

លីមីតនៃអនុគមន៍

១ និយមន័យ

និយមន័យ ១. ដេរីវេនៃអនុគមន៍ $y = f(x)$ ត្រង់ x_0 កំណត់ដោយ

$$\frac{dy}{dx} = y' = f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

សំគាល់ ១. គេអាចសរសេរដេរីវេដោយ y' , $f'(x)$ ឬ $\frac{dy}{dx}$ ។

- អនុគមន៍ f មានដេរីវេត្រង់ x_0 នោះ f ជាប់ត្រង់ x_0 ។
- អនុគមន៍ f ជាប់ត្រង់ x_0 នោះ f អាចមានដេរីវេត្រង់ x_0 ឬ គ្មានដេរីវេត្រង់ x_0 ។

២ ភាពមានដេរីវេ

និយមន័យ ២. អនុគមន៍ f មានដេរីវេត្រង់ x_0 លុះត្រាតែ

- អនុគមន៍ f ជាប់ត្រង់ $x = 0$ ។
- ដេរីវេឆ្វេងស្មើដេរីវេស្តាំត្រង់ចំណុច x_0 គឺ $f'_-(x_0) = f'_+(x_0)$ ដែល

$$f'_-(x_0) = \lim_{h \rightarrow 0^-} \frac{f(x_0 + h) - f(x_0)}{h} \text{ និង } f'_+(x_0) = \lim_{h \rightarrow 0^+} \frac{f(x_0 + h) - f(x_0)}{h} \text{ ។}$$

២.១ ជាប់លើចន្លោះបើក

និយមន័យ ៣. អនុគមន៍ f ជាប់លើចន្លោះបើក (a, b) កាលណា f មានដេរីវេលើគ្រប់ចំណុច $x_0 \in (a, b)$ ។

២.២ ជាប់លើចន្លោះបិទ

និយមន័យ ៤. អនុគមន៍ f មានដេរីវេលើចន្លោះបិទ $[a, b]$ កាលណា f មានដេរីវេលើចន្លោះ (a, b) ហើយ f មានដេរីវេខាងឆ្វេងត្រង់ $x = a$ និងខាងស្តាំត្រង់ $x = b$ ។

៣ លក្ខណៈនៃដេរីវេ

ចំពោះ u, v ជាអនុគមន៍នៃ x និង k ជាចំនួនថេរ នោះគេបាន៖

១. $(ku)' = ku'$

៣. $(u - v)' = u' - v'$

៥. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$

២. $(u + v)' = u' + v'$

៤. $(uv)' = u'v + v'u$

៦. $\left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$

សម្រាយបញ្ជាក់

១. តាង $f(x) = k.u(x)$ ដែល $u = u(x)$ និង k ជាចំនួនថេរ តាមនិយមន័យ

$$\begin{aligned} f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{k u(x_0 + h) - k u(x_0)}{h} \\ &= k \cdot \lim_{h \rightarrow 0} \frac{u(x_0 + h) - u(x_0)}{h} \\ &= k \cdot u'(x_0) \end{aligned}$$

$$\therefore (k.u)' = k.u'$$

២. តាង $f(x) = u(x) + v(x)$ ដែល $u = u(x)$ និង $v = v(x)$ តាមនិយមន័យ

$$\begin{aligned} f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x_0 + h) + v(x_0 + h) - (u(x_0) + v(x_0))}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x_0 + h) - u(x_0)}{h} + \lim_{h \rightarrow 0} \frac{v(x_0 + h) - v(x_0)}{h} \\ &= u'(x_0) + v'(x_0) \end{aligned}$$

$$\therefore (u + v)' = u' + v'$$

៣. ស្រាយដូចទី២

៤. តាង $f(x) = uv$ ដែល $u = u(x)$ និង $v = v(x)$ តាមនិយមន័យគេបាន

$$\begin{aligned} f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x_0 + h).v(x_0 + h) - u(x_0).v(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{u(x_0 + h).v(x_0 + h) - u(x_0).v(x_0 + h) + u(x_0).v(x_0 + h) + u(x_0).v(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{u(x_0 + h).v(x_0 + h) - u(x_0).v(x_0 + h)}{h} + \frac{u(x_0).v(x_0 + h) + u(x_0).v(x_0)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[v(x_0 + h). \frac{u(x_0 + h) - u(x_0)}{h} + u(x_0). \frac{v(x_0 + h) - v(x_0)}{h} \right] \\ &= v(x_0). \frac{d}{dx}(u(x_0)) + u(x_0). \frac{d}{dx}(v(x_0)) \end{aligned}$$

$$\therefore (uv)' = u'v + v'u \quad (9)$$

៥. យក $u = u(x)$ និង $v = v(x)$ តាង $f(x) = \frac{u}{v} \Leftrightarrow f(x) \cdot v = u$ ធ្វើដេរីវេអង្គទាំងពីរធៀបនឹង x

នោះគេបាន $[f(x) \cdot v]' = u'$ ប្រើតាមសមីការ (១) គេបាន

$$f'(x) \cdot v + v'f(x) = u', \quad f(x) = \frac{u}{v}$$

$$f'(x) \cdot v + v' \cdot \frac{u}{v} = u'$$

$$\frac{f'(x) \cdot v^2}{v} + \frac{v'u}{v} = u'$$

$$f'(x) \cdot v^2 + v'u = u'v$$

$$f'(x) = \frac{u'v - v'u}{v^2}$$

$$\therefore \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \quad (2)$$

៦. យក $v = v(x)$ តាង $f(x) = \frac{1}{v}$ ប្រើសមីការ (២) គេបាន

$$f'(x) = \frac{(1)' \cdot v - v' \cdot (1)}{v^2}$$

$$= \frac{0 - v'}{v^2}$$

$$= -\frac{v'}{v^2}$$

$$\therefore \left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$$

៤ ដេរីវេនៃអនុគមន៍បណ្តាក់

បើ $y = f(u)$ និង $u = g(x)$ នោះ $\frac{d}{dx}(f \circ g) = \frac{dy}{du} \times \frac{du}{dx}$ ។

សម្រាយបញ្ជាក់

តាង $F(x) = f \circ g = f(g(x))$ តាមនិយមន័យភាពមានដេរីវេត្រង់ $x = x_0$ នោះគេបាន

$$\begin{aligned} F'(x_0) &= \lim_{x \rightarrow x_0} \frac{F(x) - F(x_0)}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \frac{f(g(x)) - f(g(x_0))}{x - x_0} \\ &= \lim_{x \rightarrow x_0} \left(\frac{f(g(x)) - f(g(x_0))}{g(x) - g(x_0)} \times \frac{g(x) - g(x_0)}{x - x_0} \right) \\ &= f'(g(x_0)) \times g'(x_0) \quad , u = g(x_0), y = f(x_0) \\ \therefore \frac{d}{dx}(f \circ g) &= \frac{dy}{du} \times \frac{du}{dx} \end{aligned}$$

ជំនួយទី ៤.១. បើ $y = c$ ដែល c ជាចំនួនថេរ នោះ $y' = 0$ ។

សម្រាយបញ្ជាក់

គេមាន $y = f(x_0) = c$ នោះ $f(x_0 + h) = c$, $c \in \mathbb{R}$ តាមនិយមន័យគេបាន

$$\begin{aligned} f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ \therefore \frac{d}{dx}(c) &= 0 \end{aligned}$$

ឧទាហរណ៍ ១. គណនា y' ដែល $y = (\ln x \cdot \log_a(\sqrt{3}))$ ។

ដំណោះស្រាយ

គេមាន $y = (\ln x \cdot \log_a(\sqrt{3})) \Rightarrow y' = (\ln x \cdot \log_a(\sqrt{3}))' = 0$

ឧទាហរណ៍ ២. ស្រាយបញ្ជាក់ថា បើ $y = x^n$ នោះ $y' = nx^{n-1}$ ។

សម្រាយបញ្ជាក់

គេមាន $f(x_0) = x_0^n$ នាំឲ្យ $f(x_0 + h) = (x_0 + h)^n$ តាមនិយមន័យ

$$\begin{aligned} y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x_0 + h)^n - x_0^n}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x_0 + h - x_0)(x_0^{n-1} + x_0^{n-2} \cdot x_0 + \dots + x_0 \cdot x_0^{n-2} x_0 + x_0^{n-1})}{h} \\ &= \lim_{h \rightarrow 0} (x_0^{n-1} + x_0^{n-1} + \dots + x_0^{n-1} + x_0^{n-1}) \\ &= x_0^{n-1} \underbrace{(1 + 1 + \dots + 1 + 1)}_{n \text{ គូរលេខ } 1} \\ &= n \cdot x_0^{n-1} \\ \therefore \frac{d}{dx}(x^n) &= n \cdot x^{n-1} \end{aligned}$$

ឧទាហរណ៍ ៣. គណនា $f'(x)$

១. $f(x) = x^3$

២. $f(x) = \sqrt{x}$

៣. $f(x) = \sqrt[3]{x^2}$

ដំណោះស្រាយ

១. $f(x) = x^3 \Rightarrow f'(x) = (x^3)' = 3x^{3-1} = 3x^2$

២. $f(x) = \sqrt{x} \Rightarrow f'(x) = (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

៣. $f(x) = \sqrt[3]{x^2} \Rightarrow f'(x) = (\sqrt[3]{x^2})' = (x^{\frac{2}{3}})' = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

ជំនួញ ៤.២. បើ $y = u^n$ ដែល u ជាអនុគមន៍នៃ x នោះ $y' = nu'u^{n-1}$ ។

សម្រាយបញ្ជាក់

គេមាន $y = u^n$ គេបាន $y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(u^n) \times u' = nu'u^{n-1}$

ឧទាហរណ៍ ៤. គណនា y'

១. $y = (2x + \ln 2)^4$

២. $y = \sqrt{u}$ ដែល u ជាអនុគមន៍នៃ x ។

ដំណោះស្រាយ

១. $y = (2x + \ln 2)^4 \Rightarrow y' = 4(2x + \ln 2)'(2x + \ln 2)^{4-1} = 4(2 + 0)(2x + \ln 2)^3 = 8(2x + \ln 2)^3$

២. $y = \sqrt{u} = u^{\frac{1}{2}}$ នាំឲ្យ $y' = (u^{\frac{1}{2}})' = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2}u' u^{-\frac{1}{2}} = \frac{u'}{2\sqrt{u}}$ ។

៥ ដេរីវេនៃអនុគមន៍ត្រីកោណមាត្រ

១. បើ $y = \sin x$ នោះ $y' = \cos x$

២. បើ $y = \cos x$ នោះ $y' = -\sin x$

៣. បើ $y = \tan x$ នោះ $y' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

៤. បើ $y = \cot x$ នោះ $y' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$

សម្រាយបញ្ជាក់

១. គេមាន $y = f(x_0) = \sin x_0$ នោះ $f(x_0 + h) = \sin(x_0 + h)$ តាមនិយមន័យ

$$\begin{aligned} y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x_0 + h) - \sin x_0}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin x_0 \cos h + \sin h \cdot \cos x_0 - \sin x_0}{h} \\ &= \lim_{h \rightarrow 0} \left(\cos x_0 \cdot \frac{\sin h}{h} - \sin x_0 \cdot \frac{1 - \cos h}{h} \right) \\ &= \cos x_0, \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1, \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0 \\ \therefore \frac{d}{dx}(\sin x) &= \cos x \end{aligned}$$

២. គេមាន $y = f(x_0) = \cos x_0$ នោះ $f(x_0 + h) = \cos(x_0 + h)$ តាមនិយមន័យ

$$\begin{aligned} y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos(x_0 + h) - \cos x_0}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cos x_0 \cdot \cos h - \sin x_0 \cdot \sin h - \cos x_0}{h} \\ &= \lim_{h \rightarrow 0} \left(-\frac{\sin h}{h} \cdot \sin x_0 - \cos x_0 \cdot \frac{1 - \cos h}{h} \right) \\ &= -\sin x_0, \lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0, \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \\ \therefore \frac{d}{dx}(\cos x) &= -\sin x \end{aligned}$$

៣. តាង $y = \tan x = \frac{\sin x}{\cos x}$ តាមសមីការ (២) គេបាន

$$\begin{aligned} y' &= \left(\frac{\sin x}{\cos x} \right)' = \frac{(\sin x)' \cos x - (\cos x)' \cdot \sin x}{(\cos x)^2} \\ &= \frac{\cos x \cdot \cos x - (-\sin x) \cdot \sin x}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\ &= 1 + \tan^2 x \\ &= \frac{1}{\cos^2 x}, \sin^2 x + \cos^2 x = 1 \\ \therefore (\tan x)' &= \frac{1}{\cos^2 x} = 1 + \tan^2 x \end{aligned}$$

៤. តាង $y = \cot x = \frac{\cos x}{\sin x}$ តាមសមីការ (២) គេបាន

$$\begin{aligned} y' &= \left(\frac{\cos x}{\sin x} \right)' = \frac{(\cos x)' \cdot \sin x - (\sin x)' \cdot \cos x}{(\sin^2 x)^2} \\ &= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\ &= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}, \sin^2 x + \cos^2 x = 1 \\ \therefore (\cot x)' &= -\frac{1}{\sin^2 x} = -(1 + \cot^2 x) \end{aligned}$$

ជំនួញ ៥.១. បើ u ជាអនុគមន៍នៃ x គេបាន

១. បើ $y = \sin u$ នោះ $y' = u' \cos u$

២. បើ $y = \cos u$ នោះ $y' = -u' \sin u$

៣. បើ $y = \tan u$ នោះ $y' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$

៤. បើ $y = \cot u$ នោះ $y' = -\frac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)$

សម្រាយបញ្ជាក់

១. បើ u ជាអនុគមន៍នៃ x នោះ $y = \sin u$ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\sin u) \times \frac{du}{dx} = \cos u \times u' = u' \cos u \\ \therefore \frac{d}{dx}(\sin u) &= u' \cos u\end{aligned}$$

២. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cos u$ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cos u) \times \frac{du}{dx} = -\sin u \times u' = -u' \sin u \\ \therefore \frac{d}{dx}(\cos u) &= -u' \sin u\end{aligned}$$

៣. បើ u ជាអនុគមន៍នៃ x នោះ $y = \tan u$ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\tan u) \times \frac{du}{dx} = \frac{1}{\cos^2 u} \times u' = (1 + \tan^2 u) \times u' \\ \therefore (\tan u)' &= \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)\end{aligned}$$

៤. បើ u ជាអនុគមន៍នៃ x នោះ $y = \cot u$ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\cot u) \times \frac{du}{dx} = -\frac{1}{\sin^2 u} \times u' = -(1 + \cot^2 u) \times u' \\ \therefore (\cot u)' &= -\frac{u'}{\sin^2 u} = -u'(1 + \cot^2 u)\end{aligned}$$

ឧទាហរណ៍ ៥. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $y = \sin(2x + 1)$

៣. $y = \tan(2x + 1)$

២. $y = \cos(2x + 1)$

៤. $y = \cot(2x + 1)$ ។

ដំណោះស្រាយ

១. $y = \sin(2x + 1) \Rightarrow y' = (2x + 1)' \cos(2x + 1) = 2 \cos(2x + 1)$

២. $y = \cos(2x + 1) \Rightarrow y' = -(2x + 1)' \sin(2x + 1) = -2 \sin(2x + 1)$

៣. $y = \tan(2x + 1) \Rightarrow y' = \frac{(2x + 1)'}{\cos^2(2x + 1)} = \frac{2}{\cos^2(2x + 1)} = 2[1 + \tan^2(2x + 1)]$

៤. $y = \cot(2x + 1) \Rightarrow y' = -\frac{(2x + 1)'}{\sin^2(2x + 1)} = -\frac{2}{\sin^2(2x + 1)} = -2[1 + \cot^2(2x + 1)]$ ។

៦ ដេរីវេអនុគមន៍អិចស្ប៉ូណង់ស្យែល

ស្រាយថាបើ $y = a^x$ នោះ $y' = a^x \cdot \ln a$

សម្រាយបញ្ជាក់

គេមាន $y = a^x$ តាមនិយមន័យ គេបាន

$$\begin{aligned}y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\&= \lim_{h \rightarrow 0} \frac{a^{x_0+h} - a^{x_0}}{h} \\&= \lim_{h \rightarrow 0} \frac{a^{x_0}(a^h - 1)}{h} \\&= a^{x_0} \lim_{h \rightarrow 0} \frac{a^h - 1}{h} \text{ ដោយ } \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a \\&= a^{x_0} \cdot \ln a \\ \therefore (a^x)' &= a^x \cdot \ln a\end{aligned}$$

ជំនួញ ៦.១. បើ u ជាអនុគមន៍នៃ x នោះ $(a^u)' = u' a^u \cdot \ln a$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(a^u) \times \frac{du}{dx} = a^u \cdot \ln a \times u' \\ \therefore (a^u)' &= u' \cdot a^u \cdot \ln a\end{aligned}$$

ឧទាហរណ៍ ៦. គណនា y' ចំពោះ u ជាអនុគមន៍នៃ x នៃអនុគមន៍ខាងក្រោម៖

១. $y = e^x$

២. $y = a^{x^2-1}$

៣. $y = e^u$

ដំណោះស្រាយ

១. $y = e^x$ នោះ $y' = (e^x)' = e^x \cdot \ln e = e^x, \ln e = 1$

២. $y = a^{x^2-1}$ នោះ $y' = (x^{x^2-1})' a^{x^2-1} \ln a = 2x \cdot a^{x^2-1} \ln a$

៣. $y = e^u$ នោះ $y' = (e^u)' = u' e^u \cdot \ln e = u' e^u, \ln e = 1$

៧ ដេរីវេនៃអនុគមន៍លោការីត

ស្រាយបញ្ជាក់ថា បើ $y = \log_a x$, $a > 0, a \neq 1$ នោះ $y' = \frac{1}{x \ln a}$ ។

សម្រាយបញ្ជាក់

គេមាន $y = \log_a x$ តាមនិយមន័យ គេបាន

$$\begin{aligned} y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\log_a(x_0 + h) - \log_a(x_0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \log_a \left(\frac{x_0 + h}{x_0} \right) \\ &= \lim_{h \rightarrow 0} \log_a \left(1 + \frac{h}{x_0} \right)^{\frac{1}{h}} \\ &= \log_a \left(\lim_{h \rightarrow 0} \ln \left(1 + \frac{h}{x_0} \right)^{\frac{1}{h}} \right)^{\frac{1}{x_0}} \\ &= \log_a e^{\frac{1}{x_0}} = \frac{1}{x_0} \frac{\ln e}{\ln a} \\ \therefore (\log_a x)' &= \frac{1}{x \ln a}, a > 0, a \neq 1 \end{aligned}$$

ជំនួយទៅ ៧.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\log_a u)' = \frac{u'}{u \ln a}$, $a > 0, a \neq 1$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\log_a u) \times \frac{du}{dx} = \frac{1}{u \ln a} \times u' \\ \therefore (\log_a u)' &= \frac{u'}{u \ln a}, a > 0, a \neq 1 \end{aligned}$$

៨ ដេរីវេនៃអនុគមន៍លោការីតនេពែ

ស្រាយបញ្ជាក់ថា បើ $y = \ln x$ នោះ $y' = \frac{1}{x}$ ។

សម្រាយបញ្ជាក់

គេមាន $y = \ln x$ តាមនិយមន័យ គេបាន

$$\begin{aligned}
 y' = f'(x_0) &= \lim_{h \rightarrow 0} \frac{f(x_0 + h) - f(x_0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\ln(x_0 + h) - \ln(x_0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\ln\left(\frac{x_0 + h}{x_0}\right)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \ln\left(1 + \frac{h}{x_0}\right) \\
 &= \lim_{h \rightarrow 0} \ln\left(1 + \frac{h}{x_0}\right)^{\frac{1}{h}} \\
 &= \ln \left[\lim_{h \rightarrow 0} \left(1 + \frac{1}{\frac{x_0}{h}}\right)^{\frac{x_0}{h} \times \frac{1}{x_0}} \right], \lim_{h \rightarrow 0} \left(1 + \frac{1}{\frac{x_0}{h}}\right)^{\frac{x_0}{h}} = e \\
 &= \ln e^{\frac{1}{x_0}}, \ln e = 1 \\
 \therefore (\ln x)' &= \frac{1}{x}
 \end{aligned}$$

ឧទាហរណ៍ ៧. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x^2 \cdot \log_a x, a > 0, a \neq 1$

៤. $f(x) = \log(x^2 \sqrt{x^3 - 1})$

២. $f(x) = \sin(2x) + \log_2(x^2 + 1)$

៥. $f(x) = (\sin x)^{\log x}$

៣. $f(x) = \frac{e^{2x} + \log_3 x}{x^2}$

៦. $f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1$

សម្រាយបញ្ជាក់

១. $f(x) = x^2 \cdot \log_a x, a > 0, a \neq 1 \Rightarrow f'(x) = (x^2)' \log_a x + (\log_a x)' x^2$

$$= 2x \log_a x + \frac{1}{x \ln a} x^2$$

$$\therefore f'(x) = 2x \log_a x + \frac{x}{\ln a}, a > 0, a \neq 1$$

២. $f(x) = \sin(2x) + \log_2(x^2 + 1) \Rightarrow f'(x) = -(2x)' \cos(2x) + \frac{(x^2 + 1)'}{(x^2 + 1) \ln 2}$

$$\therefore f'(x) = -2 \cos(2x) + \frac{2x}{(x^2 + 1) \ln 2}$$

$$\begin{aligned}
 \text{៣. } f(x) &= \frac{e^{2x} + \log_3 x}{x^2} \Rightarrow f'(x) = \frac{(e^{2x} + \log_3 x)'x^2 - (x^2)'(e^{2x} + \log_3 x)}{x^4} \\
 &= \frac{(2e^{2x} + \frac{1}{x \ln 3})x^2 - 2x(e^{2x} + \log_3 x)}{x^4} \\
 &= \frac{2xe^{2x} + \frac{1}{\ln 3} - 2e^{2x} - 2\log_3 x}{x^3} \\
 \therefore f'(x) &= \frac{2e^{2x}(x-1) + \frac{1}{\ln 3} - \log_3 x^2}{x^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{៤. } f(x) &= \log(x^2 \sqrt{x^3 - 1}) = \log x^2 + \log(x^3 - 1)^{\frac{1}{2}} = 2\log x + \frac{1}{2} \log(x^3 - 1) \\
 \therefore f'(x) &= \frac{2}{x \ln 10} + \frac{(x^3 - 1)'}{2(x^3 - 1) \ln 10} = \frac{2}{x \ln 10} + \frac{3x^2}{2(x^3 - 1) \ln 10}
 \end{aligned}$$

$$\text{៥. } f(x) = (\sin x)^{\log x} \Leftrightarrow \ln f(x) = \ln(\sin x)^{\log x}$$

$$\begin{aligned}
 (\ln f(x))' &= (\log x \cdot \ln(\sin x))' \\
 \frac{f'(x)}{f(x)} &= (\log x)' \ln(\sin x) + (\ln(\sin x))' \log x \\
 f'(x) &= f(x) \left(\frac{1}{x \ln 10} \ln(\sin x) + \frac{(\sin x)'}{\sin x} \cdot \log x \right) \\
 \therefore f'(x) &= (\sin x)^{\log x} \left(\frac{\ln(\sin x)}{x \ln 10} + \cot x \cdot \log x \right)
 \end{aligned}$$

$$\text{៦. } f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1 \Leftrightarrow \ln f(x) = \ln(\log_a x)^{\ln(2x)}$$

$$\begin{aligned}
 (\ln f(x))' &= (\ln(2x) \cdot \ln(\log_a x))' \\
 \frac{f'(x)}{f(x)} &= (\ln(2x))' \ln(\log_a x) + (\ln(\log_a x))' \ln(2x) \\
 f'(x) &= f(x) \left(\frac{(2x)'}{2x} \ln(\log_a x) + \frac{(\log_a x)'}{\log_a x} \ln(2x) \right) \\
 \therefore f'(x) &= (\log_a x)^{\ln(2x)} \left(\frac{\ln(\log_a x)}{x} + \frac{\ln(2x)}{x \ln a \log_a x} \right), a > 0, a \neq 1
 \end{aligned}$$

ជំនួយទៅ ៨.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\ln u)' = \frac{u'}{u}$ ។

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\ln u) \times \frac{du}{dx} = \frac{1}{u} \times u' \\ \therefore (\ln u)' &= \frac{u'}{u}\end{aligned}$$

ឧទាហរណ៍ ៨. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x \cdot \ln x$

៤. $f(x) = \ln(x^2 \sqrt{x^3 - 1})$

២. $f(x) = x^2 + \ln(x^2 + 1)$

៥. $f(x) = x^x$

៣. $f(x) = \frac{e^x + \ln x}{x^2}$

៦. $f(x) = (\sin x)^{\cos x}$

សម្រាយបញ្ជាក់

១. $f(x) = x \cdot \ln x \Rightarrow f'(x) = x' \ln x + (\ln x)'x = \ln x + \frac{1}{x} \cdot x = \ln x + 1$

២. $f(x) = x^2 + \ln(x^2 + 1) \Rightarrow f'(x) = (x^2)' + \frac{(x^2 + 1)'}{x^2 + 1} = 2x + \frac{2x}{x^2 + 1}$

៣. $f(x) = \frac{e^x + \ln x}{x^2} \Rightarrow f'(x) = \frac{(e^x + \ln x)'x^2 - (x^2)'(e^x + \ln x)}{(x^2)^2}$

$$= \frac{\left(e^x + \frac{1}{x}\right)x^2 - 2x(e^x + \ln x)}{x^4}$$

$$\therefore f'(x) = \frac{xe^x + 1 - 2e^x - 2 \ln x}{x^3}$$

៤. $f(x) = \ln(x^2 \sqrt{x^3 - 1}) = \ln x^2 + \ln \sqrt{x^3 - 1} = 2 \ln x + \ln(x^3 - 1)^{\frac{1}{2}}$

$$f'(x) = 2(\ln x)' + \frac{1}{2}[\ln(x^3 - 1)]'$$

$$= 2 \cdot \frac{1}{x} + \frac{1}{2} \cdot \frac{(x^3 - 1)'}{x^3 - 1}$$

$$\therefore f'(x) = \frac{2}{x} + \frac{3}{2} \cdot \frac{x^2}{x^3 - 1}$$

$$៥. f(x) = x^x \iff \ln f(x) = \ln x^x = x \ln x$$

$$(\ln f(x))' = (x \ln x)'$$

$$\frac{f'(x)}{f(x)} = x' \ln x + (\ln x)' x$$

$$f'(x) = f(x) \left(\ln x + \frac{1}{x} \cdot x \right)$$

$$\therefore f'(x) = x^x (\ln x + 1)$$

$$៦. f(x) = (\sin x)^{\cos x} \iff \ln f(x) = \ln(\sin x)^{\cos x}$$

$$(\ln f(x))' = (\cos x \ln \sin x)'$$

$$\frac{f'(x)}{f(x)} = (\cos x)' \ln \sin x + (\ln \sin x)' \cos x$$

$$f'(x) = f(x) \left(-\sin x \ln \sin x + \frac{(\sin x)'}{\sin x} \cdot \cos x \right)$$

$$\therefore f'(x) = (\sin x)^{\cos x} (\cos x \cot x - \sin x \ln \sin x)$$

៩ ដេរីវេនៃអនុគមន៍ Arc Sine និង Arc Tangent

$$y = \arcsin x \iff x = \sin y \text{ និង } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2},$$

$$y = \arctan x \iff x = \tan y \text{ និង } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2},$$

$$\text{បើ } y = \arcsin x \text{ នោះ } y' = \frac{1}{\sqrt{1-x^2}} \text{ ។}$$

សម្រាយបញ្ជាក់

បើ $y = \arcsin x$ នោះ $x = \sin y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$(x)' = (\sin y)' \iff 1 = y' \cos y$$

$$y' = \frac{1}{\cos y}, \sin^2 y + \cos^2 y = 1 \implies \cos y = \pm \sqrt{1 - \sin^2 y}$$

$$\text{ដោយ } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \implies \cos y \geq 0 \implies \cos y = \sqrt{1 - x^2}$$

$$\therefore (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

បើ $y = \arctan x$ នោះ $y' = \frac{1}{1+x^2}$ ។

សម្រាយបញ្ជាក់

បើ $y = \arctan x$ នោះ $x = \tan y$ ធ្វើដេរីវេអង្គសងខាងធៀបនឹង x គេបាន

$$\begin{aligned}(x)' &= (\sin y)' \Leftrightarrow 1 = y'(1 + \tan^2 y) \\ y' &= \frac{1}{1 + \tan^2 y} \\ \therefore (\arctan x)' &= \frac{1}{1 + x^2}\end{aligned}$$

ជំនួញ ៩.១. បើ u ជាអនុគមន៍នៃ x នោះ $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arcsin u) \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times u' \\ \therefore (\arcsin u)' &= \frac{u'}{\sqrt{1-u^2}}\end{aligned}$$

ជំនួញ ៩.២. បើ u ជាអនុគមន៍នៃ x នោះ $(\arctan u)' = \frac{u'}{1+u^2}$

សម្រាយបញ្ជាក់

បើ u ជាអនុគមន៍នៃ x នោះ គេបាន

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arctan u) \times \frac{du}{dx} = \frac{1}{1+u^2} \times u' \\ \therefore (\arctan u)' &= \frac{u'}{1+u^2}\end{aligned}$$

លំហាត់ ១. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = \arcsin x \cdot \sin x$

៣. $f(x) = \sin(\arcsin x)$

២. $f(x) = \arctan x \cos x$

៤. $f(x) = \arctan(\tan x)$

សម្រាយបញ្ជាក់

១. $f(x) = \arcsin x \cdot \sin x \Rightarrow f'(x) = (\arcsin x)' \sin x + (\sin x)' \arcsin x$

$$\therefore f'(x) = \frac{\sin x}{\sqrt{1-x^2}} + \cos x \cdot \arcsin x$$

២. $f(x) = \arctan x \cos x \Rightarrow f'(x) = (\arctan x)' \cos x + (\cos x)' \arctan x$

$$\therefore f'(x) = \frac{\cos x}{1+x^2} - \sin x \cdot \arctan x$$

៣. $f(x) = \sin(\arcsin x) \Rightarrow f'(x) = (\arcsin x)' \cos(\arcsin x)$

$$\therefore f'(x) = \frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$

$$\text{៤. } f(x) = \arctan(\tan x) \Rightarrow f'(x) = \frac{(\tan x)'}{1+(\tan x)^2} = \frac{1+\tan^2 x}{1+\tan^2 x}$$

$$\therefore f'(x) = 1$$

លក្ខណៈនៃដេរីវេ

បើ f, g, y, u, v ជាអនុគមន៍នៃ x និង k ជាចំនួនថេរនោះគេបាន៖

$$១. (u \pm v)' = u' \pm v'$$

$$២. (ku)' = ku'$$

$$៣. (uv)' = u'v + v'u$$

$$៤. \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$៥. \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$៦. \left(\frac{1}{v}\right)' = -\frac{v'}{v^2}$$

រូបមន្តនៃដេរីវេ

បើ C, a, b, c ជាចំនួនថេរ និង u ជាអនុគមន៍នៃ x ដែល $n \in \mathbb{N}$ គេបាន៖

$$១. (C)' = 0$$

$$២. (x)' = 1$$

$$៣. (ax + b)' = a$$

$$៤. (ax^2 + bx + c)' = 2ax + b$$

$$៥. (x^n)' = nx^{n-1}$$

$$៦. (u^n)' = n \cdot u' \cdot u^{n-1}$$

$$៧. (x)^{-n} = -\frac{n}{x^{n+1}}$$

$$៨. \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$៩. \left(\frac{1}{u}\right)' = -\frac{u'}{u^2}$$

$$១០. (\sqrt{x})' = \frac{1}{2\sqrt{x}}$$

$$១១. (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$$

$$១២. (\sqrt[n]{x})' = \frac{1}{n\sqrt[n]{x^{n-1}}}$$

$$១៣. (\ln x)' = \frac{1}{x}$$

$$១៤. (\ln u)' = \frac{u'}{u}$$

$$១៥. (\log_a x)' = \frac{1}{x \ln a}, a > 0, a \neq 1$$

$$១៦. (\log_a u)' = \frac{u'}{u \cdot \ln a}, a > 0, a \neq 1$$

$$១៧. (a^x)' = a^x \ln a, a > 0, a \neq 1$$

$$១៨. (a^u)' = u' a^u \ln a, a > 0, a \neq 1$$

$$១៩. (e^x)' = e^x$$

$$២០. (e^u)' = u' e^u$$

$$២១. (\sin x)' = \cos x$$

$$២២. (\sin u)' = u' \cos u$$

$$២៣. (\cos x)' = -\sin x$$

$$២៤. (\cos u)' = -u' \sin u$$

$$២៥. (\tan x)' = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$២៦. (\tan u)' = \frac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$$

$$២៧. (\cot x)' = -\frac{1}{\sin^2 x} = -(1 + \cot^2 x)$$

$$២៨. (\cot u)' = -\frac{u'}{\sin^2 u} = -(1 + \cot^2 u)$$

$$២៩. (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$៣០. (\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$$

$$៣១. (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$៣២. (\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$$

$$៣៣. (\arctan x)' = \frac{1}{1+x^2}$$

$$៣៤. (\arctan u)' = \frac{u'}{1+u^2}$$

$$៣៥. (\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

$$៣៦. (\operatorname{arccot} u)' = -\frac{u'}{1+u^2}$$

$$៣៧. (u^v)' = \left(v' \cdot \ln u + \frac{v \cdot u'}{u}\right) \cdot u^v$$

៩០ លំហាត់ និង ជំនោះស្រាយ

លំហាត់ ២. គណនា $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

$$១. f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$$

$$៦. \sqrt[4]{x^3 - 2x}$$

$$២. f(x) = 2x^2 - \sqrt{x} + \frac{2}{x}$$

$$៧. f(x) = (x+1)(2x-1)^2$$

$$៣. f(x) = (x^4 - 7x^2 + \sin a)^7$$

$$៨. f(x) = (x^2 + 2x + 3)(x^3 - 3x - 1)$$

$$៤. f(x) = (x^2 - \sqrt{x})^{2019}$$

$$៩. f(x) = \frac{1}{\frac{x-1}{x\sqrt{x}}}$$

$$៥. f(x) = \sqrt{x^3 - x^2 + 3}$$

$$១០. f(x) = \frac{1}{x+1}$$

សម្រាយបញ្ជាក់

$$១. f(x) = x^5 - x^4 + x^3 - x^2 + x - 1 \Rightarrow f'(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$$

$$២. f(x) = 2x^2 - \sqrt{x} - \frac{2}{x} \Rightarrow f'(x) = 4x^2 + \frac{1}{2\sqrt{x}} - \frac{2}{x^2}$$

$$f'(x) = 7(x^4 - 7x^2 + \sin a)'(x^4 - 7x^2 + \sin a)^{7-1} = 7(4x^3 - 14x)(x^4 - 7x^2 + \sin a)^6$$

$$៣. f(x) = (x^4 - 7x^2 + \sin a)^7$$

$$៤. f(x) = (x^2 - \sqrt{x})^{2019} \Rightarrow f'(x) = 2019(x^2 - \sqrt{x})'(x^2 - \sqrt{x})^{2019-1}$$

$$= 2019 \left(2x - \frac{1}{2\sqrt{x}} \right) (x^2 - \sqrt{x})^{2018}$$

$$៥. f(x) = \sqrt{x^3 - x^2 + 3} \Rightarrow f'(x) = \frac{(x^3 - x^2 + 3)'}{2\sqrt{x^3 - x^2 + 3}} = \frac{3x - 2}{2\sqrt{x^3 - x^2 + 3}}$$

$$៦. \sqrt[4]{x^3 - 2x} \Leftrightarrow f(x) = (x^3 - 2x)^{\frac{1}{4}}$$

$$f'(x) = \frac{1}{4}(x^3 - 2x)'(x^3 - 2x)^{\frac{1}{4}-1}$$

$$= \frac{1}{4}(3x^2 - 2)(x^3 - 2x)^{-\frac{3}{4}}$$

$$\therefore f'(x) = \frac{3x^2 - 2}{4\sqrt[4]{(x^3 - 2x)^3}}$$

$$៧. f(x) = (x+1)(2x-1)^2$$

$$\begin{aligned} f'(x) &= (x+1)'(2x-1)^2 + [(2x-1)^2]'(x+1) \\ &= (2x-1)^2 + 2(2x-1)'(2x-1)(x+1) \\ &= (2x-1)(2x-1+4x+4) \\ \therefore f'(x) &= (2x-1)(6x+3) \end{aligned}$$

$$៨. f(x) = (x^2+2x+3)(x^3-3x-1)$$

$$\begin{aligned} f'(x) &= (x^2+2x+3)'(x^3-3x-1) + (x^3-3x-1)'(x^2+2x+3) \\ &= (2x+2)(x^3-3x-1) + (2x-3)(x^2+2x+3) \\ &= 2x^3-6x^2-2x+2x^2-6x-2+2x^3+4x^2+6x-3x^2-6x-9 \\ \therefore f'(x) &= 4x^3-3x^2-8x-11 \end{aligned}$$

$$៩. f(x) = \frac{1}{x-1} \Rightarrow f'(x) = -\frac{(x-1)'}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

$$១០. f(x) = \frac{x\sqrt{x}}{x+1}$$

$$\begin{aligned} f'(x) &= \frac{(x\sqrt{x})'(x+1) - (x+1)'x\sqrt{x}}{(x+1)^2} \\ &= \frac{[x'\sqrt{x} + (\sqrt{x})'x](x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{\left(x + \frac{x}{2\sqrt{x}}\right)(x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{x\sqrt{x} + \sqrt{x} + \frac{x}{2\sqrt{x}}(x+1) - x\sqrt{x}}{(x+1)^2} \\ \therefore f'(x) &= \frac{x^2+3x}{2\sqrt{x}(x+1)^2} \end{aligned}$$

លំហាត់ ៣. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = x \cdot \sin x + \cos x$

៤. $f(x) = \sin^2 \sqrt{x} + \cos^2(3x)$

២. $f(x) = \sin^3 x - x \cdot \cos x$

៥. $f(x) = \cos(3x + 4) + 3 \cos x \cdot \sin x$

៣. $f(x) = \cos(x^2 + 1) + 2 \sin(x^2 - 1)$

៦. $f(x) = \sin(\sin \sqrt{x}) + \cos^3 x$

ដំណោះស្រាយ

១. $f(x) = x \cdot \sin x + \cos x$

$$f'(x) = x' \sin x + (\sin x)' \cdot x - \sin x$$

$$= \sin x + x \cdot \cos x - \sin x$$

$$\therefore f'(x) = x \cdot \cos x$$

២. $f(x) = \sin^3 x - x \cdot \cos x$

$$f'(x) = 3(\sin x)' \sin^{3-1} x - [x' \cdot \cos x + (\cos x)' \cdot x]$$

$$= 3 \cos x \cdot \sin^2 x - (\cos x - x \cdot \sin x)$$

$$\therefore f'(x) = 3 \cos x \cdot \sin^2 x - \cos x + x \sin x$$

៣. $f(x) = \cos(x^2 + 1) + 2 \sin(x^2 - 1)$

$$f'(x) = -(x^2 + 1)' \sin(x^2 + 1) + 2(x^2 - 1)' \cos(x^2 - 1)$$

$$\therefore f'(x) = -2x \sin(x^2 + 1) + 4x \cos(x^2 - 1)$$

៤. $f(x) = \sin^2 \sqrt{x} + \cos^2(3x)$

$$f'(x) = 2(\sin \sqrt{x})' \cos^{2-1} \sqrt{x} + 2(\cos(3x))' \sin(3x)$$

$$= 2(\sqrt{x})' \cdot \cos \sqrt{x} \cdot \cos \sqrt{x} - 2(3x)' \sin(3x) \cdot \sin(3x)$$

$$\therefore f'(x) = \frac{1}{\sqrt{x}} \cdot \cos^2 \sqrt{x} - 6 \sin^2(3x)$$

$$៥. f(x) = \cos(3x + 4) + 3 \cos x \cdot \sin x$$

$$\begin{aligned} f'(x) &= -(3x + 4)' \cdot \sin(3x + 4) + 3[(\cos x)' \cdot \sin x + (\sin x)' \cdot \cos x] \\ &= -3 \sin(3x + 4) + 3[-\sin x \cdot \sin x + \cos x \cdot \cos x] \\ \therefore f'(x) &= -3[\sin(3x + 4) + \sin^2 x - \cos^2 x] \end{aligned}$$

$$៦. f(x) = \sin(\sin \sqrt{x}) + \cos^3 x$$

$$\begin{aligned} f'(x) &= (\sin \sqrt{x})' \cdot \cos(\sin \sqrt{x}) + 3(\cos x) \cos^{3-1} x \\ &= (\sqrt{x})' \cdot \cos \sqrt{x} \cdot \cos(\sin \sqrt{x}) - 3 \sin x \cos^2 x \\ \therefore f'(x) &= \frac{1}{2\sqrt{x}} \cos \sqrt{x} \cdot \cos(\sin \sqrt{x}) - 3 \sin x \cdot \cos^2 x \end{aligned}$$

លំហាត់ ៤. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$១. f(x) = (1 + \tan x)^4$$

$$៣. f(x) = x \cdot \tan(x^2 - 1) + x \cot(2x^2 - 3)$$

$$២. f(x) = x^2 \tan x + (1 + \cot x)^2$$

$$៤. f(x) = \frac{\tan(2x)}{1 - \cos x}$$

ដំណោះស្រាយ

$$១. f(x) = (1 + \tan x)^4$$

$$\begin{aligned} f'(x) &= 4(1 + \tan x)'(1 + \tan^2 x)^{4-1} \\ \therefore f'(x) &= 4(1 + \tan^2 x)(1 + \tan x)^3 \end{aligned}$$

$$២. f(x) = x^2 \tan x + (1 + \cot x)^2$$

$$\begin{aligned} f'(x) &= (x^2)' \tan x + (\tan x)' x^2 + 2(1 + \cot x)'(1 + \cot x)^{2-1} \\ \therefore f'(x) &= 2x \tan x + x^2(1 + \tan^2 x) - 2(1 + \cot^2 x)(1 + \cot x) \end{aligned}$$

$$\text{៣. } f(x) = x \cdot \tan(x^2 - 1) + x \cot(2x^2 - 3)$$

$$\begin{aligned} f'(x) &= x' \tan(x^2 - 1) + [\tan(x^2 - 1)]'x + x' \cot(2x^2 - 3) + [\cot(2x^2 - 3)]'x \\ &= \tan(x^2 - 1) + (x^2 - 1)'[1 + \tan^2(x^2 - 1)]x - (2x^2 - 3)'[1 + \cot^2(2x^2 - 3)]x \\ \therefore f'(x) &= \tan(x^2 - 1) + 2x^2[1 + \tan^2(x^2 - 1)] - 4x^2[1 + \cot^2(2x^2 - 3)] \end{aligned}$$

លំហាត់ ៥. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$\begin{aligned} \text{១. } f(x) &= \frac{1 - x - 2x^2}{x^3 - \ln 3} \\ \text{២. } f(x) &= \frac{2x^2 + 3x + 4}{\sqrt{1 + 2x - x^2}} \end{aligned}$$

$$\begin{aligned} \text{៣. } f(x) &= \sin x^2 \cdot \tan(2x + 3) \\ \text{៤. } f(x) &= \sin(x^2 + 5) + \cos(\sin x) \\ \text{៥. } f(x) &= \frac{\sin(\tan \sqrt{x})}{\sin(\sqrt{x})} \end{aligned}$$

ដំណោះស្រាយ

$$\text{១. } f(x) = \frac{1 - x - 2x^2}{x^3 - \ln 3}$$

$$\begin{aligned} f'(x) &= \frac{(1 - x - 2x^2)'(x^3 - \ln 3) - (x^3 - \ln 3)'(1 - x - 2x^2)}{(x^3 - \ln 3)^2} \\ &= \frac{(-1 - 4x)(x^3 - \ln 3) - 3x^2(1 - x - 2x^2)}{(x^3 - \ln 3)^2} \\ &= \frac{-x^3 + \ln 3 - 4x^4 + 4x \ln 3 - 3x^2 + 3x^3 + 6x^4}{(x^3 - \ln 3)^2} \\ \therefore f'(x) &= \frac{2x^4 + 2x^3 - 3x^2 + 4x \cdot \ln 3 + \ln 3}{(x^3 - \ln 3)^2} \end{aligned}$$

$$\text{២. } f(x) = \frac{2x^2 + 3x + 4}{\sqrt{1 + 2x - x^2}} \Leftrightarrow f(x) \cdot \sqrt{1 + 2x - x^2} = 2x^2 + 3x + 4 \text{ ធ្វើដេរីវេអង្គសងខាង គេបាន}$$

$$[f(x)\sqrt{1 + 2x - x^2}]' = (2x^2 + 3x + 4)'$$

$$f'(x)\sqrt{1 + 2x - x^2} + (\sqrt{1 + 2x - x^2})'f(x) = 4x + 3$$

$$f'(x)\sqrt{1 + 2x - x^2} + \frac{(1 + 2x - x^2)'}{2\sqrt{1 + 2x - x^2}}f(x) = 4x + 3$$

$$f'(x)\sqrt{1 + 2x - x^2} = 4x + 3 - \frac{1 - x}{\sqrt{1 + 2x - x^2}} \cdot f(x)$$

$$\therefore f'(x) = \frac{4x + 3}{\sqrt{1 + 2x - x^2}} + \frac{(x - 1)(2x^2 + 3x + 4)}{(1 + 2x - x^2)\sqrt{1 + 2x - x^2}}$$

$$\text{៣. } f(x) = \sin x^2 \cdot \tan(2x + 3)$$

$$\begin{aligned} f'(x) &= (\sin x^2)' \tan(2x + 3) + (\tan(2x + 3))' \sin x^2 \\ &= (x^2)' \cdot \sin x^2 \cdot \tan(2x + 3) + (2x + 3)' [1 + \tan^2(2x + 3)] \sin x^2 \\ &= 2x \sin x^2 \cdot \tan(2x + 3) + 2 \sin x^2 [1 + \tan^2(2x + 3)] \\ \therefore f'(x) &= 2 \sin x^2 [\tan^2(2x + 3) + x \tan(2x + 3) + 1] \end{aligned}$$

$$\text{៤. } f(x) = \sin(x^2 + 5) + \cos(\sin x)$$

$$\begin{aligned} f'(x) &= (x^2 + 5)' \cos(x^2 + 5) - (\sin x)' \sin(\sin x) \\ \therefore f'(x) &= 2x \cos(x^2 + 5) - \cos x \sin(\sin x) \end{aligned}$$

$$\text{៥. } f(x) = \frac{\sin(\tan \sqrt{x})}{\sin(\sqrt{x})} \iff f(x) \cdot \sin \sqrt{x} = \sin(\tan \sqrt{x})$$

$$(f(x) \cdot \sin \sqrt{x})' = (\sin(\tan \sqrt{x}))'$$

$$f'(x) \cdot \sin \sqrt{x} + (\sin \sqrt{x})' f(x) = (\tan \sqrt{x})' \cos(\tan \sqrt{x})$$

$$f'(x) \cdot \sin \sqrt{x} + (\sqrt{x})' \cos \sqrt{x} \cdot f(x) = (\sqrt{x})' (1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x})$$

$$f'(x) \sin \sqrt{x} + \frac{1}{2\sqrt{x}} \cos \sqrt{x} \cdot f(x) = \frac{1}{2\sqrt{x}} (1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x})$$

$$f'(x) \sin \sqrt{x} = \frac{1}{2\sqrt{x}} [(1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) - \cos \sqrt{x} f(x)]$$

$$\therefore f'(x) = \frac{1}{2\sqrt{x} \cdot \sin \sqrt{x}} [(1 + \tan^2 \sqrt{x}) \cos(\tan \sqrt{x}) - \cos \sqrt{x} f(x)]$$

$$, f(x) = \frac{\sin(\tan \sqrt{x})}{\sin(\sqrt{x})}$$

លំហាត់ ៦. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$\text{១. } f(x) = xe^x + \frac{1}{2}x^2$$

$$\text{២. } f(x) = e^{x^2+2x+1} + (x^2-3)e^x$$

$$\text{៣. } f(x) = \frac{\sqrt{x}}{e^x}$$

$$\text{៤. } f(x) = x^3 e^{-3x}$$

$$\text{៥. } f(x) = e^{2x} 3^{x^2+1}$$

$$\text{៦. } f(x) = e^{\sin x \cos x}$$

ដំណោះស្រាយ

$$9. f(x) = xe^x + \frac{1}{2}x^2$$

$$f'(x) = x'e^x + (e^x)'x + \frac{1}{2}.2x = e^x + e^xx + x = e^x(1+x) + x$$

$$10. f(x) = e^{x^2+2x+1} + (x^2-3)e^x$$

$$\begin{aligned} f'(x) &= (x^2+2x+1)'e^{x^2+2x+1} + (x^2-3)'e^x + (e^x)'(x^2-3) \\ &= (2x+2)e^{x^2+2x+1} + 2xe^x + e^x(x^2-3) \end{aligned}$$

$$\therefore f'(x) = 2(x+1)e^{x^2+2x+1} + e^x(2x+x^2-3)$$

$$11. f(x) = \frac{\sqrt{x}}{e^x}$$

$$f'(x) = \frac{(\sqrt{x})'e^x + (e^x)'\sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}}e^x + e^x\sqrt{x}}{e^{2x}} = \frac{1+2x}{2\sqrt{x}e^x}$$

$$12. f(x) = x^3e^{-3x}$$

$$\begin{aligned} f'(x) &= (x^3)'e^{-3x} + (e^{-3x})'x^3 \\ &= 3x^2e^{-3x} + (-3x)'e^{-3x}x^3 \\ &= 3x^2e^{-3x} - 3e^3e^{-3x} \end{aligned}$$

$$\therefore f'(x) = 3x^2e^{-3x}(1-x)$$

$$13. f(x) = e^{2x}3^{x^2+1}$$

$$\begin{aligned} f'(x) &= (e^{2x})'3^{x^2+1} + (3^{x^2+1})'.e^{2x} \\ &= (2x)'e^{2x}.3^{x^2+1} + (x^2+1)'3^{x^2+1} \ln 3.e^{2x} \\ &= 2.e^{2x}3^{x^2+1} + 2x3^{x^2+1} \ln 3.e^{2x} \end{aligned}$$

$$\therefore f'(x) = 2e^{2x}3^{x^2+1}(1+x \ln 3)$$

៦. $f(x) = e^{\sin x \cos x}$

$$\begin{aligned} f'(x) &= (\sin x \cos x)' e^{\sin x \cos x} \\ &= [(\sin x)' \cos x + (\cos x)' \sin x] e^{\sin x \cos x} \\ \therefore f'(x) &= (\cos^2 x - \sin^2 x) e^{\sin x \cos x} \end{aligned}$$

លំហាត់ ៧. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

១. $f(x) = (x^2 - 1) \ln(x^2 - 1)$

៣. $f(x) = \ln(\sin x \cdot \cos(2x))$

២. $f(x) = \ln\left(\frac{x^2 - 2}{\sqrt[3]{x^2 - 2}}\right)$

៤. $f(x) = \ln\left(\sqrt{\frac{1 + \sin x}{1 - \sin x}}\right)$

ដំណោះស្រាយ

$$\begin{aligned} ១. f(x) &= (x^2 - 1) \ln(x^2 - 1) \Rightarrow f'(x) = (x^2 - 1)' \ln(x^2 - 1) + \ln(x^2 - 1)' (x^2 - 1) \\ &= 2x \ln(x^2 - 1) + \frac{(x^2 - 1)'}{x^2 - 1} \cdot (x^2 - 1) \\ &= 2x \ln(x^2 - 1) + 2x \\ \therefore f'(x) &= 2x[\ln(x^2 - 1) + 1] \end{aligned}$$

២. $f(x) = \ln\left(\frac{x^2 - 2}{\sqrt[3]{x^2 - 2}}\right) = \ln(x^2 - 2) - \ln(x^2 - 2)^{\frac{1}{3}}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - 2)'}{x^2 - 2} - \frac{1}{3} \cdot \frac{(x^2 - 2)'}{x^2 - 2} \\ &= \frac{3(2x) - 2x}{3(x^2 - 2)} \\ \therefore f'(x) &= \frac{4x}{3(x^2 - 2)} \end{aligned}$$

$$\text{៣. } f(x) = \ln(\sin x \cdot \cos(2x)) = \ln(\sin x) + \ln(\cos(2x))$$

$$\begin{aligned} f'(x) &= \frac{(\sin x)'}{\sin x} + \frac{(\cos(2x))'}{\cos(2x)} \\ &= \frac{\cos x}{\sin x} - \frac{2 \sin(2x)}{\cos(2x)} \end{aligned}$$

$$\therefore f'(x) = \cot x - 2 \tan(2x)$$

$$\text{៤. } f(x) = \ln \left(\sqrt{\frac{1 + \sin x}{1 - \sin x}} \right) = \ln \left(\frac{1 + \sin x}{1 - \sin x} \right)^{\frac{1}{2}} = \frac{1}{2} (\ln(1 + \sin x) - \ln(1 - \sin x))$$

$$\begin{aligned} f'(x) &= \frac{1}{2} \left(\frac{(1 + \sin x)'}{1 + \sin x} - \frac{(1 - \sin x)'}{1 - \sin x} \right) \\ &= \frac{1}{2} \left(\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} \right) \\ &= \frac{1}{2} \cdot \frac{(\cos x(1 - \sin x) + \cos x(1 + \sin x))}{1 - \sin^2 x} \\ \therefore f'(x) &= \frac{2 \cos x}{2 \cos^2 x} = \frac{1}{\cos x} \end{aligned}$$

បំណាច់ ៨. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$\text{១. } f(x) = \cos(\arcsin x)$$

$$\text{៥. } f(x) = \arcsin \sqrt{x}$$

$$\text{២. } f(x) = \cot(\arctan x)$$

$$\text{៦. } f(x) = \arctan(\sin x)$$

$$\text{៣. } f(x) = \tan(\arctan x)$$

$$\text{៧. } f(x) = \frac{\arctan x}{\arcsin x}$$

$$\text{៤. } f(x) = \arcsin(2x)$$

ដំណោះស្រាយ

$$\text{១. } f(x) = \cos(\arcsin x) \Rightarrow f'(x) = -(\arcsin x)' \sin(\arcsin x)$$

$$\therefore f'(x) = -\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

$$\text{២. } f(x) = \cot(\arctan x) \Rightarrow f'(x) = -(\arctan x)' [1 + \cot^2(\arctan x)]$$

$$\therefore f'(x) = -\frac{1 + \cot^2(\arctan x)}{1 + x^2}$$

$$\text{៣. } f(x) = \tan(\arctan x) \Rightarrow f'(x) = (\arctan x)' [1 + \tan^2(\arctan x)]$$

$$\therefore f'(x) = \frac{1 + \tan^2(\arctan x)}{1 + x^2}$$

$$៤. f(x) = \arcsin(2x) \Rightarrow f'(x) = \frac{(2x)'}{\sqrt{1-(2x)^2}}$$

$$\therefore f'(x) = \frac{2}{\sqrt{1-4x^2}}$$

$$៥. f(x) = \arcsin \sqrt{x} \Rightarrow f'(x) = \frac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}} = \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x^2}} = \frac{1}{2\sqrt{x}\sqrt{1-x^2}}$$

$$\therefore f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

$$៦. f(x) = \arctan(\sin x) \Rightarrow f'(x) = \frac{(\sin x)'}{1+(\sin x)^2}, \sin^2 x + \cos^2 x = 1$$

$$\therefore f'(x) = \frac{\cos x}{2-\cos^2 x}$$

$$៧. f(x) = \frac{\arctan x}{\arcsin x} \Rightarrow f'(x) = \frac{(\arctan x)' \arcsin x - (\arcsin x)' \arctan x}{(\arcsin x)^2}$$

$$\therefore f'(x) = \frac{\frac{\arcsin x}{1+x^2} - \frac{\arctan x}{\sqrt{1-x^2}}}{(\arcsin x)^2}$$

១១ លំហាត់មេរៀន

១. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

(ក) $y = x^3 + 2x^2$

(ខ) $y = x^3 - 4x^2$

(គ) $y = x^4 - 27x$

(ឃ) $y = x^4 - 5x^2 + 4$

(ង) $y = x^5 - 16x$

(ច) $y = \frac{x}{x+1}$

(ឆ) $y = \frac{x^2}{1+x^2}$

(ជ) $y = x - \frac{1}{x}$

(ឈ) $y = x^3 + 2x^2 - x$

(ញ) $y = x^4 - 2x^3 + 2x$

(ដ) $y = \sqrt{1+x^2}$

(ឋ) $y = \sqrt[4]{1+x^2}$

២. រក $f'(x)$ នៃអនុគមន៍ខាងក្រោម៖

(ក) $f(x) = \sin x + \cos x$

(ខ) $f(x) = 2 \sin x - 3 \cos x$

(គ) $f(x) = 3 \sin x + 2 \cos x$

(ឃ) $f(x) = x \sin x + \cos x$

(ង) $f(x) = x \cos x - \sin x$

(ច) $f(x) = \cos(2x)$

(ឆ) $f(x) = \frac{1 - \sin(2x)}{1 - \sin x}$

(ជ) $f(x) = 1 + \sin x^2$

(ឈ) $f(x) = \cot x - \cos x$

(ញ) $f(x) = \sin(2x) - \cos(3x)$

(ដ) $f(x) = \sin(\cos(3x))$

(ឋ) $f(x) = \frac{\sin x^2}{x^2}$

(ឌ) $f(x) = \tan(1 + x^2)$

(ឍ) $f(x) = \cos 2x - \cos x^2$

(ណ) $f(x) = (1 + \sqrt{1+x})^3$

៣. រក y' នៃអនុគមន៍ខាងក្រោម៖

(ក) $xy = \frac{\pi}{6}$

(ខ) $\sin(xy) = 1$

(គ) $xy = \frac{1}{x+y}$

(ឃ) $x + y = xy$

(ង) $(y-1)^2 + x = 0$

(ច) $(y+1)^2 + y - x = 0$

(ឆ) $(y-x)^2 + x = 0$

(ជ) $(y+x) + 2y - x = 0$

(ឈ) $(y^2 - 1)^2 + x = 0$

(ញ) $(y^2 + 1)^2 - x = 0$

(ដ) $x^3 + xy + y^3 = 3$

(ឋ) $\sin x + \sin y = 1$

(ឌ) $\sin x + xy + y^5 = \pi$

(ឍ) $\tan x + \tan y = 1$

៤. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$(ក) f(x) = \sqrt{1-x}$$

$$(ខ) f(x) = \sqrt[4]{x+x^2}$$

$$(គ) y = \sqrt{1-\sqrt{x}}$$

$$(ឃ) y = \sqrt{x-\sqrt{x}}$$

$$(ង) y = \sqrt[3]{\sqrt{2x+1}} - x^2$$

$$(ច) y = \sqrt[4]{x+x^2}x + x^2$$

$$(ឆ) y = \sqrt[3]{x-\sqrt{2x+1}}$$

$$(ជ) y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$$

៥. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$(ក) f(x) = e^x + e^{-x}$$

$$(ខ) f(x) = e^{3x} + 4e^x$$

$$(គ) f(x) = \frac{e^x}{1+e^x}$$

$$(ឃ) f(x) = \frac{2e^{2x}}{1+e^{2x}}$$

$$(ង) f(x) = xe^{-x} + x \ln x$$

$$(ច) f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2e^{x+2}$$

$$(ឆ) f(x) = x^{-\frac{1}{2}x} + \ln \sqrt{x}$$

$$(ជ) f(x) = (\ln x)^2 + \ln x + 1$$

$$(ឈ) f(x) = \frac{\ln x}{x} + \ln \frac{1}{x}$$

$$(ញ) f(x) = \ln \left(\sqrt{\frac{\sqrt{x}+1}{\sqrt{x}-1}} \right)$$

៦. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$(ក) f(x) = \tan(\arctan x)$$

$$(ខ) f(x) = \arcsin(\sin x)$$

$$(គ) f(x) = \cot(\arcsin x)$$

$$(ឃ) f(x) = \sin(\arctan x)$$

$$(ង) f(x) = (\arcsin x)^2$$

$$(ច) f(x) = \frac{1}{1+(\arctan x)^2}$$

$$(ឆ) f(x) = \sqrt{1-(\arcsin x)^2}$$

៧. គណនាដេរីវេនៃអនុគមន៍ខាងក្រោម៖

$$(ក) y = (x+1)(x-1)$$

$$(ខ) y = (x^2+1)(x^2-1)$$

$$(គ) y = \frac{1}{x+1} + \frac{1}{1+\sin x}$$

$$(ឃ) y = \frac{1}{1+x^2} + \frac{1}{1-\sin x}$$

$$(ង) y = (x-1)(x-2)(x-3)$$

$$(ច) y = x^2 \cos x + 2x \sin x$$

$$(ឆ) y = x^{\frac{1}{2}}(x + \sin x)$$

$$(ជ) y = x^{\frac{1}{2}} \sin^2 x + (\sin x)^{\frac{1}{2}}$$

$$(ឈ) y = x^4 \cos x + x \cos x$$

$$(ញ) y = \frac{1}{2}x^2 \sin x - x \cos x + \sin x$$

$$(ដ) y = \sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$$

$$(ហ) y = (x-6)^{10} + \sin^{10} x$$

$$(ខ) y = (\sin x \cos x)^3 + \sin(2x)$$

$$(ឈ) y = x^{\frac{1}{2}} \sin(2x) + (\sin x)^{\frac{1}{2}}$$

$$(ណ) y = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$(ត) y = \frac{1}{\tan x} - \frac{1}{\cot x}$$