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September 4, 2018

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1.

1.1

Definition 1.1.1 $y = f(x)$ a

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \tag{1.1}$$

- f (b,c) f $a \in (b,c)$
- f $[b,c]$ f (b,c) f $x = b$ $x = c$

■ **Example 1.1** $y = f(x) = 2x^2 + 3$ 2 ■

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \quad f(2) = 2(2)^2 + 3 = 11$$

$$\begin{aligned} f'(2) &= \lim_{x \to 2} \frac{2x^2 + 3x - 11}{x - 2} \\ &= \lim_{x \to 2} \frac{2x^2 - 8}{x - 2} \\ &= \lim_{x \to 2} \frac{2(x+2)(x-2)}{x - 2} \ , x \neq 2 \\ &= 2 \times 4 = 8 \end{aligned}$$

1.1.1

$$h = x - a \Longrightarrow x = h + a \;\; h \longrightarrow 0 \;\; x \longrightarrow a \;\; (1.1)$$

$$f'(a) = \lim_{h\to 0} \frac{f(h+a) - f(a)}{h} \tag{1.2}$$

Notation 1.1. y' , $f'(x)$ $\dfrac{dy}{dx}$

■ **Example 1.2** $y = x$ $y' = 1$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(h+x) - f(x)}{h} \quad y = f(x) = x, f(h+x) = h + x \\ &= \lim_{h=\to 0} \frac{h+x-x}{h} = \lim_{h\to 0} 1 = 1 \\ \therefore \quad \frac{dy}{dx} &= 1 \end{aligned}$$

■

Theorem 1.1.1 f x_0 f x_0

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \quad \lim_{x\to a} f(x) = f(a)$$

$$\begin{aligned} \lim_{x\to a} f(x) &= \lim_{x\to a} (f(x) - f(a) + f(a)) \\ &= \lim_{x\to a} \left(\frac{f(x) - f(a)}{x - a} \times (x - a) + f(a) \right) \\ &= \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \times \lim_{x\to a} (x - a) + f(a) \\ &= f'(a) \times 0 + f(a) \\ &= f(a) \end{aligned}$$

Notation 1.2. f x_0 f x_0 x_0

1.2

Definition 1.2.1 f x

- f x
- x $f'_{-}(x) = f'_{+}(x)$

  $f'_{-}(x) = \lim_{h\to 0^-} \dfrac{f(x+h)-f(x)}{h}$ $f'_{+}(x) = \lim_{h\to 0^+} \dfrac{f(x+h)-f(x)}{h}$

■ **Example 1.3** f $f(x) = \begin{cases} \cos x & x \leq \frac{\pi}{4} \\ a+bx & x > \frac{\pi}{4} \end{cases}$

a b f $x = \frac{\pi}{4}$ ■

- f $x = \frac{\pi}{4}$ $\lim_{x\to\frac{\pi}{4}^-} f(x) = \lim_{x\to\frac{\pi}{4}^+} f(x) = f\left(\frac{\pi}{4}\right)$

  $\lim_{x\to\frac{\pi}{4}^-} \cos x = \lim_{x\to\frac{\pi}{4}^+} (a+bx) = \cos\frac{\pi}{4} \Longleftrightarrow \dfrac{\sqrt{2}}{2} = a + b.\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2} \Longrightarrow a = \dfrac{\sqrt{2}}{2} - \dfrac{\pi}{4}.b$
- $f'_{-}(x)$

$$\begin{aligned}
f'_{-}\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^-} \frac{f(\frac{\pi}{4}+h) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = \cos x \\
&= \lim_{h\to 0^-} \frac{\cos(\frac{\pi}{4}+h) - \cos\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-} \frac{\cos\frac{\pi}{4}\cos h - \sin\frac{\pi}{4}\sin h - \cos\dfrac{\pi}{4}}{h} \\
&= \lim_{h\to 0^-} \frac{-\cos\frac{\pi}{4}(1-\cos h) - \sin\frac{\pi}{4}\sin h}{h} \\
&= \frac{\sqrt{2}}{2} \lim_{h\to 0^-} \left(-\frac{1-\cos h}{h} - \frac{\sin h}{h} \right), \lim_{h\to 0^-} \frac{1-\cos h}{h} = 0, \lim_{h\to 0^-} \frac{\sin h}{h} = 1 \\
&= \frac{\sqrt{2}}{2}(0-1) = -\frac{\sqrt{2}}{2}
\end{aligned}$$

- $f'_{+}(x)$

$$\begin{aligned}
f'_{+}\left(\frac{\pi}{4}\right) &= \lim_{h\to 0^+} \frac{f(\frac{\pi}{4}+h) - f\left(\frac{\pi}{4}\right)}{h}, f(x) = a+bx \\
&= \lim_{h\to 0^+} \frac{a + b(\frac{\pi}{4}+h) - (a + b.\frac{\pi}{4}))}{h} \\
&= \lim_{h\to 0^+} \frac{a + b.\frac{\pi}{4} + bh - a - b.\frac{\pi}{4}}{h} \\
&= \lim_{h\to 0^+} \frac{bh}{h} = b
\end{aligned}$$

$$f \quad x = \tfrac{\pi}{4} \quad f'_{-}(\tfrac{\pi}{4}) = f'_{+}(\tfrac{\pi}{4}) \Longleftrightarrow b = -\frac{\sqrt{2}}{2} \Longrightarrow a = \frac{\sqrt{2}}{2}\left(1 + \frac{\pi}{4}\right)$$

1.3

$u, v \quad x \quad k \quad 3$

1. $(ku)' = ku'$
2. $(u+v)' = u' + v$
3. $(u-v)' = u' - v'$
4. $(uv)' = u'v + v'u$
5. $\left(\dfrac{u}{v}\right)' = \dfrac{u'v - v'u}{v^2}$
6. $\left(\dfrac{1}{v}\right)' = -\dfrac{v'}{v^2}$

1. $f(x) = k.u(x) \quad u = u(x) \quad k$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{ku(x+h) - k.u(x)}{h} \\ &= k.\lim_{h\to 0} \frac{u(x+h) - u(x)}{h} \\ &= k.u'(x) \end{aligned}$$

$$\therefore \quad (k.u)' = k.u'$$

2. $f(x) = u(x) + v(x) \quad u = u(x) \quad v = v(x)$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{u(x+h) + v(x+h) - (u(x) + v(x))}{h} \\ &= \lim_{h\to 0} \frac{u(x+h) - u(x)}{h} + \lim_{h\to 0} \frac{v(x+h) - v(x)}{h} \\ &= u'(x) + v'(x) \end{aligned}$$

$$\therefore \quad (u+v)' = u' + v'$$

3.

4. $f(x)=uv \;\; u=u(x) \;\; v=v(x)$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{u(x+h).v(x+h)-u(x).v(x)}{h}\\
&= \lim_{h\to 0}\frac{u(x+h).v(x+h)-u(x).v(x+h)+u(x).v(x+h)+u(x).v(x)}{h}\\
&= \lim_{h\to 0}\left[\frac{u(x+h).v(x+h)-u(x).v(x+h)}{h}+\frac{u(x).v(x+h)+u(x).v(x)}{h}\right]\\
&= \lim_{h\to 0}\left[v(x+h).\frac{u(x+h)-u(x)}{h}+u(x).\frac{v(x+h)+v(x)}{h}\right]\\
&= v(x).\frac{d}{dx}(u(x))+u(x).\frac{d}{dx}(v(x))
\end{aligned}
$$

$$\therefore \quad (uv)'=u'v+v'u \tag{1.3}$$

5. $u=u(x) \;\; v=v(x) \;\; f(x)=\dfrac{u}{v}\Leftrightarrow f(x).v=u \;\; x$

$[f(x).v]'=u'$ (1.3)

$$
\begin{aligned}
f'(x).v+v'f(x) &= u',\; f(x)=\frac{u}{v}\\
f'(x).v+v'.\frac{u}{v} &= u'\\
\frac{f'(x).v^2}{v}+\frac{v'u}{v} &= u'\\
f'(x).v^2+v'u &= u'v\\
f'(x) &= \frac{u'v-v'u}{v^2}
\end{aligned}
$$

$$\therefore \quad \left(\frac{u}{v}\right)'=\frac{u'v-v'u}{v^2} \tag{1.4}$$

6. $v=v(x) \;\; f(x)=\dfrac{1}{v}$ (1.4)

$$
\begin{aligned}
f'(x) &= \frac{(1)'.v-v'.(1)}{v^2}\\
&= \frac{0-v'}{v^2}\\
&= -\frac{v'}{v^2}
\end{aligned}
$$

$$\therefore \quad \left(\frac{1}{v}\right)'=-\frac{v'}{v^2}$$

1.4

1 $y=f(u)$ $u=g(x)$ $\frac{d}{dx}(f\circ g)=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

$F(x)=f\circ g=f(g(x))$ $x=a$

$$
\begin{aligned}
F'(a) &= \lim_{x\to a}\frac{F(x)-F(a)}{x-a}\\
&= \lim_{x\to a}\frac{f(g(x))-f(g(a))}{x-a}\\
&= \lim_{x\to a}\left(\frac{f(g(x))-f(g(a))}{g(x)-g(a)}\times\frac{g(x)-g(a)}{x-a}\right)\\
&= f'(g(a))\times g'(a) \quad ,u=g(a),y=f(a)
\end{aligned}
$$

$$\therefore \quad \frac{d}{dx}(f\circ g)=\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

2 $y=c$ c $y'=0$

$y=f(x_0)=c$ $f(x_0+h)=c$ $,c\in\mathbb{R}$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{c-c}{h}\\
&= \lim_{h\to 0}\frac{0}{h}
\end{aligned}
$$

$$\therefore \quad \frac{d}{dx}(c)=0$$

■ **Example 1.4** y' $y=\left(\ln x.\log_a(\sqrt{3})\right)$ ■

$$y=\left(\ln x.\log_a(\sqrt{3})\right)\Rightarrow y'=\left(\ln x.\log_a(\sqrt{3})\right)'=0$$

■ **Example 1.5** $y=x^n$ $y'=nx^{n-1}$ ■

$f(x) = x^n$ $f(x+h) = (x+h)^n$

$$\begin{aligned}
y' = f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} \\
&= \lim_{h\to 0} \frac{(x+h-x)(x^{n-1}+x^{n-2}.x+\ldots+x.x^{n-2}x+x^{n-1})}{h} \\
&= \lim_{h\to 0}(x^{n-1}+x^{n-1}+\ldots+x^{n-1}+x^{n+1}) \\
&= x^{n-1}(\underbrace{1+1+\ldots+1+1}_{n\ 1}) \\
&= n.x^{n-1}
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(x^n) = n.x^{n-1}$$

■ **Example 1.6** $f'(x)$ 3

1. $f(x) = x^3$
2. $f(x) = \sqrt{x}$
3. $f(x) = \sqrt[3]{x^2}$

■

1. $f(x) = x^3 \Rightarrow f'(x) = (x^3)' = 3x^{3-1} = 3x^2$
2. $f(x) = \sqrt{x} \Rightarrow f'(x) = (\sqrt{x})' = (x^{\frac{1}{2}})' = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
3. $f(x) = \sqrt[3]{x^2} \Rightarrow f'(x) = (\sqrt[3]{x^2})' = (x^{\frac{2}{3}})' = \frac{2}{3}x^{\frac{2}{3}-1} = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

3 $y = u^n$ u x $y' = nu'u^{n-1}$

$$y = u^n \quad y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(u^n) \times u' = nu'u^{n-1}$$

■ **Example 1.7** y' 2

1. $y = (2x + \ln 2)^4$
2. $y = \sqrt{u}$ u x

■

1. $y=(2x+\ln 2)^4 \Longrightarrow y'=4(2x+\ln 2)'(2x+\ln 2)^{4-1}=4(2+0)(2x+\ln 2)^3$

$$\therefore \quad y'=8(2x+\ln 2)^3$$

2. $y=\sqrt{u}=u^{\frac{1}{x}} \Longrightarrow y'=(u^{\frac{1}{2}})'=\frac{1}{2}u'u^{\frac{1}{2}-1}=\frac{1}{2}u'u^{-\frac{1}{2}}=\frac{u'}{2\sqrt{u}}$

1.5

1. $y=\sin x \;\; y'=\cos x$
2. $y=\cos x \;\; y'=-\sin x$
3. $y=\tan x \;\; y'=\dfrac{1}{\cos^2 x}=1+\tan^2 x$
4. $y=\cot x \;\; y'=-\dfrac{1}{\sin^2 x}=-(1+\cot^2 x)$

1. $y=f(x)=\sin x \;\; f(x+h)=\sin(x+h)$

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}\\
&=\lim_{h\to 0}\frac{\sin x\cos h+\sin h.\cos x-\sin x}{h}\\
&=\lim_{h\to 0}\left(\cos x.\frac{\sin h}{h}-\sin x.\frac{1-\cos h}{h}\right)\\
&=\cos x \quad ,\lim_{h\to 0}\frac{\sin h}{h}=1,\lim_{h\to 0}\frac{1-\cos h}{h}=0
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(\sin x)=\cos x$$

2. $y=f(x)=\cos x \;\; f(x+h)=\cos(x+h)$

$$\begin{aligned}
y'=f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}\\
&= \lim_{h\to 0}\frac{\cos x.\cos h-\sin x.\sin h-\cos x}{h}\\
&= \lim_{h\to 0}\left(-\frac{\sin h}{h}.\sin x-\cos x.\frac{1-\cos h}{h}\right)\\
&= -\sin x \quad , \lim_{h\to 0}\frac{1-\cos h}{h}=0, \lim_{h\to 0}\frac{\sin h}{h}=1
\end{aligned}$$

$$\therefore \quad \frac{d}{dx}(\cos x)=-\sin x$$

3. $y=\tan x=\dfrac{\sin x}{\cos x}$ (1.4)

$$\begin{aligned}
y'=\left(\frac{\sin x}{\cos x}\right)' &= \frac{(\sin x)'\cos x-(\cos x)'.\sin x}{(\cos x)^2}\\
&= \frac{\cos x.\cos x-(-\sin x).\sin x}{\cos^2 x}\\
&= \frac{\cos^2 x+\sin^2 x}{\cos^2 x}\\
&= 1+\tan^2 x\\
&= \frac{1}{\cos^2 x},\ \sin^2 x+\cos^2 x=1
\end{aligned}$$

$$\therefore \quad (\tan x)'=\frac{1}{\cos^2}=1+\tan^2 x$$

4. $y=\cot x=\dfrac{\cos x}{\sin x}$ (1.4)

$$\begin{aligned}
y'=\left(\frac{\cos x}{\sin x}\right)' &= \frac{(\cos x)'.\sin x-(\sin x)'.\cos x}{(\sin^2 x)^2}\\
&= \frac{-\sin x.\sin x-\cos x.\cos x}{\sin^2 x}\\
&= \frac{-\sin^2 x-\cos^2 x}{\sin^2 x}\\
&= -\frac{\sin^2 x+\cos^2 x}{\sin^2 x}\ ,\sin^2 x+\cos^2 x=1
\end{aligned}$$

$$\therefore \quad (\cot x)'=-\frac{1}{\sin^2 x}=-(1+\cot^2 x)$$

4 u x

1. $y=\sin u \quad y'=u'\cos u$
2. $y=\cos u \quad y'=-u'\sin u$
3. $y=\tan u \quad y'=\dfrac{u'}{\cos^2 u}=u(1+\tan^2 u)$
4. $y=\cot u \quad y'=-\dfrac{u'}{\sin^2 u}=-u'(1+\cot^2 u)$

1. $u \ x \ y=\sin u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\sin u)\times\frac{du}{dx}=\cos u\times u'=u'\cos u$$
$$\therefore \quad \frac{d}{dx}(\sin u)=u'\cos u$$

2. $u \ x \ y=\cos u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\cos u)\times\frac{du}{dx}=-\sin u\times u'=-u'\sin u$$
$$\therefore \quad \frac{d}{dx}(\cos u)=-u'\sin u$$

3. $u \ x \ y=\tan u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\tan u)\times\frac{du}{dx}=\frac{1}{\cos^2 u}\times u'=(1+\tan^2 u)\times u'$$
$$\therefore \quad (\tan u)'=\frac{u'}{\cos^2 u}=u'(1+\tan^2 u)$$

4. $u \ x \ y=\cot u$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(\cot u)\times\frac{du}{dx}=-\frac{1}{\sin^2 u}\times u'=-(1+\cot^2 u)\times u'$$
$$\therefore \quad (\cot u)'=-\frac{u'}{\sin^2 u}=-u'(1+\cot^2 u)$$

■ **Example 1.8** 2

1. $y=\sin(2x+1)$
2. $y=\cos(2x+1)$
3. $y=\tan(2x+1)$
4. $y=\cot(2x+1)$

■

1. $y=\sin(2x+1) \Rightarrow y'=(2x+1)'\cos(2x+1)=2\cos(2x+1)$
2. $y=\cos(2x+1) \Rightarrow y'=-(2x+1)'\sin(2x+1)=-2\sin(2x+1)$
3. $y=\tan(2x+1) \Rightarrow y'=\dfrac{(2x+1)'}{\cos^2(2x+1)}=\dfrac{2}{\cos^2(2x+1)}=2[1+\tan^2(2x+1)]$
4. $y=\cot(2x+1) \Rightarrow y'=-\dfrac{(2x+1)'}{\sin^2(2x+1)}=-\dfrac{2}{\sin^2(2x+1)}=-2[1+\cot^2(2x+1)]$

1.6

$y=a^x$ $y'=a^x.\ln a$

$y=a^x$

$$\begin{aligned}
y'=f'(x)&=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to 0}\frac{a^{x+h}-a^x}{h}\\
&=\lim_{h\to 0}\frac{a^x(a^h-1)}{h}\\
&=a^x\lim_{h\to 0}\frac{a^h-1}{h} \quad \lim_{x\to 0}\frac{a^x-1}{x}=\ln a
\end{aligned}$$

$$\therefore \quad (a^x)'=a^x.\ln a$$

5 u x $(a^u)'=u'a^u.\ln a$

u x

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\frac{d}{du}(a^u)\times\frac{du}{dx}=a^u.\ln a\times u'$$

$$\therefore \quad (a^u)'=u'.a^u.\ln a)$$

■ **Example 1.9** y' u x 3

1. $y=e^x$

2. $y = a^{x^2-1}$

3. $y = e^u$

■

1. $y = e^x$ $y' = (e^x)' = e^x.\ln e = e^x$, $\ln e = 1$

2. $y = a^{x^2-1}$ $y' = (x^{x^2-1})' a^{x^2-1} \ln a = 2x.a^{x^2-1} \ln a$

3. $y = e^u$ $y' = (e^u)' = u'e^u.\ln e = u'e^u$, $\ln e = 1$

1.7

$y = \log_a x$, $a > 0, a \neq 1$ $y' = \dfrac{1}{x \ln a}$

$y = \log_a x$

$$
\begin{aligned}
y' = f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\log_a(x+h) - \log_a(x)}{h} \\
&= \lim_{h \to 0} \frac{1}{h} \log_a \left(\frac{x+h}{x} \right) \\
&= \lim_{h \to 0} \log_a \left(1 + \frac{h}{x} \right)^{\dfrac{1}{h}} \\
&= \log_a \left(\lim_{h \to 0} \left(1 + \frac{1}{\frac{x}{h}} \right)^{\dfrac{x}{h}} \right)^{\dfrac{1}{x}} \quad \lim_{x \to 0} (1 + \frac{1}{x})^x = e \\
&= \log_a e^{\dfrac{1}{x}} = \frac{1}{x} \frac{\ln e}{\ln a}
\end{aligned}
$$

$\therefore \quad (\log_a x)' = \dfrac{1}{x \ln a}$, $a > 0, a \neq 1$

6 u x $(\log_a u)' = \dfrac{u'}{u \ln a}$, $a > 0, a \neq 1$

u x

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\log_a u) \times \frac{du}{dx} = \frac{1}{u\ln a} \times u'$$

$$\therefore \quad (\log_a u)' = \frac{u'}{u\ln a}, a > 0, a \neq 1$$

1.8

$y = \ln x$ $y' = \dfrac{1}{x}$

$y = \ln x$

$$\begin{aligned} y' = f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{\ln(x+h) - \ln(x)}{h} \\ &= \lim_{h\to 0} \frac{\ln\left(\dfrac{x+h}{x}\right)}{h} \\ &= \lim_{h\to 0} \frac{1}{h} \ln\left(1 + \frac{h}{x}\right) \\ &= \lim_{h\to 0} \ln\left(1 + \frac{h}{x}\right)^{\dfrac{1}{h}} \\ &= \ln\left[\lim_{h\to 0}\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}\times\frac{1}{x}}\right], \lim_{h\to 0}\left(1 + \frac{1}{\frac{x}{h}}\right)^{\frac{x}{h}} = e \\ &= \ln e^{\dfrac{1}{x}}, \ln e = 1 \end{aligned}$$

$$\therefore \quad (\ln x)' = \frac{1}{x}$$

Example 1.10 $f'(x)$ 2

1. $f(x) = x^2.\log_a x, a > 0, a \neq 1$
2. $f(x) = \sin(2x) + \log_2(x^2+1)$
3. $f(x) = \dfrac{e^{2x} + \log_3 x}{x^2}$
4. $f(x) = \log(x^2\sqrt{x^3-1})$
5. $f(x) = (\sin x)^{\log x}$

6. $f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1$

■

1. $f(x) = x^2 . \log_a x, a > 0, a \neq 1 \Longrightarrow f'(x) = (x^2)' \log_a x + (\log_a x)' x^2$

$$= 2x \log_a x + \frac{1}{x \ln a} x^2$$

$$\therefore \quad f'(x) = 2x \log_a x + \frac{x}{\ln a}, a > 0, a \neq 1$$

2. $f(x) = \sin(2x) + \log_2(x^2+1) \Longrightarrow f'(x) = -(2x)' \cos(2x) + \dfrac{(x^2+1)'}{(x^2+1)\ln 2}$

$$\therefore \quad f'(x) = -2\cos(2x) + \frac{2x}{(x^2+1)\ln 2}$$

3. $f(x) = \dfrac{e^{2x} + \log_3 x}{x^2} \Longrightarrow f'(x) = \dfrac{(e^{2x} + \log_3 x)' x^2 - (x^2)'(e^{2x} + \log_3 x)}{x^4}$

$$= \frac{(2e^{2x} + \frac{1}{x \ln 3})x^2 - 2x(e^{2x} + \log_3 x)}{x^4}$$

$$= \frac{2xe^{2x} + \frac{1}{\ln 3} - 2e^{2x} - 2\log_3 x}{x^3}$$

$$\therefore \quad f'(x) = \frac{2e^{2x}(x-1) + \frac{1}{\ln 3} - \log_3 x^2}{x^3}$$

4. $f(x) = \log(x^2 \sqrt{x^3 - 1}) = \log x^2 + \log(x^3-1)^{\frac{1}{2}} = 2\log x + \dfrac{1}{2}\log(x^3-1)$

$$\therefore \quad f'(x) = \frac{2}{x \ln 10} + \frac{(x^3-1)'}{2(x^3-1)\ln 10} = \frac{2}{x \ln 10} + \frac{3x^2}{2(x^3-1)\ln 10}$$

5. $f(x) = (\sin x)^{\log x} \Longleftrightarrow \ln f(x) = \ln(\sin x)^{\log x}$

$$(\ln f(x))' = (\log x . \ln(\sin x))'$$

$$\frac{f'(x)}{f(x)} = (\log x)' \ln(\sin x) + (\ln(\sin x))' \log x$$

$$f'(x) = f(x)\left(\frac{1}{x \ln 10} \ln(\sin x) + \frac{(\sin x)'}{\sin x} . \log x\right)$$

$$\therefore \quad f'(x) = (\sin x)^{\log x}\left(\frac{\ln(\sin x)}{x \ln 10} + \cot x . \log x\right)$$

6. $f(x) = (\log_a x)^{\ln(2x)}, a > 0, a \neq 1 \Longleftrightarrow \ln f(x) = \ln(\log_a x)^{\ln(2x)}$

$$
\begin{aligned}
(\ln f(x))' &= (\ln(2x).\ln(\log_a x))' \\
\frac{f'(x)}{f(x)} &= (\ln(2x))'\ln(\log_a x) + (\ln(\log_a x))'\ln(2x) \\
f'(x) &= f(x)\left(\frac{(2x)'}{2x}\ln(\log_a x) + \frac{(\log_a x)'}{\log_a x}\ln(2x)\right) \\
\therefore \quad f'(x) &= (\log_a x)^{\ln(2x)}\left(\frac{\ln(\log_a x)}{x} + \frac{\ln(2x)}{x\ln a\log_a x}\right), a > 0, a \neq 1
\end{aligned}
$$

7 u x $(\ln u)' = \dfrac{u'}{u}$

u x

$$
\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du}\times\frac{du}{dx} = \frac{d}{du}(\ln u)\times\frac{du}{dx} = \frac{1}{u}\times u' \\
\therefore \quad (\ln u)' &= \frac{u'}{u}
\end{aligned}
$$

Example 1.11 $f'(x)$ 2

1. $f(x) = x.\ln x$
2. $f(x) = x^2 + \ln(x^2+1)$
3. $f(x) = \dfrac{e^x + \ln x}{x^2}$
4. $f(x) = \ln(x^2\sqrt{x^3-1})$
5. $f(x) = x^x$
6. $f(x) = (\sin x)^{\cos x}$

■

1. $f(x) = x.\ln x \Longrightarrow f'(x) = x'\ln x + (\ln x)'x = \ln x + \dfrac{1}{x}.x = \ln x + 1$
2. $f(x) = x^2 + \ln(x^2+1) \Longrightarrow f'(x) = (x^2)' + \dfrac{(x^2+1)'}{x^2+1} = 2x + \dfrac{2x}{x^2+1}$

3. $f(x)=\dfrac{e^x+\ln x}{x^2} \Longrightarrow f'(x)=\dfrac{(e^x+\ln x)'x^2-(x^2)'(e^x+\ln x)}{(x^2)^2}$

$$=\frac{\left(e^x+\dfrac{1}{x}\right)x^2-2x(e^x+\ln x)}{x^4}$$

$$\therefore \quad f'(x)=\frac{xe^x+1-2e^x-2\ln x}{x^3}$$

4. $f(x)=\ln(x^2\sqrt{x^3-1})=\ln x^2+\ln\sqrt{x^3-1}=2\ln x+\ln(x^3-1)^{\frac{1}{2}}$

$$f'(x)=2(\ln x)'+\frac{1}{2}[\ln(x^3-1)]'$$

$$=2.\frac{1}{x}+\frac{1}{2}.\frac{(x^3-1)'}{x^3-1}$$

$$\therefore \quad f'(x)=\frac{2}{x}+\frac{3}{2}.\frac{x^2}{x^3-1}$$

5. $f(x)=x^x \Longleftrightarrow \ln f(x)=\ln x^x=x\ln x$

$$(\ln f(x))'=(x\ln x)'$$

$$\frac{f'(x)}{f(x)}=x'\ln x+(\ln x)'x$$

$$f'(x)=f(x)(\ln x+\frac{1}{x}.x)$$

$$\therefore \quad f'(x)=x^x(\ln x+1)$$

6. $f(x)=(\sin x)^{\cos x} \Longleftrightarrow \ln f(x)=\ln(\sin x)^{\cos x}$

$$(\ln f(x))'=(\cos x\ln\sin x)'$$

$$\frac{f'(x)}{f(x)}=(\cos x)'\ln\sin x+(\ln\sin x)'\cos x$$

$$f'(x)=f(x)\left(-\sin x\ln sinx+\frac{(\sin x)'}{\sin x}.\cos x\right)$$

$$\therefore \quad f'(x)=(\sin x)^{\cos x}(\cos x\cot x-\sin x\ln\sin x)$$

1.9 Arc Sine Arc Tangent

$$y=\arcsin x \Longleftrightarrow x=\sin y \quad -\tfrac{\pi}{2}\le y\le\tfrac{\pi}{2},$$

$$y=\arctan x \Longleftrightarrow x=\tan y \quad -\tfrac{\pi}{2}\le y\le\tfrac{\pi}{2},$$

■ **Example 1.12** $y = \arcsin x$ $y' = \dfrac{1}{\sqrt{1-x^2}}$ ■

$y = \arcsin x$ $x = \sin y$ x

$$(x)' = (\sin y)' \Longleftrightarrow 1 = y' \cos y$$

$$y' = \frac{1}{\cos y} \quad \sin^2 y + \cos^2 y = 1$$

$$\Longrightarrow \cos y = \pm\sqrt{1-\sin^2 x}$$

$$-\frac{\pi}{2} \le y \le \frac{\pi}{2} \Longrightarrow \cos y \ge 0 \Longrightarrow \cos y = \sqrt{1-x^2}$$

$$\therefore \quad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

■ **Example 1.13** $y = \arctan x$ $y' = \dfrac{1}{1+x^2}$ ■

$y = \arctan x$ $x = \tan y$ x

$$(x)' = (\sin y)' \Longleftrightarrow 1 = y'(1+\tan^2 y)$$

$$y' = \frac{1}{1+\tan^2 y}$$

$$\therefore \quad (\arctan x)' = \frac{1}{1+x^2}$$

8 u x $(\arcsin u)' = \dfrac{u'}{\sqrt{1-x^2}}$

u x

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arcsin u) \times \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \times u'$$

$$\therefore \quad (\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$$

9 u x $(\arctan u)' = \dfrac{u'}{1+u^2}$

u x

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{d}{du}(\arctan u) \times \frac{du}{dx} = \frac{1}{1-u^2} \times u'$$

$$\therefore \quad (\arctan u)' = \frac{u'}{1+u^2}$$

Example 1.14 2

1. $f(x) = \arcsin x.\sin x$
2. $f(x) = \arctan x \cos x$
3. $f(x) = \sin(\arcsin x)$
4. $f(x) = \arctan(\tan x)$

■

1. $f(x) = \arcsin x.\sin x \Longrightarrow f'(x) = (\arcsin x)' \sin x + (\sin x)' \arcsin x$
$$\therefore \quad f'(x) = \frac{\sin x}{\sqrt{1-x^2}} + \cos x.\arcsin x$$
2. $f(x) = \arctan x \cos x \Longrightarrow f'(x) = (\arctan x)' \cos x + (\cos x)' \arctan$
$$\therefore \quad f'(x) = \frac{\cos x}{1+x^2} - \sin x.\arctan x$$
3. $f(x) = \sin(\arcsin x) \Longrightarrow f'(x) = (\arcsin x)' \cos(\arcsin x)$
$$\therefore \quad f'(x) = \frac{\cos(\arcsin x)}{\sqrt{1-x^2}}$$
4. $f(x) = \arctan(\tan x) \Longrightarrow f'(x) = \dfrac{(\tan x)'}{1+(\tan x)^2} = \dfrac{1+\tan^2 x}{1+\tan^2 x}$
$$\therefore \quad f'(x) = 1$$

1.10

$C, a, b, c \quad u \quad x \quad n \in \mathbb{N} \quad 2$

1. $(C)' = 0$
2. $(x)' = 1$
3. $(ax+b)' = a$
4. $(ax^2+bx+c)' = 2ax+b$
5. $(x^n)' = nx^{n-1}$
6. $(u^n)' = n.u'.u^{n-1}$
7. $(x)^{-n} = -\dfrac{n}{x^{n+1}}$
8. $\left(\dfrac{1}{x}\right)' = -\dfrac{1}{x^2}$
9. $\left(\dfrac{1}{u}\right)' = -\dfrac{u}{u^2}$
10. $(\sqrt{x})' = \dfrac{1}{2\sqrt{x}}$
11. $(\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$
12. $(\sqrt[n]{x})' = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$
13. $(\ln x)' = \dfrac{1}{x}$
14. $(\ln u)' = \dfrac{u'}{u}$
15. $(\log_a x)' = \dfrac{1}{x \ln a}, a > 0, a \neq 1$
16. $(\log_a u)' = \dfrac{u'}{u.\ln a}, a > 0, a \neq 1$
17. $(a^x)' = a^x \ln a, a > 0, a \neq 1$
18. $(a^u)' = u' a^u \ln a, a > 0, a \neq 1$
19. $(e^x)' = e^x$
20. $(e^u)' = u' e^u$
21. $(\sin x)' = \cos x$
22. $(\sin u)' = u' \cos u$
23. $(\cos x)' = -\sin x$
24. $(\cos u)' = -u' \sin u$

25. $(\tan x)' = \dfrac{1}{\cos^2 x} = 1 + \tan^2 x$
26. $(\tan u)' = \dfrac{u'}{\cos^2 u} = u'(1 + \tan^2 u)$
27. $(\cot x)' = -\dfrac{1}{\sin^2 x} = -(1 + \cot^2 x)$
28. $(\cot u)' = -\dfrac{u'}{\sin^2 u} = -(1 + \cot^2 u)$
29. $(\arcsin x)' = \dfrac{1}{\sqrt{1 - x^2}}$
30. $(\arcsin u)' = \dfrac{u'}{\sqrt{1 - u^2}}$
31. $(\arccos x)' = -\dfrac{1}{\sqrt{1 - x^2}}$
32. $(\arccos u)' = -\dfrac{u'}{\sqrt{1 - u^2}}$
33. $(\arctan x)' = \dfrac{1}{1 + x^2}$
34. $(\arctan u)' = \dfrac{u'}{1 + u^2}$
35. $(\operatorname{arccot} x)' = -\dfrac{1}{1 + x^2}$
36. $(\operatorname{arccot} u)' = -\dfrac{u'}{1 + u^2}$
37. $(u^v)' = \left(v'.\ln u + \dfrac{v.u'}{u}\right).u^v$

1.11

Exercise 1.1 $f'(x)$ 2

1. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$
2. $f(x) = 2x^2 - \sqrt{x} + \dfrac{2}{x}$
3. $f(x) = (x^4 - 7x^2 + \sin a)^7$
4. $f(x) = (x^2 - \sqrt{x})^{2019}$
5. $f(x) = \sqrt{x^3 - x^2 + 3}$
6. $\sqrt[4]{x^3 - 2x}$
7. $f(x) = (x+1)(2x-1)^2$
8. $f(x) = (x^2 + 2x + 3)(x^3 - 3x - 1)$
9. $f(x) = \dfrac{1}{x-1}$
10. $f(x) = \dfrac{x\sqrt{x}}{x+1}$

1. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1 \Longrightarrow f'(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$
2. $f(x) = 2x^2 - \sqrt{x} - \dfrac{2}{x} \Longrightarrow f'(x) = 4x^2 + \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^2}$

$$f'(x) = 7(x^4 - 7x^2 + \sin a)'(x^4 - 7x^2 + \sin a)^{7-1} = 7(4x^3 - 14x)(x^4 - 7x^2 + \sin a)^6$$

3. $f(x) = (x^4 - 7x^2 + \sin a)^7$
4. $f(x) = (x^2 - \sqrt{x})^{2019} \Longrightarrow f'(x) = 2019(x^2 - \sqrt{x})'(x^2 - \sqrt{x})^{2019-1}$

$$= 2019\left(2x - \frac{1}{2\sqrt{x}}\right)(x^2 - \sqrt{x})^{2018}$$

5. $f(x) = \sqrt{x^3 - x^2 + 3} \Longrightarrow f'(x) = \dfrac{(x^3 - x^2 + 3)'}{2\sqrt{x^3 - x^2 + 3}} = \dfrac{3x - 2}{2\sqrt{x^3 - x^2 + 3}}$
6. $\sqrt[4]{x^3 - 2x} \Longleftrightarrow f(x) = (x^3 - 2x)^{\frac{1}{4}}$

$$\begin{aligned} f'(x) &= \frac{1}{4}(x^3 - 2x)'(x^3 - 2x)^{\frac{1}{4}-1} \\ &= \frac{1}{4}(3x^2 - 2)(x^3 - 2x)^{-\frac{3}{4}} \\ \therefore \quad f'(x) &= \frac{3x^2 - 2}{4\sqrt[4]{(x^3 - 2x)^3}} \end{aligned}$$

7. $f(x) = (x+1)(2x-1)^2$

$$\begin{aligned} f'(x) &= (x+1)'(2x-1)^2 + [(2x-1)^2]'(x+1) \\ &= (2x-1)^2 + 2(2x-1)'(2x-1)(x+1) \\ &= (2x-1)(2x-1+4x+4) \\ \therefore \quad f'(x) &= (2x-1)(6x+3) \end{aligned}$$

8. $f(x) = (x^2+2x+3)(x^3-3x-1)$

$$\begin{aligned} f'(x) &= (x^2+2x+3)'(x^2-3x-1) + (x^2-3x-1)'(x^2+2x+3) \\ &= (2x+2)(x^2-3x-1) + (2x-3)(x^2+2x+3) \\ &= 2x^3-6x^2-2x+2x^2-6x-2+2x^3+4x^2+6x-3x^2-6x-9 \\ \therefore \quad f'(x) &= 4x^3-3x^2-8x-11 \end{aligned}$$

9. $f(x) = \dfrac{1}{x-1} \Longrightarrow f'(x) = -\dfrac{(x-1)'}{(x-1)^2} = -\dfrac{1}{(x-1)^2}$

10. $f(x) = \dfrac{x\sqrt{x}}{x+1}$

$$\begin{aligned} f'(x) &= \frac{(x\sqrt{x})'(x+1) - (x+1)'x\sqrt{x}}{(x+1)^2} \\ &= \frac{[x'\sqrt{x} + (\sqrt{x})'x](x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{\left(x + \dfrac{x}{2\sqrt{x}}\right)(x+1) - x\sqrt{x}}{(x+1)^2} \\ &= \frac{x\sqrt{x} + \sqrt{x} + \dfrac{x}{2\sqrt{x}}(x+1) - x\sqrt{x}}{(x+1)^2} \\ \therefore \quad f'(x) &= \frac{x^2+3x}{2\sqrt{x}(x+1)^2} \end{aligned}$$

Exercise 1.2 2

1. $f(x) = x.\sin x + \cos x$
2. $f(x) = \sin^3 x - x.\cos x$
3. $f(x) = \cos(x^2+1) + 2\sin(x^2-1)$
4. $f(x) = \sin^2\sqrt{x} + \cos^2(3x)$

5. $f(x) = \cos(3x+4) + 3\cos x.\sin x$

6. $f(x) = \sin(\sin\sqrt{x}) + \cos^3 x$

1. $f(x) = x.\sin x + \cos x$

$$\begin{aligned} & f'(x) = x'\sin x + (\sin x)'.x - \sin x \\ & \quad = \sin x + x.\cos x - \sin x \\ \therefore \quad & f'(x) = x.\cos x \end{aligned}$$

2. $f(x) = \sin^3 x - x.\cos x$

$$\begin{aligned} & f'(x) = 3(\sin x)'\sin^{3-1} x - [x'.\cos x + (\cos x)'.x] \\ & \quad = 3\cos x.\sin^2 x - (\cos x - x.\sin x) \\ \therefore \quad & f'(x) = 3\cos x.\sin^2 x - \cos x + x\sin x \end{aligned}$$

3. $f(x) = \cos(x^2+1) + 2\sin(x^2-1)$

$$\begin{aligned} & f'(x) = -(x^2+1)'\sin(x^2+1) + 2(x^2-1)'\cos(x^2-1) \\ \therefore \quad & f'(x) = -2x\sin(x^2+1) + 4x\cos(x^2-1) \end{aligned}$$

4. $f(x) = \sin^2\sqrt{x} + \cos^2(3x)$

$$\begin{aligned} & f'(x) = 2(\sin\sqrt{x})'\cos^{2-1}\sqrt{x} + 2(\cos(3x))'\sin(3x) \\ & \quad = 2(\sqrt{x})'.\cos\sqrt{x}.\cos\sqrt{x} - 2(3x)'\sin(3x).\sin(3x) \\ \therefore \quad & f'(x) = \frac{1}{\sqrt{x}}.\cos^2\sqrt{x} - 6\sin^2(3x) \end{aligned}$$

5. $f(x) = \cos(3x+4) + 3\cos x.\sin x$

$$\begin{aligned} & f'(x) = -(3x+4)'.\sin(3x+4) + 3[(\cos x)'.\sin x + (\sin x)'.\cos x] \\ & \quad = -3\sin(3x+4) + 3[-\sin x.\sin x + \cos x.\cos x] \\ \therefore \quad & f'(x) = -3[\sin(3x+4) + \sin^2 x - \cos^2 x] \end{aligned}$$

6. $f(x) = \sin(\sin\sqrt{x}) + \cos^3 x$

$$f'(x) = (\sin\sqrt{x})'.\cos(\sin\sqrt{x}) + 3(\cos x)\cos^{3-1} x$$
$$= (\sqrt{x})'.\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x\cos^2 x$$
$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x}}\cos\sqrt{x}.\cos(\sin\sqrt{x}) - 3\sin x.\cos^2 x$$

Exercise 1.3 2

1. $f(x) = (1+\tan x)^4$
2. $f(x) = x^2\tan x + (1+\cot x)^2$
3. $f(x) = x.\tan(x^2-1) + x\cot(2x^2)$
4. $f(x) = \dfrac{\tan(2x)}{1-\cos x}$

1. $f(x) = (1+\tan x)^4$

$$f'(x) = 4(1+\tan x)'(1+\tan^2 x)^{4-1}$$
$$\therefore \quad f'(x) = 4(1+\tan^2 x)(1+\tan x)^3$$

2. $f(x) = x^2\tan x + (1+\cot x)^2$

$$f'(x) = (x^2)'\tan x + (\tan x)'x^2 + 2(1+\cot x)'(1+\cot x)^{2-1}$$
$$\therefore \quad f'(x) = 2x\tan x + x^2(1+\tan^2 x) - 2(1+\cot^2 x)(1+\cot x)$$

3. $f(x) = x.\tan(x^2-1) + x\cot(2x^2)$

$$f'(x) = x'\tan(x^2-1) + [\tan(x^2-1)]'x + x'\cot(2x^2) + [\cot(2x^2)]'x$$
$$= \tan(x^2-1) + (x^2-1)'[1+\tan^2(x^2-1)]x - (2x^2)'[1+\cot^2(2x^2)]x$$
$$\therefore \quad f'(x) = \tan(x^2-1) + 2x^2[1+\tan^2(x^2-1)] - 4x^2[1+\cot^2(2x^2)]$$

Exercise 1.4 2

1. $f(x)=\dfrac{1-x-2x^2}{x^3-\ln 3}$
2. $f(x)=\dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}}$
3. $f(x)=\sin x^2.\tan(2x+3)$
4. $f(x)=\sin(x^2+5)+\cos(\sin x)$
5. $f(x)=\dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$

1. $f(x)=\dfrac{1-x-2x^2}{x^3-\ln 3}$

$$\begin{aligned}
f'(x)&=\frac{(1-x-2x^2)'(x^3-\ln 3)-(x^3-\ln 3)'(1-x-2x^2)}{(x^3-\ln 3)^2}\\
&=\frac{(-1-4x)(x^3-\ln 3)-3x^2(1-x-2x^2)}{(x^3-\ln 3)^2}\\
&=\frac{-x^3+\ln 3-4x^4+4x\ln 3-3x^2+3x^3+6x^4}{(x^3-\ln 3)^2}\\
\therefore\quad f'(x)&=\frac{2x^4+2x^3-3x^2+4x.\ln 3+\ln 3}{(x^3-\ln 3)^2}
\end{aligned}$$

2. $f(x)=\dfrac{2x^2+3x+4}{\sqrt{1+2x-x^2}}\iff f(x).\sqrt{1+2x-x^2}=2x^2+3x+4$

$$\begin{aligned}
[f(x)\sqrt{1+2x-x^2}]'&=(2x^2+3x+4)'\\
f'(x)\sqrt{1+2x-x^2}+(\sqrt{1+2x-x^2})'f(x)&=4x+3\\
f'(x)\sqrt{1+2x-x^2}+\frac{(1+2x-x^2)'}{2\sqrt{1+2x-x^2}}f(x)&=4x+3\\
f'(x)\sqrt{1+2x-x^2}&=4x+3-\frac{1-x}{\sqrt{1+2x-x^2}}.f(x)\\
\therefore\quad f'(x)&=\frac{4x+3}{\sqrt{1+2x-x^2}}+\frac{(x-1)(2x^2+3x+4)}{(1+2x-x^2)\sqrt{1+2x-x^2}}
\end{aligned}$$

3. $f(x) = \sin x^2 . \tan(2x+3)$

$$\begin{aligned} f'(x) &= (\sin x^2)' \tan(2x+3) + (\tan(2x+3))' \sin x^2 \\ &= (x^2)'. \sin x^2 . \tan(2x+3) + (2x+3)'[1+\tan^2(2x+3)] \sin x^2 \\ &= 2x \sin x^2 . \tan(2x+3) + 2\sin x^2 [1+\tan^2(2x+3)] \\ \therefore \quad f'(x) &= 2\sin x^2 [\tan^2(2x+3) + x\tan(2x+3) + 1] \end{aligned}$$

4. $f(x) = \sin(x^2+5) + \cos(\sin x)$

$$\begin{aligned} f'(x) &= (x^2+5)' \cos(x^2+5) - (\sin x)' \sin(\sin x) \\ \therefore \quad f'(x) &= 2x\cos(x^2+5) - \cos x \sin(\sin x) \end{aligned}$$

5. $f(x) = \dfrac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})} \iff f(x). \sin\sqrt{x} = \sin(\tan\sqrt{x})$

$$\left(f(x). \sin\sqrt{x}\right)' = \left(\sin(\tan\sqrt{x})\right)'$$

$$f'(x). \sin\sqrt{x} + (\sin\sqrt{x})' f(x) = (\tan\sqrt{x})' \cos(\tan\sqrt{x})$$

$$f'(x). \sin\sqrt{x} + (\sqrt{x})' \cos\sqrt{x}. f(x) = (\sqrt{x})'(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x})$$

$$f'(x) \sin\sqrt{x} + \frac{1}{2\sqrt{x}} \cos\sqrt{x}. f(x) = \frac{1}{2\sqrt{x}}(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x})$$

$$f'(x) \sin\sqrt{x} = \frac{1}{2\sqrt{x}} \left[(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) - \cos\sqrt{x}. f(x)\right]$$

$$\therefore \quad f'(x) = \frac{(1+\tan^2\sqrt{x}) \cos(\tan\sqrt{x}) - \cos\sqrt{x}. f(x)}{2\sqrt{x}. \sin\sqrt{x}}, f(x) = \frac{\sin(\tan\sqrt{x})}{\sin(\sqrt{x})}$$

Exercise 1.5 2

1. $f(x) = xe^x + \frac{1}{2}x^2$
2. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$
3. $f(x) = \dfrac{\sqrt{x}}{e^x}$
4. $f(x) = x^3 e^{-3x}$
5. $f(x) = e^{2x} 3^{x^2+1}$
6. $f(x) = e^{\sin x \cos x}$

1. $f(x) = xe^x + \frac{1}{2}x^2$

$$f'(x) = x'e^x + (e^x)'x + \frac{1}{2}.2x = e^x + e^x x + x = e^x(1+x) + x$$

2. $f(x) = e^{x^2+2x+1} + (x^2-3)e^x$

$$\begin{aligned} f'(x) &= (x^2+2x+1)'e^{x^2+2x+1} + (x^2-3)'e^x + (e^x)'(x^2-3) \\ &= (2x+2)e^{x^2+2x+1} + 2xe^x + e^x(x^2-3) \\ \therefore \quad f'(x) &= 2(x+1)e^{x^2+2x+1} + e^x(2x+x^2-3) \end{aligned}$$

3. $f(x) = \dfrac{\sqrt{x}}{e^x}$

$$f'(x) = \frac{(\sqrt{x})'e^x + (e^x)'\sqrt{x}}{(e^x)^2} = \frac{\frac{1}{2\sqrt{x}}e^x + e^x\sqrt{x}}{e^{2x}} = \frac{1+2x}{2\sqrt{x}e^x}$$

4. $f(x) = x^3e^{-3x}$

$$\begin{aligned} f'(x) &= (x^3)'e^{-3x} + (e^{-3x})'x^3 \\ &= 3x^2e^{-3x} + (-3x)'e^{-3x}x^3 \\ &= 3x^2e^{-3x} - 3e^3e^{-3x} \\ \therefore \quad f'(x) &= 3x^2e^{-3x}(1-x) \end{aligned}$$

5. $f(x) = e^{2x}3^{x^2+1}$

$$\begin{aligned} f'(x) &= (e^{2x})'3^{x^2+1} + (3^{x^2+1})'.e^{2x} \\ &= (2x)'e^{2x}.3^{x^2+1} + (x^2+1)'3^{x^2+1}\ln 3.e^{2x} \\ &= 2.e^{2x}3^{x^2+1} + 2x3^{x^2+1}\ln 3.e^{2x} \\ \therefore \quad f'(x) &= 2e^{2x}3^{x^2+1}(1+x\ln 3) \end{aligned}$$

6. $f(x) = e^{\sin x\cos x}$

$$\begin{aligned} f'(x) &= (\sin x\cos x)'e^{\sin x\cos x} \\ &= [(\sin x)'\cos x + (\cos x)'\cos x]e^{\sin x\cos x} \\ \therefore \quad f'(x) &= (\cos^2 x - \sin^2 x)e^{\sin x\cos x} \end{aligned}$$

Exercise 1.6 2

1. $f(x) = (x^2-1)\ln(x^2-1)$
2. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right)$
3. $f(x) = \ln(\sin x.\cos(2x))$
4. $f(x) = \ln\left(\sqrt{\dfrac{1+\sin x}{1-\sin x}}\right)$

1. $f(x) = (x^2-1)\ln(x^2-1) \Longrightarrow f'(x) = (x^2-1)'\ln(x^2-1) + \ln(x^2-1)'(x^2-1)$

$$= 2x\ln(x^2-1) + \frac{(x^2-1)'}{x^2-1}.(x^2-1)$$
$$= 2x\ln(x^2-1) + 2x$$
$$\therefore \quad f'(x) = 2x[\ln(x^2-1)+1]$$

2. $f(x) = \ln\left(\dfrac{x^2-2}{\sqrt[3]{x^2-2}}\right) = \ln(x^2-2) - \ln(x^2-2)^{\frac{1}{3}}$

$$f'(x) = \frac{(x^2-2)'}{x^2-2} - \frac{1}{3}.\frac{(x^2-2)'}{x^2-2}$$
$$= \frac{3(2x)-2x}{3(x^2-2)}$$
$$\therefore \quad f'(x) = \frac{4x}{3(x^2-2)}$$

3. $f(x) = \ln(\sin x.\cos(2x)) = \ln(\sin x) + \ln(\cos(2x))$

$$f'(x) = \frac{(\sin x)'}{\sin x} + \frac{(\cos(2x))'}{\cos(2x)}$$
$$= \frac{\cos x}{\sin x} - \frac{2\sin(2x)}{\cos(2x)}$$
$$\therefore \quad f'(x) = \cot x - 2\tan(2x)$$

4. $f(x) = \ln\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right) = \ln\left(\frac{1+\sin x}{1-\sin x}\right)^{\frac{1}{2}} = \frac{1}{2}(\ln(1+\sin x) - \ln(1-\sin x))$

$$\begin{aligned} f'(x) &= \frac{1}{2}\left(\frac{(1+\sin x)'}{1+\sin x} - \frac{(1-\sin x)'}{1-\sin x}\right) \\ &= \frac{1}{2}\left(\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}\right) \\ &= \frac{1}{2}.\frac{(\cos x(1-\sin x+1+\sin x))}{1-\sin^2 x} \\ \therefore \quad f'(x) &= \frac{2\cos x}{2\cos^2 x} = \frac{1}{\cos x} \end{aligned}$$

Exercise 1.7 2

1. $f(x) = \cos(\arcsin x)$
2. $f(x) = \cot(\arctan x)$
3. $f(x) = \tan(\arctan x)$
4. $f(x) = \arcsin(2x)$
5. $f(x) = \arcsin\sqrt{x}$
6. $f(x) = \arctan(\sin x)$
7. $f(x) = \frac{\arctan x}{\arcsin x}$

1. $f(x) = \cos(\arcsin x) \Longrightarrow f'(x) = -(\arcsin x)' \sin(\arcsin x)$

$$\therefore \quad f'(x) = -\frac{\sin(\arcsin x)}{\sqrt{1-x^2}}$$

2. $f(x) = \cot(\arctan x) \Longrightarrow f'(x) = -(\arctan x)'[1+\cot^2(\arctan x)]$

$$\therefore \quad f'(x) = -\frac{1+\cot^2(\arctan x)}{1+x^2}$$

3. $f(x) = \tan(\arctan x) \Longrightarrow f'(x) = (\arctan x)'[1+\tan^2(\arctan x)]$

$$\therefore \quad f'(x) = \frac{1+\tan^2(\arctan x)}{1+x^2}$$

4. $f(x) = \arcsin(2x) \Longrightarrow f'(x) = \frac{(2x)'}{\sqrt{1-(2x)^2}}$

$$\therefore \quad f'(x) = \frac{2}{\sqrt{1-4x^2}}$$

5. $f(x) = \arcsin\sqrt{x} \Longrightarrow f'(x) = \dfrac{(\sqrt{x})'}{\sqrt{1-(\sqrt{x})^2}} = \dfrac{\frac{1}{2\sqrt{x}}}{\sqrt{1-x^2}} = \dfrac{1}{2\sqrt{x}\sqrt{1-x^2}}$

$$\therefore \quad f'(x) = \frac{1}{2\sqrt{x-x^2}}$$

6. $f(x) = \arctan(\sin x) \Longrightarrow f'(x) = \dfrac{(\sin x)'}{1+(\sin x)^2}, \sin^2 x + \cos^2 x = 1$

$$\therefore \quad f'(x) = \frac{\cos x}{2-\cos^2 x}$$

7. $f(x) = \dfrac{\arctan x}{\arcsin x} \Longrightarrow f'(x) = \dfrac{(\arctan x)' \arcsin x - (\arcsin x)' \arctan x}{(\arcsin x)^2}$

$$\therefore \quad f'(x) = \frac{\dfrac{\arcsin x}{1+x^2} - \dfrac{\arctan x}{\sqrt{1-x^2}}}{(\arcsin x)^2}$$

1.12

1. 2

   (a) $y = x^3 + 2x^2$

   (b) $y = x^3 - 4x^2$

   (c) $y = x^4 - 27x$

   (d) $y = x^4 - 5x^2 + 4$

   (e) $y = x^5 - 16x$

   (f) $y = \dfrac{x}{x+1}$

   (g) $y = \dfrac{x^2}{1+x^2}$

   (h) $y = x - \dfrac{1}{x}$

   (i) $y = x^3 + 2x^2 - x$

   (j) $y = x^4 - 2x^3 + 2x$

   (k) $y = \sqrt{1+x^2}$

   (l) $y = \sqrt[4]{1+x^2}$

2. $f'(x)$ 2

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = 2\sin x - 3\cos x$

(c) $f(x) = 3\sin x + 2\cos x$

(d) $f(x) = x\sin x + \cos x$

(e) $f(x) = x\cos x - \sin x$

(f) $f(x) = \cos(2x)$

(g) $f(x) = \dfrac{1-\sin(2x)}{1-\sin x}$

(h) $f(x) = 1 + \sin x^2$

(i) $f(x) = \cot x - \cos x$

(j) $f(x) = \sin(2x) - \cos(3x)$

(k) $f(x) = \sin(\cos(3x))$

(l) $f(x) = \dfrac{\sin x^2}{x^2}$

(m) $f(x) = \tan(1+x^2)$

(n) $f(x) = \cos 2x - \cos x^2$

(o) $f(x) = (1+\sqrt{1+x})^3$

3. y' 2

(a) $xy = \dfrac{\pi}{6}$

(b) $\sin(xy) = 1$

(c) $xy = \dfrac{1}{x+y}$

(d) $x+y = xy$

(e) $(y-1)^2 + x = 0$

(f) $(y+1)^2 + y - x = 0$

(g) $(y-x)^2 + x = 0$

(h) $(y+x) + 2y - x = 0$

(i) $(y^2-1)^2 + x = 0$

(j) $(y^2+1)^2 - x = 0$

(k) $x^3 + xy + y^3 = 3$

(l) $\sin x + \sin y = 1$

(m) $\sin x + xy + y^5 = \pi$

(n) $\tan x + \tan y = 1$

(o) $x \ln y = e^{\ln \sin x}$

(p) $(\sin x)^{\ln y} = (\tan y)^{e^{3x}}$

4. 2

(a) $f(x) = \sqrt{1-x}$

(b) $f(x) = \sqrt[4]{x+x^2}$

(c) $y = \sqrt{1-\sqrt{x}}$

(d) $y = \sqrt{x-\sqrt{x}}$

(e) $y = \sqrt[3]{\sqrt{2x+1}} - x^2$

(f) $y = \sqrt[4]{x+x^2}x + x^2$

(g) $y = \sqrt[3]{x-\sqrt{2x+1}}$

(h) $y = \sqrt[4]{\sqrt[3]{x}} + \sqrt[3]{\sqrt{x}} + \sqrt{x}$

5. 2

(a) $f(x) = e^x + e^{-x}$

(b) $f(x) = e^{3x} + 4e^x$

(c) $f(x) = \dfrac{e^x}{1+e^x}$

(d) $f(x) = \dfrac{2e^{2x}}{1+e^{2x}}$

(e) $f(x) = xe^{-x} + x\ln x$

(f) $f(x) = \sqrt{x}e^{-\frac{x}{4}} + x^2 e^{x+2}$

(g) $f(x) = x^{-\frac{1}{2}x} + \ln\sqrt{x}$

(h) $f(x) = (\ln x)^2 + \ln x + 1$

(i) $f(x) = \dfrac{\ln x}{x} + \ln\dfrac{1}{x}$

(j) $f(x) = \ln\left(\sqrt{\dfrac{\sqrt{x}+1}{\sqrt{x}-1}}\right)$

6. 2

(a) $f(x) = \tan(\arctan x)$

(b) $f(x) = \arcsin(\sin x)$

(c) $f(x) = \sin(\arctan x)$

(d) $f(x) = (\arcsin x)^2$

(e) $f(x) = \dfrac{1}{1+(\arctan x)^2}$

(f) $f(x) = \sqrt{1-(\arcsin x)^2}$

7. 2

(a) $y = (x+1)(x-1)$

(b) $y = (x^2+1)(x^2-1)$

(c) $y = \dfrac{1}{x+1} + \dfrac{1}{1+\sin x}$

(d) $y = \dfrac{1}{1+x^2} + \dfrac{1}{1-\sin x}$

(e) $y = (x-1)(x-2)(x-3)$

(f) $y = x^2\cos x + 2x\sin x$

(g) $y = x^{\frac{1}{2}}(x+\sin x)$

(h) $y = x^{\frac{1}{2}}\sin^2 x + (\sin x)^{\frac{1}{2}}$

(i) $y = x^4\cos x + x\cos x$

(j) $y = \dfrac{1}{2}x^2\sin x - x\cos x + \sin x$

(k) $y = \sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)$

(l) $y = (x-6)^{10} + \sin^{10} x$

(m) $y = (\sin x\cos x)^3 + \sin(2x)$

(n) $y = x^{\frac{1}{2}}\sin(2x) + (\sin x)^{\frac{1}{2}}$

(o) $y = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

(p) $y = \dfrac{1}{\tan x} - \dfrac{1}{\cot x}$