Homework 1. Due Wednesday, Sept. 4.

1. (**4 pts**) (Problem 7 from Section 2.10 in D. Bindel's and J. Goodman's book "Principles of Scientific Computing") Starting with the declarations

```
      float x, y, z, w;
const float oneThird = 1/ (float) 3;
const float oneHalf  = 1/ (float) 2;
                  // const means these never are reassigned
```

we do lots of arithmetic on the variables `x, y, z, w`. In each case below, determine whether the two arithmetic expressions result in the same floating point number (down to the last bit) as long as no `NaN` or `inf` values or denormalized numbers are produced.

(a)

```
( x * y ) + ( z - w )
( z - w ) + ( y * x )
```

(b)

```
( x +   y ) + z
  x + ( y   + z )
```

(c)

```
  x * oneHalf + y * oneHalf
( x + y ) * oneHalf
```

(d)

```
  x * oneThird + y * oneThird
( x + y ) * oneThird
```

2. **(10 pts)** The *tent map* of the interval $[0, 1]$ onto itself is defined as

$$f(x) = \begin{cases} 2x, & x \in [0, 1/2), \\ 2 - 2x, & x \in [1/2, 1]. \end{cases} \tag{1}$$

Consider the iteration $x_{n+1} = f(x_n)$, $n = 0, 1, 2, \ldots$.

(a) What are the fixed points of this iteration, i.e. the points x^* such that $x^* = f(x^*)$? Show that these fixed points are unstable, i.e., if you start iteration at $x^* + \delta$ for any δ small enough then the next iterate will be farther away from x^* then $x^* + \delta$.

(b) Prove that if x_0 is rational, then the sequence of iterates generated starting from x_0 is periodic.

(c) Show that for any period length p, one can find a rational number x_0 such that the sequence of iterates generated starting from x_0 is periodic of period p.

(d) Generate several long enough sequences of iterates on a computer using any suitable language (Matlab, Python, C, etc.) starting from a pseudorandom x_0 uniformly distributed on $[0, 1)$ to observe a pattern. I checked in Python and C that 100 iterates are enough. Report what you observe. If possible, experiment with single precision and double precision.

(e) Explain the observed behavior of the generated sequences of iterates.