

**Homework 4. Due February 29.**

**Please upload a single pdf file on ELMS. Link your codes to your pdf (i.e., put your codes to dropbox, Github, google drive, etc. and place links to them in your pdf file with your solutions.**

1. **(4 pts)** Solve exercise 16 in my lecture notes ODEsolvers.pdf.
2. **(4 pts)** Solve exercise 17 in my lecture notes ODEsolvers.pdf.
3. **(4 pts)** Solve exercise 18 in my lecture notes ODEsolvers.pdf.
4. **(4 pts)** Solve exercise 19 in my lecture notes ODEsolvers.pdf.
5. **(4 pts)**

- (a) Derive the following formula for the 2-step BDF method with a variable timestep:

$$u_{n+1} - \frac{(1+\omega)^2}{1+2\omega}u_n + \frac{\omega^2}{1+2\omega}u_{n-1} = h_n \frac{1+\omega}{1+2\omega}f(t_{n+1}, u_{n+1}), \quad (1)$$

where  $h_n := t_{n+1} - t_n$ ,  $\omega = h_n/h_{n-1}$ .

- (b) Prove that this method is stable provided that  $\omega_n < 1 + \sqrt{2}$ . *Hint: use the main theorem for methods with constant stepsize and **Vieta's formulas for quadratic equations**.*