english

លីមីតនៃអនុគមន៍

១ និយមន័យ

និយមន័យ ១. ដេរីវេនៃអនុគមន៍ $y = f(x)$ ត្រង់ x_0 កំណត់ដោយ

$$\frac{dy}{dx} = y' = f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

សំគាល់ ១. គេអាចសរសេរដេរីវេដោយ y' , $f'(x)$ ឬ $\dfrac{dy}{dx}$ ។

- អនុគមន៍ f មានដេរីវេត្រង់ x_0 នោះ f ជាប់ត្រង់ x_0 ។
- អនុគមន៍ f ជាប់ត្រង់ x_0 នោះ f អាចមានដេរីវេត្រង់ x_0 ឬ គ្មានដេរីវេត្រង់ x_0 ។

២ ភាពមានដេរីវេ

និយមន័យ ២. អនុគមន៍ f មានដេរីវេត្រង់ x_0 លុះត្រាតែ

- អនុគមន៍ f ជាប់ត្រង់ $x = 0$ ។
- ដេរីវេឆ្វេងស្មើដេរីវេស្តាំត្រង់ចំណុច x_0 គឺ $f'_-(x_0) = f'_+(x_0)$ ដែល
  $f'_-(x_0) = \lim\limits_{h \to 0^-} \dfrac{f(x_0 + h) - f(x_0)}{h}$ និង $f'_+(x_0) = \lim\limits_{h \to 0^+} \dfrac{f(x_0 + h) - f(x_0)}{h}$ ។

២.១ ជាប់លើចន្លោះបើក

និយមន័យ ៣. អនុគមន៍ f ជាប់លើចន្លោះបើក (a, b) កាលណា f មានដេរីវេលើគ្រប់ចំណុច $x_0 \in (a, b)$ ។

២.២ ជាប់លើចន្លោះបិទ

និយមន័យ ៤. អនុគមន៍ f មានដេរីវេលើចន្លោះបិទ $[a, b]$ កាលណា f មានដេរីវេលើចន្លោះ (a, b) ហើយ f មានដេរីវេខាងឆ្វេងត្រង់ $x = a$ និងខាងស្តាំត្រង់ $x = b$ ។

៣ លក្ខណៈនៃដេរីវេ

ចំពោះ u, v ជាអនុគមន៍នៃ x និង k ជាចំនួនថេរ នោះគេបាន៖

១. $(ku)' = ku'$

២. $(u+v)' = u' + v$

៣. $(u-v)' = u' - v'$

៤. $(uv)' = u'v + v'u$

៥. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

៦. $\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

សម្រាយបញ្ជាក់

១. តាង $f(x) = k.u(x)$ ដែល $u = u(x)$ និង k ជាចំនួនថេ តាមនិយមន័យ

$$
\begin{aligned}
f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h) - f(x_0)}{h}\\
&= \lim_{h\to 0}\frac{ku(x_0+h) - k.u(x_0)}{h}\\
&= k.\lim_{h\to 0}\frac{u(x_0+h) - u(x_0)}{h}\\
&= k.u'(x_0)
\end{aligned}
$$

$$\therefore \quad (k.u)' = k.u'$$

២. តាង $f(x) = u(x) + v(x)$ ដែល $u = u(x)$ និង $v = v(x)$ តាមនិយមន័យ

$$
\begin{aligned}
f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h) - f(x_0)}{h}\\
&= \lim_{h\to 0}\frac{u(x_0+h) + v(x_0+h) - (u(x_0) + v(x_0))}{h}\\
&= \lim_{h\to 0}\frac{u(x_0+h) - u(x_0)}{h} + \lim_{h\to 0}\frac{v(x_0+h) - v(x_0)}{h}\\
&= u'(x_0) + v'(x_0)
\end{aligned}
$$

$$\therefore \quad (u+v)' = u' + v'$$

៣. ស្រាយដូចទី២

៤. តាង $f(x) = uv$ ដែល $u = u(x)$ និង $v = v(x)$ តាមនិយមន័យគេបាន

$$
\begin{aligned}
f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}\\
&= \lim_{h\to 0}\frac{u(x_0+h).v(x_0+h)-u(x_0).v(x_0)}{h}\\
&= \lim_{h\to 0}\frac{u(x_0+h).v(x_0+h)-u(x_0).v(x_0+h)+u(x_0).v(x_0+h)+u(x_0).v(x_0)}{h}\\
&= \lim_{h\to 0}\left[\frac{u(x_0+h).v(x_0+h)-u(x_0).v(x_0+h)}{h}+\frac{u(x_0).v(x_0+h)+u(x_0).v(x_0)}{h}\right]\\
&= \lim_{h\to 0}\left[v(x_0+h).\frac{u(x_0+h)-u(x_0)}{h}+u(x_0).\frac{v(x_0+h)+v(x_0)}{h}\right]\\
&= v(x_0).\frac{d}{dx}(u(x_0))+u(x_0).\frac{d}{dx}(v(x_0))
\end{aligned}
$$

$$\therefore \quad (uv)' = u'v + v'u \tag{១}$$

៥. យក $u = u(x)$ និង $v = v(x)$ តាង $f(x) = \frac{u}{v} \Leftrightarrow f(x).v = u$ ធ្វើដេរីវេអង្គទាំងពីរធៀបនឹង x

នោះគេបាន $[f(x).v]' = u'$ ប្រើតាមសមីការ (១) គេបាន

$$
\begin{aligned}
f'(x).v + v'f(x) &= u',\ f(x) = \frac{u}{v}\\
f'(x).v + v'.\frac{u}{v} &= u'\\
\frac{f'(x).v^2}{v}+\frac{v'u}{v} &= u'\\
f'(x).v^2 + v'u &= u'v\\
f'(x) &= \frac{u'v - v'u}{v^2}
\end{aligned}
$$

$$\therefore \quad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \tag{២}$$

៦. យក $v = v(x)$ តាង $f(x) = \dfrac{1}{v}$ ប្រើសមីការ (២) គេបាន

$$\begin{aligned} f'(x) &= \frac{(1)'.v - v'.(1)}{v^2} \\ &= \frac{0 - v'}{v^2} \\ &= -\frac{v'}{v^2} \\ \therefore \quad \left(\frac{1}{v}\right)' &= -\frac{v'}{v^2} \end{aligned}$$

៤ ដេរីវេនៃអនុគមន៍បណ្តាក់

បើ $y = f(u)$ និង $u = g(x)$ នោះ $\dfrac{d}{dx}(f \circ g) = \dfrac{dy}{du} \times \dfrac{du}{dx}$ ។

សម្រាយបញ្ជាក់

តាង $F(x) = f \circ g = f(g(x))$ តាមនិយមន័យភាពមានដេរីវេត្រង់ $x = x_0$ នោះគេបាន

$$\begin{aligned} F'(x_0) &= \lim_{x \to x_0} \frac{F(x) - F(x_0)}{x - x_0} \\ &= \lim_{x \to x_0} \frac{f(g(x)) - f(g(x_0))}{x - x_0} \\ &= \lim_{x \to x_0} \left(\frac{f(g(x)) - f(g(x_0))}{g(x) - g(x_0)} \times \frac{g(x) - g(x_0)}{x - x_0} \right) \\ &= f'(g(x_0)) \times g'(x_0) \quad , u = g(x_0), y = f(x_0) \\ \therefore \quad \frac{d}{dx}(f \circ g) &= \frac{dy}{du} \times \frac{dy}{dx} \end{aligned}$$

ជាទូទៅ ៤.១. បើ $y = c$ ដែល c ជាចំនួនថេរ នោះ $y' = 0$ ។

សម្រាយបញ្ជាក់

គេមាន $y = f(x_0) = c$ នោះ $f(x_0+h) = c\ , c \in \mathbb{R}$ តាមនិយមន័យគេបាន

$$\begin{aligned} f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0}\frac{c-c}{h} \\ &= \lim_{h\to 0}\frac{0}{h} \\ \therefore\ \frac{d}{dx}(c) &= 0 \end{aligned}$$

ឧទាហរណ៍ ១. គណនា y' ដែល $y = \left(\ln x.\log_a(\sqrt{3})\right)$ ។

ដំណោះស្រាយ

គេមាន $y = \left(\ln x.\log_a(\sqrt{3})\right) \Rightarrow y' = \left(\ln x.\log_a(\sqrt{3})\right)' = 0$

ឧទាហរណ៍ ២. ស្រាយបញ្ជាក់ថា បើ $y = x^n$ នោះ $y' = nx^{n-1}$ ។

សម្រាយបញ្ជាក់

គេមាន $f(x_0) = x_0^n$ នាំឲ្យ $f(x_0+h) = (x_0+h)^n$ តាមនិយមន័យ

$$\begin{aligned} y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0}\frac{(x_0+h)^n - x_0^n}{h} \\ &= \lim_{h\to 0}\frac{(x_0+h-x_0)(x_0^{n-1}+x^{n-2}.x_0+\ldots+x_0.x^{n-2}x_0+x_0^{n-1})}{h} \\ &= \lim_{h\to 0}(x_0^{n-1}+x_0^{n-1}+\ldots+x_0^{n-1}+x_0^{n+1}) \\ &= x_0^{n-1}(\underbrace{1+1+\ldots+1+1}_{n\text{ តួលេខ }1}) \\ &= n.x_0^{n-1} \\ \therefore\ \frac{d}{dx}(x^n) &= n.x^{n-1} \end{aligned}$$

ឧទាហរណ៍ ៣. គណនា $f'(x)$

១. $f(x) = x^3$ ២. $f(x) = \sqrt{x}$ ៣. $f(x) = \sqrt[3]{x^2}$

ដំណោះស្រាយ

១. $f(x)=x^3 \Rightarrow f'(x)=(x^3)'=3x^{3-1}=3x^2$

២. $f(x)=\sqrt{x} \Rightarrow f'(x)=(\sqrt{x})'=(x^{\frac{1}{2}})'=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

៣. $f(x)=\sqrt[3]{x^2} \Rightarrow f'(x)=(\sqrt[3]{x^2})'=(x^{\frac{2}{3}})'=\frac{2}{3}x^{\frac{2}{3}-1}=\frac{2}{3}x^{-\frac{1}{3}}=\frac{2}{3\sqrt[3]{x}}$

ជាទូទៅ ៤.២. បើ $y=u^n$ ដែល u ជាអនុគមន៍នៃ x នោះ $y'=nu'u^{n-1}$ ។

សម្រាយបញ្ជាក់

គេមាន $y=u^n$ គេបាន $y'=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(u^n)\times u'=nu'u^{n-1}$

ឧទាហរណ៍ ៤. គណនា y'

១. $y=(2x+\ln 2)^4$

២. $y=\sqrt{u}$ ដែល u ជាអនុគមន៍នៃ x ។

ដំណោះស្រាយ

១. $y=(2x+\ln 2)^4 \Rightarrow y'=4(2x+\ln 2)'(2x+\ln 2)^{4-1}=4(2+0)(2x+\ln 2)^3=8(2x+\ln 2)^3$

២. $y=\sqrt{u}=u^{\frac{1}{x}}$ នាំឲ្យ $y'=(u^{\frac{1}{2}})'=\frac{1}{2}u'u^{\frac{1}{2}-1}=\frac{1}{2}u'u^{-\frac{1}{2}}=\frac{u'}{2\sqrt{u}}$ ។

៥ ដេរីវេនៃអនុគមន៍ត្រីកោណមាត្រ

១. បើ $y=\sin x$ នោះ $y'=\cos x$

២. បើ $y=\cos x$ នោះ $y'=-\sin x$

៣. បើ $y=\tan x$ នោះ $y'=\frac{1}{\cos^2 x}=1+\tan^2 x$

៤. បើ $y=\cot x$ នោះ $y'=-\frac{1}{\sin^2 x}=-(1+\cot^2 x)$

សម្រាយបញ្ជាក់

១. គេមាន $y = f(x_0) = \sin x_0$ នោះ $f(x_0+h) = \sin(x_0+h)$ តាមនិយមន័យ

$$
\begin{aligned}
y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\
&= \lim_{h\to 0}\frac{\sin(x_0+h)-\sin x_0}{h} \\
&= \lim_{h\to 0}\frac{\sin x_0\cos h+\sin h.\cos x_0-\sin x_0}{h} \\
&= \lim_{h\to 0}\left(\cos x_0.\frac{\sin h}{h}-\sin x_0.\frac{1-\cos h}{h}\right) \\
&= \cos x_0 \quad , \lim_{h\to 0}\frac{\sin h}{h} = 1, \lim_{h\to 0}\frac{1-\cos h}{h} = 0
\end{aligned}
$$

$$\therefore \quad \frac{d}{dx}(\sin x) = \cos x$$

២. គេមាន $y = f(x_0) = \cos x_0$ នោះ $f(x_0+h) = \cos(x_0+h)$ តាមនិយមន័យ

$$
\begin{aligned}
y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\
&= \lim_{h\to 0}\frac{\cos(x_0+h)-\cos x_0}{h} \\
&= \lim_{h\to 0}\frac{\cos x_0.\cos h-\sin x_0.\sin h-\cos x_0}{h} \\
&= \lim_{h\to 0}\left(-\frac{\sin h}{h}.\sin x_0-\cos x_0.\frac{1-\cos h}{h}\right) \\
&= -\sin x_0 \quad , \lim_{h\to 0}\frac{1-\cos h}{h} = 0, \lim_{h\to 0}\frac{\sin h}{h} = 1
\end{aligned}
$$

$$\therefore \quad \frac{d}{dx}(\cos x) = -\sin x$$

៣. តាង $y = \tan x = \dfrac{\sin x}{\cos x}$ តាមសមីការ (២) គេបាន

$$
\begin{aligned}
y' = \left(\frac{\sin x}{\cos x}\right)' &= \frac{(\sin x)'\cos x-(\cos x)'.\sin x}{(\cos x)^2} \\
&= \frac{\cos x.\cos x-(-\sin x).\sin x}{\cos^2 x} \\
&= \frac{\cos^2 x+\sin^2 x}{\cos^2 x} \\
&= 1+\tan^2 x \\
&= \frac{1}{\cos^2 x}, \ \sin^2 x+\cos^2 x = 1
\end{aligned}
$$

$$\therefore \quad (\tan x)' = \frac{1}{\cos^2} = 1+\tan^2 x$$

៤. តាង $y = \cot x = \dfrac{\cos x}{\sin x}$ តាមសមីការ (២) គេបាន

$$
\begin{aligned}
y' = \left(\frac{\cos x}{\sin x}\right)' &= \frac{(\cos x)'.\sin x - (\sin x)'.\cos x}{(\sin^2 x)^2} \\
&= \frac{-\sin x.\sin x - \cos x.\cos x}{\sin^2 x} \\
&= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\
&= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}, \sin^2 x + \cos^2 x = 1 \\
\therefore \quad (\cot x)' &= -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)
\end{aligned}
$$

ជាទូទៅ ៥.១. បើ u ជាអនុគមន៍នៃ x គេបាន

១. បើ $y = \sin u$ នោះ $y' = u' \cos u$

២. បើ $y = \cos u$ នោះ $y' = -u' \sin u$

៣. បើ $y = \tan u$ នោះ $y' = \dfrac{u'}{\cos^2 u} = u(1 + \tan^2 u)$

៤. បើ $y = \cot u$ នោះ $y' = -\dfrac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)$

សម្រាយបញ្ជាក់

១. បើ u ជាអនុគមន៍នៃ x នោះ $y = \sin u$ គេបាន

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\sin u) \times \frac{du}{dx} = \cos u \times u' = u' \cos u \\
\therefore \quad \frac{d}{dx}(\sin u) &= u' \cos u
\end{aligned}
$$

២. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cos u$ គេបាន

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cos u) \times \frac{du}{dx} = -\sin u \times u' = -u' \sin u \\
\therefore \quad \frac{d}{dx}(\cos u) &= -u' \sin u
\end{aligned}
$$

៣. បើ u ជាអនុគមន៍នៃ x នោះ $y = \tan u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\tan u) \times \frac{du}{dx} = \frac{1}{\cos^2 u} \times u' = (1 + \tan^2 u) \times u'$$

$$\therefore \quad (\tan u)' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$$

៤. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cot u$ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cot u) \times \frac{du}{dx} = -\frac{1}{\sin^2 u} \times u' = -(1 + \cot^2 u) \times u'$$

$$\therefore \quad (\cot u)' = -\frac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)$$

ឧទាហរណ៍ ៨. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $y = \sin(2x + 1)$

២. $y = \cos(2x + 1)$

៣. $y = \tan(2x + 1)$

៤. $y = \cot(2x + 1)$ ។

ដំណោះស្រាយ

១. $y = \sin(2x + 1) \Rightarrow y' = (2x + 1)' \cos(2x + 1) = 2\cos(2x + 1)$

២. $y = \cos(2x + 1) \Rightarrow y' = -(2x + 1)' \sin(2x + 1) = -2\sin(2x + 1)$

៣. $y = \tan(2x + 1) \Rightarrow y' = \dfrac{(2x + 1)'}{\cos^2(2x + 1)} = \dfrac{2}{\cos^2(2x + 1)} = 2[1 + \tan^2(2x + 1)]$

៤. $y = \cot(2x + 1) \Rightarrow y' = -\dfrac{(2x + 1)'}{\sin^2(2x + 1)} = -\dfrac{2}{\sin^2(2x + 1)} = -2[1 + \cot^2(2x + 1)]$ ។

៦ ដេរីវេអនុគមន៍អិចស្ប៉ូណង់ស្យែល

ស្រាយថាបើ $y = a^x$ នោះ $y' = a^x . \ln a$

សម្រាយបញ្ជាក់

គេមាន $y = a^x$ តាមនិយមន័យ គេបាន

$$\begin{aligned} y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0}\frac{a^{x_0+h}-a^{x_0}}{h} \\ &= \lim_{h\to 0}\frac{a^{x_0}(a^h-1)}{h} \\ &= a^{x_0}\lim_{h\to 0}\frac{a^h-1}{h} \text{ ដោយ } \lim_{x\to 0}\frac{a^x-1}{x} = \ln a \\ &= a^{x_0}.\ln a \end{aligned}$$

$$\therefore \quad (a^x)' = a^x.\ln a$$

ជាទូទៅ ៦.១. បើ u ជាអនុគមន៍នៃ x នោះ $(a^u)' = u'a^u.\ln a$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(a^u)\times\frac{du}{dx} = a^u.\ln a\times u'$$

$$\therefore \quad (a^u)' = u'.a^u.\ln a)$$

ឧទាហរណ៍ ៦. គណនា y' ចំពោះ u ជាអនុគមន៍នៃ x នៃអនុគមន៍ខាងក្រោម៖

១. $y = e^x$ ២. $y = a^{x^2-1}$ ៣. $y = e^u$

ដំណោះស្រាយ

១. $y = e^x$ នោះ $y' = (e^x)' = e^x.\ln e = e^x$, $\ln e = 1$

២. $y = a^{x^2-1}$ នោះ $y' = (x^{x^2-1})'a^{x^2-1}\ln a = 2x.a^{x^2-1}\ln a$

៣. $y = e^u$ នោះ $y' = (e^u)' = u'e^u.\ln e = u'e^u$, $\ln e = 1$

៧ ដេរីវេនៃអនុគមន៍លោការីត

ស្រាយបញ្ជាក់ថា បើ $y = \log_a x$, $a > 0, a \neq 1$ នោះ $y' = \dfrac{1}{x\ln a}$ ។

សម្រាយបញ្ជាក់

គេមាន $y = \log_a x$ តាមនិយមន័យ គេបាន

$$\begin{aligned} y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h} \\ &= \lim_{h\to 0}\frac{\log_a(x_0+h)-\log_a(x_0)}{h} \\ &= \lim_{h\to 0}\frac{1}{h}\log_a\left(\frac{x_0+h}{x_0}\right) \\ &= \lim_{h\to 0}\log_a\left(1+\frac{h}{x_0}\right)^{\frac{1}{h}} \\ &= \log_a\left(\lim_{h\to 0}\ln\left(1+\frac{h}{x_0}\right)^{\frac{1}{h}}\right)^{\frac{1}{x_0}} \\ &= \log_a e^{\frac{1}{x_0}} = \frac{1}{x_0}\frac{\ln e}{\ln a} \end{aligned}$$
$$\therefore\quad (\log_a x)' = \frac{1}{x\ln a}, a > 0, a \neq 1$$

ជាទូទៅ ៧.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\log_a u)' = \dfrac{u'}{u\ln a}, a > 0, a \neq 1$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\log_a u)\times\frac{du}{dx} = \frac{1}{u\ln a}\times u'$$
$$\therefore\quad (\log_a u)' = \frac{u'}{u\ln a}, a > 0, a \neq 1$$

៨ ដេរីវេនៃអនុគមន៍លោការីតនេពែ

ស្រាយបញ្ជាក់ថា បើ $y = \ln x$ នោះ $y' = \dfrac{1}{x}$ ។

សម្រាយបញ្ជាក់

គេមាន $y=\ln x$ តាមនិយមន័យ គេបាន

$$\begin{aligned} y' = f'(x_0) &= \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}\\ &= \lim_{h\to 0}\frac{\ln(x_0+h)-\ln(x_0)}{h}\\ &= \lim_{h\to 0}\frac{\ln\left(\dfrac{x_0+h}{x_0}\right)}{h}\\ &= \lim_{h\to 0}\frac{1}{h}\ln\left(1+\frac{h}{x_0}\right)\\ &= \lim_{h\to 0}\ln\left(1+\frac{h}{x_0}\right)^{\dfrac{1}{h}}\\ &= \ln\left[\lim_{h\to 0}\left(1+\frac{1}{\frac{x_0}{h}}\right)^{\frac{x_0}{h}\times\frac{1}{x_0}}\right], \lim_{h\to 0}\left(1+\frac{1}{\frac{x_0}{h}}\right)^{\frac{x_0}{h}} = e\\ &= \ln e^{\dfrac{1}{x_0}}, \ln e = 1 \end{aligned}$$

$$\therefore\quad (\ln x)' = \frac{1}{x}$$

ឧទាហរណ៍ ៧. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x)=x^2.\log_a x, a>0, a\neq 1$

២. $f(x)=\sin(2x)+\log_2(x^2+1)$

៣. $f(x)=\dfrac{e^{2x}+\log_3 x}{x^2}$

៤. $f(x)=\log(x^2\sqrt{x^3-1})$

៥. $f(x)=(\sin x)^{\log x}$

៦. $f(x)=(\log_a x)^{\ln(2x)}, a>0, a\neq 1$

សម្រាយបញ្ជាក់

១. $f(x)=x^2.\log_a x, a>0, a\neq 1 \Longrightarrow f'(x)=(x^2)'\log_a x+\left(\log_a x\right)' x^2$

$$= 2x\log_a x+\frac{1}{x\ln a}x^2$$

$$\therefore\quad f'(x)=2x\log_a x+\frac{x}{\ln a}, a>0, a\neq 1$$

២. $f(x)=\sin(2x)+\log_2(x^2+1) \Longrightarrow f'(x)=-(2x)'\cos(2x)+\dfrac{(x^2+1)'}{(x^2+1)\ln 2}$

$$\therefore\quad f'(x)=-2\cos(2x)+\frac{2x}{(x^2+1)\ln 2}$$

៣. $f(x)=\dfrac{e^{2x}+\log_3 x}{x^2}\Longrightarrow f'(x)=\dfrac{(e^{2x}+\log_3 x)'x^2-(x^2)'(e^{2x}+\log_3 x)}{x^4}$

$$=\frac{(2e^{2x}+\frac{1}{x\ln 3})x^2-2x(e^{2x}+\log_3 x)}{x^4}$$
$$=\frac{2xe^{2x}+\frac{1}{\ln 3}-2e^{2x}-2\log_3 x}{x^3}$$
$$\therefore\quad f'(x)=\frac{2e^{2x}(x-1)+\frac{1}{\ln 3}-\log_3 x^2}{x^3}$$

៤. $f(x)=\log(x^2\sqrt{x^3-1})=\log x^2+\log(x^3-1)^{\frac12}=2\log x+\dfrac12\log(x^3-1)$

$\therefore\quad f'(x)=\dfrac{2}{x\ln 10}+\dfrac{(x^3-1)'}{2(x^3-1)\ln 10}=\dfrac{2}{x\ln 10}+\dfrac{3x^2}{2(x^3-1)\ln 10}$

៥. $f(x)=(\sin x)^{\log x}\Longleftrightarrow \ln f(x)=\ln(\sin x)^{\log x}$

$$(\ln f(x))'=(\log x.\ln(\sin x))'$$
$$\frac{f'(x)}{f(x)}=(\log x)'\ln(\sin x)+(\ln(\sin x))'\log x$$
$$f'(x)=f(x)\left(\frac{1}{x\ln 10}\ln(\sin x)+\frac{(\sin x)'}{\sin x}.\log x\right)$$
$$\therefore\quad f'(x)=(\sin x)^{\log x}\left(\frac{\ln(\sin x)}{x\ln 10}+\cot x.\log x\right)$$

៦. $f(x)=(\log_a x)^{\ln(2x)}, a>0, a\neq 1\Longleftrightarrow \ln f(x)=\ln(\log_a x)^{\ln(2x)}$

$$(\ln f(x))'=\left(\ln(2x).\ln(\log_a x)\right)'$$
$$\frac{f'(x)}{f(x)}=(\ln(2x))'\ln(\log_a x)+(\ln(\log_a x))'\ln(2x)$$
$$f'(x)=f(x)\left(\frac{(2x)'}{2x}\ln(\log_a x)+\frac{(\log_a x)'}{\log_a x}\ln(2x)\right)$$
$$\therefore\quad f'(x)=(\log_a x)^{\ln(2x)}\left(\frac{\ln(\log_a x)}{x}+\frac{\ln(2x)}{x\ln a\log_a x}\right), a>0, a\neq 1$$

ជាទូទៅ ៨.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\ln u)'=\dfrac{u'}{u}$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\ln u) \times \frac{du}{dx} = \frac{1}{u} \times u'$$
$$\therefore \quad (\ln u)' = \frac{u'}{u}$$

ឧទាហរណ៍ ៨. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x.\ln x$

២. $f(x) = x^2 + \ln(x^2+1)$

៣. $f(x) = \dfrac{e^x + \ln x}{x^2}$

៤. $f(x) = \ln(x^2\sqrt{x^3-1})$

៥. $f(x) = x^x$

៦. $f(x) = (\sin x)^{\cos x}$

សម្រាយបញ្ជាក់

១. $f(x) = x.\ln x \Longrightarrow f'(x) = x'\ln x + (\ln x)'x = \ln x + \dfrac{1}{x}.x = \ln x + 1$

២. $f(x) = x^2 + \ln(x^2+1) \Longrightarrow f'(x) = (x^2)' + \dfrac{(x^2+1)'}{x^2+1} = 2x + \dfrac{2x}{x^2+1}$

៣. $f(x) = \dfrac{e^x + \ln x}{x^2} \Longrightarrow f'(x) = \dfrac{(e^x+\ln x)'x^2 - (x^2)'(e^x+\ln x)}{(x^2)^2}$

$$= \frac{\left(e^x + \frac{1}{x}\right)x^2 - 2x(e^x+\ln x)}{x^4}$$
$$\therefore \quad f'(x) = \frac{xe^x + 1 - 2e^x - 2\ln x}{x^3}$$

៤. $f(x) = \ln(x^2\sqrt{x^3-1}) = \ln x^2 + \ln\sqrt{x^3-1} = 2\ln x + \ln(x^3-1)^{\frac{1}{2}}$

$$f'(x) = 2(\ln x)' + \frac{1}{2}[\ln(x^3-1)]'$$
$$= 2.\frac{1}{x} + \frac{1}{2}.\frac{(x^3-1)'}{x^3-1}$$
$$\therefore \quad f'(x) = \frac{2}{x} + \frac{3}{2}.\frac{x^2}{x^3-1}$$

៥. $f(x) = x^x \Longleftrightarrow \ln f(x) = \ln x^x = x\ln x$

$$\begin{aligned}(\ln f(x))' &= (x\ln x)'\\ \frac{f'(x)}{f(x)} &= x'\ln x + (\ln x)'x\\ f'(x) &= f(x)(\ln x + \frac{1}{x}.x)\\ \therefore\quad f'(x) &= x^x(\ln x + 1)\end{aligned}$$

៦. $f(x) = (\sin x)^{\cos x} \Longleftrightarrow \ln f(x) = \ln(\sin x)^{\cos x}$

$$\begin{aligned}(\ln f(x))' &= (\cos x\ln\sin x)'\\ \frac{f'(x)}{f(x)} &= (\cos x)'\ln\sin x + (\ln\sin x)'\cos x\\ f'(x) &= f(x)\left(-\sin x\ln\sin x + \frac{(\sin x)'}{\sin x}.\cos x\right)\\ \therefore\quad f'(x) &= (\sin x)^{\cos x}(\cos x\cot x - \sin x\ln\sin x)\end{aligned}$$

៩ ដេរីវេនៃអនុគមន៍ Arc Sine និង Arc Tangent

$$\begin{aligned}y = \arcsin x &\Longleftrightarrow x = \sin y \text{ និង } -\frac{\pi}{2} \le y \le \frac{\pi}{2},\\ y = \arctan x &\Longleftrightarrow x = \tan y \text{ និង } -\frac{\pi}{2} \le y \le \frac{\pi}{2},\end{aligned}$$

បើ $y = \arcsin x$ នោះ $y' = \dfrac{1}{\sqrt{1-x^2}}$ ។

សម្រាយបញ្ជាក់

បើ $y = \arcsin x$ នោះ $x = \sin y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$\begin{aligned}(x)' = (\sin y)' &\Longleftrightarrow 1 = y'\cos y\\ y' = \frac{1}{\cos y}&, \sin^2 y + \cos^2 y = 1 \Longrightarrow \cos y = \pm\sqrt{1-\sin^2 x}\end{aligned}$$

ដោយ $-\frac{\pi}{2} \le y\frac{\pi}{2} \Longrightarrow \cos y \ge 0 \Longrightarrow \cos y = \sqrt{1-x^2}$

$$\therefore\quad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

បើ $y = \arctan x$ នោះ $y' = \dfrac{1}{1+x^2}$ ។

សម្រាយបញ្ជាក់

បើ $y = \arctan x$ នោះ $x = \tan y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$\begin{aligned} (x)' &= (\sin y)' \iff 1 = y'(1+\tan^2 y) \\ y' &= \frac{1}{1+\tan^2 y} \\ \therefore \quad (\arctan x)' &= \frac{1}{1+x^2} \end{aligned}$$

ជាទូទៅ ៩.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\arcsin u)' = \dfrac{u'}{\sqrt{1-x^2}}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\arcsin u)\times\frac{du}{dx} = \frac{1}{\sqrt{1-u^2}}\times u' \\ \therefore \quad (\arcsin u)' &= \frac{u'}{\sqrt{1-u^2}} \end{aligned}$$

ជាទូទៅ ៩.២. បើ u ជាអនុគមន៍នៃ x នោះ $(\arctan u)' = \dfrac{u'}{1+u^2}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\arctan u)\times\frac{du}{dx} = \frac{1}{1-u^2}\times u' \\ \therefore \quad (\arctan u)' &= \frac{u'}{1+u^2} \end{aligned}$$

លំហាត់ ១. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = \arcsin x . \sin x$

២. $f(x) = \arctan x \cos x$

៣. $f(x) = \sin(\arcsin x)$

៤. $f(x) = \arctan(\tan x)$

សម្រាយបញ្ជាក់

១. $f(x) = \arcsin x . \sin x \Longrightarrow f'(x) = (\arcsin x)' \sin x + (\sin x)' \arcsin x$

$$\therefore \quad f'(x) = \frac{\sin x}{\sqrt{1-x^2}} + \cos x . \arcsin x$$

២. $f(x) = \arctan x \cos x \Longrightarrow f'(x) = (\arctan x)' \cos x + (\cos x)' \arctan$

$$\therefore \quad f'(x) = \frac{\cos x}{1+x^2} - \sin x . \arctan x$$

៣. $f(x) = \sin(\arcsin x) \Longrightarrow f'(x) = (\arcsin x)' \cos(\arcsin x)$

$$\therefore \quad f'(x) = \frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$

៤. $f(x) = \arctan(\tan x) \Longrightarrow f'(x) = \dfrac{(\tan x)'}{1+(\tan x)^2} = \dfrac{1+\tan^2 x}{1+\tan^2 x}$

$$\therefore \quad f'(x) = 1$$

លក្ខណៈនៃដេរីវេ

បើ f, g, y, u, v ជាអនុគមន៍នៃ x និង k ជាចំនួនថេរនោះគេបាន៖

១. $(u \pm v)' = u' \pm v'$

២. $(ku)' = ku'$

៣. $(uv)' = u'v + v'u$

៤. $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$

៥. $\dfrac{dy}{dx} = \dfrac{dy}{du}.\dfrac{du}{dx}$

៦. $\left(\dfrac{1}{v}\right)' = -\dfrac{v'}{v^2}$

រូបមន្តនៃដេរីវេ

បើ C, a, b, c ជាចំនួនថេរ និង u ជាអនុគមន៍នៃ x ដែល $n \in \mathbb{N}$ គេបាន៖

១. $(C)' = 0$

២. $(x)' = 1$

៣. $(ax + b)' = a$

៤. $(ax^2 + bx + c)' = 2ax + b$

៥. $(x^n)' = nx^{n-1}$

៦. $(u^n)' = n.u'.u^{n-1}$

៧. $(x)^{-n} = -\dfrac{n}{x^{n+1}}$

៨. $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$

៩. $\left(\dfrac{1}{u}\right)' = -\dfrac{u}{u^2}$

១០. $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$

១១. $(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

១២. $(\sqrt[n]{x})' = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$

១៣. $(\ln x)' = \dfrac{1}{x}$

១៤. $(\ln u)' = \dfrac{u'}{u}$

១៥. $(\log_a x)' = \dfrac{1}{x \ln a}, a > 0, a \neq 1$

១៦. $(\log_a u)' = \dfrac{u'}{u. \ln a}, a > 0, a \neq 1$

១៧. $(a^x)' = a^x \ln a, a > 0, a \neq 1$

១៨. $(a^u)' = u'a^u \ln a, a > 0, a \neq 1$

១៩. $(e^x)' = e^x$

២០. $(e^u)' = u'e^u$

២១. $(\sin x)' = \cos x$

២២. $(\sin u)' = u' \cos u$

២៣. $(\cos x)' = -\sin x$

២៤. $(\cos u)' = -u' \sin u$

២៥. $(\tan x)' = \dfrac{1}{\cos^2 x} = 1 + \tan^2 x$

២៦. $(\tan u)' = \dfrac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$

២៧. $(\cot x)' = -\dfrac{1}{\sin^2 x} = -(1 + \cot^2 x)$

២៨. $(\cot u)' = -\dfrac{u'}{\sin^2 u} = -(1 + \cot^2 u)$

២៩. $(\arcsin x)' = \dfrac{1}{\sqrt{1 - x^2}}$

៣០. $(\arcsin u)' = \dfrac{u'}{\sqrt{1 - u^2}}$

៣១. $(\arccos x)' = -\dfrac{1}{\sqrt{1 - x^2}}$

៣២. $(\arccos u)' = -\dfrac{u'}{\sqrt{1 - u^2}}$

៣៣. $(\arctan x)' = \dfrac{1}{1 + x^2}$

៣៤. $(\arctan u)' = \dfrac{u'}{1 + u^2}$

៣៥. $(\text{arccot}\, x)' = -\dfrac{1}{1 + x^2}$

៣៦. $(\text{arccot}\, u)' = -\dfrac{u'}{1 + u^2}$

៣៧. $(u^v)' = \left(v'. \ln u + \dfrac{v.u'}{u}\right).u^v$

១០ លំហាត់ និង ដំណោះស្រាយ

លំហាត់ ២. គណនា $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$

២. $f(x) = 2x^2 - \sqrt{x} + \dfrac{2}{x}$

៣. $f(x) = (x^4 - 7x^2 + \sin a)^7$

៤. $f(x) = (x^2 - \sqrt{x})^{2019}$

៥. $f(x) = \sqrt{x^3 - x^2 + 3}$

៦. $\sqrt[4]{x^3 - 2x}$

៧. $f(x) = (x+1)(2x-1)^2$

៨. $f(x) = (x^2 + 2x + 3)(x^3 - 3x - 1)$

៩. $f(x) = \dfrac{1}{x-1}$

១០. $f(x) = \dfrac{x\sqrt{x}}{x+1}$

សម្រាយបញ្ជាក់

១. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1 \Longrightarrow f'(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$

២. $f(x) = 2x^2 - \sqrt{x} - \dfrac{2}{x} \Longrightarrow f'(x) = 4x^2 + \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^2}$

$$f'(x) = 7(x^4 - 7x^2 + \sin a)'(x^4 - 7x^2 + \sin a)^{7-1} = 7(4x^3 - 14x)(x^4 - 7x^2 + \sin a)^6$$

៣. $f(x) = (x^4 - 7x^2 + \sin a)^7$

៤. $f(x) = (x^2 - \sqrt{x})^{2019} \Longrightarrow f'(x) = 2019(x^2 - \sqrt{x})'(x^2 - \sqrt{x})^{2019-1}$

$$= 2019\left(2x - \frac{1}{2\sqrt{x}}\right)(x^2 - \sqrt{x})^{2018}$$

៥. $f(x) = \sqrt{x^3 - x^2 + 3} \Longrightarrow f'(x) = \dfrac{(x^3 - x^2 + 3)'}{2\sqrt{x^3 - x^2 + 3}} = \dfrac{3x - 2}{2\sqrt{x^3 - x^2 + 3}}$

៦. $\sqrt[4]{x^3 - 2x} \Longleftrightarrow f(x) = (x^3 - 2x)^{\frac{1}{4}}$

$$\begin{aligned} f'(x) &= \frac{1}{4}(x^3 - 2x)'(x^3 - 2x)^{\frac{1}{4}-1} \\ &= \frac{1}{4}(3x^2 - 2)(x^3 - 2x)^{-\frac{3}{4}} \\ \therefore\quad f'(x) &= \frac{3x^2 - 2}{4\sqrt[4]{(x^3 - 2x)^3}} \end{aligned}$$

៧. $f(x) = (x+1)(2x-1)^2$

$$\begin{aligned} f'(x) &= (x+1)'(2x-1)^2 + [(2x-1)^2]'(x+1) \\ &= (2x-1)^2 + 2(2x-1)'(2x-1)(x+1) \\ &= (2x-1)(2x-1+4x+4) \\ \therefore\ f'(x) &= (2x-1)(6x+3) \end{aligned}$$

៨. $f(x) = (x^2+2x+3)(x^3-3x-1)$

$$\begin{aligned} f'(x) &= (x^2+2x+3)'(x^2-3x-1) + (x^2-3x-1)'(x^2+2x+3) \\ &= (2x+2)(x^2-3x-1) + (2x-3)(x^2+2x+3) \\ &= 2x^3-6x^2-2x+2x^2-6x-2+2x^3+4x^2+6x-3x^2-6x-9 \\ \therefore\ f'(x) &= 4x^3-3x^2-8x-11 \end{aligned}$$

៩. $f(x) = \dfrac{1}{x-1} \Longrightarrow f'(x) = -\dfrac{(x-1)'}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

១០. $f(x) = \dfrac{x\sqrt{x}}{x+1}$

$$\begin{aligned} f'(x) &= \frac{(x\sqrt{x})'(x+1) - (x+1)'x\sqrt{x}}{(x+1)^2} \\ &= \frac{[x'\sqrt{x} + (\sqrt{x})'x](x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{\left(x + \dfrac{x}{2\sqrt{x}}\right)(x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{x\sqrt{x} + \sqrt{x} + \dfrac{x}{2\sqrt{x}}(x+1) - x\sqrt{x}}{(x+1)^2} \\ \therefore\ f'(x) &= \frac{x^2+3x}{2\sqrt{x}(x+1)^2} \end{aligned}$$

លំហាត់ ៣. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x.\sin x + \cos x$

២. $f(x) = \sin^3 x - x.\cos x$

៣. $f(x) = \cos(x^2+1) + 2\sin(x^2-1)$

៤. $f(x) = \sin^2\sqrt{x} + \cos^2(3x)$

៥. $f(x) = \cos(3x+4) + 3\cos x.\sin x$

៦. $f(x) = \sin(\sin\sqrt{x}) + \cos^3 x$

ដំណោះស្រាយ

១. $f(x) = x.\sin x + \cos x$

$$\begin{aligned} f'(x) &= x'\sin x + (\sin x)'.x - \sin x \\ &= \sin x + x.\cos x - \sin x \\ \therefore\quad f'(x) &= x.\cos x \end{aligned}$$

២. $f(x) = \sin^3 x - x.\cos x$

$$\begin{aligned} f'(x) &= 3(\sin x)'\sin^{3-1} x - [x'.\cos x + (\cos x)'.x] \\ &= 3\cos x.\sin^2 x - (\cos x - x.\sin x) \\ \therefore\quad f'(x) &= 3\cos x.\sin^2 x - \cos x + x\sin x \end{aligned}$$

៣. $f(x) = \cos(x^2+1) + 2\sin(x^2-1)$

$$\begin{aligned} f'(x) &= -(x^2+1)'\sin(x^2+1) + 2(x^2-1)'\cos(x^2-1) \\ \therefore\quad f'(x) &= -2x\sin(x^2+1) + 4x\cos(x^2-1) \end{aligned}$$

៤. $f(x) = \sin^2\sqrt{x} + \cos^2(3x)$

$$\begin{aligned} f'(x) &= 2(\sin\sqrt{x})'\cos^{2-1}\sqrt{x} + 2(\cos(3x))'\sin(3x) \\ &= 2(\sqrt{x})'.\cos\sqrt{x}.\cos\sqrt{x} - 2(3x)'\sin(3x).\sin(3x) \\ \therefore\quad f'(x) &= \frac{1}{\sqrt{x}}.\cos^2\sqrt{x} - 6\sin^2(3x) \end{aligned}$$

៥. $f(x) = \cos(3x+4) + 3\cos x.\sin x$

$$\begin{aligned} f'(x) &= -(3x+4)'.\sin(3x+4) + 3[(\cos x)'.\sin x + (\sin x)'.\cos x] \\ &= -3\sin(3x+4) + 3[-\sin x.\sin x + \cos x.\cos x] \\ \therefore\quad f'(x) &= -3[\sin(3x+4) + \sin^2 x - \cos^2 x] \end{aligned}$$

៦. $f(x) = \sin(\sin\sqrt{x}) + \cos^3 x$

$$\begin{aligned} f'(x) &= (\sin\sqrt{x})'.\cos(\sin\sqrt{x}) + 3(\cos x)\cos^{3-1} x \\ &= (\sqrt{x})'.\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x\cos^2 x \\ \therefore\quad f'(x) &= \frac{1}{2\sqrt{x}}\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x.\cos^2 x \end{aligned}$$

លំហាត់ ៤. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = (1+\tan x)^4$

២. $f(x) = x^2\tan x + (1+\cot x)^2$

៣. $f(x) = x.\tan(x^2-1) + x\cot(2x^2-3)$

៤. $f(x) = \dfrac{\tan(2x)}{1-\cos x}$

ដំណោះស្រាយ

១. $f(x) = (1+\tan x)^4$

$$\begin{aligned} f'(x) &= 4(1+\tan x)'(1+\tan^2 x)^{4-1} \\ \therefore\quad f'(x) &= 4(1+\tan^2 x)(1+\tan x)^3 \end{aligned}$$

២. $f(x) = x^2\tan x + (1+\cot x)^2$

$$\begin{aligned} f'(x) &= (x^2)'\tan x + (\tan x)'x^2 + 2(1+\cot x)'(1+\cot x)^{2-1} \\ \therefore\quad f'(x) &= 2x\tan x + x^2(1+\tan^2 x) - 2(1+\cot^2 x)(1+\cot x) \end{aligned}$$

៣. $f(x) = x.\tan(x^2-1) + x\cot(2x^2-3)$

$$f'(x) = x'\tan(x^2-1) + [\tan(x^2-1)]'x + x'\cot(2x^2-3) + [\cot(2x^2-3)]'x$$
$$= \tan(x^2-1) + (x^2-1)'[1+\tan^2(x^2-1)]x - (2x^2-3)'[1+\cot^2(2x^2-3)]x$$
$$\therefore\ f'(x) = \tan(x^2-1) + 2x^2[1+\tan^2(x^2-1)] - 4x^2[1+\cot^2(2x^2-3)]$$

លំហាត់ ៥. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = \dfrac{1-x-2x^2}{x^3-\ln 3}$

២. $f(x) = \dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}}$

៣. $f(x) = \sin x^2.\tan(2x+3)$

៤. $f(x) = \sin(x^2+5) + \cos(\sin x)$

៥. $f(x) = \dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$

ដំណោះស្រាយ

១. $f(x) = \dfrac{1-x-2x^2}{x^3-\ln 3}$

$$f'(x) = \frac{(1-x-2x^2)'(x^3-\ln 3) - (x^3-\ln 3)'(1-x-2x^2)}{(x^3-\ln 3)^2}$$
$$= \frac{(-1-4x)(x^3-\ln 3) - 3x^2(1-x-2x^2)}{(x^3-\ln 3)^2}$$
$$= \frac{-x^3+\ln 3 - 4x^4 + 4x\ln 3 - 3x^2 + 3x^3 + 6x^4}{(x^3-\ln 3)^2}$$
$$\therefore\ f'(x) = \frac{2x^4+2x^3-3x^2+4x.\ln 3+\ln 3}{(x^3-\ln 3)^2}$$

២. $f(x) = \dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}} \iff f(x).\sqrt{1+2x-x^2} = 2x^2+3x+4$ ធ្វើដេរីវេអង្គសងខាង គេបាន

$$[f(x)\sqrt{1+2x-x^2}]' = (2x^2+3x+4)'$$
$$f'(x)\sqrt{1+2x-x^2} + (\sqrt{1+2x-x^2})'f(x) = 4x+3$$
$$f'(x)\sqrt{1+2x-x^2} + \frac{(1+2x-x^2)'}{2\sqrt{1+2x-x^2}}f(x) = 4x+3$$
$$f'(x)\sqrt{1+2x-x^2} = 4x+3-\frac{1-x}{\sqrt{1+2x-x^2}}.f(x)$$
$$\therefore\ f'(x) = \frac{4x+3}{\sqrt{1+2x-x^2}} + \frac{(x-1)(2x^2+3x+4)}{(1+2x-x^2)\sqrt{1+2x-x^2}}$$

៣. $f(x) = \sin x^2 . \tan(2x+3)$

$$\begin{aligned} f'(x) &= (\sin x^2)' \tan(2x+3) + (\tan(2x+3))' \sin x^2 \\ &= (x^2)' . \sin x^2 . \tan(2x+3) + (2x+3)'[1+\tan^2(2x+3)] \sin x^2 \\ &= 2x \sin x^2 . \tan(2x+3) + 2\sin x^2 [1+\tan^2(2x+3)] \\ \therefore \quad f'(x) &= 2\sin x^2 [\tan^2(2x+3) + x\tan(2x+3) + 1] \end{aligned}$$

៤. $f(x) = \sin(x^2+5) + \cos(\sin x)$

$$\begin{aligned} f'(x) &= (x^2+5)' \cos(x^2+5) - (\sin x)' \sin(\sin x) \\ \therefore \quad f'(x) &= 2x\cos(x^2+5) - \cos x \sin(\sin x) \end{aligned}$$

៥. $f(x) = \dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})} \Longleftrightarrow f(x). \sin\sqrt{x} = \sin(\tan\sqrt{x})$

$$\begin{aligned} (f(x). \sin\sqrt{x})' &= (\sin(\tan\sqrt{x}))' \\ f'(x). \sin\sqrt{x} + (\sin\sqrt{x})' f(x) &= (\tan\sqrt{x})' \cos(\tan\sqrt{x}) \\ f'(x). \sin\sqrt{x} + (\sqrt{x})' \cos\sqrt{x}.f(x) &= (\sqrt{x})'(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) \\ f'(x) \sin\sqrt{x} + \frac{1}{2\sqrt{x}} \cos\sqrt{x}.f(x) &= \frac{1}{2\sqrt{x}}(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) \\ f'(x) \sin\sqrt{x} &= \frac{1}{2\sqrt{x}} \left[(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) - \cos\sqrt{x} f(x)\right] \\ \therefore \quad f'(x) &= \frac{1}{2\sqrt{x}. \sin\sqrt{x}} \left[(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) - \cos\sqrt{x} f(x)\right] \\ , f(x) &= \frac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})} \end{aligned}$$

លំហាត់ ៦. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = xe^x + \frac{1}{2}x^2$

២. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$

៣. $f(x) = \dfrac{\sqrt{x}}{e^x}$

៤. $f(x) = x^3 e^{-3x}$

៥. $f(x) = e^{2x} 3^{x^2+1}$

៦. $f(x) = e^{\sin x \cos x}$

ដំណោះស្រាយ

១. $f(x) = xe^x + \frac{1}{2}x^2$

$$f'(x) = x'e^x + (e^x)'x + \frac{1}{2}.2x = e^x + e^x x + x = e^x(1+x) + x$$

២. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$

$$\begin{aligned} f'(x) &= (x^2+2x+1)'e^{x^2+2x+1} + (x^2-3)'e^x + (e^x)'(x^2-3) \\ &= (2x+2)e^{x^2+2x+1} + 2xe^x + e^x(x^2-3) \\ \therefore\ f'(x) &= 2(x+1)e^{x^2+2x+1} + e^x(2x+x^2-3) \end{aligned}$$

៣. $f(x) = \frac{\sqrt{x}}{e^x}$

$$f'(x) = \frac{(\sqrt{x})'e^x + (e^x)'\sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}}e^x + e^x\sqrt{x}}{e^{2x}} = \frac{1+2x}{2\sqrt{x}e^x}$$

៤. $f(x) = x^3e^{-3x}$

$$\begin{aligned} f'(x) &= (x^3)'e^{-3x} + (e^{-3x})'x^3 \\ &= 3x^2e^{-3x} + (-3x)'e^{-3x}x^3 \\ &= 3x^2e^{-3x} - 3e^3e^{-3x} \\ \therefore\ f'(x) &= 3x^2e^{-3x}(1-x) \end{aligned}$$

៥. $f(x) = e^{2x}3^{x^2+1}$

$$\begin{aligned} f'(x) &= (e^{2x})'3^{x^2+1} + (3^{x^2+1})'.e^{2x} \\ &= (2x)'e^{2x}.3^{x^2+1} + (x^2+1)'3^{x^2+1}\ln 3.e^{2x} \\ &= 2.e^{2x}3^{x^2+1} + 2x3^{x^2+1}\ln 3.e^{2x} \\ \therefore\ f'(x) &= 2e^{2x}3^{x^2+1}(1+x\ln 3) \end{aligned}$$

៦. $f(x) = e^{\sin x\cos x}$

$$f'(x) = (\sin x\cos x)'e^{\sin x\cos x}$$
$$= [(\sin x)'\cos x + (\cos x)'\cos x]e^{\sin x\cos x}$$
$$\therefore\quad f'(x) = (\cos^2 x - \sin^2 x)e^{\sin x\cos x}$$

លំហាត់ ៧. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = (x^2-1)\ln(x^2-1)$

២. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right)$

៣. $f(x) = \ln(\sin x.\cos(2x))$

៤. $f(x) = \ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$

ដំណោះស្រាយ

១. $f(x) = (x^2-1)\ln(x^2-1) \Longrightarrow f'(x) = (x^2-1)'\ln(x^2-1) + \ln(x^2-1)'(x^2-1)$

$$= 2x\ln(x^2-1) + \frac{(x^2-1)'}{x^2-1}.(x^2-1)$$
$$= 2x\ln(x^2-1) + 2x$$
$$\therefore\quad f'(x) = 2x[\ln(x^2-1)+1]$$

២. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right) = \ln(x^2-2) - \ln(x^2-2)^{\frac{1}{3}}$

$$f'(x) = \frac{(x^2-2)'}{x^2-2} - \frac{1}{3}.\frac{(x^2-2)'}{x^2-2}$$
$$= \frac{3(2x)-2x}{3(x^2-2)}$$
$$\therefore\quad f'(x) = \frac{4x}{3(x^2-2)}$$

៣. $f(x) = \ln(\sin x.\cos(2x)) = \ln(\sin x) + \ln(\cos(2x))$

$$f'(x) = \frac{(\sin x)'}{\sin x} + \frac{(\cos(2x))'}{\cos(2x)}$$
$$= \frac{\cos x}{\sin x} - \frac{2\sin(2x)}{\cos(2x)}$$
$$\therefore\quad f'(x) = \cot x - 2\tan(2x)$$

៤. $f(x) = \ln\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right) = \ln\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}} = \frac{1}{2}(\ln(1+\sin x) - \ln(1-\sin x))$

$$\begin{aligned} f'(x) &= \frac{1}{2}\left(\frac{(1+\sin x)'}{1+\sin x} - \frac{(1-\sin x)'}{1-\sin x}\right) \\ &= \frac{1}{2}\left(\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}\right) \\ &= \frac{1}{2}.\frac{(\cos x(1-\sin x+1+\sin x))}{1-\sin^2 x} \\ \therefore\ f'(x) &= \frac{2\cos x}{2\cos^2 x} = \frac{1}{\cos x} \end{aligned}$$

លំហាត់ ៨. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = \cos(\arcsin x)$

២. $f(x) = \cot(\arctan x)$

៣. $f(x) = \tan(\arctan x)$

៤. $f(x) = \arcsin(2x)$

៥. $f(x) = \arcsin\sqrt{x}$

៦. $f(x) = \arctan(\sin x)$

៧. $f(x) = \frac{\arctan x}{\arcsin x}$

ដំណោះស្រាយ

១. $f(x) = \cos(\arcsin x) \Longrightarrow f'(x) = -(\arcsin x)'\sin(\arcsin x)$

$$\therefore\ f'(x) = -\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

២. $f(x) = \cot(\arctan x) \Longrightarrow f'(x) = -(\arctan x)'[1+\cot^2(\arctan x)]$

$$\therefore\ f'(x) = -\frac{1+\cot^2(\arctan x)}{1+x^2}$$

៣. $f(x) = \tan(\arctan x) \Longrightarrow f'(x) = (\arctan x)'[1+\tan^2(\arctan x)]$

$$\therefore\ f'(x) = \frac{1+\tan^2(\arctan x)}{1+x^2}$$

៤. $f(x) = \arcsin(2x) \Longrightarrow f'(x) = \frac{(2x)'}{\sqrt{1-(2x)^2}}$

$$\therefore\ f'(x) = \frac{2}{\sqrt{1-4x^2}}$$

៥. $f(x) = \arcsin\sqrt{x} \Longrightarrow f'(x) = \frac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}} = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x^2}} = \frac{1}{2\sqrt{x}\sqrt{1-x^2}}$

$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

៦. $f(x) = \arctan(\sin x) \Longrightarrow f'(x) = \dfrac{(\sin x)'}{1+(\sin x)^2}, \sin^2 x + \cos^2 x = 1$

$$\therefore \quad f'(x) = \frac{\cos x}{2-\cos^2 x}$$

៧. $f(x) = \dfrac{\arctan x}{\arcsin x} \Longrightarrow f'(x) = \dfrac{(\arctan x)'\arcsin x - (\arcsin x)'\arctan x}{(\arcsin x)^2}$

$$\therefore \quad f'(x) = \frac{\dfrac{\arcsin x}{1+x^2} - \dfrac{\arctan x}{\sqrt{1-x^2}}}{(\arcsin x)^2}$$

១១ លំហាត់មេរៀន

១. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $y = x^3 + 2x^2$

(ខ) $y = x^3 - 4x^2$

(គ) $y = x^4 - 27x$

(ឃ) $y = x^4 - 5x^2 + 4$

(ង) $y = x^5 - 16x$

(ច) $y = \frac{x}{x+1}$

(ឆ) $y = \frac{x^2}{1+x^2}$

(ជ) $y = x - \frac{1}{x}$

(ឈ) $y = x^3 + 2x^2 - x$

(ញ) $y = x^4 - 2x^3 + 2x$

(ដ) $y = \sqrt{1+x^2}$

(ឋ) $y = \sqrt[4]{1+x^2}$

២. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \sin x + \cos x$

(ខ) $f(x) = 2\sin x - 3\cos x$

(គ) $f(x) = 3\sin x + 2\cos x$

(ឃ) $f(x) = x\sin x + \cos x$

(ង) $f(x) = x\cos x - \sin x$

(ច) $f(x) = \cos(2x)$

(ឆ) $f(x) = \frac{1-\sin(2x)}{1-\sin x}$

(ជ) $f(x) = 1 + \sin x^2$

(ឈ) $f(x) = \cot x - \cos x$

(ញ) $f(x) = \sin(2x) - \cos(3x)$

(ដ) $f(x) = \sin(\cos(3x))$

(ឋ) $f(x) = \frac{\sin x^2}{x^2}$

(ឌ) $f(x) = \tan(1+x^2)$

(ឍ) $f(x) = \cos 2x - \cos x^2$

(ណ) $f(x) = (1+\sqrt{1+x})^3$

៣. រក y' នៃអនុគមន៍ខាងក្រោម៖

(ក) $xy = \frac{\pi}{6}$

(ខ) $\sin(xy) = 1$

(គ) $xy = \frac{1}{x+y}$

(ឃ) $x + y = xy$

(ង) $(y-1)^2 + x = 0$

(ច) $(y+1)^2 + y - x = 0$

(ឆ) $(y-x)^2 + x = 0$

(ជ) $(y+x) + 2y - x = 0$

(ឈ) $(y^2-1)^2 + x = 0$

(ញ) $(y^2+1)^2 - x = 0$

(ដ) $x^3 + xy + y^3 = 3$

(ឋ) $\sin x + \sin y = 1$

(ឌ) $\sin x + xy + y^5 = \pi$

(ឍ) $\tan x + \tan y = 1$

៤. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \sqrt{1-x}$

(ខ) $f(x) = \sqrt[4]{x+x^2}$

(គ) $y = \sqrt{1-\sqrt{x}}$

(ឃ) $y = \sqrt{x-\sqrt{x}}$

(ង) $y = \sqrt[3]{\sqrt{2x+1}} - x^2$

(ច) $y = \sqrt[4]{x+x^2}x + x^2$

(ឆ) $y = \sqrt[3]{x-\sqrt{2x+1}}$

(ជ) $y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$

៥. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = e^x + e^{-x}$

(ខ) $f(x) = e^{3x} + 4e^x$

(គ) $f(x) = \dfrac{e^x}{1+e^x}$

(ឃ) $f(x) = \dfrac{2e^{2x}}{1+e^{2x}}$

(ង) $f(x) = xe^{-x} + x\ln x$

(ច) $f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2e^{x+2}$

(ឆ) $f(x) = x^{-\frac{1}{2}x} + \ln\sqrt{x}$

(ជ) $f(x) = (\ln x)^2 + \ln x + 1$

(ឈ) $f(x) = \dfrac{\ln x}{x} + \ln\dfrac{1}{x}$

(ញ) $f(x) = \ln\left(\sqrt{\dfrac{\sqrt{x}+1}{\sqrt{x}-1}}\right)$

៦. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \tan(\arctan x)$

(ខ) $f(x) = \arcsin(\sin x)$

(គ) $f(x) = \cot(\arcsin x)$

(ឃ) $f(x) = \sin(\arctan x)$

(ង) $f(x) = (\arcsin x)^2$

(ច) $f(x) = \dfrac{1}{1+(\arctan x)^2}$

(ឆ) $f(x) = \sqrt{1-(\arcsin x)^2}$

៧. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $y = (x+1)(x-1)$

(ខ) $y = (x^2+1)(x^2-1)$

(គ) $y = \dfrac{1}{x+1} + \dfrac{1}{1+\sin x}$

(ឃ) $y = \dfrac{1}{1+x^2} + \dfrac{1}{1-\sin x}$

(ង) $y = (x-1)(x-2)(x-3)$

(ច) $y = x^2\cos x + 2x\sin x$

(ឆ) $y = x^{\frac{1}{2}}(x+\sin x)$

(ជ) $y = x^{\frac{1}{2}}\sin^2 x + (\sin x)^{\frac{1}{2}}$

(ឈ) $y = x^4\cos x + x\cos x$

(ញ) $y = \dfrac{1}{2}x^2\sin x - x\cos x + \sin x$

(ដ) $y = \sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$

(ឋ) $y = (x-6)^{10} + \sin^{10} x$

(ឌ) $y = (\sin x\cos x)^3 + \sin(2x)$

(ឍ) $y = x^{\frac{1}{2}}\sin(2x) + (\sin x)^{\frac{1}{2}}$

(ណ) $y = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

(ត) $y = \dfrac{1}{\tan x} - \dfrac{1}{\cot x}$