must not* invoke any functions of the solver, environment or task. Otherwise the state of the solver and its outcome are undefined. The only exception is the possibility to retrieve an integer solution, see below.

#### Retrieving mixed-integer solutions

If the mixed-integer optimizer is used, the callback will take place, in particular, every time an improved integer solution is found. In that case it is possible to retrieve the current values of the best integer solution from within the callback function. It can be useful for implementing complex termination criteria for integer optimization. Note that there is a specialized callback class for retrieving only the integer solution anyway.

#### 7.6.1 Data callback

In the data callback **MOSEK** passes a callback code and values of all information items to a user-defined function. The callback function is called, in particular, at the beginning of each iteration of the interior-point optimizer. For the simplex optimizers  $iparam.log\_sim\_freq$  controls how frequently the call-back is called. Note that the callback is done quite frequently, which can lead to degraded performance. If the information items are not required, the simpler progress callback may be a better choice.

The callback is set by calling the method  $Task.set\_InfoCallback$ . The callback function must be implemented by extending the abstract class DataCallback and implementing the method DataCallback.callback.

Non-zero return value of the callback function indicates that the optimizer should be terminated.

### 7.6.2 Progress callback

In the progress callback **MOSEK** provides a single code indicating the current stage of the optimization process.

The callback is set by calling the method <code>Task.set\_Progress</code>. The callback function must be implemented by extending the abstract class <code>Progress</code> and implementing the method <code>Progress.progressCB</code>. Non-zero return value of the callback function indicates that the optimizer should be terminated.

# 7.6.3 Integer solution callback

In this type of callback the user-defined callback function receives an updated solution every time the mixed-integer optimizer improves the objective value. It can be useful for implementing complex termination criteria for integer optimization.

### **Syntax**

The callback is set by calling the method  $Task.set\_ItgSolutionCallback$ . The callback function must be implemented by extending the abstract class ItgSolutionCallback and implementing the method ItgSolutionCallback.callback.

## 7.6.4 Working example: Data callback

The following example defines a data callback function that prints out some of the information items. It interrupts the solver after a certain time limit.

Listing 7.5: An example of a data callback function.

```
class myCallback : mosek.DataCallback
{
 double maxtime;
 public myCallback(double maxtime_)
   maxtime = maxtime_;
 public override int callback( callbackcode caller,
                                 double[]
                                              douinf.
                                 int[]
                                              intinf.
                                 long[]
                                              lintinf )
  {
    double opttime = 0.0;
    int itrn;
   double pobj, dobj, stime;
   switch (caller)
    {
      case callbackcode.begin_intpnt:
```

```
Console.WriteLine("Starting interior-point optimizer");
 case callbackcode.intpnt:
           = intinf[(int) iinfitem.intpnt_iter
           = douinf[(int) dinfitem.intpnt_primal_obj];
           = douinf[(int) dinfitem.intpnt_dual_obj ];
   stime = douinf[(int) dinfitem.intpnt_time
   opttime = douinf[(int) dinfitem.optimizer_time
   Console.WriteLine("Iterations: {0,-3}",itrn);
   Console.WriteLine(" Elapsed: Time: {0,6:F2}({1:F2})",opttime,stime);
   Console.WriteLine(" Primal obj.: {0,-18:E6} Dual obj.: {1,018:E6}e",pobj,dobj);
   break;
 case callbackcode.end_intpnt:
   Console.WriteLine("Interior-point optimizer finished.");
 case callbackcode.begin_primal_simplex:
   Console.WriteLine("Primal simplex optimizer started.");
   break;
 case callbackcode.update_primal_simplex:
   itrn = intinf[(int) iinfitem.sim_primal_iter ];
           = douinf[(int) dinfitem.sim_obj
                                                    ];
   stime = douinf[(int) dinfitem.sim_time
                                                    ];
   opttime = douinf[(int) dinfitem.optimizer_time ];
   Console.WriteLine("Iterations: {0,-3}}", itrn);
   Console.WriteLine(" Elapsed time: {0,6:F2}({1:F2})",opttime,stime);
   Console.WriteLine(" Obj.: {0,-18:E6}", pobj );
 case callbackcode.end_primal_simplex:
   Console.WriteLine("Primal simplex optimizer finished.");
   break:
 case callbackcode.begin_dual_simplex:
   Console.WriteLine("Dual simplex optimizer started.");
 case callbackcode.update_dual_simplex:
           = intinf[(int) iinfitem.sim_dual_iter
           = douinf[(int) dinfitem.sim_obj
                                                    ];
          = douinf[(int) dinfitem.sim_time
                                                    ];
   opttime = douinf[(int) dinfitem.optimizer_time
   Console.WriteLine("Iterations: {0,-3}}", itrn);
   Console.WriteLine(" Elapsed time: {0,6:F2}({1:F2})",opttime,stime);
   Console.WriteLine(" Obj.: {0,-18:E6}", pobj );
 case callbackcode.end_dual_simplex:
   Console.WriteLine("Dual simplex optimizer finished.");
 case callbackcode.begin_bi:
   Console.WriteLine("Basis identification started.");
   break:
 case callbackcode.end_bi:
   Console.WriteLine("Basis identification finished.");
   break:
 default:
   break;
if (opttime >= maxtime)
 // mosek is spending too much time. Terminate it.
 return 1;
```

```
return 0;
}
}
```

Assuming that we have defined a task task and a time limit maxtime, the callback function is attached as follows:

Listing 7.6: Attaching the data callback function to the model.

```
task.set_InfoCallback(new myCallback(maxtime));
```

# 7.7 MOSEK OptServer

MOSEK provides an easy way to offload optimization problem to a remote server. This section demonstrates related functionalities from the client side, i.e. sending optimization tasks to the remote server and retrieving solutions.

Setting up and configuring the remote server is described in a separate manual for the OptServer.

# 7.7.1 Synchronous Remote Optimization

In synchronous mode the client sends an optimization problem to the server and blocks, waiting for the optimization to end. Once the result has been received, the program can continue. This is the simplest mode all it takes is to provide the host and port where the server is running and listening as additional arguments. The rest of the code remains untouched.

Note that it is impossible to recover the job in case of a broken connection.

#### Source code example

Listing 7.7: Using the OptServer in synchronous mode.

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
 {
   public override void streamCB (string msg)
      Console.Write ("{0}", msg);
  }
 public class simple
   public static void Main (string[] args)
      if (args.Length == 0)
        Console.WriteLine ("Missing arguments, syntax is:");
        Console.WriteLine (" opt_server_sync inputfile host port");
      }
      else
      {
        String inputfile = args[0];
                         = args[1];
        String host
                         = args[2];
        String port
        mosek.rescode trm;
```

```
using (mosek.Env env = new mosek.Env())
{
    using (mosek.Task task = new mosek.Task(env))
    {
        task.set_Stream (mosek.streamtype.log, new msgclass ());

        task.readdata (inputfile);

        task.optimizermt (host, port, out trm);

        task.solutionsummary (mosek.streamtype.log);
    }
}
}
}
```

### 7.7.2 Asynchronous Remote Optimization

In asynchronous mode the client sends a job to the remote server and the execution of the client code continues. In particular, it is the client's responsibility to periodically check the optimization status and, when ready, fetch the results. The client can also interrupt optimization. The most relevant methods are:

- Task.asyncoptimize: Offload the optimization task to a solver server.
- Task. asyncpoll: Request information about the status of the remote job.
- Task. asyncgetresult: Request the results from a completed remote job.
- Task. asyncstop: Terminate a remote job.

#### Source code example

In the example below the program enters in a polling loop that regularly checks whether the result of the optimization is available.

Listing 7.8: Using the OptServer in asynchronous mode.

```
using System;
using System.Threading;
namespace mosek.example
{
   class msgclass : mosek.Stream
   {
      public override void streamCB (string msg)
      {
            Console.Write ("{0}", msg);
      }
   }
   public class opt_server_async
   {
      public static void Main (string[] args)
      {
         if (args.Length == 0)
            {
                  Console.WriteLine ("Missing argument, syntax is:");
            }
}
```

```
Console.WriteLine (" opt_server inputfile host port numpolls");
}
else
{
  string inputfile = args[0];
  string host = args[1];
  string port
                 = args[2];
               = Convert.ToInt32(args[3]);
  int numpolls
  using (mosek.Env env = new mosek.Env())
   string token;
    using (mosek.Task task = new mosek.Task(env))
      task.readdata (inputfile);
      token = task.asyncoptimize (host, port);
    using (mosek.Task task = new mosek.Task(env))
     task.readdata (inputfile);
      task.set_Stream (mosek.streamtype.log, new msgclass ());
      Console.WriteLine("Starting polling loop...");
     int i = 0;
      while ( true )
        Thread.Sleep(500);
        Console.WriteLine("poll {0}...\n", i);
        mosek.rescode trm;
        mosek.rescode resp;
        int respavailable = task.asyncpoll( host,
                                            port,
                                            token,
                                            out resp,
                                            out trm);
        Console.WriteLine("polling done");
        if (respavailable != 0)
          Console.WriteLine("solution available!");
          task.asyncgetresult(host,
                              port,
                              token,
                              out resp,
                              out trm);
          task.solutionsummary (mosek.streamtype.log);
          break;
        if (i == numpolls)
```

# Chapter 8

# **Debugging Tutorials**

This collection of tutorials contains basic techniques for debugging optimization problems using tools available in MOSEK: optimizer log, solution summary, infeasibility report, command-line tools. It is intended as a first line of technical help for issues such as: Why do I get solution status *unknown* and how can I fix it? Why is my model infeasible while it shouldn't be? Should I change some parameters? Can the model solve faster? etc.

#### The major steps when debugging a model are always:

• Enable log output. See Sec. 7.3.1 for how to do it. In the simplest case: Create a log handler function:

```
class myStream : mosek.Stream
{
   public override void streamCB(string msg)
      {
        Console.WriteLine("{0}", msg);
    }
}
```

attach it to the log stream:

```
task.set_Stream(mosek.streamtype.log, new myStream());
```

and include solution summary after the call to optimize:

```
task.optimize();
task.solutionsummary(mosek.streamtype.log);
```

- Run the optimization and analyze the log output, see Sec. 8.1. In particular:
  - check if the problem setup (number of constraints/variables etc.) matches your expectation.
  - check solution summary and solution status.
- Dump the problem to disk if necessary to continue analysis. See Sec. 7.3.3.
  - use a human-readable text format, such as \*.opf if you want to check the problem structure by hand. Assign names to variables and constraints to make them easier to identify.

```
task.writedata("data.opf");
```

- use the MOSEK native format \*.task.gz when submitting a bug report or support question.

```
task.writedata("data.task.gz");
```

• Fix problem setup, improve the model, locate infeasibility or adjust parameters, depending on the diagnosis.

See the following sections for details.

# 8.1 Understanding optimizer log

The optimizer produces a log which splits roughly into four sections:

- 1. summary of the input data,
- 2. presolve and other pre-optimize problem setup stages,
- 3. actual optimizer iterations,
- 4. solution summary.

In this tutorial we show how to analyze the most important parts of the log when initially debugging a model: input data (1) and solution summary (4). For the iterations log (3) see Sec. 13.3.4 or Sec. 13.4.8.

### 8.1.1 Input data

If **MOSEK** behaves very far from expectations it may be due to errors in problem setup. The log file will begin with a summary of the structure of the problem, which looks for instance like:

```
Problem
Name :
Objective sense : max
Type : CONIC (conic optimization problem)
Constraints : 20413
Cones : 2508
Scalar variables : 20414
Matrix variables : 0
Integer variables : 0
```

This can be consulted to eliminate simple errors: wrong objective sense, wrong number of variables etc. Note that Fusion, and third-party modeling tools can introduce additional variables and constraints to the model. In the remaining **MOSEK** APIs the problem dimensions should match exactly what the user specified.

If this is not sufficient a bit more information can be obtained by dumping the problem to a file (see Sec. 8) and using the anapro option of any of the command line tools. It can also be done directly with the function Task. analyzeproblem. This will produce a longer summary similar to:

```
** Variables
scalar: 20414
                  integer: 0
                                     matrix: 0
low: 2082
                  up: 5014
                                                       free: 12892
                                                                          fixed: 426
                                     ranged: 0
** Constraints
all: 20413
low: 10028
                                     ranged: 0
                                                       free: 0
                                                                          fixed: 10385
                  up: 0
** Cones
QUAD: 1
                  dims: 2865: 1
RQUAD: 2507
                  dims: 3: 2507
** Problem data (numerics)
lcl
                  nnz: 10028
                                     min=2.09e-05
                                                       max=1.00e+00
|A|
                  nnz: 597023
                                     min=1.17e-10
                                                       max=1.00e+00
                                     min=-3.60e+09
blx
                  fin: 2508
                                                       max=2.75e+05
                  fin: 5440
                                     min=0.00e+00
                                                       max=2.94e+08
bux
blc
                  fin: 20413
                                     min=-7.61e+05
                                                       max=7.61e+05
                  fin: 10385
                                     min=-5.00e-01
                                                       max=0.00e+00
buc
```

Again, this can be used to detect simple errors, such as:

- Wrong type of cone was used or it has wrong dimension.
- The bounds for variables or constraints are incorrect or incomplete. Check if you defined bound keys for all variables. A variable for which no bound was defined is by default fixed at 0.

- The model is otherwise incomplete.
- Suspicious values of coefficients.
- For various data sizes the model does not scale as expected.

Finally saving the problem in a human-friendly text format such as LP or OPF (see Sec. 8) and analyzing it by hand can reveal if the model is correct.

#### Warnings and errors

At this stage the user can encounter warnings which should not be ignored, unless they are well-understood. They can also serve as hints as to numerical issues with the problem data. A typical warning of this kind is

```
MOSEK warning 53: A numerically large upper bound value 2.9e+08 is specified for variable → 'absh[107]' (2613).
```

Warnings do not stop the problem setup. If, on the other hand, an error occurs then the model will become invalid. The user should make sure to test for errors/exceptions from all API calls that set up the problem and validate the data. See Sec. 7.2 for more details.

#### 8.1.2 Solution summary

The last item in the log is the solution summary. In the Optimizer API it is only printed by invoking the function Task.solutionsummary.

#### Continuous problem

#### **Optimal solution**

A typical solution summary for a continuous (linear, conic, quadratic) problem looks like:

```
      Problem status : PRIMAL_AND_DUAL_FEASIBLE

      Solution status : OPTIMAL

      Primal. obj: 8.7560516107e+01 nrm: 1e+02 Viol. con: 3e-12 var: 0e+00 cones: 3e-11

      Dual. obj: 8.7560521345e+01 nrm: 1e+00 Viol. con: 5e-09 var: 9e-11 cones: 0e+00
```

It contains the following elements:

- Problem and solution status. For details see Sec. 7.1.3.
- A summary of the primal solution: objective value, infinity norm of the solution vector  $\mathbf{x}\mathbf{x}$ , maximal violations of constraints, variable bounds and cones. The violation of a linear constraint such as  $a^T x < b$  is  $\max(a^T x b, 0)$ . The violation of a conic constraint  $x \in \mathcal{K}$  is the distance  $\operatorname{dist}(x, \mathcal{K})$ .
- The same for the dual solution.

The features of the solution summary which characterize a very good and accurate solution and a well-posed model are:

- Status: The solution status is OPTIMAL.
- Duality gap: The primal and dual objective values are (almost) identical, which proves the solution is (almost) optimal.
- Norms: Ideally the norms of the solution and the objective values should not be too large. This of course depends on the input data, but a huge solution norm can be an indicator of issues with the scaling, conditioning and/or well-posedness of the model. It may also indicate that the problem is borderline between feasibility and infeasibility and sensitive to small perturbations in this respect.
- Violations: The violations are close to zero, which proves the solution is (almost) feasible. Observe that due to rounding errors it can be expected that the violations are proportional to the norm (nrm:) of the solution. It is rarely the case that violations are exactly zero.

#### Solution status UNKNOWN

A typical example with solution status UNKNOWN due to numerical problems will look like:

```
Problem status
                : UNKNOWN
Solution status : UNKNOWN
         obj: 1.3821656824e+01
                                   nrm: 1e+01
                                                   Viol.
                                                          con: 2e-03
                                                                         var: 0e+00
                                                                                        cones: 0e+00
Dual.
         obj: 3.0119004098e-01
                                                                                        cones: 0e+00
                                   nrm: 5e+07
                                                   Viol.
                                                          con: 4e-16
                                                                         var: 1e-01
```

#### Note that:

- The primal and dual objective are very different.
- The dual solution has very large norm.
- There are considerable violations so the solution is likely far from feasible.

Follow the hints in Sec. 8.2 to resolve the issue.

#### Solution status UNKNOWN with a potentially useful solution

Solution status UNKNOWN does not necessarily mean that the solution is completely useless. It only means that the solver was unable to make any more progress due to numerical difficulties, and it was not able to reach the accuracy required by the termination criteria (see Sec. 13.3.2). Consider for instance:

```
Problem status : UNKNOWN
Solution status : UNKNOWN
Primal.
         obj: 3.4531019648e+04
                                   nrm: 1e+05
                                                 Viol.
                                                         con: 7e-02
                                                                       var: 0e+00
                                                                                      cones: 0e+00
Dual.
         obj: 3.4529720645e+04
                                   nrm: 8e+03
                                                 Viol.
                                                         con: 1e-04
                                                                       var: 2e-04
                                                                                      cones: 0e+00
```

Such a solution may still be useful, and it is always up to the user to decide. It may be a good enough approximation of the optimal point. For example, the large constraint violation may be due to the fact that one constraint contained a huge coefficient.

#### Infeasibility certificate

A primal infeasibility certificate is stored in the dual variables:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 2.9238975853e+02 nrm: 6e+02 Viol. con: 0e+00 var: 1e-11 cones: 0e+00
```

It is a Farkas-type certificate as described in Sec. 12.2.2. In particular, for a good certificate:

- The dual objective is positive for a minimization problem, negative for a maximization problem. Ideally it is well bounded away from zero.
- The norm is not too big and the violations are small (as for a solution).

If the model was not expected to be infeasible, the likely cause is an error in the problem formulation. Use the hints in Sec. 8.1.1 and Sec. 8.3 to locate the issue.

Just like a solution, the infeasibility certificate can be of better or worse quality. The infeasibility certificate above is very solid. However, there can be less clear-cut cases, such as for example:

```
Problem status : PRIMAL_INFEASIBLE
Solution status : PRIMAL_INFEASIBLE_CER
Dual. obj: 1.6378689238e-06 nrm: 6e+05 Viol. con: 7e-03 var: 2e-04 cones: 0e+00
```

This infeasibility certificate is more dubious because the dual objective is positive, but barely so in comparison with the large violations. It also has rather large norm. This is more likely an indication that the problem is borderline between feasibility and infeasibility or simply ill-posed and sensitive to tiny variations in input data. See Sec. 8.3 and Sec. 8.2.

The same remarks apply to dual infeasibility (i.e. unboundedness) certificates. Here the primal objective should be negative a minimization problem and positive for a maximization problem.

### 8.1.3 Mixed-integer problem

#### Optimal integer solution

For a mixed-integer problem there is no dual solution and a typical optimal solution report will look as follows:

```
Problem status : PRIMAL_FEASIBLE
Solution status : INTEGER_OPTIMAL
Primal. obj: 6.0111122960e+06 nrm: 1e+03 Viol. con: 2e-13 var: 2e-14 itg: 5e-15
```

The interpretation of all elements is as for a continuous problem. The additional field itg denotes the maximum violation of an integer variable from being an exact integer.

#### Feasible integer solution

If the solver found an integer solution but did not prove optimality, for instance because of a time limit, the solution status will be PRIMAL\_FEASIBLE:

```
Problem status : PRIMAL_FEASIBLE
Solution status : PRIMAL_FEASIBLE
Primal. obj: 6.0114607792e+06 nrm: 1e+03 Viol. con: 2e-13 var: 2e-13 itg: 4e-15
```

In this case it is valuable to go back to the optimizer summary to see how good the best solution is:

```
31 35 1 0 6.0114607792e+06 6.0078960892e+06 0.06 4.1

Objective of best integer solution : 6.011460779193e+06

Best objective bound : 6.007896089225e+06
```

In this case the best integer solution found has objective value 6.011460779193e+06, the best proved lower bound is 6.007896089225e+06 and so the solution is guaranteed to be within 0.06% from optimum. The same data can be obtained as information items through an API. See also Sec. 13.4 for more details.

#### Infeasible problem

If the problem is declared infeasible the summary is simply

```
Problem status : PRIMAL_INFEASIBLE
Solution status : UNKNOWN
Primal. obj: 0.00000000000e+00 nrm: 0e+00 Viol. con: 0e+00 var: 0e+00 itg: 0e+00
```

If infeasibility was not expected, consult Sec. 8.3.

# 8.2 Addressing numerical issues

The suggestions in this section should help diagnose and solve issues with numerical instability, in particular UNKNOWN solution status or solutions with large violations. Since numerically stable models tend to solve faster, following these hints can also dramatically shorten solution times.

We always recommend that issues of this kind are addressed by reformulating or rescaling the model, since it is the modeler who has the best insight into the structure of the problem and can fix the cause of the issue.

# 8.2.1 Formulating problems

### **Scaling**

Make sure that all the data in the problem are of comparable orders of magnitude. This applies especially to the linear constraint matrix. Use Sec. 8.1.1 if necessary. For example a report such as

```
|A| nnz: 597023 min=1.17e-6 max=2.21e+5
```

means that the ratio of largest to smallest elements in A is 10<sup>11</sup>. In this case the user should rescale or reformulate the model to avoid such spread which makes it difficult for **MOSEK** to scale the problem internally. In many cases it may be possible to change the units, i.e. express the model in terms of rescaled variables (for instance work with millions of dollars instead of dollars, etc.).

Similarly, if the objective contains very different coefficients, say

maximize 
$$10^{10}x + y$$

then it is likely to lead to inaccuracies. The objective will be dominated by the contribution from x and y will become insignificant.

#### Removing huge bounds

Never use a very large number as replacement for  $\infty$ . Instead define the variable or constraint as unbounded from below/above. Similarly, avoid artificial huge bounds if you expect they will not become tight in the optimal solution.

#### Avoiding linear dependencies

As much as possible try to avoid linear dependencies and near-linear dependencies in the model. See Example 8.3.

#### **Avoiding ill-posedness**

Avoid continuous models which are ill-posed: the solution space is degenerate, for example consists of a single point (technically, the Slater condition is not satisfied). In general, this refers to problems which are borderline between feasible and infeasible. See Example 8.1.

#### Scaling the expected solution

Try to formulate the problem in such a way that the expected solution (both primal and dual) is not very large. Consult the solution summary Sec. 8.1.2 to check the objective values or solution norms.

### 8.2.2 Further suggestions

Here are other simple suggestions that can help locate the cause of the issues. They can also be used as hints for how to tune the optimizer if fixing the root causes of the issue is not possible.

- Remove the objective and solve the feasibility problem. This can reveal issues with the objective.
- Change the objective or change the objective sense from minimization to maximization (if applicable). If the two objective values are almost identical, this may indicate that the feasible set is very small, possibly degenerate.
- Perturb the data, for instance bounds, very slightly, and compare the results.
- For linear problems: solve the problem using a different optimizer by setting the parameter *iparam*. optimizer and compare the results.
- Force the optimizer to solve the primal/dual versions of the problem by setting the parameter <code>iparam.intpnt\_solve\_form</code> or <code>iparam.sim\_solve\_form</code>. MOSEK has a heuristic to decide whether to dualize, but for some problems the guess is wrong an explicit choice may give better results.
- Solve the problem without presolve or some of its parts by setting the parameter *iparam*. *presolve\_use*, see Sec. 13.1.
- Use different numbers of threads (*iparam.num\_threads*) and compare the results. Very different results indicate numerical issues resulting from round-off errors.

If the problem was dumped to a file, experimenting with various parameters is facilitated with the **MOSEK** Command Line Tool or **MOSEK** Python Console Sec. 8.4.

### 8.2.3 Typical pitfalls

Example 8.1 (Ill-posedness). A toy example of this situation is the feasibility problem

$$(x-1)^2 \le 1$$
,  $(x+1)^2 \le 1$ 

whose only solution is x=0 and moreover replacing any 1 on the right hand side by  $1-\varepsilon$  makes the problem infeasible and replacing it by  $1+\varepsilon$  yields a problem whose solution set is an interval (fully-dimensional). This is an example of ill-posedness.

**Example 8.2** (Huge solution). If the norm of the expected solution is very large it may lead to numerical issues or infeasibility. For example the problem

$$(10^{-4}, x, 10^3) \in \mathcal{Q}_r^3$$

may be declared infeasible because the expected solution must satisfy  $x \ge 5 \cdot 10^9$ .

**Example 8.3** (Near linear dependency). Consider the following problem:

If we add the equalities together we obtain:

$$0 = \varepsilon$$

which is infeasible for any  $\varepsilon \neq 0$ . Here infeasibility is caused by a linear dependency in the constraint matrix coupled with a precision error represented by the  $\varepsilon$ . Indeed if a problem contains linear dependencies then the problem is either infeasible or contains redundant constraints. In the above case any of the equality constraints can be removed while not changing the set of feasible solutions. To summarize linear dependencies in the constraints can give rise to infeasible problems and therefore it is better to avoid them.

Example 8.4 (Presolving very tight bounds). Next consider the problem

Now the **MOSEK** presolve will, for the sake of efficiency, fix variables (and constraints) that have tight bounds where tightness is controlled by the parameter  $dparam.presolve\_tol\_x$ . Since the bounds

$$-10^{-9} < x_1 < 10^{-9}$$

are tight, presolve will set  $x_1 = 0$ . It easy to see that this implies  $x_4 = 0$ , which leads to the incorrect conclusion that the problem is infeasible. However a tiny change of the value  $10^{-9}$  makes the problem

feasible. In general it is recommended to avoid ill-posed problems, but if that is not possible then one solution is to reduce parameters such as  $dparam.presolve\_tol\_x$  to say  $10^{-10}$ . This will at least make sure that presolve does not make the undesired reduction.

# 8.3 Debugging infeasibility

This section contains hints for debugging problems that are unexpectedly infeasible. It is always a good idea to remove the objective, i.e. only solve a feasibility problem when debugging such issues.

#### 8.3.1 Numerical issues

Infeasible problem status may be just an artifact of numerical issues appearing when the problem is badly-scaled, barely feasible or otherwise ill-conditioned so that it is unstable under small perturbations of the data or round-off errors. This may be visible in the solution summary if the infeasibility certificate has poor quality. See Sec. 8.1.2 for how to diagnose that and Sec. 8.2 for possible hints. Sec. 8.2.3 contains examples of situations which may lead to infeasibility for numerical reasons.

We refer to Sec. 8.2 for further information on dealing with those sort of issues. For the rest of this section we concentrate on the case when the solution summary leaves little doubt that the problem solved by the optimizer actually is infeasible.

## 8.3.2 Locating primal infeasibility

As an example of a primal infeasible problem consider minimizing the cost of transportation between a number of production plants and stores: Each plant produces a fixed number of goods, and each store has a fixed demand that must be met. Supply, demand and cost of transportation per unit are given in Fig. 8.1.

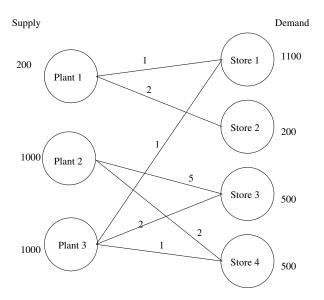


Fig. 8.1: Supply, demand and cost of transportation.

The problem represented in Fig. 8.1 is infeasible, since the total demand

$$2300 = 1100 + 200 + 500 + 500$$

exceeds the total supply

$$2200 = 200 + 1000 + 1000$$

If we denote the number of transported goods from plant i to store j by  $x_{ij}$ , the problem can be

formulated as the LP:

```
2x_{12}
minimize
                                                                                                                                      200.
subject to
                  s_0
                           x_{11}
                                          x_{12}
                                                                                                                                \leq
                                                                                                                                      1000,
                                                                          x_{24}
                                                                                                                                      1000,
                  s_2:
                                                                                         x_{31}
                  d_1:
                                                                                                                                      1100.
                          x_{11}
                                                                                         x_{31}
                                                                                                                                      200,
                  d_2:
                                          x_{12}
                  d_3:
                                                                                                                                      500,
                                                          x_{23}
                                                                                                         x_{33}
                                                                                                                                      500.
                                                                          x_{24}
                                                                                                                        x_{34}
                                                                                                                                \geq
                                                                                                                                      0.
                                                                                                                                              (8.1)
```

Solving problem (8.1) using **MOSEK** will result in an infeasibility status. The infeasibility certificate is contained in the dual variables an can be accessed from an API. The variables and constraints with nonzero solution values form an infeasible subproblem, which frequently is very small. See Sec. 12.1.2 or Sec. 12.2.2 for detailed specifications of infeasibility certificates.

A short infeasibility report can also be printed to the log stream. It can be turned on by setting the parameter <code>iparam.infeas\_report\_auto</code> to <code>onoffkey.on</code>. This causes MOSEK to print a report on variables and constraints which are involved in infeasibility in the above sense, i.e. have nonzero values in the certificate. The parameter <code>iparam.infeas\_report\_level</code> controls the amount of information presented in the infeasibility report. The default value is 1. For the above example the report is

```
MOSEK PRIMAL INFEASIBILITY REPORT.
Problem status: The problem is primal infeasible
The following constraints are involved in the primal infeasibility.
                    Lower bound
                                                                         Dual upper
Index
         Name
                                      Upper bound
                                                        Dual lower
         s0
                    NONE
                                      2.000000e+002
                                                        0.000000e+000
                                                                         1.000000e+000
0
2
         s2
                    NONE
                                      1.000000e+003
                                                        0.000000e+000
                                                                         1.000000e+000
3
                    1.100000e+003
                                      1.100000e+003
                                                        1.000000e+000
                                                                         0.000000e+000
         d1
4
                    2.000000e+002
                                      2.000000e+002
                                                        1.000000e+000
                                                                         0.000000e+000
         d2
The following bound constraints are involved in the infeasibility.
                    Lower bound
Index
         Name
                                      Upper bound
                                                        Dual lower
                                                                         Dual upper
8
         x33
                    0.000000e+000
                                     NONE
                                                        1.000000e+000
                                                                         0.000000e+000
         x34
                    0.000000e+000
                                     NONE
                                                        1.000000e+000
                                                                         0.000000e+000
10
```

The infeasibility report is divided into two sections corresponding to constraints and variables. It is a selection of those lines from the problem solution which are important in understanding primal infeasibility. In this case the constraints s0, s2, d1, d2 and variables x33, x34 are of importance because of nonzero dual values. The columns Dual lower and Dual upper contain the values of dual variables  $s_l^x$ ,  $s_u^x$ ,  $s_u^x$  and  $s_u^x$  in the primal infeasibility certificate (see Sec. 12.1.2).

In our example the certificate means that an appropriate linear combination of constraints s0, s1 with coefficient  $s_u^c = 1$ , constraints d1 and d2 with coefficient  $s_u^c - s_l^c = 0 - 1 = -1$  and lower bounds on x33 and x34 with coefficient  $-s_l^x = -1$  gives a contradiction. Indeed, the combination of the four involved constraints is  $x_{33} + x_{34} \le -100$  (as indicated in the introduction, the difference between supply and demand).

It is also possible to extract the infeasible subproblem with the command-line tool. For an infeasible problem called infeas.lp the command:

```
mosek -d MSK_IPAR_INFEAS_REPORT_AUTO MSK_ON infeas.lp -info rinfeas.lp
```

will produce the file rinfeas.bas.inf.lp which contains the infeasible subproblem. Because of its size it may be easier to work with than the original problem file.

Returning to the transportation example, we discover that removing the fifth constraint  $x_{12} = 200$  makes the problem feasible. Almost all undesired infeasibilities should be fixable at the modeling stage.

### 8.3.3 Locating dual infeasibility

A problem may also be  $dual\ infeasible$ . In this case the primal problem is usually unbounded, meaning that feasible solutions exists such that the objective tends towards infinity. For example, consider the problem

maximize 
$$200y_1 + 1000y_2 + 1000y_3 + 1100y_4 + 200y_5 + 500y_6 + 500y_7$$
 subject to 
$$y_1 + y_4 \le 1, \ y_1 + y_5 \le 2, \ y_2 + y_6 \le 5, \ y_2 + y_7 \le 2,$$
 
$$y_3 + y_4 \le 1, \ y_3 + y_6 \le 2, \ y_3 + y_7 \le 1$$
 
$$y_1, y_2, y_3 \le 0$$

which is dual to (8.1) (and therefore is dual infeasible). The dual infeasibility report may look as follows:

MOSEK DUAL INFEASIBILITY REPORT. Problem status: The problem is dual infeasible The following constraints are involved in the infeasibility. Index Name Activity Objective Lower bound Upper bound -1.000000e+00 NONE 2.000000e+00 5 x33 6 x34 -1.000000e+00 NONE 1.000000e+00 The following variables are involved in the infeasibility. Index Name Activity Objective Lower bound Upper bound 0 -1.000000e+00 2.000000e+02 NONE. 0.000000e+00 у1 уЗ 2 -1.000000e+00 1.000000e+03 NONE 0.000000e+00 3 1.000000e+00 1.100000e+03 NONE NONE y4 у5 1.000000e+00 2.000000e+02 NONE NONE Interior-point solution summary Problem status : DUAL\_INFEASIBLE Solution status : DUAL\_INFEASIBLE\_CER Primal. obj: 1.000000000e+02 nrm: 1e+00 Viol. con: 0e+00 var: 0e+00

In the report we see that the variables y1, y3, y4, y5 and two constraints contribute to infeasibility with non-zero values in the Activity column. Therefore

$$(y_1,\ldots,y_7)=(-1,0,-1,1,1,0,0)$$

is the dual infeasibility certificate as in Sec. 12.1.2. This just means, that along the ray

$$(0,0,0,0,0,0,0) + t(y_1,\ldots,y_7) = (-t,0,-t,t,t,0,0), t > 0,$$

which belongs to the feasible set, the objective value 100t can be arbitrarily large, i.e. the problem is unbounded.

In the example problem we could

- Add a lower bound on y3. This will directly invalidate the certificate of dual infeasibility.
- Increase the objective coefficient of y3. Changing the coefficients sufficiently will invalidate the inequality  $c^T y^* > 0$  and thus the certificate.

### 8.3.4 Suggestions

#### Primal infeasibility

When trying to understand what causes the unexpected primal infeasible status use the following hints:

• Remove the objective function. This does not change the infeasibility status but simplifies the problem, eliminating any possibility of issues related to the objective function.

- Remove cones, semidefinite variables and integer constraints. Solve only the linear part of the problem. Typical simple modeling errors will lead to infeasibility already at this stage.
- Consider whether your problem has some obvious necessary conditions for feasibility and examine if these are satisfied, e.g. total supply should be greater than or equal to total demand.
- Verify that coefficients and bounds are reasonably sized in your problem.
- See if there are any obvious contradictions, for instance a variable is bounded both in the variables and constraints section, and the bounds are contradictory.
- Consider replacing suspicious equality constraints by inequalities. For instance, instead of  $x_{12} = 200$  see what happens for  $x_{12} \ge 200$  or  $x_{12} \le 200$ .
- Relax bounds of the suspicious constraints or variables.
- For integer problems, remove integrality constraints on some/all variables and see if the problem solves.
- Remember that variables without explicitly initialized bounds are fixed at zero.
- Form an **elastic model**: allow to violate constraints at a cost. Introduce slack variables and add them to the objective as penalty. For instance, suppose we have a constraint

minimize 
$$c^T x$$
, subject to  $a^T x \le b$ .

which might be causing infeasibility. Then create a new variable y and form the problem which contains:

Solving this problem will reveal by how much the constraint needs to be relaxed in order to become feasible. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

- If you think you have a feasible solution or its part, fix all or some of the variables to those values. Presolve will propagate them through the model and potentially reveal more localized sources of infeasibility.
- Dump the problem in OPF or LP format and verify that the problem that was passed to the optimizer corresponds to the problem expressed in the high-level modeling language, if any such was used.

#### **Dual infeasibility**

When trying to understand what causes the unexpected dual infeasible status use the following hints:

- Verify that the objective coefficients are reasonably sized.
- Check if no bounds and constraints are missing, for example if all variables that should be nonnegative have been declared as such etc.
- Strengthen bounds of the suspicious constraints or variables.
- Remember that constraints without explicitly initialized bounds are free (no bound).
- Form an series of models with decreasing bounds on the objective, that is, instead of objective

minimize 
$$c^T x$$

solve the problem with an additional constraint such as

$$c^T x = -10^5$$

and inspect the solution to figure out the mechanism behind arbitrarily decreasing objective values. This is equivalent to inspecting the infeasibility certificate but may be more intuitive.

• Dump the problem in OPF or LP format and verify that the problem that was passed to the optimizer corresponds to the problem expressed in the high-level modeling language, if any such was used.

Please note that modifying the problem to invalidate the reported certificate does *not* imply that the problem becomes feasible — the reason for infeasibility may simply *move*, resulting a problem that is still infeasible, but for a different reason. More often, the reported certificate can be used to give a hint about errors or inconsistencies in the model that produced the problem.

# 8.4 Python Console

The **MOSEK** Python Console is an alternative to the **MOSEK** Command Line Tool. It can be used for interactive loading, solving and debugging optimization problems stored in files, for example **MOSEK** task files. It facilitates debugging techniques described in Sec. 8.

### 8.4.1 Usage

The tool requires Python 2 or 3. The **MOSEK** interface for Python must be installed following the installation instructions for Python API or Python Fusion API. In the basic case it should be sufficient to execute the script

```
python setup.py install --user
```

in the directory containing the MOSEK Python module.

The Python Console is contained in the file mosekconsole.py in the folder with MOSEK binaries. It can be copied to an arbitrary location. The file is also available for download here (mosekconsole.py).

To run the console in interactive mode use

```
python mosekconsole.py
```

To run the console in batch mode provide a semicolon-separated list of commands as the second argument of the script, for example:

```
python mosekconsole.py "read data.task.gz; solve form=dual; writesol data"
```

The script is written using the **MOSEK** Python API and can be extended by the user if more specific functionality is required. We refer to the documentation of the Python API.

#### 8.4.2 Examples

To read a problem from data.task.gz, solve it, and write solutions to data.sol, data.bas or data.itg:

```
read data.task.gz; solve; writesol data
```

To convert between file formats:

```
read data.task.gz; write data.mps
```

To set a parameter before solving:

```
read data.task.gz; param INTPNT_CO_TOL_DFEAS 1e-9; solve"
```

To list parameter values related to the mixed-integer optimizer in the task file:

```
read data.task.gz; param MIO
```

To print a summary of problem structure:

```
read data.task.gz; anapro
```

To solve a problem forcing the dual and switching off presolve:

```
read data.task.gz; solve form=dual presolve=no
```

To write an infeasible subproblem to a file for debugging purposes:

read data.task.gz; solve; infsub; write inf.opf

# 8.4.3 Full list of commands

Below is a brief description of all the available commands. Detailed information about a specific command cmd and its options can be obtained with

help cmd

Table 8.1: List of commands of the MOSEK Python Console.

Command	Description		
help [command]	Print list of commands or info about a specific command		
log filename	Save the session to a file		
intro	Print MOSEK splashscreen		
testlic	Test the license system		
read filename	Load problem from file		
reread	Reload last problem file		
solve	Solve current problem		
[options]			
write filename	Write current problem to file		
param [name	Set a parameter or get parameter values		
[value]]			
paramdef	Set all parameters to default values		
info [name]	Get an information item		
anapro	Analyze problem data		
hist	Plot a histogram of problem data		
histsol	Plot a histogram of the solutions		
spy	Plot the sparsity pattern of the A matrix		
truncate	Truncate small coefficients down to 0		
epsilon			
anasol	Analyze solutions		
removeitg	Remove integrality constraints		
infsub	Replace current problem with its infeasible subproblem		
writesol	Write solution(s) to file(s) with given basename		
basename			
delsol	Remove all solutions from the task		
exit	Leave		

# Chapter 9

# **Advanced Numerical Tutorials**

# 9.1 Solving Linear Systems Involving the Basis Matrix

A linear optimization problem always has an optimal solution which is also a basic solution. In an optimal basic solution there are exactly m basic variables where m is the number of rows in the constraint matrix A. Define

$$B \in \mathbb{R}^{m \times m}$$

as a matrix consisting of the columns of A corresponding to the basic variables. The basis matrix B is always non-singular, i.e.

$$det(B) \neq 0$$

or, equivalently,  $B^{-1}$  exists. This implies that the linear systems

$$B\bar{x} = w \tag{9.1}$$

and

$$B^T \bar{x} = w \tag{9.2}$$

each have a unique solution for all w.

**MOSEK** provides functions for solving the linear systems (9.1) and (9.2) for an arbitrary w. In the next sections we will show how to use **MOSEK** to

- identify the solution basis,
- solve arbitrary linear systems.

#### 9.1.1 Basis identification

To use the solutions to (9.1) and (9.2) it is important to know how the basis matrix B is constructed. Internally **MOSEK** employs the linear optimization problem

maximize 
$$c^{T}x$$
subject to 
$$Ax - x^{c} = 0,$$

$$l^{x} \leq x \leq u^{x},$$

$$l^{c} \leq x^{c} \leq u^{c}.$$

$$(9.3)$$

where

$$x^c \in \mathbb{R}^m$$
 and  $x \in \mathbb{R}^n$ .

The basis matrix is constructed of m columns taken from

$$\begin{bmatrix} A & -I \end{bmatrix}$$
.

If variable  $x_j$  is a basis variable, then the j-th column of A, denoted  $a_{:,j}$ , will appear in B. Similarly, if  $x_i^c$  is a basis variable, then the i-th column of -I will appear in the basis. The ordering of the basis variables and therefore the ordering of the columns of B is arbitrary. The ordering of the basis variables may be retrieved by calling the function Task.initbasissolve. This function initializes data structures for later use and returns the indexes of the basic variables in the array basis. The interpretation of the basis is as follows. If we have

$${\tt basis}[i] < {\tt numcon}$$

then the *i*-th basis variable is

$$x_{\mathtt{basis}[i]}^c$$
.

Moreover, the *i*-th column in B will be the *i*-th column of -I. On the other hand if

$$basis[i] \ge numcon$$
,

then the i-th basis variable is the variable

$$x_{\mathtt{basis}[i]-\mathtt{numcon}}$$

and the i-th column of B is the column

$$A_{:,(\mathtt{basis}[i]-\mathtt{numcon})}$$
.

For instance if basis[0] = 4 and numcon = 5, then since basis[0] < numcon, the first basis variable is  $x_4^c$ . Therefore, the first column of B is the fourth column of -I. Similarly, if basis[1] = 7, then the second variable in the basis is  $x_{basis[1]-numcon} = x_2$ . Hence, the second column of B is identical to  $a_{:,2}$ .

#### An example

Consider the linear optimization problem:

minimize 
$$x_0 + x_1$$
  
subject to  $x_0 + 2x_1 \le 2$ ,  
 $x_0 + x_1 \le 6$ ,  
 $x_0, x_1 \ge 0$ .  $(9.4)$ 

Suppose a call to Task. initbasissolve returns an array basis so that

```
basis[0] = 1,
basis[1] = 2.
```

Then the basis variables are  $x_1^c$  and  $x_0$  and the corresponding basis matrix B is

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right].$$

Please note the ordering of the columns in B .

Listing 9.1: A program showing how to identify the basis.

```
using System;
namespace mosek.example
{
  class msgclass : mosek.Stream
  {
    string prefix;
    public msgclass (string prfx)
    {
```

```
prefix = prfx;
 public override void streamCB (string msg)
    Console.Write ("{0}{1}", prefix, msg);
  }
}
public class solvebasis
 public static void Main ()
    const int numcon = 2;
    const int numvar = 2;
    // Since the value infinity is never used, we define
    // 'infinity' symbolic purposes only
    double
    infinity = 0;
    double[] c = \{1.0, 1.0\};
    int[] ptrb = {0, 2};
            ptre = \{2, 3\};
    int[]
    int[]
             asub = \{0, 1,
                    0, 1
                    };
    double[] aval = {1.0, 1.0,
                    2.0, 1.0
                    };
    mosek.boundkey[] bkc = {mosek.boundkey.up,
                            mosek.boundkey.up
    double[] blc = { -infinity,
                      -infinity
    double[] buc = \{2.0,
                    6.0
                    };
    mosek.boundkey[] bkx = {mosek.boundkey.lo,
                             mosek.boundkey.lo
    double[] blx = \{0.0,
                     0.0
                     };
    double[] bux = { +infinity,
                      +infinity
                     };
    double[] w1 = {2.0, 6.0};
    double[] w2 = {1.0, 0.0};
    try
    {
     using (mosek.Env env = new mosek.Env())
       using (mosek.Task task = new mosek.Task(env))
          task.set_Stream (mosek.streamtype.log, new msgclass ("[task]"));
          task.inputdata(numcon, numvar,
```

```
С,
               0.0,
               ptrb,
               ptre,
               asub,
               aval,
               bkc,
               blc,
               buc,
               bkx,
               blx,
               bux);
task.putobjsense(mosek.objsense.maximize);
try
  task.optimize();
catch (mosek.Warning w)
  Console.WriteLine("Mosek warning:");
  Console.WriteLine (w.Code);
  Console.WriteLine (w);
int[] basis = new int[numcon];
task.initbasissolve(basis);
//List basis variables corresponding to columns of B
int[] varsub = {0, 1};
for (int i = 0; i < numcon; i++) {</pre>
  if (basis[varsub[i]] < numcon)</pre>
    Console.WriteLine ("Basis variable no {0} is xc{1}",
                        basis[i]);
    Console.WriteLine ("Basis variable no {0} is x{1}",
                        basis[i] - numcon);
}
// solve Bx = w1
// varsub contains index of non-zeros in b.
// On return b contains the solution x and
// varsub the index of the non-zeros in x.
int nz = 2;
task.solvewithbasis(0, ref nz, varsub, w1);
Console.WriteLine ("nz = {0}", nz);
Console.WriteLine ("Solution to Bx = w1:\n");
for (int i = 0; i < nz; i++) {
  if (basis[varsub[i]] < numcon)</pre>
    Console.WriteLine ("xc \{0\} = \{1\}",
                        basis[varsub[i]],
                        w1[varsub[i]] );
    Console.WriteLine (x{0} = {1},
                        basis[varsub[i]] - numcon,
                        w1[varsub[i]]);
}
```

```
// Solve B^Tx = w2
            nz = 1;
            varsub[0] = 0; // Only w2[0] is nonzero.
            task.solvewithbasis(1, ref nz, varsub, w2);
            Console.WriteLine ("\nSolution to B^Ty = w2:\n");
            for (int i = 0; i < nz; i++)
              Console.WriteLine ("y \{0\} = \{1\}",
                                  varsub[i],
                                  w2[varsub[i]]);
        }
      }
      catch (mosek.Exception e)
        Console.WriteLine (e.Code);
        Console.WriteLine (e);
        throw;
    }
 }
}
```

In the example above the linear system is solved using the optimal basis for (9.4) and the original right-hand side of the problem. Thus the solution to the linear system is the optimal solution to the problem. When running the example program the following output is produced.

```
basis[0] = 1
Basis variable no 0 is xc1.
basis[1] = 2
Basis variable no 1 is x0.

Solution to Bx = b:

x0 = 2.000000e+00
xc1 = -4.000000e+00

Solution to B^Tx = c:

x1 = -1.000000e+00
x0 = 1.000000e+00
```

Please note that the ordering of the basis variables is

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right]$$

and thus the basis is given by:

$$B = \left[ \begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array} \right]$$

It can be verified that

$$\left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} -4 \\ 2 \end{array}\right]$$

is a solution to

$$\left[\begin{array}{cc} 0 & 1 \\ -1 & 1 \end{array}\right] \left[\begin{array}{c} x_1^c \\ x_0 \end{array}\right] = \left[\begin{array}{c} 2 \\ 6 \end{array}\right].$$

### 9.1.2 Solving arbitrary linear systems

MOSEK can be used to solve an arbitrary (rectangular) linear system

$$Ax = b$$

using the Task.solvewithbasis function without optimizing the problem as in the previous example. This is done by setting up an A matrix in the task, setting all variables to basic and calling the Task.solvewithbasis function with the b vector as input. The solution is returned by the function.

#### An example

Below we demonstrate how to solve the linear system

$$\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
 (9.5)

with two inputs b = (1, -2) and b = (7, 0).

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
   string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
   }
 }
 public class solvelinear
    static public void setup(mosek.Task task,
                             double[][] aval,
                              int[][] asub,
                              int[] ptrb,
                             int[] ptre,
                             int numvar,
                             int[] basis
      // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
      double
      infinity = 0;
      mosek.stakey[] skx = new mosek.stakey [numvar];
      mosek.stakey[] skc = new mosek.stakey [numvar];
      for (int i = 0; i < numvar ; ++i)</pre>
        skx[i] = mosek.stakey.bas;
        skc[i] = mosek.stakey.fix;
```

```
task.appendvars(numvar);
  task.appendcons(numvar);
  for (int i = 0; i < numvar ; ++i)</pre>
    task.putacol(i,
                 asub[i],
                 aval[i]);
  for (int i = 0 ; i < numvar ; ++i)</pre>
    task.putconbound(
      i,
      mosek.boundkey.fx,
      0.0,
      0.0);
  for (int i = 0 ; i < numvar ; ++i)</pre>
    task.putvarbound(
     mosek.boundkey.fr,
      -infinity,
      infinity);
  /* Define a basic solution by specifying
     status keys for variables & constraints. */
  task.deletesolution(mosek.soltype.bas);
  task.putskcslice(mosek.soltype.bas, 0, numvar, skc);
  task.putskxslice(mosek.soltype.bas, 0, numvar, skx);
  task.initbasissolve(basis);
}
public static void Main ()
  const int numcon = 2;
  const int numvar = 2;
  double[][]
  aval = new double[numvar][];
  aval[0] = new double[] { -1.0 };
  aval[1] = new double[] {1.0, 1.0};
  int[][]
  asub = new int[numvar][];
  asub[0] = new int[] {1};
  asub[1] = new int[] {0, 1};
  int []
              ptrb = {0, 1};
              ptre = \{1, 3\};
  int []
              bsub = new int[numvar];
  int[]
  double[]
              b = new double[numvar];
              basis = new int[numvar];
  int[]
  try
  {
    using (mosek.Env env = new mosek.Env())
```

```
{
    using (mosek.Task task = new mosek.Task(env))
      // Directs the log task stream to the user specified
      // method task_msg_obj.streamCB
      task.set_Stream(mosek.streamtype.log, new msgclass ("[task]"));
      /* Put A matrix and factor A.
         Call this function only once for a given task. */
      setup(
        task,
        aval,
        asub,
        ptrb,
        ptre,
        numvar,
        basis
      );
      /* now solve rhs */
      b[0] = 1;
      b[1] = -2;
      bsub[0] = 0;
      bsub[1] = 1;
      int nz = 2;
      task.solvewithbasis(0, ref nz, bsub, b);
      Console.WriteLine ("\nSolution to Bx = b: \n\n");
      \slash * Print solution and show correspondents
         to original variables in the problem */
      for (int i = 0; i < nz; ++i)
        if (basis[bsub[i]] < numcon)</pre>
          Console.WriteLine ("This should never happen\n");
        else
          Console.WriteLine ("x{0} = {1}\n", basis[bsub[i]] - numcon , b[bsub[i]] );
      }
      b[0] = 7;
      bsub[0] = 0;
      nz = 1;
      task.solvewithbasis(0, ref nz, bsub, b);
      Console.WriteLine ("\nSolution to Bx = b: \n\n");
      /* Print solution and show correspondents
        to original variables in the problem */
      for (int i = 0; i < nz; ++i)
        if (basis[bsub[i]] < numcon)</pre>
          Console.WriteLine ("This should never happen\n");
         Console.WriteLine ("x{0} = {1}\n", basis[bsub[i]] - numcon , b[bsub[i]] );
    }
 }
}
catch (mosek.Exception e)
{
  Console.WriteLine (e.Code);
```

```
Console.WriteLine (e);
    throw;
}
}
}
```

The most important step in the above example is the definition of the basic solution, where we define the status key for each variable. The actual values of the variables are not important and can be selected arbitrarily, so we set them to zero. All variables corresponding to columns in the linear system we want to solve are set to basic and the slack variables for the constraints, which are all non-basic, are set to their bound.

The program produces the output:

```
Solution to Bx = b:

x1 = 1
x0 = 3

Solution to Bx = b:

x1 = 7
x0 = 7
```

# 9.2 Calling BLAS/LAPACK Routines from MOSEK

Sometimes users need to perform linear algebra operations that involve dense matrices and vectors. Also **MOSEK** extensively uses high-performance linear algebra routines from the BLAS and LAPACK packages and some of these routines are included in the package shipped to the users.

The MOSEK versions of BLAS/LAPACK routines:

- use MOSEK data types and return value conventions,
- $\bullet$  preserve the BLAS/LAPACK naming convention.

Therefore the user can leverage on efficient linear algebra routines, with a simplified interface, with no need for additional packages.

#### List of available routines

BLAS Name	MOSEK function	Math Expression
AXPY	Env. axpy	$y = \alpha x + y$
DOT	Env. dot	$x^Ty$
GEMV	Env.gemv	$y = \alpha Ax + \beta y$
GEMM	Env.gemm	$C = \alpha AB + \beta C$
SYRK	Env.syrk	$C = \alpha A A^T + \beta C$

Table 9.1: BLAS routines available.

Table 9.2: LAPACK routines available.

LAPACK Name	MOSEK function	Description
POTRF	Env.potrf	Cholesky factorization of a semidefinite symmetric matrix
SYEVD	Env.syevd	Eigenvalues and eigenvectors of a symmetric matrix
SYEIG	Env.syeig	Eigenvalues of a symmetric matrix

### Source code examples

In Listing 9.2 we provide a simple working example. It has no practical meaning except showing how to organize the input and call the methods.

Listing 9.2: Calling BLAS and LAPACK routines from Optimizer API for .NET.

```
using System;
namespace mosek.example
 public class blas_lapack
   public static void Main ()
      const int n = 3, m = 2, k = 2;
      double alpha = 2.0, beta = 0.5;
      double[] x = \{1.0, 1.0, 1.0\};
      double[] y = \{1.0, 2.0, 3.0\};
      double[] z = \{1.0, 1.0\};
      /*A has m=2 rows and k=3 cols*/
      double[] A = \{1.0, 1.0, 2.0, 2.0, 3.0, 3.0\};
      /*B has k=3 rows and n=3 cols*/
      double[] B = \{1.0, 1.0, 1.0,
                    1.0, 1.0, 1.0,
                   1.0, 1.0, 1.0
                   };
      double[] C = \{1.0, 2.0, 3.0,
                   4.0, 5.0, 6.0
                   };
      double[] D = {1.0, 1.0, 1.0, 1.0};
      double[] Q = \{1.0, 0.0, 0.0, 2.0\};
      double[] v = new double[2];
      double xy;
      using (mosek.Env env = new mosek.Env())
        /* BLAS routines */
        try
          env.dot(n, x, y, out xy);
          env.axpy(n, alpha, x, y);
          env.gemv(mosek.transpose.no, m, n, alpha, A, x, beta, z);
          env.gemm(mosek.transpose.no, mosek.transpose.no, m, n, k, alpha, A, B, beta, C);
          env.syrk(mosek.uplo.lo, mosek.transpose.no, m, k, alpha, A, beta, D);
          /* LAPACK routines*/
          env.potrf(mosek.uplo.lo, m, Q);
```

```
env.syeig(mosek.uplo.lo, m, Q, v);

env.syevd(mosek.uplo.lo, m, Q, v);
}
catch (mosek.Exception e)
{
    Console.WriteLine (e.Code);
    Console.WriteLine (e);
}
finally
{
    if (env != null) env.Dispose ();
}
}
}
}
```

# 9.3 Computing a Sparse Cholesky Factorization

Given a positive semidefinite symmetric (PSD) matrix

$$A \in \mathbb{R}^{n \times n}$$

it is well known there exists a matrix L such that

$$A = LL^T$$
.

If the matrix L is lower triangular then it is called a *Cholesky factorization*. Given A is positive definite (nonsingular) then L is also nonsingular. A Cholesky factorization is useful for many reasons:

- A system of linear equations Ax = b can be solved by first solving the lower triangular system Ly = b followed by the upper triangular system  $L^Tx = y$ .
- A quadratic term  $x^T A x$  in a constraint or objective can be replaced with  $y^T y$  for  $y = L^T x$ , potentially leading to a more robust formulation (see [And13]).

Therefore, **MOSEK** provides a function that can compute a Cholesky factorization of a PSD matrix. In addition a function for solving linear systems with a nonsingular lower or upper triangular matrix is available.

In practice A may be very large with n is in the range of millions. However, then A is typically sparse which means that most of the elements in A are zero, and sparsity can be exploited to reduce the cost of computing the Cholesky factorization. The computational savings depend on the positions of zeros in A. For example, below a matrix A is given together with a Cholesky factor up to 5 digits of accuracy:

$$A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad L = \begin{bmatrix} 2.0000 & 0 & 0 & 0 \\ 0.5000 & 0.8660 & 0 & 0 \\ 0.5000 & -0.2887 & 0.8165 & 0 \\ 0.5000 & -0.2887 & -0.4082 & 0.7071 \end{bmatrix}.$$
(9.6)

However, if we symmetrically permute the rows and columns of A using a permutation matrix P

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad A' = PAP^T = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 4 \end{bmatrix},$$

then the Cholesky factorization of  $A' = L'L'^T$  is

$$L' = \left[ \begin{array}{rrrr} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{array} \right]$$

which is sparser than L.

Computing a permutation matrix that leads to the sparsest Cholesky factorization or the minimal amount of work is NP-hard. Good permutations can be chosen by using heuristics, such as the minimum degree heuristic and variants. The function <code>Env.computesparsecholesky</code> provided by **MOSEK** for computing a Cholesky factorization has a build in permutation aka. reordering heuristic. The following code illustrates the use of <code>Env.computesparsecholesky</code> and <code>Env.sparsetriangularsolvedense</code>.

Listing 9.3: How to use the sparse Cholesky factorization routine available in **MOSEK**.

```
env.computesparsecholesky(0,
                                    //Disable multithreading
                                    //Apply reordering heuristic
                          1.0e-14, //Singularity tolerance
                          anzc, aptrc, asubc, avalc,
                          out perm, out diag,
                          out lnzc, out lptrc, out lensubnval, out lsubc, out lvalc);
printsparse(n, perm, diag, lnzc, lptrc, lensubnval, lsubc, lvalc);
/* Permuted b is stored as x. */
double[] x = new double[n];
for (int i = 0; i < n; i++) x[i] = b[perm[i]];
/*Compute inv(L)*x.*/
env.sparsetriangularsolvedense(mosek.transpose.no, lnzc, lptrc, lsubc, lvalc, x);
/*Compute inv(L^T)*x.*/
env.sparsetriangularsolvedense(mosek.transpose.yes, lnzc, lptrc, lsubc, lvalc, x);
Console.Write("\nSolution A x = b, x = [ ");
for (int i = 0; i < n; i++)
  for (int j = 0; j < n; j++) if (perm[j] == i) Console.Write("{0} ", x[j]);
Console.WriteLine("]\n");
```

We can set up the data to recreate the matrix A from (9.6):

```
//Observe that anzc, aptrc, asubc and avalc only specify the lower triangular part.

const int n = 4;
int[] anzc = {4, 1, 1, 1};
int[] asubc = {0, 1, 2, 3, 1, 2, 3};
long[] aptrc = {0, 4, 5, 6};
double[] avalc = {4.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0};
double[] b = {13.0, 3.0, 4.0, 5.0};
```

and we obtain the following output:

The output indicates that with the permutation matrix

$$P = \left[ \begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array} \right]$$

there is a Cholesky factorization  $PAP^T = LL^T$ , where

$$L = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1.4142 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \end{array} \right]$$

The remaining part of the code solvers the linear system Ax = b for  $b = [13, 3, 4, 5]^T$ . The solution is reported to be  $x = [1, 2, 3, 4]^T$ , which is correct.

The second example shows what happens when we compute a sparse Cholesky factorization of a singular matrix. In this example A is a rank 1 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}^{T}$$

$$(9.7)$$

Now we get the output

```
P = [ 0 2 1 ]
diag(D) = [ 0.00e+00 1.00e-14 1.00e-14 ]
L=
1.00e+00 0.00e+00 0.00e+00
1.00e+00 1.00e-07 0.00e+00
1.00e+00 0.00e+00 1.00e-07
```

which indicates the decomposition

$$PAP^T = LL^T - D$$

where

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 10^{-7} & 0 \\ 1 & 0 & 10^{-7} \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 10^{-14} & 0 \\ 0 & 0 & 10^{-14} \end{bmatrix}.$$

Since A is only positive semdefinite, but not of full rank, some of diagonal elements of A are boosted to make it truely positive definite. The amount of boosting is passed as an argument to Env. computesparsecholesky, in this case  $10^{-14}$ . Note that

$$PAP^T = LL^T - D$$

where D is a small matrix so the computed Cholesky factorization is exact of slightly perturbed A. In general this is the best we can hope for in finite precision and when A is singular or close to being singular.

We will end this section by a word of caution. Computing a Cholesky factorization of a matrix that is not of full rank and that is not suffciently well conditioned may lead to incorrect results i.e. a matrix that is indefinite may declared positive semidefinite and vice versa.

# 9.4 Converting a quadratically constrained problem to conic form

MOSEK employs the following form of quadratic problems:

minimize 
$$\frac{1}{2}x^{T}Q^{o}x + c^{T}x + c^{f}$$
 subject to 
$$l_{k}^{c} \leq \frac{1}{2}x^{T}Q^{k}x + \sum_{j=0}^{n-1}a_{k,j}x_{j} \leq u_{k}^{c}, \quad k = 0, \dots, m-1,$$
 
$$l_{j}^{x} \leq x_{j} \leq u_{j}^{x}, \quad j = 0, \dots, n-1.$$
 (9.8)

A conic quadratic constraint has the form

$$x \in \mathcal{Q}^n$$

in its most basic form where

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

A quadratic problem such as (9.8), if convex, can be reformulated in conic form. This is in fact the reformulation **MOSEK** performs internally. It has many advantages:

- elegant duality theory for conic problems,
- reporting accurate dual information for quadratic inequalities is hard and/or computational expensive,
- it certifies that the original quadratic problem is indeed convex,
- modeling directly in conic form usually leads to a better model [And13] i.e. a faster solution time and better numerical properties.

In addition, there are more types of conic constraints that can be combined with a quadratic cone, for example semidefinite cones.

**MOSEK** offers a function that performs the conversion from quadratic to conic quadratic form explicitly. Note that the reformulation is not unique. The approach followed by **MOSEK** is to introduce additional variables, linear constraints and quadratic cones to obtain a larger but equivalent problem in which the original variables are preserved.

In particular:

- all variables and constraints are kept in the problem,
- each quadratic constraint and quadratic terms in the objective generate one rotated quadratic cone,
- each quadratic constraint will contain no coefficients and upper/lower bounds will be set to  $\infty$ ,  $-\infty$  respectively.

This allows the user to recover the original variable and constraint values, as well as their dual values, with no conversion or additional effort.

**Note:** Task.toconic modifies the input task in-place: this means that if the reformulation is not possible, i.e. the problem is not conic representable, the state of the task is in general undefined. The user should consider cloning the original task.

# 9.4.1 Quadratic Constraint Reformulation

Let us assume we want to convert the following quadratic constraint

$$l \le \frac{1}{2}x^T Q x + \sum_{j=0}^{n-1} a_j x_j \le u$$

to conic form. We first check whether  $l = -\infty$  or  $u = \infty$ , otherwise either the constraint can be dropped, or the constraint is not convex. Thus let us consider the case

$$\frac{1}{2}x^T Q x + \sum_{j=0}^{n-1} a_j^T x_j \le u. \tag{9.9}$$

Introducing an additional variable w such that

$$w = u - \sum_{j=0}^{n-1} a_j^T x_j \tag{9.10}$$

we obtain the equivalent form

$$\begin{array}{rcl} \frac{1}{2}x^TQx & \leq & w, \\ u - \sum_{j=0}^{n-1} a_j x_j & = & w. \end{array}$$

If Q is positive semidefinite, then there exists a matrix F such that

$$Q = FF^T (9.11)$$

and therefore we can write

$$||Fx||^2 \le 2w,$$
  
 $u - \sum_{j=0}^{n-1} a_j^T x_j = w.$ 

Introducing an additional variable z=1, and setting y=Fx we obtain the conic formulation

$$(w, z, y) \in \mathcal{Q}_r,$$

$$z = 1$$

$$y = Fx$$

$$w = u - a^T x.$$

$$(9.12)$$

Summarizing, for each quadratic constraint involving t variables, MOSEK introduces

- 1. a rotated quadratic cone of dimension t+2,
- 2. two additional variables for the cone roots,
- 3. t additional variables to map the remaining part of the cone,
- 4. t linear constraints.

A quadratic term in the objective is reformulated in a similar fashion. We refer to [And13] for a more thorough discussion.

# **Example**

Next we consider a simple problem with quadratic objective function:

```
minimize  \begin{array}{l} \text{minimize} & \frac{1}{2}(13x_0^2+17x_1^2+12x_2^2+24x_0x_1+12x_1x_2-4x_0x_2)-22x_0-14.5x_1+12x_2+1\\ \text{subject to} & -1\leq x_0,x_1,x_2\leq 1 \end{array}
```

We can specify it in the human-readable OPF format.

```
[comment]
An example of small QO problem from Boyd and Vandenberghe, "Convex Optimization", page 189 exu 4.3
The solution is (1,0.5,-1)
[/comment]
[variables]
x0 x1 x2
[/variables]
[objective min]
0.5 (13 x0^2 + 17 x1^2 + 12 x2^2 + 24 x0 * x1 + 12 x1 * x2 - 4 x0 * x2 ) - 22 x0 - 14.5 x1 + 12 x1 + 12 x2 + 1
[/objective]
[bounds]
[b] -1 <= * <= 1 [/b]
[/bounds]
```

The objective function is convex, the minimum is attained for  $x^* = (1, 0.5, -1)$ . The conversion will introduce first a variable  $x_3$  in the objective function such that  $x_3 \ge 1/2x^TQx$  and then convert the latter directly in conic form. The converted problem follows:

```
minimize -22x_0 - 14.5x_1 + 12x_2 + x_3 + 1 subject to 3.61x_0 + 3.33x_1 - 0.55x_2 - x_6 = 0 +2.29x_1 + 3.42x_2 - x_7 = 0 0.81x_1 - x_8 = 0 -x_3 + x_4 = 0 x_5 = 1 (x_4, x_5, x_6, x_7, x_8) \in \mathcal{Q}_{\nabla} -1 \le x_0, x_1, x_2 \le 1
```

The model generated by Task. toconic is

```
[comment]
   Written by MOSEK version 8.1.0.19
   Date 21-08-17
   Time 10:53:36
[/comment]
[hints]
  [hint NUMVAR] 9 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 11 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
 x0000_x0 x0001_x1 x0002_x2 x0003 x0004
 x0005 x0006 x0007 x0008
[/variables]
[objective minimize]
   - 2.2e+01 x0000_x0 - 1.45e+01 x0001_x1 + 1.2e+01 x0002_x2 + x0003
   + 1e+00
[/objective]
[constraints]
  [con c0000]
              3.605551275463989e+00 x0000_x0 - 5.547001962252291e-01 x0002_x2 + 3.
328201177351375e+00 x0001_x1 - x0006 = 0e+00 [/con]
  [con c0001] 3.419401657060442e+00 \times 0002_x2 + 2.294598480395823e+00 \times 0001_x1 - \times 0007 = 0e+00_u
  [con c0002] 8.111071056538127e-01 \times 0001_x1 - x0008 = 0e+00 [/con]
  [con c0003]
               - x0003 + x0004 = 0e+00 [/con]
[/constraints]
[bounds]
  [b] -1e+00
                 <= x0000_x0,x0001_x1,x0002_x2 <= 1e+00 [/b]
  ГъТ
                    x0003,x0004 free [/b]
                    x0005 = 1e+00 [/b]
                    x0006,x0007,x0008 free [/b]
  [cone rquad k0000] x0004, x0005, x0006, x0007, x0008 [/cone]
[/bounds]
```

We can clearly see that constraints c0000, c0001 and c0002 represent the original linear constraints as in (9.11), while c0003 corresponds to (9.10). The cone roots are x0005 and x0004.

# Chapter 10

# Technical guidelines

This section contains some more in-depth technical guidelines for Optimizer API for .NET, not strictly necessary for basic use of **MOSEK**.

# 10.1 Memory management and garbage collection

Users who experience memory leaks, especially:

- memory usage not decreasing after the solver terminates,
- memory usage increasing when solving a sequence of problems,

should make sure that the *Task* objects are properly garbage collected. Since each *Task* object links to a **MOSEK** task resource in a linked library, it is sometimes the case that the garbage collector is unable to reclaim it automatically. This means that substantial amounts of memory may be leaked. For this reason it is very important to make sure that the *Task* object is disposed of, either automatically or manually, when it is not used any more.

The *Task* class supports the *Context Manager* protocol, so it will be destroyed properly when used in a using statement:

```
using (env = new mosek.Env())
{
   // Create a task object.
   using (task = new mosek.Task(env, 0, 0))
   {
        // Construct the problem
        // ...
```

If this is not possible, then the necessary cleanup is performed by the methods Task. Dispose and Env. Dispose which should be called explicitly.

# 10.2 Names

All elements of an optimization problem in **MOSEK** (objective, constraints, variables, etc.) can be given names. Assigning meaningful names to variables and constraints makes it much easier to understand and debug optimization problems dumped to a file. On the other hand, note that assigning names can substantially increase setup time, so it should be avoided in time-critical applications.

Names of various elements of the problem can be set and retrieved using various functions listed in the **Names** section of Sec. 15.2.

# 10.3 Multithreading

## Thread safety

Sharing a task between threads is safe, as long as it is not accessed from more than one thread at a time. Multiple tasks can be created and used in parallel without any problems.

#### **Parallelization**

The interior-point and mixed-integer optimizers in **MOSEK** are parallelized. By default **MOSEK** will automatically select the number of threads. However, the maximum number of threads allowed can be changed by setting the parameter *iparam.num\_threads* and related parameters. This should never exceed the number of cores. See Sec. 13 and Sec. 13.4 for more details.

The speed-up obtained when using multiple threads is highly problem and hardware dependent. We recommend experimenting with various thread numbers to determine the optimal settings. For small problems using multiple threads may be counter-productive because of the associated overhead.

#### **Determinism**

By default the optimizer is run-to-run deterministic, which means that it will return the same answer each time it is run on the same machine with the same input, the same parameter settings (including number of threads) and no time limits.

### Setting the number of threads

The number of threads the optimizer uses can be changed with the parameter <code>iparam.num\_threads</code>. For conic problems (when the conic optimizer is used) the value set at the first optimization will remain fixed through the lifetime of the process. The thread pool will be reserved once for all and subsequent changes to <code>iparam.num\_threads</code> will have no effect. The only possibility at that point is to switch between multi-threaded and single-threaded interior-point optimization using the parameter <code>iparam.intpnt\_multi\_thread</code>.

# 10.4 Efficiency

Although MOSEK is implemented to handle memory efficiently, the user may have valuable knowledge about a problem, which could be used to improve the performance of MOSEK This section discusses some tricks and general advice that hopefully make MOSEK process your problem faster.

### Reduce the number of function calls and avoid input loops

For example, instead of setting the entries in the linear constraint matrix one by one (Task.putaij) define them all at once (Task.putaijlist) or in convenient large chunks (Task.putacollist etc.)

### Use one environment only

If possible share the environment between several tasks. For most applications you need to create only a single environment.

### Read part of the solution

When fetching the solution, data has to be copied from the optimizer to the user's data structures. Instead of fetching the whole solution, consider fetching only the interesting part (see for example Task. getxxslice and similar).

### Avoiding memory fragmentation

MOSEK stores the optimization problem in internal data structures in the memory. Initially MOSEK will allocate structures of a certain size, and as more items are added to the problem the structures are reallocated. For large problems the same structures may be reallocated many times causing memory fragmentation. One way to avoid this is to give MOSEK an estimated size of your problem using the functions:

- Task.putmaxnumvar. Estimate for the number of variables.
- Task.putmaxnumcon. Estimate for the number of constraints.

- Task.putmaxnumcone. Estimate for the number of cones.
- Task.putmaxnumbarvar. Estimate for the number of semidefinite matrix variables.
- Task.putmaxnumanz. Estimate for the number of non-zeros in A.
- Task.putmaxnumqnz. Estimate for the number of non-zeros in the quadratic terms.

None of these functions changes the problem, they only serve as hints. If the problem ends up growing larger, the estimates are automatically increased.

### Do not mix put- and get- functions

MOSEK will queue put- requests internally until a get- function is called. If put- and get- calls are interleaved, the queue will have to be flushed more frequently, decreasing efficiency.

In general get- commands should not be called often (or at all) during problem setup.

## Use the LIFO principle

When removing constraints and variables, try to use a LIFO (Last In First Out) approach. **MOSEK** can more efficiently remove constraints and variables with a high index than a small index.

An alternative to removing a constraint or a variable is to fix it at 0, and set all relevant coefficients to 0. Generally this will not have any impact on the optimization speed.

## Add more constraints and variables than you need (now)

The cost of adding one constraint or one variable is about the same as adding many of them. Therefore, it may be worthwhile to add many variables instead of one. Initially fix the unused variable at zero, and then later unfix them as needed. Similarly, you can add multiple free constraints and then use them as needed.

#### Do not remove basic variables

When performing re-optimizations, instead of removing a basic variable it may be more efficient to fix the variable at zero and then remove it when the problem is re-optimized and it has left the basis. This makes it easier for **MOSEK** to restart the simplex optimizer.

# 10.5 The license system

MOSEK is a commercial product that always needs a valid license to work. MOSEK uses a third party license manager to implement license checking. The number of license tokens provided determines the number of optimizations that can be run simultaneously.

By default a license token remains checked out from the first optimization until the end of the MOSEK session, i.e.

- a license token is checked out when Task. optimize is first called, and
- it is returned when the **MOSEK** environment is deleted.

Calling Task. optimize from different threads using the same MOSEK environment only consumes one license token.

Starting the optimization when no license tokens are available will result in an error.

Default behaviour of the license system can be changed in several ways:

- Setting the parameter *iparam.cache\_license* to *onoffkey.off* will force **MOSEK** to return the license token immediately after the optimization completed.
- Setting the license wait flag with the parameter <code>iparam.license\_wait</code> will force <code>MOSEK</code> to wait until a license token becomes available instead of returning with an error. The wait time between checks can be set with <code>Env.putlicensewait</code>.

- Additional license checkouts and checkins can be performed with the functions *Env. checkinlicense* and *Env. checkoutlicense*.
- Usually the license system is stopped automatically when the MOSEK library is unloaded. However, when the user explicitly unloads the library (using e.g. FreeLibrary), the license system must be stopped before the library is unloaded. This can be done by calling the function <code>Env.licensecleanup</code> as the last function call to MOSEK.

# 10.6 Deployment

When redistributing a .NET application using the  $\mathbf{MOSEK}$  Optimizer API for .NET 9.1.13, the following libraries must be included:

64-bit Windows	32-bit Windows
mosek64_9_1.dll	mosek9_1.dll
cilkrts20.dll	cilkrts20.dll
mosekxx9_1.dll	mosekxx9_1.dll
mosekdotnet9_1.dll	mosekdotnet9_1.dll

# Chapter 11

# Case Studies

In this section we present some case studies in which the Optimizer API for .NET is used to solve real-life applications. These examples involve some more advanced modeling skills and possibly some input data. The user is strongly recommended to first read the basic tutorials of Sec. 6 before going through these advanced case studies.

- Portfolio Optimization
  - Keywords: Markowitz model, variance, risk, efficient frontier, transaction cost, market impact cost
  - Type: Conic Quadratic, Power Cone, Mixed-Integer Optimization
- Logistic regression
  - **Keywords:** machine learning, logistic regression, classifier, log-sum-exp, softplus, regularization
  - **Type:** Exponential Cone, Quadratic Cone
- Concurrent Optimizer
  - **Keywords:** Concurrent optimization
  - Type: Linear Optimization, Mixed-Integer Optimization

# 11.1 Portfolio Optimization

In this section the Markowitz portfolio optimization problem and variants are implemented using the MOSEK optimizer API.

- Basic Markowitz model
- Efficient frontier
- Factor model and efficiency
- Market impact costs
- Transaction costs
- Cardinality constraints

# 11.1.1 A Basic Portfolio Optimization Model

The classical Markowitz portfolio optimization problem considers investing in n stocks or assets held over a period of time. Let  $x_j$  denote the amount invested in asset j, and assume a stochastic model where the return of the assets is a random variable r with known mean

$$\mu = \mathbf{E}r$$

and covariance

$$\Sigma = \mathbf{E}(r - \mu)(r - \mu)^T.$$

The return of the investment is also a random variable  $y = r^T x$  with mean (or expected return)

$$\mathbf{E}y = \mu^T x$$

and variance

$$\mathbf{E}(y - \mathbf{E}y)^2 = x^T \Sigma x.$$

The standard deviation

$$\sqrt{x^T \Sigma x}$$

is usually associated with risk.

The problem facing the investor is to rebalance the portfolio to achieve a good compromise between risk and expected return, e.g., maximize the expected return subject to a budget constraint and an upper bound (denoted  $\gamma$ ) on the tolerable risk. This leads to the optimization problem

$$\begin{array}{lll} \text{maximize} & \mu^T x \\ \text{subject to} & e^T x & = & w + e^T x^0, \\ & x^T \Sigma x & \leq & \gamma^2, \\ & x & \geq & 0. \end{array} \tag{11.1}$$

The variables x denote the investment i.e.  $x_j$  is the amount invested in asset j and  $x_j^0$  is the initial holding of asset j. Finally, w is the initial amount of cash available.

A popular choice is  $x^0 = 0$  and w = 1 because then  $x_j$  may be interpreted as the relative amount of the total portfolio that is invested in asset j.

Since e is the vector of all ones then

$$e^T x = \sum_{j=1}^n x_j$$

is the total investment. Clearly, the total amount invested must be equal to the initial wealth, which is

$$w + e^T x^0$$
.

This leads to the first constraint

$$e^T x = w + e^T x^0.$$

The second constraint

$$x^T \Sigma x \le \gamma^2$$

ensures that the variance, is bounded by the parameter  $\gamma^2$ . Therefore,  $\gamma$  specifies an upper bound of the standard deviation (risk) the investor is willing to undertake. Finally, the constraint

$$x_j \geq 0$$

excludes the possibility of short-selling. This constraint can of course be excluded if short-selling is allowed.

The covariance matrix  $\Sigma$  is positive semidefinite by definition and therefore there exist a matrix G such that

$$\Sigma = GG^T. (11.2)$$

In general the choice of G is **not** unique and one possible choice of G is the Cholesky factorization of  $\Sigma$ . However, in many cases another choice is better for efficiency reasons as discussed in Sec. 11.1.3. For a given G we have that

$$x^{T} \Sigma x = x^{T} G G^{T} x$$
$$= \|G^{T} x\|^{2}.$$

Hence, we may write the risk constraint as

$$\gamma \ge \|G^T x\|$$

or equivalently

$$(\gamma, G^T x) \in \mathcal{Q}^{n+1},$$

where  $Q^{n+1}$  is the (n+1)-dimensional quadratic cone. Therefore, problem (11.1) can be written as

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $(\gamma, G^T x) \in \mathcal{Q}^{n+1}$ ,  
 $x \geq 0$ , (11.3)

which is a conic quadratic optimization problem that can easily be formulated and solved with Optimizer API for .NET. Subsequently we will use the example data

$$\mu = \left[ \begin{array}{c} 0.1073 \\ 0.0737 \\ 0.0627 \end{array} \right]$$

and

$$\Sigma = 0.1 \cdot \left[ \begin{array}{ccc} 0.2778 & 0.0387 & 0.0021 \\ 0.0387 & 0.1112 & -0.0020 \\ 0.0021 & -0.0020 & 0.0115 \end{array} \right].$$

This implies

$$G^T = \sqrt{0.1} \begin{bmatrix} 0.5271 & 0.0734 & 0.0040 \\ 0 & 0.3253 & -0.0070 \\ 0 & 0 & 0.1069 \end{bmatrix}$$

# Why a Conic Formulation?

Problem (11.1) is a convex quadratically constrained optimization problem that can be solved directly using MOSEK. Why then reformulate it as a conic quadratic optimization problem (11.3)? The main reason for choosing a conic model is that it is more robust and usually solves faster and more reliably. For instance it is not always easy to numerically validate that the matrix  $\Sigma$  in (11.1) is positive semidefinite

due to the presence of rounding errors. It is also very easy to make a mistake so  $\Sigma$  becomes indefinite. These problems are completely eliminated in the conic formulation.

Moreover, observe the constraint

$$\left\|G^Tx\right\| \leq \gamma$$

more numerically robust than

$$x^T \Sigma x < \gamma^2$$

for very small and very large values of  $\gamma$ . Indeed, if say  $\gamma \approx 10^4$  then  $\gamma^2 \approx 10^8$ , which introduces a scaling issue in the model. Hence, using conic formulation we work with the standard deviation instead of variance, which usually gives rise to a better scaled model.

## Implementing the Portfolio Model

### Creating a matrix formulation

The Optimizer API for .NET requires that an optimization problem is entered in the following standard form:

We refer to  $\hat{x}$  as the API variable. It means we need to reformulate (11.3). The first step is to introduce auxiliary variables so that the conic constraint involves only unique variables:

maximize 
$$\mu^T x$$
  
subject to  $e^T x = w + e^T x^0$ ,  
 $G^T x - t = 0$ ,  
 $[s;t] \in \mathcal{Q}^{n+1}$ ,  
 $x \geq 0$ ,  
 $s = \gamma$ . (11.5)

Here s is an additional scalar variable and t is a vector variable of dimension n. The next step is to concatenate all the variables into one long variable vector:

$$\hat{x} = [x; s; t] = \begin{bmatrix} x \\ s \\ t \end{bmatrix}$$
 (11.6)

The details of the concatenation are specified below.

Table 11.1: Storage layout of the  $\hat{x}$  variable.

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1

The offset determines where the variable starts. (Note that all variables are indexed from 0). For instance

$$\hat{x}_{n+1+i} = t_i.$$

because the offset of the t variable is n+1.

Given the ordering of the variables specified by (11.6) it is useful to visualize the linear constraints (11.4) in an explicit block matrix form:

$$\begin{bmatrix}
 & 1 & 0 & 0 \\
\hline
 & G^T & 0 & -1 \\
\hline
 & & & -1
\end{bmatrix} \cdot \begin{bmatrix} x \\
\hline
 & s \\
\hline
 & t \end{bmatrix} = \begin{bmatrix} w + e^T x_0 \\
\hline
 & 0 \end{bmatrix}.$$
(11.7)

In other words, we should define the specific components of the problem description as follows:

$$c = \begin{bmatrix} \mu^{T} & 0 & 0_{n} \end{bmatrix}^{T}, 
A = \begin{bmatrix} e^{T} & 0 & 0_{n} \\ G^{T} & 0_{n} & -I_{n} \end{bmatrix}, 
l^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T}, 
u^{c} = \begin{bmatrix} w + e^{T}x^{0} & 0_{n} \end{bmatrix}^{T}, 
l^{x} = \begin{bmatrix} 0_{n} & \gamma & -\infty_{n} \end{bmatrix}^{T}, 
u^{x} = \begin{bmatrix} \infty_{n} & \gamma & \infty_{n} \end{bmatrix}^{T}.$$
(11.8)

### Source code example

From the block matrix form (11.7) and the explicit specification (11.8), using the offset information in Table 11.1 it is easy to calculate the index and value of each entry of the linear constraint matrix. The code below sets up the general optimization problem (11.5) and solves it for the example data. Of course it is only necessary to set non-zero entries of the linear constraint matrix.

Listing 11.1: Code implementing model (11.5).

```
using System;
namespace mosek.example
 /* Log handler class */
 class msgclass : mosek.Stream
    string prefix;
   public msgclass (string prfx) { prefix = prfx; }
   public override void streamCB (string msg)
     Console.Write ("{0}{1}", prefix, msg);
 }
 public class portfolio_1_basic
   public static void Main (String[] args)
     const int n = 3;
      // Since the value infinity is never used, we define
      // 'infinity' for symbolic purposes only
     double infinity = 0.0;
     double gamma = 0.05;
     double[] mu = {0.1073, 0.0737, 0.0627};
     double[,] GT = {
        {0.1667, 0.0232, 0.0013},
        {0.0000, 0.1033, -0.0022},
```

```
{0.0000, 0.0000, 0.0338}
};
double[] x0 = \{0.0, 0.0, 0.0\};
double w = 1.0;
double
       totalBudget;
int numvar = 2 * n + 1;
int numcon = n + 1;
// Offset of variables into the API variable.
int offsetx = 0;
int offsets = n;
int offsett = n + 1;
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
  // Create a task object.
  using (mosek.Task task = new mosek.Task(env, 0, 0))
    // Directs the log task stream
    task.set_Stream(mosek.streamtype.log, new msgclass (""));
    // Constraints.
    task.appendcons(numcon);
    // Constraint bounds. Compute total budget.
    totalBudget = w;
    for (int i = 0; i < n; ++i)
    {
     totalBudget += x0[i];
      /* Constraint bounds c^l = c^u = 0 */
     task.putconbound(i + 1, mosek.boundkey.fx, 0.0, 0.0);
     task.putconname(i + 1, "GT[" + (i + 1) + "]");
    /* The total budget constraint c^l = c^u = totalBudget in first row of A. */
    task.putconbound(0, mosek.boundkey.fx, totalBudget, totalBudget);
    task.putconname(0, "budget");
    // Variables.
    task.appendvars(numvar);
    /* x variables. */
    for (int j = 0; j < n; ++j)
      /* Return of asset j in the objective */
     task.putcj(offsetx + j, mu[j]);
      /* Coefficients in the first row of A */
     task.putaij(0, offsetx + j, 1.0);
      /* No short-selling - x^l = 0, x^u = inf */
     task.putvarbound(offsetx + j, mosek.boundkey.lo, 0.0, infinity);
      task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
    /* s variable is a constant equal to gamma. */
    task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
    task.putvarname(offsets, "s");
    /* t variables (t = GT*x). */
    for (int j = 0; j < n; ++j)
```

```
/* Copying the GT matrix in the appropriate block of A */
            for (int k = 0; k < n; ++k)
              if (GT[k,j] != 0.0)
                task.putaij(1 + k, offsetx + j, GT[k,j]);
            /* Diagonal -1 entries in a block of A */
            task.putaij(1 + j, offsett + j, -1.0);
            /* Free - no bounds */
            task.putvarbound(offsett + j, mosek.boundkey.fr, -infinity, infinity);
            task.putvarname(offsett + j, "t[" + (j + 1) + "]");
          /* Define the cone spanned by (s, t), i.e. of dimension n + 1 */
          int[] csub = new int[n + 1];
          csub[0] = offsets;
          for(int j = 0; j < n; j++) csub[j + 1] = offsett + j;
          task.appendcone( mosek.conetype.quad,
                           0.0, /* For future use only, can be set to 0.0 */
                           csub );
          task.putconename(0, "stddev");
          /* A maximization problem */
          task.putobjsense(mosek.objsense.maximize);
          task.optimize();
          /* Display solution summary for quick inspection of results */
          task.solutionsummary(mosek.streamtype.log);
          task.writedata("dump.opf");
          /* Read the results */
          double expret = 0.0;
          double[] xx = new double[n + 1];
          task.getxxslice(mosek.soltype.itr, 0, offsets + 1, xx);
          for (int j = 0; j < n; ++j)
            expret += mu[j] * xx[j + offsetx];
          Console WriteLine("\nExpected return {0:E} for gamma {1:E}", expret, xx[offsets]);
        }
     }
   }
 }
}
```

The above code produces the result:

Listing 11.2: Output from the solver.

```
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 7.4766507287e-02
                                     nrm: 1e+00
                                                   Viol.
                                                          con: 2e-08
                                                                         var: 0e+00
                                                                                        cones: 2e-
<u></u> 08
 Dual.
           obj: 7.4766554102e-02
                                                   Viol. con: 0e+00
                                     nrm: 3e-01
                                                                         var: 3e-08
                                                                                        cones:
-0e+00
Expected return 7.476651e-02 for gamma 5.000000e-02
```

#### Source code comments

The source code is a direct translation of the model (11.5) using the explicit block matrix specification (11.8) but a few comments are nevertheless in place.

In the lines

```
// Offset of variables into the API variable.
int offsetx = 0;
int offsets = n;
int offsett = n + 1;
```

offsets into the MOSEK API variable are stored as in Table 11.1. The code

```
/* x variables. */
for (int j = 0; j < n; ++j)
{
    /* Return of asset j in the objective */
    task.putcj(offsetx + j, mu[j]);
    /* Coefficients in the first row of A */
    task.putaij(0, offsetx + j, 1.0);
    /* No short-selling - x^l = 0, x^u = inf */
    task.putvarbound(offsetx + j, mosek.boundkey.lo, 0.0, infinity);
    task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
}</pre>
```

sets up the data for x variables. For instance

```
/* Return of asset j in the objective */
task.putcj(offsetx + j, mu[j]);
```

inputs the objective coefficients for the x variables. Moreover, the code

```
task.putvarname(offsetx + j, "x[" + (j + 1) + "]");
```

assigns meaningful names to the API variables. This is not needed but it makes debugging easier. Note that the solution values are only accessed for the interesting variables; for instance the auxiliary variable t is omitted from this process.

### **Debugging Tips**

Implementing an optimization model in Optimizer API for .NET can be error-prone. In order to check the code for accidental errors it is very useful to dump the problem to a file in a human readable form for visual inspection. The line

```
task.writedata("dump.opf");
```

does that and it produces a file with the content:

Listing 11.3: Problem (11.5) stored in OPF format.

```
[comment]
   Written by MOSEK version 8.1.0.24
   Date 11-09-17
   Time 14:34:24
[/comment]
[hints]
  [hint NUMVAR] 7 [/hint]
  [hint NUMCON] 4 [/hint]
  [hint NUMANZ] 12 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 1 [/hint]
[/hints]
[variables disallow_new_variables]
  'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]'
[/variables]
[objective maximize]
   1.073e-01 'x[1]' + 7.37e-02 'x[2]' + 6.27000000000001e-02 'x[3]'
[/objective]
[constraints]
  [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' = 1e+00 [/con]
   [con 'GT[1]'] \quad 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00 [/con] 
   [con 'GT[2]'] \quad 1.033e-01 \ 'x[2]' - 2.2e-03 \ 'x[3]' - 't[2]' = 0e+00 \ [/con] 
  [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
[/constraints]
[bounds]
                 <= 'x[1]','x[2]','x[3]' [/b]
  [b] 0e+00
  ГъТ
                    s = 5e-02 [/b]
                     't[1]', 't[2]', 't[3]' free [/b]
  [b]
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
[/bounds]
```

Since the API variables have been given meaningful names it is easy to verify by hand that the model is correct.

## 11.1.2 The efficient Frontier

The portfolio computed by the Markowitz model is efficient in the sense that there is no other portfolio giving a strictly higher return for the same amount of risk. An efficient portfolio is also sometimes called a Pareto optimal portfolio. Clearly, an investor should only invest in efficient portfolios and therefore it may be relevant to present the investor with all efficient portfolios so the investor can choose the portfolio that has the desired tradeoff between return and risk.

Given a nonnegative  $\alpha$  the problem

maximize 
$$\mu^T x - \alpha x^T \Sigma x$$
  
subject to  $e^T x = w + e^T x^0$ , (11.9)  
 $x > 0$ .

is one standard way to trade the expected return against penalizing variance. Note that, in contrast to the previous example, we explicitly use the variance ( $\|G^Tx\|_2^2$ ) rather than standard deviation ( $\|G^Tx\|_2^2$ ),

therefore the conic model includes a rotated quadratic cone:

```
maximize \mu^T x - \alpha s

subject to e^T x = w + e^T x^0,

G^T x - t = 0,

u = 0.5,

(s, u, t) \in Q_r^{n+2} (evaluates to s \ge \|G^T x\|_2^2 = x^T \Sigma x),

x \ge 0. (11.10)
```

Ideally the problem (11.9) should be solved for all values  $\alpha \geq 0$  but in practice it is impossible. Using the example data as before, the optimal values of return and variance for several values of  $\alpha$  are shown below:

Listing 11.4: Results obtained solving problem (11.9) for different values of  $\alpha$ .

```
alpha
              exp ret
                             variance
0.000e+00
              1.073e-01
                             2.779e-02
2.500e-01
              1.073e-01
                             2.779e-02
5.000e-01
              1.073e-01
                             2.779e-02
7.500e-01
              1.057e-01
                             2.554e-02
1.000e+00
              9.965e-02
                             1.851e-02
1.500e+00
              8.802e-02
                             8.850e-03
2.000e+00
              8.213e-02
                             5.415e-03
2.500e+00
              7.860e-02
                             3.826e-03
3.000e+00
              7.625e-02
                             2.963e-03
3.500e+00
              7.457e-02
                             2.442e-03
4.000e+00
                             2.104e-03
              7.331e-02
4.500e+00
              7.232e-02
                             1.873e-03
```

### Source code example

The example code in Listing 11.5 demonstrates how to compute the efficient portfolios for several values of  $\alpha$ . The code is mostly similar to the one in Sec. 11.1.1, except the problem is re-optimized in a loop for varying  $\alpha$ .

Listing 11.5: Code implementing model (11.9).

```
namespace mosek.example
{
    class msgclass : mosek.Stream
    {
        string prefix;
        public msgclass (string prfx)
        {
            prefix = prfx;
        }

        public override void streamCB (string msg)
        {
                Console.Write ("{0}{1}", prefix, msg);
        }
    }

    public class portfolio_2_frontier
    {
        public static void Main (String[] args)
        {
            const int n = 3;
            // Since the value infinity is never used, we define
```

```
// 'infinity' symbolic purposes only
double infinity = 0;
double[] mu = {0.1073, 0.0737, 0.0627};
double[,] GT = {
 {0.1667, 0.0232, 0.0013},
 {0.0000, 0.1033, -0.0022},
 {0.0000, 0.0000, 0.0338}
};
double[] x0 = \{0.0, 0.0, 0.0\};
double w = 1.0;
double[] alphas = {0.0, 0.25, 0.5, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5};
int numalphas = 12;
int numvar = 2 * n + 2;
int numcon = n + 1;
//Offset of variables into the API variable.
int offsetx = 0;
int offsets = n;
int offsett = n + 1;
int offsetu = 2 * n + 1;
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
  // Create a task object.
  using (mosek.Task task = new mosek.Task(env, 0, 0))
    // Directs the log task stream to the user specified
    // method msgclass.streamCB
   task.set_Stream (mosek.streamtype.log, new msgclass (""));
    task.appendvars(numvar);
    task.appendcons(numcon);
    //Constraints.
    for (int i = 1; i <= n; ++i)
      w += x0[i - 1];
      task.putconbound(i, mosek.boundkey.fx, 0.0, 0.0);
      task.putconname(i, "GT[" + i + "]");
    for (int i = 0; i < n; ++i)
      task.putaij(0, offsetx + i, 1.0);
    task.putconbound(0, mosek.boundkey.fx, w, w);
    task.putconname(0, "budget");
    // Objective
    for(int i = 0; i < n; i++)</pre>
      task.putcj(offsetx + i, mu[i]);
    for (int i = 0; i < n; ++i)
      // t = GT * x
      for (int j = i; j < n; ++j)
        task.putaij(i + 1, offsetx + j, GT[i, j]);
      // Budget constraint
      task.putaij(i + 1, offsett + i, -1.0);
      // x variable
```

```
task.putvarbound(offsetx + i, mosek.boundkey.lo, 0.0, 0.0);
      task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
      task.putvarname(offsett + i, "t[" + (i + 1) + "]");
      // t variable
      task.putvarbound(offsett + i, mosek.boundkey.fr, -infinity, infinity);
    }
    // s variable
    task.putvarbound(offsets, mosek.boundkey.fr, -infinity, infinity);
    task.putvarname(offsets, "s");
    // u variable
    task.putvarbound(offsetu, mosek.boundkey.fx, 0.5, 0.5);
    task.putvarname(offsetu, "u");
    //Cones.
    int[] csub = new int[n+2];
    csub[0] = offsets;
    csub[1] = offsetu;
    for(int i = 0; i < n; i++)</pre>
      csub[i+2] = offsett + i;
    task.appendcone( mosek.conetype.rquad,
                     0.0, /* For future use only, can be set to 0.0 */
                     csub);
    task.putconename(0, "variance");
    /* A maximization problem */
    task.putobjsense(mosek.objsense.maximize);
    //task.writedata("dump.opf");
    //Turn all log output off.
    task.putintparam(mosek.iparam.log, 0);
    Console.WriteLine("{0,-15}{1,-15}{2,-15}", "alpha", "exp ret", "variance");
    for (int k = 0; k < numalphas; ++k)</pre>
      task.putcj(offsets, -alphas[k]);
      task.optimize();
      task.solutionsummary(mosek.streamtype.log);
      double expret = 0.0;
      double[] xx = new double[numvar];
      task.getxx(mosek.soltype.itr, xx);
      for (int j = 0; j < n; ++j)
        expret += mu[j] * xx[j + offsetx];
      Console.WriteLine("{0:E6} {1:E} {2:E}", alphas[k], expret, xx[offsets]);
    Console.WriteLine("\n");
 }
}
```

```
}
}
}
```

# 11.1.3 Factor model and efficiency

In practice it is often important to solve the portfolio problem very quickly. Therefore, in this section we discuss how to improve computational efficiency at the modeling stage.

The computational cost is of course to some extent dependent on the number of constraints and variables in the optimization problem. However, in practice a more important factor is the sparsity: the number of nonzeros used to represent the problem. Indeed it is often better to focus on the number of nonzeros in G see (11.2) and try to reduce that number by for instance changing the choice of G.

In other words if the computational efficiency should be improved then it is always good idea to start with focusing at the covariance matrix. As an example assume that

$$\Sigma = D + VV^T$$

where D is a positive definite diagonal matrix. Moreover, V is a matrix with n rows and p columns. Such a model for the covariance matrix is called a factor model and usually p is much smaller than n. In practice p tends to be a small number independent of n, say less than 100.

One possible choice for G is the Cholesky factorization of  $\Sigma$  which requires storage proportional to n(n+1)/2. However, another choice is

$$G^T = \left[ \begin{array}{c} D^{1/2} \\ V^T \end{array} \right]$$

because then

$$GG^T = D + VV^T.$$

This choice requires storage proportional to n + pn which is much less than for the Cholesky choice of G. Indeed assuming p is a constant storage requirements are reduced by a factor of n.

The example above exploits the so-called factor structure and demonstrates that an alternative choice of G may lead to a significant reduction in the amount of storage used to represent the problem. This will in most cases also lead to a significant reduction in the solution time.

The lesson to be learned is that it is important to investigate how the covariance matrix is formed. Given this knowledge it might be possible to make a special choice for G that helps reducing the storage requirements and enhance the computational efficiency. More details about this process can be found in [And13].

# 11.1.4 Slippage Cost

The basic Markowitz model assumes that there are no costs associated with trading the assets and that the returns of the assets are independent of the amount traded. Neither of those assumptions is usually valid in practice. Therefore, a more realistic model is

where the function

$$C_j|x_j-x_j^0|$$

specifies the transaction costs when the holding of asset j is changed from its initial value. In the next two sections we show two different variants of this problem with two nonlinear cost functions T.

## 11.1.5 Market Impact Costs

If the initial wealth is fairly small and no short selling is allowed, then the holdings will be small and the traded amount of each asset must also be small. Therefore, it is reasonable to assume that the prices of the assets are independent of the amount traded. However, if a large volume of an asset is sold or purchased, the price, and hence return, can be expected to change. This effect is called market impact costs. It is common to assume that the market impact cost for asset j can be modeled by

$$C_j = m_j \sqrt{|x_j - x_j^0|}$$

where  $m_j$  is a constant that is estimated in some way by the trader. See [GK00] [p. 452] for details. Hence, we have

$$C_j(x_j - x_j^0) = m_j |x_j - x_j^0| \sqrt{|x_j - x_j^0|} = m_j |x_j - x_j^0|^{3/2}.$$

From the Modeling Cookbook we know that  $c \geq z^{3/2}$  can be modeled directly using the power cone  $\mathcal{P}_3^{2/3,1/3}$ :

$$\{(c,z):c\geq z^{3/2},z\geq 0\}=\{(c,z):(c,1,z)\in \mathcal{P}_3^{2/3,1/3}\}$$

Hence, it follows that we can write the model as

$$\begin{aligned}
z_j &= |x_j - x_j^0|, \\
(c_j, 1, z_j) &\in \mathcal{P}_3^{2/3, 1/3}, \\
\sum_{j=1}^n C_j |x_j - x_j^0| &= \sum_{j=1}^n c_j m_j.
\end{aligned}$$

Unfortunately this set of constraints is nonconvex due to the constraint

$$z_j = |x_j - x_j^0| (11.12)$$

but in many cases the constraint may be replaced by the relaxed constraint

$$z_i \ge |x_i - x_i^0|,\tag{11.13}$$

which is equivalent to

$$z_{j} \ge x_{j} - x_{j}^{0}, z_{j} \ge -(x_{j} - x_{j}^{0}).$$
(11.14)

For instance if the universe of assets contains a risk free asset then

$$z_j > |x_j - x_j^0| (11.15)$$

cannot hold for an optimal solution.

If the optimal solution has the property (11.15) then the market impact cost within the model is larger than the true market impact cost and hence money are essentially considered garbage and removed by generating transaction costs. This may happen if a portfolio with very small risk is requested because the only way to obtain a small risk is to get rid of some of the assets by generating transaction costs. We generally assume that this is not the case and hence the models (11.12) and (11.13) are equivalent.

The above observations lead to

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + m^{T}c = w + e^{T}x^{0}$ ,  
 $[\gamma; G^{T}x] \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} \geq x_{j} - x_{j}^{0}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \geq x_{j}^{0} - x_{j}$ ,  $j = 1, \dots, n$ ,  
 $(c_{j}, 1, z_{j}) \in \mathcal{P}_{3}^{2/3, 1/3}$ ,  $j = 1, \dots, n$ ,  
 $x > 0$ . (11.16)

The revised budget constraint

$$e^T x + m^T c = w + e^T x^0$$

specifies that the initial wealth covers the investment and the transaction costs. It should be mentioned that transaction costs of the form

$$t_j \geq z_i^p$$

where p > 1 is a real number can be modeled with the power cone as

$$(t_j, 1, z_j) \in \mathcal{P}_3^{1/p, 1 - 1/p}.$$

See Modeling Cookbook for details.

### Creating a matrix formulation

One more reformulation of (11.16) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^T x$$
  
subject to  $e^T x + m^T c = w + e^T x^0$ ,  
 $G^T x - t = 0$ ,  
 $z_j - x_j \geq -x_j^0$ ,  $j = 1, ..., n$ ,  
 $z_j + x_j \geq x_j^0$ ,  $j = 1, ..., n$ ,  
 $(s,t) \in \mathcal{Q}^{n+1}$ ,  
 $(c_j, f_j, z_j) \in \mathcal{P}_3^{2/3, 1/3}$ ,  $j = 1, ..., n$ ,  
 $x \geq 0$ ,  
 $f_j = 1$ ,  $j = 1, ..., n$ ,  
 $s = \gamma$ , (11.17)

where  $f \in \mathbb{R}^n$  is an additional variable representing the unused coordinate in the power cone. The formulation (11.17) is not the most compact possible, but it is easy to implement. **MOSEK** presolve will automatically simplify it.

The first step in developing the implementation is to chose an ordering of the variables. We will choose the following ordering:

$$\hat{x} = [x; s; t; c; z; f]$$

Table 11.2 shows the mapping between the  $\hat{x}$  vector and the model variables.

	_	
Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1
c	n	2n+1
~	n	$3n \pm 1$

Table 11.2: Storage layout for  $\hat{x}$ 

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^Tx - t = 0$  from Sec. 11.1.1 is shown below.

### Source code example

The example code in Listing 11.6 demonstrates how to implement the model (11.17).

Listing 11.6: Code implementing model (11.17).

```
using System;
namespace mosek.example {
```

```
class msgclass : mosek.Stream
 string prefix;
 public msgclass (string prfx)
   prefix = prfx;
 public override void streamCB (string msg)
    Console.Write ("{0}{1}", prefix, msg);
 }
}
public class portfolio_3_impact
 public static void Main (String[] args)
    const int n = 3;
    // Since the value infinity is never used, we define
    // 'infinity' symbolic purposes only
    double infinity = 0;
    double gamma = 0.05;
    double[] mu = {0.1073, 0.0737, 0.0627};
    double[,] GT = {
     {0.1667, 0.0232, 0.0013},
     {0.0000, 0.1033, -0.0022},
     {0.0000, 0.0000, 0.0338}
    };
    double[] x0 = \{0.0, 0.0, 0.0\};
    double w = 1.0;
    double[] m = {0.01, 0.01, 0.01};
    int offsetx = 0;
    int offsets = offsetx + n;
    int offsett = offsets + 1;
    int offsetc = offsett + n;
    int offsetz = offsetc + n;
    int offsetf = offsetz + n;
    int numvar = 5 * n + 1;
    int offset_con_budget = 0;
    int offset_con_gx_t = offset_con_budget + 1;
    int offset_con_abs1 = offset_con_gx_t + n;
    int offset_con_abs2 = offset_con_abs1 + n;
    int numcon = 3 * n + 1;
    // Make mosek environment.
    using (mosek.Env env = new mosek.Env())
      // Create a task object.
      using (mosek.Task task = new mosek.Task(env, 0, 0))
        // Directs the log task stream to the user specified
        // method msqclass.streamCB
        task.set_Stream(mosek.streamtype.log, new msgclass(""));
        //Set up constraint bounds, names and variable coefficients
```

```
task.appendcons(numcon);
for (int i = 0; i < n; ++i)
  w += x0[i];
  task.putconbound(offset_con_gx_t + i, mosek.boundkey.fx, 0.0, 0.0);
  task.putconname(offset_con_gx_t + i, "GT[" + (i + 1) + "]");
  task.putconbound(offset_con_abs1 + i, mosek.boundkey.lo, -x0[i], infinity);
 task.putconname(offset_con_abs1 + i, "zabs1[" + (i + 1) + "]");
 task.putconbound(offset_con_abs2 + i, mosek.boundkey.lo, x0[i], infinity);
  task.putconname(offset_con_abs2 + i, "zabs2[" + (i + 1) + "]");
// e x = w + e x0
task.putconbound(offset_con_budget, mosek.boundkey.fx, w, w);
task.putconname(offset_con_budget, "budget");
//Variables.
task.appendvars(numvar);
//the objective function coefficients
for (int i = 0; i < n; i++)
  task.putcj(offsetx + i, mu[i]);
double[] one_m_one = { 1.0, -1.0 };
double[] one_one = { 1.0, 1.0 };
//set up variable bounds and names
for (int i = 0; i < n; ++i)
{
  task.putvarbound(offsetx + i, mosek.boundkey.lo, 0.0, infinity);
  task.putvarbound(offsett + i, mosek.boundkey.fr, -infinity);
  task.putvarbound(offsetc + i, mosek.boundkey.fr, -infinity);
  task.putvarbound(offsetz + i, mosek.boundkey.fr, -infinity, infinity);
  task.putvarbound(offsetf + i, mosek.boundkey.fx, 1.0, 1.0);
  task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
  task.putvarname(offsett + i, "t[" + (i + 1) + "]");
  task.putvarname(offsetc + i, "c[" + (i + 1) + "]");
  task.putvarname(offsetz + i, "z[" + (i + 1) + "]");
  task.putvarname(offsetf + i, "f[" + (i + 1) + "]");
  for (int j = i; j < n; ++j)
    task.putaij(offset_con_gx_t + i, j, GT[i, j]);
  task.putaij(offset_con_gx_t + i, offsett + i, -1.0);
  task.putaij(offset_con_budget, offsetx + i, 1.0);
  task.putaij(offset_con_budget, offsetc + i, m[i]);
  // z_{-}j - x_{-}j >= -x0_{-}j
  int[] indx1 = { offsetz + i, offsetx + i };
  task.putarow(offset_con_abs1 + i, indx1, one_m_one);
  // z_{-}j + x_{-}j >= +x0_{-}j
  int[] indx2 = { offsetz + i, offsetx + i };
  task.putarow(offset_con_abs2 + i, indx2, one_one);
task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
```

```
task.putvarname(offsets, "s");
          // Quaadratic cone
          int[] csub = new int[n+1];
          csub[0] = offsets;
          for(int i = 0; i < n; i++) csub[i + 1] = offsett + i;
          task.appendcone(mosek.conetype.quad, 0.0, csub);
          task.putconename(0, "stddev");
          // Power cones
          for (int j = 0; j < n; ++j)
            int[] coneindx = { offsetc + j, offsetf + j, offsetz + j };
           task.appendcone(mosek.conetype.ppow, 2.0/3.0, coneindx);
            task.putconename(1 + j, "trans[" + (j + 1) + "]");
          /* A maximization problem */
          task.putobjsense(mosek.objsense.maximize);
          //Turn all log output off.
          //task.putintparam(mosek.iparam.log,0);
          //task.writedata("dump.opf");
          /* Solve the problem */
          task.optimize();
          task.solutionsummary(mosek.streamtype.log);
          double expret = 0.0;
          double[] xx = new double[numvar];
          task.getxx(mosek.soltype.itr, xx);
          for (int j = 0; j < n; ++j)
           expret += mu[j] * xx[j + offsetx];
          Console.WriteLine("Expected return {0:E6} for gamma {1:E6}\n\n", expret,
}
      }
   }
 }
}
```

The example code above produces the result

```
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 7.4390639578e-02
                                   nrm: 1e+00
                                                 Viol. con: 1e-08
                                                                     var: 0e+00
                                                                                   cones: 7e-
⇔09
 Dual.
          obj: 7.4390755614e-02
                                   nrm: 3e-01
                                                 Viol. con: 1e-19
                                                                      var: 3e-08
                                                                                   cones:
-0e+00
Expected return 7.439064e-02 for gamma 5.000000e-02
```

If the problem is dumped to an OPF file, it has the following content.

Listing 11.7: OPF file for problem (11.17).

```
[comment]
   Written by MOSEK version 9.0.0.31
   Date 10-01-18
   Time 12:10:24
[/comment]
[hints]
  [hint NUMVAR] 16 [/hint]
  [hint NUMCON] 10 [/hint]
  [hint NUMANZ] 27 [/hint]
  [hint NUMQNZ] 0 [/hint]
  [hint NUMCONE] 4 [/hint]
[/hints]
[variables disallow_new_variables]
  'x[1]' 'x[2]' 'x[3]' s 't[1]'
  't[2]' 't[3]' 'c[1]' 'c[2]' 'c[3]'
  'z[1]' 'z[2]' 'z[3]' 'f[1]' 'f[2]'
  'f[3]'
[/variables]
[objective maximize]
   1.073e-01 \ 'x[1]' + 7.37e-02 \ 'x[2]' + 6.27000000000001e-02 \ 'x[3]'
[/objective]
[constraints]
  [con 'budget'] 'x[1]' + 'x[2]' + 'x[3]' + 1e-02 'c[1]' + 1e-02 'c[2]'
     + 1e-02 'c[3]' = 1e+00 [/con]
 [con 'GT[1]'] 1.667e-01 'x[1]' + 2.32e-02 'x[2]' + 1.3e-03 'x[3]' - 't[1]' = 0e+00 [/con] [con 'GT[2]'] 1.033e-01 'x[2]' - 2.2e-03 'x[3]' - 't[2]' = 0e+00 [/con] [con 'GT[3]'] 3.38e-02 'x[3]' - 't[3]' = 0e+00 [/con]
  [con 'zabs1[1]'] 0e+00 <= - 'x[1]' + 'z[1]' [/con]
  [con 'zabs1[2]'] 0e+00 \le - 'x[2]' + 'z[2]' [/con]
  [con 'zabs1[3]'] 0e+00 <= - 'x[3]' + 'z[3]' [/con]
  [con 'zabs2[1]'] Oe+OO <= 'x[1]' + 'z[1]' [/con]
  [con 'zabs2[2]'] 0e+00 \le 'x[2]' + 'z[2]' [/con]
  [con 'zabs2[3]'] 0e+00 \le 'x[3]' + 'z[3]' [/con]
[/constraints]
[bounds]
  [b] 0e+00
                  <= 'x[1]','x[2]','x[3]' [/b]
  [b]
                      s = 5e-02 [/b]
  [b]
                      't[1]','t[2]','t[3]','c[1]','c[2]','c[3]' free [/b]
                      'z[1]','z[2]','z[3]' free [/b]
  ГъЪ
                      'f[1]', 'f[2]', 'f[3]' = 1e+00 [/b]
  ГъЪ
  [cone quad 'stddev'] s, 't[1]', 't[2]', 't[3]' [/cone]
  [cone ppow '6.666666666666666666001' 'trans[1]'] 'c[1]', 'f[1]', 'z[1]' [/cone]
  [cone ppow '6.66666666666666666001' 'trans[2]'] 'c[2]', 'f[2]', 'z[2]' [/cone]
  [cone ppow '6.6666666666666666e-01' 'trans[3]'] 'c[3]', 'f[3]', 'z[3]' [/cone]
[/bounds]
```

The file verifies that the correct problem has been set up.

### 11.1.6 Transaction Costs

Now assume there is a cost associated with trading asset j given by

$$T_j(\Delta x_j) = \begin{cases} 0, & \Delta x_j = 0, \\ f_j + g_j |\Delta x_j|, & \text{otherwise.} \end{cases}$$

Here  $\Delta x_j$  is the change in the holding of asset j i.e.

$$\Delta x_j = x_j - x_j^0.$$

Hence, whenever asset j is traded we pay a fixed setup cost  $f_j$  and a variable cost of  $g_j$  per unit traded. This sort of cost function can be modeled using mixed-integer optimization, in particular using a binary variable  $y_i$  to indicate if asset j is traded. Given the assumptions about transaction costs in this section problem (11.11) may be formulated as

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x + f^{T}y + g^{T}z = w + e^{T}x^{0}$ ,  
 $[\gamma; G^{T}x] \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} \geq x_{j} - x_{j}^{0}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \geq x_{j}^{0} - x_{j}$ ,  $j = 1, \dots, n$ ,  
 $z_{j} \leq U_{j}y_{j}$ ,  $j = 1, \dots, n$ ,  
 $y_{j} \in \{0, 1\}$ ,  $j = 1, \dots, n$ ,  
 $x \geq 0$ . (11.18)

First observe that

$$z_j \ge |x_j - x_j^0| = |\Delta x_j|.$$

Here  $U_j$  is some a priori chosen upper bound on the amount of trading in asset j and therefore if  $z_j > 0$ then  $y_j = 1$  has to be the case. This implies that the transaction cost for asset j is given by

$$f_i y_i + g_i z_i$$
.

In our problem a safe bound for each  $U_j$  is the total initial wealth  $w + e^T x^0$ , however knowing a tighter bound may lead to shorter solution times.

## Creating a matrix formulation

One more reformulation of (11.18) is needed to bring it to the standard form (11.4).

mulation of (11.18) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^T x$$
subject to  $e^T x + f^T y + g^T z = w + e^T x^0$ ,

 $G^T x - t = 0$ ,
 $z_j - x_j \geq -x_j^0, \quad j = 1, \dots, n$ ,
 $z_j + x_j \geq x_j^0, \quad j = 1, \dots, n$ ,
 $(s,t) \in \mathcal{Q}^{n+1}, \quad (11.19)$ 
 $z_j - U_j y_j \leq 0, \quad j = 1, \dots, n$ ,
 $x \geq 0$ ,
 $y_j \in [0,1], \quad j = 1, \dots, n$ ,
 $y_j \in \mathbb{Z}, \quad j = 1, \dots, n$ ,
 $s = \gamma$ .

We will choose the following ordering of variables:

$$\hat{x} = [x; s; t; z; y]$$

Table 11.3 shows the mapping between the  $\hat{x}$  vector and the model variables.

Table 11.3: Storage layout for  $\hat{x}$ 

Variable	Length	Offset
x	n	0
s	1	n
t	n	n+1
z	n	2n+1
y	n	3n + 1

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^Tx - t = 0$  from Sec. 11.1.1 is shown below.

### Example code

The following example code demonstrates how to compute an optimal portfolio when transaction setup costs are included. Note that we are now solving a problem with integer variables, and therefore the solution must be retrieved from soltype.itg rather than soltype.itr.

Listing 11.8: Code solving problem (11.18).

```
using System;
namespace mosek.example {
 class msgclass : mosek.Stream
   string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
   }
 }
 public class portfolio_4_transcost
   public static void Main (String[] args)
      const int n = 3;
      // Since the value infinity is never used, we define
      // 'infinity' symbolic purposes only
      double infinity = 0;
      double gamma = 0.05;
      double[] mu = {0.1073, 0.0737, 0.0627};
      double[,] GT = {
        {0.1667, 0.0232, 0.0013},
        {0.0000, 0.1033, -0.0022},
        {0.0000, 0.0000, 0.0338}
      };
      double[] x0 = \{0.0, 0.0, 0.0\};
      double w = 1.0;
      double[] f = {0.01, 0.01, 0.01};
      double[] g = {0.001, 0.001, 0.001};
      // Total initial wealth
      double U = w;
      for(int i=0; i< n; i++) U += x0[i];</pre>
      int offsetx = 0;
      int offsets = offsetx + n;
      int offsett = offsets + 1;
      int offsetz = offsett + n;
      int offsety = offsetz + n;
      int numvar = 4 * n + 1;
```

```
int offset_con_budget = 0;
int offset_con_gx_t = offset_con_budget + 1;
int offset_con_abs1 = offset_con_gx_t + n;
int offset_con_abs2 = offset_con_abs1 + n;
int offset_con_ind = offset_con_abs2 + n;
int numcon = 4 * n + 1;
// Make mosek environment.
using (mosek.Env env = new mosek.Env())
  // Create a task object.
  using (mosek.Task task = new mosek.Task(env, 0, 0))
    // Directs the log task stream to the user specified
    // method msgclass.streamCB
    task.set_Stream(mosek.streamtype.log, new msgclass(""));
    //Set up constraint bounds, names and variable coefficients
    task.appendcons(numcon);
    for (int i = 0; i < n; ++i)
      task.putconbound(offset_con_gx_t + i, mosek.boundkey.fx, 0.0, 0.0);
      task.putconname(offset_con_gx_t + i, "GT[" + (i + 1) + "]");
      task.putconbound(offset_con_abs1 + i, mosek.boundkey.lo, -x0[i], infinity);
      task.putconname(offset_con_abs1 + i, "zabs1[" + (i + 1) + "]");
      task.putconbound(offset_con_abs2 + i, mosek.boundkey.lo, x0[i], infinity);
      task.putconname(offset_con_abs2 + i, "zabs2[" + (i + 1) + "]");
      task.putconbound(offset_con_ind + i, mosek.boundkey.up, -infinity, 0.0);
      task.putconname(offset_con_ind + i, "ind[" + (i + 1) + "]");
    // e x = w + e x0
    task.putconbound(offset_con_budget, mosek.boundkey.fx, U, U);
    task.putconname(offset_con_budget, "budget");
    //Variables.
    task.appendvars(numvar);
    //the objective function coefficients
    for (int i = 0; i < n; i++)
    {
      task.putcj(offsetx + i, mu[i]);
    double[] one_m_one = { 1.0, -1.0 };
    double[] one_one = { 1.0, 1.0 };
    //set up variable bounds and names
    for (int i = 0; i < n; ++i)
      task.putvarbound(offsetx + i, mosek.boundkey.lo, 0.0, infinity);
      task.putvarbound(offsett + i, mosek.boundkey.fr, -infinity, infinity);
      task.putvarbound(offsetz + i, mosek.boundkey.fr, -infinity, infinity);
      task.putvarbound(offsety + i, mosek.boundkey.ra, 0.0, 1.0);
      task.putvartype(offsety + i, mosek.variabletype.type_int);
      task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
```

```
task.putvarname(offsett + i, "t[" + (i + 1) + "]");
           task.putvarname(offsetz + i, "z[" + (i + 1) + "]");
           task.putvarname(offsety + i, "y[" + (i + 1) + "]");
           for (int j = i; j < n; ++j)
             task.putaij(offset_con_gx_t + i, j, GT[i, j]);
           task.putaij(offset_con_gx_t + i, offsett + i, -1.0);
           task.putaij(offset_con_budget, offsetx + i, 1.0);
           task.putaij(offset_con_budget, offsetz + i, g[i]);
           task.putaij(offset_con_budget, offsety + i, f[i]);
           // z_{j} - x_{j} > = -x0_{j}
           int[] indx1 = { offsetz + i, offsetx + i };
           task.putarow(offset_con_abs1 + i, indx1, one_m_one);
           // z_{-}j + x_{-}j >= +x0_{-}j
           int[] indx2 = { offsetz + i, offsetx + i };
           task.putarow(offset_con_abs2 + i, indx2, one_one);
           // z_{j} - U*y_{j} <= 0
           int[] indx3 = {offsetz + i, offsety + i};
           double[] va = {1.0, -U};
           task.putarow(offset_con_ind + i, indx3, va);
         task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
         task.putvarname(offsets, "s");
         // Quaadratic cone
         int[] csub = new int[n+1];
         csub[0] = offsets;
         for(int i = 0; i < n; i++) csub[i + 1] = offsett + i;
         task.appendcone(mosek.conetype.quad, 0.0, csub);
         task.putconename(0, "stddev");
         /* A maximization problem */
         task.putobjsense(mosek.objsense.maximize);
         //Turn all log output off.
         //task.putintparam(mosek.iparam.log,0);
         //task.writedata("dump.opf");
         /* Solve the problem */
         task.optimize();
         task.solutionsummary(mosek.streamtype.log);
         double expret = 0.0;
         double[] xx = new double[numvar];
         task.getxx(mosek.soltype.itg, xx);
         for (int j = 0; j < n; ++j)
           expret += mu[j] * xx[j + offsetx];
         Console.WriteLine("Expected return {0:E6} for gamma {1:E6}\n\n", expret,
}
   }
 }
```

# 11.1.7 Cardinality constraints

Another method to reduce costs involved with processing transactions is to only change positions in a small number of assets. In other words, at most k of the differences  $|\Delta x_j| = |x_j - x_j^0|$  are allowed to be non-zero, where k is (much) smaller than the total number of assets n.

This type of constraint can be again modeled by introducing a binary variable  $y_j$  which indicates if  $\Delta x_i \neq 0$  and bounding the sum of  $y_i$ . The basic Markowitz model then gets updated as follows:

maximize 
$$\mu^T x$$
 subject to  $e^T x = w + e^T x^0$ ,  $[\gamma; G^T x] \in \mathcal{Q}^{n+1}$ ,  $U_j y_j \geq |x_j - x_j^0|, \quad j = 1, \dots, n,$   $y_j \in \{0, 1\}, \quad j = 1, \dots, n,$   $e^T y \leq k,$   $x \geq 0,$  ori chosen upper bound on the amount of trading in asset  $j$ . This guara

were  $U_j$  is some a priori chosen upper bound on the amount of trading in asset j. This guarantees that  $|x_j - x_j^0|$  forces  $y_j = 1$  and therefore  $e^T y$  counts the number of assets in which we trade. In our problem a safe bound for each  $U_j$  is the total initial wealth  $w + e^T x^0$ , however knowing a tighter bound may lead to shorter solution times.

## Creating a matrix formulation

One more reformulation of (11.20) is needed to bring it to the standard form (11.4).

maximize 
$$\mu^{T}x$$
  
subject to  $e^{T}x = w + e^{T}x^{0}$ ,  
 $G^{T}x - t = 0$ ,  
 $z_{j} - x_{j} \geq -x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $z_{j} + x_{j} \geq x_{j}^{0}$ ,  $j = 1, ..., n$ ,  
 $(s,t) \in \mathcal{Q}^{n+1}$ ,  
 $z_{j} - U_{j}y_{j} \leq 0$ ,  $j = 1, ..., n$ ,  
 $e^{T}y \leq k$ ,  
 $x \geq 0$ ,  
 $y_{j} \in [0,1]$ ,  $j = 1, ..., n$ ,  
 $y_{j} \in \mathbb{Z}$ ,  $j = 1, ..., n$ ,  
 $s = \gamma$ . (11.21)

We will choose the following ordering of variables:

$$\hat{x} = [x; s; t; z; y]$$

Table 11.4 shows the mapping between the  $\hat{x}$  vector and the model variables.

 $\begin{tabular}{|c|c|c|c|c|c|c|c|} \hline Variable & Length & Offset \\ \hline $x$ & $n$ & $0$ \\ \hline $s$ & $1$ & $n$ \\ \hline $t$ & $n$ & $n+1$ \\ \hline $z$ & $n$ & $2n+1$ \\ \hline $y$ & $n$ & $3n+1$ \\ \hline \end{tabular}$ 

Table 11.4: Storage layout for  $\hat{x}$ 

The next step is to consider how the linear constraint matrix A and the remaining data vectors are laid out. Reusing the idea in Sec. 11.1.1 we can write the data in block matrix form and read off all the required coordinates. This extension of the code setting up the constraint  $G^Tx - t = 0$  from Sec. 11.1.1 is shown below.

### Example code

The following example code demonstrates how to compute an optimal portfolio with cardinality bounds. Note that we are now solving a problem with integer variables, and therefore the solution must be retrieved from <code>soltype.itg</code>.

Listing 11.9: Code solving problem (11.20).

```
public static double[] markowitz_with_card(int
                                            double[]
                                                       x0,
                                            double
                                                       W.
                                            double
                                                       gamma,
                                            double[]
                                                      mu,
                                            double[,] GT,
                                                       k)
  // Total initial wealth
  double U = w;
  for(int i=0; i< n; i++) U += x0[i];
  int offsetx = 0;
  int offsets = offsetx + n;
  int offsett = offsets + 1;
  int offsetz = offsett + n;
  int offsety = offsetz + n;
  int numvar = 4 * n + 1;
  int offset_con_budget = 0;
  int offset_con_gx_t = offset_con_budget + 1;
  int offset_con_abs1 = offset_con_gx_t + n;
  int offset_con_abs2 = offset_con_abs1 + n;
  int offset_con_ind = offset_con_abs2 + n;
  int offset_con_card = offset_con_ind + n;
  int numcon = 4 * n + 2;
  // Make mosek environment.
  using (mosek.Env env = new mosek.Env())
    // Create a task object.
    using (mosek.Task task = new mosek.Task(env, 0, 0))
      // Directs the log task stream to the user specified
      // method msgclass.streamCB
      task.set_Stream(mosek.streamtype.log, new msgclass(""));
      //Set up constraint bounds, names and variable coefficients
      task.appendcons(numcon);
      for (int i = 0; i < n; ++i)
        task.putconbound(offset_con_gx_t + i, mosek.boundkey.fx, 0.0, 0.0);
        task.putconname(offset\_con\_gx\_t + i, "GT[" + (i + 1) + "]");
        task.putconbound(offset_con_abs1 + i, mosek.boundkey.lo, -x0[i], infinity);
        task.putconname(offset_con_abs1 + i, "zabs1[" + (i + 1) + "]");
        task.putconbound(offset_con_abs2 + i, mosek.boundkey.lo, x0[i], infinity);
        task.putconname(offset_con_abs2 + i, "zabs2[" + (i + 1) + "]");
        task.putconbound(offset_con_ind + i, mosek.boundkey.up, -infinity, 0.0);
        task.putconname(offset_con_ind + i, "ind[" + (i + 1) + "]");
```

```
// e x = w + e x0
task.putconbound(offset_con_budget, mosek.boundkey.fx, U, U);
task.putconname(offset_con_budget, "budget");
// sum(y) \ll k
task.putconbound(offset_con_card, mosek.boundkey.up, -infinity, k);
task.putconname(offset_con_card, "cardinality");
//Variables.
task.appendvars(numvar);
//the objective function coefficients
for (int i = 0; i < n; i++)
  task.putcj(offsetx + i, mu[i]);
double[] one_m_one = { 1.0, -1.0 };
double[] one_one = { 1.0, 1.0 };
//set up variable bounds and names
for (int i = 0; i < n; ++i)
  task.putvarbound(offsetx + i, mosek.boundkey.lo, 0.0, infinity);
 task.putvarbound(offsett + i, mosek.boundkey.fr, -infinity, infinity);
  task.putvarbound(offsetz + i, mosek.boundkey.fr, -infinity, infinity);
  task.putvarbound(offsety + i, mosek.boundkey.ra, 0.0, 1.0);
  task.putvartype(offsety + i, mosek.variabletype.type_int);
  task.putvarname(offsetx + i, "x[" + (i + 1) + "]");
  task.putvarname(offsett + i, "t[" + (i + 1) + "]");
  task.putvarname(offsetz + i, "z[" + (i + 1) + "]");
  task.putvarname(offsety + i, "y[" + (i + 1) + "]");
  for (int j = i; j < n; ++j)
    task.putaij(offset_con_gx_t + i, j, GT[i, j]);
  task.putaij(offset_con_gx_t + i, offsett + i, -1.0);
  task.putaij(offset_con_budget, offsetx + i, 1.0);
  // z_{j} - x_{j} > = -x0_{j}
  int[] indx1 = { offsetz + i, offsetx + i };
  task.putarow(offset_con_abs1 + i, indx1, one_m_one);
  // z_{-}j + x_{-}j > = +x0_{-}j
  int[] indx2 = { offsetz + i, offsetx + i };
  task.putarow(offset_con_abs2 + i, indx2, one_one);
  // z_{j} - U*y_{j} <= 0
  int[] indx3 = {offsetz + i, offsety + i};
  double[] va = {1.0, -U};
 task.putarow(offset_con_ind + i, indx3, va);
  // sum(y)
  task.putaij(offset_con_card, offsety + i, 1.0);
task.putvarbound(offsets, mosek.boundkey.fx, gamma, gamma);
task.putvarname(offsets, "s");
// Quaadratic cone
int[] csub = new int[n+1];
```

```
csub[0] = offsets;
      for(int i = 0; i < n; i++) csub[i + 1] = offsett + i;
      task.appendcone(mosek.conetype.quad, 0.0, csub);
      task.putconename(0, "stddev");
      /* A maximization problem */
      task.putobjsense(mosek.objsense.maximize);
      //Turn all log output off.
      task.putintparam(mosek.iparam.log,0);
      //task.writedata("dump.opf");
      /* Solve the problem */
      task.optimize();
      task.solutionsummary(mosek.streamtype.log);
      double[] xx = new double[n];
      task.getxxslice(mosek.soltype.itg, offsetx, offsetx + n, xx);
      return xx;
  }
}
```

If we solve our running example with k = 1, 2, 3 then we get the following solutions, with increasing expected returns:

```
Bound 1: x = 0.00000 0.00000 1.00000 Return: x = 0.06270
Bound 2: x = 0.25286 0.00000 0.74714 Return: x = 0.07398
Bound 3: x = 0.23639 0.13850 0.62511 Return: x = 0.07477
```

# 11.2 Logistic regression

Logistic regression is an example of a binary classifier, where the output takes one two values 0 or 1 for each data point. We call the two values *classes*.

#### Formulation as an optimization problem

Define the sigmoid function

$$S(x) = \frac{1}{1 + \exp(-x)}.$$

Next, given an observation  $x \in \mathbb{R}^d$  and a weights  $\theta \in \mathbb{R}^d$  we set

$$h_{\theta}(x) = S(\theta^T x) = \frac{1}{1 + \exp(-\theta^T x)}.$$

The weights vector  $\theta$  is part of the setup of the classifier. The expression  $h_{\theta}(x)$  is interpreted as the probability that x belongs to class 1. When asked to classify x the returned answer is

$$x \mapsto \begin{cases} 1 & h_{\theta}(x) \ge 1/2, \\ 0 & h_{\theta}(x) < 1/2. \end{cases}$$

When training a logistic regression algorithm we are given a sequence of training examples  $x_i$ , each labelled with its class  $y_i \in \{0,1\}$  and we seek to find the weights  $\theta$  which maximize the likelihood function

$$\prod_{i} h_{\theta}(x_{i})^{y_{i}} (1 - h_{\theta}(x_{i}))^{1 - y_{i}}.$$

Of course every single  $y_i$  equals 0 or 1, so just one factor appears in the product for each training data point. By taking logarithms we can define the logistic loss function:

$$J(\theta) = -\sum_{i:y_i=1} \log(h_{\theta}(x_i)) - \sum_{i:y_i=0} \log(1 - h_{\theta}(x_i)).$$

The training problem with regularization (a standard technique to prevent overfitting) is now equivalent to

$$\min_{\theta} J(\theta) + \lambda \|\theta\|_2.$$

This can equivalently be phrased as

minimize 
$$\sum_{i} t_{i} + \lambda r$$
subject to 
$$\begin{aligned} t_{i} &\geq -\log(h_{\theta}(x)) &= \log(1 + \exp(-\theta^{T} x_{i})) & \text{if } y_{i} = 1, \\ t_{i} &\geq -\log(1 - h_{\theta}(x)) &= \log(1 + \exp(\theta^{T} x_{i})) & \text{if } y_{i} = 0, \\ r &\geq \|\theta\|_{2}. \end{aligned}$$

$$(11.22)$$

#### **Implementation**

As can be seen from (11.22) the key point is to implement the softplus bound  $t \ge \log(1 + e^u)$ , which is the simplest example of a log-sum-exp constraint for two scalar variables t, u. This is equivalent to

$$\exp(u-t) + \exp(-t) \le 1$$

and further to

$$(z_1, 1, u - t) \in K_{\exp} (z_1 \ge \exp(u - t)),$$
  
 $(z_2, 1, -t) \in K_{\exp} (z_2 \ge \exp(-t)),$   
 $z_1 + z_2 \le 1.$  (11.23)

To feed these constraints into **MOSEK** we add more auxiliary variables  $q_1, q_2, v_1, v_2$  with constraints  $(z_1, q_1, v_1) \in K_{\text{exp}}, (z_2, q_2, v_2) \in K_{\text{exp}}, q_1 = q_2 = 1, v_1 = u - t \text{ and } v_2 = -t.$ 

Listing 11.10: Implementation of  $t \ge \log(1 + e^u)$  as in (11.23).

```
//t \geq \log(1 + \exp(u))
// t_i >= log(1 + exp(u_i)), i = 0..n-1
// Adds auxiliary variables and constraints
public static void softplus(Task task, int t, int u, int n)
  int nvar = task.getnumvar();
  int ncon = task.getnumcon();
  task.appendvars(6*n);
  task.appendcons(3*n);
  int z1 = nvar, z2 = nvar+n, v1 = nvar+2*n, v2 = nvar+3*n, q1 = nvar+4*n, q2 = nvar+5*n;
  int zcon = ncon, v1con = ncon+n, v2con=ncon+2*n;
           subi = new int[7*n];
          subj = new int[7*n];
  double[] aval = new double[7*n];
             k = 0:
  // Linear constraints
  for(int i = 0; i < n; i++)
    // z1 + z2 = 1
    subi[k] = zcon+i; subj[k] = z1+i; aval[k] = 1; k++;
    subi[k] = zcon+i; subj[k] = z2+i; aval[k] = 1; k++;
    // u - t - v1 = 0
    subi[k] = v1con+i; subj[k] = u+i; aval[k] = 1; k++;
```

```
subi[k] = v1con+i; subj[k] = t+i; aval[k] = -1; k++;
  subi[k] = v1con+i; subj[k] = v1+i; aval[k] = -1; k++;
  // - t - v2 = 0
  subi[k] = v2con+i; subj[k] = t+i; aval[k] = -1; k++;
  subi[k] = v2con+i; subj[k] = v2+i; aval[k] = -1; k++;
task.putaijlist(subi, subj, aval);
task.putconboundsliceconst(ncon, ncon+n, boundkey.fx, 1, 1);
task.putconboundsliceconst(ncon+n, ncon+3*n, boundkey.fx, 0, 0);
// Bounds for variables
task.putvarboundsliceconst(nvar, nvar+4*n, boundkey.fr, -inf, inf);
task.putvarboundsliceconst(nvar+4*n, nvar+6*n, boundkey.fx, 1, 1);
// Cones
for(int i = 0; i < n; i++)</pre>
  task.appendcone(conetype.pexp, 0.0, new int[]{z1+i, q1+i, v1+i});
  task.appendcone(conetype.pexp, 0.0, new int[]{z2+i, q2+i, v2+i});
}
```

Once we have this subroutine, it is easy to implement a function that builds the regularized loss function model (11.22).

Listing 11.11: Implementation of (11.22).

```
// Model logistic regression (regularized with full 2-norm of theta)
// X - n x d matrix of data points
// y - length n vector classifying training points
// lamb - regularization parameter
public static double[] logisticRegression(Env
                                                      env.
                                           double[,]
                                                      Х,
                                           bool[]
                                                      у,
                                           double
                                                      lamb)
{
  int n = X.GetLength(0);
  int d = X.GetLength(1);
                                // num samples, dimension
  using (Task task = new Task(env, 0, 0))
    // Variables [r; theta; t; u]
    int nvar = 1+d+2*n;
    task.appendvars(nvar);
    task.putvarboundsliceconst(0, nvar, boundkey.fr, -inf, inf);
    int r = 0, theta = 1, t = 1+d, u = 1+d+n;
    // Constraints: theta'*X +/- u = 0
    task.appendcons(n);
    task.putconboundsliceconst(0, n, boundkey.fx, 0, 0);
    // Objective lambda*r + sum(t)
    task.putcj(r, lamb);
    for(int i = 0; i < n; i++)</pre>
      task.putcj(t+i, 1.0);
    // The X block in theta'*X +/- u = 0
    int[]
             subi = new int[d*n+n];
             subj
                   = new int[d*n+n];
    int[]
    double[] aval
                   = new double[d*n+n];
    int
                   = 0;
```

```
for(int i = 0; i < n; i++)
      for(int j = 0; j < d; j++)
        subi[k] = i; subj[k] = theta+j; aval[k] = X[i,j]; k++;
      subi[d*n+i] = i; subj[d*n+i] = u+i;
      if (y[i]) aval[d*n+i] = 1; else aval[d*n+i] = -1;
    task.putaijlist(subi, subj, aval);
    // Softplus function constraints
    softplus(task, t, u, n);
    // Regularization
    task.appendconeseq(conetype.quad, 0.0, 1+d, r);
    // Solution
    task.optimize();
    double[] xx = new double[d];
    task.getxxslice(soltype.itr, theta, theta+d, xx);
    return xx;
}
```

## Example: 2D dataset fitting

In the next figure we apply logistic regression to the training set of 2D points taken from the example ex2data2.txt. The two-dimensional dataset was converted into a feature vector  $x \in \mathbb{R}^{28}$  using monomial coordinates of degrees at most 6.

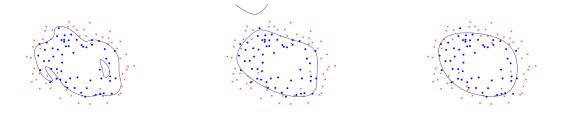


Fig. 11.1: Logistic regression example with none, medium and strong regularization (small, medium, large  $\lambda$ ). Without regularization we get obvious overfitting.

# 11.3 Concurrent optimizer

The idea of the concurrent optimizer is to run multiple optimizations of **the same problem** simultaneously, and pick the one that provides the fastest or best answer. This approach is especially useful for problems which require a very long time and it is hard to say in advance which optimizer or algorithm will perform best.

The major applications of concurrent optimization we describe in this section are:

• Using the interior-point and simplex optimizers simultaneously on a linear problem. Note that any solution present in the task will also be used for hot-starting the simplex algorithms. One possible scenario would therefore be running a hot-start simplex in parallel with interior point,

taking advantage of both the stability of the interior-point method and the ability of the simplex method to use an initial solution.

• Using multiple instances of the mixed-integer optimizer to solve many copies of one mixed-integer problem. This is not in contradiction with the run-to-run determinism of **MOSEK** if a different value of the MIO seed parameter <code>iparam.mio\_seed</code> is set in each instance. As a result each setting leads to a different optimizer run (each of them being deterministic in its own right).

The downloadable file contains usage examples of both kinds.

## 11.3.1 Common setup

We first define a method that runs a number of optimization tasks in parallel, using the standard multithreading setup available in the language. All tasks register for a callback function which will signal them to interrupt as soon as the first task completes successfully (with response code rescode. ok).

Listing 11.12: Simple callback function which signals the optimizer to stop.

```
/**
    Defines a Mosek callback function whose only function
    is to indicate if the optimizer should be stopped.
    */
private class CallbackProxy : mosek.Progress
{
    private bool stop;
    public CallbackProxy()
    {
        stop = false;
    }

    public override int progressCB(mosek.callbackcode caller)
    {
            // Return non-zero implies terminate the optimizer
            return stop ? 1 : 0;
    }

    public bool Stop
    {
        get { return stop; }
        set { stop = value; }
    }
}
```

When all remaining tasks respond to the stop signal, response codes and statuses are returned to the caller, together with the index of the task which won the race.

Listing 11.13: A routine for parallel task race.

```
// Initialize
for (var i = 0; i < n; ++i)
 res[i] = mosek.rescode.err_unknown;
 trm[i] = mosek.rescode.err_unknown;
}
// Start parallel optimizations, one per task
for (var i = 0; i < n; ++i)
  int num = i;
  jobs[i] = System.Threading.Tasks.Task.Factory.StartNew( () => {
      trm[num] = tasks[num].optimize();
     res[num] = mosek.rescode.ok;
    catch (mosek.Exception e)
      trm[num] = mosek.rescode.err_unknown;
     res[num] = e.Code;
    finally
      // If this finished with success, inform other tasks to interrupt
      if (res[num] == mosek.rescode.ok)
        if (!cb.Stop) firstStop = num;
        cb.Stop = true;
    }
 });
// Join all threads
foreach (var j in jobs)
  j.Wait();
// For debugging, print res and trm codes for all optimizers
for (var i = 0; i < n; ++i)
  Console.WriteLine("Optimizer {0} res {1} trm {2}", i, res[i], trm[i]);
firstOK = firstStop;
return cb.Stop;
```

## 11.3.2 Linear optimization

We use the multithreaded setup to run the interior-point and simplex optimizers simultaneously on a linear problem. The next methods simply clones the given task and sets a different optimizer for each. The result is the clone which finished first.

Listing 11.14: Concurrent optimization with different optimizers.

```
var n = optimizers.Length;
 var tasks = new mosek.Task[n];
  var res = new mosek.rescode[n];
          = new mosek.rescode[n];
  // Clone tasks and choose various optimizers
 for (var i = 0; i < n; ++i)
    tasks[i] = new mosek.Task(task);
    tasks[i].putintparam(mosek.iparam.optimizer, optimizers[i]);
  // Solve tasks in parallel
 bool success;
  int firstOK;
  success = optimize(tasks, res, trm, out firstOK);
 if (success)
 {
    winTask = tasks[firstOK];
   winTrm = trm[firstOK];
           = res[firstOK];
    winRes
    return firstOK;
 }
 else
  {
    winTask = null;
    winTrm = 0;
    winRes = 0;
    return -1;
}
```

It remains to call the method with a choice of optimizers, for example:

Listing 11.15: Calling concurrent linear optimization.

```
int[] optimizers = {
    mosek.optimizertype.conic,
    mosek.optimizertype.dual_simplex,
    mosek.optimizertype.primal_simplex
};
idx = optimizeconcurrent(task, optimizers, out t, out trm, out res);
```

## 11.3.3 Mixed-integer optimization

We use the multithreaded setup to run many, differently seeded copies of the mixed-integer optimizer. This approach is most useful for hard problems where we don't expect an optimal solution in reasonable time. The input task would typically contain a time limit. It is possible that all the cloned tasks reach the time limit, in which case it doesn't really mater which one terminated first. Instead we examine all the task clones for the best objective value.

Listing 11.16: Concurrent optimization of a mixed-integer problem.

```
var n = seeds.Length;
var tasks = new mosek.Task[n];
var res = new mosek.rescode[n];
var trm = new mosek.rescode[n];
// Clone tasks and choose various seeds for the optimizer
for (var i = 0; i < n; ++i)
 tasks[i] = new mosek.Task(task);
 tasks[i].putintparam(mosek.iparam.mio_seed, seeds[i]);
// Solve tasks in parallel
bool success;
int firstOK;
success = optimize(tasks, res, trm, out firstOK);
if (success)
{
  // Pick the task that ended with res = ok
  // and contains an integer solution with best objective value
 mosek.objsense sense = task.getobjsense();
  double bestObj = (sense == mosek.objsense.minimize) ? 1.0e+10 : -1.0e+10;
  int bestPos = -1;
  for (var i = 0; i < n; ++i)
   Console.WriteLine("{0} {1} ", i, tasks[i].getprimalobj(mosek.soltype.itg));
  for (var i = 0; i < n; ++i)
    if ((res[i] == mosek.rescode.ok) &&
        (tasks[i].getsolsta(mosek.soltype.itg) == mosek.solsta.prim_feas ||
        tasks[i].getsolsta(mosek.soltype.itg) == mosek.solsta.integer_optimal) &&
        ((sense == mosek.objsense.minimize) ?
            (tasks[i].getprimalobj(mosek.soltype.itg) < bestObj) :</pre>
            (tasks[i].getprimalobj(mosek.soltype.itg) > best0bj) )
      bestObj = tasks[i].getprimalobj(mosek.soltype.itg);
      bestPos = i;
   }
 if (bestPos != -1)
   winTask = tasks[bestPos];
   winTrm = trm[bestPos];
   winRes = res[bestPos];
   return bestPos;
 }
}
winTask = null;
winTrm = 0;
winRes
       = 0;
return -1;
```

It remains to call the method with a choice of seeds, for example:

Listing 11.17: Calling concurrent integer optimization.

```
int[] seeds = { 42, 13, 71749373 };
idx = optimizeconcurrentMIO(task, seeds, out t, out trm, out res);
```

# Chapter 12

# Problem Formulation and Solutions

In this chapter we will discuss the following issues:

- The formal, mathematical formulations of the problem types that MOSEK can solve and their duals.
- The solution information produced by MOSEK.
- The infeasibility certificate produced by MOSEK if the problem is infeasible.

For the underlying mathematical concepts, derivations and proofs see the Modeling Cookbook or any book on convex optimization. This chapter explains how the related data is organized specifically within the **MOSEK** API.

## 12.1 Linear Optimization

MOSEK accepts linear optimization problems of the form

where

- *m* is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted. A primal solution (x) is (primal) feasible if it satisfies all constraints in (12.1). If (12.1) has at least one primal feasible solution, then (12.1) is said to be (primal) feasible. In case (12.1) does not have a feasible solution, the problem is said to be (primal) infeasible

## 12.1.1 Duality for Linear Optimization

Corresponding to the primal problem (12.1), there is a dual problem

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$

$$A^T y + s_l^x - s_u^x = c,$$
subject to 
$$-y + s_l^c - s_u^c = 0,$$

$$s_l^c, s_u^c, s_l^x, s_u^x \geq 0.$$
 (12.2)

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem. In other words:

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_j = 0 \text{ and } l_i^x \cdot (s_l^x)_j = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x)$$

to the dual problem is feasible if it satisfies all the constraints in (12.2). If (12.2) has at least one feasible solution, then (12.2) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

is denoted a primal-dual feasible solution, if  $(x^*)$  is a solution to the primal problem (12.1) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (12.2). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the  $duality\ gap$  as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] \geq 0$$

$$(12.3)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (12.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that a linear optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity* conditions

$$\begin{array}{rclcrcl} (s_l^c)_i^*((x_i^c)^*-l_i^c) & = & 0, & i=0,\dots,m-1, \\ (s_u^c)_i^*(u_i^c-(x_i^c)^*) & = & 0, & i=0,\dots,m-1, \\ (s_l^x)_j^*(x_j^*-l_j^x) & = & 0, & j=0,\dots,n-1, \\ (s_u^x)_j^*(u_j^x-x_j^*) & = & 0, & j=0,\dots,n-1, \end{array}$$

are satisfied.

If (12.1) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

## 12.1.2 Infeasibility for Linear Optimization

#### **Primal Infeasible Problems**

If the problem (12.1) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*)$$

to (12.4) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (12.4) is unbounded, and that (12.1) is infeasible.

#### **Dual Infeasible Problems**

If the problem (12.2) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
  
subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ ,  $\hat{l}^x \leq x \leq \hat{u}^x$ , (12.5)

where

$$\hat{l}_i^c = \left\{ \begin{array}{ll} 0 & \text{if } l_i^c > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_i^c := \left\{ \begin{array}{ll} 0 & \text{if } u_i^c < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{ll} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{ll} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$

such that

$$c^T x < 0.$$

Such a solution implies that (12.5) is unbounded, and that (12.2) is infeasible.

In case that both the primal problem (12.1) and the dual problem (12.2) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

## 12.1.3 Minimalization vs. Maximalization

When the objective sense of problem (12.1) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (12.2). The dual problem thus takes the form

This means that the duality gap, defined in (12.3) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$\begin{split} A^T y + s_l^x - s_u^x &= 0, \\ - y + s_l^c - s_u^c &= 0, \\ s_l^c, s_u^x, s_l^x, s_u^x &\leq 0, \end{split} \tag{12.6}$$

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (12.5) such that  $c^T x > 0$ .

## 12.2 Conic Optimization

Conic optimization is an extension of linear optimization (see Sec. 12.1) allowing conic domains to be specified for subsets of the problem variables. A conic optimization problem to be solved by **MOSEK** can be written as

minimize 
$$c^T x + c^f$$
  
subject to  $l^c \le Ax \le u^c$ ,  
 $l^x \le x \le u^x$ , (12.7)

where

- m is the number of constraints.
- $\bullet$  *n* is the number of decision variables.
- $x \in \mathbb{R}^n$  is a vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear part of the objective function.
- $c^f \in \mathbb{R}$  is a constant term in the objective
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.

Lower and upper bounds can be infinite, or in other words the corresponding bound may be omitted. The set  $\mathcal{K}$  is a Cartesian product of convex cones, namely  $\mathcal{K} = \mathcal{K}_1 \times \cdots \times \mathcal{K}_p$ . Having the domain restriction  $x \in \mathcal{K}$ , is thus equivalent to

$$x^t \in \mathcal{K}_t \subset \mathbb{R}^{n_t}$$
,

where  $x = (x^1, ..., x^p)$  is a partition of the problem variables. Please note that the *n*-dimensional Euclidean space  $\mathbb{R}^n$  is a cone itself, so simple linear variables are still allowed. The user only needs to specify subsets of variables which belong to non-trivial cones.

In this section we discuss the formulations which apply to the following cones supported by MOSEK:

- The set  $\mathbb{R}^n$ .
- The zero cone  $\{(0, ..., 0)\}.$
- Quadratic cone

$$Q^{n} = \left\{ x \in \mathbb{R}^{n} : x_{1} \ge \sqrt{\sum_{j=2}^{n} x_{j}^{2}} \right\}.$$

• Rotated quadratic cone

$$Q_r^n = \left\{ x \in \mathbb{R}^n : 2x_1 x_2 \ge \sum_{j=3}^n x_j^2, \quad x_1 \ge 0, \quad x_2 \ge 0 \right\}.$$

• Primal exponential cone

$$K_{\text{exp}} = \{ x \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0 \}$$

as well as its dual

$$K_{\text{exp}}^* = \left\{ x \in \mathbb{R}^3 : x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0 \right\}.$$

• Primal power cone (with parameter  $0 < \alpha < 1$ )

$$\mathcal{P}_n^{\alpha, 1 - \alpha} = \left\{ x \in \mathbb{R}^n : x_1^{\alpha} x_2^{1 - \alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0 \right\}$$

as well as its dual

$$(\mathcal{P}_n^{\alpha,1-\alpha})^* = \left\{ x \in \mathbb{R}^n : \left(\frac{x_1}{\alpha}\right)^\alpha \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$

**MOSEK** supports also the cone of positive semidefinite matrices. Since that is handled through a separate interface, we discuss it in Sec. 12.3.

## 12.2.1 Duality for Conic Optimization

Corresponding to the primal problem (12.7), there is a dual problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & \\ & A^T y + s_l^x - s_u^x + s_n^x = c \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_u^s, s_u^s, s_u^s \geq 0, \\ & s_n^x \in \mathcal{K}^*, \end{array}$$

where the dual cone  $\mathcal{K}^*$  is a Cartesian product of the cones dual to  $\mathcal{K}_t$ . In practice this means that  $s_n^x$  has one entry for each entry in x. Please note that the dual problem of the dual problem is identical to the original primal problem.

If a bound in the primal problem is plus or minus infinity, the corresponding dual variable is fixed at 0, and we use the convention that the product of the bound value and the corresponding dual variable is 0. This is equivalent to removing variable  $(s_l^x)_j$  from the dual problem. In other words:

$$l_i^x = -\infty \quad \Rightarrow \quad (s_l^x)_i = 0 \text{ and } l_i^x \cdot (s_l^x)_i = 0.$$

A solution

$$(y, s_l^c, s_u^c, s_l^x, s_u^x, s_n^x)$$

to the dual problem is feasible if it satisfies all the constraints in (12.8). If (12.8) has at least one feasible solution, then (12.8) is (dual) feasible, otherwise the problem is (dual) infeasible.

A solution

$$(x^*, y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (s_u^x)^*)$$

is denoted a primal-dual feasible solution, if  $(x^*)$  is a solution to the primal problem (12.7) and  $(y^*, (s_l^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*, (s_u^c)^*$  is a solution to the corresponding dual problem (12.8). We also define an auxiliary vector

$$(x^c)^* := Ax^*$$

containing the activities of linear constraints.

For a primal-dual feasible solution we define the *duality gap* as the difference between the primal and the dual objective value,

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] + \sum_{j=0}^{m-1} (s_{n}^{x})_{j}^{*}x_{j}^{*} \ge 0$$

$$(12.9)$$

where the first relation can be obtained by transposing and multiplying the dual constraints (12.2) by  $x^*$  and  $(x^c)^*$  respectively, and the second relation comes from the fact that each term in each sum is nonnegative. It follows that the primal objective will always be greater than or equal to the dual objective.

It is well-known that, under some non-degeneracy assumptions that exclude ill-posed cases, a conic optimization problem has an optimal solution if and only if there exist feasible primal-dual solution so that the duality gap is zero, or, equivalently, that the *complementarity conditions* 

$$(s_l^c)_i^*((x_i^c)^* - l_i^c) = 0, \quad i = 0, \dots, m - 1,$$

$$(s_u^c)_i^*(u_i^c - (x_i^c)^*) = 0, \quad i = 0, \dots, m - 1,$$

$$(s_l^T)_j^*(x_j^* - l_j^T) = 0, \quad j = 0, \dots, n - 1,$$

$$(s_u^x)_j^*(u_j^T - x_j^*) = 0, \quad j = 0, \dots, n - 1,$$

$$\sum_{j=0}^{n-1} (s_n^x)_j^* x_j^* = 0.$$

$$(12.10)$$

are satisfied.

If (12.7) has an optimal solution and **MOSEK** solves the problem successfully, both the primal and dual solution are reported, including a status indicating the exact state of the solution.

## 12.2.2 Infeasibility for Conic Optimization

## **Primal Infeasible Problems**

If the problem (12.7) is infeasible (has no feasible solution), **MOSEK** will report a certificate of primal infeasibility: The dual solution reported is the certificate of infeasibility, and the primal solution is undefined.

A certificate of primal infeasibility is a feasible solution to the modified dual problem

$$\begin{array}{ll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x \\ \text{subject to} & \\ A^T y + s_l^x - s_u^x + s_n^x = 0, \\ & -y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0, \\ & s_n^x \in \mathcal{K}^*, \end{array}$$

such that the objective value is strictly positive, i.e. a solution

$$(y^*, (s_l^c)^*, (s_u^c)^*, (s_l^x)^*, (s_u^x)^*, (s_u^x)^*)$$

to (12.11) so that

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* > 0.$$

Such a solution implies that (12.11) is unbounded, and that (12.7) is infeasible.

#### **Dual Infeasible Problems**

If the problem (12.8) is infeasible (has no feasible solution), **MOSEK** will report a certificate of dual infeasibility: The primal solution reported is the certificate of infeasibility, and the dual solution is undefined.

A certificate of dual infeasibility is a feasible solution to the modified primal problem

minimize 
$$c^T x$$
 subject to  $\hat{l}^c \leq Ax \leq \hat{u}^c$ , 
$$\hat{l}^x \leq x \leq \hat{u}^x$$
, 
$$x \in K$$
, 
$$(12.12)$$

where

$$\hat{l}_{i}^{c} = \left\{ \begin{array}{ll} 0 & \text{if } l_{i}^{c} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{i}^{c} := \left\{ \begin{array}{ll} 0 & \text{if } u_{i}^{c} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
(12.13)

and

$$\hat{l}_{j}^{x} = \left\{ \begin{array}{cc} 0 & \text{if } l_{j}^{x} > -\infty, \\ -\infty & \text{otherwise,} \end{array} \right\} \quad \text{and} \quad \hat{u}_{j}^{x} := \left\{ \begin{array}{cc} 0 & \text{if } u_{j}^{x} < \infty, \\ \infty & \text{otherwise,} \end{array} \right\}$$
 (12.14)

such that

$$c^T x < 0$$
.

Such a solution implies that (12.12) is unbounded, and that (12.8) is infeasible.

In case that both the primal problem (12.7) and the dual problem (12.8) are infeasible, **MOSEK** will report only one of the two possible certificates — which one is not defined (**MOSEK** returns the first certificate found).

## 12.2.3 Minimalization vs. Maximalization

When the objective sense of problem (12.7) is maximization, i.e.

the objective sense of the dual problem changes to minimization, and the domain of all dual variables changes sign in comparison to (12.2). The dual problem thus takes the form

$$\begin{array}{ll} \text{minimize} & (l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x = c, \\ & - y + s_l^c - s_u^c = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \leq 0, \\ & - s_n^x \in \mathcal{K}^* \\ \end{array}$$

This means that the duality gap, defined in (12.9) as the primal minus the dual objective value, becomes nonpositive. It follows that the dual objective will always be greater than or equal to the primal objective. The primal infeasibility certificate will be reported by **MOSEK** as a solution to the system

$$A^{T}y + s_{l}^{x} - s_{u}^{x} + s_{n}^{x} = 0,$$

$$-y + s_{l}^{c} - s_{u}^{c} = 0,$$

$$s_{l}^{c}, s_{u}^{c}, s_{l}^{x}, s_{u}^{x} \leq 0,$$

$$-s_{n}^{x} \in \mathcal{K}^{*}$$
(12.15)

such that the objective value is strictly negative

$$(l^c)^T (s_l^c)^* - (u^c)^T (s_u^c)^* + (l^x)^T (s_l^x)^* - (u^x)^T (s_u^x)^* < 0.$$

Similarly, the certificate of dual infeasibility is an x satisfying the requirements of (12.12) such that  $c^T x > 0$ .

# 12.3 Semidefinite Optimization

Semidefinite optimization is an extension of conic optimization (see Sec. 12.2) allowing positive semidefinite matrix variables to be used in addition to the usual scalar variables. All the other parts of the input are defined exactly as in Sec. 12.2, and the discussion from that section applies verbatim to all properties of problems with semidefinite variables. We only briefly indicate how the corresponding formulae should be modified with semidefinite terms.

A semidefinite optimization problem can be written as

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle + c^f$$
subject to  $l_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq u_i^c, \quad i = 0, \dots, m-1$ 

$$l_j^x \leq x_j \leq u_j^x, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K},$$

$$\overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(12.16)$$

where the problem has p symmetric positive semidefinite variables  $\overline{X}_j \in \mathcal{S}_+^{r_j}$  of dimension  $r_j$  with symmetric coefficient matrices  $\overline{C}_j \in \mathcal{S}^{r_j}$  and  $\overline{A}_{i,j} \in \mathcal{S}^{r_j}$ . We use standard notation for the matrix inner product, i.e., for  $U, V \in \mathbb{R}^{m \times n}$  we have

$$\langle U, V \rangle := \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} U_{ij} V_{ij}.$$

As always we write  $A = (a_{i,j})$  for the linear coefficient matrix.

#### **Duality**

The definition of the dual problem (12.8) becomes:

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + s_n^x = c$$
 
$$-y + s_l^c - s_u^c = 0,$$
 
$$\overline{C}_j - \sum_{i=0}^{m-1} y_i \overline{A}_{ij} = \overline{S}_j, \qquad j = 0, \dots, p-1$$
 
$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$
 
$$s_n^x \in \mathcal{K}^*,$$
 
$$\overline{S}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1.$$
 (12.17)

The duality gap (12.9) is computed as:

$$c^{T}x^{*} + c^{f} - \left\{ (l^{c})^{T}(s_{l}^{c})^{*} - (u^{c})^{T}(s_{u}^{c})^{*} + (l^{x})^{T}(s_{l}^{x})^{*} - (u^{x})^{T}(s_{u}^{x})^{*} + c^{f} \right\}$$

$$= \sum_{i=0}^{m-1} \left[ (s_{l}^{c})_{i}^{*}((x_{i}^{c})^{*} - l_{i}^{c}) + (s_{u}^{c})_{i}^{*}(u_{i}^{c} - (x_{i}^{c})^{*}) \right]$$

$$+ \sum_{j=0}^{m-1} \left[ (s_{l}^{x})_{j}^{*}(x_{j} - l_{j}^{x}) + (s_{u}^{x})_{j}^{*}(u_{j}^{x} - x_{j}^{*}) \right] + \sum_{j=0}^{m-1} (s_{n}^{x})_{j}^{*}x_{j}^{*} + \sum_{j=0}^{p-1} \langle \overline{X}_{j}, \overline{S}_{j} \rangle \ge 0.$$

$$(12.18)$$

Complementarity conditions (12.10) include the additional relation:

$$\langle \overline{X}_j, \overline{S}_j \rangle = 0 \quad j = 0, \dots, p - 1.$$
 (12.19)

## Infeasibility

A certificate of primal infeasibility (12.11) is now a feasible solution to:

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x$$
  
subject to 
$$A^T y + s_l^x - s_u^x + s_n^x = 0,$$

$$-y + s_l^c - s_u^c = 0,$$

$$\sum_{i=0}^{m-1} y_i \overline{A}_{ij} + \overline{S}_j = 0,$$

$$s_l^c, s_u^c, s_l^x, s_u^x \ge 0,$$

$$s_n^x \in \mathcal{K}^*,$$

$$\overline{S}_l^x \in S^{r_j}$$
 $i = 0, \dots, p-1$ 

$$(12.20)$$

such that the objective value is strictly positive.

Similarly, a dual infeasibility certificate (12.12) is a feasible solution to

minimize 
$$\sum_{j=0}^{n-1} c_j x_j + \sum_{j=0}^{p-1} \left\langle \overline{C}_j, \overline{X}_j \right\rangle$$
subject to  $\hat{l}_i^c \leq \sum_{j=0}^{n-1} a_{ij} x_j + \sum_{j=0}^{p-1} \left\langle \overline{A}_{ij}, \overline{X}_j \right\rangle \leq \hat{u}_i^c, \quad i = 0, \dots, m-1$ 

$$\hat{l}_j^x \leq x_j \leq \hat{u}_j^x, \quad j = 0, \dots, n-1$$

$$x \in \mathcal{K},$$

$$\overline{X}_j \in \mathcal{S}_+^{r_j}, \qquad j = 0, \dots, p-1$$

$$(12.21)$$

where the modified bounds are as in (12.13) and (12.14) and the objective value is strictly negative.

# 12.4 Quadratic and Quadratically Constrained Optimization

A convex quadratic and quadratically constrained optimization problem has the form

where all variables and bounds have the same meaning as for linear problems (see Sec. 12.1) and  $Q^o$  and all  $Q^k$  are symmetric matrices. Moreover, for convexity,  $Q^o$  must be a positive semidefinite matrix and  $Q^k$  must satisfy

$$\begin{array}{rcl} -\infty < l_k^c & \Rightarrow & Q^k \text{ is negative semidefinite,} \\ u_k^c < \infty & \Rightarrow & Q^k \text{ is positive semidefinite,} \\ -\infty < l_k^c \leq u_k^c < \infty & \Rightarrow & Q^k = 0. \end{array}$$

The convexity requirement is very important and MOSEK checks whether it is fulfilled.

## 12.4.1 A Recommendation

Any convex quadratic optimization problem can be reformulated as a conic quadratic optimization problem, see Modeling Cookbook and [And13]. In fact **MOSEK** does such conversion internally as a part of the solution process for the following reasons:

- the conic optimizer is numerically more robust than the one for quadratic problems.
- the conic optimizer is usually faster because quadratic cones are simpler than quadratic functions, even though the conic reformulation usually has more constraints and variables than the original quadratic formulation.
- it is easy to dualize the conic formulation if deemed worthwhile potentially leading to (huge) computational savings.

However, instead of relying on the automatic reformulation we recommend to formulate the problem as a conic problem from scratch because:

- it saves the computational overhead of the reformulation including the convexity check. A conic problem is convex by construction and hence no convexity check is needed for conic problems.
- usually the modeler can do a better reformulation than the automatic method because the modeler can exploit the knowledge of the problem at hand.

To summarize we recommend to formulate quadratic problems and in particular quadratically constrained problems directly in conic form.

## 12.4.2 Duality for Quadratic and Quadratically Constrained Optimization

The dual problem corresponding to the quadratic and quadratically constrained optimization problem (12.22) is given by

maximize 
$$(l^c)^T s_l^c - (u^c)^T s_u^c + (l^x)^T s_l^x - (u^x)^T s_u^x + \frac{1}{2} x^T \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x + c^f$$
 subject to 
$$A^T y + s_l^x - s_u^x + \left\{ \sum_{k=0}^{m-1} y_k Q^k - Q^o \right\} x = c,$$
 
$$-y + s_l^c - s_u^c = 0,$$
 
$$s_l^c, s_u^c, s_l^c, s_u^x, s_l^x > 0.$$
 (12.23)

The dual problem is related to the dual problem for linear optimization (see Sec. 12.1.1), but depends on the variable x which in general can not be eliminated. In the solutions reported by **MOSEK**, the value of x is the same for the primal problem (12.22) and the dual problem (12.23).

## 12.4.3 Infeasibility for Quadratic Optimization

In case **MOSEK** finds a problem to be infeasible it reports a certificate of infeasibility. We write them out explicitly for quadratic problems, that is when  $Q^k = 0$  for all k and quadratic terms appear only in the objective  $Q^o$ . In this case the constraints both in the primal and dual problem are linear, and **MOSEK** produces for them the same infeasibility certificate as for linear problems.

The certificate of primal infeasibility is a solution to the problem (12.4) such that the objective value is strictly positive.

The certificate of dual infeasibility is a solution to the problem (12.5) together with an additional constraint

$$Q^o x = 0$$

such that the objective value is strictly negative.

# Chapter 13

# **Optimizers**

The most essential part of MOSEK are the optimizers:

- primal simplex (linear problems),
- dual simplex (linear problems),
- interior-point (linear, quadratic and conic problems),
- mixed-integer (problems with integer variables).

The structure of a successful optimization process is roughly:

#### • Presolve

- 1. Elimination: Reduce the size of the problem.
- 2. Dualizer: Choose whether to solve the primal or the dual form of the problem.
- 3. Scaling: Scale the problem for better numerical stability.

#### • Optimization

- 1. Optimize: Solve the problem using selected method.
- 2. Terminate: Stop the optimization when specific termination criteria have been met.
- 3. Report: Return the solution or an infeasibility certificate.

The preprocessing stage is transparent to the user, but useful to know about for tuning purposes. The purpose of the preprocessing steps is to make the actual optimization more efficient and robust. We discuss the details of the above steps in the following sections.

## 13.1 Presolve

Before an optimizer actually performs the optimization the problem is preprocessed using the so-called presolve. The purpose of the presolve is to

- 1. remove redundant constraints,
- 2. eliminate fixed variables,
- 3. remove linear dependencies,
- 4. substitute out (implied) free variables, and
- 5. reduce the size of the optimization problem in general.

After the presolved problem has been optimized the solution is automatically postsolved so that the returned solution is valid for the original problem. Hence, the presolve is completely transparent. For further details about the presolve phase, please see [AA95] and [AGMX96].

It is possible to fine-tune the behavior of the presolve or to turn it off entirely. If presolve consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This is done by setting the parameter  $iparam.presolve\_use$  to presolvemode.off. The two most time-consuming steps of the presolve are

- the eliminator, and
- the linear dependency check.

Therefore, in some cases it is worthwhile to disable one or both of these.

#### Numerical issues in the presolve

During the presolve the problem is reformulated so that it hopefully solves faster. However, in rare cases the presolved problem may be harder to solve then the original problem. The presolve may also be infeasible although the original problem is not. If it is suspected that presolved problem is much harder to solve than the original, we suggest to first turn the eliminator off by setting the parameter <code>iparam.presolve\_eliminator\_max\_num\_tries</code> to 0. If that does not help, then trying to turn entire presolve off may help.

Since all computations are done in finite precision, the presolve employs some tolerances when concluding a variable is fixed or a constraint is redundant. If it happens that **MOSEK** incorrectly concludes a problem is primal or dual infeasible, then it is worthwhile to try to reduce the parameters <code>dparam.presolve\_tol\_x</code> and <code>dparam.presolve\_tol\_s</code>. However, if reducing the parameters actually helps then this should be taken as an indication that the problem is badly formulated.

#### Eliminator

The purpose of the eliminator is to eliminate free and implied free variables from the problem using substitution. For instance, given the constraints

$$\begin{array}{rcl} y & = & \sum_{j} x_{j}, \\ y, x & \geq & 0, \end{array}$$

y is an implied free variable that can be substituted out of the problem, if deemed worthwhile. If the eliminator consumes too much time or memory compared to the reduction in problem size gained it may be disabled. This can be done by setting the parameter <code>iparam.presolve\_eliminator\_max\_num\_tries</code> to 0. In rare cases the eliminator may cause that the problem becomes much hard to solve.

## Linear dependency checker

The purpose of the linear dependency check is to remove linear dependencies among the linear equalities. For instance, the three linear equalities

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 1, \\ x_1 + 0.5x_2 & = & 0.5, \\ 0.5x_2 + x_3 & = & 0.5. \end{array}$$

contain exactly one linear dependency. This implies that one of the constraints can be dropped without changing the set of feasible solutions. Removing linear dependencies is in general a good idea since it reduces the size of the problem. Moreover, the linear dependencies are likely to introduce numerical problems in the optimization phase. It is best practice to build models without linear dependencies, but that is not always easy for the user to control. If the linear dependencies are removed at the modeling stage, the linear dependency check can safely be disabled by setting the parameter <code>iparam.presolve\_lindep\_use</code> to <code>onoffkey.off</code>.

## Dualizer

All linear, conic, and convex optimization problems have an equivalent dual problem associated with them. **MOSEK** has built-in heuristics to determine if it is more efficient to solve the primal or dual problem. The form (primal or dual) is displayed in the **MOSEK** log and available as an information item from the solver. Should the internal heuristics not choose the most efficient form of the problem it may be worthwhile to set the dualizer manually by setting the parameters:

- iparam.intpnt\_solve\_form: In case of the interior-point optimizer.
- $iparam.sim\_solve\_form$ : In case of the simplex optimizer.

Note that currently only linear and conic (but not semidefinite) problems may be automatically dualized.

#### **Scaling**

Problems containing data with large and/or small coefficients, say 1.0e+9 or 1.0e-7, are often hard to solve. Significant digits may be truncated in calculations with finite precision, which can result in the optimizer relying on inaccurate data. Since computers work in finite precision, extreme coefficients should be avoided. In general, data around the same *order of magnitude* is preferred, and we will refer to a problem, satisfying this loose property, as being *well-scaled*. If the problem is not well scaled, **MOSEK** will try to scale (multiply) constraints and variables by suitable constants. **MOSEK** solves the scaled problem to improve the numerical properties.

The scaling process is transparent, i.e. the solution to the original problem is reported. It is important to be aware that the optimizer terminates when the termination criterion is met on the scaled problem, therefore significant primal or dual infeasibilities may occur after unscaling for badly scaled problems. The best solution of this issue is to reformulate the problem, making it better scaled.

By default **MOSEK** heuristically chooses a suitable scaling. The scaling for interior-point and simplex optimizers can be controlled with the parameters *iparam.intpnt\_scaling* and *iparam.sim\_scaling* respectively.

# 13.2 Linear Optimization

## 13.2.1 Optimizer Selection

Two different types of optimizers are available for linear problems: The default is an interior-point method, and the alternative is the simplex method (primal or dual). The optimizer can be selected using the parameter <code>iparam.optimizer</code>.

#### The Interior-point or the Simplex Optimizer?

Given a linear optimization problem, which optimizer is the best: the simplex or the interior-point optimizer? It is impossible to provide a general answer to this question. However, the interior-point optimizer behaves more predictably: it tends to use between 20 and 100 iterations, almost independently of problem size, but cannot perform warm-start. On the other hand the simplex method can take advantage of an initial solution, but is less predictable from cold-start. The interior-point optimizer is used by default.

## The Primal or the Dual Simplex Variant?

MOSEK provides both a primal and a dual simplex optimizer. Predicting which simplex optimizer is faster is impossible, however, in recent years the dual optimizer has seen several algorithmic and computational improvements, which, in our experience, make it faster on average than the primal version. Still, it depends much on the problem structure and size. Setting the <code>iparam.optimizer</code> parameter to <code>optimizertype.free\_simplex</code> instructs MOSEK to choose one of the simplex variants automatically.

To summarize, if you want to know which optimizer is faster for a given problem type, it is best to try all the options.

## 13.2.2 The Interior-point Optimizer

The purpose of this section is to provide information about the algorithm employed in the MOSEK interior-point optimizer for linear problems and about its termination criteria.

## The homogeneous primal-dual problem

In order to keep the discussion simple it is assumed that **MOSEK** solves linear optimization problems of standard form

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  $x \ge 0$ . (13.1)

This is in fact what happens inside MOSEK; for efficiency reasons MOSEK converts the problem to standard form before solving, then converts it back to the input form when reporting the solution.

Since it is not known beforehand whether problem (13.1) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason why **MOSEK** solves the so-called homogeneous model

$$\begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^{T}y + s - c\tau & = & 0, \\
-c^{T}x + b^{T}y - \kappa & = & 0, \\
x, s, \tau, \kappa & \geq & 0,
\end{array}$$
(13.2)

where y and s correspond to the dual variables in (13.1), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.2) always has solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (13.2) satisfies

$$x_i^* s_i^* = 0$$
 and  $\tau^* \kappa^* = 0$ .

Moreover, there is always a solution that has the property  $\tau^* + \kappa^* > 0$ .

First, assume that  $\tau^* > 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left\{ \frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*} \right\}$$

is a primal-dual optimal solution (see Sec. 12.1 for the mathematical background on duality and optimality).

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl} Ax^* & = & 0, \\ A^Ty^* + s^* & = & 0, \\ -c^Tx^* + b^Ty^* & = & \kappa^*, \\ x^*, s^*, \tau^*, \kappa^* & \geq & 0. \end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.3}$$

or

$$b^T y^* > 0 (13.4)$$

is satisfied. If (13.3) is satisfied then  $x^*$  is a certificate of dual infeasibility, whereas if (13.4) is satisfied then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

#### **Interior-point Termination Criterion**

For efficiency reasons it is not practical to solve the homogeneous model exactly. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In the k-th iteration of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to homogeneous model is generated, where

$$x^k, s^k, \tau^k, \kappa^k > 0.$$

#### Optimal case

Whenever the trial solution satisfies the criterion

$$\left\| A \frac{x^{k}}{\tau^{k}} - b \right\|_{\infty} \leq \epsilon_{p} (1 + \|b\|_{\infty}), 
\left\| A^{T} \frac{y^{k}}{\tau^{k}} + \frac{s^{k}}{\tau^{k}} - c \right\|_{\infty} \leq \epsilon_{d} (1 + \|c\|_{\infty}), \text{ and} 
\min \left( \frac{(x^{k})^{T} s^{k}}{(\tau^{k})^{2}}, \left| \frac{c^{T} x^{k}}{\tau^{k}} - \frac{b^{T} y^{k}}{\tau^{k}} \right| \right) \leq \epsilon_{g} \max \left( 1, \frac{\min \left( \left| c^{T} x^{k} \right|, \left| b^{T} y^{k} \right| \right)}{\tau^{k}} \right),$$
(13.5)

the interior-point optimizer is terminated and

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is reported as the primal-dual optimal solution. The interpretation of (13.5) is that the optimizer is terminated if

- $\frac{x^k}{\tau^k}$  is approximately primal feasible,
- $\left\{\frac{y^k}{\tau^k}, \frac{s^k}{\tau^k}\right\}$  is approximately dual feasible, and
- the duality gap is almost zero.

## **Dual infeasibility certificate**

On the other hand, if the trial solution satisfies

$$-\epsilon_i c^T x^k > \frac{\|c\|_{\infty}}{\max\left(1, \|b\|_{\infty}\right)} \|Ax^k\|_{\infty}$$

then the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows: First assume that  $\|Ax^k\|_{\infty} = 0$ ; then  $x^k$  is an exact certificate of dual infeasibility. Next assume that this is not the case, i.e.

$$||Ax^k||_{\infty} > 0,$$

and define

$$\bar{x} := \epsilon_i \frac{\max(1, \|b\|_{\infty})}{\|Ax^k\|_{\infty} \|c\|_{\infty}} x^k.$$

It is easy to verify that

$$\|A\bar{x}\|_{\infty} = \epsilon_i \frac{\max\left(1, \|b\|_{\infty}\right)}{\|c\|_{\infty}} \text{ and } -c^T\bar{x} > 1,$$

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation. A smaller value means a better approximation.

## Primal infeasibility certificate

Finally, if

$$\epsilon_i b^T y^k > \frac{\|b\|_{\infty}}{\max\left(1, \|c\|_{\infty}\right)} \|A^T y^k + s^k\|_{\infty}$$

then  $y^k$  is reported as a certificate of primal infeasibility.

#### Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.1: Parameters employed in termination criterion

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.5) reveals that the quality of the solution depends on  $||b||_{\infty}$  and  $||c||_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_q$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

If the optimizer terminates without locating a solution that satisfies the termination criteria, for example because of a stall or other numerical issues, then it will check if the solution found up to that point satisfies the same criteria with all tolerances multiplied by the value of <code>dparam.intpnt\_co\_tol\_near\_rel</code>. If this is the case, the solution is still declared as optimal.

The basis identification discussed in Sec. 13.2.2 requires an optimal solution to work well; hence basis identification should be turned off if the termination criterion is relaxed.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

#### **Basis Identification**

An interior-point optimizer does not return an optimal basic solution unless the problem has a unique primal and dual optimal solution. Therefore, the interior-point optimizer has an optimal post-processing step that computes an optimal basic solution starting from the optimal interior-point solution. More information about the basis identification procedure may be found in [AY96]. In the following we provide an overall idea of the procedure.

There are some cases in which a basic solution could be more valuable:

- a basic solution is often more accurate than an interior-point solution,
- a basic solution can be used to warm-start the simplex algorithm in case of reoptimization,
- a basic solution is in general more sparse, i.e. more variables are fixed to zero. This is particularly appealing when solving continuous relaxations of mixed integer problems, as well as in all applications in which sparser solutions are preferred.

To illustrate how the basis identification routine works, we use the following trivial example:

$$\begin{array}{lll} \mbox{minimize} & x+y \\ \mbox{subject to} & x+y & = & 1, \\ & x,y \geq 0. \end{array}$$

It is easy to see that all feasible solutions are also optimal. In particular, there are two basic solutions, namely

$$\begin{array}{rcl} (x_1^*,y_1^*) & = & (1,0), \\ (x_2^*,y_2^*) & = & (0,1). \end{array}$$

The interior point algorithm will actually converge to the center of the optimal set, i.e. to  $(x^*, y^*) = (1/2, 1/2)$  (to see this in **MOSEK** deactivate *Presolve*).

In practice, when the algorithm gets close to the optimal solution, it is possible to construct in polynomial time an initial basis for the simplex algorithm from the current interior point solution. This basis is used to warm-start the simplex algorithm that will provide the optimal basic solution. In most cases the constructed basis is optimal, or very few iterations are required by the simplex algorithm to make it optimal and hence the final *clean-up* phase be short. However, for some cases of ill-conditioned problems the additional simplex clean up phase may take of lot a time.

By default **MOSEK** performs a basis identification. However, if a basic solution is not needed, the basis identification procedure can be turned off. The parameters

- iparam.intpnt\_basis,
- iparam.bi\_ignore\_max\_iter, and
- iparam.bi\_ignore\_num\_error

control when basis identification is performed.

The type of simplex algorithm to be used (primal/dual) can be tuned with the parameter <code>iparam.bi\_clean\_optimizer</code>, and the maximum number of iterations can be set with <code>iparam.bi\_max\_iterations</code>.

Finally, it should be mentioned that there is no guarantee on which basic solution will be returned.

#### The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer
          - threads
                                    : 1
Optimizer
          - solved problem
                                    : the dual
Optimizer - Constraints
                                    : 2
                                    : 0
Optimizer - Cones
                                   : 6
                                                                                : 0
Optimizer - Scalar variables
                                                         conic
Optimizer - Semi-definite variables: 0
                                                         scalarized
                                                                                : 0
                                   : 0.00
Factor
           - setup time
                                                         dense det. time
                                                                                : 0.00
Factor
           - ML order time
                                    : 0.00
                                                        GP order time
                                                                                : 0.00
Factor
           - nonzeros before factor : 3
                                                        after factor
                                                                                : 3
Factor
            dense dim.
                                                         flops
                                                                                : 7.00e+001
ITE PFEAS
             DFEAS
                      GFEAS
                               PRSTATUS
                                          POBJ
                                                            DOBJ
                                                                               MU
                                                                                        TIME
                                                            -2.208000000e+003 1.0e+000 0.00
   1.0e+000 8.6e+000 6.1e+000 1.00e+000
                                          0.00000000e+000
                                          -7.901380925e+003 -7.394611417e+003 2.5e+000 0.00
    1.1e+000 2.5e+000 1.6e-001 0.00e+000
   1.4e-001 3.4e-001 2.1e-002 8.36e-001
                                          -8.113031650e+003 -8.055866001e+003 3.3e-001 0.00
   2.4e-002 5.8e-002 3.6e-003 1.27e+000
                                          -7.777530698e+003 -7.766471080e+003 5.7e-002 0.01
   1.3e-004 3.2e-004 2.0e-005 1.08e+000
                                          -7.668323435e+003 -7.668207177e+003 3.2e-004 0.01
5
   1.3e-008 3.2e-008 2.0e-009 1.00e+000
                                          -7.668000027e+003 -7.668000015e+003 3.2e-008 0.01
   1.3e-012 3.2e-012 2.0e-013 1.00e+000
                                          -7.667999994e+003 -7.667999994e+003 3.2e-012 0.01
```

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see <code>iparam.intpnt\_solve\_form</code>). The next lines display the problem dimensions as seen by the optimizer, and the <code>Factor...</code> lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.2.2 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $||A^Ty^k + s^k c\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to −1 if that is not the case.

- POBJ:  $c^T x^k / \tau^k$ . An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started.

## 13.2.3 The Simplex Optimizer

An alternative to the interior-point optimizer is the simplex optimizer. The simplex optimizer uses a different method that allows exploiting an initial guess for the optimal solution to reduce the solution time. Depending on the problem it may be faster or slower to use an initial guess; see Sec. 13.2.1 for a discussion. **MOSEK** provides both a primal and a dual variant of the simplex optimizer.

#### **Simplex Termination Criterion**

The simplex optimizer terminates when it finds an optimal basic solution or an infeasibility certificate. A basic solution is optimal when it is primal and dual feasible; see Sec. 12.1 for a definition of the primal and dual problem. Due to the fact that computations are performed in finite precision **MOSEK** allows violations of primal and dual feasibility within certain tolerances. The user can control the allowed primal and dual tolerances with the parameters <code>dparam.basis\_tol\_x</code> and <code>dparam.basis\_tol\_s</code>.

Setting the parameter *iparam.optimizer* to *optimizertype.free\_simplex* instructs **MOSEK** to select automatically between the primal and the dual simplex optimizers. Hence, **MOSEK** tries to choose the best optimizer for the given problem and the available solution. The same parameter can also be used to force one of the variants.

## Starting From an Existing Solution

When using the simplex optimizer it may be possible to reuse an existing solution and thereby reduce the solution time significantly. When a simplex optimizer starts from an existing solution it is said to perform a warm-start. If the user is solving a sequence of optimization problems by solving the problem, making modifications, and solving again, MOSEK will warm-start automatically.

By default **MOSEK** uses presolve when performing a warm-start. If the optimizer only needs very few iterations to find the optimal solution it may be better to turn off the presolve.

#### **Numerical Difficulties in the Simplex Optimizers**

Though MOSEK is designed to minimize numerical instability, completely avoiding it is impossible when working in finite precision. MOSEK treats a "numerically unexpected behavior" event inside the optimizer as a *set-back*. The user can define how many set-backs the optimizer accepts; if that number is exceeded, the optimization will be aborted. Set-backs are a way to escape long sequences where the optimizer tries to recover from an unstable situation.

Examples of set-backs are: repeated singularities when factorizing the basis matrix, repeated loss of feasibility, degeneracy problems (no progress in objective) and other events indicating numerical difficulties. If the simplex optimizer encounters a lot of set-backs the problem is usually badly scaled; in such a situation try to reformulate it into a better scaled problem. Then, if a lot of set-backs still occur, trying one or more of the following suggestions may be worthwhile:

- Raise tolerances for allowed primal or dual feasibility: increase the value of
  - dparam.basis\_tol\_x, and
     dparam.basis\_tol\_s.
- Raise or lower pivot tolerance: Change the dparam.simplex\_abs\_tol\_piv parameter.
- Switch optimizer: Try another optimizer.
- Switch off crash: Set both iparam.sim\_primal\_crash and iparam.sim\_dual\_crash to 0.
- Experiment with other pricing strategies: Try different values for the parameters

- iparam.sim\_primal\_selection and
- iparam.sim\_dual\_selection.
- If you are using warm-starts, in rare cases switching off this feature may improve stability. This is controlled by the <code>iparam.sim\_hotstart</code> parameter.
- Increase maximum number of set-backs allowed controlled by <code>iparam.sim\_max\_num\_setbacks</code>.
- If the problem repeatedly becomes infeasible try switching off the special degeneracy handling. See the parameter <code>iparam.sim\_degen</code> for details.

#### The Simplex Log

Below is a typical log output from the simplex optimizer:

Optim:	izer - solved p	problem	: the pr	rimal			
Optimizer - Constraints			: 667				
Optimizer - Scalar variables		: 1424	conic		: 0		
Optimizer - hotstart		: no					
ITER	DEGITER(%)	PFEAS	DFEAS	POBJ	DOBJ		TIME
$\hookrightarrow$	TOTTIME						
0	0.00	1.43e+05	NA	6.5584140832e+03	NA		0.00 <sub>L</sub>
$\hookrightarrow$	0.02						
1000	1.10	0.00e+00	NA	1.4588289726e+04	NA		0.13 <sub>L</sub>
$\hookrightarrow$	0.14						
2000	0.75	0.00e+00	NA	7.3705564855e+03	NA		0.21 <sub>L</sub>
$\hookrightarrow$	0.22						
3000	0.67	0.00e+00	NA	6.0509727712e+03	NA		0.29⊔
$\hookrightarrow$	0.31						
4000	0.52	0.00e+00	NA	5.5771203906e+03	NA		0.38⊔
$\hookrightarrow$	0.39						
4533	0.49	0.00e+00	NA	5.5018458883e+03	NA		0.42 <u>u</u>
$\hookrightarrow$	0.44						
1							

The first lines summarize the problem the optimizer is solving. This is followed by the iteration log, with the following meaning:

- ITER: Number of iterations.
- DEGITER(%): Ratio of degenerate iterations.
- PFEAS: Primal feasibility measure reported by the simplex optimizer. The numbers should be 0 if the problem is primal feasible (when the primal variant is used).
- DFEAS: Dual feasibility measure reported by the simplex optimizer. The number should be 0 if the problem is dual feasible (when the dual variant is used).
- POBJ: An estimate for the primal objective value (when the primal variant is used).
- DOBJ: An estimate for the dual objective value (when the dual variant is used).
- TIME: Time spent since this instance of the simplex optimizer was invoked (in seconds).
- TOTTIME: Time spent since optimization started (in seconds).

# 13.3 Conic Optimization - Interior-point optimizer

For conic optimization problems only an interior-point type optimizer is available.

## 13.3.1 The homogeneous primal-dual problem

The interior-point optimizer is an implementation of the so-called homogeneous and self-dual algorithm. For a detailed description of the algorithm, please see [ART03]. In order to keep our discussion simple we will assume that **MOSEK** solves a conic optimization problem of the form:

minimize 
$$c^T x$$
  
subject to  $Ax = b$ ,  $x \in \mathcal{K}$  (13.6)

where K is a convex cone. The corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & b^T y \\ \text{subject to} & A^T y + s & = & c, \\ & s \in \mathcal{K}^* \end{array} \tag{13.7}$$

where  $\mathcal{K}^*$  is the dual cone of  $\mathcal{K}$ . See Sec. 12.2 for definitions.

Since it is not known beforehand whether problem (13.6) has an optimal solution, is primal infeasible or is dual infeasible, the optimization algorithm must deal with all three situations. This is the reason that **MOSEK** solves the so-called homogeneous model

$$Ax - b\tau = 0,$$

$$A^{T}y + s - c\tau = 0,$$

$$-c^{T}x + b^{T}y - \kappa = 0,$$

$$x \in \mathcal{K},$$

$$s \in \mathcal{K}^{*},$$

$$\tau, \kappa \geq 0,$$

$$(13.8)$$

where y and s correspond to the dual variables in (13.6), and  $\tau$  and  $\kappa$  are two additional scalar variables. Note that the homogeneous model (13.8) always has a solution since

$$(x, y, s, \tau, \kappa) = (0, 0, 0, 0, 0)$$

is a solution, although not a very interesting one. Any solution

$$(x^*, y^*, s^*, \tau^*, \kappa^*)$$

to the homogeneous model (13.8) satisfies

$$(x^*)^T s^* + \tau^* \kappa^* = 0$$

i.e. complementarity. Observe that  $x^* \in \mathcal{K}$  and  $s^* \in \mathcal{K}^*$  implies

$$(x^*)^T s^* > 0$$

and therefore

$$\tau^* \kappa^* = 0$$

since  $\tau^*, \kappa^* \geq 0$ . Hence, at least one of  $\tau^*$  and  $\kappa^*$  is zero.

First, assume that  $\tau^* > 0$  and hence  $\kappa^* = 0$ . It follows that

$$\begin{array}{rcl} A\frac{x^*}{\tau^*} & = & b, \\ A^T\frac{y^*}{\tau^*} + \frac{s^*}{\tau^*} & = & c, \\ -c^T\frac{x^*}{\tau^*} + b^T\frac{y^*}{\tau^*} & = & 0, \\ x^*/\tau^* & \in & \mathcal{K}, \\ s^*/\tau^* & \in & \mathcal{K}^*. \end{array}$$

This shows that  $\frac{x^*}{\tau^*}$  is a primal optimal solution and  $(\frac{y^*}{\tau^*}, \frac{s^*}{\tau^*})$  is a dual optimal solution; this is reported as the optimal interior-point solution since

$$(x, y, s) = \left(\frac{x^*}{\tau^*}, \frac{y^*}{\tau^*}, \frac{s^*}{\tau^*}\right)$$

is a primal-dual optimal solution.

On other hand, if  $\kappa^* > 0$  then

$$\begin{array}{rcl}
Ax^* & = & 0, \\
A^T y^* + s^* & = & 0, \\
-c^T x^* + b^T y^* & = & \kappa^*, \\
x^* & \in & \mathcal{K}, \\
s^* & \in & \mathcal{K}^*.
\end{array}$$

This implies that at least one of

$$c^T x^* < 0 \tag{13.9}$$

or

$$b^T y^* > 0 (13.10)$$

holds. If (13.9) is satisfied, then  $x^*$  is a certificate of dual infeasibility, whereas if (13.10) holds then  $y^*$  is a certificate of primal infeasibility.

In summary, by computing an appropriate solution to the homogeneous model, all information required for a solution to the original problem is obtained. A solution to the homogeneous model can be computed using a primal-dual interior-point algorithm [And09].

## 13.3.2 Interior-point Termination Criterion

Since computations are performed in finite precision, and for efficiency reasons, it is not possible to solve the homogeneous model exactly in general. Hence, an exact optimal solution or an exact infeasibility certificate cannot be computed and a reasonable termination criterion has to be employed.

In every iteration k of the interior-point algorithm a trial solution

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

to the homogeneous model is generated, where

$$x^k \in \mathcal{K}, s^k \in \mathcal{K}^*, \tau^k, \kappa^k > 0.$$

Therefore, it is possible to compute the values:

$$\begin{array}{lll} \rho_p^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \leq \rho \varepsilon_p (1 + \|b\|_{\infty}) \right\}, \\ \rho_d^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_{\infty} \leq \rho \varepsilon_d (1 + \|c\|_{\infty}) \right\}, \\ \rho_g^k &=& \arg\min_{\rho} \left\{ \rho \mid \left( \frac{(x^k)^T s^k}{(\tau^k)^2}, |\frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k}| \right) \leq \rho \varepsilon_g \max \left( 1, \frac{\min(|c^T x^k|, |b^T y^k|)}{\tau^k} \right) \right\}, \\ \rho_{pi}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A^T y^k + s^k \right\|_{\infty} \leq \rho \varepsilon_i b^T y^k, \, b^T y^k > 0 \right\} \text{ and } \\ \rho_{di}^k &=& \arg\min_{\rho} \left\{ \rho \mid \left\| A x^k \right\|_{\infty} \leq -\rho \varepsilon_i c^T x^k, \, c^T x^k < 0 \right\}. \end{array}$$

Note  $\varepsilon_p, \varepsilon_d, \varepsilon_g$  and  $\varepsilon_i$  are nonnegative user specified tolerances.

#### **Optimal Case**

Observe  $\rho_p^k$  measures how far  $x^k/\tau^k$  is from being a good approximate primal feasible solution. Indeed if  $\rho_p^k \leq 1$ , then

$$\left\| A \frac{x^k}{\tau^k} - b \right\|_{\infty} \le \varepsilon_p (1 + \|b\|_{\infty}). \tag{13.11}$$

This shows the violations in the primal equality constraints for the solution  $x^k/\tau^k$  is small compared to the size of b given  $\varepsilon_p$  is small.

Similarly, if  $\rho_d^k \leq 1$ , then  $(y^k, s^k)/\tau^k$  is an approximate dual feasible solution. If in addition  $\rho_g \leq 1$ , then the solution  $(x^k, y^k, s^k)/\tau^k$  is approximate optimal because the associated primal and dual objective values are almost identical.

In other words if  $\max(\rho_p^k, \rho_d^k, \rho_g^k) \leq 1$ , then

$$\frac{(x^k, y^k, s^k)}{\tau^k}$$

is an approximate optimal solution.

#### **Dual Infeasibility Certificate**

Next assume that  $\rho_{di}^k \leq 1$  and hence

$$||Ax^k||_{\infty} \le -\varepsilon_i c^T x^k$$
 and  $-c^T x^k > 0$ 

holds. Now in this case the problem is declared dual infeasible and  $x^k$  is reported as a certificate of dual infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{x} := \frac{x^k}{-c^T x^k}$$

and it is easy to verify that

$$||A\bar{x}||_{\infty} \leq \varepsilon_i$$
 and  $c^T\bar{x} = -1$ 

which shows  $\bar{x}$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

#### **Primal Infeasiblity Certificate**

Next assume that  $\rho_{pi}^k \leq 1$  and hence

$$||A^T y^k + s^k||_{\infty} \le \varepsilon_i b^T y^k$$
 and  $b^T y^k > 0$ 

holds. Now in this case the problem is declared primal infeasible and  $(y^k, s^k)$  is reported as a certificate of primal infeasibility. The motivation for this stopping criterion is as follows. Let

$$\bar{y} := \frac{y^k}{b^T y^k}$$
 and  $\bar{s} := \frac{s^k}{b^T y^k}$ 

and it is easy to verify that

$$||A^T \bar{y} + \bar{s}||_{\infty} \le \varepsilon_i \text{ and } b^T \bar{y} = 1$$

which shows  $(y^k, s^k)$  is an approximate certificate of dual infeasibility, where  $\varepsilon_i$  controls the quality of the approximation.

## 13.3.3 Adjusting optimality criteria

It is possible to adjust the tolerances  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_g$  and  $\varepsilon_i$  using parameters; see table for details.

Table 13.2: Parameters employed in termination criterion

ToleranceParameter	name
$arepsilon_p$	$dparam.intpnt\_co\_tol\_pfeas$
$\varepsilon_d$	$dparam.intpnt\_co\_tol\_dfeas$
$\varepsilon_g$	$dparam.intpnt\_co\_tol\_rel\_gap$
$arepsilon_i$	$dparam.intpnt\_co\_tol\_infeas$

The default values of the termination tolerances are chosen such that for a majority of problems appearing in practice it is not possible to achieve much better accuracy. Therefore, tightening the tolerances usually is not worthwhile. However, an inspection of (13.11) reveals that the quality of the solution depends on  $\|b\|_{\infty}$  and  $\|c\|_{\infty}$ ; the smaller the norms are, the better the solution accuracy.

The interior-point method as implemented by **MOSEK** will converge toward optimality and primal and dual feasibility at the same rate [And09]. This means that if the optimizer is stopped prematurely then it is very unlikely that either the primal or dual solution is feasible. Another consequence is that in most cases all the tolerances,  $\varepsilon_p$ ,  $\varepsilon_d$ ,  $\varepsilon_q$  and  $\varepsilon_i$ , have to be relaxed together to achieve an effect.

If the optimizer terminates without locating a solution that satisfies the termination criteria, for example because of a stall or other numerical issues, then it will check if the solution found up to that point satisfies the same criteria with all tolerances multiplied by the value of <code>dparam.intpnt\_co\_tol\_near\_rel</code>. If this is the case, the solution is still declared as optimal.

To conclude the discussion in this section, relaxing the termination criterion is usually not worthwhile.

## 13.3.4 The Interior-point Log

Below is a typical log output from the interior-point optimizer:

```
Optimizer
           - threads
                                     : 20
Optimizer
          - solved problem
                                     : the primal
Optimizer
          - Constraints
                                     : 1
Optimizer
          - Cones
         - Scalar variables
Optimizer
Optimizer - Semi-definite variables: 0
                                                         scalarized
           - setup time
                                                                                 : 0.00
                                                         dense det. time
Factor
           - ML order time
                                    : 0.00
                                                         GP order time
                                                                                 : 0.00
Factor
           - nonzeros before factor : 1
                                                         after factor
                                                                                 : 1
                                    : 0
Factor
           - dense dim.
                                                         flops
                                                                                : 1.70e+01
             DFEAS
                               PRSTATUS
                                                                               MII
ITE PFEAS
                      GFEAS
                                          POR.I
                                                             DOB.T
                                                                                         TIME
                     3.4e+00 0.00e+00
                                           2.414213562e+00
                                                             0.00000000e+00
   1.0e+00 2.9e-01
                                                                                1.0e+00
                                                                                         0.01
    2.7e-01
            7.9e-02
                      2.2e+00
                               8.83e-01
                                           6.969257574e-01
                                                             -9.685901771e-03
                                                                               2.7e-01
1
                                                                                         0.01
   6.5e-02
            1.9e-02
                      1.2e+00
                               1.16e+00
                                           7.606090061e-01
                                                             6.046141322e-01
                                                                                6.5e-02
                                                                                         0.01
    1.7e-03
             5.0e-04
                      2.2e-01
                               1.12e+00
                                           7.084385672e-01
                                                             7.045122560e-01
                                                                                1.7e-03
                                                                                         0.01
    1.4e-08
             4.2e-09
                      4.9e-08
                               1.00e+00
                                           7.071067941e-01
                                                             7.071067599e-01
                                                                                1.4e-08
```

The first line displays the number of threads used by the optimizer and the second line indicates if the optimizer chose to solve the primal or dual problem (see *iparam.intpnt\_solve\_form*). The next lines display the problem dimensions as seen by the optimizer, and the Factor... lines show various statistics. This is followed by the iteration log.

Using the same notation as in Sec. 13.3.1 the columns of the iteration log have the following meaning:

- ITE: Iteration index k.
- PFEAS:  $||Ax^k b\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- DFEAS:  $||A^Ty^k + s^k c\tau^k||_{\infty}$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- GFEAS:  $|-c^Tx^k+b^Ty^k-\kappa^k|$ . The numbers in this column should converge monotonically towards zero but may stall at low level due to rounding errors.
- PRSTATUS: This number converges to 1 if the problem has an optimal solution whereas it converges to -1 if that is not the case.
- $\bullet$  POBJ:  $c^Tx^k/\tau^k.$  An estimate for the primal objective value.
- DOBJ:  $b^T y^k / \tau^k$ . An estimate for the dual objective value.
- $\bullet$  MU:  $\frac{(x^k)^T s^k + \tau^k \kappa^k}{n+1}$  . The numbers in this column should always converge to zero.
- TIME: Time spent since the optimization started (in seconds).

# 13.4 The Optimizer for Mixed-integer Problems

A problem is a mixed-integer optimization problem when one or more of the variables are constrained to be integer valued. Readers unfamiliar with integer optimization are recommended to consult some relevant literature, e.g. the book /Wol98/ by Wolsey.

## 13.4.1 The Mixed-integer Optimizer Overview

MOSEK can solve mixed-integer

- linear,
- quadratic and quadratically constrained, and
- conic

problems, except for mixed-integer semidefinite problems. The mixed-integer optimizer is specialized for solving linear and conic optimization problems. Pure quadratic and quadratically constrained problems are automatically converted to conic form.

By default the mixed-integer optimizer is run-to-run deterministic. This means that if a problem is solved twice on the same computer with identical parameter settings and no time limit then the obtained solutions will be identical. If a time limit is set then this may not be case since the time taken to solve a problem is not deterministic. The mixed-integer optimizer is parallelized i.e. it can exploit multiple cores during the optimization.

The solution process can be split into these phases:

- 1. Presolve: See Sec. 13.1.
- 2. Cut generation: Valid inequalities (cuts) are added to improve the lower bound.
- 3. **Heuristic:** Using heuristics the optimizer tries to guess a good feasible solution. Heuristics can be controlled by the parameter <code>iparam.mio\_heuristic\_level</code>.
- 4. **Search:** The optimal solution is located by branching on integer variables.

## 13.4.2 Relaxations and bounds

It is important to understand that, in a worst-case scenario, the time required to solve integer optimization problems grows exponentially with the size of the problem (solving mixed-integer problems is NP-hard). For instance, a problem with n binary variables, may require time proportional to  $2^n$ . The value of  $2^n$  is huge even for moderate values of n.

In practice this implies that the focus should be on computing a near-optimal solution quickly rather than on locating an optimal solution. Even if the problem is only solved approximately, it is important to know how far the approximate solution is from an optimal one. In order to say something about the quality of an approximate solution the concept of *relaxation* is important.

Consider for example a mixed-integer optimization problem

$$z^* = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ ,
 $x \ge 0$ 
 $x_j \in \mathbb{Z}, \quad \forall j \in \mathcal{J}.$ 

$$(13.12)$$

It has the continuous relaxation

$$\underline{z} = \text{minimize} \quad c^T x$$
subject to  $Ax = b$ ,
 $x \ge 0$  (13.13)

obtained simply by ignoring the integrality restrictions. The relaxation is a continuous problem, and therefore much faster to solve to optimality with a linear (or, in the general case, conic) optimizer. We call the optimal value  $\underline{z}$  the *objective bound*. The objective bound  $\underline{z}$  normally increases during the solution search process when the continuous relaxation is gradually refined.

Moreover, if  $\hat{x}$  is any feasible solution to (13.12) and

$$\bar{z} := c^T \hat{x}$$

then

$$\underline{z} \le z^* \le \bar{z}$$
.

These two inequalities allow us to estimate the quality of the integer solution: it is no further away from the optimum than  $\bar{z} - \underline{z}$  in terms of the objective value. Whenever a mixed-integer problem is solved **MOSEK** reports this lower bound so that the quality of the reported solution can be evaluated.

## 13.4.3 Outer approximation for mixed-integer conic problems

The relaxations of mixed integer conic problems can be solved either as a nonlinear problem with the interior point algorithm (default) or with a linear outer approximation algorithm. The type of relaxation used can be set with <code>iparam.mio\_conic\_outer\_approximation</code>. The best value for this option is highly problem dependent.

#### 13.4.4 Randomization

A number of internal algorithms of the mixed-integer solver are dependend on random tie-breaking. The random tie-breaking can have a significant impact on the path taken by the algorithm and the optimal solution returned. The random seed can be set with the parameter <code>iparam.mio\_seed</code>.

#### 13.4.5 Termination Criterion

In general, it is time consuming to find an exact feasible and optimal solution to an integer optimization problem, though in many practical cases it may be possible to find a sufficiently good solution. The issue of terminating the mixed-integer optimizer is rather delicate and the user has numerous possibilities of influencing it with various parameters. The mixed-integer optimizer employs a relaxed feasibility and optimality criterion to determine when a satisfactory solution is located.

A candidate solution that is feasible for the continuous relaxation is said to be an *integer feasible* solution if the criterion

$$\min(x_j - \lfloor x_j \rfloor, \lceil x_j \rceil - x_j) \le \delta_1 \quad \forall j \in \mathcal{J}$$

is satisfied, meaning that  $x_j$  is at most  $\delta_1$  from the nearest integer.

Whenever the integer optimizer locates an integer feasible solution it will check if the criterion

$$\bar{z} - \underline{z} \le \max(\delta_2, \delta_3 \max(\delta_4, |\bar{z}|))$$

is satisfied. If this is the case, the integer optimizer terminates and reports the integer feasible solution as an optimal solution.

All the  $\delta$  tolerances discussed above can be adjusted using suitable parameters — see Table 13.3.

Table 13.3: Tolerances for the mixed-integer optimizer.

Tolerance	Parameter name
$\delta_1$	$dparam.mio\_tol\_abs\_relax\_int$
$\delta_2$	$dparam.mio\_tol\_abs\_gap$
$\delta_3$	$dparam.mio\_tol\_rel\_gap$
$\delta_4$	$dparam.mio\_rel\_gap\_const$

In Table 13.4 some other common parameters affecting the integer optimizer termination criterion are shown.

Table 13.4: Other parameters affecting the integer optimizer termination criterion.

Parameter name	Explanation
iparam.mio_max_num_branches	Maximum number of branches allowed.
iparam.mio_max_num_relaxs	Maximum number of relaxations allowed.
iparam.mio_max_num_solutions	Maximum number of feasible integer solutions allowed.

## 13.4.6 Speeding Up the Solution Process

As mentioned previously, in many cases it is not possible to find an optimal solution to an integer optimization problem in a reasonable amount of time. Some suggestions to reduce the solution time are:

• Relax the termination criterion: In case the run time is not acceptable, the first thing to do is to relax the termination criterion — see Sec. 13.4.5 for details.

- Specify a good initial solution: In many cases a good feasible solution is either known or easily computed using problem-specific knowledge. If a good feasible solution is known, it is usually worthwhile to use this as a starting point for the integer optimizer. See Sec. 6.7.2.
- Improve the formulation: A mixed-integer optimization problem may be impossible to solve in one form and quite easy in another form. However, it is beyond the scope of this manual to discuss good formulations for mixed-integer problems. For discussions on this topic see for example [Wol98].

## 13.4.7 Understanding Solution Quality

To determine the quality of the solution one should check the following:

- The problem status and solution status returned by MOSEK, as well as constraint violations in case of suboptimal solutions.
- The optimality gap defined as

```
\epsilon = |(\text{objective value of feasible solution}) - (\text{objective bound})| = |\bar{z} - \underline{z}|.
```

which measures how much the located solution can deviate from the optimal solution to the problem. The optimality gap can be retrieved through the information item dinfitem.  $mio\_obj\_abs\_gap$ . Often it is more meaningful to look at the relative optimality gap normalized against the magnitude of the solution.

$$\epsilon_{\rm rel} = \frac{|\bar{z} - \underline{z}|}{\max(\delta_4, |\bar{z}|)}.$$

The relative optimality gap is available in the information item dinfitem.mio\_obj\_rel\_gap.

## 13.4.8 The Mixed-integer Log

Below is a typical log output from the mixed-integer optimizer:

Presolved problem: 6573 variables, 35728 constraints, 101258 non-zeros								
Presolved problem: 0 general integer, 4294 binary, 2279 continuous								
Clique	Clique table size: 1636							
BRANCHE	ES RELAXS	ACT_NDS	DEPTH	BEST_INT_OBJ	BEST_RELAX_OBJ	REL_GAP(%)	TIME	
0	1	0	0	NA	1.8218819866e+07	NA	1.6	
0	1	0	0	1.8331557950e+07	1.8218819866e+07	0.61	3.5	
0	1	0	0	1.8300507546e+07	1.8218819866e+07	0.45	4.3	
Cut ger	Cut generation started.							
0	2	0	0	1.8300507546e+07	1.8218819866e+07	0.45	5.3	
Cut generation terminated. Time = 1.43								
0	3	0	0	1.8286893047e+07	1.8231580587e+07	0.30	7.5	
15	18	1	0	1.8286893047e+07	1.8231580587e+07	0.30	10.5	
31	34	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.1	
51	54	1	0	1.8286893047e+07	1.8231580587e+07	0.30	11.6	
91	94	1	0	1.8286893047e+07	1.8231580587e+07	0.30	12.4	
171	174	1	0	1.8286893047e+07	1.8231580587e+07	0.30	14.3	
331	334	1	0	1.8286893047e+07	1.8231580587e+07	0.30	17.9	
[ ]								
Objecti	ive of bes	st integer	solution	: 1.825846762609e+07				
Best objective bound : 1.823311032986e+07								
Construct solution objective			ve	: Not employed				
Construct solution # roundings			ings.	: 0				
User objective cut value : 0				: 0				
Number of cuts generated				: 117				
Number of Gomory cuts				: 108				
Number of CMIR cuts : 9								

 $({\rm continued\ from\ previous\ page})$ 

Number	of	branches :	4425
Number	of	relaxations solved :	4410
Number	of	interior point iterations:	25
Number	of	simplex iterations :	221131

The first lines contain a summary of the problem as seen by the optimizer. This is followed by the iteration log. The columns have the following meaning:

- BRANCHES: Number of branches generated.
- RELAXS: Number of relaxations solved.
- ACT\_NDS: Number of active branch bound nodes.
- DEPTH: Depth of the recently solved node.
- $\bullet$  BEST\_INT\_OBJ: The best integer objective value,  $\bar{z}.$
- BEST\_RELAX\_OBJ: The best objective bound,  $\underline{z}$ .
- REL\_GAP(%): Relative optimality gap,  $100\% \cdot \epsilon_{\rm rel}$
- TIME: Time (in seconds) from the start of optimization.

Following that a summary of the optimization process is printed.

# Chapter 14

# Additional features

In this section we describe additional features and tools which enable more detailed analysis of optimization problems with  $\mathbf{MOSEK}$ .

# 14.1 Problem Analyzer

The problem analyzer prints a survey of the structure of the problem, with information about linear constraints and objective, quadratic constraints, conic constraints and variables.

In the initial stages of model formulation the problem analyzer may be used as a quick way of verifying that the model has been built or imported correctly. In later stages it can help revealing special structures within the model that may be used to tune the optimizer's performance or to identify the causes of numerical difficulties.

The problem analyzer is run using *Task. analyzeproblem*. It prints its output to a log stream. The output is similar to the one below (this is the problem survey of the aflow30a problem from the MIPLIB 2003 collection).

** Structural	report					
	Dimensions					
	Constraints	Variables	Matrix var.	Cones		
	479	842	0	0		
	Constraint a	nd bound types	3			
	Free	Lower	Upper	Ranged	Fixed	
Constraints:	0	0	421	0	58	
Variables:	0	0	0	842	0	
	Integer cons	traint types				
	Binary	General				
	421	0				
*** Data repor	rt					
_	Nonzeros	Min	Max			
cj :	421	1.1e+01	5.0e+02			
Aij :		1.0e+00	1.0e+02			
	# finite	Min	Max			
blci :	58	1.0e+00	1.0e+01			
buci :	479	0.0e+00	1.0e+01			
blxj :	842	0.0e+00	0.0e+00			
buxj :	842	1.0e+00	1.0e+02			

The survey is divided into a structural and numerical report. The content should be self-explanatory.

# 14.2 Automatic Repair of Infeasible Problems

**MOSEK** provides an automatic repair tool for infeasible linear problems which we cover in this section. Note that most infeasible models are so due to bugs which can (and should) be more reliably fixed manually, using the knowledge of the model structure. We discuss this approach in Sec. 8.3.

# 14.2.1 Automatic repair

The main idea can be described as follows. Consider the linear optimization problem with m constraints and n variables

which is assumed to be infeasible.

One way of making the problem feasible is to reduce the lower bounds and increase the upper bounds. If the change is sufficiently large the problem becomes feasible. Now an obvious idea is to compute the optimal relaxation by solving an optimization problem. The problem

minimize 
$$p(v_{l}^{c}, v_{u}^{c}, v_{u}^{x}, v_{u}^{x})$$
 subject to 
$$l^{c} \leq Ax + v_{l}^{c} - v_{u}^{c} \leq u^{c},$$
 
$$l^{x} \leq x + v_{l}^{x} - v_{u}^{x} \leq u^{x},$$
 
$$v_{l}^{c}, v_{u}^{c}, v_{l}^{x}, v_{u}^{x} \geq 0$$
 (14.1)

does exactly that. The additional variables  $(v_l^c)_i$ ,  $(v_u^c)_i$ ,  $(v_l^x)_j$  and  $(v_u^c)_j$  are elasticity variables because they allow a constraint to be violated and hence add some elasticity to the problem. For instance, the elasticity variable  $(v_l^c)_i$  controls how much the lower bound  $(l^c)_i$  should be relaxed to make the problem feasible. Finally, the so-called penalty function

$$p(v_l^c, v_u^c, v_l^x, v_u^x)$$

is chosen so it penalizes changes to bounds. Given the weights

- $w_l^c \in \mathbb{R}^m$  (associated with  $l^c$ ),
- $w_u^c \in \mathbb{R}^m$  (associated with  $u^c$ ),
- $w_l^x \in \mathbb{R}^n$  (associated with  $l^x$ ),
- $w_u^x \in \mathbb{R}^n$  (associated with  $u^x$ ),

a natural choice is

$$p(v_l^c, v_u^c, v_l^x, v_u^x) = (w_l^c)^T v_l^c + (w_u^c)^T v_u^c + (w_l^x)^T v_l^x + (w_u^x)^T v_u^x.$$

Hence, the penalty function p() is a weighted sum of the elasticity variables and therefore the problem (14.1) keeps the amount of relaxation at a minimum. Please observe that

- the problem (14.1) is always feasible.
- a negative weight implies problem (14.1) is unbounded. For this reason if the value of a weight is negative **MOSEK** fixes the associated elasticity variable to zero. Clearly, if one or more of the weights are negative, it may imply that it is not possible to repair the problem.

A simple choice of weights is to set them all to 1, but of course that does not take into account that constraints may have different importance.

#### **Caveats**

Observe if the infeasible problem

```
\begin{array}{rll} \text{minimize} & x+z \\ \text{subject to} & x & = & -1, \\ & x & > & 0 \end{array}
```

is repaired then it will become unbounded. Hence, a repaired problem may not have an optimal solution. Another and more important caveat is that only a minimal repair is performed i.e. the repair that barely makes the problem feasible. Hence, the repaired problem is barely feasible and that sometimes makes the repaired problem hard to solve.

#### Using the automatic repair tool

In this subsection we consider an infeasible linear optimization example:

minimize 
$$-10x_1$$
  $-9x_2$ ,  
subject to  $7/10x_1 + 1x_2 \le 630$ ,  
 $1/2x_1 + 5/6x_2 \le 600$ ,  
 $1x_1 + 2/3x_2 \le 708$ ,  
 $1/10x_1 + 1/4x_2 \le 135$ ,  
 $x_1$ ,  $x_2 \ge 0$ ,  
 $x_2 \ge 650$ . (14.2)

The function *Task.primalrepair* can be used to repair an infeasible problem. This can be used for linear and conic optimization problems, possibly with integer variables.

Listing 14.1: An example of feasibility repair applied to problem (14.2).

(continued from previous page)

```
task.primalrepair(null, null, null);

double sum_viol = task.getdouinf(mosek.dinfitem.primal_repair_penalty_obj);

Console.WriteLine("Minimized sum of violations = %{0}", sum_viol);

task.optimize();
task.solutionsummary(mosek.streamtype.msg);
}
}
}
}
}
```

The above code will produce the following log report:

```
MOSEK Version 9.0.0.25(ALPHA) (Build date: 2017-11-7 16:11:50)
Copyright (c) MOSEK ApS, Denmark. WWW: mosek.com
Platform: Linux/64-X86
Open file 'feasrepair.lp'
Reading started.
Reading terminated. Time: 0.00
Read summary
 Туре
                 : LO (linear optimization problem)
 Objective sense : min
 Scalar variables : 2
 Matrix variables : 0
 Constraints : 4
 Cones
                 : 0
 Time
                 : 0.0
Problem
 Name
  Objective sense
                       : min
 Type
                       : LO (linear optimization problem)
 Constraints
                       : 4
                       : 0
 Cones
 Scalar variables
                       : 2
 Matrix variables
                       : 0
 Integer variables
                       : 0
Primal feasibility repair started.
Optimizer started.
Presolve started.
Linear dependency checker started.
Linear dependency checker terminated.
Eliminator started.
Freed constraints in eliminator : 2
Eliminator terminated.
Eliminator - tries
                                                                            : 0.00
                                  : 1
                                                      time
Lin. dep. - tries
                                  : 1
                                                      time
                                                                            : 0.00
Lin. dep. - number
                                  : 0
Presolve terminated. Time: 0.00
Problem
 Name
 Objective sense
                       : min
                        : LO (linear optimization problem)
 Туре
  Constraints
                       : 8
                        : 0
 Cones
 Scalar variables
                        : 14
```

(continued from previous page)

```
Matrix variables
                     : 0
  Integer variables
                      : 0
Optimizer - threads
                                : 20
Optimizer - solved problem
                                : the primal
Optimizer - Constraints
                                : 2
Optimizer - Cones
                               : 0
Optimizer - Scalar variables : 5
                                                                       : 0
                                                  conic
Optimizer - Semi-definite variables: 0
                                                  scalarized
                                                                       : 0
Factor - setup time : 0.00
                                                 dense det. time
                                                                       : 0.00
Factor
         - ML order time
                               : 0.00
                                                GP order time
Factor
         - nonzeros before factor : 3
                                                 after factor
Factor
          - dense dim.
                               : 0
                                                 flops
                                                                       : 5.00e+01
ITE PFEAS
           DFEAS GFEAS
                           PRSTATUS
                                     POBJ
                                                      DOBJ
                                                                      MU
                                                                               TIME
0 2.7e+01 1.0e+00 4.0e+00 1.00e+00 3.00000000e+00 0.00000000e+00 1.0e+00 0.00
   2.5e+01 9.1e-01 1.4e+00 0.00e+00
                                      8.711262850e+00 1.115287830e+01 2.4e+00 0.00
2 2.4e+00 8.8e-02 1.4e-01 -7.33e-01 4.062505701e+01 4.422203730e+01 2.3e-01 0.00
3 9.4e-02 3.4e-03 5.5e-03 1.33e+00 4.250700434e+01 4.258548510e+01 9.1e-03 0.00
4 2.0e-05 7.2e-07 1.1e-06 1.02e+00 4.249996599e+01 4.249998669e+01 1.9e-06 0.00
5 2.0e-09 7.2e-11 1.1e-10 1.00e+00 4.250000000e+01 4.250000000e+01 1.9e-10 0.00
Basis identification started.
Basis identification terminated. Time: 0.00
Optimizer terminated. Time: 0.01
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: 4.2500000000e+01 nrm: 6e+02 Viol. con: 1e-13 var: 0e+00
 Dual. obj: 4.2499999999e+01 nrm: 2e+00 Viol. con: 0e+00 var: 9e-11
Optimal objective value of the penalty problem: 4.250000000000e+01
Repairing bounds.
Increasing the upper bound 1.35e+02 on constraint 'c4' (3) with 2.25e+01.
Decreasing the lower bound 6.50e+02 on variable 'x2' (4) with 2.00e+01.
Primal feasibility repair terminated.
Optimizer started.
Optimizer terminated. Time: 0.00
Interior-point solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                             Viol. con: 0e+00
                                                              var: 0e+00
          obj: -5.6700000000e+03 nrm: 1e+01
                                             Viol. con: 0e+00 var: 0e+00
Basic solution summary
 Problem status : PRIMAL_AND_DUAL_FEASIBLE
 Solution status : OPTIMAL
 Primal. obj: -5.6700000000e+03 nrm: 6e+02
                                             Viol. con: 0e+00
                                                              var: 0e+00
 Dual. obj: -5.6700000000e+03 nrm: 1e+01
                                             Viol. con: 0e+00 var: 0e+00
Optimizer summary
 Optimizer
                                                time: 0.00
   Interior-point
                         - iterations : 0
                                               time: 0.00
                                               time: 0.00
     Basis identification -
       Primal
                         - iterations : 0
                                               time: 0.00
       Dual
                         - iterations : 0
                                               time: 0.00
       Clean primal
                         - iterations : 0
                                               time: 0.00
       Clean dual
                         - iterations : 0
                                               time: 0.00
                                               time: 0.00
   Simplex
     Primal simplex
                         - iterations : 0
                                               time: 0.00
```

Dual simplex	- iterations : 0	time: 0.00	
Mixed integer	- relaxations: 0	time: 0.00	

It will also modify the task according to the optimal elasticity variables found. In this case the optimal repair it is to increase the upper bound on constraint c4 by 22.5 and decrease the lower bound on variable x2 by 20.

# 14.3 Sensitivity Analysis

Given an optimization problem it is often useful to obtain information about how the optimal objective value changes when the problem parameters are perturbed. E.g, assume that a bound represents the capacity of a machine. Now, it may be possible to expand the capacity for a certain cost and hence it is worthwhile knowing what the value of additional capacity is. This is precisely the type of questions the sensitivity analysis deals with.

Analyzing how the optimal objective value changes when the problem data is changed is called sensitivity analysis.

#### References

The book [Chv83] discusses the classical sensitivity analysis in Chapter 10 whereas the book [RTV97] presents a modern introduction to sensitivity analysis. Finally, it is recommended to read the short paper [Wal00] to avoid some of the pitfalls associated with sensitivity analysis.

Warning: Currently, sensitivity analysis is only available for continuous linear optimization problems. Moreover, MOSEK can only deal with perturbations of bounds and objective function coefficients.

# 14.3.1 Sensitivity Analysis for Linear Problems

# The Optimal Objective Value Function

Assume that we are given the problem

$$z(l^{c}, u^{c}, l^{x}, u^{x}, c) = \underset{\text{subject to}}{\text{minimize}} c^{T}x$$

$$l^{c} \leq Ax \leq u^{c},$$

$$l^{x} \leq x \leq u^{x},$$

$$(14.3)$$

and we want to know how the optimal objective value changes as  $l_i^c$  is perturbed. To answer this question we define the perturbed problem for  $l_i^c$  as follows

$$f_{l_i^c}(\beta) = \text{minimize} \qquad c^T x$$
  
subject to  $l^c + \beta e_i \leq Ax \leq u^c$ ,  $l^x \leq x \leq u^x$ ,

where  $e_i$  is the *i*-th column of the identity matrix. The function

$$f_{l_i^c}(\beta) \tag{14.4}$$

shows the optimal objective value as a function of  $\beta$ . Please note that a change in  $\beta$  corresponds to a perturbation in  $l_i^c$  and hence (14.4) shows the optimal objective value as a function of varying  $l_i^c$  with the other bounds fixed.

It is possible to prove that the function (14.4) is a piecewise linear and convex function, i.e. its graph may look like in Fig. 14.1 and Fig. 14.2.

Clearly, if the function  $f_{l_i^c}(\beta)$  does not change much when  $\beta$  is changed, then we can conclude that the optimal objective value is insensitive to changes in  $l_i^c$ . Therefore, we are interested in the rate of change in  $f_{l_i^c}(\beta)$  for small changes in  $\beta$ —specifically the gradient

$$f'_{l_i^c}(0),$$

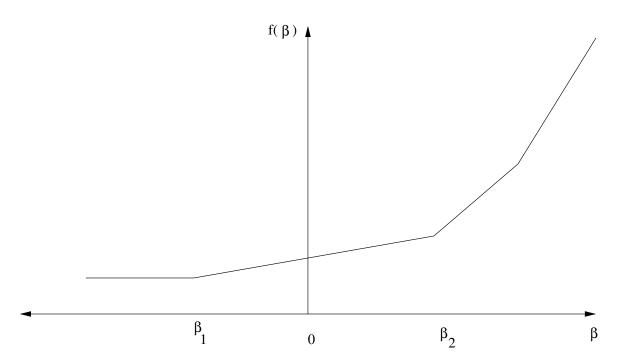


Fig. 14.1:  $\beta=0$  is in the interior of linearity interval.

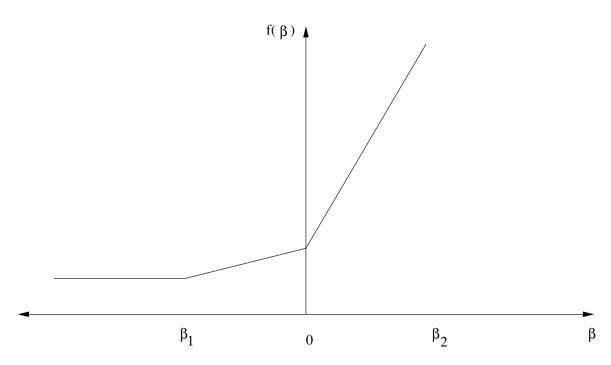


Fig. 14.2:  $\beta = 0$  is a breakpoint.

which is called the *shadow price* related to  $l_i^c$ . The shadow price specifies how the objective value changes for small changes of  $\beta$  around zero. Moreover, we are interested in the *linearity interval* 

$$\beta \in [\beta_1, \beta_2]$$

for which

$$f'_{l_i^c}(\beta) = f'_{l_i^c}(0).$$

Since  $f_{l_i^c}$  is not a smooth function  $f'_{l_i^c}$  may not be defined at 0, as illustrated in Fig. 14.2. In this case we can define a left and a right shadow price and a left and a right linearity interval.

The function  $f_{l_i^c}$  considered only changes in  $l_i^c$ . We can define similar functions for the remaining parameters of the z defined in (14.3) as well:

$$f_{l_i^c}(\beta) = z(l^c + \beta e_i, u^c, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{u_i^c}(\beta) = z(l^c, u^c + \beta e_i, l^x, u^x, c), \quad i = 1, \dots, m,$$

$$f_{l_j^x}(\beta) = z(l^c, u^c, l^x + \beta e_j, u^x, c), \quad j = 1, \dots, n,$$

$$f_{u_j^x}(\beta) = z(l^c, u^c, l^x, u^x + \beta e_j, c), \quad j = 1, \dots, n,$$

$$f_{c_j}(\beta) = z(l^c, u^c, l^x, u^x, c + \beta e_j), \quad j = 1, \dots, n.$$

Given these definitions it should be clear how linearity intervals and shadow prices are defined for the parameters  $u_i^c$  etc.

#### **Equality Constraints**

In **MOSEK** a constraint can be specified as either an equality constraint or a ranged constraint. If some constraint  $e_i^c$  is an equality constraint, we define the optimal value function for this constraint as

$$f_{e_i^c}(\beta) = z(l^c + \beta e_i, u^c + \beta e_i, l^x, u^x, c)$$

Thus for an equality constraint the upper and the lower bounds (which are equal) are perturbed simultaneously. Therefore, **MOSEK** will handle sensitivity analysis differently for a ranged constraint with  $l_i^c = u_i^c$  and for an equality constraint.

#### The Basis Type Sensitivity Analysis

The classical sensitivity analysis discussed in most textbooks about linear optimization, e.g. [Chv83], is based on an optimal basis. This method may produce misleading results [RTV97] but is computationally cheap. This is the type of sensitivity analysis implemented in **MOSEK**.

We will now briefly discuss the basis type sensitivity analysis. Given an optimal basic solution which provides a partition of variables into basic and non-basic variables, the basis type sensitivity analysis computes the linearity interval  $[\beta_1, \beta_2]$  so that the basis remains optimal for the perturbed problem. A shadow price associated with the linearity interval is also computed. However, it is well-known that an optimal basic solution may not be unique and therefore the result depends on the optimal basic solution employed in the sensitivity analysis. If the optimal objective value function has a breakpoint for  $\beta = 0$  then the basis type sensitivity method will only provide a subset of either the left or the right linearity interval.

In summary, the basis type sensitivity analysis is computationally cheap but does not provide complete information. Hence, the results of the basis type sensitivity analysis should be used with care.

#### **Example: Sensitivity Analysis**

As an example we will use the following transportation problem. Consider the problem of minimizing the transportation cost between a number of production plants and stores. Each plant supplies a number of goods and each store has a given demand that must be met. Supply, demand and cost of transportation per unit are shown in Fig. 14.3.

If we denote the number of transported goods from location i to location j by  $x_{ij}$ , problem can be formulated as the linear optimization problem of minimizing

$$1x_{11} + 2x_{12} + 5x_{23} + 2x_{24} + 1x_{31} + 2x_{33} + 1x_{34}$$

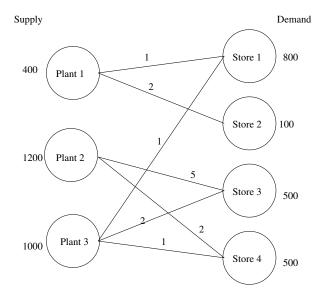


Fig. 14.3: Supply, demand and cost of transportation.

subject to

The sensitivity parameters are shown in Table 14.1 and Table 14.2.

Table 14.1: Ranges and shadow prices related to bounds on constraints and variables.

Con.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
1	-300.00	0.00	3.00	3.00
2	-700.00	$+\infty$	0.00	0.00
3	-500.00	0.00	3.00	3.00
4	-0.00	500.00	4.00	4.00
5	-0.00	300.00	5.00	5.00
6	-0.00	700.00	5.00	5.00
7	-500.00	700.00	2.00	2.00
Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$x_{11}$	$-\infty$	300.00	0.00	0.00
$x_{12}$	$-\infty$	100.00	0.00	0.00
$x_{23}$	$-\infty$	0.00	0.00	0.00
$x_{24}$	$-\infty$	500.00	0.00	0.00
$x_{31}$	$-\infty$	500.00	0.00	0.00
$x_{33}$	$-\infty$	500.00	0.00	0.00
$x_{34}$	-0.000000	500.00	2.00	2.00

Table 14.2: Ranges and shadow prices related to the objective coefficients.

Var.	$\beta_1$	$\beta_2$	$\sigma_1$	$\sigma_2$
$c_1$	$-\infty$	3.00	300.00	300.00
$c_2$	$-\infty$	$\infty$	100.00	100.00
$c_3$	-2.00	$\infty$	0.00	0.00
$c_4$	$-\infty$	2.00	500.00	500.00
$c_5$	-3.00	$\infty$	500.00	500.00
$c_6$	$-\infty$	2.00	500.00	500.00
$c_7$	-2.00	$\infty$	0.00	0.00

Examining the results from the sensitivity analysis we see that for constraint number 1 we have  $\sigma_1 = 3$  and  $\beta_1 = -300$ ,  $\beta_2 = 0$ .

If the upper bound on constraint 1 is decreased by

$$\beta \in [0,300]$$

then the optimal objective value will increase by the value

$$\sigma_1 \beta = 3\beta$$
.

# 14.3.2 Sensitivity Analysis with MOSEK

MOSEK provides the functions Task. primalsensitivity and Task. dualsensitivity for performing sensitivity analysis. The code in Listing 14.2 gives an example of its use.

Listing 14.2: Example of sensitivity analysis with the **MOSEK** Optimizer API for .NET.

```
using System;
namespace mosek.example
 class msgclass : mosek.Stream
    string prefix;
   public msgclass (string prfx)
     prefix = prfx;
   public override void streamCB (string msg)
      Console.Write ("{0}{1}", prefix, msg);
 }
 public class sensitivity
   public static void Main ()
      const double
      infinity = 0;
      mosek.boundkey[] bkc = new mosek.boundkey[] {
       mosek.boundkey.up, mosek.boundkey.up,
        mosek.boundkey.up, mosek.boundkey.fx,
```

(continued from previous page)

```
mosek.boundkey.fx, mosek.boundkey.fx,
 mosek.boundkey.fx
};
mosek.boundkey[] bkx = new mosek.boundkey[] {
 mosek.boundkey.lo, mosek.boundkey.lo,
 mosek.boundkey.lo, mosek.boundkey.lo,
 mosek.boundkey.lo, mosek.boundkey.lo,
 mosek.boundkey.lo
};
int[] ptrb = new int[] {0, 2, 4, 6, 8, 10, 12};
int[] ptre = new int[] {2, 4, 6, 8, 10, 12, 14};
int[] sub = new int[] {0, 3, 0, 4, 1, 5, 1, 6, 2, 3, 2, 5, 2, 6};
double[] blc = new double[] {
  -infinity, -infinity,
  -infinity, 800, 100, 500, 500
};
double[] buc = new double[] {400, 1200, 1000, 800, 100, 500, 500};
double[] c = new double[] {1.0, 2.0, 5.0, 2.0, 1.0, 2.0, 1.0};
double[] blx = new double[] {0.0, 0.0, 0.0, 0.0, 0.0, 0.0};
double[] bux = new double[] {infinity,
                             infinity,
                             infinity,
                             infinity,
                             infinity,
                             infinity,
                             infinity
                            };
double[] val = new double[] {1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0,
                             1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0
int numcon = 7; /* Number of constraints.
int numvar = 7; /* Number of variables.
try
 using (mosek.Env env = new mosek.Env())
   using (mosek.Task task = new mosek.Task(env))
      // Directs the log task stream to the user specified
      // method task_msq_obj.streamCB
     task.set_Stream(mosek.streamtype.log, new msgclass ("[task]"));
      task.inputdata(numcon, numvar,
                     С,
                     0.0.
                     ptrb,
                     ptre,
                     sub,
                     val,
                     bkc,
                     blc,
                     buc,
                     bkx,
                     blx,
                     bux);
```

(continued from previous page)

```
/* A maximization problem */
task.putobjsense(mosek.objsense.minimize);
try
  task.optimize();
catch (mosek.Warning w)
  Console.WriteLine("Mosek warning:");
  Console.WriteLine (w.Code);
  Console.WriteLine (w);
/* Analyze upper bound on c1 and the equality constraint on c4 */
int[] subi = new int [] {0, 3};
mosek.mark[] marki = new mosek.mark[] {mosek.mark.up,
                                       mosek.mark.up
                                      };
/* Analyze lower bound on the variables x12 and x31 */
int[] subj = new int [] {1, 4};
mosek.mark[] markj = new mosek.mark[] {mosek.mark.lo,
                                       mosek.mark.lo
                                      };
double[] leftpricei = new double[2];
double[] rightpricei = new double[2];
double[] leftrangei = new double[2];
double[] rightrangei = new double[2];
double[] leftpricej = new double[2];
double[] rightpricej = new double[2];
double[] leftrangej = new double[2];
double[] rightrangej = new double[2];
task.primalsensitivity( subi,
                        marki,
                        subj,
                        markj,
                        leftpricei,
                        rightpricei,
                        leftrangei,
                        rightrangei,
                        leftpricej,
                        rightpricej,
                        leftrangej,
                        rightrangej);
Console.Write("Results from sensitivity analysis on bounds:\n");
Console.Write("For constraints:\n");
for (int i = 0; i < 2; ++i)
  Console.Write(
    "leftprice = {0}, rightprice = {1}, leftrange = {2}, rightrange ={3}\n",
    leftpricei[i], rightpricei[i], leftrangei[i], rightrangei[i]);
Console.Write("For variables:\n");
for (int i = 0; i < 2; ++i)
  Console.Write(
```

 $({\rm continued\ from\ previous\ page})$ 

```
"leftprice = \{0\}, rightprice = \{1\}, leftrange = \{2\}, rightrange =\{3\}\n",
              leftpricej[i], rightpricej[i], leftrangej[i], rightrangej[i]);
          double[] leftprice = new double[2];
          double[] rightprice = new double[2];
          double[] leftrange = new double[2];
          double[] rightrange = new double[2];
          int[] subc = new int[] {2, 5};
          task.dualsensitivity( subc,
                                 leftprice,
                                 rightprice,
                                 leftrange,
                                 rightrange
                               );
          Console.Write("Results from sensitivity analysis on objective coefficients:");
          for (int i = 0; i < 2; ++i)
            Console.Write(
              "leftprice = {0}, rightprice = {1}, leftrange = {2}, rightrange = {3}\n",
              leftprice[i], rightprice[i], leftrange[i], rightrange[i]);
        }
      }
    }
    catch (mosek.Exception e)
      Console.WriteLine (e.Code);
      Console.WriteLine (e);
      throw;
  }
}
```

# Chapter 15

# **API** Reference

This section contains the complete reference of the **MOSEK** Optimizer API for .NET. It is organized as follows:

- General API conventions.
- Methods:
  - Class Env (The MOSEK environment)
  - Class Task (An optimization task)
  - Browse by topic
- Optimizer parameters:
  - Double, Integer, String
  - Full list
  - Browse by topic
- $\bullet$  Optimizer information items:
  - Double, Integer, Long
- Optimizer response codes
- Enumerations
- Exceptions
- User-defined class types
- Nonlinear API (SCopt)

# 15.1 API Conventions

# 15.1.1 Function arguments

## **Naming Convention**

In the definition of the **MOSEK** Optimizer API for .NET a consistent naming convention has been used. This implies that whenever for example numcon is an argument in a function definition it indicates the number of constraints. In Table 15.1 the variable names used to specify the problem parameters are listed.

Table 15.1: Naming conventions used in the **MOSEK** Optimizer API for .NET.

API name	API type	Dimension	Related problem parameter
numcon	int		m
numvar	int		n
numcone	int		t
aptrb	int[]	numvar	$a_{ij}$
aptre	int[]	numvar	$  a_{ij}  $
asub	int[]	aptre[numvar-1]	$  a_{ij}  $
aval	double[]	aptre[numvar-1]	$a_{ij}$
С	double[]	numvar	$c_j$ $c^f$
cfix	double		$c^f$
blc	double[]	numcon	$l_k^c$
buc	double[]	numcon	$u_k^c$
blx	double[]	numvar	$l_k^x$
bux	double[]	numvar	$u_k^x$
numqonz	int		$q_{ij}^o$
qosubi	int[]	numqonz	$q_{ij}^o$
qosubj	int[]	numqonz	$q_{ij}^o$
qoval	double[]	numqonz	$\mid q_{ij}^o \mid$
numqcnz	int		$q_{ij}^k$
qcsubk	int[]	numqcnz	$q_{ij}^{\vec{k}}$
qcsubi	int[]	numqcnz	$q_{ij}^{ec{k}}$
qcsubj	int[]	numqcnz	$q_{ij}^{ec{k}}$
qcval	double[]	numqcnz	$q_{ij}^{\vec{k}}$
bkc	int[]	numcon	$l_k^c$ and $u_k^c$
bkx	int[]	numvar	$l_k^x$ and $u_k^x$

The relation between the variable names and the problem parameters is as follows:

- $\bullet \ \ \text{The quadratic terms in the objective:} \ \ q^o_{\texttt{qosubi[t]},\texttt{qosubj[t]}} = \texttt{qoval[t]}, \quad t = 0, \dots, \texttt{numqonz} 1.$
- The linear terms in the objective :  $c_j = c[j], \quad j = 0, \dots, numvar 1$
- The fixed term in the objective :  $c^f = cfix$ .
- $\bullet \ \ \text{The quadratic terms in the constraints:} \ \ q_{\mathtt{qcsubi[t]},\mathtt{qcsubj[t]}}^{\mathtt{qcsubk[t]}} = \mathtt{qcval[t]}, \quad t = 0, \ldots, \mathtt{numqcnz} 1$
- The linear terms in the constraints:  $a_{\mathtt{asub[t]}, \mathtt{j}} = \mathtt{aval[t]}, \quad t = \mathtt{ptrb[j]}, \ldots, \mathtt{ptre[j]} 1, \quad j = 0, \ldots, \mathtt{numvar} 1$

#### Information about input/output arguments

The following are purely informational tags which indicate how  $\mathbf{MOSEK}$  treats a specific function argument.

- (input) An input argument. It is used to input data to MOSEK.
- (output) An output argument. It can be a user-preallocated data structure, a reference, a string buffer etc. where **MOSEK** will output some data.
- (input/output) An input/output argument. **MOSEK** will read the data and overwrite it with new/updated information.

# 15.1.2 Bounds

The bounds on the constraints and variables are specified using the variables bkc, blc, and buc. The components of the integer array bkc specify the bound type according to Table 15.2

Table 15.2: Symbolic key for variable and constraint bounds.

Symbolic constant	Lower bound	Upper bound
boundkey.fx	finite	identical to the lower bound
boundkey.fr	minus infinity	plus infinity
boundkey.lo	finite	plus infinity
boundkey.ra	finite	finite
boundkey.up	minus infinity	finite

For instance bkc[2]=boundkey. lo means that  $-\infty < l_2^c$  and  $u_2^c = \infty$ . Even if a variable or constraint is bounded only from below, e.g.  $x \ge 0$ , both bounds are inputted or extracted; the irrelevant value is ignored.

Finally, the numerical values of the bounds are given by

$$l_k^c = \texttt{blc}[\texttt{k}], \quad k = 0, \dots, \texttt{numcon} - 1$$
 
$$u_k^c = \texttt{buc}[\texttt{k}], \quad k = 0, \dots, \texttt{numcon} - 1.$$

The bounds on the variables are specified using the variables bkx, blx, and bux in the same way. The numerical values for the lower bounds on the variables are given by

$$l^x_j = \mathtt{blx[j]}, \quad j = 0, \dots, \mathtt{numvar} - 1.$$
 
$$u^x_j = \mathtt{bux[j]}, \quad j = 0, \dots, \mathtt{numvar} - 1.$$

# 15.1.3 Vector Formats

Three different vector formats are used in the **MOSEK** API:

## Full (dense) vector

This is simply an array where the first element corresponds to the first item, the second element to the second item etc. For example to get the linear coefficients of the objective in task with number variables, one would write

```
double[] c = new double[numvar];
task.getc(c);
```

## **Vector slice**

A vector slice is a range of values from first up to and **not including last** entry in the vector, i.e. for the set of indices i such that first <= i < last. For example, to get the bounds associated with constrains 2 through 9 (both inclusive) one would write

## Sparse vector

A sparse vector is given as an array of indexes and an array of values. The indexes need not be ordered. For example, to input a set of bounds associated with constraints number 1, 6, 3, and 9, one might write

#### 15.1.4 Matrix Formats

The coefficient matrices in a problem are inputted and extracted in a sparse format. That means only the nonzero entries are listed.

## **Unordered Triplets**

In unordered triplet format each entry is defined as a row index, a column index and a coefficient. For example, to input the A matrix coefficients for  $a_{1,2}=1.1, a_{3,3}=4.3$ , and  $a_{5,4}=0.2$ , one would write as follows:

```
int[] subi = { 1, 3, 5 };
int[] subj = { 2, 3, 4 };
double[] cof = { 1.1, 4.3, 0.2 };
task.putaijlist(subi,subj,cof);
```

Please note that in some cases (like Task.putaijlist) only the specified indexes are modified — all other are unchanged. In other cases (such as Task.putqconk) the triplet format is used to modify all entries — entries that are not specified are set to 0.

#### Column or Row Ordered Sparse Matrix

In a sparse matrix format only the non-zero entries of the matrix are stored. **MOSEK** uses a sparse packed matrix format ordered either by columns or rows. Here we describe the column-wise format. The row-wise format is based on the same principle.

## Column ordered sparse format

A sparse matrix in column ordered format is essentially a list of all non-zero entries read column by column from left to right and from top to bottom within each column. The exact representation uses four arrays:

- asub: Array of size equal to the number of nonzeros. List of row indexes.
- aval: Array of size equal to the number of nonzeros. List of non-zero entries of A ordered by columns.
- ptrb: Array of size numcol, where ptrb[j] is the position of the first value/index in aval/ asub for the j-th column.
- ptre: Array of size numcol, where ptre[j] is the position of the last value/index plus one in aval / asub for the j-th column.

With this representation the values of a matrix A with numcol columns are assigned using:

$$a_{\mathtt{asub}[k],j} = \mathtt{aval}[k] \quad \text{for} \quad j = 0, \dots, \mathtt{numcol} - 1, \ k = \mathtt{ptrb}[j], \dots, \mathtt{ptre}[j] - 1.$$

As an example consider the matrix

$$A = \begin{bmatrix} 1.1 & 1.3 & 1.4 \\ 2.2 & 2.5 \\ 3.1 & 3.4 \\ 4.4 \end{bmatrix}$$
 (15.1)

which can be represented in the column ordered sparse matrix format as

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{ptre} &=& [2,3,5,7,8], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}
```

Fig. 15.1 illustrates how the matrix A in (15.1) is represented in column ordered sparse matrix format.

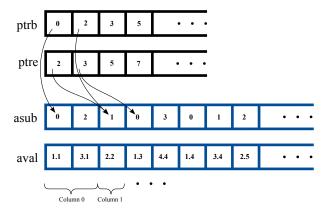


Fig. 15.1: The matrix A (15.1) represented in column ordered packed sparse matrix format.

## Column ordered sparse format with nonzeros

Note that nzc[j] := ptre[j]-ptrb[j] is exactly the number of nonzero elements in the j-th column of A. In some functions a sparse matrix will be represented using the equivalent dataset asub, aval, ptrb, nzc. The matrix A (15.1) would now be represented as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,2,3,5,7], \\ \mathtt{nzc} &=& [2,1,2,2,1], \\ \mathtt{asub} &=& [0,2,1,0,3,0,2,1], \\ \mathtt{aval} &=& [1.1,3.1,2.2,1.3,4.4,1.4,3.4,2.5]. \end{array}
```

#### Row ordered sparse matrix

The matrix A (15.1) can also be represented in the row ordered sparse matrix format as:

```
\begin{array}{lll} \mathtt{ptrb} &=& [0,3,5,7], \\ \mathtt{ptre} &=& [3,5,7,8], \\ \mathtt{asub} &=& [0,2,3,1,4,0,3,2], \\ \mathtt{aval} &=& [1.1,1.3,1.4,2.2,2.5,3.1,3.4,4.4]. \end{array}
```

# 15.2 Functions grouped by topic

#### **Callback**

- Task.set\_InfoCallback Receive callbacks with solver status and information during optimization.
- Task.set\_Progress Receive callbacks about current status of the solver during optimization.
- Task.set\_Stream Directs all output from a task stream to a callback object.
- ullet Infrequent: Task.set\_ItgSolutionCallback, Env.set\_Stream

### **Environment and task management**

- Env. Env Constructor of a new environment.
- Task. Task Constructor of a new optimization task.
- Task. puttaskname Assigns a new name to the task.
- Infrequent: Env. Dispose, Task. Dispose, Task. commitchanges, Task. deletesolution, Task. putmaxnumanz, Task. putmaxnumbarvar, Task. putmaxnumcon, Task. putmaxnumcone, Task. putmaxnumqnz, Task. putmaxnumvar, Task. resizetask

## Infeasibility diagnostic

- Task.getinfeasiblesubproblem Obtains an infeasible subproblem.
- Task.primalrepair Repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables.

#### Information items and statistics

- Task. getdouinf Obtains a double information item.
- Task. getintinf Obtains an integer information item.
- Task.getlintinf Obtains a long integer information item.
- Task.updatesolutioninfo Update the information items related to the solution.
- Infrequent: Task.getinfindex

## Input/Output

- Task.writedata Writes problem data to a file.
- Task.writesolution Write a solution to a file.
- Infrequent: Task.readdata, Task.readdataformat, Task.readjsonstring, Task.readlpstring, Task.readopfstring, Task.readparamfile, Task.readptfstring, Task.readsolution, Task.readsummary, Task.readtask, Task.writejsonsol, Task.writeparamfile, Task.writetask

# Inspecting the task

- Task. analyzeproblem Analyze the data of a task.
- Task. getnumcon Obtains the number of constraints.
- Task. getnumcone Obtains the number of cones.
- Task. getnumvar Obtains the number of variables.
- Infrequent: Task.analyzesolution, Task.getacol, Task.getacolnumnz, Task.getacolslice, Task.getacolslicenumnz, Task.getacolslicetrip, Task.getaij, Task.getapiecenumnz, Task.getarow, Task.getarownumnz, Task.getarowslice, Task.getarowslicenumnz, Task.getarowslicetrip, Task.getbarablocktriplet, Task.getbaraidx, Task.getbaraidxij, Task.getbaraidxinfo, Task.getbarasparsity, Task.getbarcblocktriplet, Task.getbarcidx, Task.getbarcidxinfo, Task.getbarcidxj, Task.getbarcsparsity, Task.getbarvarname, Task.getbarvarnameindex, Task.getbarvarnamelen, Task.getc, Task.getcfix, Task.getcj, Task.getclist, Task.getconbound, Task.getconboundslice,

Task.getcone, Task.getconeinfo, Task.getconename, Task.getconenameindex, Task.getconenamelen, Task.getconname, Task.getconnameindex, Task.getconnamelen, Task.getconnamelen, Task.getconnameindex, Task.getmaxnumanz, Task.getmaxnumbarvar, Task.getmaxnumcon, Task.getmaxnumcone, Task.getmaxnumqnz, Task.getmaxnumvar, Task.getnumanz, Task.getnumanz64, Task.getnumbarablocktriplets, Task.getnumbarvar, Task.getnumbarcblocktriplets, Task.getnumbarcnz, Task.getnumbarvar, Task.getnumconemem, Task.getnumintvar, Task.getnumparam, Task.getnumqconknz, Task.getnumqobjnz, Task.getnumsymmat, Task.getobjname, Task.getobjnamelen, Task.getprobtype, Task.getqconk, Task.getqobj, Task.getqobjij, Task.getsparsesymmat, Task.getsymmatinfo, Task.gettaskname, Task.gettasknamelen, Task.getvarbound, Task.getvarboundslice, Task.getvarname, Task.getvarnameindex, Task.getvarnamelen, Task.getvarnamelen, Task.getvartype, Task.getvartypelist, Task.readsummary

## License system

- Env. checkoutlicense Check out a license feature from the license server ahead of time.
- Env. putlicensedebug Enables debug information for the license system.
- Env. putlicensepath Set the path to the license file.
- Env. putlicensewait Control whether mosek should wait for an available license if no license is available.
- $\begin{array}{lll} \bullet \ \, \textit{Infrequent:} & \textit{Env.checkinall} \,, & \textit{Env.checkinlicense} \,, & \textit{Env.licensecleanup} \,, & \textit{Env.} \\ \, \textit{putlicensecode} & & \end{array}$

## Linear algebra

• Infrequent: Env.axpy, Env.computesparsecholesky, Env.dot, Env.gemm, Env.gemv, Env. potrf, Env.sparsetriangularsolvedense, Env.syeiq, Env.syevd, Env.syrk

## Logging

- Task. linkfiletostream Directs all output from a task stream to a file.
- Task.onesolutionsummary Prints a short summary of a specified solution.
- Task.optimizersummary Prints a short summary with optimizer statistics from last optimization
- Task.set\_Stream Directs all output from a task stream to a callback object.
- Task. solutionsummary Prints a short summary of the current solutions.
- Infrequent: Env.echointro, Env.linkfiletostream, Env.set\_Stream

#### **Names**

- Env. getcodedesc Obtains a short description of a response code.
- Task. putbarvarname Sets the name of a semidefinite variable.
- Task.putconename Sets the name of a cone.
- Task.putconname Sets the name of a constraint.
- Task. putobjname Assigns a new name to the objective.
- Task. puttaskname Assigns a new name to the task.

- Task. putvarname Sets the name of a variable.
- Infrequent: Task.analyzenames, Task.generateconenames, Task.generateconnames, Task.generatevarnames, Task.getbarvarname, Task.getbarvarnameindex, Task.getbarvarnameindex, Task.getbarvarnameindex, Task.getconenamelen, Task.getconname, Task.getconnameindex, Task.getconnamelen, Task.getobjname, Task.getobjnamelen, Task.getstrparam, Task.getstrparamlen, Task.gettaskname, Task.gettasknamelen, Task.getvarname, Task.getvarnameindex, Task.getvarnamelen, Task.isdouparname, Task.isintparname, Task.isstrparname, Task.strtoconetype, Task.strtosk

#### **Optimization**

• Task. optimize - Optimizes the problem.

#### **Parameters**

- Task. putdouparam Sets a double parameter.
- Task.putintparam Sets an integer parameter.
- Task.putparam Modifies the value of parameter.
- Task. putstrparam Sets a string parameter.
- Task.setdefaults Resets all parameter values.
- Infrequent: Task.getatruncatetol, Task.getdouparam, Task.getintparam, Task.getnumparam, Task.getstrparam, Task.getstrparamlen, Task.isdouparname, Task.isintparname, Task.isstrparname, Task.putnadouparam, Task.putnaintparam, Task.putnastrparam, Task.writeparamfile

## Problem data - bounds

- $\bullet$   $\it Task.putconbound$  Changes the bound for one constraint.
- Task.putconboundslice Changes the bounds for a slice of the constraints.
- Task.putvarbound Changes the bounds for one variable.
- Task.putvarboundslice Changes the bounds for a slice of the variables.
- Infrequent: Task.chgconbound, Task.chgvarbound, Task.getconbound, Task.getconboundslice, Task.getvarbound, Task.getvarboundslice, Task.inputdata, Task.putconboundlist, Task.putconboundlistconst, Task.putconboundsliceconst, Task.putvarboundlist, Task.putvarboundlistconst, Task.putvarboundsliceconst

#### Problem data - cones

- Task.appendcone Appends a new conic constraint to the problem.
- Task.appendconesseq Appends multiple conic constraints to the problem.
- Task. getnumcone Obtains the number of cones.
- Task.putcone Replaces a conic constraint.
- Task.putconename Sets the name of a cone.
- Task. removecones Removes a number of conic constraints from the problem.
- Infrequent: Task.appendconeseq, Task.generateconenames, Task.getcone, Task.getconeinfo, Task.getconename, Task.getconenameindex, Task.getconenamelen, Task.getmaxnumcone, Task.getnumconemem, Task.putmaxnumcone

## Problem data - constraints

- Task. appends Appends a number of constraints to the optimization task.
- Task. getnumcon Obtains the number of constraints.
- Task.putconbound Changes the bound for one constraint.
- Task.putconboundslice Changes the bounds for a slice of the constraints.
- Task.putconname Sets the name of a constraint.
- Task. removecons Removes a number of constraints.
- Infrequent: Task.chgconbound, Task.generateconnames, Task.getconbound, Task.getconboundslice, Task.getconname, Task.getconnameindex, Task.getconnamelen, Task.getmaxnumcon, Task.getnumqconknz, Task.getqconk, Task.inputdata, Task.putconboundlist, Task.putconboundlistconst, Task.putconboundsliceconst, Task.putmaxnumcon

# Problem data - linear part

- Task. appended Appends a number of constraints to the optimization task.
- Task. appendvars Appends a number of variables to the optimization task.
- Task. getnumcon Obtains the number of constraints.
- Task.putacol Replaces all elements in one column of the linear constraint matrix.
- Task. putaij Changes a single value in the linear coefficient matrix.
- $\bullet$   $\it Task.putaijlist$  Changes one or more coefficients in the linear constraint matrix.
- $\bullet$   $Task.\,putarow$  Replaces all elements in one row of the linear constraint matrix.
- $\bullet$   $\it Task.putcfix$  Replaces the fixed term in the objective.
- Task.putcj Modifies one linear coefficient in the objective.
- Task.putconbound Changes the bound for one constraint.
- Task.putconboundslice Changes the bounds for a slice of the constraints.
- Task.putconname Sets the name of a constraint.
- Task.putcslice Modifies a slice of the linear objective coefficients.
- Task.putobjname Assigns a new name to the objective.
- Task. putobjsense Sets the objective sense.
- Task. putvarbound Changes the bounds for one variable.
- Task. putvarboundslice Changes the bounds for a slice of the variables.
- Task.putvarname Sets the name of a variable.
- Task. removecons Removes a number of constraints.
- Task. removevars Removes a number of variables.

• Infrequent: Task.chgconbound, Task.chgvarbound, Task.generateconnames, Task.generatevarnames, Task.getacol, Task.getacolnumnz, Task.getacolslice, Task.getacolslicenumnz, Task.getacolslicetrip, Task.getaij, Task.getapiecenumnz, Task.getarow, Task.getarownumnz, Task.getarowslice, Task.getarowslicenumnz, Task.getarowslicetrip, Task.getatruncatetol, Task.getc, Task.getcfix, Task.getcj, Task.getclist, Task.getconbound, Task.getconboundslice, Task.getconname, Task.getconnameindex, Task.getconnamelen, Task.getcslice, Task.getmaxnumnz, Task.getmaxnumcon, Task.getmaxnumvar, Task.getnumnz, Task.getnumnz64, Task.getobjsense, Task.getvarbound, Task.getvarboundslice, Task.getvarname, Task.getvarnameindex, Task.getvarnamelen, Task.inputdata, Task.putacollist, Task.putarowlist, Task.putaroundslicetonst, Task.putconboundsliceconst, Task.putvarboundslicetonst, Task.putvarboundsliceconst

# Problem data - objective

- Task.putbarcj Changes one element in barc.
- Task.putcfix Replaces the fixed term in the objective.
- Task. putcj Modifies one linear coefficient in the objective.
- Task.putcslice Modifies a slice of the linear objective coefficients.
- Task. putobjname Assigns a new name to the objective.
- Task. putobjsense Sets the objective sense.
- Task.putqobj Replaces all quadratic terms in the objective.
- Task. putqobjij Replaces one coefficient in the quadratic term in the objective.
- Infrequent: Task.putclist

## Problem data - quadratic part

- Task.putqcon Replaces all quadratic terms in constraints.
- Task.putqconk Replaces all quadratic terms in a single constraint.
- Task.putqobj Replaces all quadratic terms in the objective.
- Task.putqobjij Replaces one coefficient in the quadratic term in the objective.
- Infrequent: Task.getmaxnumqnz, Task.getnumqconknz, Task.getnumqobjnz, Task.getqconk, Task.getqobj, Task.getqobjij, Task.putmaxnumqnz, Task.toconic

#### Problem data - semidefinite

- Task. appendbarvars Appends semidefinite variables to the problem.
- Task.appendsparsesymmat Appends a general sparse symmetric matrix to the storage of symmetric matrices.
- Task.appendsparsesymmatlist Appends a general sparse symmetric matrix to the storage of symmetric matrices.
- Task.putbaraij Inputs an element of barA.
- Task. putbaraijlist Inputs list of elements of barA.
- Task.putbararowlist Replace a set of rows of barA

- Task. putbarcj Changes one element in barc.
- Task.putbarvarname Sets the name of a semidefinite variable.
- Infrequent: Task.getbarablocktriplet, Task.getbaraidx, Task.getbaraidxij, Task. getbaraidxinfo, Task.getbarasparsity, Task.getbarcblocktriplet, Task.getbarcidx, Task.getbarcidxinfo, Task.getbarcidxj, Task.getbarcsparsity, Task.getdimbarvarj, Task.getlenbarvarj, Task.getmaxnumbarvar, Task.getnumbarablocktriplets, Task.getnumbaranz, Task.getnumbarcblocktriplets, Task.getnumbarcraz, Task. getnumbarvar, Task.getnumsymmat, Task.getsparsesymmat, Task.getsymmatinfo, Task.putbarablocktriplet, Task.putbarcblocktriplet, Task.putmaxnumanz, Task. putmaxnumbarvar, Task.removebarvars

#### Problem data - variables

- Task. appendvars Appends a number of variables to the optimization task.
- Task.getnumvar Obtains the number of variables.
- Task.putvarbound Changes the bounds for one variable.
- Task.putvarboundslice Changes the bounds for a slice of the variables.
- Task.putvarname Sets the name of a variable.
- Task.putvartype Sets the variable type of one variable.
- Task. removevars Removes a number of variables.
- Infrequent: Task.chgvarbound, Task.generatevarnames, Task.getc, Task.getcj, Task.getmaxnumvar, Task.getnumintvar, Task.getvarbound, Task.getvarboundslice, Task.getvarname, Task.getvarnameindex, Task.getvarnamelen, Task.getvartype, Task.getvartypelist, Task.putclist, Task.putmaxnumvar, Task.putvarboundlist, Task.putvarboundsliceconst, Task.putvartypelist

## Remote optimization

- $\bullet$   $Task.\,asyncgetresult$  Request a response from a remote job.
- $\bullet$   $Task.\,asyncoptimize$  Offload the optimization task to a solver server.
- Task. asyncpoll Requests information about the status of the remote job.
- Task. asyncstop Request that the job identified by the token is terminated.
- Task. optimizermt Offload the optimization task to a solver server.

# Responses, errors and warnings

• Env. getcodedesc - Obtains a short description of a response code.

### Sensitivity analysis

- $\bullet$   $Task.\, duals ensitivity$  Performs sensitivity analysis on objective coefficients.
- Task.primalsensitivity Perform sensitivity analysis on bounds.
- Task. sensitivityreport Creates a sensitivity report.

#### Solution - dual

- Task. qetdualobj Computes the dual objective value associated with the solution.
- Task. gety Obtains the y vector for a solution.
- Task. getyslice Obtains a slice of the y vector for a solution.
- Infrequent: Task.getreducedcosts, Task.getslc, Task.getslcslice, Task.getslx, Task. getslxslice, Task.getsnx, Task.getsnxslice, Task.getsolution, Task.getsolutionslice, Task.getsuc, Task.getsucslice, Task.getsux, Task.getsuxslice, Task.putconsolutioni, Task.putslc, Task.putslcslice, Task.putslx, Task.putslxslice, Task.putsnx, Task.putsnxslice, Task.putsolution, Task.putsolutionyi, Task.putsuc, Task.putsucslice, Task.putsux, Task.putsuxslice, Task.putvarsolutionj, Task.putyslice

#### Solution - primal

- Task. getprimalobj Computes the primal objective value for the desired solution.
- Task. getxx Obtains the xx vector for a solution.
- Task.getxxslice Obtains a slice of the xx vector for a solution.
- Task.putxx Sets the xx vector for a solution.
- Task.putxxslice Sets a slice of the xx vector for a solution.
- Infrequent: Task.getsolution, Task.getsolutionslice, Task.getxc, Task.getxcslice, Task.putconsolutioni, Task.putsolution, Task.putvarsolutionj, Task.putxc, Task.putxcslice, Task.puty

### Solution - semidefinite

- Task. qetbars j Obtains the dual solution for a semidefinite variable.
- Task. getbarsslice Obtains the dual solution for a sequence of semidefinite variables.
- Task. getbarxj Obtains the primal solution for a semidefinite variable.
- Task. qetbarxslice Obtains the primal solution for a sequence of semidefinite variables.
- Infrequent: Task.putbarsj, Task.putbarxj

## Solution information

- Task. getdualobj Computes the dual objective value associated with the solution.
- Task. qetprimalob i Computes the primal objective value for the desired solution.
- Task. getprosta Obtains the problem status.
- Task. getpviolcon Computes the violation of a primal solution associated to a constraint.
- Task. getpviolvar Computes the violation of a primal solution for a list of scalar variables.
- Task. qetsolsta Obtains the solution status.
- Task. getsolutioninfo Obtains information about of a solution.
- Task.onesolutionsummary Prints a short summary of a specified solution.
- Task. solutiondef Checks whether a solution is defined.
- Task. solutionsummary Prints a short summary of the current solutions.

• Infrequent: Task.analyzesolution, Task.deletesolution, Task.getdualsolutionnorms, Task.getduiolbarvar, Task.getduiolcon, Task.getduiolcones, Task.getduiolvar, Task.getprimalsolutionnorms, Task.getpviolbarvar, Task.getpviolcones, Task.getskc, Task.getskcslice, Task.getskn, Task.getskx, Task.getskxslice, Task.getsolution, Task.getsolutionslice, Task.putconsolutioni, Task.putskc, Task.putskcslice, Task.putskx, Task.putskxslice, Task.putsolutionj

## Solving systems with basis matrix

• Infrequent: Task.basiscond, Task.initbasissolve, Task.solvewithbasis

#### System, memory and debugging

• Infrequent: Task.checkmem, Task.getmemusage, Env.setupthreads

#### **Versions**

- Env. getversion Obtains MOSEK version information.
- Infrequent: Env. getbuildinfo

# 15.3 Class Env

mosek.Env

The **MOSEK** global environment.

Env. Env

```
Env()
```

```
Env(string dbgfile)
```

Constructor of a new environment.

Parameters dbgfile (string) – File where the memory debugging log is written. (input)

Env.Dispose

```
void Dispose ()
```

Free the underlying native allocation.

Env.axpy

```
void axpy
  (int n,
   double alpha,
   double[] x,
   double[] y)
```

Adds  $\alpha x$  to y, i.e. performs the update

```
y := \alpha x + y.
```

Note that the result is stored overwriting y.

#### Parameters

```
• n (int) - Length of the vectors. (input)
```

- alpha (double) The scalar that multiplies x. (input)
- x (double[]) The x vector. (input)
- y (double[]) The y vector. (input/output)

Groups Linear algebra

Env.checkinall

```
void checkinall ()
```

Check in all unused license features to the license token server.

Groups License system

Env.checkinlicense

```
void checkinlicense (feature)
```

Check in a license feature to the license server. By default all licenses consumed by functions using a single environment are kept checked out for the lifetime of the **MOSEK** environment. This function checks in a given license feature back to the license server immediately.

If the given license feature is not checked out at all, or it is in use by a call to <code>Task.optimize</code>, calling this function has no effect.

Please note that returning a license to the license server incurs a small overhead, so frequent calls to this function should be avoided.

Parameters feature (feature) – Feature to check in to the license system. (input) Groups License system

Env.checkoutlicense

```
void checkoutlicense (feature feature)
```

Checks out a license feature from the license server. Normally the required license features will be automatically checked out the first time they are needed by the function <code>Task.optimize</code>. This function can be used to check out one or more features ahead of time.

The feature will remain checked out until the environment is deleted or the function  ${\it Env.}$  checkinlicense is called.

If a given feature is already checked out when this function is called, the call has no effect.

Parameters feature (feature) – Feature to check out from the license system. (input) Groups  $License\ system$ 

Env.computesparsecholesky

```
void computesparsecholesky
  (int multithread,
   int ordermethod,
   double tolsingular,
   int[] anzc,
   long[] aptrc,
   int[] asubc,
   double[] avalc,
   out int[] perm,
   out double[] diag,
   out int[] lnzc,
```

(continued from previous page)

```
out long[] lptrc,
out long lensubnval,
out int[] lsubc,
out double[] lvalc)
```

The function computes a Cholesky factorization of a sparse positive semidefinite matrix. Sparsity is exploited during the computations to reduce the amount of space and work required. Both the input and output matrices are represented using the sparse format.

To be precise, given a symmetric matrix  $A \in \mathbb{R}^{n \times n}$  the function computes a nonsingular lower triangular matrix L, a diagonal matrix D and a permutation matrix P such that

$$LL^T - D = PAP^T.$$

If ordermethod is zero then reordering heuristics are not employed and P is the identity.

If a pivot during the computation of the Cholesky factorization is less than

$$-\rho \cdot \max((PAP^T)_{jj}, 1.0)$$

then the matrix is declared negative semidefinite. On the hand if a pivot is smaller than

$$\rho \cdot \max((PAP^T)_{jj}, 1.0),$$

then  $D_{ij}$  is increased from zero to

$$\rho \cdot \max((PAP^T)_{jj}, 1.0).$$

Therefore, if A is sufficiently positive definite then D will be the zero matrix. Here  $\rho$  is set equal to value of tolsingular.

# Parameters

- multithread (int) If nonzero then the function may exploit multiple threads. (input)
- ordermethod (int) If nonzero, then a sparsity preserving ordering will be employed. (input)
- tolsingular (double) A positive parameter controlling when a pivot is declared zero. (input)
- anzc (int[]) anzc[j] is the number of nonzeros in the j-th column of A. (input)
- aptrc (long[]) aptrc[j] is a pointer to the first element in column j of A. (input)
- asubc (int[]) Row indexes for each column stored in increasing order. (input)
- avalc (double[]) The value corresponding to row indexed stored in asubc. (input)
- perm (int[]) Permutation array used to specify the permutation matrix P computed by the function. (output)
- diag (double[]) The diagonal elements of matrix D. (output)
- lnzc(int[]) lnzc[j] is the number of non zero elements in column j of L. (output)
- lptrc (long[]) lptrc[j] is a pointer to the first row index and value in column j of L. (output)
- lensubnval (long) Number of elements in lsubc and lvalc. (output)
- lsubc (int[]) Row indexes for each column stored in increasing order. (output)
- lvalc (double[]) The values corresponding to row indexed stored in lsubc. (output)

Groups Linear algebra

#### Env.dot

```
void dot
  (int n,
   double[] x,
   double[] y,
   out double xty)
```

Computes the inner product of two vectors x, y of length  $n \geq 0$ , i.e

$$x \cdot y = \sum_{i=1}^{n} x_i y_i.$$

Note that if n = 0, then the result of the operation is 0.

#### **Parameters**

- n (int) Length of the vectors. (input)
- x (double[]) The x vector. (input)
- y (double[]) The y vector. (input)
- xty (double) The result of the inner product between x and y. (output)

Groups Linear algebra

Env.echointro

```
void echointro (int longver)
```

Prints an intro to message stream.

Parameters longver (int) – If non-zero, then the intro is slightly longer. (input) Groups Logging

Env.gemm

```
void gemm
  (transpose transa,
   transpose transb,
   int m,
   int n,
   int k,
   double alpha,
   double[] a,
   double[] b,
   double beta,
   double[] c)
```

Performs a matrix multiplication plus addition of dense matrices. Given A, B and C of compatible dimensions, this function computes

$$C := \alpha op(A)op(B) + \beta C$$

where  $\alpha, \beta$  are two scalar values. The function op(X) denotes X if transX is transpose.no, or  $X^T$  if set to transpose.yes. The matrix C has m rows and n columns, and the other matrices must have compatible dimensions.

The result of this operation is stored in C.

#### Parameters

• transa (transpose) - Indicates whether the matrix A must be transposed. (input)

- transb (transpose) Indicates whether the matrix B must be transposed. (input)
- m (int) Indicates the number of rows of matrix C. (input)
- n (int) Indicates the number of columns of matrix C. (input)
- k (int) Specifies the common dimension along which op(A) and op(B) are multiplied. For example, if neither A nor B are transposed, then this is the number of columns in A and also the number of rows in B. (input)
- alpha (double) A scalar value multiplying the result of the matrix multiplication. (input)
- a (double[]) The pointer to the array storing matrix A in a column-major format. (input)
- b (double[]) The pointer to the array storing matrix B in a column-major format. (input)
- beta (double) A scalar value that multiplies C. (input)
- c (double[]) The pointer to the array storing matrix C in a column-major format. (input/output)

Groups Linear algebra

Env.gemv

```
void gemv
  (transpose transa,
  int m,
  int n,
  double alpha,
  double[] a,
  double[] x,
  double beta,
  double[] y)
```

Computes the multiplication of a scaled dense matrix times a dense vector, plus a scaled dense vector. Precisely, if trans is transpose.no then the update is

$$y := \alpha Ax + \beta y,$$

and if trans is transpose. yes then

$$y := \alpha A^T x + \beta y$$
,

where  $\alpha, \beta$  are scalar values and A is a matrix with m rows and n columns.

Note that the result is stored overwriting y.

#### Parameters

- $\bullet$  transa (transpose) Indicates whether the matrix A must be transposed. (input)
- m (int) Specifies the number of rows of the matrix A. (input)
- n (int) Specifies the number of columns of the matrix A. (input)
- alpha (double) A scalar value multiplying the matrix A. (input)
- a (double[]) A pointer to the array storing matrix A in a column-major format. (input)
- x (double[]) A pointer to the array storing the vector x. (input)
- beta (double) A scalar value multiplying the vector y. (input)
- y (double[]) A pointer to the array storing the vector y. (input/output)

Groups Linear algebra

Env.getbuildinfo

```
static void getbuildinfo
(StringBuilder buildstate,
StringBuilder builddate)
```

Obtains build information.

#### Parameters

- buildstate (StringBuilder) State of binaries, i.e. a debug, release candidate or final release. (output)
- builddate (StringBuilder) Date when the binaries were built. (output)

Groups Versions

Env.getcodedesc

```
static void getcodedesc
(rescode code,
StringBuilder symname,
StringBuilder str)
```

Obtains a short description of the meaning of the response code given by code.

#### **Parameters**

- code (rescode) A valid MOSEK response code. (input)
- symname (StringBuilder) Symbolic name corresponding to code. (output)
- str (StringBuilder) Obtains a short description of a response code. (output)

Groups Names, Responses, errors and warnings

Env.getversion

```
static void getversion
(out int major,
out int minor,
out int revision)
```

Obtains MOSEK version information.

### Parameters

- major (int) Major version number. (output)
- minor (int) Minor version number. (output)
- revision (int) Revision number. (output)

Groups Versions

Env.licensecleanup

```
static void licensecleanup ()
```

Stops all threads and deletes all handles used by the license system. If this function is called, it must be called as the last **MOSEK** API call. No other **MOSEK** API calls are valid after this.

Groups License system

Env.linkfiletostream

```
void linkfiletostream
(streamtype whichstream,
string filename,
int append)
```

Sends all output from the stream defined by whichstream to the file given by filename.

#### **Parameters**

- whichstream (streamtype) Index of the stream. (input)
- filename (string) A valid file name. (input)
- append (int) If this argument is 0 the file will be overwritten, otherwise it will be appended to. (input)

Groups Logging

Env.potrf

```
void potrf
  (uplo uplo,
   int n,
   double[] a)
```

Computes a Cholesky factorization of a real symmetric positive definite dense matrix.

#### Parameters

- uplo (uplo) Indicates whether the upper or lower triangular part of the matrix is stored. (input)
- n (int) Dimension of the symmetric matrix. (input)
- a (double[]) A symmetric matrix stored in column-major order. Only the lower or the upper triangular part is used, accordingly with the uplo parameter. It will contain the result on exit. (input/output)

Groups Linear algebra

Env.putlicensecode

```
void putlicensecode (int[] code)
```

Input a runtime license code.

```
Parameters code (int[]) – A runtime license code. (input)
Groups License system
```

Env.putlicensedebug

```
void putlicensedebug (int licdebug)
```

Enables debug information for the license system. If licdebug is non-zero, then MOSEK will print debug info regarding the license checkout.

Parameters licdebug (int) – Whether license checkout debug info should be printed. (input)

Groups License system

Env.putlicensepath

```
void putlicensepath (string licensepath)
```

Set the path to the license file.

Parameters licensepath (string) – A path specifying where to search for the license. (input)

Groups License system

```
void putlicensewait (int licwait)
```

Control whether **MOSEK** should wait for an available license if no license is available. If licwait is non-zero, then **MOSEK** will wait for licwait-1 milliseconds between each check for an available license.

Parameters licwait (int) – Whether MOSEK should wait for a license if no license is available. (input)

Groups License system

Env.set\_Stream

```
void set_Stream
(streamtype whichstream,
Stream callback)
```

Directs all output from an environment stream to a callback object.

#### **Parameters**

- whichstream (streamtype) Index of the stream. (input)
- callback (Stream) The callback object. (input)

Env.setupthreads

```
void setupthreads (int numthreads)
```

Preallocates a thread pool for the interior-point and conic optimizers in the current process. This function should only be called once per process, before first optimization. Future settings of the parameter <code>iparam.num\_threads</code> will be irrelevant for the conic optimizer.

Parameters numthreads (int) – Number of threads. (input) Groups System, memory and debugging

Env.sparsetriangularsolvedense

```
void sparsetriangularsolvedense
  (transpose transposed,
  int[] lnzc,
  long[] lptrc,
  int[] lsubc,
  double[] lvalc,
  double[] b)
```

The function solves a triangular system of the form

Lx = b

or

$$L^T x = b$$

where L is a sparse lower triangular nonsingular matrix. This implies in particular that diagonals in L are nonzero.

## Parameters

- transposed (transpose) Controls whether to use with L or  $L^T$ . (input)
- lnzc (int[]) lnzc[j] is the number of nonzeros in column j. (input)

- lptrc (long[]) lptrc[j] is a pointer to the first row index and value in column j. (input)
- lsubc (int[]) Row indexes for each column stored sequentially. Must be stored in increasing order for each column. (input)
- lvalc (double[]) The value corresponding to the row index stored in lsubc. (input)
- b (double[]) The right-hand side of linear equation system to be solved as a dense vector. (input/output)

Groups Linear algebra

#### Env.syeig

```
void syeig
  (uplo uplo,
  int n,
  double[] a,
  double[] w)
```

Computes all eigenvalues of a real symmetric matrix A. Given a matrix  $A \in \mathbb{R}^{n \times n}$  it returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A.

## Parameters

- uplo (uplo) Indicates whether the upper or lower triangular part is used. (input)
- n (int) Dimension of the symmetric input matrix. (input)
- a (double[]) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. (input)
- w (double[]) Array of length at least n containing the eigenvalues of A. (output)

Groups Linear algebra

# Env.syevd

```
void syevd
  (uplo uplo,
   int n,
   double[] a,
   double[] w)
```

Computes all the eigenvalues and eigenvectors a real symmetric matrix. Given the input matrix  $A \in \mathbb{R}^{n \times n}$ , this function returns a vector  $w \in \mathbb{R}^n$  containing the eigenvalues of A and it also computes the eigenvectors of A. Therefore, this function computes the eigenvalue decomposition of A as

$$A = UVU^T$$
,

where  $V = \mathbf{diag}(w)$  and U contains the eigenvectors of A.

Note that the matrix U overwrites the input data A.

#### Parameters

- uplo (uplo) Indicates whether the upper or lower triangular part is used. (input)
- n (int) Dimension of the symmetric input matrix. (input)
- a (double[]) A symmetric matrix A stored in column-major order. Only the part indicated by uplo is used. On exit it will be overwritten by the matrix U. (input/output)
- w (double[]) Array of length at least n containing the eigenvalues of A. (output)

### Groups Linear algebra

#### Env.syrk

```
void syrk
  (uplo uplo,
    transpose trans,
    int n,
    int k,
    double alpha,
    double[] a,
    double beta,
    double[] c)
```

Performs a symmetric rank-k update for a symmetric matrix.

Given a symmetric matrix  $C \in \mathbb{R}^{n \times n}$ , two scalars  $\alpha, \beta$  and a matrix A of rank  $k \leq n$ , it computes either

$$C := \alpha A A^T + \beta C,$$

when trans is set to transpose.no and  $A \in \mathbb{R}^{n \times k}$ , or

$$C := \alpha A^T A + \beta C$$
,

when trans is set to transpose. yes and  $A \in \mathbb{R}^{k \times n}$ .

Only the part of C indicated by uplo is used and only that part is updated with the result.

#### **Parameters**

- uplo (uplo) Indicates whether the upper or lower triangular part of C is used. (input)
- trans (transpose) Indicates whether the matrix A must be transposed. (input)
- n (int) Specifies the order of C. (input)
- k (int) Indicates the number of rows or columns of A, depending on whether or not it is transposed, and its rank. (input)
- alpha (double) A scalar value multiplying the result of the matrix multiplication. (input)
- a (double[]) The pointer to the array storing matrix A in a column-major format. (input)
- beta (double) A scalar value that multiplies C. (input)
- c (double[]) The pointer to the array storing matrix C in a column-major format. (input/output)

Groups Linear algebra

# 15.4 Class Task

mosek.Task

Represents an optimization task.

Task.Task

```
Task(Env env)
```

```
Task(
Env env,
int numcon,
int numvar)
```

```
Task(Task task)
```

Constructor of a new optimization task.

#### **Parameters**

- env (Env) Parent environment. (input)
- numcon (int) An optional hint about the maximal number of constraints in the task. (input)
- numvar (int) An optional hint about the maximal number of variables in the task. (input)
- task (Task) A task that will be cloned. (input)

Task.Dispose

```
void Dispose()
```

Free the underlying native allocation.

Task.analyzenames

```
void analyzenames
(streamtype whichstream,
nametype nametype)
```

The function analyzes the names and issues an error if a name is invalid.

#### Parameters

- whichstream (streamtype) Index of the stream. (input)
- nametype (nametype) The type of names e.g. valid in MPS or LP files. (input)

Groups Names

Task.analyzeproblem

```
void analyzeproblem (streamtype whichstream)
```

The function analyzes the data of a task and writes out a report.

```
Parameters whichstream (streamtype) - Index of the stream. (input) Groups Inspecting the task
```

Task.analyzesolution

```
void analyzesolution
(streamtype whichstream,
soltype whichsol)
```

Print information related to the quality of the solution and other solution statistics.

By default this function prints information about the largest infeasibilities in the solution, the primal (and possibly dual) objective value and the solution status.

Following parameters can be used to configure the printed statistics:

- *iparam.ana\_sol\_basis* enables or disables printing of statistics specific to the basis solution (condition number, number of basic variables etc.). Default is on.
- *iparam.ana\_sol\_print\_violated* enables or disables listing names of all constraints (both primal and dual) which are violated by the solution. Default is off.
- dparam.ana\_sol\_infeas\_tol is the tolerance defining when a constraint is considered violated. If a constraint is violated more than this, it will be listed in the summary.

#### **Parameters**

- whichstream (streamtype) Index of the stream. (input)
- whichsol (soltype) Selects a solution. (input)

Groups Solution information, Inspecting the task

# Task.appendbarvars

```
void appendbarvars (int[] dim)
```

Appends positive semidefinite matrix variables of dimensions given by dim to the problem.

Parameters dim (int[]) - Dimensions of symmetric matrix variables to be added. (input)

Groups Problem data - semidefinite

#### Task.appendcone

```
void appendcone
  (conetype ct,
   double conepar,
   int[] submem)
```

```
void appendcone
  (conetype ct,
   double conepar,
   int nummem,
   int[] submem)
```

Appends a new conic constraint to the problem. Hence, add a constraint

$$\hat{x} \in \mathcal{K}$$

to the problem, where K is a convex cone.  $\hat{x}$  is a subset of the variables which will be specified by the argument submem. Cone type is specified by ct.

Define

$$\hat{x} = x_{\mathtt{submem}[0]}, \dots, x_{\mathtt{submem}[\mathtt{nummem}-1]}.$$

Depending on the value of ct this function appends one of the constraints:

• Quadratic cone (conetype. quad, requires nummem ≥ 1):

$$\hat{x}_0 \ge \sqrt{\sum_{i=1}^{i < \text{nummem}} \hat{x}_i^2}$$

• Rotated quadratic cone (conetype.rquad, requires nummem  $\geq 2$ ):

$$2\hat{x}_0\hat{x}_1 \geq \sum_{i=2}^{i<\text{nummem}} \hat{x}_i^2, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

• Primal exponential cone (conetype.pexp, requires nummem = 3):

$$\hat{x}_0 \ge \hat{x}_1 \exp(\hat{x}_2/\hat{x}_1), \quad \hat{x}_0, \hat{x}_1 \ge 0$$

• Primal power cone (conetype.ppow, requires nummem ≥ 2):

$$\hat{x}_0^\alpha \hat{x}_1^{1-\alpha} \geq \sqrt{\sum_{i=2}^{i < \text{nummem}} \hat{x}_i^2}, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

where  $\alpha$  is the cone parameter specified by conepar.

• Dual exponential cone (conetype.dexp, requires nummem = 3):

$$\hat{x}_0 \ge -\hat{x}_2 e^{-1} \exp(\hat{x}_1/\hat{x}_2), \quad \hat{x}_2 \le 0, \hat{x}_0 \ge 0$$

• Dual power cone (conetype.dpow, requires nummem  $\geq 2$ ):

$$\left(\frac{\hat{x}_0}{\alpha}\right)^{\alpha} \left(\frac{\hat{x}_1}{1-\alpha}\right)^{1-\alpha} \geq \sqrt{\sum_{i=2}^{i < \text{nummem}} \hat{x}_i^2}, \quad \hat{x}_0, \hat{x}_1 \geq 0$$

where  $\alpha$  is the cone parameter specified by conepar.

• Zero cone (conetype.zero):

$$\hat{x}_i = 0$$
 for all  $i$ 

Please note that the sets of variables appearing in different conic constraints must be disjoint. For an explained code example see Sec. 6.3, Sec. 6.5 or Sec. 6.4.

# Parameters

- ct (conetype) Specifies the type of the cone. (input)
- conepar (double) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- submem (int[]) Variable subscripts of the members in the cone. (input)
- nummem (int) Number of member variables in the cone. (input)

Groups Problem data - cones

## Task.appendconeseq

```
void appendconeseq
  (conetype ct,
   double conepar,
   int nummem,
   int j)
```

Appends a new conic constraint to the problem, as in *Task.appendcone*. The function assumes the members of cone are sequential where the first member has index j and the last j+nummem-1.

## Parameters

- ct (conetype) Specifies the type of the cone. (input)
- conepar (double) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (int) Number of member variables in the cone. (input)
- j (int) Index of the first variable in the conic constraint. (input)

Groups Problem data - cones

## Task.appendconesseq

```
void appendconesseq
  (conetype[] ct,
  double[] conepar,
  int[] nummem,
  int j)
```

Appends a number of conic constraints to the problem, as in Task.appendcone. The kth cone is assumed to be of dimension nummem[k]. Moreover, it is assumed that the first variable of the first cone has index j and starting from there the sequentially following variables belong to the first cone, then to the second cone and so on.

#### **Parameters**

- ct (conetype[]) Specifies the type of the cone. (input)
- conepar (double[]) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- nummem (int[]) Numbers of member variables in the cones. (input)
- j (int) Index of the first variable in the first cone to be appended. (input)

Groups Problem data - cones

## Task.appendcons

```
void appendcons (int num)
```

Appends a number of constraints to the model. Appended constraints will be declared free. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional constraints.

Parameters num (int) – Number of constraints which should be appended. (input) Groups Problem data - linear part, Problem data - constraints

## Task.appendsparsesymmat

```
long appendsparsesymmat
  (int dim,
  int[] subi,
  int[] subj,
  double[] valij)
```

```
void appendsparsesymmat
  (int dim,
  int[] subi,
  int[] subj,
  double[] valij,
  out long idx)
```

**MOSEK** maintains a storage of symmetric data matrices that is used to build  $\overline{C}$  and  $\overline{A}$ . The storage can be thought of as a vector of symmetric matrices denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This function appends a general sparse symmetric matrix on triplet form to the vector E of symmetric matrices. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric, only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

## **Parameters**

- dim (int) Dimension of the symmetric matrix that is appended. (input)
- subi (int[]) Row subscript in the triplets. (input)
- subj (int[]) Column subscripts in the triplets. (input)
- valij (double[]) Values of each triplet. (input)
- idx (long) Unique index assigned to the inputted matrix that can be used for later reference. (output)

Return (long) – Unique index assigned to the inputted matrix that can be used for later reference.

Groups Problem data - semidefinite

```
void appendsparsesymmatlist
  (int[] dims,
  long[] nz,
  int[] subi,
  int[] subj,
  double[] valij,
  long[] idx)
```

**MOSEK** maintains a storage of symmetric data matrices that is used to build  $\overline{C}$  and  $\overline{A}$ . The storage can be thought of as a vector of symmetric matrices denoted E. Hence,  $E_i$  is a symmetric matrix of certain dimension.

This function appends general sparse symmetric matrixes on triplet form to the vector E of symmetric matrices. The vectors  $\mathtt{subi}$ ,  $\mathtt{subj}$ , and  $\mathtt{valij}$  contains the row subscripts, column subscripts and values of each element in the symmetric matrix to be appended. Since the matrix that is appended is symmetric, only the lower triangular part should be specified. Moreover, duplicates are not allowed.

Observe the function reports the index (position) of the appended matrix in E. This index should be used for later references to the appended matrix.

## Parameters

- dims (int[]) Dimensions of the symmetric matrixes. (input)
- nz (long[]) Number of nonzeros for each matrix. (input)
- subi (int[]) Row subscript in the triplets. (input)
- subj (int[]) Column subscripts in the triplets. (input)
- valij (double[]) Values of each triplet. (input)
- idx (long[]) Unique index assigned to the inputted matrix that can be used for later reference. (output)

Groups Problem data - semidefinite

Task.appendvars

```
void appendvars (int num)
```

Appends a number of variables to the model. Appended variables will be fixed at zero. Please note that **MOSEK** will automatically expand the problem dimension to accommodate the additional variables.

Parameters num (int) – Number of variables which should be appended. (input) Groups Problem data - linear part, Problem data - variables

Task.asyncgetresult

```
int asyncgetresult
  (string server,
   string port,
   string token,
   out rescode resp,
   out rescode trm)
```

```
void asyncgetresult
  (string server,
   string port,
   string token,
   out int respavailable,
   out rescode resp,
   out rescode trm)
```

Request a response from a remote job. If successful, solver response, termination code and solutions are retrieved.

#### **Parameters**

- server (string) Name or IP address of the solver server. (input)
- port (string) Network port of the solver service. (input)
- token (string) The task token. (input)
- resp (rescode) Is the response code from the remote solver. (output)
- trm (rescode) Is either rescode. ok or a termination response code. (output)
- respavailable (int) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)

Return (int) - Indicates if a remote response is available. If this is not true, resp and trm should be ignored.

Groups Remote optimization

Task.asyncoptimize

```
string asyncoptimize
(string server,
string port)
```

```
void asyncoptimize
(string server,
string port,
StringBuilder token)
```

Offload the optimization task to a solver server defined by server:port. The call will return immediately and not wait for the result.

If the string parameter  $sparam.remote\_access\_token$  is not blank, it will be passed to the server as authentication.

## Parameters

- server (string) Name or IP address of the solver server (input)
- port (string) Network port of the solver service (input)
- token (StringBuilder) Returns the task token (output)

Return (string) - Returns the task token

Groups Remote optimization

Task.asyncpoll

```
int asyncpoll
  (string server,
   string port,
   string token,
   out rescode resp,
   out rescode trm)
```

```
void asyncpoll
(string server,
string port,
string token,
out int respavailable,
out rescode resp,
out rescode trm)
```

Requests information about the status of the remote job.

- server (string) Name or IP address of the solver server (input)
- port (string) Network port of the solver service (input)
- token (string) The task token (input)
- resp (rescode) Is the response code from the remote solver. (output)
- trm (rescode) Is either rescode. ok or a termination response code. (output)
- respavailable (int) Indicates if a remote response is available. If this is not true, resp and trm should be ignored. (output)

Return (int) – Indicates if a remote response is available. If this is not true, resp and trm should be ignored.

**Groups** Remote optimization

## Task.asyncstop

```
void asyncstop
(string server,
string port,
string token)
```

Request that the job identified by the token is terminated.

#### **Parameters**

- server (string) Name or IP address of the solver server (input)
- port (string) Network port of the solver service (input)
- token (string) The task token (input)

Groups Remote optimization

#### Task.basiscond

```
void basiscond
(out double nrmbasis,
out double nrminvbasis)
```

If a basic solution is available and it defines a nonsingular basis, then this function computes the 1-norm estimate of the basis matrix and a 1-norm estimate for the inverse of the basis matrix. The 1-norm estimates are computed using the method outlined in [Ste98], pp. 388-391.

By definition the 1-norm condition number of a matrix B is defined as

$$\kappa_1(B) := \|B\|_1 \|B^{-1}\|_1.$$

Moreover, the larger the condition number is the harder it is to solve linear equation systems involving B. Given estimates for  $||B||_1$  and  $||B^{-1}||_1$  it is also possible to estimate  $\kappa_1(B)$ .

# Parameters

- nrmbasis (double) An estimate for the 1-norm of the basis. (output)
- nrminvbasis (double) An estimate for the 1-norm of the inverse of the basis. (output)

**Groups** Solving systems with basis matrix

## Task.checkmem

```
void checkmem
  (string file,
  int line)
```

Checks the memory allocated by the task.

- file (string) File from which the function is called. (input)
- line (int) Line in the file from which the function is called. (input)

**Groups** System, memory and debugging

# Task.chgconbound

```
void chgconbound
  (int i,
   int lower,
   int finite,
   double value)
```

Changes a bound for one constraint.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \text{ finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

## Parameters

- i (int) Index of the constraint for which the bounds should be changed. (input)
- lower (int) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)
- finite (int) If non-zero, then value is assumed to be finite. (input)
- value (double) New value for the bound. (input)

**Groups** Problem data - bounds, Problem data - constraints, Problem data - linear part

## Task.chgvarbound

```
void chgvarbound
  (int j,
   int lower,
   int finite,
   double value)
```

Changes a bound for one variable.

If lower is non-zero, then the lower bound is changed as follows:

$$\text{new lower bound} = \left\{ \begin{array}{ll} -\infty, & \text{ finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Otherwise if lower is zero, then

$$\text{new upper bound} = \left\{ \begin{array}{ll} \infty, & \text{finite} = 0, \\ \text{value} & \text{otherwise}. \end{array} \right.$$

Please note that this function automatically updates the bound key for the bound, in particular, if the lower and upper bounds are identical, the bound key is changed to fixed.

- j (int) Index of the variable for which the bounds should be changed. (input)
- lower (int) If non-zero, then the lower bound is changed, otherwise the upper bound is changed. (input)

- finite (int) If non-zero, then value is assumed to be finite. (input)
- value (double) New value for the bound. (input)

Groups Problem data - bounds, Problem data - variables, Problem data - linear part

Task.commitchanges

```
void commitchanges ()
```

Commits all cached problem changes to the task. It is usually not necessary to call this function explicitly since changes will be committed automatically when required.

Groups Environment and task management

Task.deletesolution

```
void deletesolution (soltype whichsol)
```

Undefine a solution and free the memory it uses.

```
Parameters whichsol (soltype) - Selects a solution. (input)
Groups Environment and task management, Solution information
```

Task.dualsensitivity

```
void dualsensitivity
  (int[] subj,
  double[] leftpricej,
  double[] rightpricej,
  double[] leftrangej,
  double[] rightrangej)
```

Calculates sensitivity information for objective coefficients. The indexes of the coefficients to analyze are

$$\{ \text{subj}[i] \mid i = 0, \dots, \text{numj} - 1 \}$$

The type of sensitivity analysis to perform (basis or optimal partition) is controlled by the parameter  $iparam.sensitivity\_type$ .

For an example, please see Section Example: Sensitivity Analysis.

# Parameters

- subj (int[]) Indexes of objective coefficients to analyze. (input)
- leftpricej (double[]) leftpricej[j] is the left shadow price for the coefficient with index subj[j]. (output)
- rightpricej (double[]) rightpricej[j] is the right shadow price for the coefficient with index subj[j]. (output)
- leftrangej (double[]) leftrangej[j] is the left range  $\beta_1$  for the coefficient with index subj[j]. (output)
- rightrangej (double[]) rightrangej[j] is the right range  $\beta_2$  for the coefficient with index subj[j]. (output)

 ${\bf Groups} \ \ {\it Sensitivity \ analysis}$ 

Task.generateconenames

```
void generateconenames
  (int[] subk,
   string fmt,
   int[] dims,
   long[] sp)
```

Generates systematic names for cone.

#### **Parameters**

- subk (int[]) Indexes of the cone. (input)
- fmt (string) The cone name formatting string. (input)
- dims (int[]) Dimensions in the shape. (input)
- sp (long[]) Items that should be named. (input)

Groups Names, Problem data - cones

#### Task.generateconnames

```
void generateconnames
  (int[] subi,
   string fmt,
   int[] dims,
  long[] sp)
```

Generates systematic names for constraints.

## Parameters

- subi (int[]) Indexes of the constraints. (input)
- fmt (string) The constraint name formatting string. (input)
- dims (int[]) Dimensions in the shape. (input)
- sp (long[]) Items that should be named. (input)

Groups Names, Problem data - constraints, Problem data - linear part

#### Task.generatevarnames

```
void generatevarnames
  (int[] subj,
   string fmt,
   int[] dims,
   long[] sp)
```

Generates systematic names for variables.

## Parameters

- subj (int[]) Indexes of the variables. (input)
- fmt (string) The variable name formatting string. (input)
- dims (int[]) Dimensions in the shape. (input)
- sp (long[]) Items that should be named. (input)

Groups Names, Problem data - variables, Problem data - linear part

## Task.getacol

```
void getacol
  (int j,
  out int nzj,
  int[] subj,
  double[] valj)
```

Obtains one column of A in a sparse format.

- j (int) Index of the column. (input)
- nzj (int) Number of non-zeros in the column obtained. (output)
- subj (int[]) Row indices of the non-zeros in the column obtained. (output)

• valj (double[]) - Numerical values in the column obtained. (output) Groups Problem data - linear part, Inspecting the task

Task.getacolnumnz

```
void getacolnumnz
(int i,
   out int nzj)
```

Obtains the number of non-zero elements in one column of A.

#### Parameters

- i (int) Index of the column. (input)
- nzj (int) Number of non-zeros in the j-th column of A. (output)

**Return** (int) – Number of non-zeros in the j-th column of A.

Groups Problem data - linear part, Inspecting the task

Task.getacolslice

```
void getacolslice
  (int first,
   int last,
  ref long surp,
  long[] ptrb,
  long[] ptre,
  int[] sub,
  double[] val)
```

Obtains a sequence of columns from A in sparse format.

#### Parameters

- first (int) Index of the first column in the sequence. (input)
- last (int) Index of the last column in the sequence plus one. (input)
- surp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (long[]) ptrb[t] is an index pointing to the first element in the t-th column obtained. (output)
- ptre (long[]) ptre[t] is an index pointing to the last element plus one in the t-th column obtained. (output)
- sub (int[]) Contains the row subscripts. (output)
- val (double[]) Contains the coefficient values. (output)

Groups Problem data - linear part, Inspecting the task

Task.getacolslicenumnz

```
long getacolslicenumnz
  (int first,
  int last)
```

```
void getacolslicenumnz
  (int first,
   int last,
   out long numnz)
```

Obtains the number of non-zeros in a slice of columns of A.

## Parameters

- first (int) Index of the first column in the sequence. (input)
- last (int) Index of the last column plus one in the sequence. (input)
- numnz (long) Number of non-zeros in the slice. (output)

Return (long) – Number of non-zeros in the slice.

Groups Problem data - linear part, Inspecting the task

Task.getacolslicetrip

```
void getacolslicetrip
  (int first,
  int last,
  ref long surp,
  int[] subi,
  int[] subj,
  double[] val)
```

Obtains a sequence of columns from A in sparse triplet format. The function returns the content of all columns whose index j satisfies first  $\leq j \leq last$ . The triplets corresponding to nonzero entries are stored in the arrays subj, subj and val.

#### **Parameters**

- first (int) Index of the first column in the sequence. (input)
- last (int) Index of the last column in the sequence plus one. (input)
- surp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in subi, subj and val starting from position surp away from the end of the arrays. On return surp will be decremented by the total number of non-zeros written. (input/output)
- subi (int[]) Constraint subscripts. (output)
- subj (int[]) Column subscripts. (output)
- val (double[]) Values. (output)

**Groups** Problem data - linear part, Inspecting the task

Task.getaij

```
double getaij
  (int i,
   int j)
```

```
void getaij
  (int i,
   int j,
  out double aij)
```

Obtains a single coefficient in A.

# **Parameters**

- i (int) Row index of the coefficient to be returned. (input)
- j (int) Column index of the coefficient to be returned. (input)
- aij (double) The required coefficient  $a_{i,j}$ . (output)

**Return** (double) – The required coefficient  $a_{i,j}$ .

**Groups** Problem data - linear part, Inspecting the task

Task.getapiecenumnz

```
int getapiecenumnz
  (int firsti,
   int lasti,
   int firstj,
   int lastj)
```

```
void getapiecenumnz
  (int firsti,
   int lasti,
   int firstj,
   int lastj,
   out int numnz)
```

Obtains the number non-zeros in a rectangular piece of A, i.e. the number of elements in the set

```
\{(i,j): a_{i,j} \neq 0, \text{ firsti } \leq i \leq \text{lasti} - 1, \text{ firstj } \leq j \leq \text{lastj} - 1\}
```

This function is not an efficient way to obtain the number of non-zeros in one row or column. In that case use the function Task.getarounumnz or Task.getacolnumnz.

#### Parameters

- firsti (int) Index of the first row in the rectangular piece. (input)
- lasti (int) Index of the last row plus one in the rectangular piece. (input)
- first j (int) Index of the first column in the rectangular piece. (input)
- lastj (int) Index of the last column plus one in the rectangular piece. (input)
- numnz (int) Number of non-zero A elements in the rectangular piece. (output)

**Return** (int) – Number of non-zero A elements in the rectangular piece.

**Groups** Problem data - linear part, Inspecting the task

# Task.getarow

```
void getarow
  (int i,
   out int nzi,
   int[] subi,
   double[] vali)
```

Obtains one row of A in a sparse format.

# Parameters

- i (int) Index of the row. (input)
- nzi (int) Number of non-zeros in the row obtained. (output)
- subi (int[]) Column indices of the non-zeros in the row obtained. (output)
- vali (double[]) Numerical values of the row obtained. (output)

Groups Problem data - linear part, Inspecting the task

## Task.getarownumnz

```
int getarownumnz (int i)
```

```
void getarownumnz
  (int i,
  out int nzi)
```

Obtains the number of non-zero elements in one row of A.

# **Parameters**

• i (int) – Index of the row. (input)

• nzi (int) – Number of non-zeros in the *i*-th row of A. (output)

**Return** (int) – Number of non-zeros in the i-th row of A.

Groups Problem data - linear part, Inspecting the task

Task.getarowslice

```
void getarowslice
  (int first,
   int last,
   ref long surp,
   long[] ptrb,
   long[] ptre,
   int[] sub,
   double[] val)
```

Obtains a sequence of rows from A in sparse format.

## **Parameters**

- first (int) Index of the first row in the sequence. (input)
- last (int) Index of the last row in the sequence plus one. (input)
- surp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in sub and val starting from position surp away from the end of the arrays. Upon return surp will be decremented by the total number of non-zeros written. (input/output)
- ptrb (long[]) ptrb[t] is an index pointing to the first element in the t-th row obtained. (output)
- ptre (long[]) ptre[t] is an index pointing to the last element plus one in the t-th row obtained. (output)
- sub (int[]) Contains the column subscripts. (output)
- val (double[]) Contains the coefficient values. (output)

**Groups** Problem data - linear part, Inspecting the task

Task.getarowslicenumnz

```
long getarowslicenumnz
(int first,
int last)
```

```
void getarowslicenumnz
  (int first,
  int last,
  out long numnz)
```

Obtains the number of non-zeros in a slice of rows of A.

## **Parameters**

- first (int) Index of the first row in the sequence. (input)
- last (int) Index of the last row plus one in the sequence. (input)
- numnz (long) Number of non-zeros in the slice. (output)

**Return** (long) – Number of non-zeros in the slice.

Groups Problem data - linear part, Inspecting the task

Task.getarowslicetrip

```
void getarowslicetrip
  (int first,
   int last,
   ref long surp,
   int[] subi,
   int[] subj,
   double[] val)
```

Obtains a sequence of rows from A in sparse triplet format. The function returns the content of all rows whose index i satisfies first <= i < last. The triplets corresponding to nonzero entries are stored in the arrays subi, subj and val.

#### **Parameters**

- first (int) Index of the first row in the sequence. (input)
- last (int) Index of the last row in the sequence plus one. (input)
- surp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in subi, subj and val starting from position surp away from the end of the arrays. On return surp will be decremented by the total number of non-zeros written. (input/output)
- subi (int[]) Constraint subscripts. (output)
- subj (int[]) Column subscripts. (output)
- val (double[]) Values. (output)

Groups Problem data - linear part, Inspecting the task

Task.getatruncatetol

```
void getatruncatetol (double[] tolzero)
```

Obtains the tolerance value set with Task.putatruncatetol.

**Parameters** tolzero (double[]) – All elements  $|a_{i,j}|$  less than this tolerance is truncated to zero. (output)

Groups Parameters, Problem data - linear part

Task.getbarablocktriplet

```
long getbarablocktriplet
  (int[] subi,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valijkl)
```

```
void getbarablocktriplet
  (out long num,
   int[] subi,
   int[] subj,
   int[] subk,
   int[] subl,
   double[] valijkl)
```

Obtains  $\overline{A}$  in block triplet form.

- subi (int[]) Constraint index. (output)
- subj (int[]) Symmetric matrix variable index. (output)
- subk (int[]) Block row index. (output)
- subl (int[]) Block column index. (output)

- valijkl (double[]) The numerical value associated with each block triplet. (output)
- $\bullet\,$  num (long) Number of elements in the block triplet form. (output)

Return (long) – Number of elements in the block triplet form.

**Groups** Problem data - semidefinite, Inspecting the task

Task.getbaraidx

```
long getbaraidx
  (long idx,
   out int i,
   out int j,
   long[] sub,
   double[] weights)
```

```
void getbaraidx
  (long idx,
   out int i,
   out int j,
   out long num,
   long[] sub,
   double[] weights)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please observe if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

## Parameters

- idx (long) Position of the element in the vectorized form. (input)
- i (int) Row index of the element at position idx. (output)
- j (int) Column index of the element at position idx. (output)
- sub (long[]) A list indexes of the elements from symmetric matrix storage that appear in the weighted sum. (output)
- weights (double[]) The weights associated with each term in the weighted sum. (output)
- num (long) Number of terms in weighted sum that forms the element. (output)

Return (long) – Number of terms in weighted sum that forms the element.

**Groups** Problem data - semidefinite, Inspecting the task

Task.getbaraidxij

```
void getbaraidxij
  (long idx,
   out int i,
   out int j)
```

Obtains information about an element in  $\overline{A}$ . Since  $\overline{A}$  is a sparse matrix of symmetric matrices, only the nonzero elements in  $\overline{A}$  are stored in order to save space. Now  $\overline{A}$  is stored vectorized i.e. as one long vector. This function makes it possible to obtain information such as the row index and the column index of a particular element of the vectorized form of  $\overline{A}$ .

Please note that if one element of  $\overline{A}$  is inputted multiple times then it may be stored several times in vectorized form. In that case the element with the highest index is the one that is used.

- idx (long) Position of the element in the vectorized form. (input)
- i (int) Row index of the element at position idx. (output)
- j (int) Column index of the element at position idx. (output)

**Groups** Problem data - semidefinite, Inspecting the task

Task.getbaraidxinfo

```
long getbaraidxinfo (long idx)
```

```
void getbaraidxinfo
(long idx,
out long num)
```

Each nonzero element in  $\overline{A}_{ij}$  is formed as a weighted sum of symmetric matrices. Using this function the number of terms in the weighted sum can be obtained. See description of Task. appendsparsesymmat for details about the weighted sum.

#### **Parameters**

- idx (long) The internal position of the element for which information should be obtained. (input)
- num (long) Number of terms in the weighted sum that form the specified element in  $\overline{A}$ . (output)

**Return** (long) – Number of terms in the weighted sum that form the specified element in  $\overline{A}$ .

Groups Problem data - semidefinite, Inspecting the task

Task.getbarasparsity

```
void getbarasparsity
  (out long numnz,
    long[] idxij)
```

The matrix  $\overline{A}$  is assumed to be a sparse matrix of symmetric matrices. This implies that many of the elements in  $\overline{A}$  are likely to be zero matrices. Therefore, in order to save space, only nonzero elements in  $\overline{A}$  are stored on vectorized form. This function is used to obtain the sparsity pattern of  $\overline{A}$  and the position of each nonzero element in the vectorized form of  $\overline{A}$ . From the index detailed information about each nonzero  $\overline{A}_{i,j}$  can be obtained using Task.getbaraidxinfo and Task.getbaraidx.

## Parameters

- numnz (long) Number of nonzero elements in  $\overline{A}$ . (output)
- idxij (long[]) Position of each nonzero element in the vectorized form of  $\overline{A}$ . (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getbarcblocktriplet

```
long getbarcblocktriplet
  (int[] subj,
   int[] subk,
   int[] subl,
   double[] valjkl)
```

```
void getbarcblocktriplet
  (out long num,
  int[] subj,
```

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```
int[] subk,
int[] subl,
double[] valjkl)
```

Obtains  $\overline{C}$  in block triplet form.

## Parameters

- subj (int[]) Symmetric matrix variable index. (output)
- subk (int[]) Block row index. (output)
- subl (int[]) Block column index. (output)
- valjkl (double[]) The numerical value associated with each block triplet. (output)
- num (long) Number of elements in the block triplet form. (output)

Return (long) – Number of elements in the block triplet form.

**Groups** Problem data - semidefinite, Inspecting the task

Task.getbarcidx

```
void getbarcidx
  (long idx,
   out int j,
   out long num,
   long[] sub,
   double[] weights)
```

Obtains information about an element in  $\overline{C}$ .

### Parameters

- idx (long) Index of the element for which information should be obtained. (input)
- j (int) Row index in  $\overline{C}$ . (output)
- num (long) Number of terms in the weighted sum. (output)
- sub (long[]) Elements appearing the weighted sum. (output)
- weights (double[]) Weights of terms in the weighted sum. (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getbarcidxinfo

```
long getbarcidxinfo (long idx)
```

```
void getbarcidxinfo
(long idx,
out long num)
```

Obtains the number of terms in the weighted sum that forms a particular element in  $\overline{C}$ .

## **Parameters**

- idx (long) Index of the element for which information should be obtained. The value is an index of a symmetric sparse variable. (input)
- num (long) Number of terms that appear in the weighted sum that forms the requested element. (output)

**Return** (long) – Number of terms that appear in the weighted sum that forms the requested element.

**Groups** Problem data - semidefinite, Inspecting the task

Task.getbarcidxj

```
void getbarcidxj
  (long idx,
  out int j)
```

Obtains the row index of an element in  $\overline{C}$ .

#### Parameters

- idx (long) Index of the element for which information should be obtained. (input)
- j (int) Row index in  $\overline{C}$ . (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getbarcsparsity

```
void getbarcsparsity
  (out long numnz,
   long[] idxj)
```

Internally only the nonzero elements of  $\overline{C}$  are stored in a vector. This function is used to obtain the nonzero elements of  $\overline{C}$  and their indexes in the internal vector representation (in idx). From the index detailed information about each nonzero  $\overline{C}_j$  can be obtained using Task.getbarcidxinfo and Task.getbarcidx.

#### Parameters

- numnz (long) Number of nonzero elements in  $\overline{C}$ . (output)
- idxj (long[]) Internal positions of the nonzeros elements in  $\overline{C}$ . (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getbarsj

```
void getbarsj
  (soltype whichsol,
  int j,
  double[] barsj)
```

Obtains the dual solution for a semidefinite variable. Only the lower triangular part of  $\overline{S}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

## Parameters

- whichsol (soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barsj (double[]) Value of  $\overline{S}_i$ . (output)

 ${\bf Groups} \ \ Solution \ - \ semidefinite$ 

Task.getbarsslice

```
void getbarsslice
  (soltype whichsol,
   int first,
   int last,
   long slicesize,
   double[] barsslice)
```

Obtains the dual solution for a sequence of semidefinite variables. The format is that matrices are stored sequentially, and in each matrix the columns are stored as in *Task.getbarsj*.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) Index of the first semidefinite variable in the slice. (input)
- last (int) Index of the last semidefinite variable in the slice plus one. (input)
- slicesize (long) Denotes the length of the array barsslice. (input)
- barsslice (double[]) Dual solution values of symmetric matrix variables in the slice, stored sequentially. (output)

Groups Solution - semidefinite

## Task.getbarvarname

```
string getbarvarname (int i)
```

```
void getbarvarname
(int i,
StringBuilder name)
```

Obtains the name of a semidefinite variable.

### **Parameters**

- i (int) Index of the variable. (input)
- name (StringBuilder) The requested name is copied to this buffer. (output)

Return (string) - The requested name is copied to this buffer.

**Groups** Names, Inspecting the task

## Task.getbarvarnameindex

```
int getbarvarnameindex
  (string somename,
   out int asgn)
```

```
void getbarvarnameindex
(string somename,
out int asgn,
out int index)
```

Obtains the index of semidefinite variable from its name.

## **Parameters**

- somename (string) The name of the variable. (input)
- asgn (int) Non-zero if the name somename is assigned to some semidefinite variable. (output)
- index (int) The index of a semidefinite variable with the name somename (if one exists). (output)

**Return** (int) – The index of a semidefinite variable with the name somename (if one exists).

**Groups** Names, Inspecting the task

## Task.getbarvarnamelen

```
int getbarvarnamelen (int i)
```

```
void getbarvarnamelen
  (int i,
  out int len)
```

Obtains the length of the name of a semidefinite variable.

#### Parameters

- i (int) Index of the variable. (input)
- len (int) Returns the length of the indicated name. (output)

Return (int) - Returns the length of the indicated name.

Groups Names, Inspecting the task

Task.getbarxj

```
void getbarxj
  (soltype whichsol,
  int j,
  double[] barxj)
```

Obtains the primal solution for a semidefinite variable. Only the lower triangular part of  $\overline{X}_j$  is returned because the matrix by construction is symmetric. The format is that the columns are stored sequentially in the natural order.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barxj (double[]) Value of  $\overline{X}_j$ . (output)

Groups Solution - semidefinite

Task.getbarxslice

```
void getbarxslice
  (soltype whichsol,
  int first,
  int last,
  long slicesize,
  double[] barxslice)
```

Obtains the primal solution for a sequence of semidefinite variables. The format is that matrices are stored sequentially, and in each matrix the columns are stored as in *Task.getbarxj*.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) Index of the first semidefinite variable in the slice. (input)
- last (int) Index of the last semidefinite variable in the slice plus one. (input)
- slicesize (long) Denotes the length of the array barxslice. (input)
- barxslice (double[]) Solution values of symmetric matrix variables in the slice, stored sequentially. (output)

Groups Solution - semidefinite

Task.getc

```
void getc (double[] c)
```

Obtains all objective coefficients c.

**Parameters** c (double[]) – Linear terms of the objective as a dense vector. The length is the number of variables. (output)

Groups Problem data - linear part, Inspecting the task, Problem data - variables

Task.getcfix

```
double getcfix ()
```

```
void getcfix (out double cfix)
```

Obtains the fixed term in the objective.

```
Parameters cfix (double) - Fixed term in the objective. (output)
```

Return (double) - Fixed term in the objective.

Groups Problem data - linear part, Inspecting the task

Task.getcj

```
void getcj
  (int j,
  out double cj)
```

Obtains one coefficient of c.

## Parameters

- j (int) Index of the variable for which the c coefficient should be obtained. (input)
- cj (double) The value of  $c_i$ . (output)

Groups Problem data - linear part, Inspecting the task, Problem data - variables

Task.getclist

```
void getclist
  (int[] subj,
  double[] c)
```

Obtains a sequence of elements in c.

#### **Parameters**

- subj (int[]) A list of variable indexes. (input)
- c (double[]) Linear terms of the requested list of the objective as a dense vector. (output)

Groups Inspecting the task, Problem data - linear part

Task.getconbound

```
void getconbound
  (int i,
   out boundkey bk,
   out double bl,
   out double bu)
```

Obtains bound information for one constraint.

# Parameters

- i (int) Index of the constraint for which the bound information should be obtained. (input)
- bk (boundkey) Bound keys. (output)
- bl (double) Values for lower bounds. (output)
- bu (double) Values for upper bounds. (output)

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - constraints

#### Task.getconboundslice

```
void getconboundslice
  (int first,
   int last,
   boundkey[] bk,
   double[] bl,
   double[] bu)
```

Obtains bounds information for a slice of the constraints.

#### Parameters

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (boundkey []) Bound keys. (output)
- bl (double[]) Values for lower bounds. (output)
- bu (double[]) Values for upper bounds. (output)

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - constraints

## Task.getcone

```
void getcone
  (int k,
   out conetype ct,
   out double conepar,
   out int nummem,
   int[] submem)
```

Obtains a cone.

#### **Parameters**

- k (int) Index of the cone. (input)
- ct (conetype) Specifies the type of the cone. (output)
- conepar (double) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (output)
- nummem (int) Number of member variables in the cone. (output)
- submem (int[]) Variable subscripts of the members in the cone. (output)

Groups Inspecting the task, Problem data - cones

## Task.getconeinfo

```
void getconeinfo
  (int k,
  out conetype ct,
  out double conepar,
  out int nummem)
```

Obtains information about a cone.

# Parameters

- k (int) Index of the cone. (input)
- ct (conetype) Specifies the type of the cone. (output)
- conepar (double) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (output)
- nummem (int) Number of member variables in the cone. (output)

Groups Inspecting the task, Problem data - cones

```
string getconename (int i)
```

```
void getconename
(int i,
StringBuilder name)
```

Obtains the name of a cone.

# **Parameters**

- i (int) Index of the cone. (input)
- name (StringBuilder) The required name. (output)

**Return** (string) – The required name.

Groups Names, Problem data - cones, Inspecting the task

Task.getconenameindex

```
int getconenameindex
  (string somename,
  out int asgn)
```

```
void getconenameindex
  (string somename,
  out int asgn,
  out int index)
```

Checks whether the name somename has been assigned to any cone. If it has been assigned to a cone, then the index of the cone is reported.

## **Parameters**

- somename (string) The name which should be checked. (input)
- asgn (int) Is non-zero if the name somename is assigned to some cone. (output)
- index (int) If the name somename is assigned to some cone, then index is the index of the cone. (output)

Return (int) – If the name somename is assigned to some cone, then index is the index of the cone.

Groups Names, Problem data - cones, Inspecting the task

Task.getconenamelen

```
int getconenamelen (int i)
```

```
void getconenamelen
  (int i,
   out int len)
```

Obtains the length of the name of a cone.

# Parameters

- i (int) Index of the cone. (input)
- len (int) Returns the length of the indicated name. (output)

**Return** (int) – Returns the length of the indicated name.

Groups Names, Problem data - cones, Inspecting the task

Task.getconname

```
string getconname (int i)
```

```
void getconname
(int i,
StringBuilder name)
```

Obtains the name of a constraint.

#### **Parameters**

- i (int) Index of the constraint. (input)
- name (StringBuilder) The required name. (output)

**Return** (string) – The required name.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the task

Task.getconnameindex

```
int getconnameindex
  (string somename,
  out int asgn)
```

```
void getconnameindex
(string somename,
out int asgn,
out int index)
```

Checks whether the name somename has been assigned to any constraint. If so, the index of the constraint is reported.

# **Parameters**

- somename (string) The name which should be checked. (input)
- asgn (int) Is non-zero if the name somename is assigned to some constraint. (output)
- index (int) If the name somename is assigned to a constraint, then index is the index of the constraint. (output)

Return (int) - If the name somename is assigned to a constraint, then index is the index of the constraint.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the

Task.getconnamelen

```
int getconnamelen (int i)
```

```
void getconnamelen
  (int i,
   out int len)
```

Obtains the length of the name of a constraint.

### **Parameters**

- i (int) Index of the constraint. (input)
- len (int) Returns the length of the indicated name. (output)

Return (int) - Returns the length of the indicated name.

**Groups** Names, Problem data - linear part, Problem data - constraints, Inspecting the task

Task.getcslice

```
void getcslice
  (int first,
   int last,
   double[] c)
```

Obtains a sequence of elements in c.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- c (double[]) Linear terms of the requested slice of the objective as a dense vector. The length is last-first. (output)

Groups Inspecting the task, Problem data - linear part

Task.getdimbarvarj

```
int getdimbarvarj (int j)
```

```
void getdimbarvarj
  (int j,
  out int dimbarvarj)
```

Obtains the dimension of a symmetric matrix variable.

#### **Parameters**

- j (int) Index of the semidefinite variable whose dimension is requested. (input)
- dimbarvarj (int) The dimension of the j-th semidefinite variable. (output)

**Return** (int) – The dimension of the j-th semidefinite variable.

Groups Inspecting the task, Problem data - semidefinite

Task.getdouinf

```
double getdouinf (dinfitem whichdinf)
```

```
void getdouinf
(dinfitem whichdinf,
out double dvalue)
```

Obtains a double information item from the task information database.

## Parameters

- $\bullet$  which dinf (dinfitem) – Specifies a double information item. (input)
- dvalue (double) The value of the required double information item. (output)

**Return** (double) – The value of the required double information item.

Groups Information items and statistics

Task.getdouparam

```
double getdouparam (dparam param)
```

```
void getdouparam
(dparam param,
out double parvalue)
```

Obtains the value of a double parameter.

#### **Parameters**

- param (dparam) Which parameter. (input)
- parvalue (double) Parameter value. (output)

Return (double) - Parameter value.

Groups Parameters

Task.getdualobj

```
void getdualobj
(soltype whichsol,
out double dualobj)
```

Computes the dual objective value associated with the solution. Note that if the solution is a primal infeasibility certificate, then the fixed term in the objective value is not included.

Moreover, since there is no dual solution associated with an integer solution, an error will be reported if the dual objective value is requested for the integer solution.

## **Parameters**

- whichsol (soltype) Selects a solution. (input)
- dualobj (double) Objective value corresponding to the dual solution. (output)

Groups Solution information, Solution - dual

Task.getdualsolutionnorms

```
void getdualsolutionnorms
  (soltype whichsol,
   out double nrmy,
   out double nrmslc,
   out double nrmsuc,
   out double nrmslx,
   out double nrmsux,
   out double nrmsnx,
   out double nrmsnx,
```

Compute norms of the dual solution.

## Parameters

- whichsol (soltype) Selects a solution. (input)
- nrmy (double) The norm of the y vector. (output)
- nrmslc (double) The norm of the  $s_l^c$  vector. (output)
- nrmsuc (double) The norm of the  $s_u^c$  vector. (output)
- nrmslx (double) The norm of the  $s_l^x$  vector. (output)
- nrmsux (double) The norm of the  $s_u^x$  vector. (output)
- nrmsnx (double) The norm of the  $s_n^x$  vector. (output)
- nrmbars (double) The norm of the  $\overline{S}$  vector. (output)

**Groups** Solution information

Task.getdviolbarvar

```
void getdviolbarvar
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

Let  $(\overline{S}_j)^*$  be the value of variable  $\overline{S}_j$  for the specified solution. Then the dual violation of the solution associated with variable  $\overline{S}_j$  is given by

$$\max(-\lambda_{\min}(\overline{S}_j), 0.0).$$

Both when the solution is a certificate of primal infeasibility and when it is dual feasible solution the violation should be small.

## Parameters

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of  $\overline{X}$  variables. (input)
- viol (double[]) viol[k] is the violation of the solution for the constraint  $\overline{S}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

Groups Solution information

Task.getdviolcon

```
void getdviolcon
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

The violation of the dual solution associated with the *i*-th constraint is computed as follows

$$\max(\rho((s_l^c)_i^*, (b_l^c)_i), \ \rho((s_u^c)_i^*, -(b_u^c)_i), \ |-y_i + (s_l^c)_i^* - (s_u^c)_i^*|)$$

where

$$\rho(x, l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of primal infeasibility or it is a dual feasible solution the violation should be small.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of constraints. (input)
- viol (double[]) viol[k] is the violation of dual solution associated with the constraint sub[k]. (output)

Groups Solution information

Task.getdviolcones

```
void getdviolcones
  (soltype whichsol,
   int[] sub,
  double[] viol)
```

Let  $(s_n^x)^*$  be the value of variable  $(s_n^x)$  for the specified solution. For simplicity let us assume that  $s_n^x$  is a member of a quadratic cone, then the violation is computed as follows

$$\left\{ \begin{array}{ll} \max(0, (\|s_n^x\|_{2:n}^* - (s_n^x)_1^*)/\sqrt{2}, & (s_n^x)^* \geq -\|(s_n^x)_{2:n}^*\|, \\ \|(s_n^x)^*\|, & \text{otherwise.} \end{array} \right.$$

Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

#### **Parameters**

• whichsol (soltype) - Selects a solution. (input)

- sub (int[]) An array of indexes of conic constraints. (input)
- viol (double[]) viol[k] is the violation of the dual solution associated with the conic constraint sub[k]. (output)

Groups Solution information

Task.getdviolvar

```
void getdviolvar
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

The violation of the dual solution associated with the j-th variable is computed as follows

$$\max \left( \rho((s_l^x)_j^*, (b_l^x)_j), \ \rho((s_u^x)_j^*, -(b_u^x)_j), \ | \sum_{i=0}^{numcon-1} a_{ij} y_i + (s_l^x)_j^* - (s_u^x)_j^* - \tau c_j | \right)$$

where

$$\rho(x,l) = \begin{cases} -x, & l > -\infty, \\ |x|, & \text{otherwise} \end{cases}$$

and  $\tau=0$  if the solution is a certificate of primal infeasibility and  $\tau=1$  otherwise. The formula for computing the violation is only shown for the linear case but is generalized appropriately for the more general problems. Both when the solution is a certificate of primal infeasibility or when it is a dual feasible solution the violation should be small.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of x variables. (input)
- viol (double[]) viol[k] is the violation of dual solution associated with the variable sub[k]. (output)

Groups Solution information

Task.getinfeasiblesubproblem

```
Task getinfeasiblesubproblem (soltype whichsol)
```

```
void getinfeasiblesubproblem
(soltype whichsol,
out Task inftask)
```

Given the solution is a certificate of primal or dual infeasibility then a primal or dual infeasible subproblem is obtained respectively. The subproblem tends to be much smaller than the original problem and hence it is easier to locate the infeasibility inspecting the subproblem than the original problem.

For the procedure to be useful it is important to assign meaningful names to constraints, variables etc. in the original task because those names will be duplicated in the subproblem.

The function is only applicable to linear and conic quadratic optimization problems.

For more information see Sec. 8.3 and Sec. 14.2.

## Parameters

- whichsol (soltype) Which solution to use when determining the infeasible subproblem. (input)
- inftask (Task) A new task containing the infeasible subproblem. (output)

**Return** (*Task*) – A new task containing the infeasible subproblem.

Groups Infeasibility diagnostic

## Task.getinfindex

```
void getinfindex
  (inftype inftype,
   string infname,
   out int infindex)
```

Obtains the index of a named information item.

#### **Parameters**

- inftype (inftype) Type of the information item. (input)
- infname (string) Name of the information item. (input)
- infindex (int) The item index. (output)

Groups Information items and statistics

Task.getintinf

```
int getintinf (iinfitem whichiinf)
```

```
void getintinf
  (iinfitem whichiinf,
  out int ivalue)
```

Obtains an integer information item from the task information database.

#### Parameters

- $\bullet$  which iinf (iinfitem) – Specifies an integer information item. (input)
- ivalue (int) The value of the required integer information item. (output)

Return (int) - The value of the required integer information item.

Groups Information items and statistics

Task.getintparam

```
int getintparam (iparam param)
```

```
void getintparam
(iparam param,
out int parvalue)
```

Obtains the value of an integer parameter.

# Parameters

- param (*iparam*) Which parameter. (input)
- parvalue (int) Parameter value. (output)

Return (int) - Parameter value.

Groups Parameters

Task.getlenbarvarj

```
long getlenbarvarj (int j)
```

```
void getlenbarvarj
  (int j,
  out long lenbarvarj)
```

Obtains the length of the j-th semidefinite variable i.e. the number of elements in the lower triangular part.

#### **Parameters**

- j (int) Index of the semidefinite variable whose length if requested. (input)
- lenbarvarj (long) Number of scalar elements in the lower triangular part of the semidefinite variable. (output)

**Return** (long) – Number of scalar elements in the lower triangular part of the semidefinite variable.

Groups Inspecting the task, Problem data - semidefinite

Task.getlintinf

```
long getlintinf (liinfitem whichliinf)
```

```
void getlintinf
  (liinfitem whichliinf,
  out long ivalue)
```

Obtains a long integer information item from the task information database.

## **Parameters**

- whichliinf (liinfitem) Specifies a long information item. (input)
- ivalue (long) The value of the required long integer information item. (output)

Return (long) - The value of the required long integer information item.

Groups Information items and statistics

Task.getmaxnumanz

```
long getmaxnumanz ()
```

```
void getmaxnumanz (out long maxnumanz)
```

Obtains number of preallocated non-zeros in A. When this number of non-zeros is reached **MOSEK** will automatically allocate more space for A.

Parameters maxnumanz (long) – Number of preallocated non-zero linear matrix elements. (output)

Return (long) – Number of preallocated non-zero linear matrix elements.

Groups Inspecting the task, Problem data - linear part

Task.getmaxnumbarvar

```
int getmaxnumbarvar ()
```

```
void getmaxnumbarvar (out int maxnumbarvar)
```

Obtains maximum number of symmetric matrix variables for which space is currently preallocated.

Parameters maxnumbarvar (int) – Maximum number of symmetric matrix variables for which space is currently preallocated. (output)

**Return** (int) – Maximum number of symmetric matrix variables for which space is currently preallocated.

Groups Inspecting the task, Problem data - semidefinite

Task.getmaxnumcon

```
void getmaxnumcon (out int maxnumcon)
```

Obtains the number of preallocated constraints in the optimization task. When this number of constraints is reached **MOSEK** will automatically allocate more space for constraints.

Parameters maxnumcon (int) – Number of preallocated constraints in the optimization task. (output)

Groups Inspecting the task, Problem data - linear part, Problem data - constraints

Task.getmaxnumcone

```
void getmaxnumcone (out int maxnumcone)
```

Obtains the number of preallocated cones in the optimization task. When this number of cones is reached **MOSEK** will automatically allocate space for more cones.

Parameters maxnumcone (int) – Number of preallocated conic constraints in the optimization task. (output)

Groups Inspecting the task, Problem data - cones

Task.getmaxnumqnz

```
void getmaxnumqnz (out long maxnumqnz)
```

Obtains the number of preallocated non-zeros for Q (both objective and constraints). When this number of non-zeros is reached **MOSEK** will automatically allocate more space for Q.

Parameters maxnumqnz (long) - Number of non-zero elements preallocated in quadratic coefficient matrices. (output)

Groups Inspecting the task, Problem data - quadratic part

Task.getmaxnumvar

```
void getmaxnumvar (out int maxnumvar)
```

Obtains the number of preallocated variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

Parameters maxnumvar (int) – Number of preallocated variables in the optimization task. (output)

Groups Inspecting the task, Problem data - linear part, Problem data - variables

Task.getmemusage

```
void getmemusage
(out long meminuse,
out long maxmemuse)
```

Obtains information about the amount of memory used by a task.

# Parameters

- meminuse (long) Amount of memory currently used by the task. (output)
- maxmemuse (long) Maximum amount of memory used by the task until now. (output)

Groups System, memory and debugging

Task.getnumanz

```
int getnumanz ()
```

void getnumanz (out int numanz)

Obtains the number of non-zeros in A.

Parameters numanz (int) – Number of non-zero elements in the linear constraint matrix. (output)

Return (int) – Number of non-zero elements in the linear constraint matrix.

Groups Inspecting the task, Problem data - linear part

Task.getnumanz64

```
long getnumanz64 ()
```

void getnumanz64 (out long numanz)

Obtains the number of non-zeros in A.

Parameters numanz (long) - Number of non-zero elements in the linear constraint matrix. (output)

Return (long) - Number of non-zero elements in the linear constraint matrix.

Groups Inspecting the task, Problem data - linear part

Task.getnumbarablocktriplets

## long getnumbarablocktriplets ()

void getnumbarablocktriplets (out long num)

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{A}$ .

**Parameters** num (long) – An upper bound on the number of elements in the block triplet form of  $\overline{A}$ . (output)

**Return** (long) – An upper bound on the number of elements in the block triplet form of  $\overline{A}$ .

**Groups** Problem data - semidefinite, Inspecting the task

Task.getnumbaranz

## long getnumbaranz ()

void getnumbaranz (out long nz)

Get the number of nonzero elements in  $\overline{A}$ .

**Parameters nz** (long) – The number of nonzero block elements in  $\overline{A}$  i.e. the number of  $\overline{A}_{ij}$  elements that are nonzero. (output)

**Return** (long) – The number of nonzero block elements in  $\overline{A}$  i.e. the number of  $\overline{A}_{ij}$  elements that are nonzero.

Groups Problem data - semidefinite, Inspecting the task

Task.getnumbarcblocktriplets

long getnumbarcblocktriplets ()

```
void getnumbarcblocktriplets (out long num)
```

Obtains an upper bound on the number of elements in the block triplet form of  $\overline{C}$ .

**Parameters** num (long) – An upper bound on the number of elements in the block triplet form of  $\overline{C}$ . (output)

**Return** (long) – An upper bound on the number of elements in the block triplet form of  $\overline{C}$ .

Groups Problem data - semidefinite, Inspecting the task

Task.getnumbarcnz

```
long getnumbarcnz ()
```

```
void getnumbarcnz (out long nz)
```

Obtains the number of nonzero elements in  $\overline{C}$ .

**Parameters nz** (long) – The number of nonzeros in  $\overline{C}$  i.e. the number of elements  $\overline{C}_j$  that are nonzero. (output)

**Return** (long) – The number of nonzeros in  $\overline{C}$  i.e. the number of elements  $\overline{C}_j$  that are nonzero

Groups Problem data - semidefinite, Inspecting the task

Task.getnumbarvar

```
int getnumbarvar ()
```

```
void getnumbarvar (out int numbarvar)
```

Obtains the number of semidefinite variables.

Parameters numbarvar (int) - Number of semidefinite variables in the problem. (output)

Return (int) – Number of semidefinite variables in the problem.

Groups Inspecting the task, Problem data - semidefinite

Task.getnumcon

```
int getnumcon ()
```

```
void getnumcon (out int numcon)
```

Obtains the number of constraints.

Parameters numcon (int) – Number of constraints. (output)

Return (int) – Number of constraints.

Groups Problem data - linear part, Problem data - constraints, Inspecting the task

Task.getnumcone

```
int getnumcone ()
```

```
void getnumcone (out int numcone)
```

Obtains the number of cones.

```
Parameters numcone (int) - Number of conic constraints. (output)
Return (int) - Number of conic constraints.
Groups Problem data - cones, Inspecting the task
```

Task.getnumconemem

```
void getnumconemem
  (int k,
   out int nummem)
```

Obtains the number of members in a cone.

## **Parameters**

- k (int) Index of the cone. (input)
- nummem (int) Number of member variables in the cone. (output)

Groups Problem data - cones, Inspecting the task

Task.getnumintvar

```
void getnumintvar (out int numintvar)
```

Obtains the number of integer-constrained variables.

Parameters numintvar (int) – Number of integer variables. (output) Groups Inspecting the task, Problem data - variables

Task.getnumparam

```
void getnumparam
(parametertype partype,
out int numparam)
```

Obtains the number of parameters of a given type.

## Parameters

- partype (parameter type) Parameter type. (input)
- numparam (int) The number of parameters of type partype. (output)

Groups Inspecting the task, Parameters

Task.getnumqconknz

```
long getnumqconknz (int k)
```

```
void getnumqconknz
  (int k,
  out long numqcnz)
```

Obtains the number of non-zero quadratic terms in a constraint.

## **Parameters**

- k (int) Index of the constraint for which the number quadratic terms should be obtained. (input)
- numqcnz (long) Number of quadratic terms. (output)

**Return** (long) – Number of quadratic terms.

 $\textbf{Groups} \ \textit{Inspecting the task, Problem data - constraints, Problem data - quadratic part}$ 

Task.getnumqobjnz

long getnumqobjnz ()

void getnumqobjnz (out long numqonz)

Obtains the number of non-zero quadratic terms in the objective.

**Parameters** numqonz (long) – Number of non-zero elements in the quadratic objective terms. (output)

Return (long) – Number of non-zero elements in the quadratic objective terms.

Groups Inspecting the task, Problem data - quadratic part

Task.getnumsymmat

void getnumsymmat (out long num)

Obtains the number of symmetric matrices stored in the vector E.

Parameters num (long) - The number of symmetric sparse matrices. (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getnumvar

int getnumvar ()

void getnumvar (out int numvar)

Obtains the number of variables.

Parameters number (int) - Number of variables. (output)

Return (int) – Number of variables.

Groups Inspecting the task, Problem data - variables

Task.getobjname

string getobjname ()

void getobjname (StringBuilder objname)

Obtains the name assigned to the objective function.

Parameters objname (StringBuilder) - Assigned the objective name. (output)

Return (string) - Assigned the objective name.

Groups Inspecting the task, Names

Task.getobjnamelen

int getobjnamelen ()

void getobjnamelen (out int len)

Obtains the length of the name assigned to the objective function.

Parameters len (int) - Assigned the length of the objective name. (output)

Return (int) - Assigned the length of the objective name.

Groups Inspecting the task, Names

### Task.getobjsense

```
objsense getobjsense ()
```

```
void getobjsense (out objsense sense)
```

Gets the objective sense of the task.

```
Parameters sense (objsense) – The returned objective sense. (output)
Return (objsense) – The returned objective sense.
Groups Problem data - linear part
```

Task.getprimalobj

```
double getprimalobj (soltype whichsol)
```

```
void getprimalobj
  (soltype whichsol,
  out double primalobj)
```

Computes the primal objective value for the desired solution. Note that if the solution is an infeasibility certificate, then the fixed term in the objective is not included.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- primalobj (double) Objective value corresponding to the primal solution. (output)

Return (double) - Objective value corresponding to the primal solution.

Groups Solution information, Solution - primal

Task.getprimalsolutionnorms

```
void getprimalsolutionnorms
  (soltype whichsol,
  out double nrmxc,
  out double nrmxx,
  out double nrmxx)
```

Compute norms of the primal solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- nrmxc (double) The norm of the  $x^c$  vector. (output)
- nrmxx (double) The norm of the *x* vector. (output)
- nrmbarx (double) The norm of the  $\overline{X}$  vector. (output)

**Groups** Solution information

Task.getprobtype

```
problemtype getprobtype ()
```

```
void getprobtype (out problemtype probtype)
```

Obtains the problem type.

Parameters probtype (problemtype) - The problem type. (output)

```
Return (problemtype) – The problem type.
Groups Inspecting the task
```

Task.getprosta

```
prosta getprosta (soltype whichsol)
```

```
void getprosta
(soltype whichsol,
out prosta prosta)
```

Obtains the problem status.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- prosta (prosta) Problem status. (output)

Return (prosta) - Problem status.

**Groups** Solution information

Task.getpviolbarvar

```
void getpviolbarvar
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation for a set of semidefinite variables. Let  $(\overline{X}_j)^*$  be the value of the variable  $\overline{X}_j$  for the specified solution. Then the primal violation of the solution associated with variable  $\overline{X}_j$  is given by

$$\max(-\lambda_{\min}(\overline{X}_j), 0.0).$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of  $\overline{X}$  variables. (input)
- viol (double[]) viol[k] is how much the solution violates the constraint  $\overline{X}_{\text{sub}[k]} \in \mathcal{S}_+$ . (output)

Groups Solution information

Task.getpviolcon

```
void getpviolcon
  (soltype whichsol,
   int[] sub,
   double[] viol)
```

Computes the primal solution violation for a set of constraints. The primal violation of the solution associated with the i-th constraint is given by

$$\max(\tau l_i^c - (x_i^c)^*, \ (x_i^c)^* - \tau u_i^c), \ | \sum_{j=0}^{numvar-1} a_{ij} x_j^* - x_i^c |)$$

where  $\tau=0$  if the solution is a certificate of dual infeasibility and  $\tau=1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small. The above formula applies for the linear case but is appropriately generalized in other cases.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of constraints. (input)
- viol (double[]) viol[k] is the violation associated with the solution for the constraint sub[k]. (output)

Groups Solution information

Task.getpviolcones

```
void getpviolcones
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation for a set of conic constraints. Let  $x^*$  be the value of the variable x for the specified solution. For simplicity let us assume that x is a member of a quadratic cone, then the violation is computed as follows

$$\begin{cases} \max(0, ||x_{2:n}|| - x_1)/\sqrt{2}, & x_1 \ge -||x_{2:n}||, \\ ||x||, & \text{otherwise.} \end{cases}$$

Both when the solution is a certificate of dual infeasibility or when it is primal feasible the violation should be small.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of conic constraints. (input)
- viol (double[]) viol[k] is the violation of the solution associated with the conic constraint number sub[k]. (output)

Groups Solution information

Task.getpviolvar

```
void getpviolvar
  (soltype whichsol,
  int[] sub,
  double[] viol)
```

Computes the primal solution violation associated to a set of variables. Let  $x_j^*$  be the value of  $x_j$  for the specified solution. Then the primal violation of the solution associated with variable  $x_j$  is given by

$$\max(\tau l_j^x - x_j^*, \ x_j^* - \tau u_j^x, \ 0).$$

where  $\tau = 0$  if the solution is a certificate of dual infeasibility and  $\tau = 1$  otherwise. Both when the solution is a certificate of dual infeasibility and when it is primal feasible the violation should be small.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- sub (int[]) An array of indexes of x variables. (input)
- viol (double[]) viol[k] is the violation associated with the solution for the variable  $x_{sub[k]}$ . (output)

Groups Solution information

Task.getqconk

```
long getqconk
  (int k,
  ref long qcsurp,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

```
void getqconk
  (int k,
  ref long qcsurp,
  out long numqcnz,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

Obtains all the quadratic terms in a constraint. The quadratic terms are stored sequentially in qcsubj, qcsubj, and qcval.

# **Parameters**

- k (int) Which constraint. (input)
- qcsurp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qcsubi, qcsubj and qcval starting from position qcsurp away from the end of the arrays. On return qcsurp will be decremented by the total number of non-zeros written. (input/output)
- qcsubi (int[]) Row subscripts for quadratic constraint matrix. (output)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (output)
- qcval (double[]) Quadratic constraint coefficient values. (output)
- numqcnz (long) Number of quadratic terms. (output)

Return (long) - Number of quadratic terms.

Groups Inspecting the task, Problem data - quadratic part, Problem data - constraints

Task.getqobj

```
void getqobj
  (ref long qosurp,
   out long numqonz,
   int[] qosubi,
   int[] qosubj,
   double[] qoval)
```

Obtains the quadratic terms in the objective. The required quadratic terms are stored sequentially in qosubj, and qoval.

#### **Parameters**

- qosurp (long) Surplus of subscript and coefficient arrays. The required entries are stored sequentially in qosubi, qosubj and qoval starting from position qosurp away from the end of the arrays. On return qosurp will be decremented by the total number of non-zeros written. (input/output)
- numqonz (long) Number of non-zero elements in the quadratic objective terms. (output)
- qosubi (int[]) Row subscripts for quadratic objective coefficients. (output)
- qosubj (int[]) Column subscripts for quadratic objective coefficients. (output)
- qoval (double[]) Quadratic objective coefficient values. (output)

Groups Inspecting the task, Problem data - quadratic part

Task.getqobjij

```
void getqobjij
  (int i,
   int j,
   out double qoij)
```

Obtains one coefficient  $q_{ij}^o$  in the quadratic term of the objective.

#### **Parameters**

- i (int) Row index of the coefficient. (input)
- j (int) Column index of coefficient. (input)
- qoij (double) The required coefficient. (output)

Groups Inspecting the task, Problem data - quadratic part

Task.getreducedcosts

```
void getreducedcosts
  (soltype whichsol,
  int first,
  int last,
  double[] redcosts)
```

Computes the reduced costs for a slice of variables and returns them in the array redcosts i.e.

$$redcosts[j-first] = (s_l^x)_j - (s_u^x)_j, \ j = first, \dots, last - 1$$
 (15.2)

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) The index of the first variable in the sequence. (input)
- last (int) The index of the last variable in the sequence plus 1. (input)
- redcosts (double[]) The reduced costs for the required slice of variables. (output)

Groups Solution - dual

Task.getskc

```
void getskc
(soltype whichsol,
stakey[] skc)
```

Obtains the status keys for the constraints.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- skc (stakey []) Status keys for the constraints. (output)

Groups Solution information

Task.getskcslice

```
void getskcslice
  (soltype whichsol,
  int first,
  int last,
  stakey[] skc)
```

Obtains the status keys for a slice of the constraints.

# **Parameters**

• whichsol (soltype) - Selects a solution. (input)

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skc (stakey[]) Status keys for the constraints. (output)

Groups Solution information

Task.getskn

```
void getskn
  (soltype whichsol,
   stakey[] skn)
```

Obtains the status keys for the conic constraints.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- skn (stakey []) Status keys for the conic constraints. (output)

**Groups** Solution information

Task.getskx

```
void getskx
  (soltype whichsol,
   stakey[] skx)
```

Obtains the status keys for the scalar variables.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- skx (stakey []) Status keys for the variables. (output)

**Groups** Solution information

Task.getskxslice

```
void getskxslice
  (soltype whichsol,
  int first,
  int last,
  stakey[] skx)
```

Obtains the status keys for a slice of the scalar variables.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skx (stakey []) Status keys for the variables. (output)

Groups Solution information

Task.getslc

```
void getslc
  (soltype whichsol,
  double[] slc)
```

Obtains the  $s_l^c$  vector for a solution.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)

Groups Solution - dual

Task.getslcslice

```
void getslcslice
  (soltype whichsol,
  int first,
  int last,
  double[] slc)
```

Obtains a slice of the  $s_I^c$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)

Groups Solution - dual

Task.getslx

```
void getslx
  (soltype whichsol,
  double[] slx)
```

Obtains the  $s_l^x$  vector for a solution.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)

Groups Solution - dual

Task.getslxslice

```
void getslxslice
  (soltype whichsol,
  int first,
  int last,
  double[] slx)
```

Obtains a slice of the  $s_l^x$  vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)

Groups Solution - dual

Task.getsnx

```
void getsnx
  (soltype whichsol,
  double[] snx)
```

Obtains the  $s_n^x$  vector for a solution.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution - dual

Task.getsnxslice

```
void getsnxslice
  (soltype whichsol,
  int first,
  int last,
  double[] snx)
```

Obtains a slice of the  $s_n^x$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution - dual

Task.getsolsta

```
solsta getsolsta (soltype whichsol)
```

```
void getsolsta
(soltype whichsol,
out solsta solsta)
```

Obtains the solution status.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- ullet solsta (solsta) Solution status. (output)

Return (solsta) - Solution status.

**Groups** Solution information

Task.getsolution

```
void getsolution
  (soltype whichsol,
   out prosta prosta,
   out solsta solsta,
   stakey[] skc,
   stakey[] skx,
   stakey[] skn,
   double[] xc,
```

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```
double[] xx,
double[] y,
double[] slc,
double[] suc,
double[] slx,
double[] sux,
double[] snx)
```

Obtains the complete solution.

Consider the case of linear programming. The primal problem is given by

and the corresponding dual problem is

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x & = c, \\ & -y + s_l^c - s_u^c & = 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x \geq 0. \end{array}$$

A conic optimization problem has the same primal variables as in the linear case. Recall that the dual of a conic optimization problem is given by:

$$\begin{array}{lll} \text{maximize} & (l^c)^T s_l^c - (u^c)^T s_u^c \\ & + (l^x)^T s_l^x - (u^x)^T s_u^x + c^f \\ \text{subject to} & A^T y + s_l^x - s_u^x + s_n^x & = & c, \\ & -y + s_l^c - s_u^c & = & 0, \\ & s_l^c, s_u^c, s_l^x, s_u^x & \geq & 0, \\ & s_n^x \in \mathcal{K}^* & \end{array}$$

The mapping between variables and arguments to the function is as follows:

- xx: Corresponds to variable x (also denoted  $x^x$ ).
- xc: Corresponds to  $x^c := Ax$ .
- y: Corresponds to variable y.
- slc: Corresponds to variable  $s_i^c$ .
- suc: Corresponds to variable  $s_u^c$ .
- slx: Corresponds to variable  $s_l^x$ .
- sux: Corresponds to variable  $s_u^x$ .
- snx: Corresponds to variable  $s_n^x$ .

The meaning of the values returned by this function depend on the *solution status* returned in the argument solsta. The most important possible values of solsta are:

- solsta.optimal: An optimal solution satisfying the optimality criteria for continuous problems is returned.
- solsta.integer\_optimal: An optimal solution satisfying the optimality criteria for integer problems is returned.
- solsta.prim\_feas: A solution satisfying the feasibility criteria.
- solsta.prim\_infeas\_cer : A primal certificate of infeasibility is returned.
- solsta.dual\_infeas\_cer: A dual certificate of infeasibility is returned.

In order to retrieve the primal and dual values of semidefinite variables see *Task.getbarxj* and *Task.getbarxj*.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- prosta (prosta) Problem status. (output)
- solsta (solsta) Solution status. (output)
- skc (stakey []) Status keys for the constraints. (output)
- skx (stakey []) Status keys for the variables. (output)
- skn (stakey []) Status keys for the conic constraints. (output)
- xc (double[]) Primal constraint solution. (output)
- xx (double[]) Primal variable solution. (output)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (output)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (output)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (output)

Groups Solution information, Solution - primal, Solution - dual

### Task.getsolutioninfo

```
void getsolutioninfo
  (soltype whichsol,
   out double pobj,
  out double priolcon,
  out double priolvar,
  out double priolbarvar,
  out double priolcone,
  out double priolitg,
  out double dobj,
  out double dviolcon,
  out double dviolvar,
  out double dviolbarvar,
  out double dviolbarvar,
  out double dviolcone)
```

Obtains information about a solution.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- pobj (double) The primal objective value as computed by *Task*. getprimalobj. (output)
- pviolcon (double) Maximal primal violation of the solution associated with the  $x^c$  variables where the violations are computed by Task.getpviolcon. (output)
- pviolvar (double) Maximal primal violation of the solution for the x variables where the violations are computed by Task.getpviolvar. (output)
- pviolbarvar (double) Maximal primal violation of solution for the  $\overline{X}$  variables where the violations are computed by Task.getpviolbarvar. (output)
- pviolcone (double) Maximal primal violation of solution for the conic constraints where the violations are computed by *Task.getpviolcones*. (output)
- pviolitg (double) Maximal violation in the integer constraints. The violation for an integer variable  $x_j$  is given by  $\min(x_j \lfloor x_j \rfloor, \lceil x_j \rceil x_j)$ . This number is always zero for the interior-point and basic solutions. (output)

- dobj (double) Dual objective value as computed by *Task.getdualobj*. (output)
- dviolcon (double) Maximal violation of the dual solution associated with the  $x^c$  variable as computed by Task. getdviolcon. (output)
- dviolvar (double) Maximal violation of the dual solution associated with the x variable as computed by Task.qetdviolvar. (output)
- dviolbarvar (double) Maximal violation of the dual solution associated with the  $\overline{S}$  variable as computed by Task.getdviolbarvar. (output)
- dviolcone (double) Maximal violation of the dual solution associated with the dual conic constraints as computed by Task. getdviolcones. (output)

Groups Solution information

Task.getsolutionslice

```
void getsolutionslice
  (soltype whichsol,
   solitem solitem,
  int first,
  int last,
  double[] values)
```

Obtains a slice of one item from the solution. The format of the solution is exactly as in *Task*. *getsolution*. The parameter solitem determines which of the solution vectors should be returned.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- solitem (solitem) Which part of the solution is required. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- values (double[]) The values in the required sequence are stored sequentially in values. (output)

Groups Solution - primal, Solution - dual, Solution information

Task.getsparsesymmat

```
void getsparsesymmat
  (long idx,
   int[] subi,
   int[] subj,
   double[] valij)
```

Get a single symmetric matrix from the matrix store.

#### Parameters

- idx (long) Index of the matrix to retrieve. (input)
- subi (int[]) Row subscripts of the matrix non-zero elements. (output)
- subj (int[]) Column subscripts of the matrix non-zero elements. (output)
- valij (double[]) Coefficients of the matrix non-zero elements. (output)

Groups Problem data - semidefinite, Inspecting the task

Task.getstrparam

```
string getstrparam
(sparam param,
out int len)
```

```
void getstrparam
(sparam param,
out int len,
StringBuilder parvalue)
```

Obtains the value of a string parameter.

### **Parameters**

- param (sparam) Which parameter. (input)
- len (int) The length of the parameter value. (output)
- parvalue (StringBuilder) Parameter value. (output)

Return (string) - Parameter value.

Groups Names, Parameters

Task.getstrparamlen

```
int getstrparamlen (sparam param)
```

```
void getstrparamlen
(sparam param,
out int len)
```

Obtains the length of a string parameter.

### **Parameters**

- param (sparam) Which parameter. (input)
- len (int) The length of the parameter value. (output)

**Return** (int) – The length of the parameter value.

Groups Names, Parameters

Task.getsuc

```
void getsuc
  (soltype whichsol,
  double[] suc)
```

Obtains the  $s_u^c$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)

Groups Solution - dual

Task.getsucslice

```
void getsucslice
  (soltype whichsol,
   int first,
   int last,
   double[] suc)
```

Obtains a slice of the  $s_u^c$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)

- last (int) Last index plus 1 in the sequence. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (output)

Groups Solution - dual

Task.getsux

```
void getsux
  (soltype whichsol,
  double[] sux)
```

Obtains the  $s_n^x$  vector for a solution.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)

Groups Solution - dual

Task.getsuxslice

```
void getsuxslice
  (soltype whichsol,
  int first,
  int last,
  double[] sux)
```

Obtains a slice of the  $s_u^x$  vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (output)

Groups Solution - dual

Task.getsymmatinfo

```
void getsymmatinfo
  (long idx,
   out int dim,
   out long nz,
   out symmattype type)
```

**MOSEK** maintains a vector denoted by E of symmetric data matrices. This function makes it possible to obtain important information about a single matrix in E.

# **Parameters**

- idx (long) Index of the matrix for which information is requested. (input)
- dim (int) Returns the dimension of the requested matrix. (output)
- nz (long) Returns the number of non-zeros in the requested matrix. (output)
- type (symmattype) Returns the type of the requested matrix. (output)

Groups Problem data - semidefinite, Inspecting the task

Task.gettaskname

```
string gettaskname ()
```

```
void gettaskname (StringBuilder taskname)
```

Obtains the name assigned to the task.

```
Parameters taskname (StringBuilder) – Returns the task name. (output) Return (string) – Returns the task name.
```

Groups Names, Inspecting the task

Task.gettasknamelen

```
int gettasknamelen ()
```

```
void gettasknamelen (out int len)
```

Obtains the length the task name.

```
Parameters len (int) - Returns the length of the task name. (output)
```

Return (int) – Returns the length of the task name.

Groups Names, Inspecting the task

Task.getvarbound

```
void getvarbound
  (int i,
   out boundkey bk,
   out double bl,
   out double bu)
```

Obtains bound information for one variable.

### Parameters

- i (int) Index of the variable for which the bound information should be obtained. (input)
- bk (boundkey) Bound keys. (output)
- bl (double) Values for lower bounds. (output)
- bu (double) Values for upper bounds. (output)

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - variables

Task.getvarboundslice

```
void getvarboundslice
  (int first,
  int last,
  boundkey[] bk,
  double[] bl,
  double[] bu)
```

Obtains bounds information for a slice of the variables.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bk (boundkey []) Bound keys. (output)
- bl (double[]) Values for lower bounds. (output)

• bu (double[]) - Values for upper bounds. (output)

**Groups** Problem data - linear part, Inspecting the task, Problem data - bounds, Problem data - variables

Task.getvarname

```
string getvarname (int j)
```

```
void getvarname
(int j,
StringBuilder name)
```

Obtains the name of a variable.

### Parameters

- j (int) Index of a variable. (input)
- name (StringBuilder) Returns the required name. (output)

Return (string) - Returns the required name.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the task

Task.getvarnameindex

```
int getvarnameindex
  (string somename,
  out int asgn)
```

```
void getvarnameindex
(string somename,
out int asgn,
out int index)
```

Checks whether the name somename has been assigned to any variable. If so, the index of the variable is reported.

### **Parameters**

- somename (string) The name which should be checked. (input)
- asgn (int) Is non-zero if the name somename is assigned to a variable. (output)
- index (int) If the name somename is assigned to a variable, then index is the index of the variable. (output)

Return (int) - If the name somename is assigned to a variable, then index is the index of the variable.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the task

Task.getvarnamelen

```
int getvarnamelen (int i)
```

```
void getvarnamelen
  (int i,
   out int len)
```

Obtains the length of the name of a variable.

# Parameters

• i (int) – Index of a variable. (input)

• len (int) - Returns the length of the indicated name. (output)

Return (int) - Returns the length of the indicated name.

**Groups** Names, Problem data - linear part, Problem data - variables, Inspecting the task

Task.getvartype

```
variabletype getvartype (int j)
```

```
void getvartype
  (int j,
   out variabletype vartype)
```

Gets the variable type of one variable.

### **Parameters**

- j (int) Index of the variable. (input)
- vartype (variable type) Variable type of the j-th variable. (output)

**Return** (variable type) – Variable type of the j-th variable.

Groups Inspecting the task, Problem data - variables

Task.getvartypelist

```
void getvartypelist
  (int[] subj,
  variabletype[] vartype)
```

Obtains the variable type of one or more variables. Upon return vartype[k] is the variable type of variable subj[k].

### Parameters

- subj (int[]) A list of variable indexes. (input)
- vartype (variable type []) The variables types corresponding to the variables specified by subj. (output)

Groups Inspecting the task, Problem data - variables

Task.getxc

```
void getxc
  (soltype whichsol,
  double[] xc)
```

Obtains the  $x^c$  vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- xc (double[]) Primal constraint solution. (output)

Groups Solution - primal

Task.getxcslice

```
void getxcslice
  (soltype whichsol,
  int first,
  int last,
  double[] xc)
```

Obtains a slice of the  $x^c$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xc (double[]) Primal constraint solution. (output)

Groups Solution - primal

Task.getxx

```
void getxx
  (soltype whichsol,
  double[] xx)
```

Obtains the  $x^x$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- xx (double[]) Primal variable solution. (output)

Groups Solution - primal

Task.getxxslice

```
void getxxslice
  (soltype whichsol,
  int first,
  int last,
  double[] xx)
```

Obtains a slice of the  $x^x$  vector for a solution.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xx (double[]) Primal variable solution. (output)

Groups Solution - primal

Task.gety

```
void gety
  (soltype whichsol,
   double[] y)
```

Obtains the y vector for a solution.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)

Groups Solution - dual

Task.getyslice

```
void getyslice
  (soltype whichsol,
  int first,
  int last,
  double[] y)
```

Obtains a slice of the y vector for a solution.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (output)

Groups Solution - dual

#### Task.initbasissolve

```
void initbasissolve (int[] basis)
```

Prepare a task for use with the Task. solvewithbasis function.

This function should be called

- immediately before the first call to Task. solvewithbasis, and
- immediately before any subsequent call to Task. solvewithbasis if the task has been modified.

If the basis is singular i.e. not invertible, then the error  $rescode.err\_basis\_singular$  is reported.

**Parameters** basis (int[]) – The array of basis indexes to use. The array is interpreted as follows: If basis[i]  $\leq numcon - 1$ , then  $x_{basis[i]}^c$  is in the basis at position i, otherwise  $x_{basis[i]-numcon}$  is in the basis at position i. (output)

Groups Solving systems with basis matrix

# Task.inputdata

```
void inputdata
  (int maxnumcon,
   int maxnumvar,
   double[] c,
   double cfix,
   int[] aptrb,
   int[] aptre,
   int[] asub,
   double[] aval,
   boundkey[] bkc,
   double[] blc,
   double[] buc,
   boundkey[] bkx,
   double[] blx,
   double[] blx,
   double[] blx,
```

```
void inputdata
  (int maxnumcon,
   int maxnumvar,
   double[] c,
   double cfix,
   long[] aptrb,
```

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```
long[] aptre,
int[] asub,
double[] aval,
boundkey[] bkc,
double[] blc,
double[] buc,
boundkey[] bkx,
double[] blx,
double[] blx,
```

```
void inputdata
  (int maxnumcon,
   int maxnumvar,
  int numcon,
  int numvar.
  double[] c,
  double cfix,
  long[] aptrb,
  long[] aptre,
  int[] asub,
  double[] aval,
  boundkey[] bkc,
  double[] blc,
  double[] buc,
  boundkey[] bkx,
  double[] blx,
  double[] bux)
```

Input the linear part of an optimization task in one function call.

#### **Parameters**

- maxnumcon (int) Number of preallocated constraints in the optimization task. (input)
- maxnumvar (int) Number of preallocated variables in the optimization task.
   (input)
- c (double[]) Linear terms of the objective as a dense vector. The length is the number of variables. (input)
- cfix (double) Fixed term in the objective. (input)
- aptrb (int[]) Row or column start pointers. (input)
- aptrb (long[]) Row or column start pointers. (input)
- aptre (int[]) Row or column end pointers. (input)
- aptre (long[]) Row or column end pointers. (input)
- asub (int[]) Coefficient subscripts. (input)
- aval (double[]) Coefficient values. (input)
- bkc (boundkey []) Bound keys for the constraints. (input)
- blc (double[]) Lower bounds for the constraints. (input)
- buc (double[]) Upper bounds for the constraints. (input)
- bkx (boundkey []) Bound keys for the variables. (input)
- blx (double[]) Lower bounds for the variables. (input)
- bux (double[]) Upper bounds for the variables. (input)
- numcon (int) Number of constraints. (input)
- numvar (int) Number of variables. (input)

Groups Problem data - linear part, Problem data - bounds, Problem data - constraints

Task.isdouparname

```
void isdouparname
(string parname,
out dparam param)
```

Checks whether parname is a valid double parameter name.

### Parameters

- parname (string) Parameter name. (input)
- param (dparam) Returns the parameter corresponding to the name, if one exists. (output)

Groups Parameters, Names

# Task.isintparname

```
void isintparname
(string parname,
out iparam param)
```

Checks whether parname is a valid integer parameter name.

#### Parameters

- parname (string) Parameter name. (input)
- param (*iparam*) Returns the parameter corresponding to the name, if one exists. (output)

Groups Parameters, Names

# Task.isstrparname

```
void isstrparname
(string parname,
out sparam param)
```

Checks whether parname is a valid string parameter name.

# Parameters

- parname (string) Parameter name. (input)
- param (sparam) Returns the parameter corresponding to the name, if one exists. (output)

Groups Parameters, Names

# Task.linkfiletostream

```
void linkfiletostream
(streamtype whichstream,
string filename,
int append)
```

Directs all output from a task stream whichstream to a file filename.

# Parameters

- whichstream (streamtype) Index of the stream. (input)
- filename (string) A valid file name. (input)
- append (int) If this argument is 0 the output file will be overwritten, otherwise it will be appended to. (input)

Groups Logging

Task.onesolutionsummary

```
void onesolutionsummary
(streamtype whichstream,
soltype whichsol)
```

Prints a short summary of a specified solution.

### Parameters

- whichstream (streamtype) Index of the stream. (input)
- whichsol (soltype) Selects a solution. (input)

**Groups** Logging, Solution information

Task.optimize

```
rescode optimize ()
```

```
void optimize (out rescode trmcode)
```

Calls the optimizer. Depending on the problem type and the selected optimizer this will call one of the optimizers in **MOSEK**. By default the interior point optimizer will be selected for continuous problems. The optimizer may be selected manually by setting the parameter *iparam.optimizer*.

Parameters trmcode (rescode) - Is either rescode.ok or a termination response code. (output)

**Return** (rescode) – Is either rescode.ok or a termination response code.

Groups Optimization

Task.optimizermt

```
void optimizermt
  (string server,
   string port,
  out rescode trmcode)
```

Offload the optimization task to a solver server defined by server:port. The call will block until a result is available or the connection closes.

If the string parameter *sparam.remote\_access\_token* is not blank, it will be passed to the server as authentication.

# Parameters

- server (string) Name or IP address of the solver server. (input)
- port (string) Network port of the solver server. (input)
- trmcode (rescode) Is either rescode.ok or a termination response code. (output)

Groups Remote optimization

Task.optimizersummary

```
void optimizersummary (streamtype whichstream)
```

Prints a short summary with optimizer statistics from last optimization.

```
Parameters whichstream (streamtype) - Index of the stream. (input)
Groups Logging
```

Task.primalrepair

```
void primalrepair
  (double[] wlc,
  double[] wuc,
  double[] wlx,
  double[] wux)
```

The function repairs a primal infeasible optimization problem by adjusting the bounds on the constraints and variables where the adjustment is computed as the minimal weighted sum of relaxations to the bounds on the constraints and variables. Observe the function only repairs the problem but does not solve it. If an optimal solution is required the problem should be optimized after the repair.

The function is applicable to linear and conic problems possibly with integer variables.

Observe that when computing the minimal weighted relaxation the termination tolerance specified by the parameters of the task is employed. For instance the parameter <code>iparam.mio\_mode</code> can be used to make **MOSEK** ignore the integer constraints during the repair which usually leads to a much faster repair. However, the drawback is of course that the repaired problem may not have an integer feasible solution.

Note the function modifies the task in place. If this is not desired, then apply the function to a cloned task.

#### Parameters

- wlc (double[])  $(w_l^c)_i$  is the weight associated with relaxing the lower bound on constraint i. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wuc (double[])  $-(w_u^c)_i$  is the weight associated with relaxing the upper bound on constraint i. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wlx (double[])  $-(w_l^x)_j$  is the weight associated with relaxing the lower bound on variable j. If the weight is negative, then the lower bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)
- wux (double[])  $(w_l^x)_i$  is the weight associated with relaxing the upper bound on variable j. If the weight is negative, then the upper bound is not relaxed. Moreover, if the argument is NULL, then all the weights are assumed to be 1. (input)

**Groups** Infeasibility diagnostic

Task.primalsensitivity

```
void primalsensitivity
  (int[] subi,
  mark[] marki,
  int[] subj,
  mark[] markj,
  double[] leftpricei,
  double[] rightpricei,
  double[] leftrangei,
  double[] rightrangei,
  double[] rightpricej,
  double[] leftpricej,
  double[] rightpricej,
  double[] rightpricej,
  double[] leftrangej,
  double[] rightrangej)
```

```
void primalsensitivity
  (int numi,
   int[] subi,
   mark[] marki,
   int numj,
   int[] subj,
   mark[] markj,
   double[] leftpricei,
   double[] rightpricei,
   double[] rightrangei,
   double[] rightrangei,
   double[] rightpricej,
   double[] leftpricej,
   double[] rightpricej,
   double[] rightpricej,
   double[] rightpricej,
   double[] rightrangej)
```

Calculates sensitivity information for bounds on variables and constraints. For details on sensitivity analysis, the definitions of *shadow price* and *linearity interval* and an example see Section *Sensitivity Analysis*.

The type of sensitivity analysis to be performed (basis or optimal partition) is controlled by the parameter *iparam.sensitivity\_type*.

#### **Parameters**

- subi (int[]) Indexes of constraints to analyze. (input)
- marki (mark []) The value of marki [i] indicates for which bound of constraint subi [i] sensitivity analysis is performed. If marki [i] = mark.up the upper bound of constraint subi [i] is analyzed, and if marki [i] = mark.lo the lower bound is analyzed. If subi [i] is an equality constraint, either mark.lo or mark.up can be used to select the constraint for sensitivity analysis. (input)
- subj (int[]) Indexes of variables to analyze. (input)
- markj (mark[]) The value of markj[j] indicates for which bound of variable subj[j] sensitivity analysis is performed. If markj[j] = mark.up the upper bound of variable subj[j] is analyzed, and if markj[j] = mark.lo the lower bound is analyzed. If subj[j] is a fixed variable, either mark.lo or mark.up can be used to select the bound for sensitivity analysis. (input)
- leftpricei (double[]) leftpricei[i] is the left shadow price for the bound marki[i] of constraint subi[i]. (output)
- rightpricei (double[]) rightpricei[i] is the right shadow price for the bound marki[i] of constraint subi[i]. (output)
- leftrangei (double[]) leftrangei[i] is the left range  $\beta_1$  for the bound marki[i] of constraint subi[i]. (output)
- rightrangei (double[]) rightrangei[i] is the right range  $\beta_2$  for the bound marki[i] of constraint subi[i]. (output)
- leftpricej (double[]) leftpricej[j] is the left shadow price for the bound markj[j] of variable subj[j]. (output)
- rightpricej (double[]) rightpricej[j] is the right shadow price for the bound markj[j] of variable subj[j]. (output)
- leftrangej (double[]) leftrangej[j] is the left range  $\beta_1$  for the bound markj[j] of variable subj[j]. (output)
- rightrangej (double[]) rightrangej[j] is the right range  $\beta_2$  for the bound markj[j] of variable subj[j]. (output)
- numi (int) Number of bounds on constraints to be analyzed. Length of subi and marki. (input)
- numj (int) Number of bounds on variables to be analyzed. Length of subj and markj. (input)

Groups Sensitivity analysis

```
void putacol
  (int j,
   int[] subj,
   double[] valj)
```

```
void putacol
  (int j,
  int nzj,
  int[] subj,
  double[] valj)
```

Change one column of the linear constraint matrix A. Resets all the elements in column j to zero and then sets

$$a_{\mathtt{subj}[k],j} = \mathtt{valj}[k], \quad k = 0, \dots, \mathtt{nzj} - 1.$$

#### **Parameters**

- j (int) Index of a column in A. (input)
- subj (int[]) Row indexes of non-zero values in column j of A. (input)
- valj (double[]) New non-zero values of column j in A. (input)
- nzj (int) Number of non-zeros in column j of A. (input)

Groups Problem data - linear part

Task.putacollist

```
void putacollist
  (int[] sub,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

```
void putacollist
  (int num,
  int[] sub,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

Change a set of columns in the linear constraint matrix A with data in sparse triplet format. The requested columns are set to zero and then updated with:

$$\begin{split} \text{for} \quad i = 0, \dots, num - 1 \\ \quad a_{\texttt{asub}[k], \texttt{sub}[i]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{split}$$

#### **Parameters**

- sub (int[]) Indexes of columns that should be replaced, no duplicates. (input)
- ptrb (long[]) Array of pointers to the first element in each column. (input)
- ptre (long[]) Array of pointers to the last element plus one in each column. (input)
- asub (int[]) Row indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)
- num (int) Number of columns of A to replace. (input)

Groups Problem data - linear part

# Task.putaij

```
void putaij
  (int i,
   int j,
   double aij)
```

Changes a coefficient in the linear coefficient matrix A using the method

$$a_{i,j} = aij.$$

# **Parameters**

- i (int) Constraint (row) index. (input)
- j (int) Variable (column) index. (input)
- aij (double) New coefficient for  $a_{i,j}$ . (input)

Groups Problem data - linear part

Task.putaijlist

```
void putaijlist
  (int[] subi,
  int[] subj,
  double[] valij)
```

```
void putaijlist
  (long num,
  int[] subi,
  int[] subj,
  double[] valij)
```

Changes one or more coefficients in A using the method

$$a_{\mathtt{subi}[\mathtt{k}],\mathtt{subj}[\mathtt{k}]} = \mathtt{valij}[\mathtt{k}], \quad k = 0, \dots, num - 1.$$

Duplicates are not allowed.

#### **Parameters**

- subi (int[]) Constraint (row) indices. (input)
- subj (int[]) Variable (column) indices. (input)
- valij (double[]) New coefficient values for  $a_{i,j}$ . (input)
- num (long) Number of coefficients that should be changed. (input)

Groups Problem data - linear part

Task.putarow

```
void putarow
  (int i,
   int[] subi,
   double[] vali)
```

```
void putarow
  (int i,
   int nzi,
   int[] subi,
   double[] vali)
```

Change one row of the linear constraint matrix A. Resets all the elements in row i to zero and then sets

$$a_{\mathtt{i},\mathtt{subi}[k]} = \mathtt{vali}[k], \quad k = 0, \dots, \mathtt{nzi} - 1.$$

### Parameters

- i (int) Index of a row in A. (input)
- subi (int[]) Column indexes of non-zero values in row i of A. (input)
- vali (double[]) New non-zero values of row i in A. (input)
- nzi (int) Number of non-zeros in row i of A. (input)

Groups Problem data - linear part

# Task.putarowlist

```
void putarowlist
  (int[] sub,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

```
void putarowlist
  (int num,
  int[] sub,
  long[] ptrb,
  long[] ptre,
  int[] asub,
  double[] aval)
```

Change a set of rows in the linear constraint matrix A with data in sparse triplet format. The requested rows are set to zero and then updated with:

$$\label{eq:constraints} \begin{array}{ll} \texttt{for} & i = 0, \dots, num - 1 \\ & a_{\texttt{sub}[i], \texttt{asub}[k]} = \texttt{aval}[k], \quad k = \texttt{ptrb}[i], \dots, \texttt{ptre}[i] - 1. \end{array}$$

### **Parameters**

- sub (int[]) Indexes of rows that should be replaced, no duplicates. (input)
- ptrb (long[]) Array of pointers to the first element in each row. (input)
- ptre (long[]) Array of pointers to the last element plus one in each row. (input)
- asub (int[]) Column indexes of new elements. (input)
- aval (double[]) Coefficient values. (input)
- num (int) Number of rows of A to replace. (input)

Groups Problem data - linear part

#### Task.putatruncatetol

```
void putatruncatetol (double tolzero)
```

Truncates (sets to zero) all elements in A that satisfy

$$|a_{i,j}| \leq \text{tolzero}.$$

Parameters tolzero (double) - Truncation tolerance. (input) Groups Problem data - linear part

Task.putbarablocktriplet

```
void putbarablocktriplet
  (long num,
   int[] subi,
   int[] subj,
   int[] subk,
   int[] subl,
   double[] valijkl)
```

Inputs the  $\overline{A}$  matrix in block triplet form.

### **Parameters**

- num (long) Number of elements in the block triplet form. (input)
- subi (int[]) Constraint index. (input)
- subj (int[]) Symmetric matrix variable index. (input)
- subk (int[]) Block row index. (input)
- subl (int[]) Block column index. (input)
- valijkl (double[]) The numerical value associated with each block triplet. (input)

Groups Problem data - semidefinite

Task.putbaraij

```
void putbaraij
  (int i,
   int j,
   long[] sub,
   double[] weights)
```

This function sets one element in the  $\overline{A}$  matrix.

Each element in the  $\overline{A}$  matrix is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{A}_{ij}$  is a symmetric matrix. By default all elements in  $\overline{A}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function Task. appendsparsesymmat.

# Parameters

- i (int) Row index of  $\overline{A}$ . (input)
- j (int) Column index of  $\overline{A}$ . (input)
- sub (long[]) Indices in E of the matrices appearing in the weighted sum for  $\overline{A}_{ij}$ . (input)
- weights (double[]) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{A}_{ij}$ . (input)

**Groups** Problem data - semidefinite

Task.putbaraijlist

```
void putbaraijlist
  (int[] subi,
   int[] subj,
  long[] alphaptrb,
  long[] alphaptre,
  long[] matidx,
  double[] weights)
```

This function sets a list of elements in the  $\overline{A}$  matrix.

Each element in the  $\overline{A}$  matrix is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{A}_{ij}$  is a symmetric matrix. By default all elements in  $\overline{A}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function Task. appendsparsesymmat.

#### **Parameters**

- subi (int[]) Row index of  $\overline{A}$ . (input)
- subj (int[]) Column index of  $\overline{A}$ . (input)
- alphaptrb (long[]) Start entries for terms in the weighted sum that forms  $\overline{A}_{ij}$ . (input)
- alphaptre (long[]) End entries for terms in the weighted sum that forms  $\overline{A}_{ij}$ . (input)
- matidx (long[]) Indices in E of the matrices appearing in the weighted sum for  $\overline{A}_{ij}$ . (input)
- weights (double[]) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{A}_{ij}$ . (input)

Groups Problem data - semidefinite

# Task.putbararowlist

```
void putbararowlist
  (int[] subi,
  long[] ptrb,
  long[] ptre,
  int[] subj,
  long[] nummat,
  long[] matidx,
  double[] weights)
```

This function replaces a list of rows in the  $\overline{A}$  matrix.

### Parameters

- subi (int[]) Row indexes of  $\overline{A}$ . (input)
- ptrb (long[]) Start of rows in  $\overline{A}$ . (input)
- ptre (long[]) End of rows in  $\overline{A}$ . (input)
- subj (int[]) Column index of  $\overline{A}$ . (input)
- nummat (long[]) Number of entries in weighted sum of matrixes. (input)
- matidx (long[]) Matrix indexes for weighted sum of matrixes. (input)
- weights (double[]) Weights for weighted sum of matrixes. (input)

Groups Problem data - semidefinite

# Task.putbarcblocktriplet

```
void putbarcblocktriplet
  (long num,
  int[] subj,
  int[] subk,
  int[] subl,
  double[] valjkl)
```

Inputs the  $\overline{C}$  matrix in block triplet form.

# Parameters

- num (long) Number of elements in the block triplet form. (input)
- subj (int[]) Symmetric matrix variable index. (input)
- subk (int[]) Block row index. (input)
- subl (int[]) Block column index. (input)
- valjkl (double[]) The numerical value associated with each block triplet. (input)

# Groups Problem data - semidefinite

Task.putbarcj

```
void putbarcj
  (int j,
  long[] sub,
  double[] weights)
```

This function sets one entry in the  $\overline{C}$  vector.

Each element in the  $\overline{C}$  vector is a weighted sum of symmetric matrices from the symmetric matrix storage E, so  $\overline{C}_j$  is a symmetric matrix. By default all elements in  $\overline{C}$  are 0, so only non-zero elements need be added. Setting the same element again will overwrite the earlier entry.

The symmetric matrices from E are defined separately using the function Task. appendsparsesymmat.

#### **Parameters**

- j (int) Index of the element in  $\overline{C}$  that should be changed. (input)
- sub (long[]) Indices in E of matrices appearing in the weighted sum for  $\overline{C}_j$  (input)
- weights (double[]) weights [k] is the coefficient of the sub[k]-th element of E in the weighted sum forming  $\overline{C}_i$ . (input)

Groups Problem data - semidefinite, Problem data - objective

Task.putbarsj

```
void putbarsj
  (soltype whichsol,
  int j,
  double[] barsj)
```

Sets the dual solution for a semidefinite variable.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barsj (double[]) Value of  $\overline{S}_i$ . Format as in Task. getbarsj. (input)

Groups Solution - semidefinite

Task.putbarvarname

```
void putbarvarname
(int j,
string name)
```

Sets the name of a semidefinite variable.

# Parameters

- j (int) Index of the variable. (input)
- name (string) The variable name. (input)

Groups Names, Problem data - semidefinite

Task.putbarxj

```
void putbarxj
  (soltype whichsol,
   int j,
   double[] barxj)
```

Sets the primal solution for a semidefinite variable.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- j (int) Index of the semidefinite variable. (input)
- barxj (double[]) Value of  $\overline{X}_j$ . Format as in Task. getbarxj. (input)

Groups Solution - semidefinite

Task.putcfix

```
void putcfix (double cfix)
```

Replaces the fixed term in the objective by a new one.

```
Parameters cfix (double) – Fixed term in the objective. (input)
Groups Problem data - linear part, Problem data - objective
```

Task.putcj

```
void putcj
  (int j,
  double cj)
```

Modifies one coefficient in the linear objective vector c, i.e.

$$c_{i} = c_{j}$$
.

If the absolute value exceeds  $dparam.data\_tol\_c\_huge$  an error is generated. If the absolute value exceeds  $dparam.data\_tol\_cj\_large$ , a warning is generated, but the coefficient is inputted as specified.

# Parameters

- j (int) Index of the variable for which c should be changed. (input)
- cj (double) New value of  $c_i$ . (input)

Groups Problem data - linear part, Problem data - objective

Task.putclist

```
void putclist
  (int[] subj,
  double[] val)
```

```
void putclist
  (int num,
  int[] subj,
  double[] val)
```

Modifies the coefficients in the linear term c in the objective using the principle

$$c_{\mathtt{subi[t]}} = \mathtt{val[t]}, \quad t = 0, \dots, num - 1.$$

If a variable index is specified multiple times in subj only the last entry is used. Data checks are performed as in *Task.putcj*.

# Parameters

- subj (int[]) Indices of variables for which the coefficient in c should be changed. (input)
- val (double[]) New numerical values for coefficients in c that should be modified. (input)

• num (int) - Number of coefficients that should be changed. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - objective

Task.putconbound

```
void putconbound
  (int i,
   boundkey bkc,
   double blc,
   double buc)
```

Changes the bounds for one constraint.

If the bound value specified is numerically larger than  $dparam.data\_tol\_bound\_inf$  it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than  $dparam.data\_tol\_bound\_wrn$ , a warning will be displayed, but the bound is inputted as specified.

#### **Parameters**

- i (int) Index of the constraint. (input)
- bkc (boundkey) New bound key. (input)
- blc (double) New lower bound. (input)
- buc (double) New upper bound. (input)

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

Task.putconboundlist

```
void putconboundlist
  (int[] sub,
  boundkey[] bkc,
  double[] blc,
  double[] buc)
```

```
void putconboundlist
  (int num,
  int[] sub,
  boundkey[] bkc,
  double[] blc,
  double[] buc)
```

Changes the bounds for a list of constraints. If multiple bound changes are specified for a constraint, then only the last change takes effect. Data checks are performed as in *Task.putconbound*.

### Parameters

- sub (int[]) List of constraint indexes. (input)
- bkc (boundkey []) Bound keys for the constraints. (input)
- blc (double[]) Lower bounds for the constraints. (input)
- buc (double[]) Upper bounds for the constraints. (input)
- num (int) Number of bounds that should be changed. (input)

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

Task.putconboundlistconst

```
void putconboundlistconst
  (int[] sub,
  boundkey bkc,
  double blc,
  double buc)
```

```
void putconboundlistconst
  (int num,
  int[] sub,
  boundkey bkc,
  double blc,
  double buc)
```

Changes the bounds for one or more constraints. Data checks are performed as in Task. putconbound.

#### **Parameters**

- sub (int[]) List of constraint indexes. (input)
- bkc (boundkey) New bound key for all constraints in the list. (input)
- blc (double) New lower bound for all constraints in the list. (input)
- buc (double) New upper bound for all constraints in the list. (input)
- num (int) Number of bounds that should be changed. (input)

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

# Task.putconboundslice

```
void putconboundslice
  (int first,
   int last,
   boundkey[] bkc,
   double[] blc,
   double[] buc)
```

Changes the bounds for a slice of the constraints. Data checks are performed as in Task. putconbound.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bkc (boundkey []) Bound keys for the constraints. (input)
- blc (double[]) Lower bounds for the constraints. (input)
- buc (double[]) Upper bounds for the constraints. (input)

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

# Task.putconboundsliceconst

```
void putconboundsliceconst
  (int first,
   int last,
  boundkey bkc,
  double blc,
  double buc)
```

Changes the bounds for a slice of the constraints. Data checks are performed as in Task. putconbound.

### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bkc (boundkey) New bound key for all constraints in the slice. (input)
- blc (double) New lower bound for all constraints in the slice. (input)
- buc (double) New upper bound for all constraints in the slice. (input)

Groups Problem data - linear part, Problem data - constraints, Problem data - bounds

# Task.putcone

```
void putcone
  (int k,
   conetype ct,
   double conepar,
   int[] submem)
```

```
void putcone
  (int k,
  conetype ct,
  double conepar,
  int nummem,
  int[] submem)
```

Replaces a conic constraint.

### Parameters

- k (int) Index of the cone. (input)
- ct (conetype) Specifies the type of the cone. (input)
- conepar (double) For the power cone it denotes the exponent alpha. For other cone types it is unused and can be set to 0. (input)
- submem (int[]) Variable subscripts of the members in the cone. (input)
- nummem (int) Number of member variables in the cone. (input)

Groups Problem data - cones

### Task.putconename

```
void putconename
  (int j,
    string name)
```

Sets the name of a cone.

#### **Parameters**

- j (int) Index of the cone. (input)
- name (string) The name of the cone. (input)

Groups Names, Problem data - cones

### Task.putconname

```
void putconname
  (int i,
   string name)
```

Sets the name of a constraint.

#### **Parameters**

- i (int) Index of the constraint. (input)
- name (string) The name of the constraint. (input)

Groups Names, Problem data - constraints, Problem data - linear part

Task.putconsolutioni

```
void putconsolutioni
  (int i,
    soltype whichsol,
    stakey sk,
    double x,
    double sl,
    double su)
```

Sets the primal and dual solution information for a single constraint.

# Parameters

- i (int) Index of the constraint. (input)
- whichsol (soltype) Selects a solution. (input)
- sk (stakey) Status key of the constraint. (input)
- x (double) Primal solution value of the constraint. (input)
- sl (double) Solution value of the dual variable associated with the lower bound. (input)
- su (double) Solution value of the dual variable associated with the upper bound. (input)

Groups Solution information, Solution - primal, Solution - dual

Task.putcslice

```
void putcslice
  (int first,
   int last,
   double[] slice)
```

Modifies a slice in the linear term c in the objective using the principle

$$c_{i} = slice[j - first], \quad j = first, ..., last - 1$$

Data checks are performed as in Task.putcj.

### **Parameters**

- first (int) First element in the slice of c. (input)
- last (int) Last element plus 1 of the slice in c to be changed. (input)
- slice (double[]) New numerical values for coefficients in c that should be modified. (input)

Groups Problem data - linear part, Problem data - objective

Task.putdouparam

```
void putdouparam
(dparam param,
double parvalue)
```

Sets the value of a double parameter.

#### **Parameters**

- param (dparam) Which parameter. (input)
- parvalue (double) Parameter value. (input)

Groups Parameters

Task.putintparam

```
void putintparam
(iparam param,
int parvalue)
```

Sets the value of an integer parameter.

#### **Parameters**

- param (*iparam*) Which parameter. (input)
- parvalue (int) Parameter value. (input)

Groups Parameters

Task.putmaxnumanz

```
void putmaxnumanz (long maxnumanz)
```

Sets the number of preallocated non-zero entries in A.

**MOSEK** stores only the non-zero elements in the linear coefficient matrix A and it cannot predict how much storage is required to store A. Using this function it is possible to specify the number of non-zeros to preallocate for storing A.

If the number of non-zeros in the problem is known, it is a good idea to set maxnumanz slightly larger than this number, otherwise a rough estimate can be used. In general, if A is inputted in many small chunks, setting this value may speed up the data input phase.

It is not mandatory to call this function, since **MOSEK** will reallocate internal structures whenever it is necessary.

The function call has no effect if both maxnumcon and maxnumvar are zero.

Parameters maxnumanz (long) – Number of preallocated non-zeros in A. (input) Groups Environment and task management, Problem data - semidefinite

Task.putmaxnumbarvar

```
void putmaxnumbarvar (int maxnumbarvar)
```

Sets the number of preallocated symmetric matrix variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumbarvar must be larger than the current number of symmetric matrix variables in the task.

Parameters maxnumbarvar (int) – Number of preallocated symmetric matrix variables. (input)

Groups Environment and task management, Problem data - semidefinite

Task.putmaxnumcon

```
void putmaxnumcon (int maxnumcon)
```

Sets the number of preallocated constraints in the optimization task. When this number of constraints is reached **MOSEK** will automatically allocate more space for constraints.

It is never mandatory to call this function, since MOSEK will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of constraints in the task.

Parameters maxnumcon (int) – Number of preallocated constraints in the optimization task. (input)

### Groups Environment and task management, Problem data - constraints

#### Task.putmaxnumcone

```
void putmaxnumcone (int maxnumcone)
```

Sets the number of preallocated conic constraints in the optimization task. When this number of conic constraints is reached **MOSEK** will automatically allocate more space for conic constraints.

It is not mandatory to call this function, since **MOSEK** will reallocate any internal structures whenever it is required.

Please note that maxnumcon must be larger than the current number of conic constraints in the task.

Parameters maxnumcone (int) – Number of preallocated conic constraints in the optimization task. (input)

**Groups** Environment and task management, Problem data - cones

#### Task.putmaxnumqnz

```
void putmaxnumqnz (long maxnumqnz)
```

Sets the number of preallocated non-zero entries in quadratic terms.

**MOSEK** stores only the non-zero elements in Q. Therefore, **MOSEK** cannot predict how much storage is required to store Q. Using this function it is possible to specify the number non-zeros to preallocate for storing Q (both objective and constraints).

It may be advantageous to reserve more non-zeros for Q than actually needed since it may improve the internal efficiency of  $\mathbf{MOSEK}$ , however, it is never worthwhile to specify more than the double of the anticipated number of non-zeros in Q.

It is not mandatory to call this function, since  $\mathbf{MOSEK}$  will reallocate internal structures whenever it is necessary.

Parameters maxnumqnz (long) - Number of non-zero elements preallocated in quadratic coefficient matrices. (input)

Groups Environment and task management, Problem data - quadratic part

# Task.putmaxnumvar

```
void putmaxnumvar (int maxnumvar)
```

Sets the number of preallocated variables in the optimization task. When this number of variables is reached **MOSEK** will automatically allocate more space for variables.

It is not mandatory to call this function. It only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that maxnumvar must be larger than the current number of variables in the task.

Parameters maxnumvar (int) – Number of preallocated variables in the optimization task. (input)

**Groups** Environment and task management, Problem data - variables

### Task.putnadouparam

```
void putnadouparam
  (string paramname,
   double parvalue)
```

Sets the value of a named double parameter.

#### **Parameters**

- paramname (string) Name of a parameter. (input)
- parvalue (double) Parameter value. (input)

Groups Parameters

Task.putnaintparam

```
void putnaintparam
(string paramname,
int parvalue)
```

Sets the value of a named integer parameter.

### Parameters

- paramname (string) Name of a parameter. (input)
- parvalue (int) Parameter value. (input)

Groups Parameters

Task.putnastrparam

```
void putnastrparam
(string paramname,
string parvalue)
```

Sets the value of a named string parameter.

#### Parameters

- paramname (string) Name of a parameter. (input)
- parvalue (string) Parameter value. (input)

Groups Parameters

Task.putobjname

```
void putobjname (string objname)
```

Assigns a new name to the objective.

```
Parameters objname (string) - Name of the objective. (input)
Groups Problem data - linear part, Names, Problem data - objective
```

Task.putobjsense

```
void putobjsense (objsense sense)
```

Sets the objective sense of the task.

Parameters sense (objsense) – The objective sense of the task. The values objsense.

maximize and objsense.minimize mean that the problem is maximized or minimized respectively. (input)

Groups Problem data - linear part, Problem data - objective

Task.putparam

```
void putparam
(string parname,
string parvalue)
```

Checks if parname is valid parameter name. If it is, the parameter is assigned the value specified by parvalue.

#### **Parameters**

- $\bullet \ \mathtt{parname} \ (\mathtt{string}) \mathrm{Parameter} \ \mathrm{name}. \ (\mathrm{input})$
- parvalue (string) Parameter value. (input)

Groups Parameters

Task.putqcon

```
void putqcon
  (int[] qcsubk,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

```
void putqcon
  (int numqcnz,
   int[] qcsubk,
   int[] qcsubi,
   int[] qcsubj,
   double[] qcval)
```

Replace all quadratic entries in the constraints. The list of constraints has the form

$$l_k^c \le \frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^k x_i x_j + \sum_{j=0}^{numvar-1} a_{kj} x_j \le u_k^c, \quad k = 0, \dots, m-1.$$

This function sets all the quadratic terms to zero and then performs the update:

$$q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubj}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} = q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubi}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} = q_{\mathtt{qcsubh}[\mathtt{t}],\mathtt{qcsubi}[\mathtt{t}]}^{\mathtt{qcsubk}[\mathtt{t}]} + \mathtt{qcval}[\mathtt{t}],$$

for  $t = 0, \dots, numqcnz - 1$ .

Please note that:

- For large problems it is essential for the efficiency that the function *Task.putmaxnumqnz* is employed to pre-allocate space.
- Only the lower triangular parts should be specified because the Q matrices are symmetric. Specifying entries where i < j will result in an error.
- Only non-zero elements should be specified.
- The order in which the non-zero elements are specified is insignificant.
- Duplicate elements are added together as shown above. Hence, it is usually not recommended to specify the same entry multiple times.

For a code example see Section Quadratic Optimization

# Parameters

- qcsubk (int[]) Constraint subscripts for quadratic coefficients. (input)
- qcsubi (int[]) Row subscripts for quadratic constraint matrix. (input)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (input)
- qcval (double[]) Quadratic constraint coefficient values. (input)
- numqcnz (int) Number of quadratic terms. (input)

Groups Problem data - quadratic part

Task.putqconk

```
void putqconk
  (int k,
  int[] qcsubi,
  int[] qcsubj,
  double[] qcval)
```

```
void putqconk
  (int k,
   int numqcnz,
   int[] qcsubi,
   int[] qcsubj,
   double[] qcval)
```

Replaces all the quadratic entries in one constraint. This function performs the same operations as *Task.putqcon* but only with respect to constraint number **k** and it does not modify the other constraints. See the description of *Task.putqcon* for definitions and important remarks.

# Parameters

- k (int) The constraint in which the new Q elements are inserted. (input)
- qcsubi (int[]) Row subscripts for quadratic constraint matrix. (input)
- qcsubj (int[]) Column subscripts for quadratic constraint matrix. (input)
- qcval (double[]) Quadratic constraint coefficient values. (input)
- numqcnz (int) Number of quadratic terms. (input)

Groups Problem data - quadratic part

Task.putqobj

```
void putqobj
(int[] qosubi,
int[] qosubj,
double[] qoval)
```

```
void putqobj
  (int numqonz,
   int[] qosubi,
   int[] qosubj,
   double[] qoval)
```

Replace all quadratic terms in the objective. If the objective has the form

$$\frac{1}{2} \sum_{i=0}^{numvar-1} \sum_{j=0}^{numvar-1} q_{ij}^{o} x_i x_j + \sum_{j=0}^{numvar-1} c_j x_j + c^f$$

then this function sets all the quadratic terms to zero and then performs the update:

$$q^o_{\texttt{qosubi[t]},\texttt{qosubj[t]}} = q^o_{\texttt{qosubj[t]},\texttt{qosubi[t]}} = q^o_{\texttt{qosubj[t]},\texttt{qosubi[t]}} + \texttt{qoval[t]},$$

for  $t = 0, \ldots, numgon z - 1$ .

See the description of Task.putqcon for important remarks and example.

# Parameters

- qosubi (int[]) Row subscripts for quadratic objective coefficients. (input)
- qosubj (int[]) Column subscripts for quadratic objective coefficients. (input)
- qoval (double[]) Quadratic objective coefficient values. (input)
- numqonz (int) Number of non-zero elements in the quadratic objective terms. (input)

Groups Problem data - quadratic part, Problem data - objective

Task.putqobjij

```
void putqobjij
  (int i,
  int j,
  double qoij)
```

Replaces one coefficient in the quadratic term in the objective. The function performs the assignment

$$q_{ij}^o = q_{ji}^o = \text{qoij}.$$

Only the elements in the lower triangular part are accepted. Setting  $q_{ij}$  with j > i will cause an error.

Please note that replacing all quadratic elements one by one is more computationally expensive than replacing them all at once. Use Task. putqobj instead whenever possible.

## **Parameters**

- i (int) Row index for the coefficient to be replaced. (input)
- j (int) Column index for the coefficient to be replaced. (input)
- qoij (double) The new value for  $q_{ij}^o$ . (input)

Groups Problem data - quadratic part, Problem data - objective

Task.putskc

```
void putskc
  (soltype whichsol,
  stakey[] skc)
```

Sets the status keys for the constraints.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- skc (stakey []) Status keys for the constraints. (input)

Groups Solution information

Task.putskcslice

```
void putskcslice
  (soltype whichsol,
  int first,
  int last,
  stakey[] skc)
```

Sets the status keys for a slice of the constraints.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skc (stakey []) Status keys for the constraints. (input)

Groups Solution information

Task.putskx

```
void putskx
  (soltype whichsol,
   stakey[] skx)
```

Sets the status keys for the scalar variables.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- skx (stakey[]) Status keys for the variables. (input)

**Groups** Solution information

Task.putskxslice

```
void putskxslice
  (soltype whichsol,
  int first,
  int last,
  stakey[] skx)
```

Sets the status keys for a slice of the variables.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- skx (stakey[]) Status keys for the variables. (input)

Groups Solution information

Task.putslc

```
void putslc
  (soltype whichsol,
  double[] slc)
```

Sets the  $s_l^c$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)

Groups Solution - dual

Task.putslcslice

```
void putslcslice
  (soltype whichsol,
  int first,
  int last,
  double[] slc)
```

Sets a slice of the  $s_l^c$  vector for a solution.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)

# Groups Solution - dual

 ${\tt Task.putslx}$ 

```
void putslx
  (soltype whichsol,
  double[] slx)
```

Sets the  $s_l^x$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)

Groups Solution - dual

Task.putslxslice

```
void putslxslice
  (soltype whichsol,
  int first,
  int last,
  double[] slx)
```

Sets a slice of the  $s_I^x$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)

Groups Solution - dual

Task.putsnx

```
void putsnx
(soltype whichsol,
double[] sux)
```

Sets the  $s_n^x$  vector for a solution.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution - dual

Task.putsnxslice

```
void putsnxslice
  (soltype whichsol,
  int first,
  int last,
  double[] snx)
```

Sets a slice of the  $s_n^x$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (input)

Groups Solution - dual

Task.putsolution

```
void putsolution
  (soltype whichsol,
   stakey[] skc,
   stakey[] skx,
   stakey[] skn,
   double[] xc,
   double[] xx,
   double[] y,
   double[] slc,
   double[] suc,
   double[] slx,
   double[] slx,
   double[] slx,
   double[] slx,
   double[] slx,
```

Inserts a solution into the task.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- skc (stakey []) Status keys for the constraints. (input)
- skx (stakey[]) Status keys for the variables. (input)
- skn (stakey []) Status keys for the conic constraints. (input)
- xc (double[]) Primal constraint solution. (input)
- xx (double[]) Primal variable solution. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (input)
- slc (double[]) Dual variables corresponding to the lower bounds on the constraints. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)
- slx (double[]) Dual variables corresponding to the lower bounds on the variables. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)
- snx (double[]) Dual variables corresponding to the conic constraints on the variables. (input)

Groups Solution information, Solution - primal, Solution - dual

Task.putsolutionyi

```
void putsolutionyi
  (int i,
   soltype whichsol,
   double y)
```

Inputs the dual variable of a solution.

# **Parameters**

• i (int) – Index of the dual variable. (input)

```
• whichsol (soltype) - Selects a solution. (input)
```

• y (double) - Solution value of the dual variable. (input)

Groups Solution information, Solution - dual

Task.putstrparam

```
void putstrparam
(sparam param,
string parvalue)
```

Sets the value of a string parameter.

#### **Parameters**

- param (sparam) Which parameter. (input)
- parvalue (string) Parameter value. (input)

Groups Parameters

Task.putsuc

```
void putsuc
  (soltype whichsol,
   double[] suc)
```

Sets the  $s_u^c$  vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)

Groups Solution - dual

Task.putsucslice

```
void putsucslice
  (soltype whichsol,
  int first,
  int last,
  double[] suc)
```

Sets a slice of the  $s_u^c$  vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- suc (double[]) Dual variables corresponding to the upper bounds on the constraints. (input)

Groups Solution - dual

Task.putsux

```
void putsux
(soltype whichsol,
double[] sux)
```

Sets the  $s_u^x$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution - dual

Task.putsuxslice

```
void putsuxslice
  (soltype whichsol,
  int first,
  int last,
  double[] sux)
```

Sets a slice of the  $s_u^x$  vector for a solution.

### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- sux (double[]) Dual variables corresponding to the upper bounds on the variables. (input)

Groups Solution - dual

Task.puttaskname

```
void puttaskname (string taskname)
```

Assigns a new name to the task.

Parameters taskname (string) – Name assigned to the task. (input) Groups Names, Environment and task management

Task.putvarbound

```
void putvarbound
  (int j,
   boundkey bkx,
   double blx,
   double bux)
```

Changes the bounds for one variable.

If the bound value specified is numerically larger than  $dparam.data\_tol\_bound\_inf$  it is considered infinite and the bound key is changed accordingly. If a bound value is numerically larger than  $dparam.data\_tol\_bound\_wrn$ , a warning will be displayed, but the bound is inputted as specified.

#### **Parameters**

- j (int) Index of the variable. (input)
- bkx (boundkey) New bound key. (input)
- blx (double) New lower bound. (input)
- bux (double) New upper bound. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

 ${\tt Task.putvarboundlist}$ 

```
void putvarboundlist
  (int[] sub,
  boundkey[] bkx,
  double[] blx,
  double[] bux)
```

```
void putvarboundlist
  (int num,
  int[] sub,
  boundkey[] bkx,
  double[] blx,
  double[] bux)
```

Changes the bounds for one or more variables. If multiple bound changes are specified for a variable, then only the last change takes effect. Data checks are performed as in *Task.putvarbound*.

#### **Parameters**

- sub (int[]) List of variable indexes. (input)
- bkx (boundkey []) Bound keys for the variables. (input)
- blx (double[]) Lower bounds for the variables. (input)
- bux (double[]) Upper bounds for the variables. (input)
- num (int) Number of bounds that should be changed. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

# Task.putvarboundlistconst

```
void putvarboundlistconst
  (int[] sub,
  boundkey bkx,
  double blx,
  double bux)
```

```
void putvarboundlistconst
  (int num,
  int[] sub,
  boundkey bkx,
  double blx,
  double bux)
```

Changes the bounds for one or more variables. Data checks are performed as in Task. putvarbound.

#### **Parameters**

- sub (int[]) List of variable indexes. (input)
- bkx (boundkey) New bound key for all variables in the list. (input)
- blx (double) New lower bound for all variables in the list. (input)
- bux (double) New upper bound for all variables in the list. (input)
- num (int) Number of bounds that should be changed. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

### Task.putvarboundslice

```
void putvarboundslice
  (int first,
   int last,
   boundkey[] bkx,
   double[] blx,
   double[] bux)
```

Changes the bounds for a slice of the variables. Data checks are performed as in Task. putvarbound.

#### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bkx (boundkey []) Bound keys for the variables. (input)
- blx (double[]) Lower bounds for the variables. (input)
- bux (double[]) Upper bounds for the variables. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

### Task.putvarboundsliceconst

```
void putvarboundsliceconst
  (int first,
   int last,
  boundkey bkx,
  double blx,
  double bux)
```

Changes the bounds for a slice of the variables. Data checks are performed as in Task. putvarbound.

### **Parameters**

- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- bkx (boundkey) New bound key for all variables in the slice. (input)
- blx (double) New lower bound for all variables in the slice. (input)
- bux (double) New upper bound for all variables in the slice. (input)

Groups Problem data - linear part, Problem data - variables, Problem data - bounds

# Task.putvarname

```
void putvarname
  (int j,
   string name)
```

Sets the name of a variable.

# Parameters

- j (int) Index of the variable. (input)
- name (string) The variable name. (input)

Groups Names, Problem data - variables, Problem data - linear part

# Task.putvarsolutionj

```
void putvarsolutionj
  (int j,
    soltype whichsol,
    stakey sk,
    double x,
    double sl,
    double su,
    double sn)
```

Sets the primal and dual solution information for a single variable.

#### **Parameters**

- j (int) Index of the variable. (input)
- whichsol (soltype) Selects a solution. (input)
- sk (stakey) Status key of the variable. (input)
- x (double) Primal solution value of the variable. (input)
- $\bullet\,$  s1 (double) Solution value of the dual variable associated with the lower bound. (input)
- su (double) Solution value of the dual variable associated with the upper bound. (input)
- sn (double) Solution value of the dual variable associated with the conic constraint. (input)

Groups Solution information, Solution - primal, Solution - dual

#### Task.putvartype

```
void putvartype
  (int j,
  variabletype vartype)
```

Sets the variable type of one variable.

#### **Parameters**

- j (int) Index of the variable. (input)
- vartype (variable type) The new variable type. (input)

Groups Problem data - variables

Task.putvartypelist

```
void putvartypelist
  (int[] subj,
  variabletype[] vartype)
```

```
void putvartypelist
  (int num,
  int[] subj,
  variabletype[] vartype)
```

Sets the variable type for one or more variables. If the same index is specified multiple times in subj only the last entry takes effect.

# Parameters

- subj (int[]) A list of variable indexes for which the variable type should be changed. (input)
- vartype (variable type []) A list of variable types that should be assigned to the variables specified by subj. (input)
- num (int) Number of variables for which the variable type should be set. (input)

Groups Problem data - variables

#### Task.putxc

```
void putxc
  (soltype whichsol,
  double[] xc)
```

Sets the  $x^c$  vector for a solution.

### **Parameters**

```
• whichsol (soltype) - Selects a solution. (input)
```

• xc (double[]) - Primal constraint solution. (output)

Groups Solution - primal

Task.putxcslice

```
void putxcslice
  (soltype whichsol,
  int first,
  int last,
  double[] xc)
```

Sets a slice of the  $x^c$  vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xc (double[]) Primal constraint solution. (input)

Groups Solution - primal

Task.putxx

```
void putxx
  (soltype whichsol,
  double[] xx)
```

Sets the  $x^x$  vector for a solution.

# **Parameters**

- ullet whichsol (soltype) Selects a solution. (input)
- xx (double[]) Primal variable solution. (input)

Groups Solution - primal

Task.putxxslice

```
void putxxslice
  (soltype whichsol,
  int first,
  int last,
  double[] xx)
```

Sets a slice of the  $x^x$  vector for a solution.

# **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- xx (double[]) Primal variable solution. (input)

Groups Solution - primal

Task.puty

```
void puty
  (soltype whichsol,
  double[] y)
```

Sets the y vector for a solution.

#### Parameters

- whichsol (soltype) Selects a solution. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (input)

Groups Solution - primal

Task.putyslice

```
void putyslice
  (soltype whichsol,
  int first,
  int last,
  double[] y)
```

Sets a slice of the y vector for a solution.

#### **Parameters**

- whichsol (soltype) Selects a solution. (input)
- first (int) First index in the sequence. (input)
- last (int) Last index plus 1 in the sequence. (input)
- y (double[]) Vector of dual variables corresponding to the constraints. (input)

Groups Solution - dual

Task.readdata

```
void readdata (string filename)
```

Reads an optimization problem and associated data from a file.

```
Parameters filename (string) – A valid file name. (input)
Groups Input/Output
```

Task.readdataformat

```
void readdataformat
  (string filename,
  int format,
  int compress)
```

Reads an optimization problem and associated data from a file.

#### Parameters

- filename (string) A valid file name. (input)
- format (int) File data format. (input)
- compress (int) File compression type. (input)

Groups Input/Output

Task.readjsonstring

```
void readjsonstring (string data)
```

Load task data from a JSON string, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the string contains solutions, the solution status after loading a file is set to unknown, even if it is optimal or otherwise well-defined.

```
Parameters data (string) – Problem data in text format. (input)
Groups Input/Output
```

### Task.readlpstring

```
void readlpstring (string data)
```

Load task data from a string in LP format, replacing any data that already exists in the task object.

Parameters data (string) – Problem data in text format. (input) Groups Input/Output

Task.readopfstring

```
void readopfstring (string data)
```

Load task data from a string in OPF format, replacing any data that already exists in the task object.

Parameters data (string) – Problem data in text format. (input) Groups Input/Output

Task.readparamfile

```
void readparamfile (string filename)
```

Reads **MOSEK** parameters from a file. Data is read from the file filename if it is a nonempty string. Otherwise data is read from the file specified by <code>sparam.param\_read\_file\_name</code>.

Parameters filename (string) – A valid file name. (input) Groups Input/Output

Task.readptfstring

```
void readptfstring (string data)
```

Load task data from a PTF string, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the string contains solutions, the solution status after loading a file is set to unknown, even if it is optimal or otherwise well-defined.

Parameters data (string) – Problem data in text format. (input) Groups Input/Output

Task.readsolution

```
void readsolution
(soltype whichsol,
string filename)
```

Reads a solution file and inserts it as a specified solution in the task. Data is read from the file filename if it is a nonempty string. Otherwise data is read from one of the files specified by sparam.  $bas\_sol\_file\_name$ ,  $sparam.itr\_sol\_file\_name$  or  $sparam.int\_sol\_file\_name$  depending on which solution is chosen.

### Parameters

- whichsol (soltype) Selects a solution. (input)
- filename (string) A valid file name. (input)

Groups Input/Output

#### Task.readsummary

```
void readsummary (streamtype whichstream)
```

Prints a short summary of last file that was read.

```
Parameters whichstream (streamtype) - Index of the stream. (input)
Groups Input/Output, Inspecting the task
```

Task.readtask

```
void readtask (string filename)
```

Load task data from a file, replacing any data that already exists in the task object. All problem data, parameters and other settings are resorted, but if the file contains solutions, the solution status after loading a file is set to unknown, even if it was optimal or otherwise well-defined when the file was dumped.

See section The Task Format for a description of the Task format.

```
Parameters filename (string) – A valid file name. (input)
Groups Input/Output
```

Task.removebarvars

```
void removebarvars (int[] subset)
```

```
void removebarvars
  (int num,
  int[] subset)
```

The function removes a subset of the symmetric matrices from the optimization task. This implies that the remaining symmetric matrices are renumbered.

### **Parameters**

- subset (int[]) Indexes of symmetric matrices which should be removed. (input)
- num (int) Number of symmetric matrices which should be removed. (input)

Groups Problem data - semidefinite

Task.removecones

```
void removecones (int[] subset)
```

Removes a number of conic constraints from the problem. This implies that the remaining conic constraints are renumbered. In general, it is much more efficient to remove a cone with a high index than a low index.

```
Parameters subset (int[]) – Indexes of cones which should be removed. (input) Groups Problem data - cones
```

Task.removecons

```
void removecons (int[] subset)
```

```
void removecons
  (int num,
  int[] subset)
```

The function removes a subset of the constraints from the optimization task. This implies that the remaining constraints are renumbered.

#### **Parameters**

- subset (int[]) Indexes of constraints which should be removed. (input)
- num (int) Number of constraints which should be removed. (input)

Groups Problem data - constraints, Problem data - linear part

#### Task.removevars

```
void removevars (int[] subset)

void removevars
  (int num,
  int[] subset)
```

The function removes a subset of the variables from the optimization task. This implies that the remaining variables are renumbered.

# Parameters

- subset (int[]) Indexes of variables which should be removed. (input)
- num (int) Number of variables which should be removed. (input)

Groups Problem data - variables, Problem data - linear part

#### Task.resizetask

```
void resizetask
(int maxnumcon,
int maxnumvar,
int maxnumcone,
long maxnumanz,
long maxnumqnz)
```

Sets the amount of preallocated space assigned for each type of data in an optimization task.

It is never mandatory to call this function, since it only gives a hint about the amount of data to preallocate for efficiency reasons.

Please note that the procedure is **destructive** in the sense that all existing data stored in the task is destroyed.

#### **Parameters**

- maxnumcon (int) New maximum number of constraints. (input)
- maxnumvar (int) New maximum number of variables. (input)
- maxnumcone (int) New maximum number of cones. (input)
- maxnumanz (long) New maximum number of non-zeros in A. (input)
- maxnumqnz (long) New maximum number of non-zeros in all Q matrices. (input)

Groups Environment and task management

# ${\tt Task.sensitivityreport}$

```
void sensitivityreport (streamtype whichstream)
```

Reads a sensitivity format file from a location given by <code>sparam.sensitivity\_file\_name</code> and writes the result to the stream <code>whichstream</code>. If <code>sparam.sensitivity\_res\_file\_name</code> is set to a non-empty string, then the sensitivity report is also written to a file of this name.

Parameters whichstream (streamtype) - Index of the stream. (input)

# Groups Sensitivity analysis

Task.set\_InfoCallback

```
void set_InfoCallback (DataCallback callback)
```

Receive callbacks with solver status and information during optimization.

Parameters callback (DataCallback) - The callback object. (input)

Task.set\_ItgSolutionCallback

```
void set_ItgSolutionCallback (ItgSolutionCallback callback)
```

Receive callbacks with solution updates from the mixed-integer optimizer.

Parameters callback (ItgSolutionCallback) - The callback object. (input)

Task.set\_Progress

```
void set_Progress (Progress callback)
```

Receive callbacks about current status of the solver during optimization.

Parameters callback (*Progress*) – The callback object. (input)

Task.set\_Stream

```
void set_Stream
(streamtype whichstream,
Stream callback)
```

Directs all output from a task stream to a callback object.

# Parameters

- whichstream (streamtype) Index of the stream. (input)
- callback (Stream) The callback object. (input)

Task.setdefaults

```
void setdefaults ()
```

Resets all the parameters to their default values.

Groups Parameters

Task.solutiondef

```
int solutiondef (soltype whichsol)
```

```
void solutiondef
  (soltype whichsol,
  out int isdef)
```

Checks whether a solution is defined.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- isdef (int) Is non-zero if the requested solution is defined. (output)

Return (int) – Is non-zero if the requested solution is defined. Groups Solution information

Task.solutionsummary

```
void solutionsummary (streamtype whichstream)
```

Prints a short summary of the current solutions.

Parameters whichstream (streamtype) - Index of the stream. (input) Groups Logging, Solution information

Task.solvewithbasis

```
void solvewithbasis
  (int transp,
   ref int numnz,
   int[] sub,
   double[] val)
```

If a basic solution is available, then exactly numcon basis variables are defined. These numcon basis variables are denoted the basis. Associated with the basis is a basis matrix denoted B. This function solves either the linear equation system

$$B\overline{X} = b \tag{15.3}$$

or the system

$$B^T \overline{X} = b \tag{15.4}$$

for the unknowns  $\overline{X}$ , with b being a user-defined vector. In order to make sense of the solution  $\overline{X}$  it is important to know the ordering of the variables in the basis because the ordering specifies how B is constructed. When calling Task.initbasissolve an ordering of the basis variables is obtained, which can be used to deduce how MOSEK has constructed B. Indeed if the k-th basis variable is variable  $x_j$  it implies that

$$B_{i,k} = A_{i,j}, i = 0, \dots, numcon - 1.$$

Otherwise if the k-th basis variable is variable  $x_i^c$  it implies that

$$B_{i,k} = \begin{cases} -1, & i = j, \\ 0, & i \neq j. \end{cases}$$

The function Task.initbasissolve must be called before a call to this function. Please note that this function exploits the sparsity in the vector b to speed up the computations.

#### **Parameters**

- transp (int) If this argument is zero, then (15.3) is solved, if non-zero then (15.4) is solved. (input)
- numnz (int) As input it is the number of non-zeros in b. As output it is the number of non-zeros in  $\overline{X}$ . (input/output)
- sub (int[]) As input it contains the positions of non-zeros in b. As output it contains the positions of the non-zeros in  $\overline{X}$ . It must have room for numcon elements. (input/output)
- val (double[]) As input it is the vector b as a dense vector (although the positions of non-zeros are specified in sub it is required that val[i] = 0 when b[i] = 0). As output val is the vector  $\overline{X}$  as a dense vector. It must have length numcon. (input/output)

Groups Solving systems with basis matrix

#### Task.strtoconetype

```
void strtoconetype
(string str,
out conetype conetype)
```

Obtains cone type code corresponding to a cone type string.

#### **Parameters**

- str (string) String corresponding to the cone type code conetype. (input)
- conetype (conetype) The cone type corresponding to the string str. (output)

Groups Names

Task.strtosk

```
void strtosk
(string str,
out stakey sk)
```

Obtains the status key corresponding to an abbreviation string.

#### **Parameters**

- str (string) A status key abbreviation string. (input)
- sk (stakey) Status key corresponding to the string. (output)

Groups Names

Task.toconic

```
void toconic ()
```

This function tries to reformulate a given Quadratically Constrained Quadratic Optimization problem (QCQP) as a Conic Quadratic Optimization problem (CQO). The first step of the reformulation is to convert the quadratic term of the objective function, if any, into a constraint. Then the following steps are repeated for each quadratic constraint:

- a conic constraint is added along with a suitable number of auxiliary variables and constraints;
- the original quadratic constraint is not removed, but all its coefficients are zeroed out.

Note that the reformulation preserves all the original variables.

The conversion is performed in-place, i.e. the task passed as argument is modified on exit. That also means that if the reformulation fails, i.e. the given QCQP is not representable as a CQO, then the task has an undefined state. In some cases, users may want to clone the task to ensure a clean copy is preserved.

Groups Problem data - quadratic part

 ${\tt Task.update solution info}$ 

```
void updatesolutioninfo (soltype whichsol)
```

Update the information items related to the solution.

```
Parameters whichsol (soltype) - Selects a solution. (input)
Groups Information items and statistics
```

Task.writedata

```
void writedata (string filename)
```

Writes problem data associated with the optimization task to a file in one of the supported formats. See Section Supported File Formats for the complete list.

The data file format is determined by the file name extension. To write in compressed format append the extension .gz. E.g to write a gzip compressed MPS file use the extension mps.gz.

Please note that MPS, LP and OPF files require all variables to have unique names. If a task contains no names, it is possible to write the file with automatically generated anonymous names by setting the <code>iparam.write\_generic\_names</code> parameter to <code>onoffkey.on</code>.

Data is written to the file filename if it is a nonempty string. Otherwise data is written to the file specified by  $sparam.data_file_name$ .

```
Parameters filename (string) – A valid file name. (input) Groups Input/Output
```

Task.writejsonsol

```
void writejsonsol (string filename)
```

Saves the current solutions and solver information items in a JSON file.

```
Parameters filename (string) – A valid file name. (input)
Groups Input/Output
```

Task.writeparamfile

```
void writeparamfile (string filename)
```

Writes all the parameters to a parameter file.

```
Parameters filename (string) – A valid file name. (input)
Groups Input/Output, Parameters
```

Task.writesolution

```
void writesolution
(soltype whichsol,
string filename)
```

Saves the current basic, interior-point, or integer solution to a file.

# Parameters

- whichsol (soltype) Selects a solution. (input)
- filename (string) A valid file name. (input)

Groups Input/Output

Task.writetask

```
void writetask (string filename)
```

Write a binary dump of the task data. This format saves all problem data, coefficients and parameter settings. See section *The Task Format* for a description of the Task format.

```
Parameters filename (string) – A valid file name. (input)
Groups Input/Output
```

# 15.5 Exceptions

## MosekException

The base class for all exceptions in MOSEK.

# Implements Exception

### Exception

Base class for exceptions that correspond to MOSEK response codes.

Implements MosekException

#### Error

Exception class used for all error response codes from MOSEK.

Implements Exception

### Warning

Exception class used for all warning response codes from MOSEK.

Implements MosekException

# NullArrayException

Exception thrown when null was passed to a method that expected non-null array argument.

Implements MosekException

### ArrayLengthException

Exception thrown the length of an array was smaller than required. This will happen, for example, if requesting a list of N values, but the array passed to the method is less than N elements long.

Implements MosekException

# 15.6 Parameters grouped by topic

# **Analysis**

- dparam.ana\_sol\_infeas\_tol
- ullet  $iparam.ana\_sol\_basis$
- $\bullet \ \ iparam. \ ana\_sol\_print\_violated$
- iparam.log\_ana\_pro

# **Basis identification**

- dparam.sim\_lu\_tol\_rel\_piv
- $\bullet \ iparam.bi\_clean\_optimizer$
- $\bullet \ iparam.bi\_ignore\_max\_iter$
- iparam.bi\_ignore\_num\_error
- iparam.bi\_max\_iterations
- $ullet \ iparam.intpnt\_basis$
- iparam.log\_bi
- iparam.log\_bi\_freq

# Conic interior-point method

- ullet dparam.intpnt\_co\_tol\_dfeas
- dparam.intpnt\_co\_tol\_infeas
- dparam.intpnt\_co\_tol\_mu\_red
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_near\_rel$
- dparam.intpnt\_co\_tol\_pfeas
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_rel\_gap$

#### Data check

- $\bullet \ dparam.\, data\_sym\_mat\_tol \\$
- $\bullet \ dparam.\, data\_sym\_mat\_tol\_huge$
- $\bullet \ dparam.\, data\_sym\_mat\_tol\_large$
- dparam.data\_tol\_aij\_huge
- dparam.data\_tol\_aij\_large
- $\bullet \ \textit{dparam.data\_tol\_bound\_inf}$
- $\bullet \quad dparam.\, data\_tol\_bound\_wrn$
- $\bullet$  dparam.data\_tol\_c\_huge
- dparam.data\_tol\_cj\_large
- dparam.data\_tol\_qij
- $\bullet$  dparam.data\_tol\_x
- dparam.semidefinite\_tol\_approx
- iparam.check\_convexity
- iparam.log\_check\_convexity

# Data input/output

- $\bullet$  iparam.infeas\_report\_auto
- iparam.log\_file
- iparam.opf\_write\_header
- iparam.opf\_write\_hints
- $\bullet \ \ iparam.opf\_write\_line\_length$
- iparam.opf\_write\_parameters
- $\bullet$  iparam.opf\_write\_problem
- iparam.opf\_write\_sol\_bas
- iparam.opf\_write\_sol\_itg
- iparam.opf\_write\_sol\_itr
- $\bullet \ \ iparam.opf\_write\_solutions$

- $\bullet \ iparam.param\_read\_case\_name$
- iparam.param\_read\_ign\_error
- iparam.ptf\_write\_transform
- iparam.read\_debug
- $\bullet \ iparam.read\_keep\_free\_con$
- $\bullet \ iparam.read\_lp\_drop\_new\_vars\_in\_bou$
- $\bullet \quad iparam.\, read\_\, lp\_\, quoted\_\, names$
- iparam.read\_mps\_format
- $\bullet \quad iparam.read\_mps\_width$
- $\bullet \ iparam.read\_task\_ignore\_param$
- iparam.sol\_read\_name\_width
- $\bullet \quad iparam.sol\_read\_width$
- $\bullet \ \ iparam.write\_bas\_constraints$
- $\bullet \quad iparam.write\_bas\_head$
- $\bullet \ \ iparam.write\_bas\_variables$
- iparam.write\_compression
- $\bullet$   $iparam.write\_data\_param$
- iparam.write\_free\_con
- iparam.write\_generic\_names
- $\bullet \quad iparam.write\_generic\_names\_io$
- $\bullet \ iparam.write\_ignore\_incompatible\_items$
- iparam.write\_int\_constraints
- $\bullet$  iparam.write\_int\_head
- iparam.write\_int\_variables
- $\bullet$  iparam.write\_lp\_full\_obj
- $\bullet \ \ iparam.write\_lp\_line\_width$
- iparam.write\_lp\_quoted\_names
- iparam.write\_lp\_strict\_format
- iparam.write\_lp\_terms\_per\_line
- $\bullet \ iparam.write\_mps\_format$
- ullet  $iparam.write\_mps\_int$
- iparam.write\_precision
- $\bullet \ \ iparam.write\_sol\_barvariables$
- $\bullet \ \ iparam.write\_sol\_constraints$
- ullet  $iparam.write\_sol\_head$
- $\bullet \ iparam.write\_sol\_ignore\_invalid\_names$

- $\bullet$  iparam.write\_sol\_variables
- $\bullet \ \ iparam.write\_task\_inc\_sol$
- iparam.write\_xml\_mode
- $\bullet$  sparam.bas\_sol\_file\_name
- sparam.data\_file\_name
- $\bullet$  sparam.debug\_file\_name
- sparam.int\_sol\_file\_name
- $\bullet$  sparam.itr\_sol\_file\_name
- sparam.mio\_debug\_string
- sparam.param\_comment\_sign
- sparam.param\_read\_file\_name
- $\bullet \ \ sparam.param\_write\_file\_name$
- $\bullet \ sparam.read\_mps\_bou\_name$
- $\bullet \ \textit{sparam.read\_mps\_obj\_name}$
- sparam.read\_mps\_ran\_name
- sparam.read\_mps\_rhs\_name
- sparam.sensitivity\_file\_name
- $\bullet \ \ sparam.sensitivity\_res\_file\_name$
- $\bullet$  sparam.sol\_filter\_xc\_low
- $\bullet$  sparam.sol\_filter\_xc\_upr
- $\bullet \ \ sparam.sol\_filter\_xx\_low$
- $\bullet$  sparam.sol\_filter\_xx\_upr
- $\bullet \ \ sparam.stat\_file\_name$
- sparam.stat\_key
- sparam.stat\_name
- sparam.write\_lp\_gen\_var\_name

# **Debugging**

 $\bullet$  iparam.auto\_sort\_a\_before\_opt

# **Dual simplex**

- $\bullet \ iparam.sim\_dual\_crash$
- $\bullet \ iparam.sim\_dual\_restrict\_selection$
- $\bullet \ iparam.sim\_dual\_selection$

# Infeasibility report

- iparam.infeas\_generic\_names
- $\bullet$  iparam.infeas\_report\_level
- iparam.log\_infeas\_ana

# Interior-point method

- dparam.check\_convexity\_rel\_tol
- $\bullet \ \ dparam. intpnt\_co\_tol\_dfeas$
- dparam.intpnt\_co\_tol\_infeas
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_mu\_red$
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_near\_rel$
- dparam.intpnt\_co\_tol\_pfeas
- dparam.intpnt\_co\_tol\_rel\_gap
- dparam.intpnt\_qo\_tol\_dfeas
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_infeas$
- dparam.intpnt\_qo\_tol\_mu\_red
- $\bullet \ \ dparam. \ intpnt\_qo\_tol\_near\_rel$
- dparam.intpnt\_qo\_tol\_pfeas
- $\bullet$  dparam.intpnt\_qo\_tol\_rel\_gap
- ullet dparam.intpnt\_tol\_dfeas
- dparam.intpnt\_tol\_dsafe
- $\bullet \ \ dparam. \ intpnt\_tol\_infeas$
- dparam.intpnt\_tol\_mu\_red
- $\bullet \ dparam. intpnt\_tol\_path$
- $\bullet \ \ dparam. intpnt\_tol\_pfeas$
- $\bullet \ \ dparam. \ intpnt\_tol\_psafe$
- $\bullet \ dparam. intpnt\_tol\_rel\_gap$
- $\bullet \ \ dparam. \ intpnt\_tol\_rel\_step$
- $\bullet \ dparam. intpnt\_tol\_step\_size$
- $\bullet \ \ dparam. \ qcqo\_reformulate\_rel\_drop\_tol$
- $\bullet \ iparam.bi\_ignore\_max\_iter$
- iparam.bi\_ignore\_num\_error
- iparam.intpnt\_basis
- iparam.intpnt\_diff\_step
- $\bullet \ iparam. \ intpnt\_hotstart$
- $ullet \ iparam.intpnt\_max\_iterations$

- iparam.intpnt\_max\_num\_cor
- $\bullet \ iparam. intpnt\_max\_num\_refinement\_steps$
- $\bullet$  iparam.intpnt\_off\_col\_trh
- $\bullet \ iparam. intpnt\_order\_gp\_num\_seeds$
- iparam.intpnt\_order\_method
- iparam.intpnt\_purify
- $\bullet \ iparam.intpnt\_regularization\_use$
- iparam.intpnt\_scaling
- iparam.intpnt\_solve\_form
- $\bullet \ \ iparam. \ intpnt\_starting\_point$
- $\bullet$  iparam.log\_intpnt

# License manager

- iparam.cache\_license
- iparam.license\_debug
- $\bullet \ iparam. \ license\_pause\_time$
- $\bullet \ \ iparam.\ license\_suppress\_expire\_wrns$
- $\bullet \ \ iparam. \ license\_trh\_expiry\_wrn$
- $\bullet \ \ iparam. \ license\_wait$

# Logging

- iparam.log
- $\bullet$  iparam.log\_ana\_pro
- iparam.log\_bi
- iparam.log\_bi\_freq
- $\bullet \ iparam. \ log\_cut\_second\_opt$
- $\bullet$  iparam.log\_expand
- $\bullet$  iparam.log\_feas\_repair
- iparam.log\_file
- $\bullet \ iparam. log\_include\_summary$
- $\bullet \ iparam. log\_infeas\_ana$
- $\bullet$  iparam.log\_intpnt
- $\bullet$   $iparam.log_local_info$
- iparam.log\_mio
- $\bullet \ iparam. log\_mio\_freq$
- $\bullet$  iparam.log\_order

- $\bullet$   $iparam.log\_presolve$
- iparam.log\_response
- iparam.log\_sensitivity
- ullet iparam.log\_sensitivity\_opt
- iparam.log\_sim
- iparam.log\_sim\_freq
- $\bullet$  iparam.log\_storage

# Mixed-integer optimization

- dparam.mio\_max\_time
- $\bullet$  dparam.mio\_rel\_gap\_const
- dparam.mio\_tol\_abs\_gap
- dparam.mio\_tol\_abs\_relax\_int
- dparam.mio\_tol\_feas
- $\bullet \ \ dparam.mio\_tol\_rel\_dual\_bound\_improvement$
- dparam.mio\_tol\_rel\_gap
- iparam.log\_mio
- iparam.log\_mio\_freq
- iparam.mio\_branch\_dir
- iparam.mio\_conic\_outer\_approximation
- iparam.mio\_cut\_clique
- iparam.mio\_cut\_cmir
- iparam.mio\_cut\_gmi
- iparam.mio\_cut\_implied\_bound
- iparam.mio\_cut\_knapsack\_cover
- iparam.mio\_cut\_selection\_level
- iparam.mio\_feaspump\_level
- iparam.mio\_heuristic\_level
- iparam.mio\_max\_num\_branches
- iparam.mio\_max\_num\_relaxs
- $\bullet \quad iparam.mio\_max\_num\_root\_cut\_rounds$
- iparam.mio\_max\_num\_solutions
- iparam.mio\_node\_optimizer
- iparam.mio\_node\_selection
- $\bullet \ iparam.mio\_perspective\_reformulate$
- iparam.mio\_probing\_level

- $\bullet \ iparam.mio\_propagate\_objective\_constraint$
- iparam.mio\_rins\_max\_nodes
- $\bullet \ iparam.mio\_root\_optimizer$
- $\bullet \ iparam.mio\_root\_repeat\_presolve\_level$
- iparam.mio\_seed
- $\bullet \ \ iparam.mio\_vb\_detection\_level$

# **Output** information

- iparam.infeas\_report\_level
- $\bullet \ iparam.\ license\_suppress\_expire\_wrns$
- $\bullet \ \ iparam. \ license\_trh\_expiry\_wrn$
- iparam.log
- iparam.log\_bi
- iparam.log\_bi\_freq
- iparam.log\_cut\_second\_opt
- iparam.log\_expand
- $\bullet \ iparam. log\_feas\_repair$
- $\bullet$  iparam.log\_file
- iparam.log\_include\_summary
- iparam.log\_infeas\_ana
- $\bullet \ iparam. log\_intpnt$
- $\bullet$  iparam.log\_local\_info
- iparam.log\_mio
- iparam.log\_mio\_freq
- $\bullet$  iparam.log\_order
- iparam.log\_response
- iparam.log\_sensitivity
- $\bullet \ iparam. \ log\_sensitivity\_opt$
- $\bullet$  iparam.log\_sim
- iparam.log\_sim\_freq
- iparam.log\_sim\_minor
- iparam.log\_storage
- iparam.max\_num\_warnings

#### Overall solver

- iparam.bi\_clean\_optimizer
- iparam.infeas\_prefer\_primal
- iparam.license\_wait
- iparam.mio\_mode
- iparam.optimizer
- iparam.presolve\_level
- iparam.presolve\_max\_num\_reductions
- iparam.presolve\_use
- iparam.primal\_repair\_optimizer
- iparam.sensitivity\_all
- iparam.sensitivity\_optimizer
- iparam.sensitivity\_type
- $\bullet$  iparam.solution\_callback

# Overall system

- $\bullet \ \ iparam. \ auto\_update\_sol\_info$
- $\bullet$  iparam.intpnt\_multi\_thread
- iparam.license\_wait
- $\bullet$  iparam.log\_storage
- iparam.mt\_spincount
- iparam.num\_threads
- iparam.remove\_unused\_solutions
- iparam.timing\_level
- sparam.remote\_access\_token

#### **Presolve**

- dparam.presolve\_tol\_abs\_lindep
- dparam.presolve\_tol\_aij
- $\bullet \ \ dparam.presolve\_tol\_rel\_lindep$
- dparam.presolve\_tol\_s
- dparam.presolve\_tol\_x
- iparam.presolve\_eliminator\_max\_fill
- iparam.presolve\_eliminator\_max\_num\_tries
- iparam.presolve\_level
- $\bullet \ \ iparam.presolve\_lindep\_abs\_work\_trh$

- iparam.presolve\_lindep\_rel\_work\_trh
- $\bullet$  iparam.presolve\_lindep\_use
- iparam.presolve\_max\_num\_pass
- iparam.presolve\_max\_num\_reductions
- iparam.presolve\_use

# **Primal simplex**

- iparam.sim\_primal\_crash
- iparam.sim\_primal\_restrict\_selection
- iparam.sim\_primal\_selection

# Progress callback

• iparam.solution\_callback

# Simplex optimizer

- $\bullet$  dparam.basis\_rel\_tol\_s
- dparam.basis\_tol\_s
- $\bullet$  dparam.basis\_tol\_x
- $\bullet \ dparam.sim\_lu\_tol\_rel\_piv$
- $\bullet \ \ dparam.simplex\_abs\_tol\_piv$
- iparam.basis\_solve\_use\_plus\_one
- iparam.log\_sim
- iparam.log\_sim\_freq
- iparam.log\_sim\_minor
- iparam.sensitivity\_optimizer
- iparam.sim\_basis\_factor\_use
- iparam.sim\_degen
- iparam.sim\_dual\_phaseone\_method
- iparam.sim\_exploit\_dupvec
- iparam.sim\_hotstart
- iparam.sim\_hotstart\_lu
- iparam.sim\_max\_iterations
- iparam.sim\_max\_num\_setbacks
- $\bullet \ iparam.sim\_non\_singular$
- $\bullet \ iparam.sim\_primal\_phaseone\_method$
- iparam.sim\_refactor\_freq

- $\bullet$  iparam.sim\_reformulation
- $\bullet$   $iparam.sim\_save\_lu$
- iparam.sim\_scaling
- iparam.sim\_scaling\_method
- iparam.sim\_seed
- $\bullet \ \ iparam.sim\_solve\_form$
- iparam.sim\_stability\_priority
- iparam.sim\_switch\_optimizer

# Solution input/output

- iparam.infeas\_report\_auto
- iparam.sol\_filter\_keep\_basic
- iparam.sol\_filter\_keep\_ranged
- iparam.sol\_read\_name\_width
- iparam.sol\_read\_width
- $\bullet \ \ iparam.write\_bas\_constraints$
- $\bullet \ \ iparam.write\_bas\_head$
- iparam.write\_bas\_variables
- iparam.write\_int\_constraints
- $\bullet$  iparam.write\_int\_head
- $\bullet$  iparam.write\_int\_variables
- $\bullet \ \ iparam.write\_sol\_barvariables$
- iparam.write\_sol\_constraints
- $\bullet$  iparam.write\_sol\_head
- iparam.write\_sol\_ignore\_invalid\_names
- iparam.write\_sol\_variables
- $\bullet \ sparam.bas\_sol\_file\_name$
- $\bullet \ sparam.int\_sol\_file\_name$
- $\bullet$  sparam.itr\_sol\_file\_name
- $\bullet$  sparam.sol\_filter\_xc\_low
- sparam.sol\_filter\_xc\_upr
- $\bullet$  sparam.sol\_filter\_xx\_low
- $\bullet \ \ sparam.sol\_filter\_xx\_upr$

#### Termination criteria

- $\bullet \ \ dparam.basis\_rel\_tol\_s$
- dparam.basis\_tol\_s
- $\bullet$  dparam.basis\_tol\_x
- dparam.intpnt\_co\_tol\_dfeas
- dparam.intpnt\_co\_tol\_infeas
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_mu\_red$
- $\bullet \ \ dparam. \ intpnt\_co\_tol\_near\_rel$
- dparam.intpnt\_co\_tol\_pfeas
- dparam.intpnt\_co\_tol\_rel\_gap
- dparam.intpnt\_qo\_tol\_dfeas
- $\bullet \ dparam. intpnt\_qo\_tol\_infeas$
- $\bullet$  dparam.intpnt\_qo\_tol\_mu\_red
- dparam.intpnt\_qo\_tol\_near\_rel
- $\bullet \ \ dparam. intpnt\_qo\_tol\_pfeas$
- dparam.intpnt\_qo\_tol\_rel\_gap
- dparam.intpnt\_tol\_dfeas
- $\bullet$  dparam.intpnt\_tol\_infeas
- dparam.intpnt\_tol\_mu\_red
- $\bullet$  dparam.intpnt\_tol\_pfeas
- dparam.intpnt\_tol\_rel\_gap
- dparam.lower\_obj\_cut
- $\bullet \ dparam. \ lower\_obj\_cut\_finite\_trh$
- dparam.mio\_max\_time
- dparam.mio\_rel\_gap\_const
- dparam.mio\_tol\_rel\_gap
- dparam.optimizer\_max\_time
- dparam.upper\_obj\_cut
- $\bullet \ \ dparam.upper\_obj\_cut\_finite\_trh$
- iparam.bi\_max\_iterations
- $\bullet$  iparam.intpnt\_max\_iterations
- iparam.mio\_max\_num\_branches
- iparam.mio\_max\_num\_root\_cut\_rounds
- iparam.mio\_max\_num\_solutions
- $\bullet \ iparam.sim\_max\_iterations$

# Other

• iparam.compress\_statfile

# 15.7 Parameters (alphabetical list sorted by type)

- Double parameters
- Integer parameters
- String parameters

# 15.7.1 Double parameters

### dparam

The enumeration type containing all double parameters.

```
dparam.ana_sol_infeas_tol
```

If a constraint violates its bound with an amount larger than this value, the constraint name, index and violation will be printed by the solution analyzer.

```
Default 1e-6
Accepted [0.0; +inf]
Example task.putdouparam(dparam.ana_sol_infeas_tol, 1e-6)
Generic name MSK_DPAR_ANA_SOL_INFEAS_TOL
Groups Analysis
```

dparam.basis\_rel\_tol\_s

Maximum relative dual bound violation allowed in an optimal basic solution.

```
Default 1.0e-12
Accepted [0.0; +inf]
Example task.putdouparam(dparam.basis_rel_tol_s, 1.0e-12)
Generic name MSK_DPAR_BASIS_REL_TOL_S
Groups Simplex optimizer, Termination criteria
```

dparam.basis\_tol\_s

Maximum absolute dual bound violation in an optimal basic solution.

```
Default 1.0e-6
Accepted [1.0e-9; +inf]
Example task.putdouparam(dparam.basis_tol_s, 1.0e-6)
Generic name MSK_DPAR_BASIS_TOL_S
Groups Simplex optimizer, Termination criteria
```

dparam.basis\_tol\_x

Maximum absolute primal bound violation allowed in an optimal basic solution.

```
Default 1.0e-6
Accepted [1.0e-9; +inf]
Example task.putdouparam(dparam.basis_tol_x, 1.0e-6)
Generic name MSK_DPAR_BASIS_TOL_X
Groups Simplex optimizer, Termination criteria
```

### dparam.check\_convexity\_rel\_tol

This parameter controls when the full convexity check declares a problem to be non-convex. Increasing this tolerance relaxes the criteria for declaring the problem non-convex.

A problem is declared non-convex if negative (positive) pivot elements are detected in the Cholesky factor of a matrix which is required to be PSD (NSD). This parameter controls how much this non-negativity requirement may be violated.

If  $d_i$  is the pivot element for column i, then the matrix Q is considered to not be PSD if:

```
d_i \leq -|Q_{ii}| check convexity rel tol
```

Default 1e-10

Accepted [0; +inf]

Example task.putdouparam(dparam.check\_convexity\_rel\_tol, 1e-10)

Generic name MSK\_DPAR\_CHECK\_CONVEXITY\_REL\_TOL

Groups Interior-point method

### dparam.data\_sym\_mat\_tol

Absolute zero tolerance for elements in in symmetric matrices. If any value in a symmetric matrix is smaller than this parameter in absolute terms **MOSEK** will treat the values as zero and generate a warning.

**Default** 1.0e-12

**Accepted** [1.0e-16; 1.0e-6]

Example task.putdouparam(dparam.data\_sym\_mat\_tol, 1.0e-12)

Generic name MSK DPAR DATA SYM MAT TOL

Groups Data check

# dparam.data\_sym\_mat\_tol\_huge

An element in a symmetric matrix which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_sym\_mat\_tol\_huge, 1.0e20)

Generic name MSK DPAR DATA SYM MAT TOL HUGE

Groups Data check

# dparam.data\_sym\_mat\_tol\_large

An element in a symmetric matrix which is larger than this value in absolute size causes a warning message to be printed.

Default 1.0e10

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_sym\_mat\_tol\_large, 1.0e10)

Generic name MSK DPAR DATA SYM MAT TOL LARGE

Groups Data check

# dparam.data\_tol\_aij\_huge

An element in A which is larger than this value in absolute size causes an error.

**Default** 1.0e20

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_aij\_huge, 1.0e20)

Generic name MSK\_DPAR\_DATA\_TOL\_AIJ\_HUGE

Groups Data check

# dparam.data\_tol\_aij\_large

An element in A which is larger than this value in absolute size causes a warning message to be printed.

```
Default 1.0e10
```

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_aij\_large, 1.0e10)

Generic name MSK\_DPAR\_DATA\_TOL\_AIJ\_LARGE

Groups Data check

#### dparam.data\_tol\_bound\_inf

Any bound which in absolute value is greater than this parameter is considered infinite.

**Default** 1.0e16

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_bound\_inf, 1.0e16)

Generic name MSK DPAR DATA TOL BOUND INF

Groups Data check

#### dparam.data\_tol\_bound\_wrn

If a bound value is larger than this value in absolute size, then a warning message is issued.

Default 1.0e8

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_bound\_wrn, 1.0e8)

Generic name MSK DPAR DATA TOL BOUND WRN

Groups Data check

# dparam.data\_tol\_c\_huge

An element in c which is larger than the value of this parameter in absolute terms is considered to be huge and generates an error.

Default 1.0e16

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_c\_huge, 1.0e16)

Generic name MSK DPAR DATA TOL C HUGE

Groups Data check

# dparam.data\_tol\_cj\_large

An element in c which is larger than this value in absolute terms causes a warning message to be printed.

 $\textbf{Default} \ \ 1.0e8$ 

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_cj\_large, 1.0e8)

Generic name MSK DPAR DATA TOL CJ LARGE

Groups Data check

# dparam.data\_tol\_qij

Absolute zero tolerance for elements in Q matrices.

**Default** 1.0e-16

Accepted [0.0; +inf]

Example task.putdouparam(dparam.data\_tol\_qij, 1.0e-16)

Generic name MSK DPAR DATA TOL QIJ

Groups Data check

# dparam.data\_tol\_x

Zero tolerance for constraints and variables i.e. if the distance between the lower and upper bound is less than this value, then the lower and upper bound is considered identical.

**Default** 1.0e-8

Accepted [0.0; +inf]

```
Example task.putdouparam(dparam.data_tol_x, 1.0e-8)
```

Generic name MSK DPAR DATA TOL X

Groups Data check

# dparam.intpnt\_co\_tol\_dfeas

Dual feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_co\_tol\_dfeas, 1.0e-8)

See also dparam.intpnt\_co\_tol\_near\_rel

Generic name MSK DPAR INTPNT CO TOL DFEAS

Groups Interior-point method, Termination criteria, Conic interior-point method

# dparam.intpnt\_co\_tol\_infeas

Infeasibility tolerance used by the interior-point optimizer for conic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_co\_tol\_infeas, 1.0e-12)

Generic name MSK DPAR INTPNT CO TOL INFEAS

Groups Interior-point method, Termination criteria, Conic interior-point method

# dparam.intpnt\_co\_tol\_mu\_red

Relative complementarity gap tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_co\_tol\_mu\_red, 1.0e-8)

Generic name MSK DPAR INTPNT CO TOL MU RED

Groups Interior-point method, Termination criteria, Conic interior-point method

## dparam.intpnt\_co\_tol\_near\_rel

Optimality tolerance used by the interior-point optimizer for conic problems. If **MOSEK** cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example task.putdouparam(dparam.intpnt\_co\_tol\_near\_rel, 1000)

 ${\bf Generic\ name\ MSK\_DPAR\_INTPNT\_CO\_TOL\_NEAR\_REL}$ 

Groups Interior-point method, Termination criteria, Conic interior-point method

## dparam.intpnt\_co\_tol\_pfeas

Primal feasibility tolerance used by the interior-point optimizer for conic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_co\_tol\_pfeas, 1.0e-8)

See also dparam.intpnt\_co\_tol\_near\_rel

Generic name MSK\_DPAR\_INTPNT\_CO\_TOL\_PFEAS

**Groups** Interior-point method, Termination criteria, Conic interior-point method

# dparam.intpnt\_co\_tol\_rel\_gap

Relative gap termination tolerance used by the interior-point optimizer for conic problems.

```
Default 1.0e-8
```

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_co\_tol\_rel\_gap, 1.0e-8)

See also dparam.intpnt\_co\_tol\_near\_rel

Generic name MSK DPAR INTPNT CO TOL REL GAP

Groups Interior-point method, Termination criteria, Conic interior-point method

## dparam.intpnt\_qo\_tol\_dfeas

Dual feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_qo\_tol\_dfeas, 1.0e-8)

See also dparam.intpnt\_qo\_tol\_near\_rel

Generic name MSK DPAR INTPNT QO TOL DFEAS

Groups Interior-point method, Termination criteria

## dparam.intpnt\_qo\_tol\_infeas

Infeasibility tolerance used by the interior-point optimizer for quadratic problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-12

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_qo\_tol\_infeas, 1.0e-12)

Generic name MSK\_DPAR\_INTPNT\_QO\_TOL\_INFEAS

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_qo\_tol\_mu\_red

Relative complementarity gap tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_qo\_tol\_mu\_red, 1.0e-8)

Generic name MSK DPAR INTPNT QO TOL MU RED

 ${\bf Groups} \ \ {\it Interior-point method}, \ {\it Termination \ criteria}$ 

# ${\tt dparam.intpnt\_qo\_tol\_near\_rel}$

Optimality tolerance used by the interior-point optimizer for quadratic problems. If MOSEK cannot compute a solution that has the prescribed accuracy then it will check if the solution found satisfies the termination criteria with all tolerances multiplied by the value of this parameter. If yes, then the solution is also declared optimal.

Default 1000

Accepted [1.0; +inf]

Example task.putdouparam(dparam.intpnt\_go\_tol\_near\_rel, 1000)

Generic name MSK\_DPAR\_INTPNT\_QO\_TOL\_NEAR\_REL

 ${\bf Groups} \ \ Interior\mbox{-}point \ method, \ Termination \ criteria$ 

# dparam.intpnt\_qo\_tol\_pfeas

Primal feasibility tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

Accepted [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_qo\_tol\_pfeas, 1.0e-8)

See also dparam.intpnt\_qo\_tol\_near\_rel

Generic name MSK DPAR INTPNT QO TOL PFEAS

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_qo\_tol\_rel\_gap

Relative gap termination tolerance used by the interior-point optimizer for quadratic problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_qo\_tol\_rel\_gap, 1.0e-8)

See also dparam.intpnt\_qo\_tol\_near\_rel

Generic name MSK DPAR INTPNT QO TOL REL GAP

Groups Interior-point method, Termination criteria

## dparam.intpnt\_tol\_dfeas

Dual feasibility tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_tol\_dfeas, 1.0e-8)

Generic name MSK DPAR INTPNT TOL DFEAS

Groups Interior-point method, Termination criteria

#### dparam.intpnt\_tol\_dsafe

Controls the initial dual starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it might be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Example task.putdouparam(dparam.intpnt\_tol\_dsafe, 1.0)

Generic name MSK DPAR INTPNT TOL DSAFE

**Groups** Interior-point method

# dparam.intpnt\_tol\_infeas

Infeasibility tolerance used by the interior-point optimizer for linear problems. Controls when the interior-point optimizer declares the model primal or dual infeasible. A small number means the optimizer gets more conservative about declaring the model infeasible.

**Default** 1.0e-10

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_tol\_infeas, 1.0e-10)

Generic name MSK DPAR INTPNT TOL INFEAS

Groups Interior-point method, Termination criteria

# dparam.intpnt\_tol\_mu\_red

Relative complementarity gap tolerance used by the interior-point optimizer for linear problems.

**Default** 1.0e-16

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_tol\_mu\_red, 1.0e-16)

Generic name MSK DPAR INTPNT TOL MU RED

Groups Interior-point method, Termination criteria

# ${\tt dparam.intpnt\_tol\_path}$

Controls how close the interior-point optimizer follows the central path. A large value of this parameter means the central path is followed very closely. On numerically unstable problems it may be worthwhile to increase this parameter.

Default 1.0e-8

**Accepted** [0.0; 0.9999]

Example task.putdouparam(dparam.intpnt\_tol\_path, 1.0e-8)

```
Generic name MSK_DPAR_INTPNT_TOL_PATH
```

**Groups** Interior-point method

#### dparam.intpnt\_tol\_pfeas

Primal feasibility tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

**Accepted** [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_tol\_pfeas, 1.0e-8)

Generic name MSK DPAR INTPNT TOL PFEAS

Groups Interior-point method, Termination criteria

# dparam.intpnt\_tol\_psafe

Controls the initial primal starting point used by the interior-point optimizer. If the interior-point optimizer converges slowly and/or the constraint or variable bounds are very large, then it may be worthwhile to increase this value.

Default 1.0

Accepted [1.0e-4; +inf]

Example task.putdouparam(dparam.intpnt\_tol\_psafe, 1.0)

Generic name MSK DPAR INTPNT TOL PSAFE

Groups Interior-point method

#### dparam.intpnt\_tol\_rel\_gap

Relative gap termination tolerance used by the interior-point optimizer for linear problems.

Default 1.0e-8

Accepted [1.0e-14; +inf]

Example task.putdouparam(dparam.intpnt\_tol\_rel\_gap, 1.0e-8)

Generic name MSK DPAR INTPNT TOL REL GAP

Groups Termination criteria, Interior-point method

# dparam.intpnt\_tol\_rel\_step

Relative step size to the boundary for linear and quadratic optimization problems.

**Default** 0.9999

**Accepted** [1.0e-4; 0.999999]

Example task.putdouparam(dparam.intpnt\_tol\_rel\_step, 0.9999)

Generic name MSK DPAR INTPNT TOL REL STEP

 ${\bf Groups} \ \ Interior\mbox{-}point \ method$ 

# dparam.intpnt\_tol\_step\_size

Minimal step size tolerance. If the step size falls below the value of this parameter, then the interior-point optimizer assumes that it is stalled. In other words the interior-point optimizer does not make any progress and therefore it is better to stop.

Default 1.0e-6

Accepted [0.0; 1.0]

Example task.putdouparam(dparam.intpnt\_tol\_step\_size, 1.0e-6)

Generic name MSK DPAR INTPNT TOL STEP SIZE

Groups Interior-point method

# dparam.lower\_obj\_cut

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [ dparam.lower\_obj\_cut, dparam.upper\_obj\_cut ], then MOSEK is terminated.

**Default** -1.0e30

Accepted  $[-\inf; +\inf]$ 

Example task.putdouparam(dparam.lower\_obj\_cut, -1.0e30)

```
See also dparam.lower_obj_cut_finite_trh
         Generic name MSK DPAR LOWER OBJ CUT
         Groups Termination criteria
dparam.lower_obj_cut_finite_trh
     If the lower objective cut is less than the value of this parameter value, then the lower objective
     cut i.e. dparam.lower_obj_cut is treated as -\infty.
         Default -0.5e30
         Accepted [-\inf; +\inf]
         Example task.putdouparam(dparam.lower_obj_cut_finite_trh, -0.5e30)
         Generic name MSK DPAR LOWER OBJ CUT FINITE TRH
         Groups Termination criteria
dparam.mio_max_time
     This parameter limits the maximum time spent by the mixed-integer optimizer. A negative number
     means infinity.
         Default -1.0
         Accepted [-\inf; +\inf]
         Example task.putdouparam(dparam.mio_max_time, -1.0)
         Generic name MSK DPAR MIO MAX TIME
         Groups Mixed-integer optimization, Termination criteria
dparam.mio_rel_gap_const
     This value is used to compute the relative gap for the solution to an integer optimization problem.
         Default 1.0e-10
         Accepted [1.0e-15; +inf]
         Example task.putdouparam(dparam.mio_rel_gap_const, 1.0e-10)
         Generic name MSK DPAR MIO REL GAP CONST
         Groups Mixed-integer optimization, Termination criteria
dparam.mio_tol_abs_gap
     Absolute optimality tolerance employed by the mixed-integer optimizer.
         Default 0.0
         Accepted [0.0; +inf]
         Example task.putdouparam(dparam.mio_tol_abs_gap, 0.0)
         Generic name MSK DPAR MIO TOL ABS GAP
         Groups Mixed-integer optimization
dparam.mio_tol_abs_relax_int
     Absolute integer feasibility tolerance. If the distance to the nearest integer is less than this tolerance
     then an integer constraint is assumed to be satisfied.
         Default 1.0e-5
         Accepted [1e-9; +inf]
         Example task.putdouparam(dparam.mio_tol_abs_relax_int, 1.0e-5)
         Generic name MSK_DPAR_MIO_TOL_ABS_RELAX_INT
         Groups Mixed-integer optimization
dparam.mio_tol_feas
     Feasibility tolerance for mixed integer solver.
```

Example task.putdouparam(dparam.mio\_tol\_feas, 1.0e-6)

Generic name MSK DPAR MIO TOL FEAS

Default 1.0e-6

**Accepted** [1e-9; 1e-3]

# **Groups** Mixed-integer optimization

# dparam.mio\_tol\_rel\_dual\_bound\_improvement

If the relative improvement of the dual bound is smaller than this value, the solver will terminate the root cut generation. A value of 0.0 means that the value is selected automatically.

Default 0.0

**Accepted** [0.0; 1.0]

**Groups** Mixed-integer optimization

## dparam.mio\_tol\_rel\_gap

Relative optimality tolerance employed by the mixed-integer optimizer.

Default 1.0e-4

Accepted [0.0; +inf]

Example task.putdouparam(dparam.mio\_tol\_rel\_gap, 1.0e-4)

Generic name MSK DPAR MIO TOL REL GAP

Groups Mixed-integer optimization, Termination criteria

#### dparam.optimizer\_max\_time

Maximum amount of time the optimizer is allowed to spent on the optimization. A negative number means infinity.

Default -1.0

Accepted  $[-\inf; +\inf]$ 

Example task.putdouparam(dparam.optimizer\_max\_time, -1.0)

Generic name MSK DPAR OPTIMIZER MAX TIME

Groups Termination criteria

# ${\tt dparam.presolve\_tol\_abs\_lindep}$

Absolute tolerance employed by the linear dependency checker.

Default 1.0e-6

Accepted [0.0; +inf]

Example task.putdouparam(dparam.presolve\_tol\_abs\_lindep, 1.0e-6)

Generic name MSK DPAR PRESOLVE TOL ABS LINDEP

Groups Presolve

# dparam.presolve\_tol\_aij

Absolute zero tolerance employed for  $a_{ij}$  in the presolve.

**Default** 1.0e-12

Accepted [1.0e-15; +inf]

Example task.putdouparam(dparam.presolve\_tol\_aij, 1.0e-12)

 ${\bf Generic\ name\ MSK\_DPAR\_PRESOLVE\_TOL\_AIJ}$ 

Groups Presolve

# dparam.presolve\_tol\_rel\_lindep

Relative tolerance employed by the linear dependency checker.

**Default** 1.0e-10

Accepted [0.0; +inf]

Example task.putdouparam(dparam.presolve\_tol\_rel\_lindep, 1.0e-10)

Generic name MSK DPAR PRESOLVE TOL REL LINDEP

Groups Presolve

```
dparam.presolve_tol_s
```

Absolute zero tolerance employed for  $s_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example task.putdouparam(dparam.presolve\_tol\_s, 1.0e-8)

Generic name MSK\_DPAR\_PRESOLVE\_TOL\_S

Groups Presolve

## dparam.presolve\_tol\_x

Absolute zero tolerance employed for  $x_i$  in the presolve.

Default 1.0e-8

Accepted [0.0; +inf]

Example task.putdouparam(dparam.presolve\_tol\_x, 1.0e-8)

Generic name MSK DPAR PRESOLVE TOL X

Groups Presolve

#### dparam.qcqo\_reformulate\_rel\_drop\_tol

This parameter determines when columns are dropped in incomplete Cholesky factorization during reformulation of quadratic problems.

Default 1e-15

Accepted [0; +inf]

Example task.putdouparam(dparam.qcqo\_reformulate\_rel\_drop\_tol, 1e-15)

Generic name MSK DPAR QCQO REFORMULATE REL DROP TOL

Groups Interior-point method

# dparam.semidefinite\_tol\_approx

Tolerance to define a matrix to be positive semidefinite.

**Default** 1.0e-10

Accepted [1.0e-15; +inf]

Example task.putdouparam(dparam.semidefinite\_tol\_approx, 1.0e-10)

Generic name MSK DPAR SEMIDEFINITE TOL APPROX

Groups Data check

# dparam.sim\_lu\_tol\_rel\_piv

Relative pivot tolerance employed when computing the LU factorization of the basis in the simplex optimizers and in the basis identification procedure. A value closer to 1.0 generally improves numerical stability but typically also implies an increase in the computational work.

Default 0.01

**Accepted** [1.0e-6; 0.999999]

Example task.putdouparam(dparam.sim\_lu\_tol\_rel\_piv, 0.01)

Generic name MSK DPAR SIM LU TOL REL PIV

Groups Basis identification, Simplex optimizer

# dparam.simplex\_abs\_tol\_piv

Absolute pivot tolerance employed by the simplex optimizers.

Default 1.0e-7

Accepted [1.0e-12; +inf]

Example task.putdouparam(dparam.simplex\_abs\_tol\_piv, 1.0e-7)

Generic name MSK DPAR SIMPLEX ABS TOL PIV

Groups Simplex optimizer

```
dparam.upper_obj_cut
```

If either a primal or dual feasible solution is found proving that the optimal objective value is outside the interval [ dparam.lower\_obj\_cut, dparam.upper\_obj\_cut ], then MOSEK is terminated.

```
Default 1.0e30
Accepted [-inf; +inf]
Example task.putdouparam(dparam.upper_obj_cut, 1.0e30)
See also dparam.upper_obj_cut_finite_trh
Generic name MSK_DPAR_UPPER_OBJ_CUT
Groups Termination criteria
```

# dparam.upper\_obj\_cut\_finite\_trh

If the upper objective cut is greater than the value of this parameter, then the upper objective cut  $dparam.upper_obj_cut$  is treated as  $\infty$ .

```
Default 0.5e30
Accepted [-inf; +inf]
Example task.putdouparam(dparam.upper_obj_cut_finite_trh, 0.5e30)
Generic name MSK_DPAR_UPPER_OBJ_CUT_FINITE_TRH
Groups Termination criteria
```

# 15.7.2 Integer parameters

iparam

The enumeration type containing all integer parameters.

iparam.ana\_sol\_basis

Controls whether the basis matrix is analyzed in solution analyzer.

```
Default on
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.ana_sol_basis, onoffkey.on)
Generic name MSK_IPAR_ANA_SOL_BASIS
Groups Analysis
```

#### iparam.ana\_sol\_print\_violated

A parameter of the problem analyzer. Controls whether a list of violated constraints is printed. All constraints violated by more than the value set by the parameter <code>dparam.ana\_sol\_infeas\_tol</code> will be printed.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.ana_sol_print_violated, onoffkey.off)
Generic name MSK_IPAR_ANA_SOL_PRINT_VIOLATED
Groups Analysis
```

## iparam.auto\_sort\_a\_before\_opt

Controls whether the elements in each column of A are sorted before an optimization is performed. This is not required but makes the optimization more deterministic.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.auto_sort_a_before_opt, onoffkey.off)
Generic name MSK_IPAR_AUTO_SORT_A_BEFORE_OPT
Groups Debugging
```

#### iparam.auto\_update\_sol\_info

Controls whether the solution information items are automatically updated after an optimization is performed.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.auto_update_sol_info, onoffkey.off)
Generic name MSK_IPAR_AUTO_UPDATE_SOL_INFO
Groups Overall system
```

## iparam.basis\_solve\_use\_plus\_one

If a slack variable is in the basis, then the corresponding column in the basis is a unit vector with -1 in the right position. However, if this parameter is set to <code>onoffkey.on</code>, -1 is replaced by 1.

This has significance for the results returned by the Task. solvewithbasis function.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.basis_solve_use_plus_one, onoffkey.off)
Generic name MSK_IPAR_BASIS_SOLVE_USE_PLUS_ONE
Groups Simplex optimizer
```

# iparam.bi\_clean\_optimizer

Controls which simplex optimizer is used in the clean-up phase. Anything else than optimizertype.primal\_simplex or optimizertype.dual\_simplex is equivalent to optimizertype.free\_simplex.

```
Default free
Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex,
    mixed_int (see optimizertype)
Example task.putintparam(iparam.bi_clean_optimizer, optimizertype.free)
Generic name MSK_IPAR_BI_CLEAN_OPTIMIZER
Groups Basis identification, Overall solver
```

# iparam.bi\_ignore\_max\_iter

If the parameter <code>iparam.intpnt\_basis</code> has the value <code>basindtype.no\_error</code> and the interior-point optimizer has terminated due to maximum number of iterations, then basis identification is performed if this parameter has the value <code>onoffkey.on</code>.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.bi_ignore_max_iter, onoffkey.off)
Generic name MSK_IPAR_BI_IGNORE_MAX_ITER
Groups Interior-point method, Basis identification
```

# iparam.bi\_ignore\_num\_error

If the parameter <code>iparam.intpnt\_basis</code> has the value <code>basindtype.no\_error</code> and the interior-point optimizer has terminated due to a numerical problem, then basis identification is performed if this parameter has the value <code>onoffkey.on</code>.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.bi_ignore_num_error, onoffkey.off)
Generic name MSK_IPAR_BI_IGNORE_NUM_ERROR
Groups Interior-point method, Basis identification
```

# iparam.bi\_max\_iterations

Controls the maximum number of simplex iterations allowed to optimize a basis after the basis identification.

```
Default 1000000
Accepted [0; +inf]
Example task.putintparam(iparam.bi_max_iterations, 1000000)
```

```
Generic name MSK IPAR BI MAX ITERATIONS
```

Groups Basis identification, Termination criteria

```
iparam.cache_license
```

Specifies if the license is kept checked out for the lifetime of the  $\mathbf{MOSEK}$  environment/model/process (onoffkey.on) or returned to the server immediately after the optimization (onoffkey.off).

By default the license is checked out for the lifetime of the  $\mathbf{MOSEK}$  environment by the first call to Task.optimize.

Check-in and check-out of licenses have an overhead. Frequent communication with the license server should be avoided.

```
Default on
```

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.cache\_license, onoffkey.on)

Generic name MSK IPAR CACHE LICENSE

Groups License manager

# iparam.check\_convexity

Specify the level of convexity check on quadratic problems.

Default full

Accepted none, simple, full (see checkconvexitytype)

Example task.putintparam(iparam.check\_convexity, checkconvexitytype.

Generic name MSK IPAR CHECK CONVEXITY

Groups Data check

# iparam.compress\_statfile

Control compression of stat files.

Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.compress\_statfile, onoffkey.on)

Generic name MSK IPAR COMPRESS STATFILE

#### iparam.infeas\_generic\_names

Controls whether generic names are used when an infeasible subproblem is created.

Default off

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.infeas\_generic\_names, onoffkey.off)

 ${\bf Generic\ name\ MSK\_IPAR\_INFEAS\_GENERIC\_NAMES}$ 

Groups Infeasibility report

# iparam.infeas\_prefer\_primal

If both certificates of primal and dual infeasibility are supplied then only the primal is used when this option is turned on.

Default on

 $\mathbf{Accepted} \ \mathit{on} \,, \, \mathit{off} \ (\mathrm{see} \ \mathit{onoffkey})$ 

Example task.putintparam(iparam.infeas\_prefer\_primal, onoffkey.on)

Generic name MSK IPAR INFEAS PREFER PRIMAL

Groups Overall solver

## iparam.infeas\_report\_auto

Controls whether an infeasibility report is automatically produced after the optimization if the problem is primal or dual infeasible.

```
Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.infeas_report_auto, onoffkey.off)
         Generic name MSK IPAR INFEAS REPORT AUTO
         Groups Data input/output, Solution input/output
iparam.infeas_report_level
     Controls the amount of information presented in an infeasibility report. Higher values imply more
    information.
         Default 1
         Accepted [0; +inf]
         Example task.putintparam(iparam.infeas_report_level, 1)
         Generic name MSK IPAR INFEAS REPORT LEVEL
         Groups Infeasibility report, Output information
iparam.intpnt_basis
     Controls whether the interior-point optimizer also computes an optimal basis.
         Default always
         Accepted never, always, no_error, if_feasible, reservered (see basindtype)
         Example task.putintparam(iparam.intpnt_basis, basindtype.always)
         See also iparam.bi_iqnore_max_iter, iparam.bi_iqnore_num_error,
             bi_max_iterations, iparam.bi_clean_optimizer
         Generic name MSK IPAR INTPNT BASIS
         Groups Interior-point method, Basis identification
iparam.intpnt_diff_step
     Controls whether different step sizes are allowed in the primal and dual space.
         Default on
         Accepted
              • on: Different step sizes are allowed.
              • off: Different step sizes are not allowed.
         Example task.putintparam(iparam.intpnt_diff_step, onoffkey.on)
         Generic name MSK IPAR INTPNT DIFF STEP
         Groups Interior-point method
iparam.intpnt_hotstart
     Currently not in use.
         Default none
         Accepted none, primal, dual, primal_dual (see intpnthotstart)
         Example task.putintparam(iparam.intpnt_hotstart, intpnthotstart.none)
         Generic name MSK IPAR INTPNT HOTSTART
         Groups Interior-point method
iparam.intpnt_max_iterations
     Controls the maximum number of iterations allowed in the interior-point optimizer.
         Default 400
         Accepted [0; +inf]
         Example task.putintparam(iparam.intpnt_max_iterations, 400)
         Generic name MSK IPAR INTPNT MAX ITERATIONS
         Groups Interior-point method, Termination criteria
```

## iparam.intpnt\_max\_num\_cor

Controls the maximum number of correctors allowed by the multiple corrector procedure. A negative value means that **MOSEK** is making the choice.

#### Default -1

Accepted  $[-1; +\inf]$ 

Example task.putintparam(iparam.intpnt\_max\_num\_cor, -1)

Generic name MSK IPAR INTPNT MAX NUM COR

Groups Interior-point method

# iparam.intpnt\_max\_num\_refinement\_steps

Maximum number of steps to be used by the iterative refinement of the search direction. A negative value implies that the optimizer chooses the maximum number of iterative refinement steps.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.intpnt\_max\_num\_refinement\_steps, -1)

Generic name MSK\_IPAR\_INTPNT\_MAX\_NUM\_REFINEMENT\_STEPS

Groups Interior-point method

# iparam.intpnt\_multi\_thread

Controls whether the interior-point optimizers are allowed to employ multiple threads if more threads is available.

## Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.intpnt\_multi\_thread, onoffkey.on)

Generic name MSK IPAR INTPNT MULTI THREAD

Groups Overall system

# iparam.intpnt\_off\_col\_trh

Controls how many offending columns are detected in the Jacobian of the constraint matrix.

0	no detection
1	aggressive detection
> 1	higher values mean less aggressive detection

# Default 40

**Accepted**  $[0; +\inf]$ 

Example task.putintparam(iparam.intpnt\_off\_col\_trh, 40)

Generic name MSK\_IPAR\_INTPNT\_OFF\_COL\_TRH

**Groups** Interior-point method

# iparam.intpnt\_order\_gp\_num\_seeds

The GP ordering is dependent on a random seed. Therefore, trying several random seeds may lead to a better ordering. This parameter controls the number of random seeds tried.

A value of 0 means that MOSEK makes the choice.

# **Default** 0

Accepted [0; +inf]

Example task.putintparam(iparam.intpnt\_order\_gp\_num\_seeds, 0)

Generic name MSK IPAR INTPNT ORDER GP NUM SEEDS

**Groups** Interior-point method

## iparam.intpnt\_order\_method

Controls the ordering strategy used by the interior-point optimizer when factorizing the Newton equation system.

```
Default free
         Accepted free, appminloc, experimental, try_graphpar, force_graphpar, none
            (see orderingtype)
         Example task.putintparam(iparam.intpnt_order_method, orderingtype.free)
         Generic name MSK IPAR INTPNT ORDER METHOD
         Groups Interior-point method
iparam.intpnt_purify
    Currently not in use.
         Default none
         Accepted none, primal, dual, primal_dual, auto (see purify)
         Example task.putintparam(iparam.intpnt_purify, purify.none)
         Generic name MSK IPAR INTPNT PURIFY
         Groups Interior-point method
iparam.intpnt_regularization_use
    Controls whether regularization is allowed.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.intpnt_regularization_use, onoffkey.on)
         Generic name MSK IPAR INTPNT REGULARIZATION USE
         Groups Interior-point method
iparam.intpnt_scaling
    Controls how the problem is scaled before the interior-point optimizer is used.
         Default free
         Accepted free, none, moderate, aggressive (see scalingtype)
         Example task.putintparam(iparam.intpnt_scaling, scalingtype.free)
         Generic name MSK IPAR INTPNT SCALING
         Groups Interior-point method
iparam.intpnt_solve_form
    Controls whether the primal or the dual problem is solved.
         Default free
         Accepted free, primal, dual (see solveform)
         Example task.putintparam(iparam.intpnt_solve_form, solveform.free)
         Generic name MSK IPAR INTPNT SOLVE FORM
         Groups Interior-point method
iparam.intpnt_starting_point
    Starting point used by the interior-point optimizer.
         Default free
         Accepted free, quess, constant, satisfy_bounds (see startpointtype)
         Example task.putintparam(iparam.intpnt_starting_point, startpointtype.
            free)
         Generic name MSK IPAR INTPNT STARTING POINT
         Groups Interior-point method
iparam.license_debug
    This option is used to turn on debugging of the license manager.
         Default off
         Accepted on, off (see onoffkey)
```

```
Example task.putintparam(iparam.license_debug, onoffkey.off)
Generic name MSK_IPAR_LICENSE_DEBUG
Groups License manager
```

# iparam.license\_pause\_time

If *iparam.license\_wait* is *onoffkey.on* and no license is available, then **MOSEK** sleeps a number of milliseconds between each check of whether a license has become free.

Default 100
Accepted [0; 1000000]
Example task.putintparam(iparam.license\_pause\_time, 100)
Generic name MSK\_IPAR\_LICENSE\_PAUSE\_TIME
Groups License manager

# iparam.license\_suppress\_expire\_wrns

Controls whether license features expire warnings are suppressed.

## iparam.license\_trh\_expiry\_wrn

If a license feature expires in a numbers of days less than the value of this parameter then a warning will be issued.

Default 7
Accepted [0; +inf]
Example task.putintparam(iparam.license\_trh\_expiry\_wrn, 7)
Generic name MSK\_IPAR\_LICENSE\_TRH\_EXPIRY\_WRN
Groups License manager, Output information

## iparam.license\_wait

If all licenses are in use **MOSEK** returns with an error code. However, by turning on this parameter **MOSEK** will wait for an available license.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.license_wait, onoffkey.off)
Generic name MSK_IPAR_LICENSE_WAIT
Groups Overall solver, Overall system, License manager
```

# iparam.log

Controls the amount of log information. The value 0 implies that all log information is suppressed. A higher level implies that more information is logged.

Please note that if a task is employed to solve a sequence of optimization problems the value of this parameter is reduced by the value of  $iparam.log\_cut\_second\_opt$  for the second and any subsequent optimizations.

```
Default 10
Accepted [0; +inf]
Example task.putintparam(iparam.log, 10)
See also iparam.log_cut_second_opt
Generic name MSK_IPAR_LOG
Groups Output information, Logging
```

#### iparam.log\_ana\_pro

Controls amount of output from the problem analyzer.

#### Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_ana\_pro, 1)

Generic name MSK IPAR LOG ANA PRO

Groups Analysis, Logging

#### iparam.log\_bi

Controls the amount of output printed by the basis identification procedure. A higher level implies that more information is logged.

#### Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_bi, 1)

Generic name MSK IPAR LOG BI

**Groups** Basis identification, Output information, Logging

# iparam.log\_bi\_freq

Controls how frequently the optimizer outputs information about the basis identification and how frequent the user-defined callback function is called.

Default 2500

Accepted [0; +inf]

Example task.putintparam(iparam.log\_bi\_freq, 2500)

Generic name MSK IPAR LOG BI FREQ

**Groups** Basis identification, Output information, Logging

# iparam.log\_check\_convexity

Controls logging in convexity check on quadratic problems. Set to a positive value to turn logging on. If a quadratic coefficient matrix is found to violate the requirement of PSD (NSD) then a list of negative (positive) pivot elements is printed. The absolute value of the pivot elements is also shown.

# Default 0

Accepted [0; +inf]

Example task.putintparam(iparam.log\_check\_convexity, 0)

Generic name MSK\_IPAR\_LOG\_CHECK\_CONVEXITY

Groups Data check

# iparam.log\_cut\_second\_opt

If a task is employed to solve a sequence of optimization problems, then the value of the log levels is reduced by the value of this parameter. E.g *iparam.log* and *iparam.log\_sim* are reduced by the value of this parameter for the second and any subsequent optimizations.

# Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_cut\_second\_opt, 1)

See also iparam.log, iparam.log\_intpnt, iparam.log\_mio, iparam.log\_sim

 ${\bf Generic\ name\ MSK\_IPAR\_LOG\_CUT\_SECOND\_OPT}$ 

Groups Output information, Logging

# iparam.log\_expand

Controls the amount of logging when a data item such as the maximum number constrains is expanded.

# **Default** 0

Accepted [0; +inf]

```
Example task.putintparam(iparam.log_expand, 0)
Generic name MSK_IPAR_LOG_EXPAND
Groups Output information, Logging
```

## iparam.log\_feas\_repair

Controls the amount of output printed when performing feasibility repair. A value higher than one means extensive logging.

Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_feas\_repair, 1)

Generic name MSK IPAR LOG FEAS REPAIR

Groups Output information, Logging

## iparam.log\_file

If turned on, then some log info is printed when a file is written or read.

Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_file, 1)

Generic name MSK IPAR LOG FILE

Groups Data input/output, Output information, Logging

#### iparam.log\_include\_summary

If on, then the solution summary will be printed by *Task.optimize*, so a separate call to *Task. solutionsummary* is not necessary.

Default off

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.log\_include\_summary, onoffkey.off)

Generic name MSK\_IPAR\_LOG\_INCLUDE\_SUMMARY

Groups Output information, Logging

## iparam.log\_infeas\_ana

Controls amount of output printed by the infeasibility analyzer procedures. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_infeas\_ana, 1)

Generic name MSK IPAR LOG INFEAS ANA

Groups Infeasibility report, Output information, Logging

# iparam.log\_intpnt

Controls amount of output printed by the interior-point optimizer. A higher level implies that more information is logged.

Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_intpnt, 1)

Generic name MSK IPAR LOG INTPNT

Groups Interior-point method, Output information, Logging

# iparam.log\_local\_info

Controls whether local identifying information like environment variables, filenames, IP addresses etc. are printed to the log.

Note that this will only affect some functions. Some functions that specifically emit system information will not be affected.

```
Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.log_local_info, onoffkey.on)
         Generic name MSK IPAR LOG LOCAL INFO
         Groups Output information, Logging
iparam.log_mio
     Controls the log level for the mixed-integer optimizer. A higher level implies that more information
     is logged.
         Default 4
         Accepted [0; +\inf]
         Example task.putintparam(iparam.log_mio, 4)
         Generic name MSK IPAR LOG MIO
         Groups Mixed-integer optimization, Output information, Logging
iparam.log_mio_freq
     Controls how frequent the mixed-integer optimizer prints the log line. It will print line every time
     iparam. log_mio_freq relaxations have been solved.
         Default 10
         Accepted [-\inf; +\inf]
         Example task.putintparam(iparam.log_mio_freq, 10)
         Generic name MSK IPAR LOG MIO FREQ
         Groups Mixed-integer optimization, Output information, Logging
iparam.log_order
     If turned on, then factor lines are added to the log.
         Default 1
         Accepted [0; +\inf]
         Example task.putintparam(iparam.log_order, 1)
         Generic name MSK IPAR LOG ORDER
         Groups Output information, Logging
iparam.log_presolve
     Controls amount of output printed by the presolve procedure. A higher level implies that more
     information is logged.
         Default 1
         Accepted [0; +inf]
         Example task.putintparam(iparam.log_presolve, 1)
         Generic name MSK IPAR LOG PRESOLVE
         Groups Logging
iparam.log_response
     Controls amount of output printed when response codes are reported. A higher level implies that
     more information is logged.
         Default 0
         Accepted [0; +inf]
         Example task.putintparam(iparam.log_response, 0)
         Generic name MSK IPAR LOG RESPONSE
         Groups Output information, Logging
```

# iparam.log\_sensitivity

Controls the amount of logging during the sensitivity analysis.

• 0. Means no logging information is produced.

- 1. Timing information is printed.
- 2. Sensitivity results are printed.

## Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_sensitivity, 1)

Generic name MSK IPAR LOG SENSITIVITY

Groups Output information, Logging

## iparam.log\_sensitivity\_opt

Controls the amount of logging from the optimizers employed during the sensitivity analysis. 0 means no logging information is produced.

# Default 0

Accepted [0; +inf]

Example task.putintparam(iparam.log\_sensitivity\_opt, 0)

Generic name MSK IPAR LOG SENSITIVITY OPT

Groups Output information, Logging

#### iparam.log\_sim

Controls amount of output printed by the simplex optimizer. A higher level implies that more information is logged.

#### Default 4

Accepted [0; +inf]

Example task.putintparam(iparam.log\_sim, 4)

Generic name MSK IPAR LOG SIM

Groups Simplex optimizer, Output information, Logging

# iparam.log\_sim\_freq

Controls how frequent the simplex optimizer outputs information about the optimization and how frequent the user-defined callback function is called.

# Default 1000

Accepted [0; +inf]

Example task.putintparam(iparam.log\_sim\_freq, 1000)

Generic name MSK\_IPAR\_LOG\_SIM\_FREQ

Groups Simplex optimizer, Output information, Logging

## iparam.log\_sim\_minor

Currently not in use.

## Default 1

Accepted [0; +inf]

Example task.putintparam(iparam.log\_sim\_minor, 1)

Generic name MSK IPAR LOG SIM MINOR

**Groups** Simplex optimizer, Output information

# iparam.log\_storage

When turned on, MOSEK prints messages regarding the storage usage and allocation.

#### Default 0

Accepted [0; +inf]

Example task.putintparam(iparam.log\_storage, 0)

Generic name MSK\_IPAR\_LOG\_STORAGE

Groups Output information, Overall system, Logging

```
iparam.max_num_warnings
```

Each warning is shown a limited number of times controlled by this parameter. A negative value is identical to infinite number of times.

```
Default 10
```

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.max\_num\_warnings, 10)

Generic name MSK IPAR MAX NUM WARNINGS

Groups Output information

#### iparam.mio\_branch\_dir

Controls whether the mixed-integer optimizer is branching up or down by default.

# Default free

Accepted free, up, down, near, far, root\_lp, guided, pseudocost (see branchdir)

Example task.putintparam(iparam.mio\_branch\_dir, branchdir.free)

Generic name MSK\_IPAR\_MIO\_BRANCH\_DIR

Groups Mixed-integer optimization

# iparam.mio\_conic\_outer\_approximation

If this option is turned on outer approximation is used when solving relaxations of conic problems; otherwise interior point is used.

Default off

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_conic\_outer\_approximation, onoffkey.off)

Generic name MSK\_IPAR\_MIO\_CONIC\_OUTER\_APPROXIMATION

**Groups** Mixed-integer optimization

# iparam.mio\_cut\_clique

Controls whether clique cuts should be generated.

Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_cut\_clique, onoffkey.on)

Generic name MSK IPAR MIO CUT CLIQUE

**Groups** Mixed-integer optimization

#### iparam.mio\_cut\_cmir

Controls whether mixed integer rounding cuts should be generated.

Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_cut\_cmir, onoffkey.on)

Generic name MSK IPAR MIO CUT CMIR

**Groups** Mixed-integer optimization

#### iparam.mio\_cut\_gmi

Controls whether GMI cuts should be generated.

Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_cut\_gmi, onoffkey.on)

 ${\bf Generic\ name\ MSK\_IPAR\_MIO\_CUT\_GMI}$ 

**Groups** Mixed-integer optimization

#### iparam.mio\_cut\_implied\_bound

Controls whether implied bound cuts should be generated.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.mio_cut_implied_bound, onoffkey.off)
Generic name MSK_IPAR_MIO_CUT_IMPLIED_BOUND
Groups Mixed-integer optimization
```

## iparam.mio\_cut\_knapsack\_cover

Controls whether knapsack cover cuts should be generated.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.mio_cut_knapsack_cover, onoffkey.off)
Generic name MSK_IPAR_MIO_CUT_KNAPSACK_COVER
Groups Mixed-integer optimization
```

# iparam.mio\_cut\_selection\_level

Controls how aggressively generated cuts are selected to be included in the relaxation.

- $\bullet$  -1. The optimizer chooses the level of cut selection
- ullet 0. Generated cuts less likely to be added to the relaxation
- 1. Cuts are more aggressively selected to be included in the relaxation

```
Default -1
Accepted [-1; +1]
Example task.putintparam(iparam.mio_cut_selection_level, -1)
Generic name MSK_IPAR_MIO_CUT_SELECTION_LEVEL
Groups Mixed-integer optimization
```

# iparam.mio\_feaspump\_level

Controls the way the Feasibility Pump heuristic is employed by the mixed-integer optimizer.

- $\bullet$  -1. The optimizer chooses how the Feasibility Pump is used
- 0. The Feasibility Pump is disabled
- 1. The Feasibility Pump is enabled with an effort to improve solution quality
- 2. The Feasibility Pump is enabled with an effort to reach feasibility early

```
Default -1
Accepted [-1; 2]
Example task.putintparam(iparam.mio_feaspump_level, -1)
Generic name MSK_IPAR_MIO_FEASPUMP_LEVEL
Groups Mixed-integer optimization
```

# iparam.mio\_heuristic\_level

Controls the heuristic employed by the mixed-integer optimizer to locate an initial good integer feasible solution. A value of zero means the heuristic is not used at all. A larger value than 0 means that a gradually more sophisticated heuristic is used which is computationally more expensive. A negative value implies that the optimizer chooses the heuristic. Normally a value around 3 to 5 should be optimal.

```
Default -1
Accepted [-inf; +inf]
Example task.putintparam(iparam.mio_heuristic_level, -1)
Generic name MSK_IPAR_MIO_HEURISTIC_LEVEL
Groups Mixed-integer optimization
```

## iparam.mio\_max\_num\_branches

Maximum number of branches allowed during the branch and bound search. A negative value means infinite.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.mio\_max\_num\_branches, -1)

Generic name MSK IPAR MIO MAX NUM BRANCHES

Groups Mixed-integer optimization, Termination criteria

#### iparam.mio\_max\_num\_relaxs

Maximum number of relaxations allowed during the branch and bound search. A negative value means infinite.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.mio\_max\_num\_relaxs, -1)

Generic name MSK IPAR MIO MAX NUM RELAXS

**Groups** Mixed-integer optimization

## iparam.mio\_max\_num\_root\_cut\_rounds

Maximum number of cut separation rounds at the root node.

#### Default 100

Accepted  $[0; +\inf]$ 

Example task.putintparam(iparam.mio\_max\_num\_root\_cut\_rounds, 100)

Generic name MSK IPAR MIO MAX NUM ROOT CUT ROUNDS

Groups Mixed-integer optimization, Termination criteria

# iparam.mio\_max\_num\_solutions

The mixed-integer optimizer can be terminated after a certain number of different feasible solutions has been located. If this parameter has the value n > 0, then the mixed-integer optimizer will be terminated when n feasible solutions have been located.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.mio\_max\_num\_solutions, -1)

Generic name MSK IPAR MIO MAX NUM SOLUTIONS

Groups Mixed-integer optimization, Termination criteria

# ${\tt iparam.mio\_mode}$

Controls whether the optimizer includes the integer restrictions when solving a (mixed) integer optimization problem.

# Default satisfied

Accepted ignored, satisfied (see miomode)

Example task.putintparam(iparam.mio\_mode, miomode.satisfied)

Generic name MSK IPAR MIO MODE

Groups Overall solver

# iparam.mio\_node\_optimizer

Controls which optimizer is employed at the non-root nodes in the mixed-integer optimizer.

# Default free

```
 \begin{array}{lll} \textbf{Accepted} & \textit{free}, & \textit{intpnt}, & \textit{conic}, & \textit{primal\_simplex}, & \textit{dual\_simplex}, & \textit{free\_simplex}, \\ & & \textit{mixed\_int} & (\text{see } \textit{optimizertype}) \end{array}
```

Example task.putintparam(iparam.mio\_node\_optimizer, optimizertype.free)

Generic name MSK IPAR MIO NODE OPTIMIZER

# **Groups** Mixed-integer optimization

## iparam.mio\_node\_selection

Controls the node selection strategy employed by the mixed-integer optimizer.

Default free

Accepted free, first, best, pseudo (see mionodeseltype)

Example task.putintparam(iparam.mio\_node\_selection, mionodeseltype. free)

Generic name MSK\_IPAR\_MIO\_NODE\_SELECTION

**Groups** Mixed-integer optimization

#### iparam.mio\_perspective\_reformulate

Enables or disables perspective reformulation in presolve.

Default on

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_perspective\_reformulate, onoffkey. on)

Generic name MSK\_IPAR\_MIO\_PERSPECTIVE\_REFORMULATE

**Groups** Mixed-integer optimization

# iparam.mio\_probing\_level

Controls the amount of probing employed by the mixed-integer optimizer in presolve.

- $\bullet$  -1. The optimizer chooses the level of probing employed
- 0. Probing is disabled
- 1. A low amount of probing is employed
- 2. A medium amount of probing is employed
- 3. A high amount of probing is employed

Default -1

Accepted [-1; 3]

Example task.putintparam(iparam.mio\_probing\_level, -1)

Generic name MSK\_IPAR\_MIO\_PROBING\_LEVEL

**Groups** Mixed-integer optimization

# iparam.mio\_propagate\_objective\_constraint

Use objective domain propagation.

Default off

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.mio\_propagate\_objective\_constraint, onoffkey.off)

Generic name MSK\_IPAR\_MIO\_PROPAGATE\_OBJECTIVE\_CONSTRAINT

**Groups** Mixed-integer optimization

# ${\tt iparam.mio\_rins\_max\_nodes}$

Controls the maximum number of nodes allowed in each call to the RINS heuristic. The default value of -1 means that the value is determined automatically. A value of zero turns off the heuristic.

Default -1

Accepted [-1; +inf]

Example task.putintparam(iparam.mio\_rins\_max\_nodes, -1)

Generic name MSK\_IPAR\_MIO\_RINS\_MAX\_NODES

**Groups** Mixed-integer optimization

#### iparam.mio\_root\_optimizer

Controls which optimizer is employed at the root node in the mixed-integer optimizer.

```
Default free
```

```
Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex, mixed_int (see optimizertype)
```

Example task.putintparam(iparam.mio\_root\_optimizer, optimizertype.free)

Generic name MSK IPAR MIO ROOT OPTIMIZER

**Groups** Mixed-integer optimization

## iparam.mio\_root\_repeat\_presolve\_level

Controls whether presolve can be repeated at root node.

- $\bullet$  -1. The optimizer chooses whether presolve is repeated
- 0. Never repeat presolve
- 1. Always repeat presolve

## Default -1

Accepted [-1; 1]

Example task.putintparam(iparam.mio\_root\_repeat\_presolve\_level, -1)

Generic name MSK\_IPAR\_MIO\_ROOT\_REPEAT\_PRESOLVE LEVEL

**Groups** Mixed-integer optimization

#### iparam.mio\_seed

Sets the random seed used for randomization in the mixed integer optimizer. Selecting a different seed can change the path the optimizer takes to the optimal solution.

#### Default 42

Accepted [0; +inf]

Example task.putintparam(iparam.mio\_seed, 42)

Generic name MSK IPAR MIO SEED

**Groups** Mixed-integer optimization

#### iparam.mio\_vb\_detection\_level

Controls how much effort is put into detecting variable bounds.

- -1. The optimizer chooses
- 0. No variable bounds are detected
- 1. Only detect variable bounds that are directly represented in the problem
- 2. Detect variable bounds in probing

#### Default -1

Accepted [-1; +2]

Example task.putintparam(iparam.mio\_vb\_detection\_level, -1)

Generic name MSK IPAR MIO VB DETECTION LEVEL

 ${\bf Groups}\ {\it Mixed-integer\ optimization}$ 

# ${\tt iparam.mt\_spincount}$

Set the number of iterations to spin before sleeping.

#### Default 0

**Accepted** [0; 1000000000]

Example task.putintparam(iparam.mt\_spincount, 0)

Generic name MSK IPAR MT SPINCOUNT

Groups Overall system

```
iparam.num_threads
```

Controls the number of threads employed by the optimizer. If set to 0 the number of threads used will be equal to the number of cores detected on the machine.

If using the conic optimizer, the value of this parameter set at first optimization remains constant through the lifetime of the process. **MOSEK** will allocate a thread pool of given size, and changing the parameter value later will have no effect. It will, however, remain possible to demand single-threaded execution by setting <code>iparam.intpnt\_multi\_thread</code>.

For the mixed-integer optimizer and interior-point linear optimizer there is no such restriction.

```
Default 0
         Accepted [0; +\inf]
         Example task.putintparam(iparam.num_threads, 0)
         Generic name MSK IPAR NUM THREADS
         Groups Overall system
iparam.opf_write_header
    Write a text header with date and MOSEK version in an OPF file.
        Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_header, onoffkey.on)
         Generic name MSK_IPAR_OPF WRITE HEADER
         Groups Data input/output
iparam.opf_write_hints
    Write a hint section with problem dimensions in the beginning of an OPF file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_hints, onoffkey.on)
         Generic name MSK IPAR OPF WRITE HINTS
         Groups Data input/output
iparam.opf_write_line_length
     Aim to keep lines in OPF files not much longer than this.
        Default 80
         Accepted [0; +inf]
        Example task.putintparam(iparam.opf_write_line_length, 80)
         Generic name MSK IPAR OPF WRITE LINE LENGTH
         Groups Data input/output
iparam.opf_write_parameters
    Write a parameter section in an OPF file.
        Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_parameters, onoffkey.off)
         Generic name MSK IPAR OPF WRITE PARAMETERS
         Groups Data input/output
iparam.opf_write_problem
    Write objective, constraints, bounds etc. to an OPF file.
        Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_problem, onoffkey.on)
         Generic name MSK IPAR OPF WRITE PROBLEM
```

```
Groups Data input/output
iparam.opf_write_sol_bas
    If iparam. opf_write_solutions is onoffkey. on and a basic solution is defined, include the basic
    solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_sol_bas, onoffkey.on)
         Generic name MSK IPAR OPF WRITE SOL BAS
         Groups Data input/output
iparam.opf_write_sol_itg
    If iparam.opf_write_solutions is onoffkey.on and an integer solution is defined, write the
    integer solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_sol_itg, onoffkey.on)
         Generic name MSK IPAR OPF WRITE SOL ITG
         Groups Data input/output
iparam.opf_write_sol_itr
    If iparam.opf_write_solutions is onoffkey.on and an interior solution is defined, write the
    interior solution in OPF files.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_sol_itr, onoffkey.on)
         Generic name MSK IPAR OPF WRITE SOL ITR
         Groups Data input/output
iparam.opf_write_solutions
    Enable inclusion of solutions in the OPF files.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.opf_write_solutions, onoffkey.off)
         Generic name MSK IPAR OPF WRITE SOLUTIONS
         Groups Data input/output
iparam.optimizer
    The parameter controls which optimizer is used to optimize the task.
         Default free
         Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex,
             mixed_int (see optimizertype)
         Example task.putintparam(iparam.optimizer, optimizertype.free)
         Generic name MSK IPAR OPTIMIZER
         Groups Overall solver
iparam.param_read_case_name
    If turned on, then names in the parameter file are case sensitive.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.param_read_case_name, onoffkey.on)
         Generic name MSK IPAR PARAM READ CASE NAME
         Groups Data input/output
```

```
iparam.param_read_ign_error
    If turned on, then errors in parameter settings is ignored.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.param_read_ign_error, onoffkey.off)
         Generic name MSK IPAR PARAM READ IGN ERROR
         Groups Data input/output
iparam.presolve_eliminator_max_fill
     Controls the maximum amount of fill-in that can be created by one pivot in the elimination phase
    of the presolve. A negative value means the parameter value is selected automatically.
         Default -1
         Accepted [-\inf; +\inf]
         Example task.putintparam(iparam.presolve_eliminator_max_fill, -1)
         Generic name MSK IPAR PRESOLVE ELIMINATOR MAX FILL
         Groups Presolve
iparam.presolve_eliminator_max_num_tries
     Control the maximum number of times the eliminator is tried. A negative value implies MOSEK
     decides.
         Default -1
         Accepted [-\inf; +\inf]
         Example task.putintparam(iparam.presolve_eliminator_max_num_tries, -1)
         Generic name MSK IPAR PRESOLVE ELIMINATOR MAX NUM TRIES
         Groups Presolve
iparam.presolve_level
     Currently not used.
         Default -1
         Accepted [-\inf; +\inf]
         Example task.putintparam(iparam.presolve_level, -1)
         Generic name MSK IPAR PRESOLVE LEVEL
         Groups Overall solver, Presolve
iparam.presolve_lindep_abs_work_trh
     Controls linear dependency check in presolve. The linear dependency check is potentially compu-
     tationally expensive.
         Default 100
         Accepted [-\inf; +\inf]
         Example task.putintparam(iparam.presolve_lindep_abs_work_trh, 100)
         Generic name MSK IPAR PRESOLVE LINDEP ABS WORK TRH
         Groups Presolve
iparam.presolve_lindep_rel_work_trh
     Controls linear dependency check in presolve. The linear dependency check is potentially compu-
     tationally expensive.
         Default 100
         Accepted [-inf; +inf]
```

Example task.putintparam(iparam.presolve\_lindep\_rel\_work\_trh, 100) Generic name MSK IPAR PRESOLVE LINDEP REL WORK TRH

Groups Presolve

```
iparam.presolve_lindep_use
```

Controls whether the linear constraints are checked for linear dependencies.

```
Default on
```

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.presolve\_lindep\_use, onoffkey.on)

Generic name MSK IPAR PRESOLVE LINDEP USE

Groups Presolve

## iparam.presolve\_max\_num\_pass

Control the maximum number of times presolve passes over the problem. A negative value implies **MOSEK** decides.

#### Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.presolve\_max\_num\_pass, -1)

Generic name MSK IPAR PRESOLVE MAX NUM PASS

Groups Presolve

#### iparam.presolve\_max\_num\_reductions

Controls the maximum number of reductions performed by the presolve. The value of the parameter is normally only changed in connection with debugging. A negative value implies that an infinite number of reductions are allowed.

## Default -1

Accepted  $[-\inf; +\inf]$ 

Example task.putintparam(iparam.presolve\_max\_num\_reductions, -1)

Generic name MSK IPAR PRESOLVE MAX NUM REDUCTIONS

Groups Overall solver, Presolve

#### iparam.presolve\_use

Controls whether the presolve is applied to a problem before it is optimized.

```
Default free
```

Accepted off, on, free (see presolvemode)

Example task.putintparam(iparam.presolve\_use, presolvemode.free)

Generic name MSK\_IPAR\_PRESOLVE\_USE

Groups Overall solver, Presolve

# iparam.primal\_repair\_optimizer

Controls which optimizer that is used to find the optimal repair.

# Default free

```
 \begin{array}{lll} \textbf{Accepted} & free, & intpnt, & conic, & primal\_simplex, & dual\_simplex, & free\_simplex, \\ & & mixed\_int & (see & optimizertype) \end{array}
```

Example task.putintparam(iparam.primal\_repair\_optimizer, optimizertype. free)

Generic name MSK IPAR PRIMAL REPAIR OPTIMIZER

Groups Overall solver

# iparam.ptf\_write\_transform

If *iparam.ptf\_write\_transform* is *onoffkey.on*, constraint blocks with identifiable conic slacks are transformed into conic constraints and the slacks are eliminated.

```
Default on
```

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.ptf\_write\_transform, onoffkey.on)

Generic name MSK IPAR PTF WRITE TRANSFORM

```
iparam.read_debug
    Turns on additional debugging information when reading files.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.read_debug, onoffkey.off)
         Generic name MSK IPAR READ DEBUG
         Groups Data input/output
iparam.read_keep_free_con
     Controls whether the free constraints are included in the problem.
         Default off
         Accepted
              • on: The free constraints are kept.
              • off: The free constraints are discarded.
         Example task.putintparam(iparam.read_keep_free_con, onoffkey.off)
         Generic name MSK IPAR READ KEEP FREE CON
         Groups Data input/output
iparam.read_lp_drop_new_vars_in_bou
    If this option is turned on, MOSEK will drop variables that are defined for the first time in the
    bounds section.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.read_lp_drop_new_vars_in_bou, onoffkey.
            off)
         Generic name MSK IPAR READ LP DROP NEW VARS IN BOU
         Groups Data input/output
iparam.read_lp_quoted_names
    If a name is in quotes when reading an LP file, the quotes will be removed.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.read_lp_quoted_names, onoffkey.on)
         Generic name MSK IPAR READ LP QUOTED NAMES
         Groups Data input/output
iparam.read_mps_format
     Controls how strictly the MPS file reader interprets the MPS format.
         Default free
         Accepted strict, relaxed, free, cplex (see mpsformat)
         Example task.putintparam(iparam.read_mps_format, mpsformat.free)
         Generic name MSK IPAR READ MPS FORMAT
         Groups Data input/output
iparam.read_mps_width
     Controls the maximal number of characters allowed in one line of the MPS file.
         Default 1024
         Accepted [80; +inf]
         Example task.putintparam(iparam.read_mps_width, 1024)
         Generic name MSK IPAR READ MPS WIDTH
```

Groups Data input/output

```
Groups Data input/output
```

Groups Data input/output

## iparam.read\_task\_ignore\_param

Controls whether **MOSEK** should ignore the parameter setting defined in the task file and use the default parameter setting instead.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.read_task_ignore_param, onoffkey.off)
Generic name MSK_IPAR_READ_TASK_IGNORE_PARAM
```

# iparam.remove\_unused\_solutions

Removes unused solutions before the optimization is performed.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.remove_unused_solutions, onoffkey.off)
Generic name MSK_IPAR_REMOVE_UNUSED_SOLUTIONS
Groups Overall system
```

## iparam.sensitivity\_all

If set to onoffkey.on, then Task.sensitivityreport analyzes all bounds and variables instead of reading a specification from the file.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.sensitivity_all, onoffkey.off)
Generic name MSK_IPAR_SENSITIVITY_ALL
Groups Overall solver
```

# iparam.sensitivity\_optimizer

Controls which optimizer is used for optimal partition sensitivity analysis.

```
Default free_simplex

Accepted free, intpnt, conic, primal_simplex, dual_simplex, free_simplex, mixed_int (see optimizertype)

Example task.putintparam(iparam.sensitivity_optimizer, optimizertype. free_simplex)

Generic name MSK_IPAR_SENSITIVITY_OPTIMIZER

Groups Overall solver, Simplex optimizer
```

# iparam.sensitivity\_type

Controls which type of sensitivity analysis is to be performed.

```
Default basis
Accepted basis (see sensitivitytype)
Example task.putintparam(iparam.sensitivity_type, sensitivitytype.
basis)
Generic name MSK_IPAR_SENSITIVITY_TYPE
Groups Overall solver
```

# iparam.sim\_basis\_factor\_use

Controls whether an LU factorization of the basis is used in a hot-start. Forcing a refactorization sometimes improves the stability of the simplex optimizers, but in most cases there is a performance penalty.

```
Default on
Accepted on, off (see onoffkey)
```

```
Example task.putintparam(iparam.sim_basis_factor_use, onoffkey.on)
Generic name MSK_IPAR_SIM_BASIS_FACTOR_USE
Groups Simplex optimizer
```

## iparam.sim\_degen

Controls how aggressively degeneration is handled.

```
Default free
```

Accepted none, free, aggressive, moderate, minimum (see simdegen)

Example task.putintparam(iparam.sim\_degen, simdegen.free)

Generic name MSK IPAR SIM DEGEN

Groups Simplex optimizer

# iparam.sim\_dual\_crash

Controls whether crashing is performed in the dual simplex optimizer. If this parameter is set to x, then a crash will be performed if a basis consists of more than  $(100 - x) \mod f_v$  entries, where  $f_v$  is the number of fixed variables.

Default 90

Accepted [0; +inf]

Example task.putintparam(iparam.sim\_dual\_crash, 90)

Generic name MSK IPAR SIM DUAL CRASH

Groups Dual simplex

## iparam.sim\_dual\_phaseone\_method

An experimental feature.

Default 0

**Accepted** [0; 10]

Example task.putintparam(iparam.sim\_dual\_phaseone\_method, 0)

Generic name MSK IPAR SIM DUAL PHASEONE METHOD

Groups Simplex optimizer

## iparam.sim\_dual\_restrict\_selection

The dual simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the dual simplex optimizer first choose a subset of all the potential outgoing variables. Next, for some time it will choose the outgoing variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

**Default** 50

**Accepted** [0; 100]

Example task.putintparam(iparam.sim\_dual\_restrict\_selection, 50)

Generic name MSK IPAR SIM DUAL RESTRICT SELECTION

 ${\bf Groups}\ \ {\it Dual\ simplex}$ 

# ${\tt iparam.sim\_dual\_selection}$

Controls the choice of the incoming variable, known as the selection strategy, in the dual simplex optimizer.

```
Default free
```

Accepted free, full, ase, devex, se, partial (see simseltype)

Example task.putintparam(iparam.sim\_dual\_selection, simseltype.free)

Generic name MSK IPAR SIM DUAL SELECTION

Groups Dual simplex

# ${\tt iparam.sim\_exploit\_dupvec}$

Controls if the simplex optimizers are allowed to exploit duplicated columns.

```
Default off
Accepted on, off, free (see simdupvec)
Example task.putintparam(iparam.sim_exploit_dupvec, simdupvec.off)
Generic name MSK_IPAR_SIM_EXPLOIT_DUPVEC
Groups Simplex optimizer
```

#### iparam.sim\_hotstart

Controls the type of hot-start that the simplex optimizer perform.

```
Default free
Accepted none, free, status_keys (see simhotstart)
Example task.putintparam(iparam.sim_hotstart, simhotstart.free)
Generic name MSK_IPAR_SIM_HOTSTART
Groups Simplex optimizer
```

## iparam.sim\_hotstart\_lu

Determines if the simplex optimizer should exploit the initial factorization.

# Default on Accepted

- on: Factorization is reused if possible.
- *off*: Factorization is recomputed.

 $\label{lem:condition} \textbf{Example task.putintparam(iparam.sim\_hotstart\_lu, onoffkey.on)} \\ \textbf{Generic name } \mbox{MSK\_IPAR\_SIM\_HOTSTART\_LU}$ 

Groups Simplex optimizer

## iparam.sim\_max\_iterations

Maximum number of iterations that can be used by a simplex optimizer.

```
Default 100000000
Accepted [0; +inf]
Example task.putintparam(iparam.sim_max_iterations, 10000000)
Generic name MSK_IPAR_SIM_MAX_ITERATIONS
Groups Simplex optimizer, Termination criteria
```

#### iparam.sim\_max\_num\_setbacks

Controls how many set-backs are allowed within a simplex optimizer. A set-back is an event where the optimizer moves in the wrong direction. This is impossible in theory but may happen due to numerical problems.

```
\begin{tabular}{ll} \textbf{Default} & 250 \\ \textbf{Accepted} & [0;+\inf] \\ \textbf{Example} & \texttt{task.putintparam(iparam.sim\_max\_num\_setbacks, 250)} \\ \textbf{Generic name} & \texttt{MSK\_IPAR\_SIM\_MAX\_NUM\_SETBACKS} \\ \textbf{Groups} & \textit{Simplex optimizer} \\ \end{tabular}
```

# ${\tt iparam.sim\_non\_singular}$

Controls if the simplex optimizer ensures a non-singular basis, if possible.

```
Default on
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.sim_non_singular, onoffkey.on)
Generic name MSK_IPAR_SIM_NON_SINGULAR
Groups Simplex optimizer
```

# iparam.sim\_primal\_crash

Controls whether crashing is performed in the primal simplex optimizer. In general, if a basis consists of more than (100-this parameter value)% fixed variables, then a crash will be performed.

```
Default 90
```

Accepted [0; +inf]

Example task.putintparam(iparam.sim\_primal\_crash, 90)

Generic name MSK IPAR SIM PRIMAL CRASH

Groups Primal simplex

#### iparam.sim\_primal\_phaseone\_method

An experimental feature.

# Default 0

**Accepted** [0; 10]

 $\mathbf{Example} \ \mathtt{task.putintparam(iparam.sim\_primal\_phaseone\_method,\ 0)}$ 

Generic name MSK IPAR SIM PRIMAL PHASEONE METHOD

Groups Simplex optimizer

# iparam.sim\_primal\_restrict\_selection

The primal simplex optimizer can use a so-called restricted selection/pricing strategy to choose the outgoing variable. Hence, if restricted selection is applied, then the primal simplex optimizer first choose a subset of all the potential incoming variables. Next, for some time it will choose the incoming variable only among the subset. From time to time the subset is redefined. A larger value of this parameter implies that the optimizer will be more aggressive in its restriction strategy, i.e. a value of 0 implies that the restriction strategy is not applied at all.

Default 50

**Accepted** [0; 100]

Example task.putintparam(iparam.sim\_primal\_restrict\_selection, 50)

Generic name MSK IPAR SIM PRIMAL RESTRICT SELECTION

Groups Primal simplex

# ${\tt iparam.sim\_primal\_selection}$

Controls the choice of the incoming variable, known as the selection strategy, in the primal simplex optimizer.

Default free

Accepted free, full, ase, devex, se, partial (see simseltype)

Example task.putintparam(iparam.sim\_primal\_selection, simseltype.free)

 ${\bf Generic\ name\ MSK\_IPAR\_SIM\_PRIMAL\_SELECTION}$ 

Groups Primal simplex

#### iparam.sim\_refactor\_freq

Controls how frequent the basis is refactorized. The value 0 means that the optimizer determines the best point of refactorization. It is strongly recommended NOT to change this parameter.

Default 0

Accepted [0; +inf]

Example task.putintparam(iparam.sim\_refactor\_freq, 0)

Generic name MSK IPAR SIM REFACTOR FREQ

Groups Simplex optimizer

#### iparam.sim\_reformulation

Controls if the simplex optimizers are allowed to reformulate the problem.

Default off

Accepted on, off, free, aggressive (see simreform)

Example task.putintparam(iparam.sim\_reformulation, simreform.off)

Generic name MSK IPAR SIM REFORMULATION

Groups Simplex optimizer

```
iparam.sim_save_lu
```

Controls if the LU factorization stored should be replaced with the LU factorization corresponding to the initial basis.

Default off

Accepted on, off (see onoffkey)

Example task.putintparam(iparam.sim\_save\_lu, onoffkey.off)

Generic name MSK IPAR SIM SAVE LU

Groups Simplex optimizer

#### iparam.sim\_scaling

Controls how much effort is used in scaling the problem before a simplex optimizer is used.

Default free

Accepted free, none, moderate, aggressive (see scalingtype)

Example task.putintparam(iparam.sim\_scaling, scalingtype.free)

Generic name MSK IPAR SIM SCALING

Groups Simplex optimizer

## iparam.sim\_scaling\_method

Controls how the problem is scaled before a simplex optimizer is used.

Default pow2

Accepted pow2, free (see scalingmethod)

Example task.putintparam(iparam.sim\_scaling\_method, scalingmethod.pow2)

Generic name MSK IPAR SIM SCALING METHOD

Groups Simplex optimizer

# iparam.sim\_seed

Sets the random seed used for randomization in the simplex optimizers.

Default 23456

**Accepted** [0; 32749]

Example task.putintparam(iparam.sim\_seed, 23456)

Generic name MSK IPAR SIM SEED

Groups Simplex optimizer

# ${\tt iparam.sim\_solve\_form}$

Controls whether the primal or the dual problem is solved by the primal-/dual-simplex optimizer.

Default free

Accepted free, primal, dual (see solveform)

Example task.putintparam(iparam.sim\_solve\_form, solveform.free)

Generic name MSK IPAR SIM SOLVE FORM

Groups Simplex optimizer

# iparam.sim\_stability\_priority

Controls how high priority the numerical stability should be given.

**Default** 50

**Accepted** [0; 100]

Example task.putintparam(iparam.sim\_stability\_priority, 50)

Generic name MSK\_IPAR\_SIM\_STABILITY\_PRIORITY

Groups Simplex optimizer

#### iparam.sim\_switch\_optimizer

The simplex optimizer sometimes chooses to solve the dual problem instead of the primal problem. This implies that if you have chosen to use the dual simplex optimizer and the problem is dualized, then it actually makes sense to use the primal simplex optimizer instead. If this parameter is on and the problem is dualized and furthermore the simplex optimizer is chosen to be the primal (dual) one, then it is switched to the dual (primal).

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.sim_switch_optimizer, onoffkey.off)
Generic name MSK_IPAR_SIM_SWITCH_OPTIMIZER
Groups Simplex optimizer
```

#### iparam.sol\_filter\_keep\_basic

If turned on, then basic and super basic constraints and variables are written to the solution file independent of the filter setting.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.sol_filter_keep_basic, onoffkey.off)
Generic name MSK_IPAR_SOL_FILTER_KEEP_BASIC
Groups Solution input/output
```

# iparam.sol\_filter\_keep\_ranged

If turned on, then ranged constraints and variables are written to the solution file independent of the filter setting.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.sol_filter_keep_ranged, onoffkey.off)
Generic name MSK_IPAR_SOL_FILTER_KEEP_RANGED
Groups Solution input/output
```

#### iparam.sol\_read\_name\_width

When a solution is read by **MOSEK** and some constraint, variable or cone names contain blanks, then a maximum name width much be specified. A negative value implies that no name contain blanks.

```
Default -1
Accepted [-inf; +inf]
Example task.putintparam(iparam.sol_read_name_width, -1)
Generic name MSK_IPAR_SOL_READ_NAME_WIDTH
Groups Data input/output, Solution input/output
```

# iparam.sol\_read\_width

Controls the maximal acceptable width of line in the solutions when read by MOSEK.

```
Default 1024
Accepted [80; +inf]
Example task.putintparam(iparam.sol_read_width, 1024)
Generic name MSK_IPAR_SOL_READ_WIDTH
Groups Data input/output, Solution input/output
```

#### iparam.solution\_callback

Indicates whether solution callbacks will be performed during the optimization.

```
Default off
Accepted on, off (see onoffkey)
Example task.putintparam(iparam.solution_callback, onoffkey.off)
```

```
Generic name MSK IPAR SOLUTION CALLBACK
         Groups Progress callback, Overall solver
iparam.timing_level
     Controls the amount of timing performed inside MOSEK.
         Default 1
         Accepted [0; +inf]
         Example task.putintparam(iparam.timing_level, 1)
         Generic name MSK IPAR TIMING LEVEL
         Groups Overall system
iparam.write_bas_constraints
     Controls whether the constraint section is written to the basic solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_bas_constraints, onoffkey.on)
         Generic name MSK IPAR WRITE BAS CONSTRAINTS
         Groups Data input/output, Solution input/output
iparam.write_bas_head
     Controls whether the header section is written to the basic solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_bas_head, onoffkey.on)
         Generic name MSK IPAR WRITE BAS HEAD
         Groups Data input/output, Solution input/output
iparam.write_bas_variables
     Controls whether the variables section is written to the basic solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_bas_variables, onoffkey.on)
         Generic name MSK IPAR WRITE BAS VARIABLES
         Groups Data input/output, Solution input/output
iparam.write_compression
     Controls whether the data file is compressed while it is written. 0 means no compression while
    higher values mean more compression.
         Default 9
         Accepted [0; +inf]
         Example task.putintparam(iparam.write_compression, 9)
         Generic name MSK IPAR WRITE COMPRESSION
         Groups Data input/output
iparam.write_data_param
    If this option is turned on the parameter settings are written to the data file as parameters.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_data_param, onoffkey.off)
         Generic name MSK IPAR WRITE DATA PARAM
         Groups Data input/output
```

```
Controls whether the free constraints are written to the data file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_free_con, onoffkey.on)
         Generic name MSK IPAR WRITE FREE CON
         Groups Data input/output
iparam.write_generic_names
     Controls whether generic names should be used instead of user-defined names when writing to the
    data file.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_generic_names, onoffkey.off)
         Generic name MSK IPAR WRITE GENERIC NAMES
         Groups Data input/output
iparam.write_generic_names_io
    Index origin used in generic names.
         Default 1
         Accepted [0; +inf]
         Example task.putintparam(iparam.write_generic_names_io, 1)
         Generic name MSK IPAR WRITE GENERIC NAMES IO
         Groups Data input/output
iparam.write_ignore_incompatible_items
     Controls if the writer ignores incompatible problem items when writing files.
         Default off
         Accepted
              • on: Ignore items that cannot be written to the current output file format.
              • off: Produce an error if the problem contains items that cannot the written to
                the current output file format.
         Example task.putintparam(iparam.write_ignore_incompatible_items,
             onoffkey.off)
         Generic name MSK IPAR WRITE IGNORE INCOMPATIBLE ITEMS
         Groups Data input/output
iparam.write_int_constraints
     Controls whether the constraint section is written to the integer solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_int_constraints, onoffkey.on)
         Generic name MSK IPAR WRITE INT CONSTRAINTS
         Groups Data input/output, Solution input/output
iparam.write_int_head
     Controls whether the header section is written to the integer solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_int_head, onoffkey.on)
         Generic name MSK IPAR WRITE INT HEAD
         Groups Data input/output, Solution input/output
```

iparam.write\_free\_con

```
iparam.write_int_variables
     Controls whether the variables section is written to the integer solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_int_variables, onoffkey.on)
         Generic name MSK IPAR WRITE INT VARIABLES
         Groups Data input/output, Solution input/output
iparam.write_lp_full_obj
     Write all variables, including the ones with 0-coefficients, in the objective.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_lp_full_obj, onoffkey.on)
         Generic name MSK IPAR WRITE LP FULL OBJ
         Groups Data input/output
iparam.write_lp_line_width
    Maximum width of line in an LP file written by MOSEK.
         Default 80
         Accepted [40; +inf]
         Example task.putintparam(iparam.write_lp_line_width, 80)
         Generic name MSK IPAR WRITE LP LINE WIDTH
         Groups Data input/output
iparam.write_lp_quoted_names
    If this option is turned on, then MOSEK will quote invalid LP names when writing an LP file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_lp_quoted_names, onoffkey.on)
         Generic name MSK IPAR WRITE LP QUOTED NAMES
         Groups Data input/output
iparam.write_lp_strict_format
     Controls whether LP output files satisfy the LP format strictly.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_lp_strict_format, onoffkey.off)
         Generic name MSK IPAR WRITE LP STRICT FORMAT
         Groups Data input/output
iparam.write_lp_terms_per_line
    Maximum number of terms on a single line in an LP file written by \mathbf{MOSEK}. 0 means unlimited.
         Default 10
         Accepted [0; +\inf]
         Example task.putintparam(iparam.write_lp_terms_per_line, 10)
         Generic name MSK IPAR WRITE LP TERMS PER LINE
         Groups Data input/output
iparam.write_mps_format
     Controls in which format the MPS is written.
         Default free
         Accepted strict, relaxed, free, cplex (see mpsformat)
```

```
Generic name MSK IPAR WRITE MPS FORMAT
         Groups Data input/output
iparam.write_mps_int
     Controls if marker records are written to the MPS file to indicate whether variables are integer
    restricted.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_mps_int, onoffkey.on)
         Generic name MSK IPAR WRITE MPS INT
         Groups Data input/output
iparam.write_precision
     Controls the precision with which double numbers are printed in the MPS data file. In general it
    is not worthwhile to use a value higher than 15.
         Default 15
         Accepted [0; +\inf]
         Example task.putintparam(iparam.write_precision, 15)
         Generic name MSK IPAR WRITE PRECISION
         Groups Data input/output
iparam.write_sol_barvariables
     Controls whether the symmetric matrix variables section is written to the solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_sol_barvariables, onoffkey.on)
         Generic name MSK IPAR_WRITE_SOL_BARVARIABLES
         Groups Data input/output, Solution input/output
iparam.write_sol_constraints
     Controls whether the constraint section is written to the solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_sol_constraints, onoffkey.on)
         Generic name MSK IPAR WRITE SOL CONSTRAINTS
         Groups Data input/output, Solution input/output
iparam.write sol head
     Controls whether the header section is written to the solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_sol_head, onoffkey.on)
         Generic name MSK IPAR WRITE SOL HEAD
         Groups Data input/output, Solution input/output
iparam.write_sol_ignore_invalid_names
    Even if the names are invalid MPS names, then they are employed when writing the solution file.
         Default off
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_sol_ignore_invalid_names,
            onoffkey.off)
         Generic name MSK IPAR WRITE SOL IGNORE INVALID NAMES
```

Example task.putintparam(iparam.write\_mps\_format, mpsformat.free)

```
Groups Data input/output, Solution input/output
iparam.write_sol_variables
     Controls whether the variables section is written to the solution file.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_sol_variables, onoffkey.on)
         Generic name MSK IPAR WRITE SOL VARIABLES
         Groups Data input/output, Solution input/output
iparam.write_task_inc_sol
     Controls whether the solutions are stored in the task file too.
         Default on
         Accepted on, off (see onoffkey)
         Example task.putintparam(iparam.write_task_inc_sol, onoffkey.on)
         Generic name MSK IPAR WRITE TASK INC SOL
         Groups Data input/output
iparam.write_xml_mode
     Controls if linear coefficients should be written by row or column when writing in the XML file
     format.
         Default row
         Accepted row, col (see xmlwriteroutputtype)
         Example task.putintparam(iparam.write_xml_mode, xmlwriteroutputtype.
         Generic name MSK IPAR WRITE XML MODE
         Groups Data input/output
15.7.3 String parameters
sparam
     The enumeration type containing all string parameters.
sparam.bas_sol_file_name
    Name of the bas solution file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.bas_sol_file_name, "somevalue")
         Generic name MSK SPAR BAS SOL FILE NAME
         Groups Data input/output, Solution input/output
sparam.data_file_name
    Data are read and written to this file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.data_file_name, "somevalue")
         Generic name MSK SPAR DATA FILE NAME
         Groups Data input/output
sparam.debug_file_name
    MOSEK debug file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.debug_file_name, "somevalue")
         Generic name MSK SPAR DEBUG FILE NAME
         Groups Data input/output
```

```
sparam.int_sol_file_name
    Name of the int solution file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.int_sol_file_name, "somevalue")
         Generic name MSK SPAR INT SOL FILE NAME
         Groups Data input/output, Solution input/output
sparam.itr_sol_file_name
    Name of the itr solution file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.itr_sol_file_name, "somevalue")
         Generic name MSK SPAR ITR SOL FILE NAME
         Groups Data input/output, Solution input/output
sparam.mio_debug_string
    For internal debugging purposes.
         Accepted Any valid string.
         Example task.putstrparam(sparam.mio_debug_string, "somevalue")
         Generic name MSK SPAR MIO DEBUG STRING
         Groups Data input/output
sparam.param_comment_sign
     Only the first character in this string is used. It is considered as a start of comment sign in the
     MOSEK parameter file. Spaces are ignored in the string.
         Default
            %%
         Accepted Any valid string.
         Example task.putstrparam(sparam.param_comment_sign, "%%")
         Generic name MSK SPAR PARAM COMMENT SIGN
         Groups Data input/output
sparam.param_read_file_name
    Modifications to the parameter database is read from this file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.param_read_file_name, "somevalue")
         Generic name MSK SPAR PARAM READ FILE NAME
         Groups Data input/output
sparam.param_write_file_name
    The parameter database is written to this file.
         Accepted Any valid file name.
         Example task.putstrparam(sparam.param_write_file_name, "somevalue")
         Generic name MSK SPAR PARAM WRITE FILE NAME
         Groups Data input/output
sparam.read_mps_bou_name
     Name of the BOUNDS vector used. An empty name means that the first BOUNDS vector is used.
         Accepted Any valid MPS name.
```

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Example task.putstrparam(sparam.read\_mps\_bou\_name, "somevalue")

Generic name MSK SPAR READ MPS BOU NAME

Groups Data input/output

## sparam.read\_mps\_obj\_name

Name of the free constraint used as objective function. An empty name means that the first constraint is used as objective function.

**Accepted** Any valid MPS name.

Example task.putstrparam(sparam.read\_mps\_obj\_name, "somevalue")

Generic name MSK SPAR READ MPS OBJ NAME

Groups Data input/output

## sparam.read\_mps\_ran\_name

Name of the RANGE vector used. An empty name means that the first RANGE vector is used.

Accepted Any valid MPS name.

Example task.putstrparam(sparam.read\_mps\_ran\_name, "somevalue")

Generic name MSK\_SPAR\_READ\_MPS\_RAN\_NAME

Groups Data input/output

## sparam.read\_mps\_rhs\_name

Name of the RHS used. An empty name means that the first RHS vector is used.

Accepted Any valid MPS name.

Example task.putstrparam(sparam.read\_mps\_rhs\_name, "somevalue")

Generic name MSK\_SPAR\_READ\_MPS\_RHS\_NAME

Groups Data input/output

## sparam.remote\_access\_token

An access token used to submit tasks to a remote **MOSEK** server. An access token is a random 32-byte string encoded in base64, i.e. it is a 44 character ASCII string.

Accepted Any valid string.

Example task.putstrparam(sparam.remote\_access\_token, "somevalue")

Generic name MSK\_SPAR\_REMOTE\_ACCESS\_TOKEN

Groups Overall system

#### sparam.sensitivity\_file\_name

If defined *Task.sensitivityreport* reads this file as a sensitivity analysis data file specifying the type of analysis to be done.

Accepted Any valid string.

Example task.putstrparam(sparam.sensitivity\_file\_name, "somevalue")

Generic name MSK SPAR SENSITIVITY FILE NAME

Groups Data input/output

## sparam.sensitivity\_res\_file\_name

If this is a nonempty string, then Task.sensitivityreport writes results to this file.

**Accepted** Any valid string.

Example task.putstrparam(sparam.sensitivity\_res\_file\_name, "somevalue")

Generic name MSK SPAR SENSITIVITY RES FILE NAME

Groups Data input/output

## sparam.sol\_filter\_xc\_low

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]>0.5 should be listed, whereas +0.5 means that all constraints having xc[i]>=blc[i]+0.5 should be listed. An empty filter means that no filter is applied.

**Accepted** Any valid filter.

Example task.putstrparam(sparam.sol\_filter\_xc\_low, "somevalue")

Generic name MSK SPAR SOL FILTER XC LOW

Groups Data input/output, Solution input/output

```
sparam.sol_filter_xc_upr
```

A filter used to determine which constraints should be listed in the solution file. A value of 0.5 means that all constraints having xc[i]<0.5 should be listed, whereas -0.5 means all constraints having xc[i]<-buc[i]-0.5 should be listed. An empty filter means that no filter is applied.

Accepted Any valid filter.

Example task.putstrparam(sparam.sol\_filter\_xc\_upr, "somevalue")

Generic name MSK SPAR SOL FILTER XC UPR

Groups Data input/output, Solution input/output

## sparam.sol\_filter\_xx\_low

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j] >= 0.5 should be listed, whereas "+0.5" means that all constraints having xx[j] >= blx[j] + 0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid filter.

Example task.putstrparam(sparam.sol\_filter\_xx\_low, "somevalue")

Generic name MSK SPAR SOL FILTER XX LOW

Groups Data input/output, Solution input/output

## sparam.sol\_filter\_xx\_upr

A filter used to determine which variables should be listed in the solution file. A value of "0.5" means that all constraints having xx[j]<0.5 should be printed, whereas "-0.5" means all constraints having xx[j]<=bux[j]-0.5 should be listed. An empty filter means no filter is applied.

Accepted Any valid file name.

Example task.putstrparam(sparam.sol\_filter\_xx\_upr, "somevalue")

Generic name MSK\_SPAR\_SOL\_FILTER\_XX\_UPR

Groups Data input/output, Solution input/output

## sparam.stat\_file\_name

Statistics file name.

Accepted Any valid file name.

Example task.putstrparam(sparam.stat\_file\_name, "somevalue")

Generic name MSK SPAR STAT FILE NAME

Groups Data input/output

## sparam.stat\_key

Key used when writing the summary file.

Accepted Any valid string.

Example task.putstrparam(sparam.stat\_key, "somevalue")

Generic name MSK SPAR STAT KEY

 ${\bf Groups}\ {\it Data\ input/output}$ 

# sparam.stat\_name

Name used when writing the statistics file.

Accepted Any valid XML string.

Example task.putstrparam(sparam.stat\_name, "somevalue")

Generic name MSK SPAR STAT NAME

Groups Data input/output

## sparam.write\_lp\_gen\_var\_name

Sometimes when an LP file is written additional variables must be inserted. They will have the prefix denoted by this parameter.

Default xmskgen

Accepted Any valid string.

Example task.putstrparam(sparam.write\_lp\_gen\_var\_name, "xmskgen")

Generic name MSK SPAR WRITE LP GEN VAR NAME

Groups Data input/output

# 15.8 Response codes

Response codes include:

- Termination codes
- Warnings
- Errors

The numerical code (in brackets) identifies the response in error messages and in the log output.

The enumeration type containing all response codes.

## 15.8.1 Termination

rescode.ok (0)

No error occurred.

rescode.trm\_max\_iterations (10000)

The optimizer terminated at the maximum number of iterations.

rescode.trm\_max\_time (10001)

The optimizer terminated at the maximum amount of time.

rescode.trm\_objective\_range (10002)

The optimizer terminated with an objective value outside the objective range.

rescode.trm\_mio\_num\_relaxs (10008)

The mixed-integer optimizer terminated as the maximum number of relaxations was reached.

rescode.trm\_mio\_num\_branches (10009)

The mixed-integer optimizer terminated as the maximum number of branches was reached.

rescode.trm\_num\_max\_num\_int\_solutions (10015)

The mixed-integer optimizer terminated as the maximum number of feasible solutions was reached.
rescode.trm\_stall (10006)

The optimizer is terminated due to slow progress.

Stalling means that numerical problems prevent the optimizer from making reasonable progress and that it makes no sense to continue. In many cases this happens if the problem is badly scaled or otherwise ill-conditioned. There is no guarantee that the solution will be feasible or optimal. However, often stalling happens near the optimum, and the returned solution may be of good quality. Therefore, it is recommended to check the status of the solution. If the solution status is optimal the solution is most likely good enough for most practical purposes.

Please note that if a linear optimization problem is solved using the interior-point optimizer with basis identification turned on, the returned basic solution likely to have high accuracy, even though the optimizer stalled.

Some common causes of stalling are a) badly scaled models, b) near feasible or near infeasible problems.

rescode.trm\_user\_callback (10007)

The optimizer terminated due to the return of the user-defined callback function.

rescode.trm\_max\_num\_setbacks (10020)

The optimizer terminated as the maximum number of set-backs was reached. This indicates serious numerical problems and a possibly badly formulated problem.

rescode.trm\_numerical\_problem (10025)

The optimizer terminated due to numerical problems.

rescode.trm\_internal (10030)

The optimizer terminated due to some internal reason. Please contact MOSEK support. rescode.trm\_internal\_stop (10031)

The optimizer terminated for internal reasons. Please contact  $\mathbf{MOSEK}$  support.

# 15.8.2 Warnings

rescode.wrn\_open\_param\_file (50)

The parameter file could not be opened.

rescode.wrn\_large\_bound (51)

A numerically large bound value is specified.

rescode.wrn\_large\_lo\_bound (52)

A numerically large lower bound value is specified.

rescode.wrn\_large\_up\_bound (53)

A numerically large upper bound value is specified.

rescode.wrn\_large\_con\_fx (54)

An equality constraint is fixed to a numerically large value. This can cause numerical problems.

rescode.wrn\_large\_cj (57)

A numerically large value is specified for one  $c_j$ .

rescode.wrn\_large\_aij (62)

A numerically large value is specified for an  $a_{i,j}$  element in A. The parameter dparam.  $data\_tol\_aij\_large$  controls when an  $a_{i,j}$  is considered large.

rescode.wrn\_zero\_aij (63)

One or more zero elements are specified in A.

rescode.wrn\_name\_max\_len (65)

A name is longer than the buffer that is supposed to hold it.

rescode.wrn\_spar\_max\_len (66)

A value for a string parameter is longer than the buffer that is supposed to hold it.

rescode.wrn\_mps\_split\_rhs\_vector (70)

An RHS vector is split into several nonadjacent parts in an MPS file.

rescode.wrn\_mps\_split\_ran\_vector (71)

A RANGE vector is split into several nonadjacent parts in an MPS file.

rescode.wrn\_mps\_split\_bou\_vector (72)

A BOUNDS vector is split into several nonadjacent parts in an MPS file.

rescode.wrn\_lp\_old\_quad\_format (80)

Missing '/2' after quadratic expressions in bound or objective.

rescode.wrn\_lp\_drop\_variable (85)

Ignored a variable because the variable was not previously defined. Usually this implies that a variable appears in the bound section but not in the objective or the constraints.

rescode.wrn\_nz\_in\_upr\_tri (200)

Non-zero elements specified in the upper triangle of a matrix were ignored.

rescode.wrn\_dropped\_nz\_qobj (201)

One or more non-zero elements were dropped in the Q matrix in the objective.

rescode.wrn\_ignore\_integer (250)

Ignored integer constraints.

rescode.wrn\_no\_global\_optimizer (251)

No global optimizer is available.

rescode.wrn\_mio\_infeasible\_final (270)

The final mixed-integer problem with all the integer variables fixed at their optimal values is infeasible.

rescode.wrn\_sol\_filter (300)

Invalid solution filter is specified.

rescode.wrn\_undef\_sol\_file\_name (350)

Undefined name occurred in a solution.

rescode.wrn\_sol\_file\_ignored\_con (351)

One or more lines in the constraint section were ignored when reading a solution file.

rescode.wrn\_sol\_file\_ignored\_var (352)

One or more lines in the variable section were ignored when reading a solution file.

rescode.wrn\_too\_few\_basis\_vars (400)

An incomplete basis has been specified. Too few basis variables are specified.

rescode.wrn\_too\_many\_basis\_vars (405)

A basis with too many variables has been specified.

rescode.wrn\_license\_expire (500)

The license expires.

#### rescode.wrn\_license\_server (501)

The license server is not responding.

## rescode.wrn\_empty\_name (502)

A variable or constraint name is empty. The output file may be invalid.

## rescode.wrn\_using\_generic\_names (503)

Generic names are used because a name is not valid. For instance when writing an LP file the names must not contain blanks or start with a digit.

## rescode.wrn\_license\_feature\_expire (505)

The license expires.

## rescode.wrn\_param\_name\_dou (510)

The parameter name is not recognized as a double parameter.

#### rescode.wrn\_param\_name\_int (511)

The parameter name is not recognized as a integer parameter.

## rescode.wrn\_param\_name\_str (512)

The parameter name is not recognized as a string parameter.

## rescode.wrn\_param\_str\_value (515)

The string is not recognized as a symbolic value for the parameter.

# rescode.wrn\_param\_ignored\_cmio (516)

A parameter was ignored by the conic mixed integer optimizer.

## rescode.wrn\_zeros\_in\_sparse\_row (705)

One or more (near) zero elements are specified in a sparse row of a matrix. Since, it is redundant to specify zero elements then it may indicate an error.

#### rescode.wrn\_zeros\_in\_sparse\_col (710)

One or more (near) zero elements are specified in a sparse column of a matrix. It is redundant to specify zero elements. Hence, it may indicate an error.

## rescode.wrn\_incomplete\_linear\_dependency\_check (800)

The linear dependency check(s) is incomplete. Normally this is not an important warning unless the optimization problem has been formulated with linear dependencies. Linear dependencies may prevent **MOSEK** from solving the problem.

## rescode.wrn\_eliminator\_space (801)

The eliminator is skipped at least once due to lack of space.

# rescode.wrn\_presolve\_outofspace (802)

The presolve is incomplete due to lack of space.

## rescode.wrn\_write\_changed\_names (803)

Some names were changed because they were invalid for the output file format.

## rescode.wrn\_write\_discarded\_cfix (804)

The fixed objective term could not be converted to a variable and was discarded in the output file.

## rescode.wrn\_duplicate\_constraint\_names (850)

Two constraint names are identical.

## rescode.wrn\_duplicate\_variable\_names (851)

Two variable names are identical.

#### rescode.wrn\_duplicate\_barvariable\_names (852)

Two barvariable names are identical.

## rescode.wrn\_duplicate\_cone\_names (853)

Two cone names are identical.

# rescode.wrn\_ana\_large\_bounds (900)

This warning is issued by the problem analyzer, if one or more constraint or variable bounds are very large. One should consider omitting these bounds entirely by setting them to +inf or -inf.

# rescode.wrn\_ana\_c\_zero (901)

This warning is issued by the problem analyzer, if the coefficients in the linear part of the objective are all zero.

# rescode.wrn\_ana\_empty\_cols (902)

This warning is issued by the problem analyzer, if columns, in which all coefficients are zero, are found.

## rescode.wrn\_ana\_close\_bounds (903)

This warning is issued by problem analyzer, if ranged constraints or variables with very close upper and lower bounds are detected. One should consider treating such constraints as equalities and such variables as constants.

```
rescode.wrn_ana_almost_int_bounds (904)
```

This warning is issued by the problem analyzer if a constraint is bound nearly integral.

## rescode.wrn\_quad\_cones\_with\_root\_fixed\_at\_zero (930)

For at least one quadratic cone the root is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

## rescode.wrn\_rquad\_cones\_with\_root\_fixed\_at\_zero (931)

For at least one rotated quadratic cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

#### rescode.wrn\_exp\_cones\_with\_variables\_fixed\_at\_zero (932)

For at least one exponential cone  $x \ge y \exp(z/y)$  either the variable x or y is fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

## rescode.wrn\_pow\_cones\_with\_root\_fixed\_at\_zero (933)

For at least one power cone at least one of the root variables are fixed at (nearly) zero. This may cause problems such as a very large dual solution. Therefore, it is recommended to remove such cones before optimizing the problem, or to fix all the variables in the cone to 0.

#### rescode.wrn\_no\_dualizer (950)

No automatic dualizer is available for the specified problem. The primal problem is solved.

## rescode.wrn\_sym\_mat\_large (960)

A numerically large value is specified for an  $e_{i,j}$  element in E. The parameter dparam.  $data\_sym\_mat\_tol\_large$  controls when an  $e_{i,j}$  is considered large.

## 15.8.3 Errors

rescode.err\_license (1000)

Invalid license.

rescode.err\_license\_expired (1001)

The license has expired.

rescode.err\_license\_version (1002)

The license is valid for another version of **MOSEK**.

rescode.err\_size\_license (1005)

The problem is bigger than the license.

rescode.err\_prob\_license (1006)

The software is not licensed to solve the problem.

rescode.err\_file\_license (1007)

Invalid license file.

rescode.err\_missing\_license\_file (1008)

MOSEK cannot find license file or a token server. See the MOSEK licensing manual for details. rescode.err\_size\_license\_con (1010)

The problem has too many constraints to be solved with the available license.

rescode.err\_size\_license\_var (1011)

The problem has too many variables to be solved with the available license.

rescode.err\_size\_license\_intvar (1012)

The problem contains too many integer variables to be solved with the available license.

rescode.err\_optimizer\_license (1013)

The optimizer required is not licensed.

rescode.err\_flexlm (1014)

The FLEXIm license manager reported an error.

rescode.err\_license\_server (1015)

The license server is not responding.

rescode.err\_license\_max (1016)

Maximum number of licenses is reached.

rescode.err\_license\_moseklm\_daemon (1017)

The MOSEKLM license manager daemon is not up and running.

```
rescode.err_license_feature (1018)
```

A requested feature is not available in the license file(s). Most likely due to an incorrect license system setup.

## rescode.err\_platform\_not\_licensed (1019)

A requested license feature is not available for the required platform.

rescode.err\_license\_cannot\_allocate (1020)

The license system cannot allocate the memory required.

rescode.err\_license\_cannot\_connect (1021)

MOSEK cannot connect to the license server. Most likely the license server is not up and running. rescode.err\_license\_invalid\_hostid (1025)

The host ID specified in the license file does not match the host ID of the computer.

rescode.err\_license\_server\_version (1026)

The version specified in the checkout request is greater than the highest version number the daemon supports.

## rescode.err\_license\_no\_server\_support (1027)

The license server does not support the requested feature. Possible reasons for this error include:

- The feature has expired.
- The feature's start date is later than today's date.
- The version requested is higher than feature's the highest supported version.
- A corrupted license file.

Try restarting the license and inspect the license server debug file, usually called lmgrd.log. rescode.err\_license\_no\_server\_line (1028)

There is no SERVER line in the license file. All non-zero license count features need at least one SERVER line.

rescode.err\_older\_dll (1035)

The dynamic link library is older than the specified version.

rescode.err\_newer\_dll (1036)

The dynamic link library is newer than the specified version.

rescode.err\_link\_file\_dll (1040)

A file cannot be linked to a stream in the DLL version.

rescode.err\_thread\_mutex\_init (1045)

Could not initialize a mutex.

rescode.err\_thread\_mutex\_lock (1046)

Could not lock a mutex.

rescode.err\_thread\_mutex\_unlock (1047)

Could not unlock a mutex.

rescode.err\_thread\_create (1048)

Could not create a thread. This error may occur if a large number of environments are created and not deleted again. In any case it is a good practice to minimize the number of environments created.

rescode.err\_thread\_cond\_init (1049)

Could not initialize a condition.

rescode.err\_unknown (1050)

Unknown error.

rescode.err\_space (1051)

Out of space.

rescode.err\_file\_open (1052)

Error while opening a file.

rescode.err\_file\_read (1053)

File read error.

rescode.err\_file\_write (1054)

File write error.

rescode.err\_data\_file\_ext (1055)

The data file format cannot be determined from the file name.

rescode.err\_invalid\_file\_name (1056)

An invalid file name has been specified.

```
rescode.err_invalid_sol_file_name (1057)
```

An invalid file name has been specified.

rescode.err\_end\_of\_file (1059)

End of file reached.

rescode.err\_null\_env (1060)

env is a NULL pointer.

rescode.err\_null\_task (1061)

task is a NULL pointer.

rescode.err\_invalid\_stream (1062)

An invalid stream is referenced.

rescode.err\_no\_init\_env (1063)

env is not initialized.

rescode.err\_invalid\_task (1064)

The task is invalid.

rescode.err\_null\_pointer (1065)

An argument to a function is unexpectedly a NULL pointer.

rescode.err\_living\_tasks (1066)

All tasks associated with an environment must be deleted before the environment is deleted. There are still some undeleted tasks.

rescode.err\_blank\_name (1070)

An all blank name has been specified.

rescode.err\_dup\_name (1071)

The same name was used multiple times for the same problem item type.

rescode.err\_format\_string (1072)

The name format string is invalid.

rescode.err\_invalid\_obj\_name (1075)

An invalid objective name is specified.

rescode.err\_invalid\_con\_name (1076)

An invalid constraint name is used.

rescode.err\_invalid\_var\_name (1077)

An invalid variable name is used.

rescode.err\_invalid\_cone\_name (1078)

An invalid cone name is used.

rescode.err\_invalid\_barvar\_name (1079)

An invalid symmetric matrix variable name is used.

rescode.err\_space\_leaking (1080)

MOSEK is leaking memory. This can be due to either an incorrect use of MOSEK or a bug.

rescode.err\_space\_no\_info (1081)

No available information about the space usage.

rescode.err\_read\_format (1090)

The specified format cannot be read.

rescode.err\_mps\_file (1100)

An error occurred while reading an MPS file.

rescode.err\_mps\_inv\_field (1101)

A field in the MPS file is invalid. Probably it is too wide.

rescode.err\_mps\_inv\_marker (1102)

An invalid marker has been specified in the MPS file.

rescode.err\_mps\_null\_con\_name (1103)

An empty constraint name is used in an MPS file.

rescode.err\_mps\_null\_var\_name (1104)

An empty variable name is used in an MPS file.

rescode.err\_mps\_undef\_con\_name (1105)

An undefined constraint name occurred in an MPS file.

rescode.err\_mps\_undef\_var\_name (1106)

An undefined variable name occurred in an MPS file.

rescode.err\_mps\_inv\_con\_key (1107)

An invalid constraint key occurred in an MPS file.

```
rescode.err_mps_inv_bound_key (1108)
     An invalid bound key occurred in an MPS file.
rescode.err_mps_inv_sec_name (1109)
     An invalid section name occurred in an MPS file.
rescode.err_mps_no_objective (1110)
     No objective is defined in an MPS file.
rescode.err_mps_splitted_var (1111)
     All elements in a column of the A matrix must be specified consecutively. Hence, it is illegal to
     specify non-zero elements in A for variable 1, then for variable 2 and then variable 1 again.
rescode.err_mps_mul_con_name (1112)
     A constraint name was specified multiple times in the ROWS section.
rescode.err_mps_mul_qsec (1113)
     Multiple QSECTIONs are specified for a constraint in the MPS data file.
rescode.err_mps_mul_qobj (1114)
     The Q term in the objective is specified multiple times in the MPS data file.
rescode.err_mps_inv_sec_order (1115)
     The sections in the MPS data file are not in the correct order.
rescode.err_mps_mul_csec (1116)
     Multiple CSECTIONs are given the same name.
rescode.err_mps_cone_type (1117)
     Invalid cone type specified in a CSECTION.
rescode.err_mps_cone_overlap (1118)
     A variable is specified to be a member of several cones.
rescode.err_mps_cone_repeat (1119)
     A variable is repeated within the CSECTION.
rescode.err_mps_non_symmetric_q (1120)
     A non symmetric matrix has been speciefied.
rescode.err_mps_duplicate_q_element (1121)
     Duplicate elements is specified in a Q matrix.
rescode.err_mps_invalid_objsense (1122)
     An invalid objective sense is specified.
rescode.err_mps_tab_in_field2 (1125)
     A tab char occurred in field 2.
rescode.err_mps_tab_in_field3 (1126)
     A tab char occurred in field 3.
rescode.err_mps_tab_in_field5 (1127)
     A tab char occurred in field 5.
rescode.err_mps_invalid_obj_name (1128)
     An invalid objective name is specified.
rescode.err_lp_incompatible (1150)
     The problem cannot be written to an LP formatted file.
rescode.err_lp_empty (1151)
     The problem cannot be written to an LP formatted file.
rescode.err_lp_dup_slack_name (1152)
     The name of the slack variable added to a ranged constraint already exists.
rescode.err_write_mps_invalid_name (1153)
     An invalid name is created while writing an MPS file. Usually this will make the MPS file unread-
rescode.err_lp_invalid_var_name (1154)
     A variable name is invalid when used in an LP formatted file.
```

rescode.err\_lp\_free\_constraint (1155)

Free constraints cannot be written in LP file format.

rescode.err\_write\_opf\_invalid\_var\_name (1156)

Empty variable names cannot be written to OPF files.

rescode.err\_lp\_file\_format (1157)

Syntax error in an LP file.

rescode.err\_write\_lp\_format (1158)

Problem cannot be written as an LP file.

```
rescode.err_read_lp_missing_end_tag (1159)
Syntax error in LP file. Possibly missing End tag.
```

rescode.err\_lp\_format (1160) Syntax error in an LP file.

rescode.err\_write\_lp\_non\_unique\_name (1161)

An auto-generated name is not unique.

rescode.err\_read\_lp\_nonexisting\_name (1162)

A variable never occurred in objective or constraints.

rescode.err\_lp\_write\_conic\_problem (1163)

The problem contains cones that cannot be written to an LP formatted file.

rescode.err\_lp\_write\_geco\_problem (1164)

The problem contains general convex terms that cannot be written to an LP formatted file.

rescode.err\_writing\_file (1166)

An error occurred while writing file

rescode.err\_ptf\_format (1167)

Syntax error in an PTF file

rescode.err\_opf\_format (1168)

Syntax error in an OPF file

rescode.err\_opf\_new\_variable (1169)

Introducing new variables is now allowed. When a [variables] section is present, it is not allowed to introduce new variables later in the problem.

rescode.err\_invalid\_name\_in\_sol\_file (1170)

An invalid name occurred in a solution file.

rescode.err\_lp\_invalid\_con\_name (1171)

A constraint name is invalid when used in an LP formatted file.

rescode.err\_opf\_premature\_eof (1172)

Premature end of file in an OPF file.

rescode.err\_json\_syntax (1175)

Syntax error in an JSON data

rescode.err\_json\_string (1176)

Error in JSON string.

rescode.err\_json\_number\_overflow (1177)

Invalid number entry - wrong type or value overflow.

rescode.err\_json\_format (1178)

Error in an JSON Task file

rescode.err\_json\_data (1179)

Inconsistent data in JSON Task file

rescode.err\_json\_missing\_data (1180)

Missing data section in JSON task file.

rescode.err\_argument\_lenneq (1197)

Incorrect length of arguments.

rescode.err\_argument\_type (1198)

Incorrect argument type.

rescode.err\_num\_arguments (1199)

Incorrect number of function arguments.

rescode.err\_in\_argument (1200)

A function argument is incorrect.

rescode.err\_argument\_dimension (1201)

A function argument is of incorrect dimension.

rescode.err\_shape\_is\_too\_large (1202)

The size of the n-dimensional shape is too large.

rescode.err\_index\_is\_too\_small (1203)

An index in an argument is too small.

rescode.err\_index\_is\_too\_large (1204)

An index in an argument is too large.

rescode.err\_param\_name (1205)

The parameter name is not correct.

```
rescode.err_param_name_dou (1206)
```

The parameter name is not correct for a double parameter.

rescode.err\_param\_name\_int (1207)

The parameter name is not correct for an integer parameter.

rescode.err\_param\_name\_str (1208)

The parameter name is not correct for a string parameter.

rescode.err\_param\_index (1210)

Parameter index is out of range.

rescode.err\_param\_is\_too\_large (1215)

The parameter value is too large.

rescode.err\_param\_is\_too\_small (1216)

The parameter value is too small.

rescode.err\_param\_value\_str (1217)

The parameter value string is incorrect.

rescode.err\_param\_type (1218)

The parameter type is invalid.

rescode.err\_inf\_dou\_index (1219)

A double information index is out of range for the specified type.

rescode.err\_inf\_int\_index (1220)

An integer information index is out of range for the specified type.

rescode.err\_index\_arr\_is\_too\_small (1221)

An index in an array argument is too small.

rescode.err\_index\_arr\_is\_too\_large (1222)

An index in an array argument is too large.

rescode.err\_inf\_lint\_index (1225)

A long integer information index is out of range for the specified type.

rescode.err\_arg\_is\_too\_small (1226)

The value of a argument is too small.

rescode.err\_arg\_is\_too\_large (1227)

The value of a argument is too large.

rescode.err\_invalid\_whichsol (1228)

whichsol is invalid.

rescode.err\_inf\_dou\_name (1230)

A double information name is invalid.

rescode.err\_inf\_int\_name (1231)

An integer information name is invalid.

rescode.err\_inf\_type (1232)

The information type is invalid.

rescode.err\_inf\_lint\_name (1234)

A long integer information name is invalid.

rescode.err\_index (1235)

An index is out of range.

rescode.err\_whichsol (1236)

The solution defined by whichsol does not exists.

rescode.err\_solitem (1237)

The solution item number solitem is invalid. Please note that solitem.snx is invalid for the basic solution.

rescode.err\_whichitem\_not\_allowed (1238)

whichitem is unacceptable.

rescode.err\_maxnumcon (1240)

The maximum number of constraints specified is smaller than the number of constraints in the task.

rescode.err\_maxnumvar (1241)

The maximum number of variables specified is smaller than the number of variables in the task. rescode.err\_maxnumbarvar (1242)

The maximum number of semidefinite variables specified is smaller than the number of semidefinite variables in the task.

```
rescode.err_maxnumqnz (1243)
```

The maximum number of non-zeros specified for the Q matrices is smaller than the number of non-zeros in the current Q matrices.

## rescode.err\_too\_small\_max\_num\_nz (1245)

The maximum number of non-zeros specified is too small.

rescode.err\_invalid\_idx (1246)

A specified index is invalid.

rescode.err\_invalid\_max\_num (1247)

A specified index is invalid.

rescode.err\_numconlim (1250)

Maximum number of constraints limit is exceeded.

rescode.err\_numvarlim (1251)

Maximum number of variables limit is exceeded.

rescode.err\_too\_small\_maxnumanz (1252)

The maximum number of non-zeros specified for A is smaller than the number of non-zeros in the current A.

rescode.err\_inv\_aptre (1253)

aptre[j] is strictly smaller than aptrb[j] for some j.

rescode.err\_mul\_a\_element (1254)

An element in A is defined multiple times.

rescode.err\_inv\_bk (1255)

Invalid bound key.

rescode.err\_inv\_bkc (1256)

Invalid bound key is specified for a constraint.

rescode.err\_inv\_bkx (1257)

An invalid bound key is specified for a variable.

rescode.err\_inv\_var\_type (1258)

An invalid variable type is specified for a variable.

rescode.err\_solver\_probtype (1259)

Problem type does not match the chosen optimizer.

rescode.err\_objective\_range (1260)

Empty objective range.

rescode.err\_undef\_solution (1265)

**MOSEK** has the following solution types:

- an interior-point solution,
- a basic solution,
- and an integer solution.

Each optimizer may set one or more of these solutions; e.g by default a successful optimization with the interior-point optimizer defines the interior-point solution and, for linear problems, also the basic solution. This error occurs when asking for a solution or for information about a solution that is not defined.

#### rescode.err\_basis (1266)

An invalid basis is specified. Either too many or too few basis variables are specified.

rescode.err\_inv\_skc (1267)

Invalid value in skc.

rescode.err\_inv\_skx (1268)

Invalid value in skx.

rescode.err\_inv\_skn (1274)

Invalid value in skn.

rescode.err\_inv\_sk\_str (1269)

Invalid status key string encountered.

rescode.err\_inv\_sk (1270)

Invalid status key code.

rescode.err\_inv\_cone\_type\_str (1271)

Invalid cone type string encountered.

rescode.err\_inv\_cone\_type (1272)

Invalid cone type code is encountered.

rescode.err\_invalid\_surplus (1275)

Invalid surplus.

rescode.err\_inv\_name\_item (1280)

An invalid name item code is used.

rescode.err\_pro\_item (1281)

An invalid problem is used.

rescode.err\_invalid\_format\_type (1283)

Invalid format type.

rescode.err\_firsti (1285)

Invalid firsti.

rescode.err\_lasti (1286)

Invalid lasti.

rescode.err\_firstj (1287)

Invalid firstj.

rescode.err\_lastj (1288)

Invalid lastj.

rescode.err\_max\_len\_is\_too\_small (1289)

A maximum length that is too small has been specified.

rescode.err\_nonlinear\_equality (1290)

The model contains a nonlinear equality which defines a nonconvex set.

rescode.err\_nonconvex (1291)

The optimization problem is nonconvex.

rescode.err\_nonlinear\_ranged (1292)

Nonlinear constraints with finite lower and upper bound always define a nonconvex feasible set.

rescode.err\_con\_q\_not\_psd (1293)

The quadratic constraint matrix is not positive semidefinite as expected for a constraint with finite upper bound. This results in a nonconvex problem. The parameter *dparam. check\_convexity\_rel\_tol* can be used to relax the convexity check.

rescode.err\_con\_q\_not\_nsd (1294)

The quadratic constraint matrix is not negative semidefinite as expected for a constraint with finite lower bound. This results in a nonconvex problem. The parameter *dparam*. *check\_convexity\_rel\_tol* can be used to relax the convexity check.

rescode.err\_obj\_q\_not\_psd (1295)

The quadratic coefficient matrix in the objective is not positive semidefinite as expected for a minimization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

rescode.err\_obj\_q\_not\_nsd (1296)

The quadratic coefficient matrix in the objective is not negative semidefinite as expected for a maximization problem. The parameter <code>dparam.check\_convexity\_rel\_tol</code> can be used to relax the convexity check.

rescode.err\_argument\_perm\_array (1299)

An invalid permutation array is specified.

rescode.err\_cone\_index (1300)

An index of a non-existing cone has been specified.

rescode.err\_cone\_size (1301)

A cone with incorrect number of members is specified.

rescode.err\_cone\_overlap (1302)

One or more of the variables in the cone to be added is already member of another cone. Now assume the variable is  $x_j$  then add a new variable say  $x_k$  and the constraint

$$x_i = x_k$$

and then let  $x_k$  be member of the cone to be appended.

rescode.err\_cone\_rep\_var (1303)

A variable is included multiple times in the cone.

rescode.err\_maxnumcone (1304)

The value specified for maxnumcone is too small.

```
rescode.err_cone_type (1305)
     Invalid cone type specified.
rescode.err_cone_type_str (1306)
     Invalid cone type specified.
rescode.err_cone_overlap_append (1307)
     The cone to be appended has one variable which is already member of another cone.
rescode.err_remove_cone_variable (1310)
     A variable cannot be removed because it will make a cone invalid.
rescode.err_appending_too_big_cone (1311)
     Trying to append a too big cone.
rescode.err_cone_parameter (1320)
     An invalid cone parameter.
rescode.err_sol_file_invalid_number (1350)
     An invalid number is specified in a solution file.
rescode.err_huge_c (1375)
     A huge value in absolute size is specified for one c_i.
rescode.err_huge_aij (1380)
     A numerically huge value is specified for an a_{i,j} element in A. The parameter dparam.
     data\_tol\_aij\_huge controls when an a_{i,j} is considered huge.
rescode.err_duplicate_aij (1385)
     An element in the A matrix is specified twice.
rescode.err_lower_bound_is_a_nan (1390)
     The lower bound specified is not a number (nan).
rescode.err_upper_bound_is_a_nan (1391)
     The upper bound specified is not a number (nan).
rescode.err_infinite_bound (1400)
     A numerically huge bound value is specified.
rescode.err_inv_qobj_subi (1401)
     Invalid value in qosubi.
rescode.err_inv_qobj_subj (1402)
     Invalid value in qosubj.
rescode.err_inv_qobj_val (1403)
     Invalid value in qoval.
rescode.err_inv_qcon_subk (1404)
     Invalid value in qcsubk.
rescode.err_inv_qcon_subi (1405)
     Invalid value in qcsubi.
rescode.err_inv_qcon_subj (1406)
     Invalid value in qcsubj.
rescode.err_inv_qcon_val (1407)
     Invalid value in qcval.
rescode.err_qcon_subi_too_small (1408)
     Invalid value in qcsubi.
rescode.err_qcon_subi_too_large (1409)
     Invalid value in qcsubi.
rescode.err_qobj_upper_triangle (1415)
     An element in the upper triangle of Q^o is specified. Only elements in the lower triangle should be
rescode.err_qcon_upper_triangle (1417)
     An element in the upper triangle of a Q^k is specified. Only elements in the lower triangle should
     be specified.
rescode.err_fixed_bound_values (1420)
     A fixed constraint/variable has been specified using the bound keys but the numerical value of the
     lower and upper bound is different.
rescode.err_too_small_a_truncation_value (1421)
```

A too small value for the A trucation value is specified.

rescode.err\_invalid\_objective\_sense (1445)
An invalid objective sense is specified.

```
rescode.err_undefined_objective_sense (1446)
     The objective sense has not been specified before the optimization.
rescode.err_y_is_undefined (1449)
     The solution item y is undefined.
rescode.err_nan_in_double_data (1450)
     An invalid floating point value was used in some double data.
rescode.err_nan_in_blc (1461)
     l^c contains an invalid floating point value, i.e. a NaN.
rescode.err_nan_in_buc (1462)
     u^c contains an invalid floating point value, i.e. a NaN.
rescode.err_nan_in_c (1470)
     c contains an invalid floating point value, i.e. a NaN.
rescode.err_nan_in_blx (1471)
     l^x contains an invalid floating point value, i.e. a NaN.
rescode.err_nan_in_bux (1472)
     u^x contains an invalid floating point value, i.e. a NaN.
rescode.err_invalid_aij (1473)
     a_{i,j} contains an invalid floating point value, i.e. a NaN or an infinite value.
rescode.err_sym_mat_invalid (1480)
     A symmetric matrix contains an invalid floating point value, i.e. a NaN or an infinite value.
rescode.err_sym_mat_huge (1482)
     A symmetric matrix contains a huge value in absolute size.
                                                                         The parameter dparam.
     data\_sym\_mat\_tol\_huge controls when an e_{i,j} is considered huge.
rescode.err_inv_problem (1500)
     Invalid problem type. Probably a nonconvex problem has been specified.
rescode.err_mixed_conic_and_nl (1501)
     The problem contains nonlinear terms conic constraints. The requested operation cannot be applied
     to this type of problem.
rescode.err_global_inv_conic_problem (1503)
     The global optimizer can only be applied to problems without semidefinite variables.
rescode.err_inv_optimizer (1550)
     An invalid optimizer has been chosen for the problem.
rescode.err_mio_no_optimizer (1551)
     No optimizer is available for the current class of integer optimization problems.
rescode.err_no_optimizer_var_type (1552)
     No optimizer is available for this class of optimization problems.
rescode.err_final_solution (1560)
     An error occurred during the solution finalization.
rescode.err_first (1570)
     Invalid first.
rescode.err_last (1571)
     Invalid index last. A given index was out of expected range.
rescode.err_slice_size (1572)
     Invalid slice size specified.
rescode.err_negative_surplus (1573)
     Negative surplus.
rescode.err_negative_append (1578)
     Cannot append a negative number.
rescode.err_postsolve (1580)
     An error occurred during the postsolve. Please contact MOSEK support.
rescode.err_overflow (1590)
     A computation produced an overflow i.e. a very large number.
rescode.err_no_basis_sol (1600)
     No basic solution is defined.
rescode.err_basis_factor (1610)
```

The factorization of the basis is invalid.

The basis is singular and hence cannot be factored.

rescode.err\_basis\_singular (1615)

```
rescode.err_factor (1650)
```

An error occurred while factorizing a matrix.

rescode.err\_feasrepair\_cannot\_relax (1700)

An optimization problem cannot be relaxed.

rescode.err\_feasrepair\_solving\_relaxed (1701)

The relaxed problem could not be solved to optimality. Please consult the log file for further details. rescode.err\_feasrepair\_inconsistent\_bound (1702)

The upper bound is less than the lower bound for a variable or a constraint. Please correct this before running the feasibility repair.

rescode.err\_repair\_invalid\_problem (1710)

The feasibility repair does not support the specified problem type.

rescode.err\_repair\_optimization\_failed (1711)

Computation the optimal relaxation failed. The cause may have been numerical problems.

rescode.err\_name\_max\_len (1750)

A name is longer than the buffer that is supposed to hold it.

rescode.err\_name\_is\_null (1760)

The name buffer is a NULL pointer.

rescode.err\_invalid\_compression (1800)

Invalid compression type.

rescode.err\_invalid\_iomode (1801)

Invalid io mode.

rescode.err\_no\_primal\_infeas\_cer (2000)

A certificate of primal infeasibility is not available.

rescode.err\_no\_dual\_infeas\_cer (2001)

A certificate of infeasibility is not available.

rescode.err\_no\_solution\_in\_callback (2500)

The required solution is not available.

rescode.err\_inv\_marki (2501)

Invalid value in marki.

rescode.err\_inv\_markj (2502)

Invalid value in markj.

rescode.err\_inv\_numi (2503)

Invalid numi.

rescode.err\_inv\_numj (2504)

Invalid numj.

rescode.err\_task\_incompatible (2560)

The Task file is incompatible with this platform. This results from reading a file on a 32 bit platform generated on a 64 bit platform.

rescode.err\_task\_invalid (2561)

The Task file is invalid.

rescode.err\_task\_write (2562)

Failed to write the task file.

rescode.err\_lu\_max\_num\_tries (2800)

Could not compute the LU factors of the matrix within the maximum number of allowed tries.

rescode.err\_invalid\_utf8 (2900)

An invalid UTF8 string is encountered.

rescode.err\_invalid\_wchar (2901)

An invalid wchar string is encountered.

rescode.err\_no\_dual\_for\_itg\_sol (2950)

No dual information is available for the integer solution.

rescode.err\_no\_snx\_for\_bas\_sol (2953)

 $s_n^x$  is not available for the basis solution.

rescode.err\_internal (3000)

An internal error occurred. Please report this problem.

rescode.err\_api\_array\_too\_small (3001)

An input array was too short.

rescode.err\_api\_cb\_connect (3002)

Failed to connect a callback object.

```
rescode.err_api_fatal_error (3005)
```

An internal error occurred in the API. Please report this problem.

## rescode.err\_api\_internal (3999)

An internal fatal error occurred in an interface function.

## rescode.err\_sen\_format (3050)

Syntax error in sensitivity analysis file.

## rescode.err\_sen\_undef\_name (3051)

An undefined name was encountered in the sensitivity analysis file.

## rescode.err\_sen\_index\_range (3052)

Index out of range in the sensitivity analysis file.

## rescode.err\_sen\_bound\_invalid\_up (3053)

Analysis of upper bound requested for an index, where no upper bound exists.

## rescode.err\_sen\_bound\_invalid\_lo (3054)

Analysis of lower bound requested for an index, where no lower bound exists.

## rescode.err\_sen\_index\_invalid (3055)

Invalid range given in the sensitivity file.

## rescode.err\_sen\_invalid\_regexp (3056)

Syntax error in regexp or regexp longer than 1024.

#### rescode.err\_sen\_solution\_status (3057)

No optimal solution found to the original problem given for sensitivity analysis.

## rescode.err\_sen\_numerical (3058)

Numerical difficulties encountered performing the sensitivity analysis.

## rescode.err\_sen\_unhandled\_problem\_type (3080)

Sensitivity analysis cannot be performed for the specified problem. Sensitivity analysis is only possible for linear problems.

## rescode.err\_unb\_step\_size (3100)

A step size in an optimizer was unexpectedly unbounded. For instance, if the step-size becomes unbounded in phase 1 of the simplex algorithm then an error occurs. Normally this will happen only if the problem is badly formulated. Please contact **MOSEK** support if this error occurs.

# rescode.err\_identical\_tasks (3101)

Some tasks related to this function call were identical. Unique tasks were expected.

# rescode.err\_ad\_invalid\_codelist (3102)

The code list data was invalid.

## rescode.err\_internal\_test\_failed (3500)

An internal unit test function failed.

## rescode.err\_xml\_invalid\_problem\_type (3600)

The problem type is not supported by the XML format.

# rescode.err\_invalid\_ampl\_stub (3700)

Invalid AMPL stub.

## rescode.err\_int64\_to\_int32\_cast (3800)

A 64 bit integer could not be cast to a 32 bit integer.

## rescode.err\_size\_license\_numcores (3900)

The computer contains more cpu cores than the license allows for.

## rescode.err\_infeas\_undefined (3910)

The requested value is not defined for this solution type.

# rescode.err\_no\_barx\_for\_solution (3915)

There is no  $\overline{X}$  available for the solution specified. In particular note there are no  $\overline{X}$  defined for the basic and integer solutions.

# rescode.err\_no\_bars\_for\_solution (3916)

There is no  $\bar{s}$  available for the solution specified. In particular note there are no  $\bar{s}$  defined for the basic and integer solutions.

## rescode.err\_bar\_var\_dim (3920)

The dimension of a symmetric matrix variable has to be greater than 0.

## rescode.err\_sym\_mat\_invalid\_row\_index (3940)

A row index specified for sparse symmetric matrix is invalid.

## rescode.err\_sym\_mat\_invalid\_col\_index (3941)

A column index specified for sparse symmetric matrix is invalid.

```
rescode.err_sym_mat_invalid_value (3943)
     The numerical value specified in a sparse symmetric matrix is not a floating point value.
rescode.err_sym_mat_duplicate (3944)
     A value in a symmetric matric as been specified more than once.
rescode.err_invalid_sym_mat_dim (3950)
     A sparse symmetric matrix of invalid dimension is specified.
rescode.err_invalid_file_format_for_sym_mat (4000)
     The file format does not support a problem with symmetric matrix variables.
rescode.err_invalid_file_format_for_cfix (4001)
     The file format does not support a problem with nonzero fixed term in c.
rescode.err_invalid_file_format_for_ranged_constraints (4002)
     The file format does not support a problem with ranged constraints.
rescode.err_invalid_file_format_for_free_constraints (4003)
     The file format does not support a problem with free constraints.
rescode.err_invalid_file_format_for_cones (4005)
     The file format does not support a problem with conic constraints.
rescode.err_invalid_file_format_for_nonlinear (4010)
     The file format does not support a problem with nonlinear terms.
rescode.err_duplicate_constraint_names (4500)
     Two constraint names are identical.
rescode.err_duplicate_variable_names (4501)
     Two variable names are identical.
rescode.err_duplicate_barvariable_names (4502)
     Two barvariable names are identical.
rescode.err_duplicate_cone_names (4503)
     Two cone names are identical.
rescode.err_non_unique_array (5000)
     An array does not contain unique elements.
rescode.err_argument_is_too_large (5005)
     The value of a function argument is too large.
rescode.err_mio_internal (5010)
     A fatal error occurred in the mixed integer optimizer. Please contact MOSEK support.
rescode.err_invalid_problem_type (6000)
     An invalid problem type.
rescode.err_unhandled_solution_status (6010)
     Unhandled solution status.
rescode.err_upper_triangle (6020)
     An element in the upper triangle of a lower triangular matrix is specified.
rescode.err_lau_singular_matrix (7000)
     A matrix is singular.
rescode.err_lau_not_positive_definite (7001)
     A matrix is not positive definite.
rescode.err_lau_invalid_lower_triangular_matrix (7002)
     An invalid lower triangular matrix.
rescode.err_lau_unknown (7005)
     An unknown error.
rescode.err_lau_arg_m (7010)
     Invalid argument m.
rescode.err_lau_arg_n (7011)
     Invalid argument n.
rescode.err_lau_arg_k (7012)
     Invalid argument k.
rescode.err_lau_arg_transa (7015)
     Invalid argument transa.
rescode.err_lau_arg_transb (7016)
     Invalid argument transb.
```

Only the lower triangular part of sparse symmetric matrix should be specified.

rescode.err\_sym\_mat\_not\_lower\_tringular (3942)

rescode.err\_lau\_arg\_uplo (7017) Invalid argument uplo.

rescode.err\_lau\_arg\_trans (7018)

Invalid argument trans.

rescode.err\_lau\_invalid\_sparse\_symmetric\_matrix (7019)

An invalid sparse symmetric matrix is specified. Note only the lower triangular part with no duplicates is specified.

rescode.err\_cbf\_parse (7100)

An error occurred while parsing an CBF file.

rescode.err\_cbf\_obj\_sense (7101)

An invalid objective sense is specified.

rescode.err\_cbf\_no\_variables (7102)

No variables are specified.

rescode.err\_cbf\_too\_many\_constraints (7103)

Too many constraints specified.

rescode.err\_cbf\_too\_many\_variables (7104)

Too many variables specified.

rescode.err\_cbf\_no\_version\_specified (7105)

No version specified.

rescode.err\_cbf\_syntax (7106)

Invalid syntax.

rescode.err\_cbf\_duplicate\_obj (7107)

Duplicate OBJ keyword.

rescode.err\_cbf\_duplicate\_con (7108)

Duplicate CON keyword.

rescode.err\_cbf\_duplicate\_var (7109)

Duplicate VAR keyword.

rescode.err\_cbf\_duplicate\_int (7110)

Duplicate INT keyword.

rescode.err\_cbf\_invalid\_var\_type (7111)

Invalid variable type.

rescode.err\_cbf\_invalid\_con\_type (7112)

Invalid constraint type.

rescode.err\_cbf\_invalid\_domain\_dimension (7113)

Invalid domain dimension.

 ${\tt rescode.err\_cbf\_duplicate\_objacoord}\ (7114)$ 

Duplicate index in OBJCOORD.

rescode.err\_cbf\_duplicate\_bcoord (7115)

Duplicate index in BCOORD.

rescode.err\_cbf\_duplicate\_acoord (7116)

Duplicate index in ACOORD.

rescode.err\_cbf\_too\_few\_variables (7117)

Too few variables defined.

rescode.err\_cbf\_too\_few\_constraints (7118)

Too few constraints defined.

rescode.err\_cbf\_too\_few\_ints (7119)

Too few ints are specified.

rescode.err\_cbf\_too\_many\_ints (7120)

Too many ints are specified.

rescode.err\_cbf\_invalid\_int\_index (7121)

Invalid INT index.

 ${\tt rescode.err\_cbf\_unsupported}\ (7122)$ 

Unsupported feature is present.

rescode.err\_cbf\_duplicate\_psdvar (7123)

Duplicate PSDVAR keyword.

rescode.err\_cbf\_invalid\_psdvar\_dimension (7124)

Invalid PSDVAR dimension.

```
rescode.err_cbf_too_few_psdvar (7125)
```

Too few variables defined.

rescode.err\_cbf\_invalid\_exp\_dimension (7126)

Invalid dimension of a exponential cone.

rescode.err\_cbf\_duplicate\_pow\_cones (7130)

Multiple POWCONES specified.

rescode.err\_cbf\_duplicate\_pow\_star\_cones (7131)

Multiple POW\*CONES specified.

rescode.err\_cbf\_invalid\_power (7132)

Invalid power specified.

rescode.err\_cbf\_power\_cone\_is\_too\_long (7133)

Power cone is too long.

rescode.err\_cbf\_invalid\_power\_cone\_index (7134)

Invalid power cone index.

rescode.err\_cbf\_invalid\_power\_star\_cone\_index (7135)

Invalid power star cone index.

rescode.err\_cbf\_unhandled\_power\_cone\_type (7136)

An unhandled power cone type.

rescode.err\_cbf\_unhandled\_power\_star\_cone\_type (7137)

An unhandled power star cone type.

rescode.err\_cbf\_power\_cone\_mismatch (7138)

The power cone does not match with it definition.

rescode.err\_cbf\_power\_star\_cone\_mismatch (7139)

The power star cone does not match with it definition.

rescode.err\_cbf\_invalid\_number\_of\_cones (7740)

Invalid number of cones.

rescode.err\_cbf\_invalid\_dimension\_of\_cones (7741)

Invalid dimension of cones.

rescode.err\_mio\_invalid\_root\_optimizer (7700)

An invalid root optimizer was selected for the problem type.

rescode.err\_mio\_invalid\_node\_optimizer (7701)

An invalid node optimizer was selected for the problem type.

rescode.err\_toconic\_constr\_q\_not\_psd (7800)

The matrix defining the quadratric part of constraint is not positive semidefinite.

rescode.err\_toconic\_constraint\_fx (7801)

The quadratic constraint is an equality, thus not convex.

rescode.err\_toconic\_constraint\_ra (7802)

The quadratic constraint has finite lower and upper bound, and therefore it is not convex.

rescode.err\_toconic\_constr\_not\_conic (7803)

The constraint is not conic representable.

rescode.err\_toconic\_objective\_not\_psd (7804)

The matrix defining the quadratric part of the objective function is not positive semidefinite.

rescode.err\_server\_connect (8000)

Failed to connect to remote solver server. The server string or the port string were invalid, or the server did not accept connection.

rescode.err\_server\_protocol (8001)

Unexpected message or data from solver server.

rescode.err\_server\_status (8002)

Server returned non-ok HTTP status code

rescode.err\_server\_token (8003)

The job ID specified is incorrect or invalid

# 15.9 Enumerations

## basindtype

Basis identification

## basindtype.never

Never do basis identification.

## basindtype.always

Basis identification is always performed even if the interior-point optimizer terminates abnormally.

# basindtype.no\_error

Basis identification is performed if the interior-point optimizer terminates without an error.

#### basindtype.if\_feasible

Basis identification is not performed if the interior-point optimizer terminates with a problem status saying that the problem is primal or dual infeasible.

# basindtype.reservered

Not currently in use.

#### boundkey

Bound keys

#### boundkey.lo

The constraint or variable has a finite lower bound and an infinite upper bound.

#### boundkey.up

The constraint or variable has an infinite lower bound and an finite upper bound.

## boundkey.fx

The constraint or variable is fixed.

#### boundkey.fr

The constraint or variable is free.

## boundkey.ra

The constraint or variable is ranged.

#### mark

Mark

#### mark.lo

The lower bound is selected for sensitivity analysis.

#### mark.up

The upper bound is selected for sensitivity analysis.

# $\verb|simdegen|$

Degeneracy strategies

## simdegen.none

The simplex optimizer should use no degeneration strategy.

## simdegen.free

The simplex optimizer chooses the degeneration strategy.

## simdegen.aggressive

The simplex optimizer should use an aggressive degeneration strategy.

## simdegen.moderate

The simplex optimizer should use a moderate degeneration strategy.

## simdegen.minimum

The simplex optimizer should use a minimum degeneration strategy.

#### transpose

Transposed matrix.

## transpose.no

No transpose is applied.

#### transpose.yes

A transpose is applied.

## uplo

Triangular part of a symmetric matrix.

## uplo.lo

Lower part.

```
uplo.up
          Upper part.
simreform
     Problem reformulation.
     simreform.on
          Allow the simplex optimizer to reformulate the problem.
     simreform.off
          Disallow the simplex optimizer to reformulate the problem.
     simreform.free
          The simplex optimizer can choose freely.
     simreform.aggressive
          The simplex optimizer should use an aggressive reformulation strategy.
simdupvec
     Exploit duplicate columns.
     simdupvec.on
          Allow the simplex optimizer to exploit duplicated columns.
     simdupvec.off
          Disallow the simplex optimizer to exploit duplicated columns.
     simdupvec.free
          The simplex optimizer can choose freely.
simhotstart
     Hot-start type employed by the simplex optimizer
     simhotstart.none
          The simplex optimizer performs a coldstart.
     simhotstart.free
          The simplex optimize chooses the hot-start type.
     simhotstart.status_keys
          Only the status keys of the constraints and variables are used to choose the type of hot-start.
intpnthotstart
     Hot-start type employed by the interior-point optimizers.
     intpnthotstart.none
          The interior-point optimizer performs a coldstart.
     intpnthotstart.primal
          The interior-point optimizer exploits the primal solution only.
     intpnthotstart.dual
          The interior-point optimizer exploits the dual solution only.
     intpnthotstart.primal_dual
          The interior-point optimizer exploits both the primal and dual solution.
purify
     Solution purification employed optimizer.
     purify.none
          The optimizer performs no solution purification.
     purify.primal
          The optimizer purifies the primal solution.
     purify.dual
          The optimizer purifies the dual solution.
     purify.primal_dual
          The optimizer purifies both the primal and dual solution.
     purify.auto
          TBD
callbackcode
```

Progress callback codes

#### callbackcode.begin\_bi

The basis identification procedure has been started.

## callbackcode.begin\_conic

The callback function is called when the conic optimizer is started.

## callbackcode.begin\_dual\_bi

The callback function is called from within the basis identification procedure when the dual phase is started.

## callbackcode.begin\_dual\_sensitivity

Dual sensitivity analysis is started.

# callbackcode.begin\_dual\_setup\_bi

The callback function is called when the dual BI phase is started.

## callbackcode.begin\_dual\_simplex

The callback function is called when the dual simplex optimizer started.

## callbackcode.begin\_dual\_simplex\_bi

The callback function is called from within the basis identification procedure when the dual simplex clean-up phase is started.

## callbackcode.begin\_full\_convexity\_check

Begin full convexity check.

#### callbackcode.begin\_infeas\_ana

The callback function is called when the infeasibility analyzer is started.

## callbackcode.begin\_intpnt

The callback function is called when the interior-point optimizer is started.

## callbackcode.begin\_license\_wait

Begin waiting for license.

## callbackcode.begin\_mio

The callback function is called when the mixed-integer optimizer is started.

#### callbackcode.begin\_optimizer

The callback function is called when the optimizer is started.

# ${\tt callbackcode.begin\_presolve}$

The callback function is called when the presolve is started.

#### callbackcode.begin\_primal\_bi

The callback function is called from within the basis identification procedure when the primal phase is started.

## callbackcode.begin\_primal\_repair

Begin primal feasibility repair.

## callbackcode.begin\_primal\_sensitivity

Primal sensitivity analysis is started.

## callbackcode.begin\_primal\_setup\_bi

The callback function is called when the primal BI setup is started.

# callbackcode.begin\_primal\_simplex

The callback function is called when the primal simplex optimizer is started.

# callbackcode.begin\_primal\_simplex\_bi

The callback function is called from within the basis identification procedure when the primal simplex clean-up phase is started.

## callbackcode.begin\_qcqo\_reformulate

Begin QCQO reformulation.

## callbackcode.begin\_read

MOSEK has started reading a problem file.

#### callbackcode.begin\_root\_cutgen

The callback function is called when root cut generation is started.

#### callbackcode.begin\_simplex

The callback function is called when the simplex optimizer is started.

## callbackcode.begin\_simplex\_bi

The callback function is called from within the basis identification procedure when the simplex clean-up phase is started.

## callbackcode.begin\_to\_conic

Begin conic reformulation.

## callbackcode.begin\_write

MOSEK has started writing a problem file.

#### callbackcode.conic

The callback function is called from within the conic optimizer after the information database has been updated.

## callbackcode.dual\_simplex

The callback function is called from within the dual simplex optimizer.

#### callbackcode.end\_bi

The callback function is called when the basis identification procedure is terminated.

#### callbackcode.end\_conic

The callback function is called when the conic optimizer is terminated.

#### callbackcode.end\_dual\_bi

The callback function is called from within the basis identification procedure when the dual phase is terminated.

## callbackcode.end\_dual\_sensitivity

Dual sensitivity analysis is terminated.

## callbackcode.end\_dual\_setup\_bi

The callback function is called when the dual BI phase is terminated.

# callbackcode.end\_dual\_simplex

The callback function is called when the dual simplex optimizer is terminated.

## $\verb|callbackcode.end_dual_simplex_bi|\\$

The callback function is called from within the basis identification procedure when the dual clean-up phase is terminated.

## callbackcode.end\_full\_convexity\_check

End full convexity check.

#### callbackcode.end\_infeas\_ana

The callback function is called when the infeasibility analyzer is terminated.

## callbackcode.end\_intpnt

The callback function is called when the interior-point optimizer is terminated.

## callbackcode.end\_license\_wait

End waiting for license.

## callbackcode.end\_mio

The callback function is called when the mixed-integer optimizer is terminated.

## callbackcode.end\_optimizer

The callback function is called when the optimizer is terminated.

## callbackcode.end\_presolve

The callback function is called when the presolve is completed.

## callbackcode.end\_primal\_bi

The callback function is called from within the basis identification procedure when the primal phase is terminated.

# ${\tt callbackcode.end\_primal\_repair}$

End primal feasibility repair.

# callbackcode.end\_primal\_sensitivity

Primal sensitivity analysis is terminated.

#### callbackcode.end\_primal\_setup\_bi

The callback function is called when the primal BI setup is terminated.

## callbackcode.end\_primal\_simplex

The callback function is called when the primal simplex optimizer is terminated.

## callbackcode.end\_primal\_simplex\_bi

The callback function is called from within the basis identification procedure when the primal clean-up phase is terminated.

## callbackcode.end\_qcqo\_reformulate

End QCQO reformulation.

## callbackcode.end\_read

MOSEK has finished reading a problem file.

## callbackcode.end\_root\_cutgen

The callback function is called when root cut generation is terminated.

## callbackcode.end\_simplex

The callback function is called when the simplex optimizer is terminated.

#### callbackcode.end\_simplex\_bi

The callback function is called from within the basis identification procedure when the simplex clean-up phase is terminated.

## callbackcode.end\_to\_conic

End conic reformulation.

#### callbackcode.end\_write

MOSEK has finished writing a problem file.

#### callbackcode.im\_bi

The callback function is called from within the basis identification procedure at an intermediate point.

## callbackcode.im\_conic

The callback function is called at an intermediate stage within the conic optimizer where the information database has not been updated.

## callbackcode.im\_dual\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

# ${\tt callbackcode.im\_dual\_sensivity}$

The callback function is called at an intermediate stage of the dual sensitivity analysis.

## callbackcode.im\_dual\_simplex

The callback function is called at an intermediate point in the dual simplex optimizer.

## callbackcode.im\_full\_convexity\_check

The callback function is called at an intermediate stage of the full convexity check.

## callbackcode.im\_intpnt

The callback function is called at an intermediate stage within the interior-point optimizer where the information database has not been updated.

#### callbackcode.im\_license\_wait

MOSEK is waiting for a license.

# callbackcode.im\_lu

The callback function is called from within the LU factorization procedure at an intermediate point.

# callbackcode.im\_mio

The callback function is called at an intermediate point in the mixed-integer optimizer.

#### callbackcode.im\_mio\_dual\_simplex

The callback function is called at an intermediate point in the mixed-integer optimizer while running the dual simplex optimizer.

## callbackcode.im\_mio\_intpnt

The callback function is called at an intermediate point in the mixed-integer optimizer while running the interior-point optimizer.

## callbackcode.im\_mio\_primal\_simplex

The callback function is called at an intermediate point in the mixed-integer optimizer while running the primal simplex optimizer.

## callbackcode.im\_order

The callback function is called from within the matrix ordering procedure at an intermediate point.

## callbackcode.im\_presolve

The callback function is called from within the presolve procedure at an intermediate stage.

## callbackcode.im\_primal\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

# callbackcode.im\_primal\_sensivity

The callback function is called at an intermediate stage of the primal sensitivity analysis.

## callbackcode.im\_primal\_simplex

The callback function is called at an intermediate point in the primal simplex optimizer.

#### callbackcode.im\_qo\_reformulate

The callback function is called at an intermediate stage of the conic quadratic reformulation.

## callbackcode.im\_read

Intermediate stage in reading.

## callbackcode.im\_root\_cutgen

The callback is called from within root cut generation at an intermediate stage.

## callbackcode.im\_simplex

The callback function is called from within the simplex optimizer at an intermediate point.

## callbackcode.im\_simplex\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the simplex clean-up phase. The frequency of the callbacks is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

## callbackcode.intpnt

The callback function is called from within the interior-point optimizer after the information database has been updated.

#### callbackcode.new\_int\_mio

The callback function is called after a new integer solution has been located by the mixed-integer optimizer.

## callbackcode.primal\_simplex

The callback function is called from within the primal simplex optimizer.

#### callbackcode.read\_opf

The callback function is called from the OPF reader.

## callbackcode.read\_opf\_section

A chunk of Q non-zeros has been read from a problem file.

## callbackcode.solving\_remote

The callback function is called while the task is being solved on a remote server.

#### callbackcode.update\_dual\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual phase.

## callbackcode.update\_dual\_simplex

The callback function is called in the dual simplex optimizer.

#### callbackcode.update\_dual\_simplex\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the dual simplex clean-up phase. The frequency of the callbacks is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

## callbackcode.update\_presolve

The callback function is called from within the presolve procedure.

# callbackcode.update\_primal\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the primal phase.

## callbackcode.update\_primal\_simplex

The callback function is called in the primal simplex optimizer.

# callbackcode.update\_primal\_simplex\_bi

The callback function is called from within the basis identification procedure at an intermediate point in the primal simplex clean-up phase. The frequency of the callbacks is controlled by the <code>iparam.log\_sim\_freq</code> parameter.

# callbackcode.write\_opf

The callback function is called from the OPF writer.

# ${\tt checkconvexitytype}$

Types of convexity checks.

## checkconvexitytype.none

No convexity check.

# checkconvexitytype.simple

Perform simple and fast convexity check.

## checkconvexitytype.full

Perform a full convexity check.

## compresstype

Compression types

## compresstype.none

No compression is used.

## compresstype.free

The type of compression used is chosen automatically.

## compresstype.gzip

The type of compression used is gzip compatible.

## compresstype.zstd

The type of compression used is zstd compatible.

#### conetype

Cone types

# conetype.quad

The cone is a quadratic cone.

## conetype.rquad

The cone is a rotated quadratic cone.

## conetype.pexp

A primal exponential cone.

## conetype.dexp

A dual exponential cone.

# conetype.ppow

A primal power cone.

## conetype.dpow

A dual power cone.

# conetype.zero

The zero cone.

## nametype

Name types

#### nametype.gen

General names. However, no duplicate and blank names are allowed.

```
nametype.mps
         MPS type names.
     nametype.lp
         LP type names.
scopr
     SCopt operator types
     scopr.ent
         Entropy
     scopr.exp
         Exponential
     scopr.log
         Logarithm
     scopr.pow
         Power
     scopr.sqrt
         Square root
symmattype
     Cone types
     symmattype.sparse
         Sparse symmetric matrix.
dataformat
     Data format types
     dataformat.extension
          The file extension is used to determine the data file format.
     dataformat.mps
          The data file is MPS formatted.
     dataformat.lp
          The data file is LP formatted.
     dataformat.op
         The data file is an optimization problem formatted file.
     dataformat.free_mps
         The data a free MPS formatted file.
     dataformat.task
          Generic task dump file.
     dataformat.ptf
          (P)retty (T)ext (F)format.
     dataformat.cb
          Conic benchmark format,
     dataformat.json_task
          JSON based task format.
dinfitem
     Double information items
     dinfitem.bi_clean_dual_time
         invocation.
```

Time spent within the dual clean-up optimizer of the basis identification procedure since its

#### dinfitem.bi\_clean\_primal\_time

Time spent within the primal clean-up optimizer of the basis identification procedure since its invocation.

## dinfitem.bi\_clean\_time

Time spent within the clean-up phase of the basis identification procedure since its invocation.

## dinfitem.bi\_dual\_time

Time spent within the dual phase basis identification procedure since its invocation.

## dinfitem.bi\_primal\_time

Time spent within the primal phase of the basis identification procedure since its invocation.

#### dinfitem.bi\_time

Time spent within the basis identification procedure since its invocation.

## dinfitem.intpnt\_dual\_feas

Dual feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed.)

#### dinfitem.intpnt\_dual\_obj

Dual objective value reported by the interior-point optimizer.

## dinfitem.intpnt\_factor\_num\_flops

An estimate of the number of flops used in the factorization.

## dinfitem.intpnt\_opt\_status

A measure of optimality of the solution. It should converge to +1 if the problem has a primaldual optimal solution, and converge to -1 if the problem is (strictly) primal or dual infeasible. If the measure converges to another constant, or fails to settle, the problem is usually ill-posed.

#### dinfitem.intpnt\_order\_time

Order time (in seconds).

## dinfitem.intpnt\_primal\_feas

Primal feasibility measure reported by the interior-point optimizer. (For the interior-point optimizer this measure is not directly related to the original problem because a homogeneous model is employed).

## dinfitem.intpnt\_primal\_obj

Primal objective value reported by the interior-point optimizer.

#### dinfitem.intpnt\_time

Time spent within the interior-point optimizer since its invocation.

# dinfitem.mio\_clique\_separation\_time

Separation time for clique cuts.

## dinfitem.mio\_cmir\_separation\_time

Separation time for CMIR cuts.

# dinfitem.mio\_construct\_solution\_obj

If **MOSEK** has successfully constructed an integer feasible solution, then this item contains the optimal objective value corresponding to the feasible solution.

## dinfitem.mio\_dual\_bound\_after\_presolve

Value of the dual bound after presolve but before cut generation.

#### dinfitem.mio\_gmi\_separation\_time

Separation time for GMI cuts.

## dinfitem.mio\_implied\_bound\_time

Separation time for implied bound cuts.

## dinfitem.mio\_knapsack\_cover\_separation\_time

Separation time for knapsack cover.

# ${\tt dinfitem.mio\_obj\_abs\_gap}$

Given the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the absolute gap defined by

|(objective value of feasible solution) – (objective bound)|.

Otherwise it has the value -1.0.

## dinfitem.mio\_obj\_bound

The best known bound on the objective function. This value is undefined until at least one relaxation has been solved: To see if this is the case check that  $iinfitem.mio\_num\_relax$  is strictly positive.

#### dinfitem.mio\_obj\_int

The primal objective value corresponding to the best integer feasible solution. Please note that at least one integer feasible solution must have been located i.e. check iinfitem.  $mio\_num\_int\_solutions$ .

#### dinfitem.mio\_obj\_rel\_gap

Given that the mixed-integer optimizer has computed a feasible solution and a bound on the optimal objective value, then this item contains the relative gap defined by

 $\frac{|(\text{objective value of feasible solution}) - (\text{objective bound})|}{\max(\delta, |(\text{objective value of feasible solution})|)}$ 

where  $\delta$  is given by the parameter  $dparam.mio\_rel\_gap\_const$ . Otherwise it has the value -1.0.

## dinfitem.mio\_probing\_time

Total time for probing.

## dinfitem.mio\_root\_cutgen\_time

Total time for cut generation.

# dinfitem.mio\_root\_optimizer\_time

Time spent in the optimizer while solving the root node relaxation

## ${\tt dinfitem.mio\_root\_presolve\_time}$

Time spent presolving the problem at the root node.

#### dinfitem.mio\_time

Time spent in the mixed-integer optimizer.

## dinfitem.mio\_user\_obj\_cut

If the objective cut is used, then this information item has the value of the cut.

## dinfitem.optimizer\_time

Total time spent in the optimizer since it was invoked.

# ${\tt dinfitem.presolve\_eli\_time}$

Total time spent in the eliminator since the presolve was invoked.

## dinfitem.presolve\_lindep\_time

Total time spent in the linear dependency checker since the presolve was invoked.

#### dinfitem.presolve\_time

Total time (in seconds) spent in the presolve since it was invoked.

## dinfitem.primal\_repair\_penalty\_obj

The optimal objective value of the penalty function.

# ${\tt dinfitem.qcqo\_reformulate\_max\_perturbation}$

Maximum absolute diagonal perturbation occurring during the QCQO reformulation.

#### dinfitem.qcqo\_reformulate\_time

Time spent with conic quadratic reformulation.

## dinfitem.qcqo\_reformulate\_worst\_cholesky\_column\_scaling

Worst Cholesky column scaling.

## dinfitem.qcqo\_reformulate\_worst\_cholesky\_diag\_scaling

Worst Cholesky diagonal scaling.

## dinfitem.rd\_time

Time spent reading the data file.

## dinfitem.sim\_dual\_time

Time spent in the dual simplex optimizer since invoking it.

#### dinfitem.sim\_feas

Feasibility measure reported by the simplex optimizer.

## dinfitem.sim\_obj

Objective value reported by the simplex optimizer.

## dinfitem.sim\_primal\_time

Time spent in the primal simplex optimizer since invoking it.

#### dinfitem.sim\_time

Time spent in the simplex optimizer since invoking it.

#### dinfitem.sol\_bas\_dual\_obj

Dual objective value of the basic solution. Updated if <code>iparam.auto\_update\_sol\_info</code> is set or by the method <code>Task.updatesolutioninfo</code>.

## dinfitem.sol\_bas\_dviolcon

Maximal dual bound violation for  $x^c$  in the basic solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_bas\_dviolvar

Maximal dual bound violation for  $x^x$  in the basic solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_bas\_nrm\_barx

Infinity norm of  $\overline{X}$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_slc

Infinity norm of  $s_l^c$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_slx

Infinity norm of  $s_l^x$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_suc

Infinity norm of  $s_u^c$  in the basic solution.

# dinfitem.sol\_bas\_nrm\_sux

Infinity norm of  $s_n^X$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_xc

Infinity norm of  $x^c$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_xx

Infinity norm of  $x^x$  in the basic solution.

## dinfitem.sol\_bas\_nrm\_y

Infinity norm of y in the basic solution.

## dinfitem.sol\_bas\_primal\_obj

Primal objective value of the basic solution. Updated if  $iparam.auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_bas\_pviolcon

Maximal primal bound violation for  $x^c$  in the basic solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_bas\_pviolvar

Maximal primal bound violation for  $x^x$  in the basic solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

# ${\tt dinfitem.sol\_itg\_nrm\_barx}$

Infinity norm of  $\overline{X}$  in the integer solution.

## dinfitem.sol\_itg\_nrm\_xc

Infinity norm of  $x^c$  in the integer solution.

# ${\tt dinfitem.sol\_itg\_nrm\_xx}$

Infinity norm of  $x^x$  in the integer solution.

# dinfitem.sol\_itg\_primal\_obj

Primal objective value of the integer solution. Updated if <code>iparam.auto\_update\_sol\_info</code> is set or by the method <code>Task.updatesolutioninfo</code>.

## dinfitem.sol\_itg\_pviolbarvar

Maximal primal bound violation for  $\overline{X}$  in the integer solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itg\_pviolcon

Maximal primal bound violation for  $x^c$  in the integer solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itg\_pviolcones

Maximal primal violation for primal conic constraints in the integer solution. Updated if <code>iparam.auto\_update\_sol\_info</code> is set or by the method <code>Task.updatesolutioninfo</code>.

## dinfitem.sol\_itg\_pviolitg

Maximal violation for the integer constraints in the integer solution. Updated if *iparam*.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itg\_pviolvar

Maximal primal bound violation for  $x^x$  in the integer solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itr\_dual\_obj

Dual objective value of the interior-point solution. Updated if *iparam.*  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itr\_dviolbarvar

Maximal dual bound violation for  $\overline{X}$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itr\_dviolcon

Maximal dual bound violation for  $x^c$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itr\_dviolcones

Maximal dual violation for dual conic constraints in the interior-point solution. Updated if iparam.auto\_update\_sol\_info is set or by the method Task.updatesolutioninfo.

#### dinfitem.sol\_itr\_dviolvar

Maximal dual bound violation for  $x^x$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

## dinfitem.sol\_itr\_nrm\_bars

Infinity norm of  $\overline{S}$  in the interior-point solution.

# dinfitem.sol\_itr\_nrm\_barx

Infinity norm of  $\overline{X}$  in the interior-point solution.

## dinfitem.sol\_itr\_nrm\_slc

Infinity norm of  $s_l^c$  in the interior-point solution.

# ${\tt dinfitem.sol\_itr\_nrm\_slx}$

Infinity norm of  $s_l^x$  in the interior-point solution.

## dinfitem.sol\_itr\_nrm\_snx

Infinity norm of  $s_n^x$  in the interior-point solution.

# dinfitem.sol\_itr\_nrm\_suc

Infinity norm of  $s_u^c$  in the interior-point solution.

# ${\tt dinfitem.sol\_itr\_nrm\_sux}$

Infinity norm of  $s_u^X$  in the interior-point solution.

# dinfitem.sol\_itr\_nrm\_xc

Infinity norm of  $x^c$  in the interior-point solution.

## dinfitem.sol\_itr\_nrm\_xx

Infinity norm of  $x^x$  in the interior-point solution.

# dinfitem.sol\_itr\_nrm\_y

Infinity norm of y in the interior-point solution.

## dinfitem.sol\_itr\_primal\_obj

Primal objective value of the interior-point solution. Updated if *iparam*.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

#### dinfitem.sol\_itr\_pviolbarvar

Maximal primal bound violation for  $\overline{X}$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

# dinfitem.sol\_itr\_pviolcon

Maximal primal bound violation for  $x^c$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

# dinfitem.sol\_itr\_pviolcones

Maximal primal violation for primal conic constraints in the interior-point solution. Updated if <code>iparam.auto\_update\_sol\_info</code> is set or by the method <code>Task.updatesolutioninfo</code>.

### dinfitem.sol\_itr\_pviolvar

Maximal primal bound violation for  $x^x$  in the interior-point solution. Updated if iparam.  $auto\_update\_sol\_info$  is set or by the method Task.updatesolutioninfo.

#### dinfitem.to\_conic\_time

Time spent in the last to conic reformulation.

#### feature

License feature

# feature.pts

Base system.

#### feature.pton

Conic extension.

#### liinfitem

Long integer information items.

#### liinfitem.bi\_clean\_dual\_deg\_iter

Number of dual degenerate clean iterations performed in the basis identification.

#### liinfitem.bi\_clean\_dual\_iter

Number of dual clean iterations performed in the basis identification.

# liinfitem.bi\_clean\_primal\_deg\_iter

Number of primal degenerate clean iterations performed in the basis identification.

# liinfitem.bi\_clean\_primal\_iter

Number of primal clean iterations performed in the basis identification.

# liinfitem.bi\_dual\_iter

Number of dual pivots performed in the basis identification.

# liinfitem.bi\_primal\_iter

Number of primal pivots performed in the basis identification.

# liinfitem.intpnt\_factor\_num\_nz

Number of non-zeros in factorization.

### liinfitem.mio\_anz

Number of non-zero entries in the constraint matrix of the probelm to be solved by the mixed-integer optimizer.

#### liinfitem.mio\_intpnt\_iter

Number of interior-point iterations performed by the mixed-integer optimizer.

# ${\tt liinfitem.mio\_presolved\_anz}$

Number of non-zero entries in the constraint matrix of the problem after the mixed-integer optimizer's presolve.

# liinfitem.mio\_simplex\_iter

Number of simplex iterations performed by the mixed-integer optimizer.

#### liinfitem.rd\_numanz

Number of non-zeros in A that is read.

### liinfitem.rd\_numqnz

Number of Q non-zeros.

# iinfitem

Integer information items.

#### iinfitem.ana\_pro\_num\_con

Number of constraints in the problem. This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_con\_eq

Number of equality constraints. This value is set by Task. analyzeproblem.

#### iinfitem.ana\_pro\_num\_con\_fr

Number of unbounded constraints. This value is set by Task. analyzeproblem.

#### iinfitem.ana\_pro\_num\_con\_lo

Number of constraints with a lower bound and an infinite upper bound. This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_con\_ra

Number of constraints with finite lower and upper bounds. This value is set by Task. analyzeproblem.

#### iinfitem.ana\_pro\_num\_con\_up

Number of constraints with an upper bound and an infinite lower bound. This value is set by Task.analyzeproblem.

# iinfitem.ana\_pro\_num\_var

Number of variables in the problem. This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_bin

Number of binary (0-1) variables. This value is set by Task. analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_cont

Number of continuous variables. This value is set by Task.analyzeproblem.

# iinfitem.ana\_pro\_num\_var\_eq

Number of fixed variables. This value is set by Task.analyzeproblem.

### iinfitem.ana\_pro\_num\_var\_fr

Number of free variables. This value is set by Task. analyzeproblem.

# iinfitem.ana\_pro\_num\_var\_int

Number of general integer variables. This value is set by Task. analyzeproblem.

# iinfitem.ana\_pro\_num\_var\_lo

Number of variables with a lower bound and an infinite upper bound. This value is set by Task.analyzeproblem.

## iinfitem.ana\_pro\_num\_var\_ra

Number of variables with finite lower and upper bounds. This value is set by Task. analyzeproblem.

# iinfitem.ana\_pro\_num\_var\_up

Number of variables with an upper bound and an infinite lower bound. This value is set by Task.analyzeproblem.

#### iinfitem.intpnt\_factor\_dim\_dense

Dimension of the dense sub system in factorization.

# iinfitem.intpnt\_iter

Number of interior-point iterations since invoking the interior-point optimizer.

#### iinfitem.intpnt\_num\_threads

Number of threads that the interior-point optimizer is using.

# iinfitem.intpnt\_solve\_dual

Non-zero if the interior-point optimizer is solving the dual problem.

#### iinfitem.mio\_absgap\_satisfied

Non-zero if absolute gap is within tolerances.

# iinfitem.mio\_clique\_table\_size

Size of the clique table.

### iinfitem.mio\_construct\_solution

This item informs if  $\mathbf{MOSEK}$  constructed an initial integer feasible solution.

• -1: tried, but failed,

- 0: no partial solution supplied by the user,
- 1: constructed feasible solution.

### iinfitem.mio\_node\_depth

Depth of the last node solved.

#### iinfitem.mio\_num\_active\_nodes

Number of active branch and bound nodes.

#### iinfitem.mio\_num\_branch

Number of branches performed during the optimization.

#### iinfitem.mio\_num\_clique\_cuts

Number of clique cuts.

#### iinfitem.mio\_num\_cmir\_cuts

Number of Complemented Mixed Integer Rounding (CMIR) cuts.

#### iinfitem.mio\_num\_gomory\_cuts

Number of Gomory cuts.

#### iinfitem.mio\_num\_implied\_bound\_cuts

Number of implied bound cuts.

#### iinfitem.mio\_num\_int\_solutions

Number of integer feasible solutions that have been found.

#### iinfitem.mio\_num\_knapsack\_cover\_cuts

Number of clique cuts.

#### iinfitem.mio\_num\_relax

Number of relaxations solved during the optimization.

# iinfitem.mio\_num\_repeated\_presolve

Number of times presolve was repeated at root.

#### iinfitem.mio\_numbin

Number of binary variables in the problem to be solved by the mixed-integer optimizer.

# iinfitem.mio\_numbinconevar

Number of binary cone variables in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_numcon

Number of constraints in the problem to be solved by the mixed-integer optimizer.

# iinfitem.mio\_numcone

Number of cones in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numconevar

Number of cone variables in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numcont

Number of continuous variables in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_numcontconevar

Number of continuous cone variables in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_numdexpcones

Number of dual exponential cones in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numdpowcones

Number of dual power cones in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numint

Number of integer variables in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numintconevar

Number of integer cone variables in the problem to be solved by the mixed-integer optimizer.

# $\verb|iinfitem.mio_numpexpcones||$

Number of primal exponential cones in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_numppowcones

Number of primal power cones in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numqcones

Number of quadratic cones in the problem to be solved by the mixed-integer optimizer.

### iinfitem.mio\_numrqcones

Number of rotated quadratic cones in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_numvar

Number of variables in the problem to be solved by the mixed-integer optimizer.

#### iinfitem.mio\_obj\_bound\_defined

Non-zero if a valid objective bound has been found, otherwise zero.

#### iinfitem.mio\_presolved\_numbin

Number of binary variables in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numbinconevar

Number of binary cone variables in the problem after the mixed-integer optimizer's presolve.

#### iinfitem.mio\_presolved\_numcon

Number of constraints in the problem after the mixed-integer optimizer's presolve.

#### iinfitem.mio\_presolved\_numcone

Number of cones in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numconevar

Number of cone variables in the problem after the mixed-integer optimizer's presolve.

#### iinfitem.mio\_presolved\_numcont

Number of continuous variables in the problem after the mixed-integer optimizer's presolve.

# $\verb|iinfitem.mio_presolved_numcontconevar|\\$

Number of continuous cone variables in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numdexpcones

Number of dual exponential cones in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numdpowcones

Number of dual power cones in the problem after the mixed-integer optimizer's presolve.

### iinfitem.mio\_presolved\_numint

Number of integer variables in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numintconevar

Number of integer cone variables in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numpexpcones

Number of primal exponential cones in the problem after the mixed-integer optimizer's presolve.

# $\verb|iinfitem.mio_presolved_numppowcones|\\$

Number of primal power cones in the problem after the mixed-integer optimizer's presolve.

#### iinfitem.mio\_presolved\_numqcones

Number of quadratic cones in the problem after the mixed-integer optimizer's presolve.

# $\verb|iinfitem.mio_presolved_numrqcones|\\$

Number of rotated quadratic cones in the problem after the mixed-integer optimizer's presolve.

# iinfitem.mio\_presolved\_numvar

Number of variables in the problem after the mixed-integer optimizer's presolve.

# $\verb|iinfitem.mio_relgap_satisfied|\\$

Non-zero if relative gap is within tolerances.

# iinfitem.mio\_total\_num\_cuts

Total number of cuts generated by the mixed-integer optimizer.

## iinfitem.mio\_user\_obj\_cut

If it is non-zero, then the objective cut is used.

#### iinfitem.opt\_numcon

Number of constraints in the problem solved when the optimizer is called.

# iinfitem.opt\_numvar

Number of variables in the problem solved when the optimizer is called

#### iinfitem.optimize\_response

The response code returned by optimize.

#### iinfitem.purify\_dual\_success

Is nonzero if the dual solution is purified.

# iinfitem.purify\_primal\_success

Is nonzero if the primal solution is purified.

#### iinfitem.rd\_numbarvar

Number of symmetric variables read.

#### iinfitem.rd\_numcon

Number of constraints read.

#### iinfitem.rd\_numcone

Number of conic constraints read.

# iinfitem.rd\_numintvar

Number of integer-constrained variables read.

#### iinfitem.rd\_numq

Number of nonempty Q matrices read.

## iinfitem.rd\_numvar

Number of variables read.

### iinfitem.rd\_protype

Problem type.

# iinfitem.sim\_dual\_deg\_iter

The number of dual degenerate iterations.

# $\verb|iinfitem.sim_dual_hotstart|\\$

If 1 then the dual simplex algorithm is solving from an advanced basis.

# iinfitem.sim\_dual\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the dual simplex algorithm.

# iinfitem.sim\_dual\_inf\_iter

The number of iterations taken with dual infeasibility.

# iinfitem.sim\_dual\_iter

Number of dual simplex iterations during the last optimization.

### iinfitem.sim\_numcon

Number of constraints in the problem solved by the simplex optimizer.

### iinfitem.sim\_numvar

Number of variables in the problem solved by the simplex optimizer.

# iinfitem.sim\_primal\_deg\_iter

The number of primal degenerate iterations.

# iinfitem.sim\_primal\_hotstart

If 1 then the primal simplex algorithm is solving from an advanced basis.

# iinfitem.sim\_primal\_hotstart\_lu

If 1 then a valid basis factorization of full rank was located and used by the primal simplex algorithm.

# iinfitem.sim\_primal\_inf\_iter

The number of iterations taken with primal infeasibility.

## iinfitem.sim\_primal\_iter

Number of primal simplex iterations during the last optimization.

#### iinfitem.sim\_solve\_dual

Is non-zero if dual problem is solved.

#### iinfitem.sol\_bas\_prosta

Problem status of the basic solution. Updated after each optimization.

#### iinfitem.sol\_bas\_solsta

Solution status of the basic solution. Updated after each optimization.

#### iinfitem.sol\_itg\_prosta

Problem status of the integer solution. Updated after each optimization.

#### iinfitem.sol\_itg\_solsta

Solution status of the integer solution. Updated after each optimization.

#### iinfitem.sol\_itr\_prosta

Problem status of the interior-point solution. Updated after each optimization.

#### iinfitem.sol\_itr\_solsta

Solution status of the interior-point solution. Updated after each optimization.

#### iinfitem.sto\_num\_a\_realloc

Number of times the storage for storing A has been changed. A large value may indicates that memory fragmentation may occur.

#### inftype

Information item types

#### inftype.dou\_type

Is a double information type.

#### inftype.int\_type

Is an integer.

#### inftype.lint\_type

Is a long integer.

#### iomode

Input/output modes

# iomode.read

The file is read-only.

#### iomode.write

The file is write-only. If the file exists then it is truncated when it is opened. Otherwise it is created when it is opened.

# iomode.readwrite

The file is to read and write.

#### branchdir

Specifies the branching direction.

# branchdir.free

The mixed-integer optimizer decides which branch to choose.

#### branchdir.up

The mixed-integer optimizer always chooses the up branch first.

# branchdir.down

The mixed-integer optimizer always chooses the down branch first.

#### branchdir.near

Branch in direction nearest to selected fractional variable.

#### branchdir.far

Branch in direction farthest from selected fractional variable.

### branchdir.root\_lp

Chose direction based on root lp value of selected variable.

#### branchdir.guided

Branch in direction of current incumbent.

### branchdir.pseudocost

Branch based on the pseudocost of the variable.

#### miocontsoltype

Continuous mixed-integer solution type

### miocontsoltype.none

No interior-point or basic solution are reported when the mixed-integer optimizer is used.

#### miocontsoltype.root

The reported interior-point and basic solutions are a solution to the root node problem when mixed-integer optimizer is used.

### miocontsoltype.itg

The reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. A solution is only reported in case the problem has a primal feasible solution.

# miocontsoltype.itg\_rel

In case the problem is primal feasible then the reported interior-point and basic solutions are a solution to the problem with all integer variables fixed at the value they have in the integer solution. If the problem is primal infeasible, then the solution to the root node problem is reported.

#### miomode

Integer restrictions

#### miomode.ignored

The integer constraints are ignored and the problem is solved as a continuous problem.

#### miomode.satisfied

Integer restrictions should be satisfied.

#### mionodeseltype

Mixed-integer node selection types

#### mionodeseltype.free

The optimizer decides the node selection strategy.

### mionodeseltype.first

The optimizer employs a depth first node selection strategy.

### mionodeseltype.best

The optimizer employs a best bound node selection strategy.

# mionodeseltype.pseudo

The optimizer employs selects the node based on a pseudo cost estimate.

# mpsformat

MPS file format type

# ${\tt mpsformat.strict}$

It is assumed that the input file satisfies the MPS format strictly.

#### mpsformat.relaxed

It is assumed that the input file satisfies a slightly relaxed version of the MPS format.

# mpsformat.free

It is assumed that the input file satisfies the free MPS format. This implies that spaces are not allowed in names. Otherwise the format is free.

# mpsformat.cplex

The CPLEX compatible version of the MPS format is employed.

#### objsense

Objective sense types

### objsense.minimize

The problem should be minimized.

# objsense.maximize

The problem should be maximized.

#### onoffkey

On/off

### onoffkey.on

Switch the option on.

```
onoffkey.off
```

Switch the option off.

#### optimizertype

Optimizer types

### optimizertype.conic

The optimizer for problems having conic constraints.

# optimizertype.dual\_simplex

The dual simplex optimizer is used.

#### optimizertype.free

The optimizer is chosen automatically.

# optimizertype.free\_simplex

One of the simplex optimizers is used.

# optimizertype.intpnt

The interior-point optimizer is used.

# optimizertype.mixed\_int

The mixed-integer optimizer.

# ${\tt optimizertype.primal\_simplex}$

The primal simplex optimizer is used.

#### orderingtype

Ordering strategies

#### orderingtype.free

The ordering method is chosen automatically.

#### orderingtype.appminloc

Approximate minimum local fill-in ordering is employed.

#### orderingtype.experimental

This option should not be used.

# orderingtype.try\_graphpar

Always try the graph partitioning based ordering.

# orderingtype.force\_graphpar

Always use the graph partitioning based ordering even if it is worse than the approximate minimum local fill ordering.

# orderingtype.none

No ordering is used.

# presolvemode

Presolve method.

#### presolvemode.off

The problem is not presolved before it is optimized.

# presolvemode.on

The problem is presolved before it is optimized.

#### presolvemode.free

It is decided automatically whether to presolve before the problem is optimized.

# parametertype

Parameter type

# parametertype.invalid\_type

Not a valid parameter.

# ${\tt parametertype.dou\_type}$

Is a double parameter.

# ${\tt parametertype.int\_type}$

Is an integer parameter.

# parametertype.str\_type

Is a string parameter.

```
problemitem
```

Problem data items

problemitem.var

Item is a variable.

problemitem.con

Item is a constraint.

problemitem.cone

Item is a cone.

#### problemtype

Problem types

problemtype.lo

The problem is a linear optimization problem.

problemtype.qo

The problem is a quadratic optimization problem.

problemtype.qcqo

The problem is a quadratically constrained optimization problem.

problemtype.conic

A conic optimization.

problemtype.mixed

General nonlinear constraints and conic constraints. This combination can not be solved by MOSEK.

# prosta

Problem status keys

prosta.unknown

Unknown problem status.

prosta.prim\_and\_dual\_feas

The problem is primal and dual feasible.

prosta.prim\_feas

The problem is primal feasible.

prosta.dual\_feas

The problem is dual feasible.

 ${\tt prosta.prim\_infeas}$ 

The problem is primal infeasible.

prosta.dual\_infeas

The problem is dual infeasible.

prosta.prim\_and\_dual\_infeas

The problem is primal and dual infeasible.

prosta.ill\_posed

The problem is ill-posed. For example, it may be primal and dual feasible but have a positive duality gap.

prosta.prim\_infeas\_or\_unbounded

The problem is either primal infeasible or unbounded. This may occur for mixed-integer problems.

xmlwriteroutputtype

XML writer output mode

 $\verb|xmlwriteroutputtype.row| \\$ 

Write in row order.

xmlwriteroutputtype.col

Write in column order.

rescodetype

Response code type

#### rescodetype.ok

The response code is OK.

#### rescodetype.wrn

The response code is a warning.

### rescodetype.trm

The response code is an optimizer termination status.

#### rescodetype.err

The response code is an error.

# rescodetype.unk

The response code does not belong to any class.

# scalingtype

Scaling type

#### scalingtype.free

The optimizer chooses the scaling heuristic.

#### scalingtype.none

No scaling is performed.

#### scalingtype.moderate

A conservative scaling is performed.

#### scalingtype.aggressive

A very aggressive scaling is performed.

# scaling method

Scaling method

# scalingmethod.pow2

Scales only with power of 2 leaving the mantissa untouched.

# scalingmethod.free

The optimizer chooses the scaling heuristic.

# sensitivitytype

Sensitivity types

#### sensitivitytype.basis

Basis sensitivity analysis is performed.

# simseltype

Simplex selection strategy

# simseltype.free

The optimizer chooses the pricing strategy.

# simseltype.full

The optimizer uses full pricing.

### simseltype.ase

The optimizer uses approximate steepest-edge pricing.

### simseltype.devex

The optimizer uses devex steepest-edge pricing (or if it is not available an approximate steepedge selection).

# simseltype.se

The optimizer uses steepest-edge selection (or if it is not available an approximate steep-edge selection).

# simseltype.partial

The optimizer uses a partial selection approach. The approach is usually beneficial if the number of variables is much larger than the number of constraints.

# solitem

Solution items

## solitem.xc

Solution for the constraints.

```
solitem.xx
```

Variable solution.

#### solitem.y

Lagrange multipliers for equations.

#### solitem.slc

Lagrange multipliers for lower bounds on the constraints.

#### solitem.suc

Lagrange multipliers for upper bounds on the constraints.

#### solitem.slx

Lagrange multipliers for lower bounds on the variables.

#### solitem.sux

Lagrange multipliers for upper bounds on the variables.

#### solitem.snx

Lagrange multipliers corresponding to the conic constraints on the variables.

#### solsta

Solution status keys

#### solsta.unknown

Status of the solution is unknown.

# solsta.optimal

The solution is optimal.

# solsta.prim\_feas

The solution is primal feasible.

#### solsta.dual\_feas

The solution is dual feasible.

### solsta.prim\_and\_dual\_feas

The solution is both primal and dual feasible.

#### solsta.prim\_infeas\_cer

The solution is a certificate of primal infeasibility.

# solsta.dual\_infeas\_cer

The solution is a certificate of dual infeasibility.

#### solsta.prim\_illposed\_cer

The solution is a certificate that the primal problem is illposed.

# $solsta.dual\_illposed\_cer$

The solution is a certificate that the dual problem is illposed.

#### solsta.integer\_optimal

The primal solution is integer optimal.

# soltype

Solution types

# soltype.bas

The basic solution.

# soltype.itr

The interior solution.

# soltype.itg

The integer solution.

#### solveform

Solve primal or dual form

#### solveform.free

The optimizer is free to solve either the primal or the dual problem.

#### solveform.primal

The optimizer should solve the primal problem.

#### solveform.dual

The optimizer should solve the dual problem.

#### stakey

Status keys

#### stakey.unk

The status for the constraint or variable is unknown.

#### stakey.bas

The constraint or variable is in the basis.

#### stakey.supbas

The constraint or variable is super basic.

#### stakey.low

The constraint or variable is at its lower bound.

#### stakey.upr

The constraint or variable is at its upper bound.

#### stakey.fix

The constraint or variable is fixed.

#### stakey.inf

The constraint or variable is infeasible in the bounds.

#### startpointtype

Starting point types

#### startpointtype.free

The starting point is chosen automatically.

#### startpointtype.guess

The optimizer guesses a starting point.

#### startpointtype.constant

The optimizer constructs a starting point by assigning a constant value to all primal and dual variables. This starting point is normally robust.

# startpointtype.satisfy\_bounds

The starting point is chosen to satisfy all the simple bounds on nonlinear variables. If this starting point is employed, then more care than usual should employed when choosing the bounds on the nonlinear variables. In particular very tight bounds should be avoided.

#### streamtype

Stream types

# streamtype.log

Log stream. Contains the aggregated contents of all other streams. This means that a message written to any other stream will also be written to this stream.

# streamtype.msg

Message stream. Log information relating to performance and progress of the optimization is written to this stream.

# streamtype.err

Error stream. Error messages are written to this stream.

#### streamtype.wrn

Warning stream. Warning messages are written to this stream.

#### value

Integer values

# ${\tt value.max\_str\_len}$

Maximum string length allowed in MOSEK.

#### value.license\_buffer\_length

The length of a license key buffer.

# variabletype

Variable types

# variabletype.type\_cont

Is a continuous variable.

variabletype.type\_int
Is an integer variable.

# 15.10 Class types

mosek.ItgSolutionCallback

A handler class for integer solution call backs.

ItgSolutionCallback.callback

```
void callback (double [] xx)
```

The integer solution callback is a user-defined function which will be called by **MOSEK** when it improves the best mixed-integer solution.

The user must not call any MOSEK function directly or indirectly from the callback function.

**Parameters** xx (double[]) – An array with the values of all variables in the currently best solution. (input)

mosek.DataCallback

A handler class for data call backs.

DataCallback.callback

```
int callback
  (callbackcode code,
   double [] dinf,
   int [] iinf,
   long [] liinf)
```

The data callback is a user-defined function which will be called by **MOSEK** occasionally during the optimization process. In particular, the callback function is called at the beginning of each iteration in the interior-point optimizer. For the simplex optimizers <code>iparam.log\_sim\_freq</code> controls how frequently the callback is called.

The user *must not* call any **MOSEK** function directly or indirectly from the callback function. The only exception is the possibility to retrieve an integer solution, see *Progress and data callback*.

# Parameters

- code (callbackcode) Callback code indicating current operation of the solver. (input)
- dinf (double []) Array of double information items. (input)
- iinf (int[]) Array of integer information items. (input)
- liinf (long[]) Array of long integer information items. (input)

**Return** (int) – Non-zero if the optimizer should be stopped; zero otherwise.

mosek.Stream

A stream handler class.

Stream.streamCB

```
void streamCB (string msg)
```

The message-stream callback function is a user-defined function which can be linked to any of the MOSEK streams. Doing so, the function is called whenever MOSEK sends a message to the stream.

The user must not call any MOSEK function directly or indirectly from the callback function.

Parameters msg (string) - Text string in the stream. (input)

mosek.Progress

A handler class for progress call backs.

Progress.progressCB

```
int progressCB (callbackcode code)
```

The progress callback is a user-defined function which will be called by **MOSEK** occasionally during the optimization process. In particular, the callback function is called at the beginning of each iteration in the interior-point optimizer. For the simplex optimizers <code>iparam.log\_sim\_freq</code> controls how frequently the callback is called.

The user must not call any MOSEK function directly or indirectly from the callback function.

**Parameters** code (callbackcode) – Callback code indicating current operation of the solver. (input)

Return (int) - Non-zero if the optimizer should be stopped; zero otherwise.

# 15.11 Nonlinear interfaces (obsolete)

**Important:** This is a legacy document for users familiar with SCopt, DGopt, EXPopt, mskenopt, mskscopt and mskgpopt from previous versions of **MOSEK**. These interfaces have now been removed. We assume familiarity with documentation included in version 8. All problems expressible with this interface can (and should) be reformulated using the exponential cone and power cones.

New users should formulate problems involving powers, logarithms and exponentials directly in conic form.

#### Conversion tutorial

We recommend converting all nonlinear problems using SCopt, DGopt, EXPopt, mskenopt, mskscopt and mskgpopt into conic form. Depending on the values of f, g, h either the epigraph or hypograph of a SCopt function if convex, and a bounding variable can be introduced following the basic rules below. We assume all variables are within safe bounds where the SCopt operators are defined and convex. We also assume f > 0.

A more comprehensive modeling guide for these types of problems can be found in the MOSEK Modeling Cookbook.

### **Powers**

Consider  $f(x+h)^g$ . This can be reformulated using the power cone.

- If g > 1 then  $t \ge f(x+h)^g$  is equivalent to  $(t/f)^{1/g} \ge |x+h|$ , that is  $(t/f, 1, x+h) \in \mathcal{P}_3^{1/g, 1-1/g}$ .
- If 0 < g < 1 then  $|t| \le f(x+h)^g$  is equivalent to  $(x+h,1,t/f) \in \mathcal{P}_3^{g,1-g}$ .
- If g < 0 then  $t \ge f(x+h)^g$  is equivalent to  $(t/f)(x+h)^{-g} \ge 1$ , that is  $(t/f, x+h, 1) \in \mathcal{P}_3^{1/(1-g), -g/(1-g)}$ .

### Logarithm

The bound  $t \leq f \log(gx + h)$  is equivalent to  $(gx + h, 1, t/f) \in K_{\text{exp}}$ .

#### **Entropy**

The bound  $t \ge fx \log x$  is equivalent to  $(1, x, -t/f) \in K_{\exp}$ .

# **Exponential**

The bound  $t \ge f \exp(gx+h)$  is equivalent to  $(t/f, 1, gx+h) \in K_{\exp}$ .

# Exponential optimization (EXPopt), Geometric programming (mskgpopt)

For a basic tutorial in geometric programming (GP) see Sec. 6.8. An exponential optimization problem in standard form consists of constraints of the type:

$$t \ge \log \left( \sum_{i} \exp(a_i^T x + b_i) \right).$$

This log-sum-exp bound is equivalent to

$$\sum_{i} \exp(a_i^T x + b_i - t) \le 1$$

and requires bounding each exponential function as explained above.

# Dual geometric optimization (DGopt)

The objective function of a dual geometric problem involves maximizing expressions of the form

$$x \log \frac{c}{x}$$
 and  $x_i \log \frac{e^T x}{x_i}$ ,

which can be achieved using bounds  $t \leq x \log \frac{y}{x}$ , that is  $(t, x, y) \in K_{\exp}$ .

# Chapter 16

# Supported File Formats

MOSEK supports a range of problem and solution formats listed in Table 16.1 and Table 16.2. The **Task** format is MOSEK's native binary format and it supports all features that MOSEK supports. The **OPF** format is MOSEK's human-readable alternative that supports nearly all features (everything except semidefinite problems). In general, text formats are significantly slower to read, but can be examined and edited directly in any text editor.

#### **Problem formats**

Table 16.1: List of supported file formats for optimization problems. The column *Conic* refers to conic problems involving the quadratic, rotated quadratic, power or exponential cone. The last two columns indicate if the format supports solutions and optimizer parameters.

Format Type	Ext.	Binary/Text	LP	QO	Conic	SDP	Sol	Param
LP	lp	plain text	X	X				
MPS	mps	plain text	X	X	X			
OPF	opf	plain text	X	X	X		X	X
PTF	ptf	plain text	X	X	X	X	X	
CBF	cbf	plain text	X		X	X		
Task format	task	binary	X	X	X	X	X	X
Jtask format	jtask	text	X	X	X	X	X	X

#### **Solution formats**

Table 16.2: List of supported solution formats.

Format Type	Ext.	Binary/Text	Description
SOL	sol	plain text	Interior Solution
	bas	plain text	Basic Solution
	int	plain text	Integer
Jsol format	jsol	text	Solution

# Compression

MOSEK supports GZIP and Zstandard compression. Problem files with extension .gz (for GZIP) and .zst (for Zstandard) are assumed to be compressed when read, and are automatically compressed when written. For example, a file called

problem.mps.gz

will be considered as a GZIP compressed MPS file.

# 16.1 The LP File Format

MOSEK supports the LP file format with some extensions. The LP format is not a completely well-defined standard and hence different optimization packages may interpret the same LP file in slightly different ways. MOSEK tries to emulate as closely as possible CPLEX's behavior, but tries to stay backward compatible.

The LP file format can specify problems of the form

$$\begin{array}{lll} \text{minimize/maximize} & & c^Tx + \frac{1}{2}q^o(x) \\ \text{subject to} & l^c & \leq & Ax + \frac{1}{2}q(x) & \leq & u^c, \\ l^x & \leq & x & \leq & u^x, \\ & & & x_{\mathcal{J}} \text{ integer,} \end{array}$$

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $c \in \mathbb{R}^n$  is the linear term in the objective.
- $q^o :\in \mathbb{R}^n \to \mathbb{R}$  is the quadratic term in the objective where

$$q^o(x) = x^T Q^o x$$

and it is assumed that

$$Q^o = (Q^o)^T$$
.

- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = x^T Q^i x$$

where it is assumed that

$$Q^i = (Q^i)^T$$
.

•  $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer constrained variables.

#### 16.1.1 File Sections

An LP formatted file contains a number of sections specifying the objective, constraints, variable bounds, and variable types. The section keywords may be any mix of upper and lower case letters.

# **Objective Function**

The first section beginning with one of the keywords

max
maximum
maximize
min
minimum
minimize

defines the objective sense and the objective function, i.e.

$$c^T x + \frac{1}{2} x^T Q^o x.$$

The objective may be given a name by writing

#### myname:

before the expressions. If no name is given, then the objective is named obj.

The objective function contains linear and quadratic terms. The linear terms are written as

```
4 x1 + x2 - 0.1 x3
```

and so forth. The quadratic terms are written in square brackets ( $[\ ]/2$ ) and are either squared or multiplied as in the examples

x1^2

and

x1 \* x2

There may be zero or more pairs of brackets containing quadratic expressions. An example of an objective section is

```
minimize
myobj: 4 x1 + x2 - 0.1 x3 + [ x1^2 + 2.1 x1 * x2 ]/2
```

Please note that the quadratic expressions are multiplied with  $\frac{1}{2}$ , so that the above expression means

minimize 
$$4x_1 + x_2 - 0.1 \cdot x_3 + \frac{1}{2}(x_1^2 + 2.1 \cdot x_1 \cdot x_2)$$

If the same variable occurs more than once in the linear part, the coefficients are added, so that  $4 \times 1 + 2 \times 1$  is equivalent to  $6 \times 1$ . In the quadratic expressions  $\times 1 \times 2$  is equivalent to  $\times 2 \times 1$  and, as in the linear part, if the same variables multiplied or squared occur several times their coefficients are added.

# **Constraints**

The second section beginning with one of the keywords

```
subj to
subject to
s.t.
st
```

defines the linear constraint matrix A and the quadratic matrices  $Q^{i}$ .

A constraint contains a name (optional), expressions adhering to the same rules as in the objective and a bound:

```
subject to
con1: x1 + x2 + [ x3^2 ]/2 <= 5.1</pre>
```

The bound type (here <=) may be any of <, <=, =, >, >= (< and <= mean the same), and the bound may be any number.

In the standard LP format it is not possible to define more than one bound per line, but **MOSEK** supports defining ranged constraints by using double-colon (::) instead of a single-colon (:) after the constraint name, i.e.

$$-5 \le x_1 + x_2 \le 5 \tag{16.1}$$

may be written as

```
con:: -5 < x_1 + x_2 < 5
```

By default MOSEK writes ranged constraints this way.

If the files must adhere to the LP standard, ranged constraints must either be split into upper bounded and lower bounded constraints or be written as an equality with a slack variable. For example the expression (16.1) may be written as

$$x_1 + x_2 - sl_1 = 0, -5 \le sl_1 \le 5.$$

#### **Bounds**

Bounds on the variables can be specified in the bound section beginning with one of the keywords

bound bounds

The bounds section is optional but should, if present, follow the **subject to** section. All variables listed in the bounds section must occur in either the objective or a constraint.

The default lower and upper bounds are 0 and  $+\infty$ . A variable may be declared free with the keyword free, which means that the lower bound is  $-\infty$  and the upper bound is  $+\infty$ . Furthermore it may be assigned a finite lower and upper bound. The bound definitions for a given variable may be written in one or two lines, and bounds can be any number or  $\pm\infty$  (written as  $+\inf/-\inf/+\inf\inf$ ) as in the example

```
bounds

x1 free

x2 <= 5

0.1 <= x2

x3 = 42

2 <= x4 < +inf
```

#### Variable Types

The final two sections are optional and must begin with one of the keywords

bin
binaries
binary

and

gen general

Under general all integer variables are listed, and under binary all binary (integer variables with bounds 0 and 1) are listed:

general
x1 x2
binary
x3 x4

Again, all variables listed in the binary or general sections must occur in either the objective or a constraint.

# **Terminating Section**

Finally, an LP formatted file must be terminated with the keyword

end

# 16.1.2 LP File Examples

#### Linear example lo1.lp

```
\ File: lo1.lp
maximize
obj: 3 x1 + x2 + 5 x3 + x4
subject to
c1: 3 x1 + x2 + 2 x3 = 30
c2: 2 x1 + x2 + 3 x3 + x4 >= 15
c3: 2 x2 + 3 x4 <= 25
bounds
0 <= x1 <= +infinity
0 <= x2 <= 10
0 <= x3 <= +infinity
0 <= x4 <= +infinity
end</pre>
```

#### Mixed integer example milo1.lp

```
maximize
obj: x1 + 6.4e-01 x2
subject to
c1: 5e+01 x1 + 3.1e+01 x2 <= 2.5e+02
c2: 3e+00 x1 - 2e+00 x2 >= -4e+00
bounds
0 <= x1 <= +infinity
0 <= x2 <= +infinity
general
x1 x2
end</pre>
```

# 16.1.3 LP Format peculiarities

### **Comments**

Anything on a line after a \ is ignored and is treated as a comment.

# **Names**

A name for an objective, a constraint or a variable may contain the letters a-z, A-Z, the digits 0-9 and the characters

```
[!"#$%&()/,.;?@_'`|~
```

The first character in a name must not be a number, a period or the letter **e** or **E**. Keywords must not be used as names.

 $\mathbf{MOSEK}$  accepts any character as valid for names, except  $\0$ . A name that is not allowed in LP file will be changed and a warning will be issued.

The algorithm for making names LP valid works as follows: The name is interpreted as an utf-8 string. For a Unicode character c:

- If c==\_ (underscore), the output is \_\_ (two underscores).
- If c is a valid LP name character, the output is just c.
- If c is another character in the ASCII range, the output is <code>\_XX</code>, where <code>XX</code> is the hexadecimal code for the character.
- If c is a character in the range 127-65535, the output is \_uXXXX, where XXXX is the hexadecimal code for the character.

• If c is a character above 65535, the output is \_UXXXXXXXX, where XXXXXXXX is the hexadecimal code for the character.

Invalid  $\mathtt{utf-8}$  substrings are escaped as  $\mathtt{XX'}$ , and if a name starts with a period, e or E, that character is escaped as  $\mathtt{XX}$ .

#### Variable Bounds

Specifying several upper or lower bounds on one variable is possible but **MOSEK** uses only the tightest bounds. If a variable is fixed (with =), then it is considered the tightest bound.

#### MOSEK Extensions to the LP Format

Some optimization software packages employ a more strict definition of the LP format than the one used by **MOSEK**. The limitations imposed by the strict LP format are the following:

- Quadratic terms in the constraints are not allowed.
- Names can be only 16 characters long.
- Lines must not exceed 255 characters in length.

To get around some of the inconveniences converting from other problem formats, **MOSEK** allows lines to contain 1024 characters and names may have any length (shorter than the 1024 characters).

If an LP formatted file created by **MOSEK** should satisfy the strict definition, then the parameter <code>iparam.write\_lp\_strict\_format</code> should be set; note, however, that some problems cannot be written correctly as a strict LP formatted file. For instance, all names are truncated to 16 characters and hence they may lose their uniqueness and change the problem.

Internally in MOSEK names may contain any (printable) character, many of which cannot be used in LP names. Setting the parameters <code>iparam.read\_lp\_quoted\_names</code> and <code>iparam.write\_lp\_quoted\_names</code> allows MOSEK to use quoted names. The first parameter tells MOSEK to remove quotes from quoted names e.g, "x1", when reading LP formatted files. The second parameter tells MOSEK to put quotes around any semi-illegal name (names beginning with a number or a period) and fully illegal name (containing illegal characters). As double quote is a legal character in the LP format, quoting semi-illegal names makes them legal in the pure LP format as long as they are still shorter than 16 characters. Fully illegal names are still illegal in a pure LP file.

## The strict LP format

The LP format is not a formal standard and different vendors have slightly different interpretations of the LP format. To make **MOSEK**'s definition of the LP format more compatible with the definitions of other vendors set the parameter *iparam.write\_lp\_strict\_format* to *onoffkey.on*.

This setting may lead to truncation of some names and hence to an invalid LP file. The simple solution to this problem is to set the parameter <code>iparam.write\_generic\_names</code> to <code>onoffkey.on</code> which will cause all names to be renamed systematically in the output file.

# Formatting of an LP File

A few parameters control the visual formatting of LP files written by **MOSEK** in order to make it easier to read the files. These parameters are

- *iparam.write\_lp\_line\_width* sets the maximum number of characters on a single line. The default value is 80 corresponding roughly to the width of a standard text document.
- *iparam.write\_lp\_terms\_per\_line* sets the maximum number of terms per line; a term means a sign, a coefficient, and a name (for example + 42 elephants). The default value is 0, meaning that there is no maximum.

#### **Unnamed Constraints**

Reading and writing an LP file with **MOSEK** may change it superficially. If an LP file contains unnamed constraints or objective these are given their generic names when the file is read (however unnamed constraints in **MOSEK** are written without names).

# 16.2 The MPS File Format

**MOSEK** supports the standard MPS format with some extensions. For a detailed description of the MPS format see the book by Nazareth [Naz87].

#### 16.2.1 MPS File Structure

The version of the MPS format supported by  $\mathbf{MOSEK}$  allows specification of an optimization problem of the form

maximize/minimize 
$$c^T x + q_0(x)$$
  
 $l^c \le Ax + q(x) \le u^c,$   
 $l^x \le x \le u^x,$   
 $x \in \mathcal{K},$   
 $x_{\mathcal{J}}$  integer, (16.2)

where

- $x \in \mathbb{R}^n$  is the vector of decision variables.
- $A \in \mathbb{R}^{m \times n}$  is the constraint matrix.
- $l^c \in \mathbb{R}^m$  is the lower limit on the activity for the constraints.
- $u^c \in \mathbb{R}^m$  is the upper limit on the activity for the constraints.
- $l^x \in \mathbb{R}^n$  is the lower limit on the activity for the variables.
- $u^x \in \mathbb{R}^n$  is the upper limit on the activity for the variables.
- $q: \mathbb{R}^n \to \mathbb{R}$  is a vector of quadratic functions. Hence,

$$q_i(x) = \frac{1}{2}x^T Q^i x$$

where it is assumed that  $Q^i = (Q^i)^T$ . Please note the explicit  $\frac{1}{2}$  in the quadratic term and that  $Q^i$  is required to be symmetric. The same applies to  $q_0$ .

- K is a convex cone.
- $\mathcal{J} \subseteq \{1, 2, \dots, n\}$  is an index set of the integer-constrained variables.
- $\bullet$  c is the vector of objective coefficients.

An MPS file with one row and one column can be illustrated like this:

* 1	2	3	4	5	6
*234567890123	4567890123	456789012345	6789012345678	9012345678	90
NAME	[name]				
OBJSENSE					
[objsense	]				
OBJNAME	[objname]				
ROWS					
? [cname1]					
COLUMNS					
[vname1]	[cname1]	[value1]	[cname2]	[value2]	
RHS	_	_	_	_	
[name]	[cname1]	[value1]	[cname2]	[value2]	
RANGES					
[name]	[cname1]	[value1]	[cname2]	[value2]	
QSECTION	[cname1]	F 2 43	r 63	F 7 07	
[vname1]	[vname2]	[value1]	[vname3]	[value2]	
QMATRIX	[ ··· - 0]	[14]			
[vname1]	[vname2]	[value1]			

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```
QUADOBJ
               [vname2]
                          [value1]
    [vname1]
QCMATRIX
               [cname1]
    [vname1]
               [vname2]
                          [value1]
BOUNDS
                          [value1]
?? [name]
               [vname1]
CSECTION
               [kname1]
                          [value1]
                                           [ktype]
    [vname1]
ENDATA
```

Here the names in capitals are keywords of the MPS format and names in brackets are custom defined names or values. A couple of notes on the structure:

• Fields: All items surrounded by brackets appear in *fields*. The fields named "valueN" are numerical values. Hence, they must have the format

```
[+|-]XXXXXXX.XXXXXX[[e|E][+|-]XXX]
```

where

```
X = [0|1|2|3|4|5|6|7|8|9].
```

- Sections: The MPS file consists of several sections where the names in capitals indicate the beginning of a new section. For example, COLUMNS denotes the beginning of the columns section.
- Comments: Lines starting with an \* are comment lines and are ignored by MOSEK.
- Keys: The question marks represent keys to be specified later.
- Extensions: The sections QSECTION and CSECTION are specific MOSEK extensions of the MPS format. The sections QMATRIX, QUADOBJ and QCMATRIX are included for sake of compatibility with other vendors extensions to the MPS format.
- The standard MPS format is a fixed format, i.e. everything in the MPS file must be within certain fixed positions. **MOSEK** also supports a *free format*. See Sec. 16.2.5 for details.

### Linear example lo1.mps

A concrete example of a MPS file is presented below:

```
* File: lo1.mps
NAME
OBJSENSE
    MAX
ROWS
N obj
E c1
G c2
L c3
COLUMNS
               obj
    x1
                          3
                          3
    x1
               c1
                          2
    x1
    x2
               obj
    x2
               c1
                          1
    x2
               c2
                          1
                          2
    x2
               сЗ
                          5
    x.3
               obj
                          2
    x3
               c1
    x3
               c2
                          3
    x4
                          1
               obi
    x4
```

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(continued from previous page)

3
30
15
25
10

Subsequently each individual section in the MPS format is discussed.

# NAME (optional)

In this section a name ([name]) is assigned to the problem.

# OBJSENSE (optional)

This is an optional section that can be used to specify the sense of the objective function. The OBJSENSE section contains one line at most which can be one of the following:

MIN	
MINIMIZE	
MAX	
MAXIMIZE	

It should be obvious what the implication is of each of these four lines.

# OBJNAME (optional)

This is an optional section that can be used to specify the name of the row that is used as objective function. objname should be a valid row name.

# ROWS

A record in the ROWS section has the form

#### ? [cname1]

where the requirements for the fields are as follows:

Field	Starting Position	Max Width	required	Description
?	2	1	Yes	Constraint key
[cname1]	5	8	Yes	Constraint name

Hence, in this section each constraint is assigned a unique name denoted by [cname1]. Please note that [cname1] starts in position 5 and the field can be at most 8 characters wide. An initial key ? must be present to specify the type of the constraint. The key can have values E, G, L, or N with the following interpretation:

Constraint type	$l_i^c$	$u_i^c$
E (equal)	finite	$=l_i^c$
G (greater)	finite	$\infty$
L (lower)	$-\infty$	finite
N (none)	$-\infty$	$\infty$

In the MPS format the objective vector is not specified explicitly, but one of the constraints having the key N will be used as the objective vector c. In general, if multiple N type constraints are specified, then the first will be used as the objective vector c, unless something else was specified in the section OBJNAME.

#### COLUMNS

In this section the elements of A are specified using one or more records having the form:

ne2]	[cname2]	[value1]	[cname1]	e1] [cname1] [value1] [cname2] [value2
[va	[va	[cname2] [va	[value1] [cname2] [value1]	alue2
[cname1] [value1] [cname1]	[cname1] [value1]	[cname1]		e1]

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

Hence, a record specifies one or two elements  $a_{ij}$  of A using the principle that [vname1] and [cname1] determines j and i respectively. Please note that [cname1] must be a constraint name specified in the ROWS section. Finally, [value1] denotes the numerical value of  $a_{ij}$ . Another optional element is specified by [cname2], and [value2] for the variable specified by [vname1]. Some important comments are:

- All elements belonging to one variable must be grouped together.
- $\bullet$  Zero elements of A should not be specified.
- At least one element for each variable should be specified.

### RHS (optional)

A record in this section has the format

me2] [value:	cname2]	[	value1]	[value	1]	[cname	ame]	П
--------------	---------	---	---------	--------	----	--------	------	---

where the requirements for each field are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RHS vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The interpretation of a record is that [name] is the name of the RHS vector to be specified. In general, several vectors can be specified. [cname1] denotes a constraint name previously specified in the ROWS section. Now, assume that this name has been assigned to the i-h constraint and  $v_1$  denotes the value specified by [value1], then the interpretation of  $v_1$  is:

Constraint	$l_i^c$	$u_i^c$
E	$v_1$	$v_1$
G	$v_1$	
L		$v_1$
N		

An optional second element is specified by [cname2] and [value2] and is interpreted in the same way. Please note that it is not necessary to specify zero elements, because elements are assumed to be zero.

# RANGES (optional)

A record in this section has the form

value2]	.e2]	[cname2]	alue1]	ame1]	[name] [cname1] [value1] [cname2] [value2]
[cname1] [value1] [cname2] [	[cname1] [value1] [cnam	[cname1] [value1]	[cname1] [v	[cna	

where the requirements for each fields are as follows:

Field	Starting Position	Max Width	required	Description
[name]	5	8	Yes	Name of the RANGE vector
[cname1]	15	8	Yes	Constraint name
[value1]	25	12	Yes	Numerical value
[cname2]	40	8	No	Constraint name
[value2]	50	12	No	Numerical value

The records in this section are used to modify the bound vectors for the constraints, i.e. the values in  $l^c$  and  $u^c$ . A record has the following interpretation: [name] is the name of the RANGE vector and [cname1] is a valid constraint name. Assume that [cname1] is assigned to the *i*-th constraint and let  $v_1$  be the value specified by [value1], then a record has the interpretation:

Constraint type	Sign of $v_1$	$l_i^c$	$u_i^c$
E	_	$u_i^c + v_1$	
E	+		$l_i^c + v_1$
G	- or +		$l_i^c +  v_1 $
L	- or +	$u_i^c -  v_1 $	
N			

Another constraint bound can optionally be modified using [cname2] and [value2] the same way.

### QSECTION (optional)

Within the QSECTION the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic terms belong. A record in the QSECTION has the form

[vname1] [vname2] [value1] [vname3] [value2]
--

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value
[vname3]	40	8	No	Variable name
[value2]	50	12	No	Numerical value

A record specifies one or two elements in the lower triangular part of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1] An optional second element is specified in the same way by the fields [vname1], [vname3], and [value2].

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation

```
* File: qo1.mps
NAME qo1
ROWS
N obj
G c1
COLUMNS
x1 c1 1.0
x2 obj -1.0
x2 c1 1.0
```

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х3	c1	1.0
RHS		
rhs	c1	1.0
QSECTION	obj	
x1	x1	2.0
x1	x3	-1.0
x2	x2	0.2
x3	x3	2.0
ENDATA		

Regarding the QSECTIONs please note that:

- Only one QSECTION is allowed for each constraint.
- The QSECTIONs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- ullet All entries specified in a QSECTION are assumed to belong to the lower triangular part of the quadratic term of Q.

#### QMATRIX/QUADOBJ (optional)

The QMATRIX and QUADOBJ sections allow to define the quadratic term of the objective function. They differ in how the quadratic term of the objective function is stored:

- QMATRIX stores all the nonzeros coefficients, without taking advantage of the symmetry of the Q
  matrix
- QUADOBJ stores the upper diagonal nonzero elements of the Q matrix.

A record in both sections has the form:

|--|

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies one elements of the Q matrix in the objective function. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q_{kj}$  is assigned the value given by [value1]. Note that a line must appear for each off-diagonal coefficient if using a QMATRIX section, while only one entry is required in a QUADOBJ section. The quadratic part of the objective function will be evaluated as  $1/2x^TQx$ .

The example

$$\begin{array}{ll} \text{minimize} & -x_2 + \frac{1}{2}(2x_1^2 - 2x_1x_3 + 0.2x_2^2 + 2x_3^2) \\ \text{subject to} & x_1 + x_2 + x_3 & \geq & 1, \\ & x \geq 0 & \end{array}$$

has the following MPS file representation using QMATRIX

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					•	
x2	c1	1.0	 			
x3	c1	1.0				
RHS						
rhs	c1	1.0				
QMATRIX						
x1	x1	2.0				
x1	x3	-1.0				
x3	x1	-1.0				
x2	x2	0.2				
x3	x3	2.0				
ENDATA						

or the following using QUADOBJ

* F	ile:	qo1_quadobj.n	nps
	NAME qo1_quadobj		
ROW	S		·
N	obj		
	c1		
COL	UMNS		
	x1	c1	1.0
	x2	obj	-1.0
	x2	c1	1.0
	x3	c1	1.0
RHS			
	rhs	c1	1.0
QUA	DOBJ		
	x1	x1	2.0
	x1	x3	-1.0
	x2	x2	0.2
	x3	x3	2.0
END	ATA		

Please also note that:

- A QMATRIX/QUADOBJ section can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QMATRIX/QUADOBJ section must already be specified in the COLUMNS section.

# QCMATRIX (optional)

A QCMATRIX section allows to specify the quadratic part of a given constraint. Within the QCMATRIX the label [cname1] must be a constraint name previously specified in the ROWS section. The label [cname1] denotes the constraint to which the quadratic term belongs. A record in the QSECTION has the form

[vname1]
----------

where the requirements for each field are:

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	Variable name
[vname2]	15	8	Yes	Variable name
[value1]	25	12	Yes	Numerical value

A record specifies an entry of the  $Q^i$  matrix where [cname1] specifies the i. Hence, if the names [vname1] and [vname2] have been assigned to the k-th and j-th variable, then  $Q^i_{kj}$  is assigned the value given by [value1]. Moreover, the quadratic term is represented as  $1/2x^TQx$ .

The example

has the following MPS file representation

```
* File: qo1.mps
NAME
               qo1
ROWS
N obj
G
   c1
   q1
COLUMNS
                         1.0
    x1
               c1
               obj
                         -1.0
    x2
    x2
               c1
                         1.0
    xЗ
               c1
                         1.0
RHS
                          1.0
               c1
    rhs
                          10.0
    rhs
               q1
QCMATRIX
               q1
               x1
                         2.0
    x1
    x1
               xЗ
                          -1.0
    хЗ
               x1
                          -1.0
                         0.2
    x2
               x2
    xЗ
               xЗ
                         2.0
ENDATA
```

Regarding the QCMATRIXs please note that:

- Only one QCMATRIX is allowed for each constraint.
- The QCMATRIXs can appear in an arbitrary order after the COLUMNS section.
- All variable names occurring in the QSECTION must already be specified in the COLUMNS section.
- QCMATRIX does not exploit the symmetry of Q: an off-diagonal entry (i,j) should appear twice.

# BOUNDS (optional)

In the BOUNDS section changes to the default bounds vectors  $l^x$  and  $u^x$  are specified. The default bounds vectors are  $l^x=0$  and  $u^x=\infty$ . Moreover, it is possible to specify several sets of bound vectors. A record in this section has the form

where the requirements for each field are:

Field	Starting Position	Max Width	Required	Description
??	2	2	Yes	Bound key
[name]	5	8	Yes	Name of the BOUNDS vector
[vname1]	15	8	Yes	Variable name
[value1]	25	12	No	Numerical value

Hence, a record in the BOUNDS section has the following interpretation: [name] is the name of the bound vector and [vname1] is the name of the variable for which the bounds are modified by the record. ?? and [value1] are used to modify the bound vectors according to the following table:

??	$l_j^x$	$u_j^x$	Made integer (added to $\mathcal{J}$ )
FR	$-\infty$	$\infty$	No
FX	$v_1$	$v_1$	No
LO	$v_1$	unchanged	No
MI	$-\infty$	unchanged	No
PL	unchanged	$\infty$	No
UP	unchanged	$v_1$	No
BV	0	1	Yes
LI	$\lceil v_1 \rceil$	unchanged	Yes
UI	unchanged	$\lfloor v_1 \rfloor$	Yes

Here  $v_1$  is the value specified by [value1].

# CSECTION (optional)

The purpose of the CSECTION is to specify the conic constraint

$$x \in \mathcal{K}$$

in (16.2). It is assumed that K satisfies the following requirements. Let

$$x^t \in \mathbb{R}^{n^t}, \quad t = 1, \dots, k$$

be vectors comprised of parts of the decision variables x so that each decision variable is a member of exactly **one** vector  $x^t$ , for example

$$x^1 = \begin{bmatrix} x_1 \\ x_4 \\ x_7 \end{bmatrix}$$
 and  $x^2 = \begin{bmatrix} x_6 \\ x_5 \\ x_3 \\ x_2 \end{bmatrix}$ .

Next define

$$\mathcal{K} := \left\{ x \in \mathbb{R}^n : \quad x^t \in \mathcal{K}_t, \quad t = 1, \dots, k \right\}$$

where  $\mathcal{K}_t$  must have one of the following forms:

•  $\mathbb{R}$  set:

$$\mathcal{K}_t = \mathbb{R}^{n^t}$$
.

• Zero cone:

$$\mathcal{K}_t = \{0\} \subseteq \mathbb{R}^{n^t}. \tag{16.3}$$

• Quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : x_1 \ge \sqrt{\sum_{j=2}^{n^t} x_j^2} \right\}. \tag{16.4}$$

• Rotated quadratic cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : 2x_1 x_2 \ge \sum_{j=3}^{n^t} x_j^2, \quad x_1, x_2 \ge 0 \right\}.$$
 (16.5)

• Primal exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0 \right\}. \tag{16.6}$$

• Primal power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_{t} = \left\{ x \in \mathbb{R}^{n^{t}} : x_{1}^{\alpha} x_{2}^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^{t}} x_{j}^{2}}, \quad x_{1}, x_{2} \ge 0 \right\}.$$
 (16.7)

• Dual exponential cone:

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^3 : x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0 \right\}.$$
 (16.8)

• Dual power cone (with parameter  $0 < \alpha < 1$ ):

$$\mathcal{K}_t = \left\{ x \in \mathbb{R}^{n^t} : \left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0 \right\}.$$
 (16.9)

In general, membership in the  $\mathbb{R}$  set is not specified. If a variable is not a member of any other cone then it is assumed to be a member of the  $\mathbb{R}$  cone.

Next, let us study an example. Assume that the power cone

$$x_4^{1/3} x_5^{2/3} \ge |x_8|$$

and the rotated quadratic cone

$$2x_3x_7 \ge x_1^2 + x_0^2, \quad x_3, x_7 \ge 0,$$

should be specified in the MPS file. One CSECTION is required for each cone and they are specified as follows:

*	1	2	3	4	5	6
*234567	78901234	567890123	4567890123	4567890123456	78901234	567890
CSECTIO	ON 1	konea	3e-1	PPOW		
x4						
x5						
x8						
CSECTIO	ON 1	koneb	0.0	RQUAD		
x7						
хЗ						
x1						
х0						

In general, a CSECTION header has the format

oe]			
-----	--	--	--

where the requirements for each field are as follows:

Field	Starting Position	Max Width	Required	Description
[kname1]	15	8	Yes	Name of the cone
[value1]	25	12	No	Cone parameter
[ktype]	40		Yes	Type of the cone.

The possible cone type keys are:

[ktype]	Members	[value1]	Interpretation.
ZERO	$\geq 0$	unused	Zero cone (16.3).
QUAD	$\geq 1$	unused	Quadratic cone (16.4).
RQUAD	$\geq 2$	unused	Rotated quadratic cone (16.5).
PEXP	3	unused	Primal exponential cone (16.6).
PPOW	$\geq 2$	α	Primal power cone (16.7).
DEXP	3	unused	Dual exponential cone (16.8).
DPOW	$\geq 2$	α	Dual power cone (16.9).

A record in the CSECTION has the format

## [vname1]

where the requirements for each field are

Field	Starting Position	Max Width	required	Description
[vname1]	5	8	Yes	A valid variable name

A variable must occur in at most one CSECTION.

#### **ENDATA**

This keyword denotes the end of the MPS file.

# 16.2.2 Integer Variables

Using special bound keys in the BOUNDS section it is possible to specify that some or all of the variables should be integer-constrained i.e. be members of  $\mathcal{J}$ . However, an alternative method is available. This method is available only for backward compatibility and we recommend that it is not used. This method requires that markers are placed in the COLUMNS section as in the example:

```
COLUMNS
           obj
x1
                      -10.0
                                       c1
                                                  0.7
x1
           c2
                      0.5
                                       с3
                                                  1.0
x1
           c4
                      0.1
* Start of integer-constrained variables.
           'MARKER'
MARK000
                                       'TNTORG'
x2
           obj
                      -9.0
                                       c1
                                                  1.0
x2
           c2
                      0.8333333333
                                       с3
                                                  0.6666667
x2
           c4
                      0.25
xЗ
           obj
                      1.0
                                       c6
                                                  2.0
MARKO01
                                       'INTEND'
           'MARKER
* End of integer-constrained variables.
```

Please note that special marker lines are used to indicate the start and the end of the integer variables. Furthermore be aware of the following

- All variables between the markers are assigned a default lower bound of 0 and a default upper bound of 1. This may not be what is intended. If it is not intended, the correct bounds should be defined in the BOUNDS section of the MPS formatted file.
- MOSEK ignores field 1, i.e. MARKO001 and MARKO01, however, other optimization systems require
  them.
- Field 2, i.e. MARKER, must be specified including the single quotes. This implies that no row can be assigned the name MARKER.
- Field 3 is ignored and should be left blank.
- Field 4, i.e. INTORG and INTEND, must be specified.
- It is possible to specify several such integer marker sections within the COLUMNS section.

# 16.2.3 General Limitations

• An MPS file should be an ASCII file.

# 16.2.4 Interpretation of the MPS Format

Several issues related to the MPS format are not well-defined by the industry standard. However,  $\mathbf{MOSEK}$  uses the following interpretation:

- If a matrix element in the COLUMNS section is specified multiple times, then the multiple entries are added together.
- If a matrix element in a QSECTION section is specified multiple times, then the multiple entries are added together.

#### 16.2.5 The Free MPS Format

**MOSEK** supports a free format variation of the MPS format. The free format is similar to the MPS file format but less restrictive, e.g. it allows longer names. However, a name must not contain any blanks.

Moreover, by default a line in the MPS file must not contain more than 1024 characters. By modifying the parameter <code>iparam.read\_mps\_width</code> an arbitrary large line width will be accepted.

The free MPS format is default. To change to the strict and other formats use the parameter  $iparam.read\_mps\_format$ .

# 16.3 The OPF Format

The Optimization Problem Format (OPF) is an alternative to LP and MPS files for specifying optimization problems. It is row-oriented, inspired by the CPLEX LP format.

Apart from containing objective, constraints, bounds etc. it may contain complete or partial solutions, comments and extra information relevant for solving the problem. It is designed to be easily read and modified by hand and to be forward compatible with possible future extensions.

#### Intended use

The OPF file format is meant to replace several other files:

- The LP file format: Any problem that can be written as an LP file can be written as an OPF file too; furthermore it naturally accommodates ranged constraints and variables as well as arbitrary characters in names, fixed expressions in the objective, empty constraints, and conic constraints.
- Parameter files: It is possible to specify integer, double and string parameters along with the problem (or in a separate OPF file).
- Solution files: It is possible to store a full or a partial solution in an OPF file and later reload it.

# 16.3.1 The File Format

The format uses tags to structure data. A simple example with the basic sections may look like this:

```
[comment]
This is a comment. You may write almost anything here...
[/comment]
# This is a single-line comment.

[objective min 'myobj']
x + 3 y + x^2 + 3 y^2 + z + 1
[/objective]

[constraints]
[con 'con01'] 4 <= x + y [/con]
[/constraints]
[bounds]
[b] -10 <= x,y <= 10 [/b]

[cone quad] x,y,z [/cone]
[/bounds]</pre>
```

A scope is opened by a tag of the form [tag] and closed by a tag of the form [/tag]. An opening tag may accept a list of unnamed and named arguments, for examples:

```
[tag value] tag with one unnamed argument [/tag]
[tag arg=value] tag with one named argument [/tag]
```

Unnamed arguments are identified by their order, while named arguments may appear in any order, but never before an unnamed argument. The value can be a quoted, single-quoted or double-quoted text string, i.e.

```
[tag 'value'] single-quoted value [/tag]
[tag arg='value'] single-quoted value [/tag]
[tag "value"] double-quoted value [/tag]
[tag arg="value"] double-quoted value [/tag]
```

# 16.3.2 Sections

The recognized tags are

#### [comment]

A comment section. This can contain *almost* any text: Between single quotes (') or double quotes (") any text may appear. Outside quotes the markup characters ([ and ]) must be prefixed by backslashes. Both single and double quotes may appear alone or inside a pair of quotes if it is prefixed by a backslash.

# [objective]

The objective function: This accepts one or two parameters, where the first one (in the above example min) is either min or max (regardless of case) and defines the objective sense, and the second one (above myobj), if present, is the objective name. The section may contain linear and quadratic expressions.

If several objectives are specified, all but the last are ignored.

#### [constraints]

This does not directly contain any data, but may contain subsections con defining a linear constraint.

#### [con]

Defines a single constraint; if an argument is present ([con NAME]) this is used as the name of the constraint, otherwise it is given a null-name. The section contains a constraint definition written as linear and quadratic expressions with a lower bound, an upper bound, with both or with an equality. Examples:

Constraint names are unique. If a constraint is specified which has the same name as a previously defined constraint, the new constraint replaces the existing one.

# [bounds]

This does not directly contain any data, but may contain subsections b (linear bounds on variables) and cone (cones).

### [b]

Bound definition on one or several variables separated by comma (,). An upper or lower bound on a variable replaces any earlier defined bound on that variable. If only one bound (upper or lower) is given only this bound is replaced. This means that upper and lower bounds can be specified separately. So the OPF bound definition:

```
[b] x,y >= -10 [/b]
[b] x,y <= 10 [/b]
```

results in the bound  $-10 \le x, y \le 10$ .

#### [cone]

Specifies a cone. A cone is defined as a sequence of variables which belong to a single unique cone. The supported cone types are:

• quad: a quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^2 \ge \sum_{i=2}^n x_i^2, \quad x_1 \ge 0.$$

• rquad: a rotated quadratic cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$2x_1x_2 \ge \sum_{i=3}^n x_i^2, \quad x_1, x_2 \ge 0.$$

• pexp: primal exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge x_2 \exp(x_3/x_2), \quad x_1, x_2 \ge 0.$$

• ppow with parameter  $0 < \alpha < 1$ : primal power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1^{\alpha} x_2^{1-\alpha} \ge \sqrt{\sum_{j=3}^n x_j^2}, \quad x_1, x_2 \ge 0.$$

• dexp: dual exponential cone of 3 variables  $x_1, x_2, x_3$  defines a constraint of the form

$$x_1 \ge -x_3 e^{-1} \exp(x_2/x_3), \quad x_3 \le 0, x_1 \ge 0.$$

• dpow with parameter  $0 < \alpha < 1$ : dual power cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$\left(\frac{x_1}{\alpha}\right)^{\alpha} \left(\frac{x_2}{1-\alpha}\right)^{1-\alpha} \ge \sqrt{\sum_{j=3}^{n^t} x_j^2}, \quad x_1, x_2 \ge 0.$$

• zero: zero cone of n variables  $x_1, \ldots, x_n$  defines a constraint of the form

$$x_1 = \dots = x_n = 0$$

A [bounds]-section example:

```
[bounds]

[b] 0 <= x,y <= 10 [/b] # ranged bound

[b] 10 >= x,y >= 0 [/b] # ranged bound

[b] 0 <= x,y <= inf [/b] # using inf

[b] x,y free [/b] # free variables

# Let (x,y,z,w) belong to the cone K

[cone rquad] x,y,z,w [/cone] # rotated quadratic cone

[cone ppow '3e-01' 'a'] x1, x2, x3 [/cone] # power cone with alpha=1/3 and name 'a'

[/bounds]
```

By default all variables are free.

#### [variables]

This defines an ordering of variables as they should appear in the problem. This is simply a space-separated list of variable names.

# [integer]

This contains a space-separated list of variables and defines the constraint that the listed variables must be integer-valued.

#### [hints]

This may contain only non-essential data; for example estimates of the number of variables, constraints and non-zeros. Placed before all other sections containing data this may reduce the time spent reading the file.

In the hints section, any subsection which is not recognized by MOSEK is simply ignored. In this section a hint is defined as follows:

```
[hint ITEM] value [/hint]
```

The hints recognized by **MOSEK** are:

- number of variables),
- numcon (number of linear/quadratic constraints),
- numanz (number of linear non-zeros in constraints),
- numqnz (number of quadratic non-zeros in constraints).

#### [solutions]

This section can contain a set of full or partial solutions to a problem. Each solution must be specified using a [solution]-section, i.e.

```
[solutions]
[solution]...[/solution] #solution 1
[solution]...[/solution] #solution 2
#other solutions....
[solution]...[/solution] #solution n
[/solutions]
```

The syntax of a [solution]-section is the following:

```
[solution SOLTYPE status=STATUS]...[/solution]
```

where SOLTYPE is one of the strings

- interior, a non-basic solution,
- basic, a basic solution,
- integer, an integer solution,

and STATUS is one of the strings

- UNKNOWN,
- OPTIMAL,
- INTEGER\_OPTIMAL,
- PRIM\_FEAS,
- DUAL\_FEAS,
- PRIM\_AND\_DUAL\_FEAS,
- NEAR\_OPTIMAL,
- NEAR\_PRIM\_FEAS,

- NEAR\_DUAL\_FEAS,
- NEAR\_PRIM\_AND\_DUAL\_FEAS,
- PRIM\_INFEAS\_CER,
- DUAL\_INFEAS\_CER,
- NEAR\_PRIM\_INFEAS\_CER,
- NEAR\_DUAL\_INFEAS\_CER,
- NEAR\_INTEGER\_OPTIMAL.

Most of these values are irrelevant for input solutions; when constructing a solution for simplex hot-start or an initial solution for a mixed integer problem the safe setting is UNKNOWN.

A [solution]-section contains [con] and [var] sections. Each [con] and [var] section defines solution information for a single variable or constraint, specified as list of KEYWORD/value pairs, in any order, written as

#### KEYWORD=value

Allowed keywords are as follows:

- sk. The status of the item, where the value is one of the following strings:
  - LOW, the item is on its lower bound.
  - UPR, the item is on its upper bound.
  - FIX, it is a fixed item.
  - BAS, the item is in the basis.
  - SUPBAS, the item is super basic.
  - UNK, the status is unknown.
  - INF, the item is outside its bounds (infeasible).
- 1v1 Defines the level of the item.
- sl Defines the level of the dual variable associated with its lower bound.
- su Defines the level of the dual variable associated with its upper bound.
- sn Defines the level of the variable associated with its cone.
- y Defines the level of the corresponding dual variable (for constraints only).

A [var] section should always contain the items sk, lvl, sl and su. Items sl and su are not required for integer solutions.

A [con] section should always contain sk, lvl, sl, su and y.

An example of a solution section

• [vendor] This contains solver/vendor specific data. It accepts one argument, which is a vendor ID – for MOSEK the ID is simply mosek – and the section contains the subsection parameters defining solver parameters. When reading a vendor section, any unknown vendor can be safely ignored. This is described later.

Comments using the # may appear anywhere in the file. Between the # and the following line-break any text may be written, including markup characters.

## 16.3.3 Numbers

Numbers, when used for parameter values or coefficients, are written in the usual way by the printf function. That is, they may be prefixed by a sign (+ or -) and may contain an integer part, decimal part and an exponent. The decimal point is always . (a dot). Some examples are

```
1
1.0
.0
1.
1e10
1e+10
1e-10
```

Some *invalid* examples are

```
e10  # invalid, must contain either integer or decimal part
.  # invalid
.e10  # invalid
```

More formally, the following standard regular expression describes numbers as used:

```
[+|-]?([0-9]+[.][0-9]*|[.][0-9]+)([eE][+|-]?[0-9]+)?
```

#### 16.3.4 Names

Variable names, constraint names and objective name may contain arbitrary characters, which in some cases must be enclosed by quotes (single or double) that in turn must be preceded by a backslash. Unquoted names must begin with a letter (a-z or A-Z) and contain only the following characters: the letters a-z and A-Z, the digits 0-9, braces ({ and }) and underscore (\_).

Some examples of legal names:

```
an_unquoted_name
another_name{123}
'single quoted name'
"double quoted name"
"name with \\"quote\\" in it"
"name with []s in it"
```

## 16.3.5 Parameters Section

In the vendor section solver parameters are defined inside the parameters subsection. Each parameter is written as

```
[p PARAMETER_NAME] value [/p]
```

where PARAMETER\_NAME is replaced by a **MOSEK** parameter name, usually of the form MSK\_IPAR\_..., MSK\_DPAR\_..., or MSK\_SPAR\_..., and the value is replaced by the value of that parameter; both integer values and named values may be used. Some simple examples are

## 16.3.6 Writing OPF Files from MOSEK

To write an OPF file then make sure the file extension is .opf.

Then modify the following parameters to define what the file should contain:

$iparam.opf\_write\_sol\_bas$	Include basic solution, if defined.
$iparam.opf\_write\_sol\_itg$	Include integer solution, if defined.
$iparam.opf\_write\_sol\_itr$	Include interior solution, if defined.
iparam.	Include solutions if they are defined. If this is off, no solutions are
$opf\_write\_solutions$	included.
$iparam.opf\_write\_header$	Include a small header with comments.
$iparam.opf\_write\_problem$	Include the problem itself — objective, constraints and bounds.
iparam.	Include all parameter settings.
opf_write_parameters	
iparam.opf_write_hints	Include hints about the size of the problem.

## 16.3.7 Examples

This section contains a set of small examples written in OPF and describing how to formulate linear, quadratic and conic problems.

## Linear Example 1o1.opf

Consider the example:

having the bounds

In the  $\mathtt{OPF}$  format the example is displayed as shown in Listing 16.1.

Listing 16.1: Example of an OPF file for a linear problem.

```
[comment]
 The lo1 example in OPF format
[/comment]
[hints]
  [hint NUMVAR] 4 [/hint]
  [hint NUMCON] 3 [/hint]
  [hint NUMANZ] 9 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4
[/variables]
[objective maximize 'obj']
  3 x1 + x2 + 5 x3 + x4
[/objective]
[constraints]
 [con 'c1'] 3 x1 + x2 + 2 x3
                                        = 30 [/con]
  [con 'c2'] 2 x1 + x2 + 3 x3 + x4 >= 15 [/con]
  [con 'c3']
                   2 x2
                               + 3 x4 <= 25 [/con]
[/constraints]
```

```
[bounds]
[b] 0 <= * [/b]
[b] 0 <= x2 <= 10 [/b]
[/bounds]
```

## Quadratic Example qo1.opf

An example of a quadratic optimization problem is

$$\begin{array}{ll} \text{minimize} & x_1^2 + 0.1x_2^2 + x_3^2 - x_1x_3 - x_2 \\ \text{subject to} & 1 & \leq & x_1 + x_2 + x_3, \\ & & x > 0. \end{array}$$

This can be formulated in opf as shown below.

Listing 16.2: Example of an OPF file for a quadratic problem.

```
[comment]
 The qo1 example in OPF format
[/comment]
[hints]
 [hint NUMVAR] 3 [/hint]
 [hint NUMCON] 1 [/hint]
 [hint NUMANZ] 3 [/hint]
 [hint NUMQNZ] 4 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3
[/variables]
[objective minimize 'obj']
 # The quadratic terms are often written with a factor of 1/2 as here,
 # but this is not required.
  - x2 + 0.5 ( 2.0 x1 ^ 2 - 2.0 x3 * x1 + 0.2 x2 ^ 2 + 2.0 x3 ^ 2 )
[/objective]
[constraints]
 [con 'c1'] 1.0 \le x1 + x2 + x3 [/con]
[/constraints]
[bounds]
 [b] 0 \ll * [/b]
[/bounds]
```

## Conic Quadratic Example cqo1.opf

Consider the example:

minimize 
$$x_3 + x_4 + x_5$$
  
subject to  $x_0 + x_1 + 2x_2 = 1$ ,  
 $x_0, x_1, x_2 \ge 0$ ,  
 $x_3 \ge \sqrt{x_0^2 + x_1^2}$ ,  
 $2x_4x_5 \ge x_2^2$ .

Please note that the type of the cones is defined by the parameter to [cone ...]; the content of the cone-section is the names of variables that belong to the cone. The resulting OPF file is in Listing 16.3.

Listing 16.3: Example of an OPF file for a conic quadratic problem.

```
[comment]
 The cqo1 example in OPF format.
[/comment]
[hints]
  [hint NUMVAR] 6 [/hint]
  [hint NUMCON] 1 [/hint]
  [hint NUMANZ] 3 [/hint]
[/hints]
[variables disallow_new_variables]
 x1 x2 x3 x4 x5 x6
[/variables]
[objective minimize 'obj']
  x4 + x5 + x6
[/objective]
[constraints]
  [con 'c1'] x1 + x2 + 2e+00 x3 = 1e+00 [/con]
[/constraints]
[bounds]
 # We let all variables default to the positive orthant
  [b] 0 <= * [/b]
  # ...and change those that differ from the default
  [b] x4,x5,x6 free [/b]
  # Define quadratic cone: x4 \ge sqrt(x1^2 + x2^2)
  [cone quad 'k1'] x4, x1, x2 [/cone]
  # Define rotated quadratic cone: 2 x5 x6 >= x3^2
  [cone rquad 'k2'] x5, x6, x3 [/cone]
[/bounds]
```

#### Mixed Integer Example milo1.opf

Consider the mixed integer problem:

This can be implemented in OPF with the file in Listing 16.4.

Listing 16.4: Example of an OPF file for a mixed-integer linear problem.

```
[comment]
  The milo1 example in OPF format
[/comment]

[hints]
  [hint NUMVAR] 2 [/hint]
  [hint NUMCON] 2 [/hint]
  [hint NUMANZ] 4 [/hint]
[/hints]
```

```
[variables disallow_new_variables]
 x1 x2
[/variables]
[objective maximize 'obj']
  x1 + 6.4e-1 x2
[/objective]
[constraints]
  [con 'c1'] 5e+1 x1 + 3.1e+1 x2 <= 2.5e+2 [/con]
  [con 'c2'] -4 \le 3 x1 - 2 x2 [/con]
[/constraints]
[bounds]
 [b] 0 <= * [/b]
[/bounds]
[integer]
 x1 x2
[/integer]
```

## 16.4 The CBF Format

This document constitutes the technical reference manual of the *Conic Benchmark Format* with file extension: .cbf or .CBF. It unifies linear, second-order cone (also known as conic quadratic) and semidefinite optimization with mixed-integer variables. The format has been designed with benchmark libraries in mind, and therefore focuses on compact and easily parsable representations. The problem structure is separated from the problem data, and the format moreover facilitates benchmarking of hotstart capability through sequences of changes.

## 16.4.1 How Instances Are Specified

This section defines the spectrum of conic optimization problems that can be formulated in terms of the keywords of the CBF format.

In the CBF format, conic optimization problems are considered in the following form:

$$\min / \max \quad g^{obj} 
g_i \in \mathcal{K}_i, \quad i \in \mathcal{I}, 
\text{s.t.} \quad G_i \in \mathcal{K}_i, \quad i \in \mathcal{I}^{PSD}, 
x_j \in \mathcal{K}_j, \quad j \in \mathcal{J}, 
\overline{X}_j \in \mathcal{K}_j, \quad j \in \mathcal{J}^{PSD}.$$

$$(16.10)$$

- Variables are either scalar variables,  $x_j$  for  $j \in \mathcal{J}$ , or variables,  $\overline{X}_j$  for  $j \in \mathcal{J}^{PSD}$ . Scalar variables can also be declared as integer.
- Constraints are affine expressions of the variables, either scalar-valued  $g_i$  for  $i \in \mathcal{I}$ , or matrix-valued  $G_i$  for  $i \in \mathcal{I}^{PSD}$

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$
$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i.$$

• The **objective function** is a scalar-valued affine expression of the variables, either to be minimized or maximized. We refer to this expression as  $g^{obj}$ 

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj}.$$

CBF format can represent the following cones  $\mathcal{K}$ :

• Free domain - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n\}, \text{ for } n \ge 1.$$

• Positive orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_i \ge 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Negative orthant - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j \le 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Fixpoint zero - A cone in the linear family defined by

$$\{x \in \mathbb{R}^n \mid x_j = 0 \text{ for } j = 1, \dots, n\}, \text{ for } n \ge 1.$$

• Quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \left( \begin{array}{c} p \\ x \end{array} \right) \in \mathbb{R} \times \mathbb{R}^{n-1}, \ p^2 \ge x^T x, \ p \ge 0 \right\}, \ \text{for } n \ge 2.$$

• Rotated quadratic cone - A cone in the second-order cone family defined by

$$\left\{ \begin{pmatrix} p \\ q \\ x \end{pmatrix} \in \mathbb{R} \times \mathbb{R} \times \mathbb{R}^{n-2}, \ 2pq \ge x^T x, \ p \ge 0, \ q \ge 0 \right\}, \text{ for } n \ge 3.$$

## 16.4.2 The Structure of CBF Files

This section defines how information is written in the CBF format, without being specific about the type of information being communicated.

All information items belong to exactly one of the three groups of information. These information groups, and the order they must appear in, are:

- 1. File format.
- 2. Problem structure.
- 3. Problem data.

The first group, file format, provides information on how to interpret the file. The second group, problem structure, provides the information needed to deduce the type and size of the problem instance. Finally, the third group, problem data, specifies the coefficients and constants of the problem instance.

## Information items

The format is composed as a list of information items. The first line of an information item is the KEYWORD, revealing the type of information provided. The second line - of some keywords only - is the HEADER, typically revealing the size of information that follows. The remaining lines are the BODY holding the actual information to be specified.



The KEYWORD determines how each line in the HEADER and BODY is structured. Moreover, the number of lines in the BODY follows either from the KEYWORD, the HEADER, or from another information item required to precede it.

#### **Embedded hotstart-sequences**

A sequence of problem instances, based on the same problem structure, is within a single file. This is facilitated via the CHANGE within the problem data information group, as a separator between the information items of each instance. The information items following a CHANGE keyword are appending to, or changing (e.g., setting coefficients back to their default value of zero), the problem data of the preceding instance.

The sequence is intended for benchmarking of hotstart capability, where the solvers can reuse their internal state and solution (subject to the achieved accuracy) as warmpoint for the succeeding instance. Whenever this feature is unsupported or undesired, the keyword CHANGE should be interpreted as the end of file.

### File encoding and line width restrictions

The format is based on the US-ASCII printable character set with two extensions as listed below. Note, by definition, that none of these extensions can be misinterpreted as printable US-ASCII characters:

- A line feed marks the end of a line, carriage returns are ignored.
- Comment-lines may contain unicode characters in UTF-8 encoding.

The line width is restricted to 512 bytes, with 3 bytes reserved for the potential carriage return, line feed and null-terminator.

Integers and floating point numbers must follow the ISO C decimal string representation in the standard C locale. The format does not impose restrictions on the magnitude of, or number of significant digits in numeric data, but the use of 64-bit integers and 64-bit IEEE 754 floating point numbers should be sufficient to avoid loss of precision.

## Comment-line and whitespace rules

The format allows single-line comments respecting the following rule:

• Lines having first byte equal to '#' (US-ASCII 35) are comments, and should be ignored. Comments are only allowed between information items.

Given that a line is not a comment-line, whitespace characters should be handled according to the following rules:

- Leading and trailing whitespace characters should be ignored.
  - The seperator between multiple pieces of information on one line, is either one or more whitespace characters.
- Lines containing only whitespace characters are empty, and should be ignored. Empty lines are only allowed between information items.

## 16.4.3 Problem Specification

#### The problem structure

The problem structure defines the objective sense, whether it is minimization and maximization. It also defines the index sets,  $\mathcal{J}$ ,  $\mathcal{J}^{PSD}$ ,  $\mathcal{I}$  and  $\mathcal{I}^{PSD}$ , which are all numbered from zero,  $\{0, 1, \ldots\}$ , and empty until explicitly constructed.

• Scalar variables are constructed in vectors restricted to a conic domain, such as  $(x_0, x_1) \in \mathbb{R}^2_+$ ,  $(x_2, x_3, x_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$x \in \mathcal{K}_1^{n_1} \times \mathcal{K}_2^{n_2} \times \dots \times \mathcal{K}_k^{n_k}$$

which in the CBF format becomes:

```
VAR
n k
K1 n1
K2 n2
...
Kk nk
```

where  $\sum_{i} n_{i} = n$  is the total number of scalar variables. The list of supported cones is found in Table 16.3. Integrality of scalar variables can be specified afterwards.

• **PSD variables** are constructed one-by-one. That is,  $X_j \succeq \mathbf{0}^{n_j \times n_j}$  for  $j \in \mathcal{J}^{PSD}$ , constructs a matrix-valued variable of size  $n_j \times n_j$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes:

```
PSDVAR
N
n1
n2
...
nN
```

where N is the total number of PSD variables.

• Scalar constraints are constructed in vectors restricted to a conic domain, such as  $(g_0, g_1) \in \mathbb{R}^2_+$ ,  $(g_2, g_3, g_4) \in \mathcal{Q}^3$ , etc. In terms of the Cartesian product, this generalizes to

$$g \in \mathcal{K}_1^{m_1} \times \mathcal{K}_2^{m_2} \times \dots \times \mathcal{K}_k^{m_k}$$

which in the CBF format becomes:

```
CON
m k
K1 m1
K2 m2
..
Kk mk
```

where  $\sum_{i} m_{i} = m$  is the total number of scalar constraints. The list of supported cones is found in Table 16.3

• **PSD constraints** are constructed one-by-one. That is,  $G_i \succeq \mathbf{0}^{m_i \times m_i}$  for  $i \in \mathcal{I}^{PSD}$ , constructs a matrix-valued affine expressions of size  $m_i \times m_i$  restricted to be symmetric positive semidefinite. In the CBF format, this list of constructions becomes

```
PSDCON
M
m1
m2
..
mM
```

where M is the total number of PSD constraints.

With the objective sense, variables (with integer indications) and constraints, the definitions of the many affine expressions follow in problem data.

#### Problem data

The problem data defines the coefficients and constants of the affine expressions of the problem instance. These are considered zero until explicitly defined, implying that instances with no keywords from this information group are, in fact, valid. Duplicating or conflicting information is a failure to comply with the standard. Consequently, two coefficients written to the same position in a matrix (or to transposed positions in a symmetric matrix) is an error.

The affine expressions of the objective,  $g^{obj}$ , of the scalar constraints,  $g_i$ , and of the PSD constraints,  $G_i$ , are defined separately. The following notation uses the standard trace inner product for matrices,  $\langle X, Y \rangle = \sum_{i,j} X_{ij} Y_{ij}$ .

• The affine expression of the objective is defined as

$$g^{obj} = \sum_{j \in \mathcal{J}^{PSD}} \langle F_j^{obj}, X_j \rangle + \sum_{j \in \mathcal{J}} a_j^{obj} x_j + b^{obj},$$

in terms of the symmetric matrices,  $F_j^{obj}$ , and scalars,  $a_j^{obj}$  and  $b^{obj}$ .

• The affine expressions of the scalar constraints are defined, for  $i \in \mathcal{I}$ , as

$$g_i = \sum_{j \in \mathcal{J}^{PSD}} \langle F_{ij}, X_j \rangle + \sum_{j \in \mathcal{J}} a_{ij} x_j + b_i,$$

in terms of the symmetric matrices,  $F_{ij}$ , and scalars,  $a_{ij}$  and  $b_i$ .

• The affine expressions of the PSD constraints are defined, for  $i \in \mathcal{I}^{PSD}$ , as

$$G_i = \sum_{j \in \mathcal{J}} x_j H_{ij} + D_i,$$

in terms of the symmetric matrices,  $H_{ij}$  and  $D_i$ .

## List of cones

The format uses an explicit syntax for symmetric positive semidefinite cones as shown above. For scalar variables and constraints, constructed in vectors, the supported conic domains and their minimum sizes are given as follows.

Table 16.3: Cones available in the CBF format

Name	CBF keyword	Cone family
Free domain	F	linear
Positive orthant	L+	linear
Negative orthant	L-	linear
Fixpoint zero	L=	linear
Quadratic cone	Q	second-order
Rotated quadratic cone	QR	second-order

## 16.4.4 File Format Keywords

#### **VER**

Description: The version of the Conic Benchmark Format used to write the file.

HEADER: None

BODY: One line formatted as:

INT

This is the version number.

Must appear exactly once in a file, as the first keyword.

## **OBJSENSE**

Description: Define the objective sense.

**HEADER:** None

BODY: One line formatted as:

STR

having MIN indicates minimize, and MAX indicates maximize. Capital letters are required. Must appear exactly once in a file.

#### **PSDVAR**

Description: Construct the PSD variables.

**HEADER**: One line formatted as:

INT

This is the number of PSD variables in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued PSD variable. The number of lines should match the number stated in the header.

#### **VAR**

Description: Construct the scalar variables.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar variables, followed by the number of conic domains they are restricted to.

BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 16.3), and the number of scalar variables restricted to this cone. These numbers should add up to the number of scalar variables stated first in the header. The number of lines should match the second number stated in the header.

## INT

Description: Declare integer requirements on a selected subset of scalar variables.

HEADER: one line formatted as:

INT

This is the number of integer scalar variables in the problem.

BODY: a list of lines formatted as:

INT

This indicates the scalar variable index  $j \in \mathcal{J}$ . The number of lines should match the number stated in the header.

Can only be used after the keyword VAR.

## **PSDCON**

Description: Construct the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of PSD constraints in the problem.

BODY: A list of lines formatted as:

INT

This indicates the number of rows (equal to the number of columns) in the matrix-valued affine expression of the PSD constraint. The number of lines should match the number stated in the header. Can only be used after these keywords: PSDVAR, VAR.

#### CON

Description: Construct the scalar constraints.

**HEADER**: One line formatted as:

INT INT

This is the number of scalar constraints, followed by the number of conic domains they restrict to. BODY: A list of lines formatted as:

STR INT

This indicates the cone name (see Table 16.3), and the number of affine expressions restricted to this cone. These numbers should add up to the number of scalar constraints stated first in the header. The number of lines should match the second number stated in the header.

Can only be used after these keywords: PSDVAR, VAR

## **OBJFCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices  $F_j^{obj}$ , as used in the objective.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

## **OBJACOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $a_j^{obj}$ , as used in the objective. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

#### **OBJBCOORD**

Description: Input the scalar,  $b^{obj}$ , as used in the objective.

HEADER: None.

BODY: One line formatted as:

#### REAL

This indicates the coefficient value.

#### **FCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $F_{ij}$ , as used in the scalar constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

#### INT INT INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the PSD variable index  $j \in \mathcal{J}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **ACOORD**

Description: Input sparse coordinates (triplets) to define the scalars,  $a_{ij}$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

### INT INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$ , the scalar variable index  $j \in \mathcal{J}$  and the coefficient value. The number of lines should match the number stated in the header.

## **BCOORD**

Description: Input sparse coordinates (pairs) to define the scalars,  $b_i$ , as used in the scalar constraints. HEADER: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT REAL

This indicates the scalar constraint index  $i \in \mathcal{I}$  and the coefficient value. The number of lines should match the number stated in the header.

## **HCOORD**

Description: Input sparse coordinates (quintuplets) to define the symmetric matrices,  $H_{ij}$ , as used in the PSD constraints.

**HEADER**: One line formatted as:

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as

#### INT INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the scalar variable index  $j \in \mathcal{J}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **DCOORD**

Description: Input sparse coordinates (quadruplets) to define the symmetric matrices,  $D_i$ , as used in the PSD constraints.

**HEADER:** One line formatted as

INT

This is the number of coordinates to be specified.

BODY: A list of lines formatted as:

INT INT INT REAL

This indicates the PSD constraint index  $i \in \mathcal{I}^{PSD}$ , the row index, the column index and the coefficient value. The number of lines should match the number stated in the header.

#### **CHANGE**

Start of a new instance specification based on changes to the previous. Can be interpreted as the end of file when the hotstart-sequence is unsupported or undesired.

BODY: None Header: None

## 16.4.5 CBF Format Examples

## Minimal Working Example

The conic optimization problem (16.11), has three variables in a quadratic cone - first one is integer - and an affine expression in domain 0 (equality constraint).

minimize 
$$5.1 x_0$$
  
subject to  $6.2 x_1 + 7.3 x_2 - 8.4 \in \{0\}$   
 $x \in \mathcal{Q}^3, x_0 \in \mathbb{Z}.$  (16.11)

Its formulation in the Conic Benchmark Format begins with the version of the CBF format used, to safeguard against later revisions.

VER. 1

Next follows the problem structure, consisting of the objective sense, the number and domain of variables, the indices of integer variables, and the number and domain of scalar-valued affine expressions (i.e., the equality constraint).

```
OBJSENSE
MIN

VAR
3 1
Q 3

INT
1
0
```

```
CON
1 1
L= 1
```

Finally follows the problem data, consisting of the coefficients of the objective, the coefficients of the constraints, and the constant terms of the constraints. All data is specified on a sparse coordinate form.

```
OBJACOORD

1
0 5.1

ACOORD
2
0 1 6.2
0 2 7.3

BCOORD
1
0 -8.4
```

This concludes the example.

## Mixing Linear, Second-order and Semidefinite Cones

The conic optimization problem (16.12), has a semidefinite cone, a quadratic cone over unordered subindices, and two equality constraints.

The equality constraints are easily rewritten to the conic form,  $(g_0, g_1) \in \{0\}^2$ , by moving constants such that the right-hand-side becomes zero. The quadratic cone does not fit under the VAR keyword in this variable permutation. Instead, it takes a scalar constraint  $(g_2, g_3, g_4) = (x_1, x_0, x_2) \in \mathcal{Q}^3$ , with scalar variables constructed as  $(x_0, x_1, x_2) \in \mathbb{R}^3$ . Its formulation in the CBF format is reported in the following list

 $({\rm continued\ from\ previous\ page})$ 

```
# Three scalar variables in this one conic domain:
      | Three are free.
VAR
3 1
F 3
# Five scalar constraints with affine expressions in two conic domains:
# | Two are fixed to zero.
#
      | Three are in conic quadratic domain.
CON
5 2
L= 2
QЗ
\# Five coordinates in F^{obj}_j coefficients:
    | F^{obj}[0][0,0] = 2.0
     | F^{obj}[0][1,0] = 1.0
     and more...
OBJFCOORD
0 0 0 2.0
0 1 0 1.0
0 1 1 2.0
0 2 1 1.0
0 2 2 2.0
# One coordinate in a^{obj}_j coefficients:
\# | a^{obj}[1] = 1.0
OBJACOORD
1
1 1.0
# Nine coordinates in F_ij coefficients:
     | F[0,0][0,0] = 1.0
      | F[0,0][1,1] = 1.0
#
     | and more...
#
FCOORD
0 0 0 0 1.0
0 0 1 1 1.0
0 0 2 2 1.0
1 0 0 0 1.0
1 0 1 0 1.0
1 0 2 0 1.0
1 0 1 1 1.0
1 0 2 1 1.0
1 0 2 2 1.0
# Six coordinates in a_ij coefficients:
\# | a[0,1] = 1.0
     | a[1,0] = 1.0
     and more...
ACOORD
0 1 1.0
1 0 1.0
1 2 1.0
2 1 1.0
3 0 1.0
4 2 1.0
```

```
# Two coordinates in b_i coefficients:
#  | b[0] = -1.0
#  | b[1] = -0.5
BCOORD
2
0 -1.0
1 -0.5
```

## Mixing Semidefinite Variables and Linear Matrix Inequalities

The standard forms in semidefinite optimization are usually based either on semidefinite variables or linear matrix inequalities. In the CBF format, both forms are supported and can even be mixed as shown in.

minimize 
$$\left\langle \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle + x_1 + x_2 + 1$$
  
subject to  $\left\langle \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, X_1 \right\rangle - x_1 - x_2 \geq 0.0,$   
 $x_1 \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \succeq \mathbf{0},$   
 $X_1 \succeq \mathbf{0}.$  (16.13)

Its formulation in the CBF format is written in what follows

```
# File written using this version of the Conic Benchmark Format:
#
     | Version 1.
VER
# The sense of the objective is:
     | Minimize.
OBJSENSE
MTN
# One PSD variable of this size:
# | Two times two.
PSDVAR
1
# Two scalar variables in this one conic domain:
     | Two are free.
VAR
2 1
# One PSD constraint of this size:
# | Two times two.
PSDCON
1
# One scalar constraint with an affine expression in this one conic domain:
     | One is greater than or equal to zero.
CON
1 1
L+ 1
# Two coordinates in F^{obj}_j coefficients:
```

```
| F^{obj}[0][0,0] = 1.0
     | F^{obj}[0][1,1] = 1.0
OBJFCOORD
0 0 0 1.0
0 1 1 1.0
# Two coordinates in a^{obj}_j coefficients:
\# | a^{obj}[0] = 1.0
#
     | a^{obj}[1] = 1.0
OBJACOORD
0 1.0
1 1.0
# One coordinate in b^{obj} coefficient:
# | b^{obj} = 1.0
OBJBCOORD
1.0
# One coordinate in F_ij coefficients:
# | F[0,0][1,0] = 1.0
FCOORD
0 0 1 0 1.0
# Two coordinates in a_ij coefficients:
\# | a[0,0] = -1.0
     | a[0,1] = -1.0
ACOORD
2
0 0 -1.0
0 1 -1.0
# Four coordinates in H_ij coefficients:
# | H[0,0][1,0] = 1.0
     | H[0,0][1,1] = 3.0
#
     and more...
HCOORD
0 0 1 0 1.0
0 0 1 1 3.0
0 1 0 0 3.0
0 1 1 0 1.0
# Two coordinates in D_i coefficients:
# | D[0][0,0] = -1.0
     | D[0][1,1] = -1.0
DCOORD
0 0 0 -1.0
0 1 1 -1.0
```

## Optimization Over a Sequence of Objectives

The linear optimization problem (16.14), is defined for a sequence of objectives such that hotstarting from one to the next might be advantages.

$$\begin{array}{lll} \text{maximize}_k & g_k^{obj} \\ \text{subject to} & 50 \, x_0 + 31 & \leq & 250 \,, \\ & 3 \, x_0 - 2 x_1 & \geq & -4 \,, \\ & x \in \mathbb{R}_+^2, \end{array} \tag{16.14}$$

given, 1.  $g_0^{obj} = x_0 + 0.64x_1$ . 2.  $g_1^{obj} = 1.11x_0 + 0.76x_1$ . 3.  $g_2^{obj} = 1.11x_0 + 0.85x_1$ .

Its formulation in the CBF format is reported in Listing 16.5.

Listing 16.5: Problem (16.14) in CBF format.

```
# File written using this version of the Conic Benchmark Format:
     | Version 1.
VER
# The sense of the objective is:
# | Maximize.
OBJSENSE
MAX
# Two scalar variables in this one conic domain:
     | Two are nonnegative.
VAR
2 1
L+ 2
# Two scalar constraints with affine expressions in these two conic domains:
# | One is in the nonpositive domain.
      | One is in the nonnegative domain.
CON
2 2
L- 1
# Two coordinates in a^{obj}_j coefficients:
     | a^{obj}[0] = 1.0
     | a^{obj}[1] = 0.64
OBJACOORD
0 1.0
1 0.64
# Four coordinates in a_ij coefficients:
     | a[0,0] = 50.0
     | a[1,0] = 3.0
      and more...
ACOORD
0 0 50.0
1 0 3.0
0 1 31.0
1 1 -2.0
# Two coordinates in b_i coefficients:
     | b[0] = -250.0
     | b[1] = 4.0
BCOORD
0 -250.0
1 4.0
```

```
# New problem instance defined in terms of changes.
CHANGE

# Two coordinate changes in a^{obj}_j coefficients. Now it is:
# | a^{obj}_[0] = 1.11
# | a^{obj}_[1] = 0.76

OBJACOORD
2
0 1.11
1 0.76

# New problem instance defined in terms of changes.
CHANGE

# One coordinate change in a^{obj}_j coefficients. Now it is:
# | a^{obj}_[0] = 1.11
# | a^{obj}_[1] = 0.85

OBJACOORD
1
1 0.85
```

## 16.5 The PTF Format

The PTF format is a new human-readable, natural text format. Its features and structure are similar to the *OPF* format, with the difference that the PTF format **does** support semidefinite terms.

#### 16.5.1 The overall format

The format is indentation based, where each section is started by a head line and followed by a section body with deeper indentation that the head line. For example:

```
Header line
Body line 1
Body line 1
Body line 1
```

Section can also be nested:

```
Header line A

Body line in A

Header line A.1

Body line in A.1

Body line in A.1

Body line in A.1
```

The indentation of blank lines is ignored, so a subsection can contain a blank line with no indentation. The character # defines a line comment and anything between the # character and the end of the line is ignored.

In a PTF file, the first section must be a Task section. The order of the remaining section is arbitrary, and sections may occur multiple times or not at all.

MOSEK will ignore any top-level section it does not recognize.

#### **Names**

In the description of the format we use following definitions for name strings:

```
NAME: PLAIN_NAME | QUOTED_NAME
PLAIN_NAME: [a-zA-Z_] [a-zA-Z0-9_-.!|]
QUOTED_NAME: "'" ( [^'\\r\n] | "\\" ( [\\rn] | "x" [0-9a-fA-F] [0-9a-fA-F] ) )* "'"
```

#### **Expressions**

An expression is a sum of terms. A term is either a linear term (a coefficient and a variable name, where the coefficient can be left out if it is 1.0), or a matrix inner product.

An expression:

```
EXPR: EMPTY | [+-]? TERM ( [+-] TERM )*
TERM: LINEAR_TERM | MATRIX_TERM
```

A linear term

```
LINEAR_TERM: FLOAT? NAME
```

A matrix term

```
MATRIX_TERM: "<" FLOAT? NAME ( [+-] FLOAT? NAME)* ";" NAME ">"
```

Here the right-hand name is the name of a (semidefinite) matrix variable, and the left-hand side is a sum of symmetric matrixes. The actual matrixes are defined in a separate section.

Expressions can span multiple lines by giving subsequent lines a deeper indentation.

For example following two section are equivalent:

## 16.5.2 Task section

The first section of the file must be a Task. The text in this section is not used and may contain comments, or meta-information from the writer or about the content.

Format:

```
Task NAME
Anything goes here...
```

NAME is a the task name.

## 16.5.3 Objective section

The Objective section defines the objective name, sense and function. The format:

```
"Objective" NAME?
( "Minimize" | "Maximize" ) EXPR
```

For example:

```
Objective 'obj'
Minimize x1 + 0.2 x2 + < M1 ; X1 >
```

### 16.5.4 Constraints section

The constraints section defines a series of constraints. A constraint defines a term  $A \cdot x + b \in K$ . For linear constraints A is just one row, while for conic constraints it can be multiple rows. If a constraint spans multiple rows these can either be written inline separated by semi-colons, or each expression in a separete sub-section.

Simple linear constraints:

```
"Constraints"
NAME? "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]" EXPR
```

If the brackets contain two values, they are used as upper and lower bounds. It they contain one value the constraint is an equality.

For example:

```
Constraints
'c1' [0;10] x1 + x2 + x3
[0] x1 + x2 + x3
```

Constraint blocks put the expression either in a subsection or inline. The cone type (domain) is written in the brackets, and **MOSEK** currently supports following types:

- SOC(N) Second order cone of dimension N
- RSOC(N) Rotated second order cone of dimension N
- PSD(N) Symmetric positive semidefinite cone of dimension N. This contains N\*(N+1)/2 elements.
- PEXP Primal exponential cone of dimension 3
- DEXP Dual exponential cone of dimension 3
- PPOW(N,P) Primal power cone of dimension N with parameter P
- DPOW(N,P) Dual power cone of dimension N with parameter P
- ZERO(N) The zero-cone of dimension N.

```
"Constraints"
NAME? "[" DOMAIN "]" EXPR_LIST
```

For example:

```
Constraints
'K1' [SOC(3)] x1 + x2 ; x2 + x3 ; x3 + x1
'K2' [RSOC(3)]
x1 + x2
x2 + x3
x3 + x1
```

## 16.5.5 Variables section

Any variable used in an expression must be defined in a variable section. The variable section defines each variable domain.

```
"Variables"

NAME "[" [-+] (FLOAT | "Inf") (";" [-+] (FLOAT | "Inf") )? "]"

NAME "[" DOMAIN "]" NAMES

For example, a linear variable
```

```
Variables x1 [0;Inf]
```

As with constraints, members of a conic domain can be listed either inline or in a subsection:

```
Variables
k1 [SOC(3)] x1; x2; x3
k2 [RSOC(3)]
x1
x2
x3
```

## 16.5.6 Integer section

This section contains a list of variables that are integral. For example:

```
Integer x1 x2 x3
```

## 16.5.7 SymmetricMatrixes section

This section defines the symmetric matrixes used for matrix coefficients in matrix inner product terms. The section lists named matrixes, each with a size and a number of non-zeros. Only non-zeros in the lower triangular part should be defined.

```
"SymmetricMatrixes"

NAME "SYMMAT" "(" INT ")" ( "(" INT "," INT "," FLOAT ")" )*

...
```

For example:

```
SymmetricMatrixes
M1 SYMMAT(3) (0,0,1.0) (1,1,2.0) (2,1,0.5)
M2 SYMMAT(3)
(0,0,1.0)
(1,1,2.0)
(2,1,0.5)
```

## 16.5.8 Solutions section

Each subsection defines a solution. A solution defines for each constraint and for each variable exactly one primal value and either one (for conic domains) or two (for linear domains) dual values. The values follow the same logic as in the **MOSEK** C API. A primal and a dual solution status defines the meaning of the values primal and dual (solution, certificate, unknown, etc.)

The format is this:

```
"Solutions"
   "Solution" WHICHSOL
      "ProblemStatus" PROSTA PROSTA?
  "SolutionStatus" SOLSTA SOLSTA?
  "Objective" FLOAT FLOAT
   "Variables"
     # Linear variable status: level, slx, sux
     NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
     # Conic variable status: level, snx
     NAME.
        "[" STATUS "]" FLOAT FLOAT?
  "Constraints"
     # Linear variable status: level, slx, sux
     NAME "[" STATUS "]" FLOAT (FLOAT FLOAT)?
     # Conic variable status: level, snx
         "[" STATUS "]" FLOAT FLOAT?
```

Following values for WHICHSOL are supported:

- interior Interior solution, the result of an interior-point solver.
- basic Basic solution, as produced by a simplex solver.
- integer Integer solution, the solution to a mixed-integer problem. This does not define a dual solution.

Following values for PROSTA are supported:

- unknown The problem status is unknown
- feasible The problem has been proven feasible
- infeasible The problem has been proven infeasible
- illposed The problem has been proved to be ill posed
- infeasible\_or\_unbounded The problem is infeasible or unbounded

Following values for SOLSTA are supported:

- unknown The solution status is unknown
- feasible The solution is feasible
- optimal The solution is optimal
- infeas\_cert The solution is a certificate of infeasibility
- illposed\_cert The solution is a certificate of illposedness

Following values for STATUS are supported:

- unknown The value is unknown
- super\_basic The value is super basic
- at\_lower The value is basic and at its lower bound
- at\_upper The value is basic and at its upper bound
- fixed The value is basic fixed
- infinite The value is at infinity

## 16.6 The Task Format

The Task format is **MOSEK**'s native binary format. It contains a complete image of a **MOSEK** task, i.e.

- Problem data: Linear, conic, semidefinite and quadratic data
- Problem item names: Variable names, constraints names, cone names etc.
- Parameter settings
- Solutions

There are a few things to be aware of:

- Status of a solution read from a file will *always* be unknown.
- Parameter settings in a task file *always override* any parameters set on the command line or in a parameter file.

The format is based on the TAR (USTar) file format. This means that the individual pieces of data in a .task file can be examined by unpacking it as a TAR file. Please note that the inverse may not work: Creating a file using TAR will most probably not create a valid **MOSEK** Task file since the order of the entries is important.

## 16.7 The JSON Format

MOSEK provides the possibility to read/write problems in valid JSON format.

JSON (JavaScript Object Notation) is a lightweight data-interchange format. It is easy for humans to read and write. It is easy for machines to parse and generate. It is based on a subset of the JavaScript Programming Language, Standard ECMA-262 3rd Edition - December 1999. JSON is a text format that is completely language independent but uses conventions that are familiar to programmers of the C-family of languages, including C, C++, C#, Java, JavaScript, Perl, Python, and many others. These properties make JSON an ideal data-interchange language.

The official JSON website http://www.json.org provides plenty of information along with the format definition.

**MOSEK** defines two JSON-like formats:

- jtask
- jsol

Despite being text-based human-readable formats, jtask and jsol files will include no indentation and no new-lines, in order to keep the files as compact as possible. We therefore strongly advise to use JSON viewer tools to inspect jtask and jsol files.

## 16.7.1 jtask format

It stores a problem instance. The *jtask* format contains the same information as a *task format*. Even though a *jtask* file is human-readable, we do not recommend users to create it by hand, but to rely on **MOSEK**.

## 16.7.2 jsol format

It stores a problem solution. The *jsol* format contains all solutions and information items. You can write a *jsol* file using Task.writejsonsol. You can not read a *jsol* file into MOSEK.

## 16.7.3 A jtask example

In Listing 16.6 we present a file in the jtask format that corresponds to the sample problem from lol.lp. The listing has been formatted for readability.

Listing 16.6: A formatted *jtask* file for the lol.lp example.

```
{
    "$schema": "http://mosek.com/json/schema#",
    "Task/INFO":{
        "taskname": "lo1",
        "numvar":4,
        "numcon":3,
        "numcone":0,
        "numbarvar":0,
        "numanz":9,
        "numsvmmat":0.
         "mosekver":[
            8,
             0,
            Ο,
             9
        ]
    },
    "Task/data":{
        "var":{
             "name": [
                 "x1"
                 "x2",
```

```
"x3",
        "x4"
    ],
"bk":[
        "lo",
        "ra",
        "lo",
        "lo"
    ],
    "bl":[
        0.0,
        0.0,
        0.0,
        0.0
    ],
    "bu":[
        1e+30,
        1e+1,
        1e+30,
        1e+30
    ],
    "type":[
        "cont",
        "cont",
        "cont",
        "cont"
    ]
},
"con":{
    "name":[
        "c1",
        "c2",
        "c3"
    ],
    "bk":[
       "fx",
        "lo",
        "up"
    ],
    "bl":[
        3e+1,
        1.5e+1,
            -1e+30
    ],
    "bu":[
        3e+1,
        1e+30,
        2.5e+1
    ]
},
"objective":{
    "sense":"max",
    "name":"obj",
    "c":{
        "subj":[
           0,
            1,
            2,
            3
        ],
        "val":[
```

```
3e+0,
                 1e+0,
                5e+0,
                1e+0
            ]
        },
        "cfix":0.0
    },
    "A":{
        "subi":[
            Ο,
            Ο,
            0,
            1,
            1,
            1,
            1,
            2,
            2
        ],
        "subj":[
           0,
            1,
            2,
            0,
            1,
            2,
            3,
            1,
            3
        ],
        "val":[
            3e+0,
            1e+0,
            2e+0,
            2e+0,
            1e+0,
            3e+0,
            1e+0,
            2e+0,
            3e+0
        ]
"Task/parameters":{
    "iparam":{
        "ANA_SOL_BASIS":"ON",
        "ANA_SOL_PRINT_VIOLATED":"OFF",
        "AUTO_SORT_A_BEFORE_OPT":"OFF",
        "AUTO_UPDATE_SOL_INFO":"OFF",
        "BASIS_SOLVE_USE_PLUS_ONE": "OFF",
        "BI_CLEAN_OPTIMIZER": "OPTIMIZER_FREE",
        "BI_IGNORE_MAX_ITER":"OFF",
        "BI_IGNORE_NUM_ERROR": "OFF",
        "BI_MAX_ITERATIONS":1000000,
        "CACHE_LICENSE": "ON",
        "CHECK_CONVEXITY": "CHECK_CONVEXITY_FULL",
        "COMPRESS_STATFILE": "ON",
        "CONCURRENT_NUM_OPTIMIZERS":2,
        "CONCURRENT_PRIORITY_DUAL_SIMPLEX":2,
        "CONCURRENT_PRIORITY_FREE_SIMPLEX":3,
```

```
"CONCURRENT_PRIORITY_INTPNT":4,
"CONCURRENT_PRIORITY_PRIMAL_SIMPLEX":1,
"FEASREPAIR_OPTIMIZE": "FEASREPAIR_OPTIMIZE_NONE",
"INFEAS_GENERIC_NAMES":"OFF",
"INFEAS_PREFER_PRIMAL":"ON",
"INFEAS_REPORT_AUTO":"OFF",
"INFEAS_REPORT_LEVEL":1,
"INTPNT_BASIS": "BI_ALWAYS",
"INTPNT_DIFF_STEP":"ON",
"INTPNT_FACTOR_DEBUG_LVL":0,
"INTPNT_FACTOR_METHOD":0,
"INTPNT_HOTSTART": "INTPNT_HOTSTART_NONE",
"INTPNT_MAX_ITERATIONS":400,
"INTPNT_MAX_NUM_COR":-1,
"INTPNT_MAX_NUM_REFINEMENT_STEPS":-1,
"INTPNT_OFF_COL_TRH":40,
"INTPNT_ORDER_METHOD": "ORDER_METHOD_FREE",
"INTPNT_REGULARIZATION_USE":"ON",
"INTPNT_SCALING": "SCALING_FREE",
"INTPNT_SOLVE_FORM": "SOLVE_FREE",
"INTPNT_STARTING_POINT": "STARTING_POINT_FREE",
"LIC_TRH_EXPIRY_WRN":7,
"LICENSE_DEBUG": "OFF",
"LICENSE_PAUSE_TIME":0,
"LICENSE_SUPPRESS_EXPIRE_WRNS": "OFF",
"LICENSE_WAIT": "OFF",
"LOG":10,
"LOG_ANA_PRO":1,
"LOG_BI":4,
"LOG_BI_FREQ":2500,
"LOG_CHECK_CONVEXITY":0,
"LOG_CONCURRENT":1,
"LOG_CUT_SECOND_OPT":1,
"LOG_EXPAND":0,
"LOG_FACTOR":1,
"LOG_FEAS_REPAIR":1,
"LOG_FILE":1,
"LOG_HEAD":1,
"LOG_INFEAS_ANA":1,
"LOG_INTPNT":4,
"LOG_MIO":4,
"LOG_MIO_FREQ":1000,
"LOG_OPTIMIZER":1,
"LOG_ORDER":1,
"LOG_PRESOLVE":1,
"LOG_RESPONSE":0,
"LOG_SENSITIVITY":1,
"LOG_SENSITIVITY_OPT":0,
"LOG_SIM":4,
"LOG_SIM_FREQ":1000,
"LOG_SIM_MINOR":1,
"LOG_STORAGE":1,
"MAX_NUM_WARNINGS":10,
"MIO_BRANCH_DIR": "BRANCH_DIR_FREE",
"MIO_CONSTRUCT_SOL": "OFF",
"MIO_CUT_CLIQUE": "ON",
"MIO_CUT_CMIR": "ON",
"MIO_CUT_GMI": "ON",
"MIO_CUT_KNAPSACK_COVER": "OFF",
"MIO_HEURISTIC_LEVEL":-1,
"MIO_MAX_NUM_BRANCHES":-1,
```

```
"MIO_MAX_NUM_RELAXS":-1,
"MIO_MAX_NUM_SOLUTIONS":-1,
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"MIO_MT_USER_CB":"ON",
"MIO_NODE_OPTIMIZER":"OPTIMIZER_FREE",
"MIO_NODE_SELECTION": "MIO_NODE_SELECTION_FREE",
"MIO_PERSPECTIVE_REFORMULATE": "ON",
"MIO_PROBING_LEVEL":-1,
"MIO_RINS_MAX_NODES":-1,
"MIO_ROOT_OPTIMIZER": "OPTIMIZER_FREE",
"MIO_ROOT_REPEAT_PRESOLVE_LEVEL":-1,
"MT_SPINCOUNT":0,
"NUM_THREADS":0,
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"OPF_WRITE_HEADER": "ON",
"OPF_WRITE_HINTS":"ON",
"OPF_WRITE_PARAMETERS": "OFF",
"OPF_WRITE_PROBLEM":"ON",
"OPF_WRITE_SOL_BAS":"ON",
"OPF_WRITE_SOL_ITG":"ON",
"OPF_WRITE_SOL_ITR":"ON",
"OPF_WRITE_SOLUTIONS": "OFF",
"OPTIMIZER": "OPTIMIZER_FREE",
"PARAM_READ_CASE_NAME": "ON",
"PARAM_READ_IGN_ERROR": "OFF",
"PRESOLVE_ELIMINATOR_MAX_FILL":-1,
"PRESOLVE_ELIMINATOR_MAX_NUM_TRIES":-1,
"PRESOLVE_LEVEL":-1,
"PRESOLVE_LINDEP_ABS_WORK_TRH":100,
"PRESOLVE_LINDEP_REL_WORK_TRH":100,
"PRESOLVE_LINDEP_USE": "ON",
"PRESOLVE_MAX_NUM_REDUCTIONS":-1,
"PRESOLVE_USE": "PRESOLVE_MODE_FREE",
"PRIMAL_REPAIR_OPTIMIZER": "OPTIMIZER_FREE",
"QO_SEPARABLE_REFORMULATION": "OFF",
"READ_DATA_COMPRESSED": "COMPRESS_FREE";
"READ_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
"READ_DEBUG": "OFF",
"READ_KEEP_FREE_CON":"OFF",
"READ_LP_DROP_NEW_VARS_IN_BOU":"OFF",
"READ_LP_QUOTED_NAMES": "ON",
"READ_MPS_FORMAT": "MPS_FORMAT_FREE",
"READ_MPS_WIDTH": 1024,
"READ_TASK_IGNORE_PARAM": "OFF",
"SENSITIVITY_ALL": "OFF",
"SENSITIVITY_OPTIMIZER": "OPTIMIZER_FREE_SIMPLEX",
"SENSITIVITY_TYPE": "SENSITIVITY_TYPE_BASIS",
"SIM_BASIS_FACTOR_USE": "ON",
"SIM_DEGEN": "SIM_DEGEN_FREE",
"SIM_DUAL_CRASH":90,
"SIM_DUAL_PHASEONE_METHOD":0,
"SIM_DUAL_RESTRICT_SELECTION":50,
"SIM_DUAL_SELECTION": "SIM_SELECTION_FREE",
"SIM_EXPLOIT_DUPVEC": "SIM_EXPLOIT_DUPVEC_OFF",
"SIM_HOTSTART": "SIM_HOTSTART_FREE",
"SIM_HOTSTART_LU": "ON",
"SIM_INTEGER":0,
"SIM_MAX_ITERATIONS":10000000,
"SIM_MAX_NUM_SETBACKS":250,
"SIM_NON_SINGULAR":"ON",
"SIM_PRIMAL_CRASH":90,
```

```
"SIM_PRIMAL_PHASEONE_METHOD":0,
    "SIM_PRIMAL_RESTRICT_SELECTION":50,
    "SIM_PRIMAL_SELECTION": "SIM_SELECTION_FREE",
    "SIM_REFACTOR_FREQ":0,
    "SIM_REFORMULATION": "SIM_REFORMULATION_OFF",
    "SIM_SAVE_LU":"OFF",
    "SIM_SCALING": "SCALING_FREE",
    "SIM_SCALING_METHOD": "SCALING_METHOD_POW2",
    "SIM_SOLVE_FORM": "SOLVE_FREE",
    "SIM_STABILITY_PRIORITY":50,
    "SIM_SWITCH_OPTIMIZER":"OFF",
    "SOL_FILTER_KEEP_BASIC": "OFF";
    "SOL_FILTER_KEEP_RANGED": "OFF",
    "SOL_READ_NAME_WIDTH":-1,
    "SOL_READ_WIDTH":1024,
    "SOLUTION_CALLBACK": "OFF",
    "TIMING_LEVEL":1,
    "WRITE_BAS_CONSTRAINTS":"ON",
    "WRITE_BAS_HEAD": "ON",
    "WRITE_BAS_VARIABLES": "ON",
    "WRITE_DATA_COMPRESSED":0,
    "WRITE_DATA_FORMAT": "DATA_FORMAT_EXTENSION",
    "WRITE_DATA_PARAM": "OFF",
    "WRITE_FREE_CON": "OFF",
    "WRITE_GENERIC_NAMES": "OFF",
    "WRITE_GENERIC_NAMES_IO":1,
    "WRITE_IGNORE_INCOMPATIBLE_CONIC_ITEMS":"OFF",
    "WRITE_IGNORE_INCOMPATIBLE_ITEMS":"OFF",
    "WRITE_IGNORE_INCOMPATIBLE_NL_ITEMS": "OFF"
    "WRITE_IGNORE_INCOMPATIBLE_PSD_ITEMS":"OFF",
    "WRITE_INT_CONSTRAINTS": "ON",
    "WRITE_INT_HEAD": "ON",
    "WRITE_INT_VARIABLES": "ON",
    "WRITE_LP_FULL_OBJ": "ON",
    "WRITE_LP_LINE_WIDTH":80,
    "WRITE_LP_QUOTED_NAMES": "ON",
    "WRITE_LP_STRICT_FORMAT": "OFF",
    "WRITE_LP_TERMS_PER_LINE":10,
    "WRITE_MPS_FORMAT": "MPS_FORMAT_FREE",
    "WRITE_MPS_INT":"ON",
    "WRITE_PRECISION":15,
    "WRITE_SOL_BARVARIABLES": "ON",
    "WRITE_SOL_CONSTRAINTS": "ON",
    "WRITE_SOL_HEAD": "ON",
    "WRITE_SOL_IGNORE_INVALID_NAMES": "OFF",
    "WRITE_SOL_VARIABLES": "ON",
    "WRITE_TASK_INC_SOL":"ON",
    "WRITE_XML_MODE": "WRITE_XML_MODE_ROW"
},
"dparam":{
    "ANA_SOL_INFEAS_TOL":1e-6,
    "BASIS_REL_TOL_S":1e-12,
    "BASIS_TOL_S":1e-6,
    "BASIS_TOL_X":1e-6,
    "CHECK_CONVEXITY_REL_TOL":1e-10,
    "DATA_TOL_AIJ":1e-12,
    "DATA_TOL_AIJ_HUGE": 1e+20,
    "DATA_TOL_AIJ_LARGE":1e+10,
    "DATA_TOL_BOUND_INF":1e+16,
    "DATA_TOL_BOUND_WRN":1e+8,
    "DATA_TOL_C_HUGE":1e+16,
```

```
"DATA_TOL_CJ_LARGE":1e+8,
    "DATA_TOL_QIJ":1e-16,
    "DATA_TOL_X":1e-8,
    "FEASREPAIR_TOL":1e-10,
    "INTPNT_CO_TOL_DFEAS":1e-8,
    "INTPNT_CO_TOL_INFEAS":1e-10,
    "INTPNT_CO_TOL_MU_RED":1e-8,
    "INTPNT_CO_TOL_NEAR_REL":1e+3,
    "INTPNT_CO_TOL_PFEAS":1e-8,
    "INTPNT_CO_TOL_REL_GAP":1e-7,
    "INTPNT_NL_MERIT_BAL":1e-4,
    "INTPNT_NL_TOL_DFEAS":1e-8,
    "INTPNT_NL_TOL_MU_RED":1e-12,
    "INTPNT_NL_TOL_NEAR_REL":1e+3,
    "INTPNT_NL_TOL_PFEAS":1e-8,
    "INTPNT_NL_TOL_REL_GAP":1e-6,
    "INTPNT_NL_TOL_REL_STEP":9.95e-1,
    "INTPNT_QO_TOL_DFEAS":1e-8,
    "INTPNT_QO_TOL_INFEAS":1e-10,
    "INTPNT_QO_TOL_MU_RED":1e-8,
    "INTPNT_QO_TOL_NEAR_REL":1e+3,
    "INTPNT_QO_TOL_PFEAS":1e-8,
    "INTPNT_QO_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_DFEAS":1e-8,
    "INTPNT_TOL_DSAFE":1e+0,
    "INTPNT_TOL_INFEAS": 1e-10,
    "INTPNT_TOL_MU_RED":1e-16,
    "INTPNT_TOL_PATH":1e-8,
    "INTPNT_TOL_PFEAS":1e-8,
    "INTPNT_TOL_PSAFE":1e+0,
    "INTPNT_TOL_REL_GAP":1e-8,
    "INTPNT_TOL_REL_STEP":9.999e-1,
    "INTPNT_TOL_STEP_SIZE":1e-6,
    "LOWER_OBJ_CUT":-1e+30,
    "LOWER_OBJ_CUT_FINITE_TRH":-5e+29,
    "MIO_DISABLE_TERM_TIME":-1e+0,
    "MIO_MAX_TIME":-1e+0,
    "MIO_MAX_TIME_APRX_OPT":6e+1,
    "MIO_NEAR_TOL_ABS_GAP":0.0,
    "MIO_NEAR_TOL_REL_GAP":1e-3,
    "MIO_REL_GAP_CONST":1e-10,
    "MIO_TOL_ABS_GAP":0.0,
    "MIO_TOL_ABS_RELAX_INT":1e-5,
    "MIO_TOL_FEAS":1e-6,
    "MIO_TOL_REL_DUAL_BOUND_IMPROVEMENT":0.0,
    "MIO_TOL_REL_GAP":1e-4,
    "MIO_TOL_X":1e-6,
    "OPTIMIZER_MAX_TIME":-1e+0,
    "PRESOLVE_TOL_ABS_LINDEP":1e-6,
    "PRESOLVE_TOL_AIJ":1e-12,
    "PRESOLVE_TOL_REL_LINDEP":1e-10,
    "PRESOLVE_TOL_S":1e-8,
    "PRESOLVE_TOL_X":1e-8,
    "QCQO_REFORMULATE_REL_DROP_TOL":1e-15,
    "SEMIDEFINITE_TOL_APPROX":1e-10,
    "SIM_LU_TOL_REL_PIV":1e-2,
    "SIMPLEX_ABS_TOL_PIV":1e-7,
    "UPPER_OBJ_CUT":1e+30,
    "UPPER_OBJ_CUT_FINITE_TRH":5e+29
},
"sparam":{
```

```
"BAS_SOL_FILE_NAME":"",
            "DATA_FILE_NAME": "examples/tools/data/lo1.mps",
            "DEBUG_FILE_NAME": "",
            "INT_SOL_FILE_NAME":""
            "ITR_SOL_FILE_NAME":"",
            "MIO_DEBUG_STRING":"",
            "PARAM_COMMENT_SIGN":"%%",
            "PARAM_READ_FILE_NAME":"",
            "PARAM_WRITE_FILE_NAME":"",
            "READ_MPS_BOU_NAME":"",
            "READ_MPS_OBJ_NAME":"",
            "READ_MPS_RAN_NAME":"",
            "READ_MPS_RHS_NAME":"",
            "SENSITIVITY_FILE_NAME":"",
            "SENSITIVITY_RES_FILE_NAME":"",
            "SOL_FILTER_XC_LOW":"",
            "SOL_FILTER_XC_UPR":"",
            "SOL_FILTER_XX_LOW":"",
            "SOL_FILTER_XX_UPR":"",
            "STAT_FILE_NAME":"",
            "STAT_KEY":"",
            "STAT_NAME":""
            "WRITE_LP_GEN_VAR_NAME": "XMSKGEN"
    }
}
```

## 16.8 The Solution File Format

MOSEK provides several solution files depending on the problem type and the optimizer used:

- basis solution file (extension .bas) if the problem is optimized using the simplex optimizer or basis identification is performed,
- interior solution file (extension .sol) if a problem is optimized using the interior-point optimizer and no basis identification is required,
- integer solution file (extension .int) if the problem contains integer constrained variables.

All solution files have the format:

```
NAME.
                    : <problem name>
PROBLEM STATUS
                   : <status of the problem>
SOLUTION STATUS
                   : <status of the solution>
OBJECTIVE NAME
                   : <name of the objective function>
PRIMAL OBJECTIVE
                  : <primal objective value corresponding to the solution>
DUAL OBJECTIVE
                   : <dual objective value corresponding to the solution>
CONSTRAINTS
INDEX NAME
               AT ACTIVITY
                               LOWER LIMIT
                                             UPPER LIMIT
                                                           DUAL LOWER
                                                                        DUAL UPPER
               ?? <a value>
                               <a value>
                                             <a value>
                                                           <a value>
      <name>
                                                                        <a value>
VARIABLES
INDEX NAME
                AT ACTIVITY
                               LOWER LIMIT
                                             UPPER LIMIT
                                                           DUAL LOWER
                                                                        DUAL UPPER
                                                                                       CONIC
→DUAL
                ?? <a value>
                               <a value>
                                             <a value>
                                                           <a value>
                                                                         <a value>
                                                                                       <a value>
```

In the example the fields ? and <> will be filled with problem and solution specific information. As can be observed a solution report consists of three sections, i.e.

• HEADER In this section, first the name of the problem is listed and afterwards the problem and solution status are shown. Next the primal and dual objective values are displayed.

 $\bullet$  CONSTRAINTS For each constraint i of the form

$$l_i^c \le \sum_{j=1}^n a_{ij} x_j \le u_i^c, \tag{16.15}$$

the following information is listed:

- INDEX: A sequential index assigned to the constraint by MOSEK
- NAME: The name of the constraint assigned by the user.
- AT: The status of the constraint. In Table 16.4 the possible values of the status keys and their interpretation are shown.

Table 16.4: Status keys.

Status key	Interpretation
UN	Unknown status
BS	Is basic
SB	Is superbasic
LL	Is at the lower limit (bound)
UL	Is at the upper limit (bound)
EQ	Lower limit is identical to upper limit
**	Is infeasible i.e. the lower limit is greater than the upper limit.

- ACTIVITY: the quantity  $\sum_{j=1}^{n} a_{ij} x_{j}^{*}$ , where  $x^{*}$  is the value of the primal solution. LOWER LIMIT: the quantity  $l_{i}^{c}$  (see (16.15).)
- UPPER LIMIT: the quantity  $u_i^c$  (see (16.15).)
- DUAL LOWER: the dual multiplier corresponding to the lower limit on the constraint.
- DUAL UPPER: the dual multiplier corresponding to the upper limit on the constraint.
- VARIABLES The last section of the solution report lists information about the variables. This information has a similar interpretation as for the constraints. However, the column with the header CONIC DUAL is included for problems having one or more conic constraints. This column shows the dual variables corresponding to the conic constraints.

## Example: lo1.sol

In Listing 16.7 we show the solution file for the lol.opf problem.

Listing 16.7: An example of .sol file.

NAME	:			
PROBLEM STATUS	: PRIMAL_AND_DUAL_FEASIBLE			
SOLUTION STATUS	: OPTIMAL			
OBJECTIVE NAME	: obj			
PRIMAL OBJECTIVE	: 8.33333333e+01			
DUAL OBJECTIVE	: 8.33333332e+01			
CONSTRAINTS				
INDEX NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	Ш
$\hookrightarrow$ DUAL LOWER	DUAL UPPER			
0 c1	EQ 3.00000000000000e+01	3.00000000e+01	3.00000000e+01	-0.
→00000000000000e+00	-2.4999999741653e+00			
1 c2	<b>SB</b> 5.3333333349187e+01	1.50000000e+01	NONE	2.
→09159033069640e-10	-0.0000000000000e+00			
2 <b>c3</b>	UL 2.4999999842049e+01	NONE	2.50000000e+01	-0.
	-3.33333332895108e-01			
VARIABLES				
INDEX NAME	AT ACTIVITY	LOWER LIMIT	UPPER LIMIT	ш
→DUAL LOWER	DUAL UPPER			
0 x1	LL 1.67020427038537e-09	0.00000000e+00	NONE	-4.
→49999999528054e+00	-0.000000000000e+00			
			(continues on a	

				(	1 0 /
1	x2	LL 2.93510446211883e-09	0.00000000e+00	1.00000000e+01	-2.
<b>→</b> 16	666666494915e+00	6.20868657679896e-10			
2	x3	SB 1.49999999899424e+01	0.0000000e+00	NONE	-8.
<b>∽</b> 79	123177245553e-10	-0.0000000000000e+00			
3	x4	SB 8.33333332273115e+00	0.0000000e+00	NONE	-1.
<b>⇔</b> 69	795978848200e-09	-0.0000000000000e+00			

## Chapter 17

# List of examples

List of examples shipped in the distribution of Optimizer API for . NET:  $\,$ 

Table 17.1: List of distributed examples

	Table 17.1: List of distributed examples		
File	Description		
blas_lapack.cs	Demonstrates the MOSEK interface to BLAS/LAPACK linear algebra routines		
callback.cs	An example of data/progress callback		
ceo1.cs	A simple conic exponential problem		
concurrent.cs	Implementation of a concurrent optimizer for linear and mixed-integer problems		
cqo1.cs	A simple conic quadratic problem		
feasrepairex1.	A simple example of how to repair an infeasible problem		
cs			
gp1.cs	A simple geometric program (GP) in conic form		
lo1.cs	A simple linear problem		
lo1.vb	A simple linear problem		
lo2.cs	A simple linear problem		
logistic.cs	Implements logistic regression and simple log-sum-exp (CEO)		
mico1.cs	A simple mixed-integer conic problem		
milo1.cs	A simple mixed-integer linear problem		
mioinitsol.cs	A simple mixed-integer linear problem with an initial guess		
modelLib.cs	· · · · · · · · · · · · · · · · · · ·		
opt_server_async. Uses MOSEK OptServer to solve an optimization problem asynchronously			
cs			
opt_server_sync.	Uses MOSEK OptServer to solve an optimization problem synchronously		
cs			
parallel.cs	Demonstrates parallel optimization		
parameters.cs	Shows how to set optimizer parameters and read information items		
portfolio_1_basi	c Portfolio optimization - basic Markowitz model		
cs			
portfolio_2_fron	t <b>Per</b> tfolio optimization - efficient frontier		
cs			
portfolio_3_impactPortfolio optimization - market impact costs			
cs			
portfolio_4_tran	scortfolio optimization - transaction costs		
cs			
portfolio_5_card	portfolio_5_card. Portfolio optimization - cardinality constraints		
CS			
pow1.cs	A simple power cone problem		
qcqo1.cs	A simple quadratically constrained quadratic problem		
qo1.cs	A simple quadratic problem		
reoptimization.	Demonstrate how to modify and re-optimize a linear problem		
CS			
	Continued on next page		

Continued on next page

Table 17.1 – continued from previous page

File	Description
response.cs	Demonstrates proper response handling
sdo1.cs	A simple semidefinite problem with one matrix variable and a quadratic cone
sdo2.cs	A simple semidefinite problem with two matrix variables
sensitivity.cs	Sensitivity analysis performed on a small linear problem
simple.cs	A simple I/O example: read problem from a file, solve and write solutions
solutionquality.	Demonstrates how to examine the quality of a solution
cs	
solvebasis.cs	Demonstrates solving a linear system with the basis matrix
solvelinear.cs	Demonstrates solving a general linear system
sparsecholesky.	Shows how to find a Cholesky factorization of a sparse matrix
cs	

Additional examples can be found on the  $\mathbf{MOSEK}$  website and in other  $\mathbf{MOSEK}$  publications.

# Chapter 18

# Interface changes

The section shows interface-specific changes to the **MOSEK** Optimizer API for .NET in version 9.1 compared to version 8. See the release notes for general changes and new features of the **MOSEK** Optimization Suite.

# 18.1 Backwards compatibility

- Parameters. Users who set parameters to tune the performance and numerical properties of the solver (termination criteria, tolerances, solving primal or dual, presolve etc.) are recommended to reevaluate such tuning. It may be that other, or default, parameter settings will be more beneficial in the current version. The hints in Sec. 8 may be useful for some cases.
- All functions using the enum accmode were removed. Use corresponding separate functions for manipulating variables and constraints. For example, instead of

```
task.putbound(accmode.var, ...);
task.putbound(accmode.con, ...);
```

use

```
task.putvarbound(...);
task.putconbound(...);
```

and so on.

- Removed all Near problem and solution statuses i.e. solsta.near\_optimal, solsta.near\_prim\_infeas\_cer, etc. See Sec. 13.3.3.
- All functions related to the general nonlinear optimizer and Scopt have been removed. See Sec. 15.11.

# 18.2 Functions

## Added

- Env.setupthreads
- Task.appendsparsesymmatlist
- Task.generateconenames
- Task.generateconnames
- Task.generatevarnames
- Task.getacolslice

- $\bullet \ \textit{Task.getacolslicenumnz}$
- ullet Task.getarowslice
- Task.getarowslicenumnz
- $\bullet \ \textit{Task.getatruncatetol}$
- $\bullet$  Task.getbarsslice
- Task.getbarxslice
- Task.getclist
- Task.getskn
- $\bullet \quad \textit{Task.putatruncatetol}$
- $\bullet \ \textit{Task.putbaraijlist}$
- $\bullet \ \textit{Task.putbararowlist}$
- $\bullet \ \textit{Task.putconboundlistconst}$
- $\bullet \ \textit{Task.putconboundsliceconst}$
- $\bullet \ \textit{Task.putconsolutioni}$
- $\bullet \ \textit{Task.putvarboundlistconst}$
- $\bullet \ \textit{Task.putvarboundsliceconst}$
- $\bullet \ \textit{Task.putvarsolutionj}$
- Task.readjsonstring
- Task.readlpstring
- Task.readopfstring
- Task.readptfstring

## Removed

- Task.checkconvexity
- Task.chgbound
- Task.getaslice
- Task.getaslicenumnz
- Task.getbound
- Task.getboundslice
- Task.getsolutioni
- Task.printdata
- Task.putbound
- Task.putboundlist
- Task.putboundslice
- Task.putsolutioni

## 18.3 Parameters

### Added

- iparam.intpnt\_order\_gp\_num\_seeds
- $\bullet$  iparam.intpnt\_purify
- iparam.log\_include\_summary
- iparam.log\_local\_info
- iparam.mio\_conic\_outer\_approximation
- iparam.mio\_feaspump\_level
- $\bullet \quad iparam.mio\_max\_num\_root\_cut\_rounds$
- iparam.mio\_propagate\_objective\_constraint
- iparam.mio\_seed
- iparam.opf\_write\_line\_length
- iparam.presolve\_max\_num\_pass
- $\bullet$  iparam.ptf\_write\_transform
- iparam.sim\_seed
- iparam.write\_compression

#### Removed

- dparam.data\_tol\_aij
- dparam.intpnt\_nl\_merit\_bal
- dparam.intpnt\_nl\_tol\_dfeas
- dparam.intpnt\_nl\_tol\_mu\_red
- dparam.intpnt\_nl\_tol\_near\_rel
- dparam.intpnt\_nl\_tol\_pfeas
- $\bullet \ \mathtt{dparam.intpnt\_nl\_tol\_rel\_gap}$
- dparam.intpnt\_nl\_tol\_rel\_step
- dparam.mio\_disable\_term\_time
- dparam.mio\_near\_tol\_abs\_gap
- dparam.mio\_near\_tol\_rel\_gap
- iparam.mio\_construct\_sol
- iparam.mio\_mt\_user\_cb
- iparam.opf\_max\_terms\_per\_line
- iparam.read\_data\_compressed
- iparam.read\_data\_format
- iparam.write\_data\_compressed
- iparam.write\_data\_format

# 18.4 Constants

### Added

- compresstype.zstd
- conetype.dexp
- conetype.dpow
- conetype.pexp
- conetype.ppow
- conetype.zero
- dataformat.ptf
- iinfitem.mio\_numbin
- iinfitem.mio\_numbinconevar
- iinfitem.mio\_numcone
- $\bullet \ \ \textit{iinfitem.mio\_numconevar}$
- iinfitem.mio\_numcont
- $\bullet$  iinfitem.mio\_numcontconevar
- iinfitem.mio\_numdexpcones
- iinfitem.mio\_numdpowcones
- iinfitem.mio\_numintconevar
- iinfitem.mio\_numpexpcones
- iinfitem.mio\_numppowcones
- $\bullet \ \ iinfitem.mio\_numqcones$
- $\bullet \ \ iinfitem.mio\_numrqcones$
- iinfitem.mio\_presolved\_numbinconevar
- iinfitem.mio\_presolved\_numcone
- iinfitem.mio\_presolved\_numconevar
- $\bullet \ \ iinfitem.mio\_presolved\_numcontconevar$
- $\bullet \ \ iinfitem.mio\_presolved\_numdexpcones$
- $\bullet \ \ iinfitem.mio\_presolved\_numdpowcones$
- iinfitem.mio\_presolved\_numintconevar
- $\bullet \ \ iinfitem.mio\_presolved\_numpexpcones$
- $\bullet \ \ iinfitem.mio\_presolved\_numppowcones$
- iinfitem.mio\_presolved\_numqcones
- iinfitem.mio\_presolved\_numrqcones
- iinfitem.purify\_dual\_success
- iinfitem.purify\_primal\_success
- liinfitem.mio\_anz

#### Removed

- constant.dataformat.xml
- constant.dinfitem.mio\_heuristic\_time
- $\bullet \verb| constant.dinfitem.mio_optimizer_time \\$
- constant.iinfitem.mio\_construct\_num\_roundings
- constant.iinfitem.mio\_initial\_solution
- constant.iinfitem.mio\_near\_absgap\_satisfied
- constant.iinfitem.mio\_near\_relgap\_satisfied
- constant.liinfitem.mio\_sim\_maxiter\_setbacks
- constant.mionodeseltype.hybrid
- constant.mionodeseltype.worst
- constant.problemtype.geco
- constant.prosta.near\_dual\_feas
- constant.prosta.near\_prim\_and\_dual\_feas
- constant.prosta.near\_prim\_feas
- constant.sensitivitytype.optimal\_partition
- constant.solsta.near\_dual\_feas
- constant.solsta.near\_dual\_infeas\_cer
- constant.solsta.near\_integer\_optimal
- constant.solsta.near\_optimal
- constant.solsta.near\_prim\_and\_dual\_feas
- $\bullet \verb| constant.solsta.near_prim_feas |$
- constant.solsta.near\_prim\_infeas\_cer

# 18.5 Response Codes

### Added

- rescode.err\_appending\_too\_big\_cone
- rescode.err\_cbf\_duplicate\_pow\_cones
- rescode.err\_cbf\_duplicate\_pow\_star\_cones
- rescode.err\_cbf\_invalid\_dimension\_of\_cones
- rescode.err\_cbf\_invalid\_exp\_dimension
- rescode.err\_cbf\_invalid\_number\_of\_cones
- $\bullet \ \ rescode. \ err\_cbf\_invalid\_power$
- $\bullet \ \ rescode. \ err\_cbf\_invalid\_power\_cone\_index$

- $\bullet \ \ rescode. \ err\_cbf\_invalid\_power\_star\_cone\_index$
- rescode.err\_cbf\_power\_cone\_is\_too\_long
- rescode.err\_cbf\_power\_cone\_mismatch
- $\bullet \ \ rescode. \ err\_cbf\_power\_star\_cone\_mismatch$
- rescode.err\_cbf\_unhandled\_power\_cone\_type
- rescode.err\_cbf\_unhandled\_power\_star\_cone\_type
- rescode.err\_cone\_parameter
- rescode.err\_format\_string
- $\bullet \ \textit{rescode.err\_invalid\_file\_format\_for\_cfix}$
- rescode.err\_invalid\_file\_format\_for\_free\_constraints
- $\bullet \ \ rescode. \ err\_invalid\_file\_format\_for\_nonlinear$
- $\bullet \ \ rescode. err\_invalid\_file\_format\_for\_ranged\_constraints$
- rescode.err\_num\_arguments
- rescode.err\_ptf\_format
- rescode.err\_shape\_is\_too\_large
- rescode.err\_slice\_size
- rescode.err\_too\_small\_a\_truncation\_value
- $\bullet \ \textit{rescode.wrn\_exp\_cones\_with\_variables\_fixed\_at\_zero}$
- $\bullet \ \textit{rescode.wrn\_pow\_cones\_with\_root\_fixed\_at\_zero}$

#### Removed

- rescode.err\_cannot\_clone\_nl
- rescode.err\_cannot\_handle\_nl
- rescode.err\_invalid\_accmode
- rescode.err\_invalid\_file\_format\_for\_general\_nl
- rescode.err\_nonlinear\_functions\_not\_allowed
- rescode.err\_nr\_arguments
- rescode.err\_open\_dl
- rescode.err\_user\_func\_ret
- rescode.err\_user\_func\_ret\_data
- rescode.err\_user\_nlo\_eval
- rescode.err\_user\_nlo\_eval\_hessubi
- rescode.err\_user\_nlo\_eval\_hessubj
- rescode.err\_user\_nlo\_func
- rescode.trm\_mio\_near\_abs\_gap
- rescode.trm\_mio\_near\_rel\_gap
- rescode.wrn\_construct\_invalid\_sol\_itg
- rescode.wrn\_construct\_no\_sol\_itg
- rescode.wrn\_construct\_solution\_infeas
- rescode.wrn\_no\_nonlinear\_function\_write

# **Bibliography**

- [AA95] E. D. Andersen and K. D. Andersen. Presolving in linear programming. *Math. Programming*, 71(2):221–245, 1995.
- [AGMX96] E. D. Andersen, J. Gondzio, Cs. Mészáros, and X. Xu. Implementation of interior point methods for large scale linear programming. In T. Terlaky, editor, *Interior-point methods of mathematical programming*, pages 189–252. Kluwer Academic Publishers, 1996.
- [ART03] E. D. Andersen, C. Roos, and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Math. Programming*, February 2003.
- [AY96] E. D. Andersen and Y. Ye. Combining interior-point and pivoting algorithms. *Management Sci.*, 42(12):1719–1731, December 1996.
- [And09] Erling D. Andersen. The homogeneous and self-dual model and algorithm for linear optimization. Technical Report TR-1-2009, MOSEK ApS, 2009. URL: http://docs.mosek.com/whitepapers/homolo.pdf.
- [And13] Erling D. Andersen. On formulating quadratic functions in optimization models. Technical Report TR-1-2013, MOSEK ApS, 2013. Last revised 23-feb-2016. URL: http://docs.mosek.com/whitepapers/qmodel.pdf.
- [BKVH07] S. Boyd, S.J. Kim, L. Vandenberghe, and A. Hassibi. A Tutorial on Geometric Programming. *Optimization and Engineering*, 8(1):67–127, 2007. Available at http://www.stanford.edu/~boyd/gp tutorial.html.
- [Chv83] V. Chvátal. Linear programming. W.H. Freeman and Company, 1983.
- [GK00] Richard C. Grinold and Ronald N. Kahn. *Active portfolio management*. McGraw-Hill, New York, 2 edition, 2000.
- [Naz87] J. L. Nazareth. Computer Solution of Linear Programs. Oxford University Press, New York, 1987.
- [RTV97] C. Roos, T. Terlaky, and J. -Ph. Vial. Theory and algorithms for linear optimization: an interior point approach. John Wiley and Sons, New York, 1997.
- [Ste98] G. W. Stewart. Matrix Algorithms. Volume 1: Basic decompositions. SIAM, 1998.
- [Wal00] S. W. Wallace. Decision making under uncertainty: is sensitivity of any use. *Oper. Res.*, 48(1):20–25, January 2000.
- [Wol98] L. A. Wolsey. Integer programming. John Wiley and Sons, 1998.

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