Scientific Computing HW 2

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Problem 1.

(b) Code: `https://github.com/RokettoJanpu/Scientific-Computing-2/blob/main/hw2%20problem%201%20part%20b.ipynb`

Generally, CPU time decreases as ϵ increases. Moreover, the CPU times are considerably higher for RK45 than LSODA, especially when comparing RK45 vs LSODA at $\mu = 1000$. This phenomena is likely due to the Van der Pol problem being stiff and RK45 being an explicit method, leading to restrictions on feasible step size which in turn limit the method's effectiveness. On the other hand, LSODA is an implicit method with stiffness detection, leading to increased effectiveness for this problem.

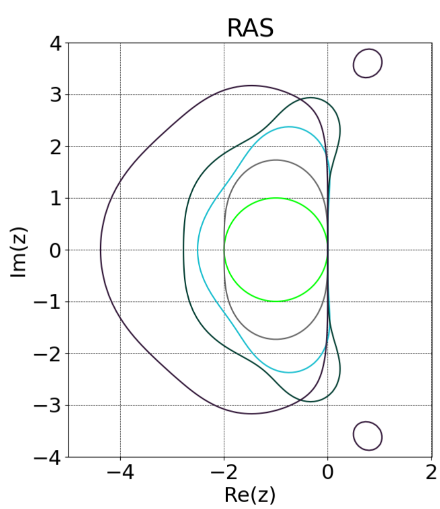
(c) Code: `https://github.com/RokettoJanpu/Scientific-Computing-2/blob/main/hw2%20problem%201%20part%20c.ipynb`

Generally, the CPU times for Radau were considerably higher than those for RK45 and DOP853. Notably for DOP853 and Radau, the satellite follows closely to the predicted trajectory for a while but eventually falls out of orbit of the Earth-Moon system. On the other hand for RK45, the satellite strays comparatively further from the predicted trajectory but does not fall out of orbit during the observed periods.

Problem 2.

Code: `https://github.com/RokettoJanpu/Scientific-Computing-2/blob/main/hw2%20problem%202.ipynb`

In order of increasing area enclosed by contour, the methods are: forward Euler, midpoint rule with Euler predictor, Kutta's method, standard Runge-Kutta, DOPRI5(4).



Problem 3. Pf. The local truncation error at step $n+1$ is

$$\tau_{n+1} = y_{n+1} - y_n - hf(t_{n+1}, y_{n+1})$$

For the numerical solution u,

$$0 = u_{n+1} - u_n - hf(t_{n+1}, u_{n+1})$$

Subtract the latter equation from the former.

$$\tau_{n+1} = (y_{n+1} - u_{n+1}) - (y_n - u_n) - h[f(t_{n+1}, y_{n+1}) - f(t_{n+1}, u_{n+1})]$$

$$\implies (y_{n+1} - u_{n+1}) = (y_n - u_n) + h[f(t_{n+1}, y_{n+1}) - f(t_{n+1}, u_{n+1})] + \tau_{n+1}$$

Set $e_n := \|y_n - u_n\|$ and $\tau := \max_n \|\tau_n\|$. Since f is L–Lipschitz in u,

$$e_{n+1} \leq e_n + hL\|y_{n+1} - u_{n+1}\| + \tau \leq e_n + hLe_{n+1} + \tau$$

$$\implies (1 - hL)e_{n+1} \leq e_n + \tau \implies e_{n+1} \leq \frac{1}{1 - hL}e_n + \frac{\tau}{1 - hL} = ae_n + b,\ a := \frac{1}{1 - hL},\ b := \frac{\tau}{1 - hL}$$