AMSC661, Spring 2023 Maria Cameron

Homework 11. Due May 3.

1. **(10 pts)** Consider the Greenberg traffic model

$$\rho_t + [-\rho \log(\rho)]_x = 0, \quad \rho(x,0) = \rho_0(x). \tag{1}$$

Here, ρ is the density of cars, and the velocity v depends on the density according to $v(\rho) = v_{\max} \log\left(\frac{\rho_{\max}}{\rho}\right)$, where $v_{\max}$ and $\rho_{\max}$ are set to be 1 for convenience.

(a) Find the formula for the characteristic $x(t)$ of Eq. (1) starting at the point $(x = x_0, t = 0)$ (the curve $x(t)$ passing through $(x = x_0, t = 0)$ along which ρ is constant, i.e., $\frac{d}{dt}\rho(x(t), t) = 0$).

(b) Plot the characteristics and the shock line on the xt-plane for the Riemann problem

$$\rho_0(x) = \begin{cases} 0.1, & x < 0, \\ 0.9, & x > 0. \end{cases}$$

(c) Suppose

$$\rho_0(x) = 0.5 + \frac{0.9}{\pi} \arctan(x). \tag{2}$$

Find the time when the shock appears. Then find the eventual shock speed.

References

[1] R. J. LeVeque, Numerical Methods for Conservation Laws, Second Edition, Birkh auser, Basel, Boston, Berlin, 1992

[2] M. Cameron's notes `burgers.pdf` available on ELMS.