Scientific Computing HW 8

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1. To show that the algorithms are equivalent, we rewrite $\alpha_k, r_{k+1}, \beta_{k+1}$. Rewrite α_k as

$$\begin{aligned}
\alpha_k &= -\frac{r_k^T p_k}{p_k^T A p_k} \\
&= -\frac{r_k^T(-r_k + \beta_k p_{k-1})}{p_k^T A p_k} && p_k = -r_k + \beta_k p_{k-1} \\
&= \frac{r_k^T r_k}{p_k^T A p_k} && r_k^T p_{k-1} = 0 \text{ by Theorem 5.2 in [NW]}
\end{aligned}$$

Rewrite r_{k+1} as

$$\begin{aligned}
r_{k+1} &= Ax_{k+1} - b \\
&= A(x_k + \alpha_k p_k) - b \\
&= Ax_k - b + \alpha_k A p_k \\
&= r_k + \alpha_k A p_k
\end{aligned}$$

The expressions for α_k, r_{k+1} give

$$A p_k = \frac{r_{k+1} - r_k}{\alpha_k} = \frac{p_k^T A p_k (r_{k+1} - r_k)}{r_k^T r_k}$$

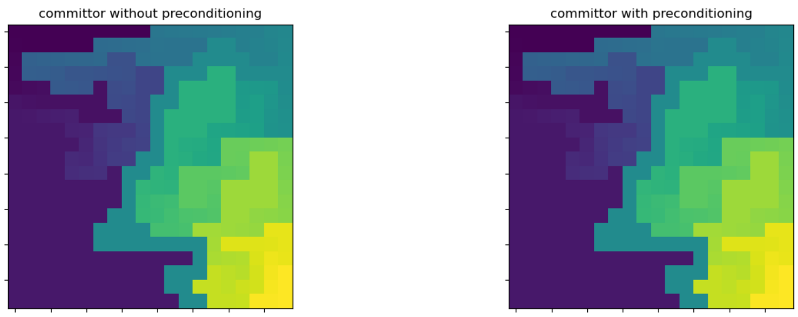
Use this to rewrite β_{k+1} as

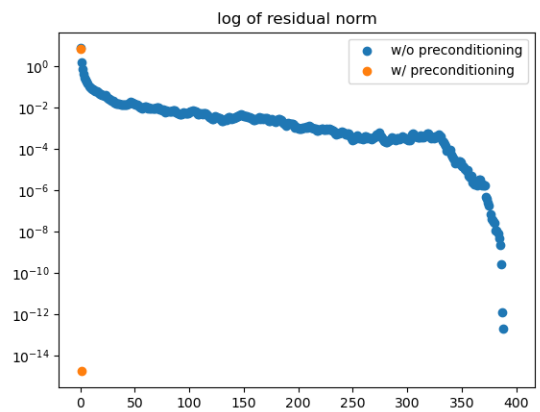
$$\begin{aligned}
\beta_{k+1} &= \frac{r_{k+1}^T A p_k}{p_k^T A p_k} \\
&= \frac{r_{k+1}^T (r_{k+1} - r_k)}{r_k^T r_k} \\
&= \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k} && r_{k+1}^T r_k = 0 \text{ by Theorem 5.3 in [NW]}
\end{aligned}$$

2. f

3. (a)
   (b)
   (c)
   (d)
   (e)
   (f)

4. Code: `https://github.com/RokettoJanpu/scientific-computing-1-redux/blob/main/hw8/hw8.ipynb`





CG with conditioning only took one iteration, resulting in only two plotted points.