Homework 6. Due March 15

Please upload a single pdf file on ELMS. Link your codes to your pdf (i.e., put your codes to dropbox, github, google drive, etc. and place links to them in your pdf file with your solutions.

1. **(10 pts.) To be done on paper.** Consider Laplace's equation $u_{xx}+u_{yy}=0$ in the given domain Ω with the given boundary conditions (BCs). Discretize the problem on the specified mesh using the 5-point stencil and write out an appropriate system of linear algebraic equations $Au = f$ for the numerical solution u as it is done in `elliptic.pdf`. Highlight the block structure of the matrix A.

   (a) $\Omega = [0,1]\times[0,1]$, the mesh and the BCs are shown in Fig. (a).

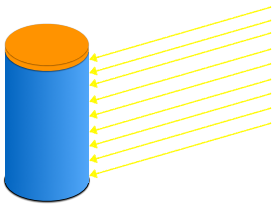
   (b) $\Omega = [0,1]\times[0,1]$, the mesh and the BCs are shown in Fig. (b).

   (c) $\Omega = [0,1]\times[0,1]$, the mesh and the BCs are shown in Fig. (c). (BCs are periodic in y and Dirichlet on the left and right boundaries.)

   (d) $\Omega = [0,1]\times[0,1]$, the mesh and the BC are shown in Fig. (d). (BCs are periodic in x and Dirichlet on thetop and bottom boundaries.)

   (e) Ω is the L-shaped domain shown in Fig. (e). Assume that the left and the bottom boundaries have length 1. The mesh and the BCs are shown in Fig. (e) as well.

2. **(10 pts.) A programming task.** Suppose an empty cylindric tin covered with an insulating top is standing on ice. The sun located low above the horizon is shining on one side of the tin as shown in the figure.



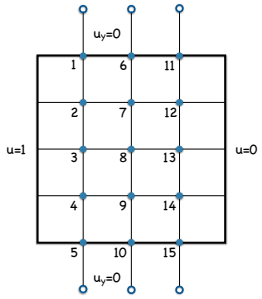
   Assume that the height of the cylinder is 2 and the radius is 1. Take all physical constants such as the heat conductance and the intensity of the sun shine equal to 1. Then the stationary heat distribution satisfies the equation

$$u_{xx}+u_{yy} = \begin{cases} -\cos(x), & -\pi/2 \le x \le \pi/2, \\ 0, & \text{otherwise}, \end{cases}$$

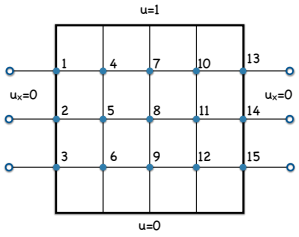
$-\pi \leq x \leq \pi$, $0 \leq y \leq 2$. Pick adequate boundary conditions and write down the boundary value problem (BVP). Solve the BVP numerically and find the stationary temperature distribution. Plot a figure showing the distribution.

Submit the formulation of the BVP, the figure with the stationary heat distribution, and a printout of your code.

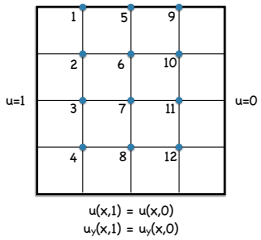
(a)



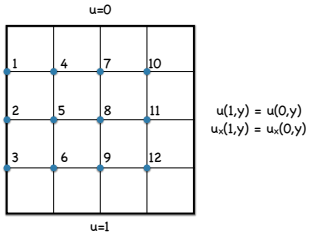
(b)



(c)



(d)



(e)