Homework 4. Due March 1

Please upload a single pdf file on ELMS. Link your codes to your pdf (i.e., put your codes to dropbox, Github, google drive, etc. and place links to them in your pdf file with your solutions.

1. **(4 pts)** Solve exercise 16 in my lecture notes `ODEsolvers.pdf`.
2. **(4 pts)** Solve exercise 17 in my lecture notes `ODEsolvers.pdf`.
3. **(4 pts)** Solve exercise 18 in my lecture notes `ODEsolvers.pdf`.
4. **(4 pts)** Solve exercise 19 in my lecture notes `ODEsolvers.pdf`.
5. **(5 pts)**
   (a) Derive the following formula for the 2-step BDF method with a variable timestep:
   $$u_{n+1} - \frac{(1+\omega)^2}{1+2\omega}u_n + \frac{\omega^2}{1+2\omega}u_{n-1} = h_n\frac{1+\omega}{1+2\omega}f(t_{n+1}, u_{n+1}), \quad (1)$$
   where $h_n := t_{n+1} - t_n$, $\omega = h_n/h_{n-1}$.

   (b) Prove that this method is stable provided that $\omega_n \le 1+\sqrt{2}$. *Hint: use the main theorem for methods with constant stepsize and Vieta's formulas for quadratic equations.*
6. **(5 pts)** Solve the stiff Robertson's problem on the time interval $[0, 40]$ using the 2-step BDF method programmed from scratch. Use fixed timestep. I am providing my code `RobertsonDIRK2.ipynb` where this task is accomplished using DIRK2 from the previous homework with a fixed timestep $h = 10^{-3}$. You can merely add to my code. If you prefer Matlab, please feel free to rewrite it in Matlab. To initialize BDF2, you will need to make the first step by DIRK2. Plot the three components of the solution in different figures as they have very different scales. Measure the CPU time for Robertson's problem for DIRK2 and BDF2 with the same timestep.