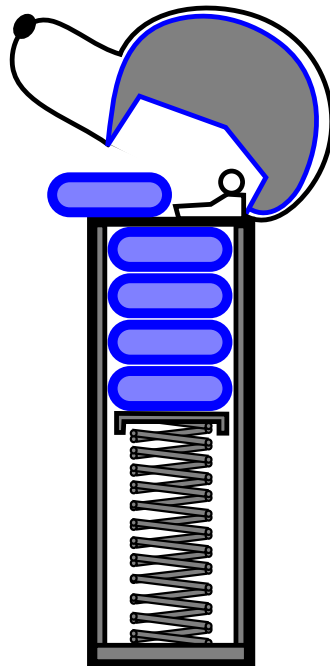


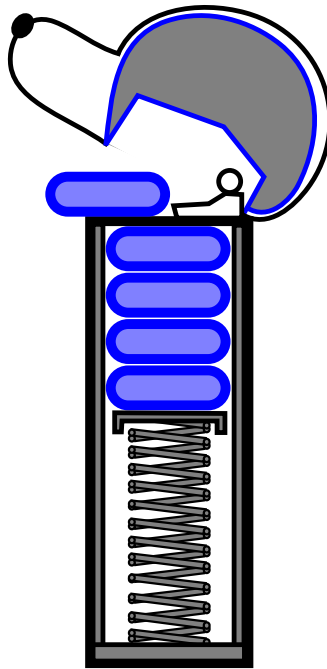
STACKS, QUEUES, AND LINKED LISTS

- Stacks
- Queues
- Linked Lists
- Double-Ended Queues
- Case Study: A Stock Analysis Applet



Stacks

- A **stack** is a container of objects that are inserted and removed according to the **last-in-first-out (LIFO)** principle.
- Objects can be inserted at any time, but only the last (the most-recently inserted) object can be removed.
- Inserting an item is known as “pushing” onto the stack. “Popping” off the stack is synonymous with removing an item.
- A PEZ[®] dispenser as an analogy:



The Stack Abstract Data Type

- A stack is an **abstract data type** (ADT) that supports two main methods:
 - **push(*o*)**: Inserts object *o* onto top of stack
Input: Object; *Output*: none
 - **pop()**: Removes the top object of stack and returns it; if stack is empty an error occurs
Input: none; *Output*: Object
- The following support methods should also be defined:
 - **size()**: Returns the number of objects in stack
Input: none; *Output*: integer
 - **isEmpty()**: Return a boolean indicating if stack is empty.
Input: none; *Output*: boolean
 - **top()**: return the top object of the stack, without removing it; if the stack is empty an error occurs.
Input: none; *Output*: Object

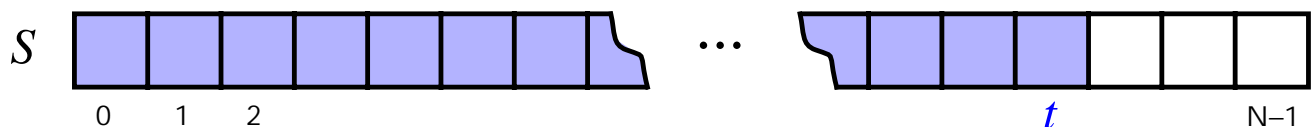
A Stack Interface in Java

- While, the stack data structure is a “built-in” class of Java’s `java.util` package, it is possible, and sometimes preferable to define your own specific one, like this:

```
public interface Stack {  
    // accessor methods  
    public int size(); // return the number of  
                       // elements in the stack  
    public boolean isEmpty(); // see if the stack  
                             // is empty  
    public Object top() // return the top element  
                       // throws StackEmptyException; // if called on  
                                                           // an empty stack  
                       // update methods  
  
    public void push (Object element); // push an  
                                       // element onto the stack  
    public Object pop() // return and remove the  
                      // top element of the stack  
                      // throws StackEmptyException; // if called on  
                                                           // an empty stack  
}
```

An Array-Based Stack

- Create a stack using an array by specifying a maximum size N for our stack, e.g. $N = 1,000$.
- The stack consists of an N -element array S and an integer variable t , the index of the top element in array S .



- Array indices start at 0, so we initialize t to -1
- Pseudo-code

Algorithm size():

return $t + 1$

Algorithm isEmpty():

return $(t < 0)$

Algorithm top():

if isEmpty() **then**

throw a StackEmptyException

return $S[t]$

...

An Array-Based Stack (contd.)

- Pseudo-Code (contd.)

Algorithm push(o):

if size() = N **then**

 throw a StackFullException

$t \leftarrow t + 1$

$S[t] \leftarrow o$

Algorithm pop():

if isEmpty() **then**

 throw a StackEmptyException

$e \leftarrow S[t]$

$S[t] \leftarrow \text{null}$

$t \leftarrow t - 1$

return e

- Each of the above method runs in constant time ($O(1)$)
- The array implementation is simple and efficient.
- There is an upper bound, N , on the size of the stack. The arbitrary value N may be too small for a given application, or a waste of memory.

Array-Based Stack: a Java Implementation

```
public class ArrayStack implements Stack {  
    // Implementation of the Stack interface  
    // using an array.  
  
    public static final int CAPACITY = 1000; // default  
        // capacity of the stack  
    private int capacity; // maximum capacity of the  
        // stack.  
    private Object S[]; // S holds the elements of  
        // the stack  
    private int top = -1; // the top element of the  
        // stack.  
  
    public ArrayStack() { // Initialize the stack  
        // with default capacity  
        this(CAPACITY);  
    }  
    public ArrayStack(int cap) { // Initialize the  
        // stack with given capacity  
        capacity = cap;  
        S = new Object[capacity];  
    }
```

Array-Based Stack in Java (contd.)

```
public int size() { //Return the current stack
                    // size
    return (top + 1);
}

public boolean isEmpty() { // Return true iff
                           // the stack is empty

    return (top < 0);
}

public void push(Object obj) { // Push a new
                              // object on the stack

    if (size() == capacity)
        throw new StackFullException("Stack overflow.");
    S[++top] = obj;
}

public Object top() // Return the top stack
                   // element

    throws StackEmptyException {
if (isEmpty())
    throw new StackEmptyException("Stack is empty.");
return S[top];
}
```

Array-Based Stack in Java (contd.)

```
public Object pop() // Pop off the stack element
    throws StackEmptyException {
    Object elem;
    if (isEmpty())
        throw new StackEmptyException("Stack is Empty.");
    elem = S[top];
    S[top--] = null; // Dereference S[top] and
                     // decrement top
    return elem;
    }
}
```

Casting With a Generic Stack

- Have an ArrayStack that can store only Integer objects or Student objects.
- In order to do so using a generic stack, the return objects must be cast to the correct data type.
- A Java code example:

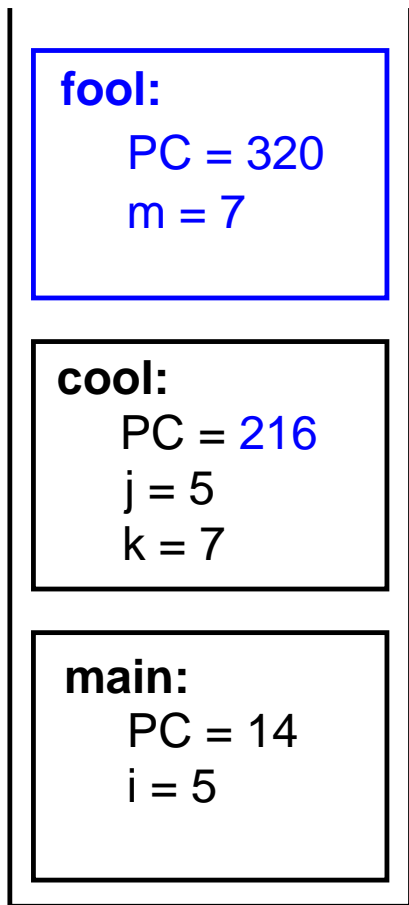
```
public static Integer[] reverse(Integer[] a) {  
    ArrayStack S = new ArrayStack(a.length);  
    Integer[] b = new Integer[a.length];  
    for (int i = 0; i < a.length; i++)  
        S.push(a[i]);  
    for (int i = 0; i < a.length; i++)  
        b[i] = (Integer)(S.pop());  
    return b;  
}
```

Stacks in the Java Virtual Machine

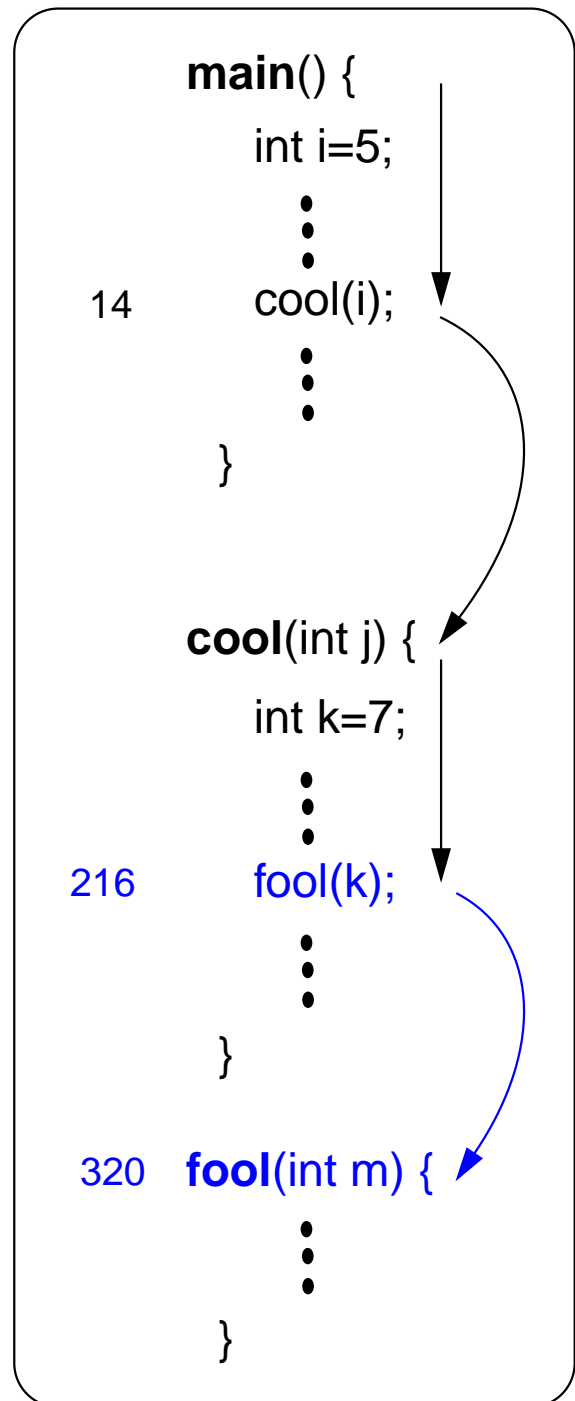
- Each process running in a Java program has its own Java Method Stack.
- Each time a method is called, it is pushed onto the stack.
- The choice of a stack for this operation allows Java to do several useful things:
 - Perform recursive method calls
 - Print stack traces to locate an error
- Java also includes an operand stack which is used to evaluate arithmetic instructions, i.e.

```
Integer add(a, b):  
    OperandStack Op  
    Op.push(a)  
    Op.push(b)  
    temp1 ← Op.pop()  
    temp2 ← Op.pop()  
    Op.push(temp1 + temp2)  
    return Op.pop()
```

Java Method Stack



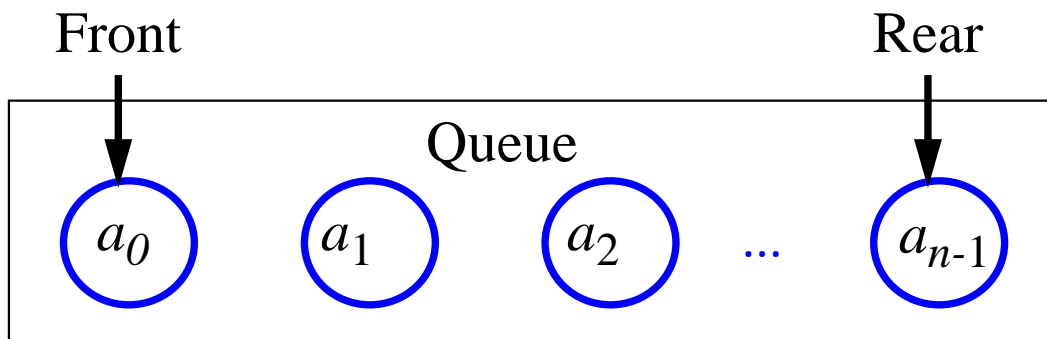
Java Stack



Java Program

Queues

- A queue differs from a stack in that its insertion and removal routines follow the **first-in-first-out (FIFO)** principle.
- Elements may be inserted at any time, but only the element which has been in the queue the longest may be removed.
- Elements are inserted at the *rear* (**enqueued**) and removed from the *front* (**dequeued**)

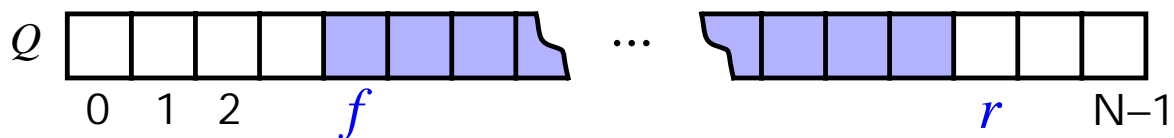


The Queue Abstract Data Type

- The queue supports two fundamental methods:
 - **enqueue(*o*)**: Insert object *o* at the rear of the queue
Input: Object; *Output*: none
 - **dequeue()**: Remove the object from the front of the queue and return it; an error occurs if the queue is empty
Input: none; *Output*: Object
- These support methods should also be defined:
 - **size()**: Return the number of objects in the queue
Input: none; *Output*: integer
 - **isEmpty()**: Return a boolean value that indicates whether the queue is empty
Input: none; *Output*: boolean
 - **front()**: Return, but do not remove, the front object in the queue; an error occurs if the queue is empty
Input: none; *Output*: Object

An Array-Based Queue

- Create a queue using an array in a circular fashion
- A maximum size N is specified, e.g. $N = 1,000$.
- The queue consists of an N -element array Q and two integer variables:
 - f , index of the front element
 - r , index of the element after the rear one
- “normal configuration”



- “wrapped around” configuration



- what does $f=r$ mean?

An Array-Based Queue (contd.)

- Pseudo-Code (contd.)

Algorithm size():

return $(N - f + r) \bmod N$

Algorithm isEmpty():

return $(f = r)$

Algorithm front():

if isEmpty() **then**

 throw a QueueEmptyException

return $Q[f]$

Algorithm dequeue():

if isEmpty() **then**

 throw a QueueEmptyException

$temp \leftarrow Q[f]$

$Q[f] \leftarrow \text{null}$

$f \leftarrow (f + 1) \bmod N$

return $temp$

Algorithm enqueue(o):

if size = $N - 1$ **then**

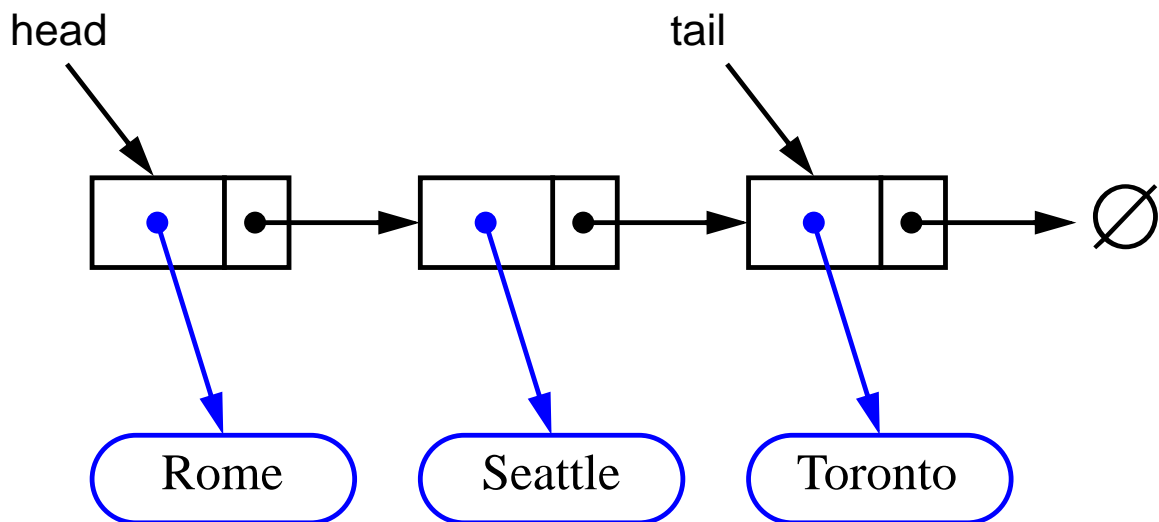
 throw a QueueFullException

$Q[r] \leftarrow o$

$r \leftarrow (r + 1) \bmod N$

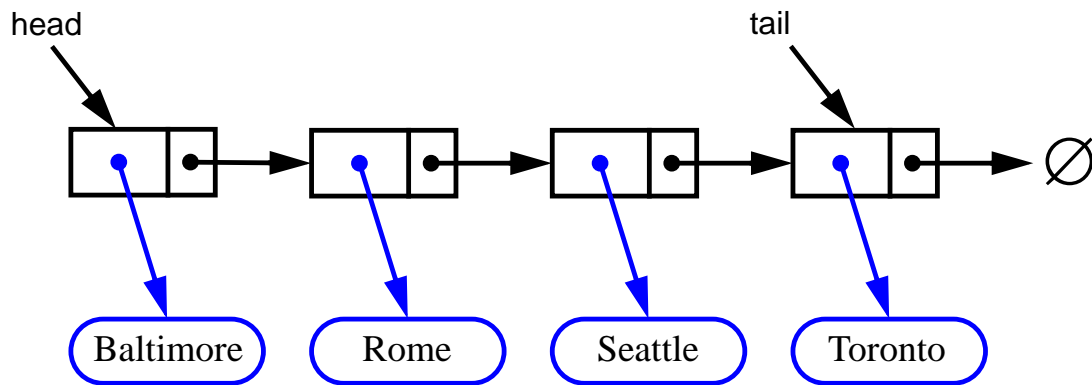
Implementing a Queue with a Singly Linked List

- nodes connected in a chain by links

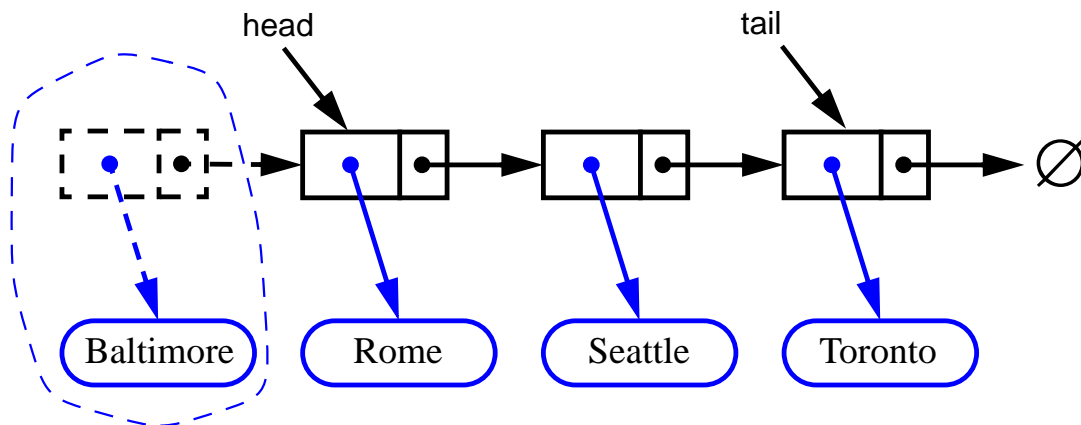


- the head of the list is the front of the queue, the tail of the list is the rear of the queue
- why not the opposite?

Removing at the Head



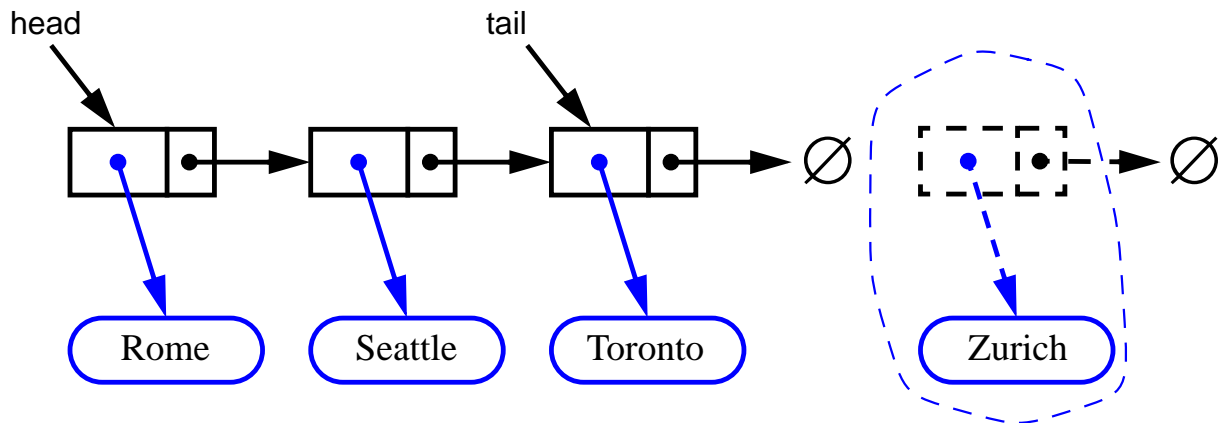
- advance head reference



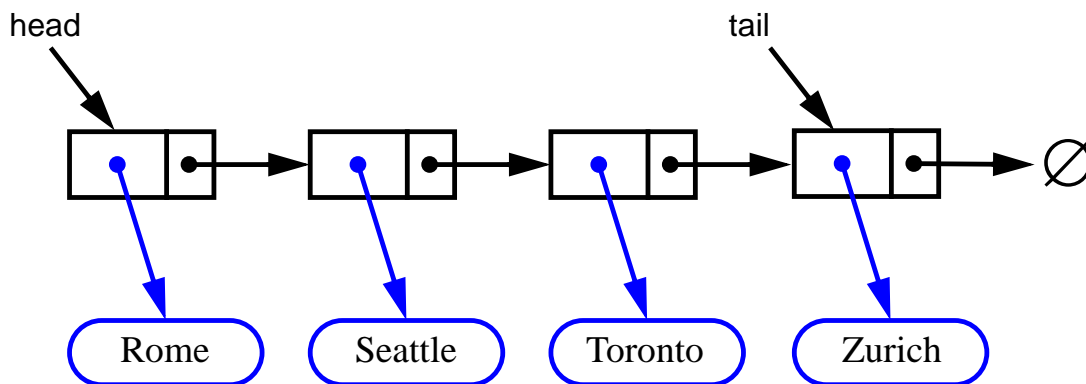
- inserting at the head is just as easy

Inserting at the Tail

- create a new node



- chain it and move the tail reference



- how about removing at the tail?

Double-Ended Queues

- A **double-ended queue**, or **deque**, supports insertion and deletion from the front and back.
- The Deque Abstract Data Type
 - **insertFirst(*e*)**: Insert *e* at the beginning of deque.
Input: Object; Output: none
 - **insertLast(*e*)**: Insert *e* at end of deque
Input: Object; Output: none
 - **removeFirst()**: Removes and returns first element
Input: none; Output: Object
 - **removeLast()**: Removes and returns last element
Input: none; Output: Object
- Additionally supported methods include:
 - **first()**
 - **last()**
 - **size()**
 - **isEmpty()**

Implementing Stacks and Queues with Deques

- Stacks with Deques:

Stack Method	Deque Implementation
size() isEmpty() top() push(e) pop()	size() isEmpty() last() insertLast(e) removeLast()

- Queues with Deques:

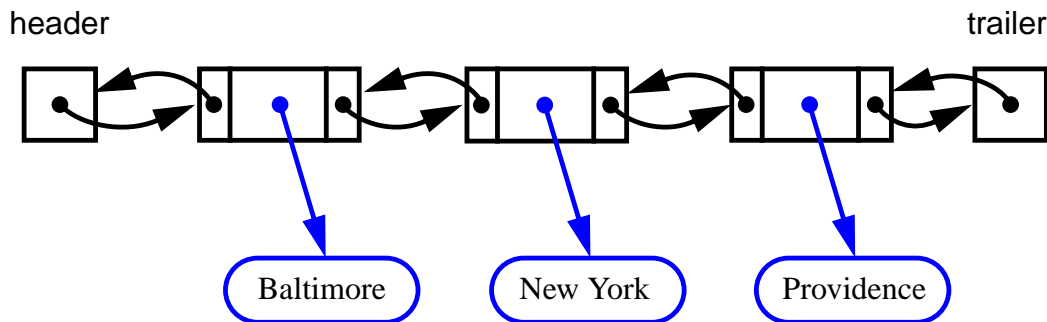
Queue Method	Deque Implementation
size() isEmpty() front() enqueue() dequeue()	size() isEmpty() first() insertLast(e) removeFirst()

The Adaptor Pattern

- Using a deque to implement a stack or queue is an example of the [adaptor pattern](#). Adaptor patterns implement a class by using methods of another class
- In general, adaptor classes specialize general classes
- Two such applications:
 - Specialize a general class by changing some methods.
Ex: implementing a stack with a deque.
 - Specialize the types of objects used by a general class.
Ex: Defining an [IntegerArrayStack](#) class that adapts [ArrayStack](#) to only store integers.

Implementing Deques with Doubly Linked Lists

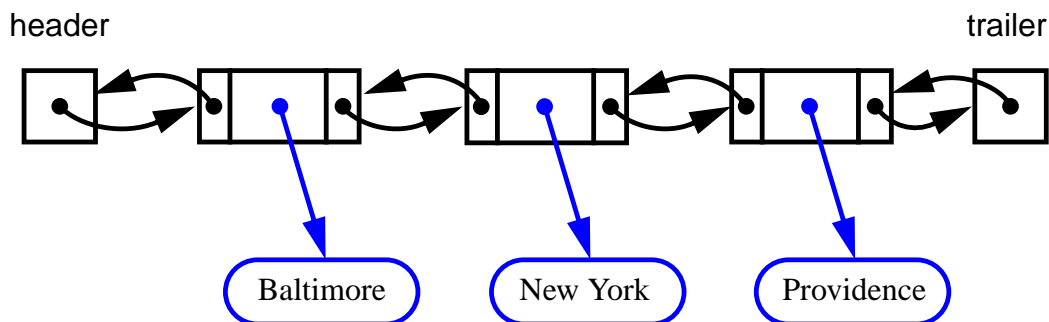
- Deletions at the tail of a singly linked list cannot be done in constant time.
- To implement a deque, we use a **doubly linked list** with special header and trailer nodes.



- A node of a doubly linked list has a **next** and a **prev** link. It supports the following methods:
 - **setElement(Object e)**
 - **setNext(Object newNext)**
 - **setPrev(Object newPrev)**
 - **getElement()**
 - **getNext()**
 - **getPrev()**
- By using a doubly linked list to, all the methods of a deque have constant (that is, $O(1)$) running time.

Implementing Deques with Doubly Linked Lists (cont.)

- When implementing a doubly linked list, we add two special nodes to the ends of the lists: the **header** and **trailer** nodes.
 - The header node goes before the first list element. It has a valid next link but a null prev link.
 - The trailer node goes after the last element. It has a valid prev reference but a null next reference.
- The header and trailer nodes are sentinel or “dummy” nodes because they do not store elements.
- Here’s a diagram of our doubly linked list:



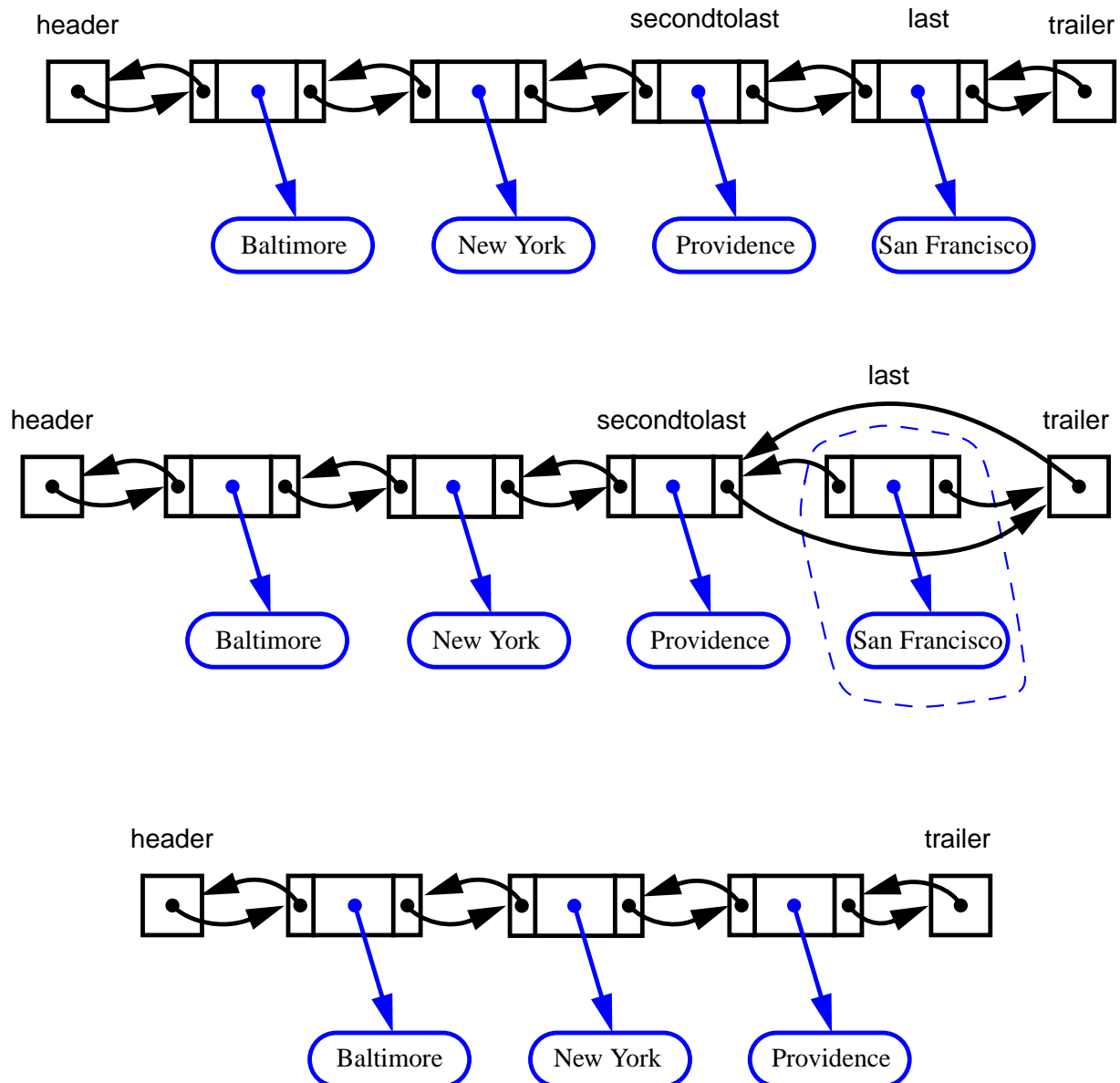
Implementing Deques with Doubly Linked Lists (cont.)

- Let's look at some code for `removeLast()`

```
public class MyDeque implements Deque{
    DLNode header_, trailer_;
    int size_;
    ...
    public Object removeLast() throws
        DequeEmptyException{
        if(isEmpty())
            throw new DequeEmptyException("Illegal
                removal request.");
        DLNode last = trailer_.getPrev();
        Object o = last.getElement();
        DLNode secondtolast = last.getPrev();
        trailer_.setPrev(secondtolast);
        secondtolast.setnext(trailer_);
        size_ --;
        return o;
    }
    ...
}
```

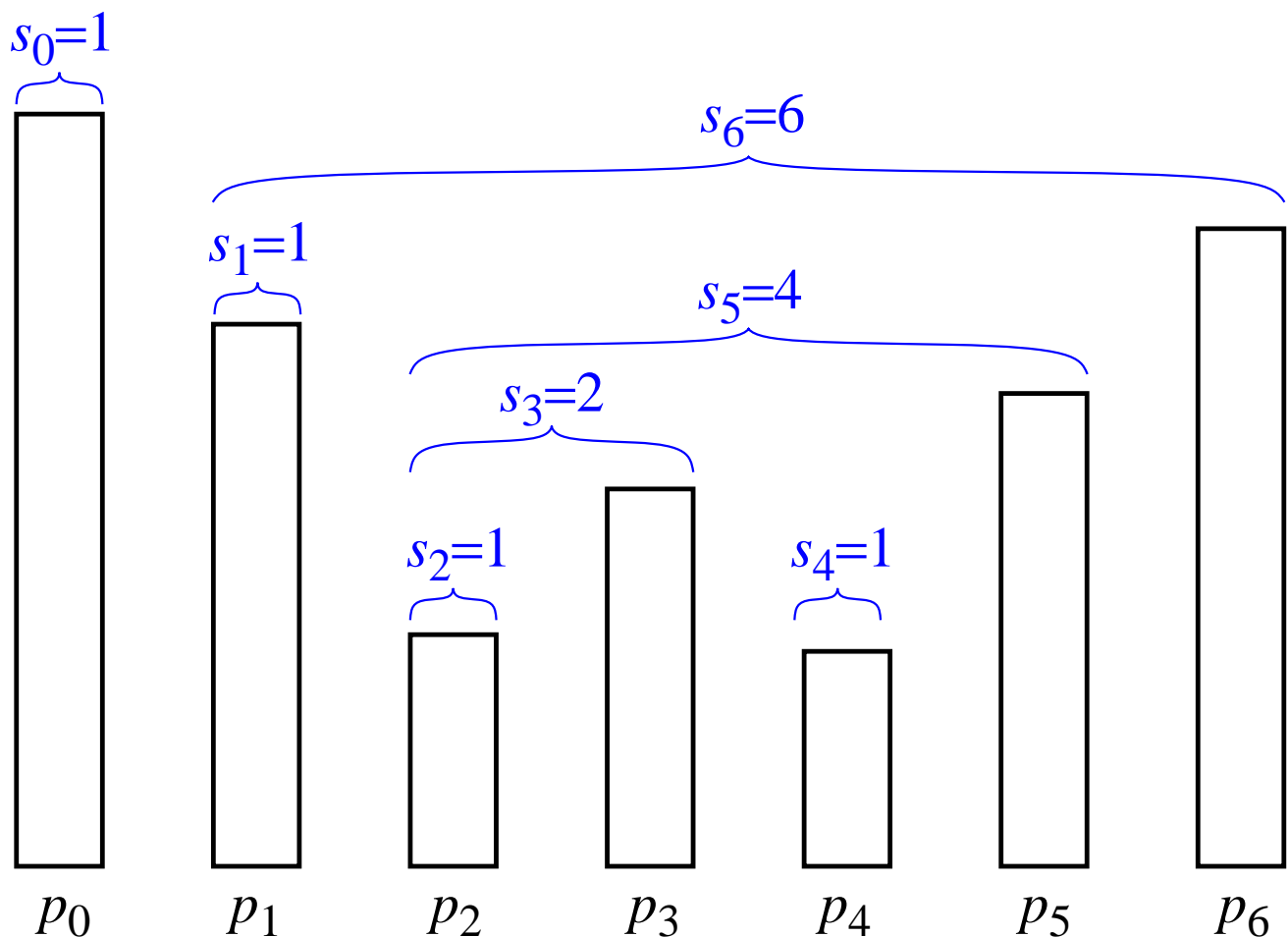
Implementing Deques with Doubly Linked Lists (cont.)

- Here's a visualization of the code for `removeLast()`.



A Stock Analysis Applet

- The span of a stock's price on a certain day, d , is the maximum number of consecutive days (up to the current day) the price of the stock has been less than or equal to its price on d .
- Below, let p_i and s_i be the span on day i



A Case Study: A Stock Analysis Applet (cont.)

- Quadratic-Time Algorithm: We can find a straightforward way to compute the span of a stock on a given day for n days:

Algorithm computeSpans1(P):

Input: An n -element array P of numbers

Output: An n -element array S of numbers such that $S[i]$ is the span of the stock on day i .

Let S be an array of n numbers

for $i=0$ **to** $n-1$ **do**

$k \leftarrow 0$

$done \leftarrow \text{false}$

repeat

if $P[i-k] \leq P[i]$ **then**

$k \leftarrow k+1$

else

$done \leftarrow \text{true}$

until $(k=i)$ **or** $done$

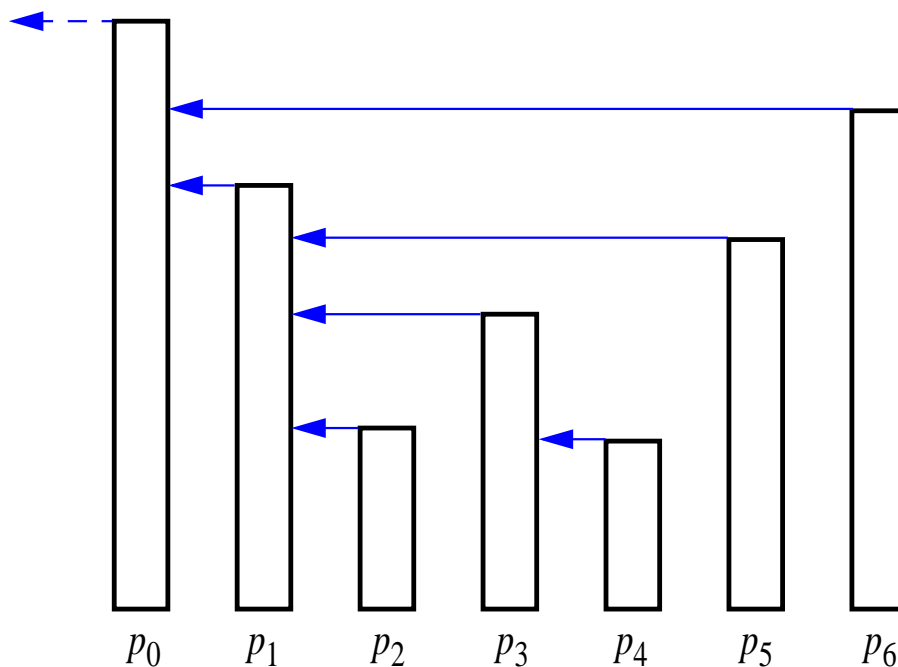
$S[i] \leftarrow k$

return array S

- The running time of this algorithm is (ugh!) $O(n^2)$. Why?

A Case Study: A Stock Analysis Applet (cont.)

- Linear-Time Algorithm: We see that s_i on day i can be easily computed if we know the closest day preceding i , such that the price is greater than on that day than the price on day i . If such a day exists let's call it $h(i)$.
- The span is now defined as $s_i = i - h(i)$



The arrows point to $h(i)$

A Case Study: A Stock Analysis Applet (cont.)

- The code for our new algorithm:

Algorithm computeSpan2(P):

Input: An n -element array P of numbers

Output: An n -element array S of numbers such that
 $S[i]$ is the span of the stock on day i .

Let S be an array of n numbers and D an empty stack

for $i=0$ **to** $n-1$ **do**

$done \leftarrow \text{false}$

while not($D.\text{isEmpty}()$ **or** $done$) **do**

if $P[i] \geq P[D.\text{top}()]$ **then**

$D.\text{pop}()$

else

$done \leftarrow \text{true}$

if $D.\text{isEmpty}()$ **then**

$h \leftarrow -1$

else

$h \leftarrow D.\text{top}()$

$S[i] \leftarrow i - h$

$D.\text{push}(i)$

return array S

- Let's analyze computeSpan2's run time...

A Case Study: A Stock Analysis Applet (cont.)

- The total running time of the while loop is

$$O\left(\sum_{i=0}^{n-1} (t_i + 1)\right)$$

- However, once an element is popped off the stack, it is never pushed on again. Therefore:

$$\sum_{i=0}^{n-1} t_i \leq n$$

- The total time spent in the while loop is $O(n)$.
- The run time of computeSpan2 is the summ of three $O(n)$ terms. Thus the run time of computeSpan2 is $O(n)$.