Scientific Computing HW 7

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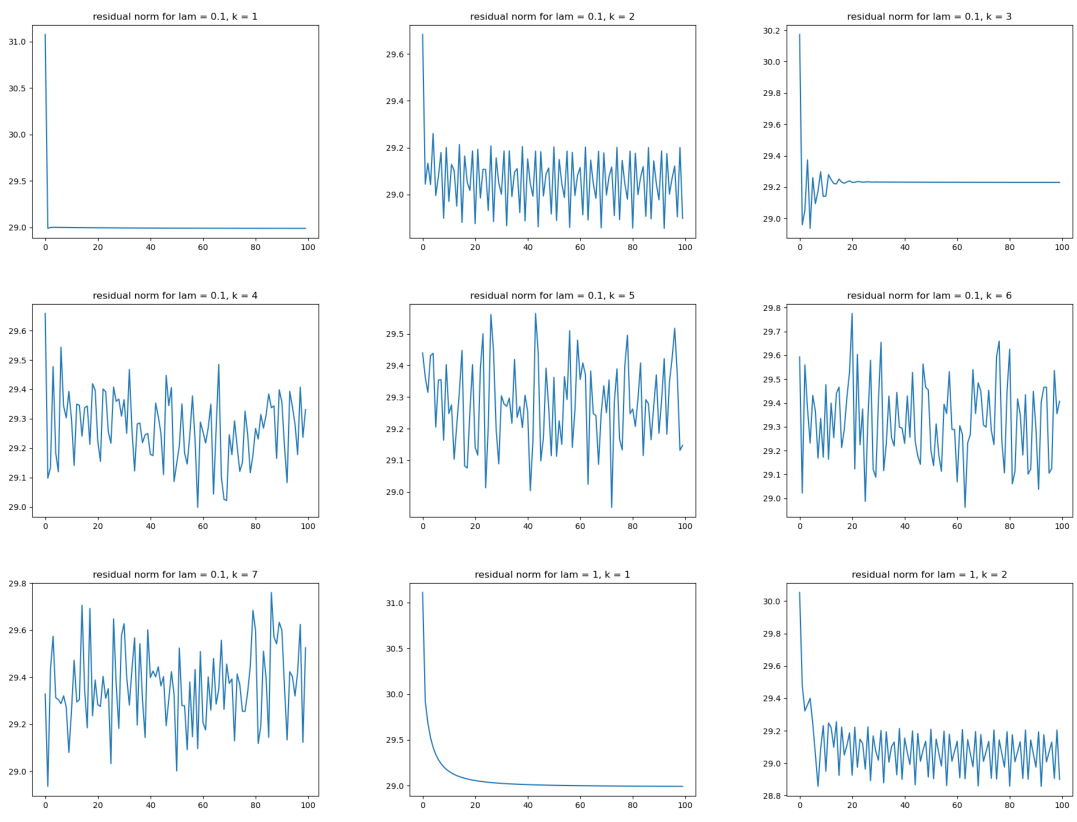
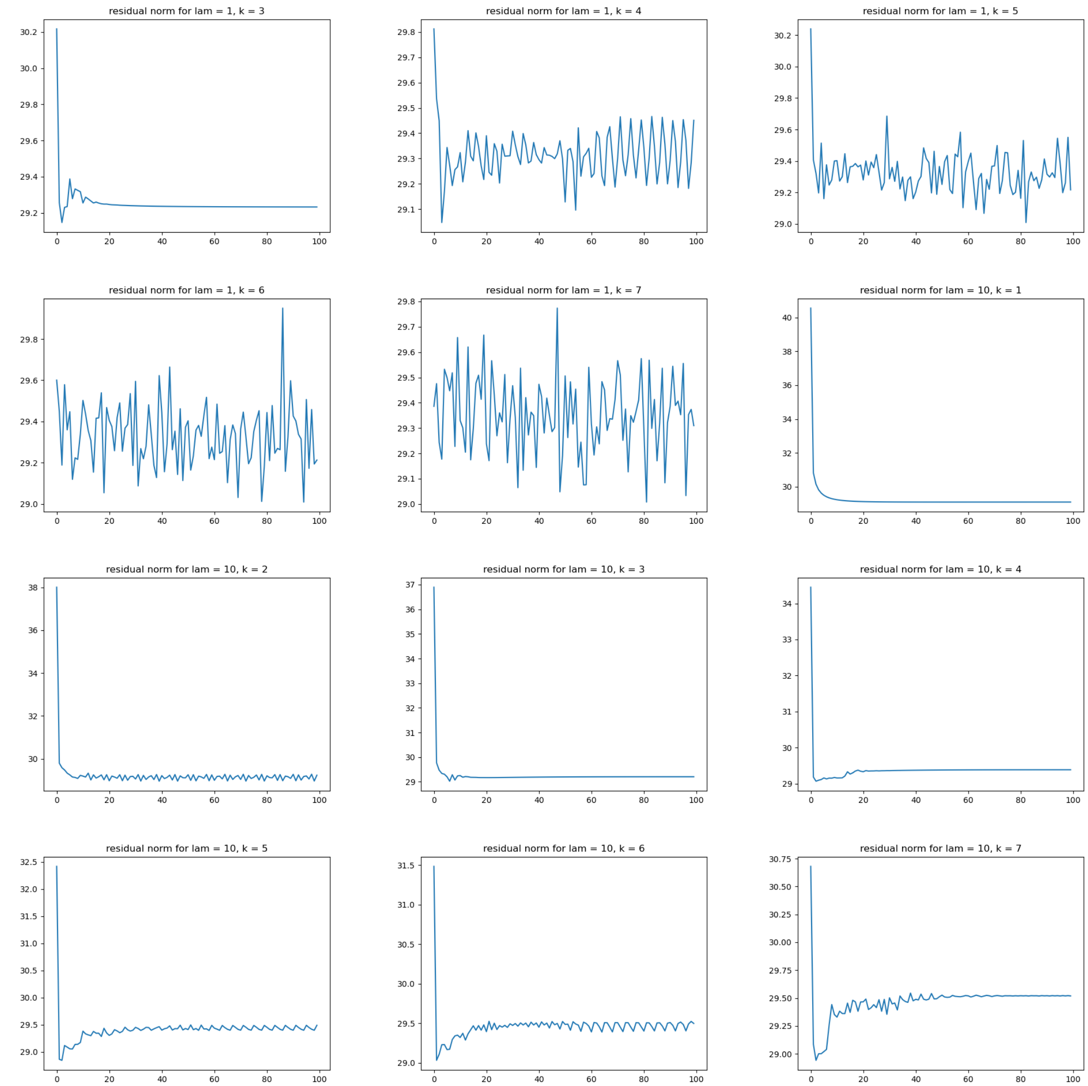
1. Code: `https://github.com/RokettoJanpu/scientific-computing-1-redux/blob/main/hw7p1.ipynb`

   (a) The low rank factorization is as follows. To update row i of X, find x_i^T as given in the lecture notes. To update column j of Y^T, i.e. row j of Y,

   $$y_j = \text{argmin}_y \left(\frac{1}{2} \| X_{\Omega^j} y - a_{\Omega^j} \|_2^2 + \frac{\lambda}{2} \|y\|_2^2 \right)$$

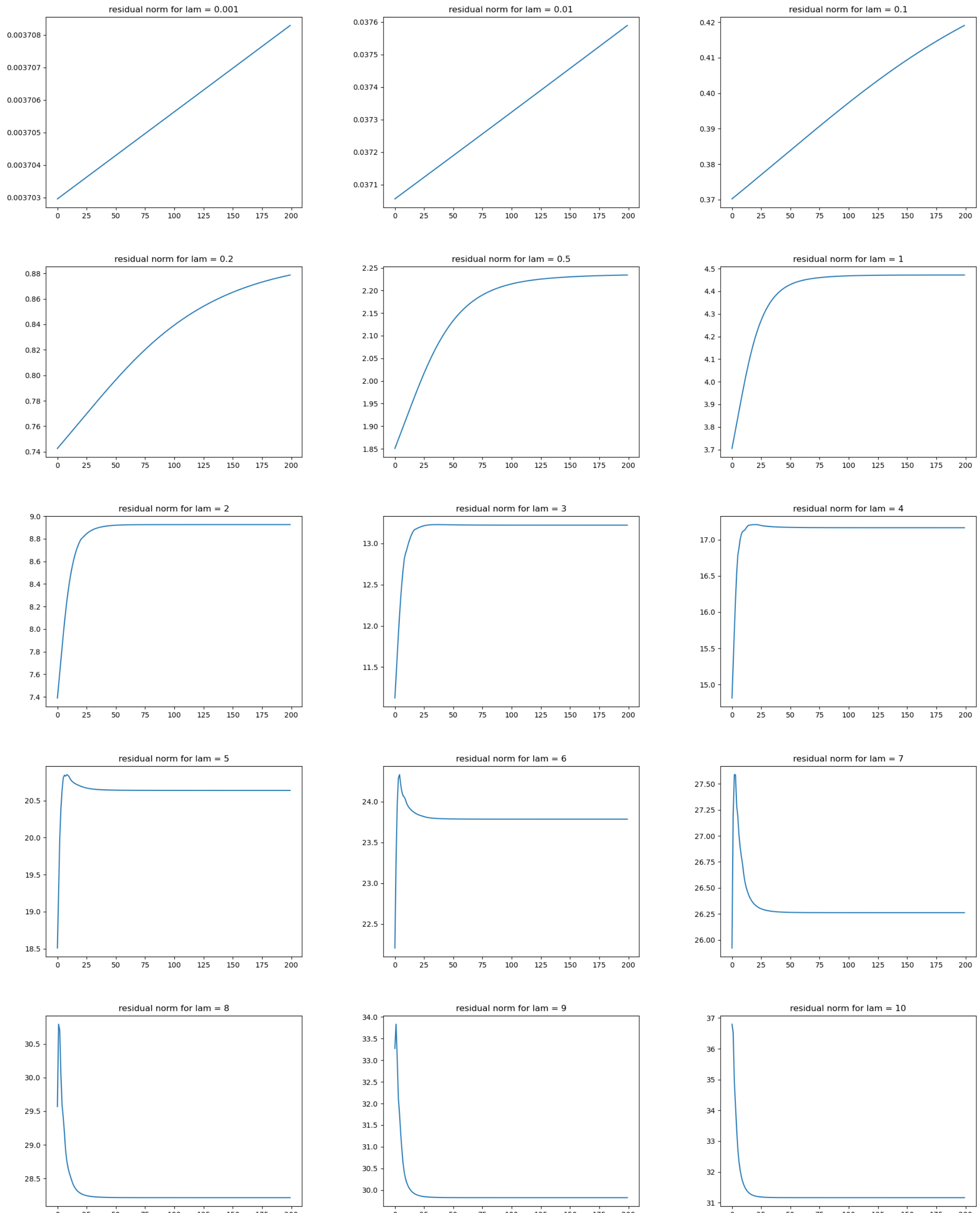
   where $\Omega^j := \{i : (i,j) \in \Omega\}$ and X_{Ω^j} is the set of rows of X with indices in Ω^j, and a_{Ω^j} is the set of known entries of A in column j.





We see that larger k results in larger fluctuations in the residual norm, while larger λ tends to dampen the fluctuations. The smallest average error between my ratings and the prediction is 0.39 which occurs at $\lambda = 0.1$ and $k = 6$, although there is lots of noise in the residual norm and this error is significant with respect to the rating system of 1–5.

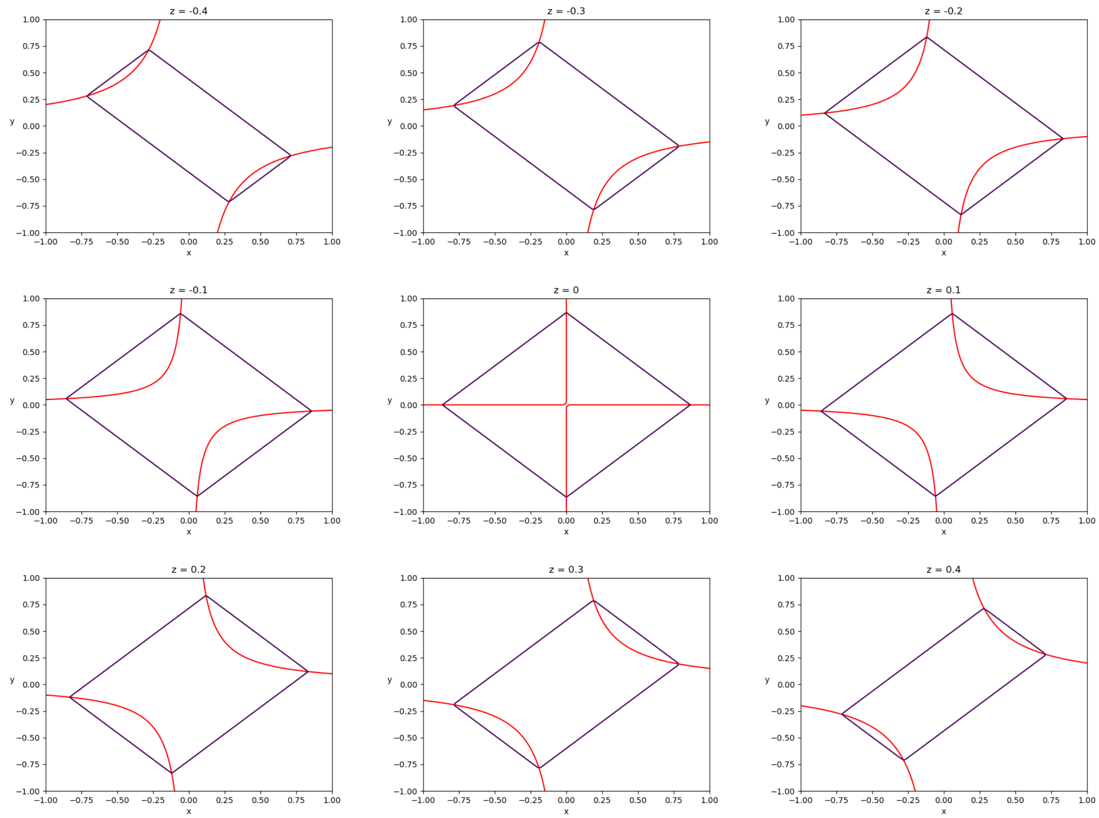
(b) Penalizing nuclear norm:



Penalizing the nuclear norm seems to give more sensible results, is easier to implement, and is faster than low rank factorization. For $\lambda = 0.5$, there is a much smaller average error of 0.04 between my ratings and the prediction.

2. Code: `https://github.com/RokettoJanpu/scientific-computing-1-redux/blob/main/hw7p2.ipynb`

   The level curve $\det A = 0$ is colored red, and the level curve $\|A\|_* = a$ is colored purple. The curves indeed intersect at the corners of $\|A\|_* = a$.



3. We will use the “direct” definition of linear independence. Let $c_0, \ldots, c_{n-1} \in \mathbb{R}$ satisfy

$$\sum_{k=0}^{n-1} c_k p_k = 0$$

Left multiply both sides by A.

$$\sum_{k=0}^{n-1} c_k A p_k = 0$$

Fix j and left multply both sides by p_j^T. Since $p_j^T A p_k = 0$ for all $k \neq j$,

$$c_j p_j^T A p_j = 0$$

Since A is SPD and $p_j \neq 0$, we have $p_j^T A p_j > 0$, hence $c_j = 0$.